$$
\begin{aligned}
J_\theta &= E_{\tau\sim p(\tau;\theta)}[r(\tau)] = \int r(\tau)p(\tau;\theta)d\tau = \int d\tau r(\tau)\int \nabla_\theta p(\tau;\theta)d\theta \\
&= \int d\tau r(\tau)\int p(\tau;\theta)\nabla_\theta \log p(\tau;\theta)d\theta = \int d\theta \int p(\tau;\theta) r(\tau)\nabla_\theta \log p(\tau;\theta)d\tau \\
&= \int d\theta E_{\tau\sim p(\tau;\theta)}[r(\tau)\nabla_\theta \log p(\tau;\theta)] = E_{\tau\sim p(\tau;\theta)}[r(\tau)\log p(\tau;\theta)] \\
&\approx \frac{1}{|batch|}\sum_{batch} r(\tau)\log p(\tau;\theta) \triangleq \frac{1}{|batch|}\sum_{batch}\sum_{t\geq 0}[r(\tau)-b_t]\log p(\tau_t;\theta)
\end{aligned}
$$