Credit Migration Matrices

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Abstract:
This entry provides a brief overview of credit migration or transition matrices, which characterize past changes in credit quality of obligors (typically firms). They are cardinal inputs to many risk management applications, including portfolio risk assessment, the pricing of bonds and credit derivatives, and the assessment of regulatory capital as is the case for the New Basel Accord. I address questions of how to estimate these matrices, how to make inference and compare them, and provide two examples of their use: the pricing of a derivative called a yield spread option, and the calculation of the value distribution for a portfolio of credit assets. The latter is especially useful for risk management of credit portfolios.

Keywords: Credit risk, credit portfolios, credit derivatives, Markov, probabilities of default

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1. Introduction

Credit migration or transition matrices, which characterize past changes in credit quality of obligors (typically firms), are cardinal inputs to many risk management applications, including portfolio risk assessment, pricing of bonds and credit derivatives, and assessment of risk capital. For example, standard bond pricing models such as [15] require a ratings projection of the bond to be priced. These matrices even play a role in regulation: in the New Basel Capital Accord [4], capital requirements are driven in part by ratings migration. This chapter provides a brief overview of credit migration matrix basics: how to compute them, how to make inference and compare them, and some examples of their use. We pay special attention to the last column of the matrix, namely the migration to default. Along the way we illustrate some of the points with data from one of the rating agencies, Standard and Poor's (S&P).

To fix ideas, suppose that there are k credit ratings for k-1 non-default states and one default state. This rating is designed to serve as a summary statistic of the credit quality of a borrower or obligor such as a firm. Ratings can be either public as provided by one of the rating agencies such as Fitch, Moody's or S&P, or they can be private such as an obligor rating internal to a bank. For firms that have issued public bonds, typically at least one rating from a rating agency is available [5]. As such the rating agencies are expected to follow the credit quality of the firm. When that changes, the agency may decide to upgrade or downgrade the credit rating of the firm. In principle the process from initial rating to the updates is the same within a financial institution when assigning so-called internal ratings, though the monitoring may be less intensive and hence the updating may be less frequent [18]. Purely as a matter of convenience, we will follow the notation used by Fitch and S&P which, from best to worst, is AAA, AA, A, BBB, BB, B, CCC, and of course D; so $k = 8$.

All three rating agencies actually provide rating modifiers (e.g. for Fitch and S&P, these are +/-, as in AA- or AA+) to arrive at a more nuanced, 17+ state rating system. But for simplicity most of the discussion in this chapter is confined to whole grades.

A concrete example of a credit migration is given below in Figure 1 where we show the one-year migration probabilities for firms, estimated using S&P ratings histories from 1981-2003. A given row denotes the probability of migrating from rating i at time T to any other rating j at time $T+1$. For example, the one-year probability that an AA rated firm is downgraded to A is 7.81%.

Several features – and these are typical – immediately stand out. First, the matrix is diagonally dominant, meaning that large values lie on the diagonal (bolded for emphasis) denoting the probability of no change or migration: for most firms ratings do not change. Such stability is by design [2]: the agencies view that investors look to them for just such stable credit rating assessments. The next largest entries are one step off the diagonal, meaning that when there are changes, they tend to be small, namely one or perhaps two rating grades.

<div align="center">T+1</div>

		AAA	AA	A	BBB	BB	B	CCC	D
	AAA	**92.29**	6.96	0.54	0.14	0.06	0.00	0.00	0.000
	AA	0.64	**90.75**	7.81	0.61	0.07	0.09	0.02	0.010
	A	0.05	2.09	**91.38**	5.77	0.45	0.17	0.03	0.051
T	BBB	0.03	0.20	4.23	**89.33**	4.74	0.86	0.23	0.376
	BB	0.03	0.08	0.39	5.68	**83.10**	8.12	1.14	1.464
	B	0.00	0.08	0.26	0.36	5.44	**82.33**	4.87	6.663
	CCC	0.10	0.00	0.29	0.57	1.52	10.84	**52.66**	34.030
	D	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**100**

Figure 1: One-year credit migration matrix using S&P rating histories, 1981-2003. Estimation method is cohort. All values in percentage points.

