2006 IEEE International Conference on
Multisensor Fusion and Integration for Intelligent Systems
September 3-6, 2006, Heidelberg, Germany

Uncertain Spatial Knowledge Management in a Mobile Robot Architecture

Piotr Skrzypczyński

***Abstract*— In this paper a new approach to spatial knowledge management for a mobile robot is proposed. The framework merges two types of maps, grid and feature based, and two uncertainty representations, probabilistic and fuzzy. It is implemented by a set of logical sensors, that extract features from range sensors data, filter out artifacts, and estimate the robot displacement. Indoor simultaneous localization and mapping is considered as the target application.**

I. INTRODUCTION

Automated building of spatial knowledge from sensory data is one of the central problems for autonomous robots. Many particular environment representations have been proposed in the literature [16]. The grid-based maps represent space as an array of equal cells. They can easily be updated with range sensors readings, tolerating data uncertainty and ambiguity, but require a large amount of memory to cover bigger areas with a dense grid. Feature-based maps contain concise and explicit representations of the geometric entities, but are less popular because of difficulties with the direct interpretation of raw sensory data. Whenever the robot pose $\mathbf{x}_R = [x_r \; y_r \; \theta_r]^T$ is unknown, the map has to be constructed while computing a pose estimate. One of the most used methods to solve this simultaneous localization and mapping (SLAM) problem is the Extended Kalman Filter (EKF), used with feature-based maps [1], [5], [15]. Unfortunately, most of the EKF-based SLAM implementations suffer from difficulties in the sensory data interpretation [14] and limitations of the EKF framework [6]. An important aspect of the spatial knowledge representations is also the mathematical framework used to handle the uncertainty.

This paper presents a new approach to spatial knowledge management for a mobile robot, that combines the most desirable properties of both the feature-based and the grid-based maps. Taking into account properties of the range sensors (2D laser scanner, sonars) and the geometric structures commonly encountered in the indoor environments we have decided to use horizontal straight line segments as the features in the global map. The line segments can be further structured into polygons and polylines, representing distinctive objects in the environment. A proper choice of the representation of geometric features has to be made, to avoid overparametrization and singularities, and to make the uncertainty independent of the used coordinates. To satisfy these requirements we have adopted the SPmap framework [5]. The grid maps are able to accumulate data taken from several consecutive poses of the robot while filtering out unreliable measurements. Because of that, a grid map is utilized here as the first-step, local representation of the spatial knowledge in the mobile robot. The occupancy grid provides means for reasoning about the range data in the temporal dimension, and can be used as a common ground for fusion of readouts from heterogenous range sensors [8].

From the other hand, we combine two frameworks for the uncertainty representation: the classical probabilistic methods to explicitly propagate the spatial uncertainty, and the fuzzy set theory to support decisions during the map building process. We propose also to extract information about the robot displacement from the laser scanner data.

Each procedure, that extracts a specific type of information from the sensory data is regarded as a logical sensor [7], and integrated within the distributed, blackboard-based architecture of the robot software.

II. FUZZY SUPPORT GRIDS

We have considered the Bayesian [8], Dempster-Shafer (D-S) [10], and fuzzy sets based [9] algorithms as the methods for grid-cell evidence update. A comparison of these three frameworks using sonars is presented in [12]. We have performed experiments with the sonars, and two different types of 2D laser scanners in a controlled environment [13]. Particularly, we sought an answer to the following questions: (i) How the conflicting evidence and lack of information are represented in the grid ? (ii) Is the algorithm able to filter out measurements originating from dynamic objects ?

Results of one of those experiments, performed with the Sick LMS 200 laser scanner in a corridor, are presented in Fig. 1. In this experiment, the robot poses have been obtained from odometry only. The dynamic object was a person walking in front of the robot, in the field of view of the scanner. In Fig. 1A, B, and C the grid maps of the occupied areas are depicted, for the Bayesian, D-S, and fuzzy method, respectively. The Bayesian method provided good representation of the corridor walls, but it failed to filter out the data produced by the moving object. In contrary, the D-S method produced a map almost free of the artifacts related to the walking person, but at the cost of eliminating many pieces of legitimate evidence. Because of that, the walls are poorly represented. The fuzzy grid of occupied cells is very conservative, indicating correctly all areas occupied by the walls, but showing also all ”traces” of the walking person. However, in this framework the occupancy and emptiness evidences are not complementary, what enables to identify the level of contradiction between the measurements. The

