$$\begin{split} J_{\theta} &= E_{\tau \sim (\tau;\theta)}[r(\tau)] = \int_{\tau} r(\tau) p(\tau;\theta) d\tau \\ \nabla_{\theta} J_{\theta} &= \int r(\tau) \nabla_{\theta} p(\tau;\theta) d\tau = \int r(\tau) p(\tau;\theta) \nabla_{\theta} \log p(\tau;\theta) d\tau \end{split}$$

$$= E_{\tau \sim p(\tau;\theta)} r(\tau) \nabla_{\theta} \log p(\tau;\theta) = E_{\tau \sim p(\tau;\theta)} r(\tau) \sum_{t \geq 0} \nabla_{\theta} \log \pi_{\theta}(a_t \mid s_t)$$

$$loss \approx \frac{1}{|batch|} \sum_{batch} r(\tau) \sum_{t \ge 0} \log \pi_{\theta}(a_t | s_t)$$
$$\approx \frac{1}{|batch|} \sum_{batch} \sum_{t \ge 0} [r(\tau) - b_t] \log \pi_{\theta}(a_t | s_t)$$