The final column, the migration to default, deserves special attention. Probabilities of default increase roughly exponentially as one descends the credit spectrum from best (AAA) to

worst (CCC). Note that in our sample period of 1981-2003, no AAA-rated firm defaulted, so that the empirical estimate of this probability of default (PD_{AAA}) is zero. But is zero an acceptable estimate of PD_{AAA}? We revisit this question below.

To square the matrix the last row is the unit vector which simply states that default is an absorbing state: once a firm is in default, it stays there. By implication it means that all firms eventually default, though it may take a (very) long time. A firm which emerges from bankruptcy (default) is typically treated as a new firm.

2. Estimation

Several approaches to estimating these migration matrices are presented and reviewed in [17] and compared extensively in [14]. Broadly there are two approaches, cohort and two variants of duration (or hazard) – parametric (imposing time homogeneity) and nonparametric (relaxing time homogeneity). The assumption of time homogeneity essentially implies that the process is time invariant: the analyst can be indifferent between two equally long samples drawn from different time periods.

The straight forward cohort approach has become the industry standard. In simple terms, the cohort approach just takes the observed proportions from the beginning of the year to the end (for the case of annual migration matrices) as estimates of migration probabilities. Suppose there are $N_i(t)$ firms in rating category i at the beginning of the year t, and $N_{ij}(t)$ migrated to grade j by year-end. An estimate of the transition probability for year t is $P_{ij}(t) = \dfrac{N_{ij}(t)}{N_i(t)}$. For example, if two firms out of 100 migrated from grade 'AA' to 'A', then $P_{AA \rightarrow A} = 2\%$. Any movements within the year are not accounted for. Typically firms whose ratings were withdrawn or migrated to Not Rated (NR) status are removed from the sample.

This approach effectively treats migrations to NR as being non-informative [6]. It is straightforward to extend this approach to multiple years. For instance, suppose that we have data for T years, then the estimate for all T years is:

$$(1) \qquad P_{ij} = \frac{N_{ij}}{N_i} = \frac{\sum_{t=1}^{T} N_{ij}(t)}{\sum_{t=1}^{T} N_i(t)}$$

Indeed this is the maximum likelihood estimate of the transition probability under a discrete time-homogeneous Markov chain. The matrix shown in Figure 1 was estimated using the cohort approach.

Any rating change activity which occurs *within* the period is ignored, unfortunately. A strength of the alternative duration approach is that it counts *all* rating changes over the course of the year (or multi-year period) and divides by the number of firm-years spent in each state or rating to obtain a matrix of migration intensities which are assumed to be time-homogenous. Under the assumption that migrations follow a Markov process, these intensities can be transformed to yield a matrix of migration probabilities.

Following [17], the $k \times k$ transition probability matrix $\mathbf{P}(t)$ can be written as

$$(2) \qquad \mathbf{P}(t) = \exp(\mathbf{\Gamma} t) \quad t \geq 0,$$

where the exponential is a matrix exponential, and the entries of the generator matrix $\mathbf{\Gamma}$ satisfy

$$(3) \qquad \begin{aligned} \gamma_{ij} &\geq 0 \text{ for } i \neq j \\ \gamma_{ii} &= -\sum_{j \neq i} \gamma_{ij}. \end{aligned}$$

The second expression in Eq. (3) merely states that the diagonal elements are such to ensure that the rows sum to zero.

The maximum likelihood estimate of an entry γ_{ij} in the intensity matrix $\mathbf{\Gamma}$ is given by

$$(4) \qquad \hat{\gamma}_{ij} = \frac{n_{ij}(T)}{\int_0^T Y_i(s)ds},$$

where $Y_i(s)$ is the number of firms with rating i at time s, and $n_{ij}(T)$ is the total number of transitions over the period from i to j where $i \neq j$. The denominator in Eq. (4) effectively is the number of "firm years" spent in state i. Thus for a horizon of one year, even if a firm spent only some of that time in transit, say from 'AA' to 'A' before ending the year in 'BBB', that portion of time spent in 'A' will contribute to the estimation of the transition probability $P_{AA \to A}$. Moreover, firms which ended the period in an 'NR' status are still counted in the denominator, at least the portion of the time they spent in state i.