P. Skrzypczyński is with Institute of Control and Information Engineering, Poznań University of Technology, ul. Piotrowo 3A, PL-60-965 Poznań, Poland. ps@ar-kari.put.poznan.pl

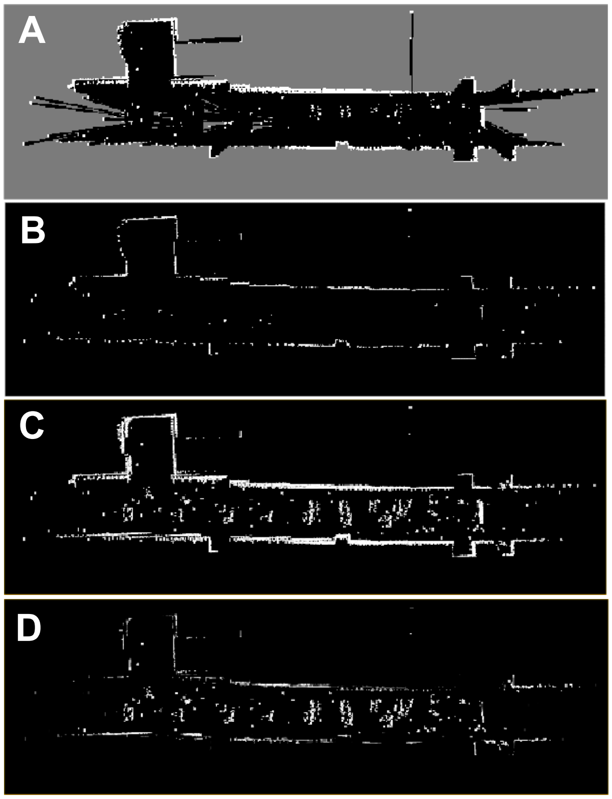

Fig. 1. Grid maps built in a dynamic environment with the frameworks under investigation: Bayesian (A), D-S (B), and fuzzy (C,D)

map of ambiguous cells, displayed in Fig. 1D, exploits this possibility, correctly identifying the areas corresponding to the measurements originating from the dynamic object.

Taking into account the experimental results, we have chosen the fuzzy-set-based framework as the one, that best fulfills the requirements for an intermediate representation of the spatial knowledge for a mobile robot.

In the fuzzy-set-based grid map the sets of occupied $\mathcal{O}_i^k$ and empty cells $\mathcal{E}_i^k$ are determined, by computing their membership functions (μ_O and μ_E) according to the sensor beam model, for every measurement r_i^k taken from the k-th robot pose. The membership functions are computed from the equations:

$$\begin{aligned}\mu_O(\rho,\varphi) &= \alpha_o(\rho)\beta_l(\varphi),\\ \mu_E(\rho,\varphi) &= \alpha_e(\rho)\beta_l(\varphi).\end{aligned} \tag{1}$$

The functions α_o, α_e and β_l, that represent the degree of membership of a given cell to the sets of occupied and empty cells, according to the laser range sensor reading r, and its uncertainty σ_r are defined as:

$$\alpha_o(\rho) = \begin{cases} 0 & 0 \le \rho < r - \Delta r,\\ k_o \frac{1}{\sqrt{2\pi}\sigma_r} e^{\frac{-(\rho-r)^2}{2\sigma_r^2}} & r - \Delta r \le \rho < r + \Delta r,\\ 0 & \rho \ge r + \Delta r, \end{cases} \tag{2}$$

$$\alpha_e(\rho) = \begin{cases} k_e & 0 \le \rho < r - \Delta r,\\ k_e\left(\frac{r-\rho}{\Delta r}\right)^2 & r - \Delta r \le \rho < r,\\ 0 & \rho \ge r, \end{cases} \tag{3}$$

$$\beta_l(\varphi) = \begin{cases} 1 & |\varphi| \le \theta_l,\\ 0 & |\varphi| > \theta_l, \end{cases} \tag{4}$$

where: ρ is the distance from the sensor to the center of the given cell, φ is the angle between the beam axis and the bearing to that cell, $\Delta r = 3\sigma_r$, $\theta_l = 0.5^o$ for the LMS 200 scanner, and k_o, k_e are the limits of μ_O and μ_E, such that $k_o + k_e < 1$.

If sonar data are available, they update the fuzzy grid as well, by using a similar sensor model, that takes into account the ultrasonic beam pattern [13].

The data gathered from a single robot pose are aggregated to the sets $\mathcal{O}^k$ and $\mathcal{E}^k$:

$$\mathcal{O}^k = \bigcup_i \mathcal{O}_i^k, \quad \mathcal{E}^k = \bigcup_i \mathcal{E}_i^k. \tag{5}$$

The fuzzy grid holds the $\mathcal{O}$ and $\mathcal{E}$ sets obtained as fuzzy union of the sets generated at the k-th pose with the previously available information. Different union operators have been proposed in the literature, e.g.: algebraic sum [12], weighted average, and Dombi operator [9]. We use the Dombi operator for map update. Two sets describing the lack of knowledge, by identifying the cells being ambiguous ($\mathcal{A}$) or indeterminate ($\mathcal{I}$) are computed:

$$\mathcal{O} = \mathcal{O} \cup \mathcal{O}^k, \quad \mathcal{E} = \mathcal{E} \cup \mathcal{E}^k, \quad \mathcal{A} = \mathcal{E} \cap \mathcal{O}, \quad \mathcal{I} = \bar{\mathcal{E}} \cap \bar{\mathcal{O}}. \tag{6}$$

Then, the sets of cells, that are useful for particular robot tasks are computed by combining the above-defined four types of evidence in a way depending on what type of information about the environment is required.

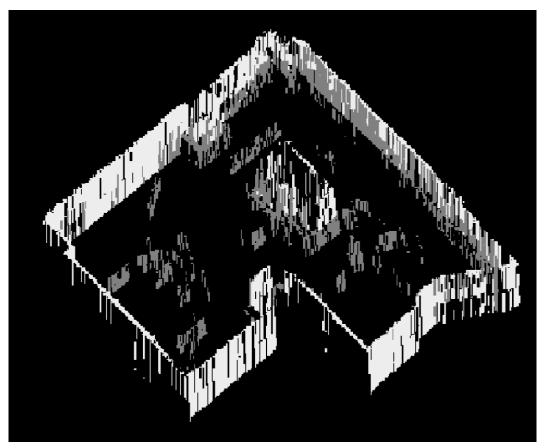

Fig. 2. An example of the fuzzy support grid (3D view)

Because our aim is to build a feature-based model of the environment, we are looking for cells providing reliable support for line segment extraction. To this end, we define a set of *support* cells, being very occupied and unambiguous. The term "very occupied" emphasizes the difference between low and high values of occupancy, and is implemented by squaring the value of the membership function:

$$\mathcal{S} = \mathcal{O}^2 \cap \bar{\mathcal{A}} \cap \bar{\mathcal{I}}. \tag{7}$$

In the local fuzzy support grid (FSG) computed according to the above formula the contours of objects are identified as aligned cells of high membership degree to the $\mathcal{S}$ set. Figure 2 shows a 3D view of the FSG resulting from an experiment with the LMS 200 scanner (cf. Fig. 6), where the higher values mean better support for line extraction, while the darker peaks are the outliers, mostly due to people walking by.

III. FEATURE-BASED LOCAL MAPS

There are several known approaches to the extraction of lines from a grid-type representation (e.g. an image). One of the most used is the Hough transform [1], [11], [13]. The Hough transform does not take uncertainty into account when estimating the line parameters, and introduces itself uncertainty depending on the level of discretization of parameter space. The loss of precision is also inherent to the grid representation, because the individual range readings are incorporated into the cells of given size, hence the information about their precise position is lost.

An alternative are the scan splitting algorithms that consider the order in which the range measurements were obtained. Such an algorithm is the Iterative End Points Fit (IEPF). This "divide and conquer" type algorithm is much faster than the Hough transform. However, the order of range measurements scanned from a particular robot pose is not preserved in the grid map. Because of that, the proposed segment extraction method considers individual scans instead of the whole fuzzy grid, while the FSG is used to judge the validity of the individual measurements, by comparing them to the evidence accumulated by the past scanner readings.

The line segments are extracted from the ordered set of laser scanner points. Only the points, that fall in the FSG cells of high membership degree to the $\mathcal{S}$ set are considered in further processing. Points found in cells of high support value, but isolated from other support cells, are also ignored to eliminate the measurements that cannot be converted into good quality segments. The spatial uncertainty of a point is represented by its covariance matrix $\mathbf{C}_p$, computed upon the laser sensor model [14].

The sonar data have to low spatial density to contribute to the line segments with the required precision. However, if they are available, they contribute to the fuzzy line support measure computed from (7), reinforcing those laser measurements that are consonant with the sonar data [13].