The Markov assumption, while convenient, may be unrealistic. A Markov process has no memory: to compute future ratings, only knowledge of the current rating is required, not the path of how the firm arrived at that rating. This makes the calculation of multi-year migration matrices quite easy. If **P** is the one-year migration matrix, then the h-year matrix is just $\mathbf{P}^h$.

A prime example of non-Markovian behavior is ratings drift, first documented in [1] and [7]. Others have documented industry heterogeneity and time variation due in particular to the business cycle [20, 3, 17]. The literature is only recently beginning to propose modeling alternatives to address these departures from the Markov assumption. For example, [8] consider the possibility of latent "excited" states for certain downgrades in an effort to address serial correlation of ratings changes (or ratings drift). A hidden Markov model is used in [11] to back out the state of the economy from ratings dynamics, [10] introduce a dynamic factor model which in turn drives rating dynamics, and [9] considers mixtures of Markov processes. Nonetheless practitioners continue to use the Markov models, and it remains an important open question just how "bad" this assumption is for practical purposes. For shorter horizons, Markov violations are likely modest, but they do increase as the forecast horizon increases [3].

3. Inference and comparison

Suppose that a new year of data becomes available, and the analyst is faced with the task of updating a migration matrix. To illustrate, consider the matrix displayed in Figure 2 which adds one more year of data (2004) to the sample used in Figure 1. Clearly most of the values are different, but are the two matrices as a whole or individual cell entries (migration probabilities) really significantly different?

		AAA	AA	A	BBB	BB	B	CCC	D
					T+1				
	AAA	**93.00**	6.18	0.66	0.07	0.08	0.01	0.00	0.000
	AA	0.61	**91.03**	7.53	0.64	0.09	0.08	0.01	0.005
	A	0.08	1.99	**91.69**	5.55	0.49	0.18	0.01	0.008
T	**BBB**	0.03	0.26	4.05	**89.70**	5.05	0.76	0.07	0.083
	BB	0.04	0.11	0.56	5.26	**83.80**	8.95	0.73	0.548
	B	0.00	0.07	0.23	0.50	4.67	**84.36**	5.71	4.448
	CCC	0.06	0.01	0.34	0.56	1.10	7.99	**47.02**	42.896
	D	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**100**

Figure 2: One-year credit migration matrix using S&P rating histories, 1981-2004. Estimation method is cohort. All values in percentage points. This figure updates Figure 1 with one more year of data.

To help answer such questions, [14] devised a scalar metric, M_{SVD}, using singular value decomposition where a larger value means the matrix is more dynamic (on average smaller entries on the diagonal). For a given migration matrix $\mathbf{P}$, first define a mobility matrix as $\mathbf{P}$ minus the identity matrix (of the same dimension), i.e. $\tilde{\mathbf{P}} = \mathbf{P} - \mathbf{I}$, thereby isolating all of the dynamics (the identity matrix denotes zero movement) in $\tilde{\mathbf{P}}$.

Then:

(5)
$$M_{SVD}(\mathbf{P}) = \frac{1}{k}\sum_{i=1}^{k}\sqrt{\lambda_i\left(\tilde{\mathbf{P}}'\tilde{\mathbf{P}}\right)},$$

where λ_i are the eigenvalues of $\tilde{\mathbf{P}}$. Using Eq. (5) we find that the "older" matrix has a value $M_{SVD} = 0.1700$ which increases to 0.1775 with the additional year of data, meaning the matrix has become more dynamic. To put this into perspective, [14] report the difference between a matrix estimated using data just during U.S. recessions and one during expansions to be 0.0434 as compared to $0.0075 = 0.1775 - 0.1700$ in our comparison. Thus the additional year of data has only a modest impact on the estimate of the migration matrix.

One may be particularly interested in the precision of default probability estimates. The first to report confidence sets for default probability estimates were [8] who used a parametric bootstrap. An interesting approach for the common case where no defaults have actually been observed was developed in [21] based on the most-prudent estimation principle, assuming that the ordinal borrower ranking is correct (i.e. monotonic). A systematic comparison of confidence intervals was provided by [13] using several analytical approaches as well as finite-sample confidence intervals obtained from parametric and nonparametric bootstrapping. They find that the bootstrapped intervals for the duration based estimates are surprisingly tight and that the less efficient cohort approach generates much wider intervals. Yet even with the tighter bootstrapped confidence intervals for the duration based estimates, it is impossible to statistically distinguish notch-level (grade with +/- modifiers) *PD*s for neighboring investment grade ratings, e.g. a PD_{AA-} from a PD_{A+} or even a PD_A. However, once the speculative grade barrier (i.e. moving from BBB- to BB+) is crossed, they are able to distinguish quite cleanly notch-level estimated default probabilities. Moreover, both [22] and [13] show that *PD* point estimates and, unsurprisingly, their confidence intervals vary substantially over time.