In the following step, the points are grouped together to form individual objects. Next, the IEPF algorithm is applied to find candidate line segments from groups of points representing particular objects. The supporting line of an extracted segment is described by the feature $\mathbf{L}_{RF}$, with regard to the robot frame. The feature parameters and covariance are computed by applying the weighted total least squares method. The endpoints of the segment are determined by projecting the laser points onto the infinite line and trimming the line at the extreme points. Finally, the center point and the length l_F are computed. There is no uncertainty of the segment length determined.

Results obtained with the FSG-supported segment extraction method have been compared in Fig. 3 to the results of the standard split-and-merge approach to laser data segmentation [5]. When a scan (Fig. 3A) containing range measurements caused by a dynamic object (indicated by "1") and some outliers due to mixed pixels ("2") is processed by the IEPF algorithm, the resulting local map (Fig. 3B) contains artifacts and segments, whose orientation is poorly estimated. If the segmentation is assisted by the FSG map (Fig. 3C), which identifies the areas, that provide reliable support for line segment extraction, the resulting local map (Fig. 3D) is better. It does not contain artifacts related to dynamic objects. The remaining line segments have more correct location, because the FSG filtered out vast majority of the outliers, that in the standard approach acted as "leverage points" during the least squares estimation of supporting lines parameters.

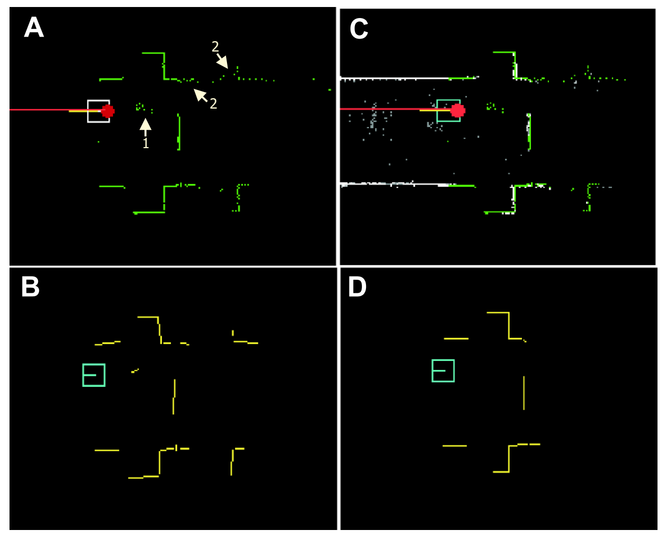

Fig. 3. Comparison of line segment extraction results: direct extraction (A,B) and extraction supported by the FSG (C,D)

The above-presented approach corresponds with the requirements of the on-line EKF-SLAM, that forms the core part of a mobile robot navigation software, and has been chosen as the "test application" of our new approach to the spatial knowledge management [14]. The LMS 200 scanner on the Labmate robot used for the experiments provides scans at the rate of about 4 Hz. Because of the high computational cost of the EKF-SLAM, the range data stream used by this algorithm has to be subsampled below the maximum possible scan acquisition rate of the LMS 200. In contrary, the FSG map can be built at the full scan rate. This results in a new set of segments being extracted in average each ten fuzzy grid updating cycles. The scanned points that are acquired between two consecutive calls to the feature extraction procedure contribute to the FSG map computed from (7), making more credible those points of the processed scans, that are consonant with the past evidence.

Unfortunately, if the FSG is built using scans registered with the odometry, between two sequential steps of SLAM pose update, it can become inconsistent due to odometry errors. Unlike the feature-based map, the grid-based paradigm provides no means to accommodate the spatial uncertainty induced by pose errors. Particularly, errors in the orientation easily cause update of incorrect FSG cells, and then, wrong decisions in the feature extraction process. To prevent such problems, the robot should keep track on its pose also between the SLAM algorithm updates.

IV. GLOBAL MAP AND LOCALIZATION

The mobile robot builds the global, feature-based map simultaneously while localizing within it. The well-known EKF-based SLAM algorithm with the stochastic map [15] is implemented, therefore the map is represented by the state vector $\hat{\mathbf{x}}$ containing the estimated pose of the robot $\hat{\mathbf{x}}_R$ and the estimated locations of n features $\hat{\mathbf{x}}_{WF_i}$, $(i = 1 \ldots n)$. The spatial uncertainty of the estimated features, the robot pose, and their correlations are represented by the covariance matrix $\mathbf{C}_x$. All locations are represented with regard to the global reference frame. We use the initial robot pose as the base coordinates.

In the EKF-SLAM prediction stage, the state vector at the k-th step is computed by estimating the robot displacement from $k-1$ to k. In wheeled robots the displacement estimate is provided by the encoder-based odometry. If the odometric uncertainty isn't modelled correctly, the pose errors, particularly in the orientation, easily lead to EKF divergence [6]. However, such sources of errors like collisions or large bumps on the floor cannot be anticipated by any statistical odometry model. Because the EKF framework assumes zero-mean Gaussian noise, a proper calibration of the systematic odometry errors is crucial [3]. One possibility to make better pose predictions in the EKF-SLAM is to improve the odometry by using displacement estimates from an exteroceptive sensor.

In the SLAM framework, the uncertainty in both the robot pose and the feature locations is decreased by re-observing the features. The new observations are the line-segment features in the current local map. They are extracted from the laser scanner data according to the approach presented in the previous section. The observations are matched with the features stored in the global map. The successfully associated features are used to update the state vector and its covariance by applying the equations of the EKF estimation stage. Unmatched features from the local map are added to the global map, augmenting the state vector [14].

The computational complexity of the EKF-SLAM estimation stage is $O(n^2 m)$, where m is the number of matched features [5]. If the feature matching fails, e.g. due to spurious features, the global map grows quickly, what makes the estimation time unacceptable and prohibits on-line operation of the robot. Robust method of feature extraction helps to slow down the state vector growing. Low quality features having poor estimates of their uncertainty, if added to the map, may also cause false matches in further time steps, thus leading to the EKF divergence.

V. ROBOT DISPLACEMENT ESTIMATION

Keeping track on the robot pose and its uncertainty during motion of the vehicle is important to both the FSG-assisted feature extraction and the EKF-SLAM itself. The encoder-based odometry alone is insufficient to accomplish this task, because of undetected bias and uncertainty due to unforeseen or ”catastrophic” errors.

One approach to overcome these problems is to employ the laser scanner readings to estimate the displacement between two sequential steps of the EKF-SLAM. The translational and rotational displacement between two robot poses can be obtained by scan matching, i.e. by finding a rotation and translation that maximize the overlap of these scans. For correction of the odometry errors we match two consecutive scans to compute the incremental displacement $\Delta \mathbf{x}_{R_i}$. Integrating these displacements, given by the translation $\Delta \mathbf{t}_i = [\Delta x_i \; \Delta y_i]^T$ and rotation $\Delta\theta_i$ for the pair of scans $i-1$ and i, we establish an alternative form of odometry.

There are a number of methods for matching scans known from the literature. Unfortunately, most of these methods do not provide a realistic covariance matrix $\mathbf{C}_\Delta$ of the displacement estimate, which is required to integrate the alternative odometry measurements in the prediction stage of the EKF-SLAM algorithm [2]. One of the scan matching methods, that satisfy this requirement is the WLSM algorithm presented in [11]. The WLSM algorithm explicitly models the sources of errors in both the range measurements and the matching process itself, hence it is able to calculate a realistic covariance of the displacement.

We have implemented the WLSM algorithm as the basis for the alternative odometry. In this implementation we profit again from the FSG map, which provides means to remove from the processed scan the measurements caused by dynamic objects. Only points located in the grid cells that have been updated by previous scans are considered for matching. Because the matched scans are taken at short relative distances, we assume that instantaneous odometry errors are inferior than the cell size. Next, for every point in the scan i we search for a corresponding point in the scan $i-1$ that lies within a given distance. When the correspondence between points is established, the estimates of the translation and rotation are computed using an iterative algorithm. Starting with an initial guess of the robot rotational displacement $\Delta\hat{\theta}_0$ provided by the encoder-based odometry a translation estimate $\Delta\hat{\mathbf{t}}$ is computed with the closed form formula (see [11] for the equations), then this estimate is used to update the current rotation estimate. The corrected $\Delta\hat{\theta}$ is used in the next iteration, until a convergence criterion is reached.

A key assumption underlying the use of Kalman filtering in the SLAM algorithm is that the robot motion estimate is statistically independent from the exteroceptive observations. Due to this reason we cannot use for scan matching the same range data, which are then used for the feature extraction. As already mentioned, the points from only one scan per ten are used for the line segment extraction, with the remaining laser data updating only the fuzzy grid values. Our alternative odometry adheres to this data processing scheme, by using only the n_u scans, taken in-between two consecutive steps of the EKF-SLAM. These data are not used directly by the SLAM algorithm [14]. The displacement estimates obtained from scan matching are compounded head-to-tail:

$$\hat{\mathbf{x}}_{R(i)} = \hat{\mathbf{x}}_{R(i-1)} \oplus \Delta\hat{\mathbf{x}}_{R_i}, \quad (8)$$

$$\mathbf{C}_{R(i)} = \mathbf{J}_{1\oplus}\mathbf{C}_{R(i-1)}\mathbf{J}_{1\oplus}^T + \mathbf{J}_{2\oplus}\mathbf{C}_{\Delta}\mathbf{J}_{2\oplus}^T, \quad (9)$$

where $i = 1 \ldots n_u$, $\oplus$ is the compounding operator, and $\mathbf{J}_{1\oplus}$, $\mathbf{J}_{2\oplus}$ are the Jacobians of this operation defined in [15].

This way separate range readings are used to estimate the vehicle motion and the features, what ensures independence of the information sources in the Kalman filter.

VI. SPATIAL KNOWLEDGE MANAGEMENT SYSTEM ARCHITECTURE

In the previous work [4] we have introduced the Multi-Agent Blackboard (MAB) architecture, which is derived from the concept of logical sensors [7]. Sensory data processing operations in the navigation system of the mobile robot are defined as black boxes loosely coupled by the data they exchange. These processing tasks are modelled as independent agents working around a blackboard accommodating spatial knowledge gathered by the robot.

The approach to spatial knowledge management presented in this paper makes use of two independent data processing modules responsible for the robust feature extraction and the vehicle displacement estimation. These software modules, as well as the EKF-SLAM algorithm, are implemented as agents within the MAB framework.

The LMS 200 scanner, the sonars, and the Labmate controller (providing the odometry) are encapsulated by the device agents *ADScan*, *ADSonar*, and *ADCtrl*, respectively. The FSG map is built upon the laser and sonar range measurements by the *AEFuzzGrid* agent, a software "expert", that implements the method described in section II. The scan matching procedure is implemented by the *AEScnMatch* agent. The line segments are extracted by the *AEFeatExt* agent feed with the current raw scan points and the FSG representing the accumulated past data. The newly extracted features are treated as observations by the EKF-SLAM algorithm implemented by the *AESlamAlg* agent. Obstacle avoidance on the robot path is a task of the pilot agent (*AEPilot*), which implements the behavioral paradigm, providing the perception-action cycle through the grid map treated as a virtual sensor [4]. The information related to motion planning is extracted from the basic $\mathcal{O}$ and $\mathcal{E}$ sets [9], what illustrates flexibility of the fuzzy grid map building method. Figure 4 depicts the blackboard structure and the collection of agents on the robot used in the experiments.

VII. EXPERIMENTS AND RESULTS

In order to verify the presented approach to the spatial knowledge management, we have performed several experiments with the Labmate robot equipped with the LMS 200 scanner. The sonars have been not used in these particular experiments, see [13] for examples of fuzzy support grids built by employing heterogenous sensors.

Figure 5 shows results of an experiment performed in a laboratory environment, to investigate how the proposed FSG-assisted feature extraction improves quality of the

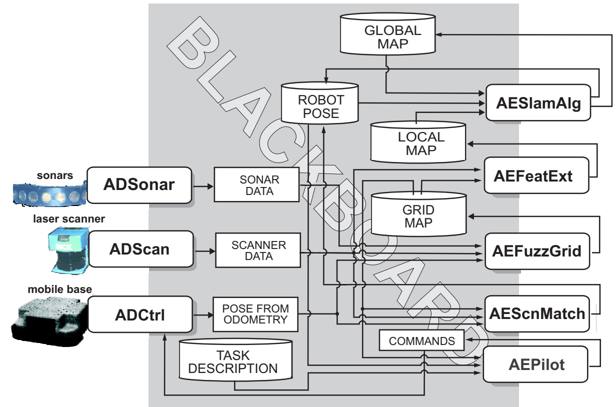

Fig. 4. Multi-agent blackboard architecture of the spatial knowledge management system in the mobile robot

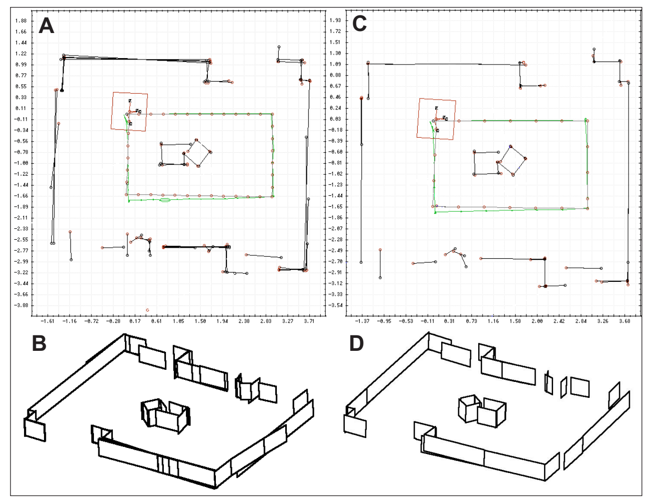

Fig. 5. Comparison of maps built by the standard EKF-SLAM (A,B) and the MAB system (C,D)

segment-based global map. In Fig. 5A a map built by the SLAM algorithm from the raw data is depicted. The green line indicates the EKF-SLAM estimated robot path, while the red one is for the odometry. The map is of reasonable quality, however, it contains a number of spurious short segments caused by persons moving close to obstacles, and some line segments have been estimated improperly. Wrong orientation estimates of the features cause inefficient matching in the EKF-SLAM, while wrong termination of segments changes shapes of the objects (e.g. the larger box in the middle of the map). A map created by the SLAM algorithm using the FSG-assisted feature extraction and improved motion estimates is shown in Fig. 5C. This map contains fewer features, there are no segments resulting from the dynamic objects, and the shapes of objects are correct. Differences in the quality and geometric consistency between the global maps can be cleary seen on their 3D (extruded) views (Fig. 5B and 5D). Better estimates of the features and the robot displacement resulted in higher ratio of successful matching between the features from local maps and the global map (Fig. 7A).

Figure 6 shows results of an experiment, in which pa-

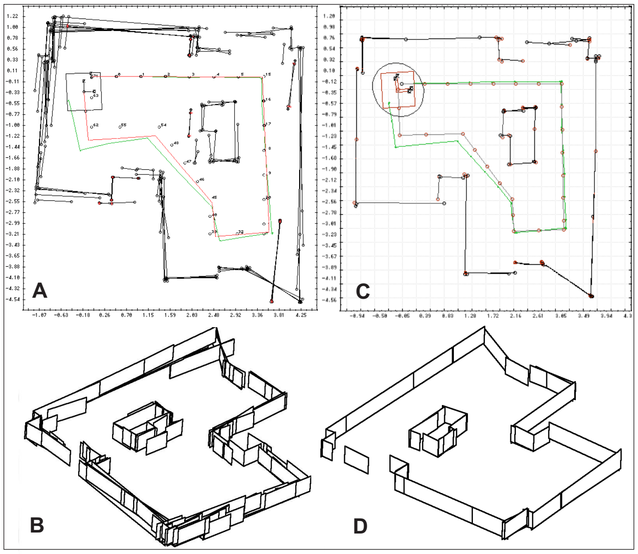

Fig. 6. Comparison of results for the experiment with poor odometry: standard EKF-SLAM (A,B) and the MAB approach (C,D)

rameters of the Labmate odometry model have been roughly identified by an ad-hoc experiment, leaving the systematic errors not calibrated. The first map was built with the basic EKF-SLAM algorithm, without use of the FSG and scan matching. As it can be seen from Fig. 6A it contains many overlapping segments and is highly inconsistent, particularly in the upper-left part, which contains many segments extracted at the end of the path. Clearly, divergence of the EKF due to the not calibrated systematic component, and inaccuracies in the model of the non-systematic odometry errors gradually prevented correct associations between the map and the segments from new observations, as shown by the plot of successful feature matching ratio in Fig. 7B. The segment-based map obtained from the scanner data gathered with the same experimental set-up, but using the MAB framework (Fig. 6C) is much more consistent. The estimates of robot displacement from scan matching prevented the EKF from large divergence in spite of the poorly modeled vehicle odometry. The ratio of feature matching is higher (Fig. 7B). Some segments are still multiplied, but the overall number of features in the second map is greatly reduced (Fig. 6B and 6D).

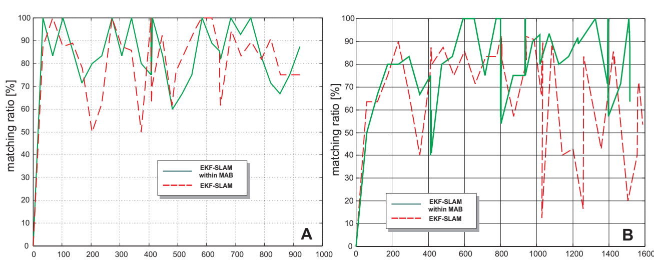

Fig. 7. Successful feature matching ratio for the experiments shown in Fig. 5 (A) and in Fig. 6 (B)

VIII. CONCLUSIONS

We have proposed a new approach to the spatial knowledge management in a mobile robot. Two types of spatial knowledge representation: feature-based and grid-based are used, what enables to mutually compensate their main drawbacks. Two frameworks for the uncertainty description: probabilistic and fuzzy are used. The former provides a way to propagate quantitative uncertainty in the global map building and localization, while the latter is used to remedy the qualitative-type uncertainty in the feature extraction. The role of reliable ego-motion estimation in both the EKF-SLAM and the grid map building has been pointed out, and the scan matching procedure has been implemented to accomplish this task. The software modules for feature extraction and robot displacement estimation, as well as the EKF-SLAM algorithm, have been implemented as agents in the MAB architecture.

REFERENCES

[1] G. C. Anousaki, K. J. Kyriakopoulos, "Simultaneous Localization and Map Building for Mobile Robot Navigation", *IEEE Robotics & Autom. Magazine*, 9, 1999, pp. 42-53.

[2] O. Bengtsson, A. Baerveldt, "Localization in Changing Environments – Estimation of Covariance Matrix for the IDC Algorithm", *Proc. IEEE/RSJ Conf. on Intell. Robots & Syst.*, Maui, 2001, pp. 1931-1937.

[3] J. Borenstein, L. Feng. "Measurement and Correction of Systematic Odometry Errors in Mobile Robots", *IEEE Trans. Robot. & Autom.*, 12(6), 1996, pp. 869-880.

[4] G. Brzykcy, J. Martinek, A. Meissner, P. Skrzypczyński, "Multi-Agent Blackboard Architecture for a Mobile Robot", *Proc. IEEE/RSJ Conf. on Intell. Robots & Syst.*, Maui, 2001, pp. 2369-2374.

[5] J. A. Castellanos, J. D. Tardós, *Mobile Robot Localization and Map Building. A Multisensor Fusion Approach*, Kluwer, 1999.

[6] J. A. Castellanos, J. Neira, J. D. Tardós, "Limits to the Consistency of the EKF-based SLAM", *Prepr. IFAC Symp. on Intell. Autonomous Vehicles*, Lisbon, 2004.

[7] T. Henderson, E. Shilcrat, "Logical Sensor Systems", *J. of Robotic Syst.*, 2(1), 1984, pp. 169-193.

[8] H. P. Moravec, "Certainty Grids for Sensor Fusion in Mobile Robots", *AI Magazine*, 9(2), 1988, pp. 61-77.

[9] G. Oriolo, G. Ulivi, M. Vendittelli, "Real-Time Map Building and Navigation for Autonomous Robots in Unknown Environments", *IEEE Trans. Syst., Man & Cybern.*, Part B, 28(3), 1998, pp. 61-70.

[10] D. Pagac, E. Nebot, H. F. Durrant-Whyte, "An Evidential Approach to Map-Building for Autonomous Vehicles", *IEEE Trans. Robot. & Autom.*, 14(3), 1998, pp. 623-629.

[11] S. Pfister, K. Kriechbaum, S. Roumeliotis, J. Burdick, "Weighted Range Sensor Matching Algorithms for Mobile Robot Displacement Estimation", *Proc. IEEE Int. Conf. Robot. & Autom.*, Washington, 2002, pp. 1667-1674.

[12] M. Ribo, A. Pinz, "A Comparison of Three Uncertainty Calculi for Building Sonar-based Occupancy Grids", *Robotics & Autonom. Syst.*, 35, 2001, pp. 201-209.

[13] P. Skrzypczyński, "Merging Probabilistic and Fuzzy Frameworks for Uncertain Spatial Knowledge Modelling", *Computer Recognition Systems*, (M. Kurzyński *et al.*, eds.), Springer, 2005, pp. 435-442.

[14] P. Skrzypczyński, "Laser-based Logical Sensors for Improved Mapping and Localization", *Proc. Eur. Conf. on Mobile Robots*, Ancona, 2005, pp. 50-55.

[15] R. Smith, M. Self, P. Cheeseman, "Estimating Uncertain Spatial Relationships in Robotics", *Autonomous Robot Vehicles*, (I. Cox, G. Wilfong, eds.), Springer, 1990, pp. 167-193.

[16] S. Thrun, *Robotic Mapping: A Survey*, Tech. Rep., CMU-CS-02-111, 2002.