An advantage of the duration over the cohort estimation approach is that it delivers non-zero default probability estimates even when no actual defaults were observed. As a result the *PD* estimates can be quite different, even taking into account the issue of estimation noise

raised above. This is shown in Figure 3 at the more granular notch-level using S&P ratings histories for 1981-2004. Note that neither method produces monotonically increasing *PD*s, though in the presence of estimation noise this non-monotonicity need not be surprising. Put differently, even if the true but unknown *PD*s are monotonically increasing, because each rating's *PD* is estimated with error, the *estimates* need not be monotonic. Since no actual defaults have been observed for AAA rated (nor AA+ or AA rated) firms over the course of the sample period, the cohort estimates must be identically equal to zero even as the duration approach generates a very small, but non-zero, estimate of 0.02bp (or 0.0002%).

	Cohort	Duration
AAA	0.00	0.02
AA+	0.00	0.05
AA	0.00	0.65
AA-	2.43	0.26
A+	5.42	0.31
A	4.00	0.88
A-	3.97	0.68
BBB+	22.99	3.22
BBB	30.42	8.48
BBB-	41.76	12.03
BB+	57.24	20.61
BB	104.63	41.41
BB-	197.34	84.32
B+	336.84	172.33
B	942.53	707.07
B-	1,384.62	1,247.41
CCC	3,253.55	4,320.35

Figure 3: Unconditional probability of default (*PD*) estimates compared using S&P rating histories, 1981-2004. All values are in basis points (bp), where 100bp = 1%.

In looking at the difference for PD_{CCC} between the two methods, [17] observe that the majority of firms default after only a brief stop in the CCC rating state. By contrast the intermediate grades generate duration *PD* estimates which are below the cohort estimates. As

argued in [13], if ratings exhibit downward persistence (firms that enter a state through a downgrade are more likely to be downgraded than other firms in the state), as shown among others in [20, 17, 3], one would expect *PD*s from the duration-based approach, which assumes that the migration process is Markov, to be downward biased. Such a bias would arise because the duration estimator ignores downward ratings momentum and consequently underestimates the probability of a chain of successive downgrades ending in default.

The New Basel Accord sets a lower bound of 0.03% on the *PD* estimate which may be used to compute regulatory capital for the internal ratings based (IRB) approach [4, §285]. Figure 3 suggests that the top two ratings, AAA and AA, would both fall under that limit and would thus be indistinguishable from a regulatory capital perspective. Indeed [13] report that once 95% confidence intervals are taken into account, the top three ratings, AAA through A, are indistinguishable from 0.03%.

4. Applications

The applications and uses of credit migration matrices are myriad, from asset pricing to portfolio choice and risk management to bank regulation. Here I present two examples, the pricing of a yield spread option and the computation of risk capital for a credit portfolio using CreditMetrics®.

4.1. Yield spread option

A yield spread option enables the buyer and seller to speculate on the evolution of the yield spread. The yield spread is defined as the difference between the continuously compounded yield of a risky and a risk-less zero-coupon bond with the same maturity. Depending on the option specifications, the relevant spread is either an individual forward

spread in case of European options, or a bundle of forward spreads in case of American options. Call (put) option buyers expect a decreasing (increasing) credit spread.

Yield spread options are priced using Markov chain models such as the one presented in [16]. To price such an option the following are needed: the yield curve of default-free zero coupon bonds, the term structure of forward credit spreads, both the option and yield spread maturity (of the underlying bond), an estimate of the recovery rate in the event of default, the current rating of the bond, and of course the migration vector of the same maturity as the option corresponding to that rating. This is illustrated in Figure 4 below for a yield spread option on a BBB-rated bond.
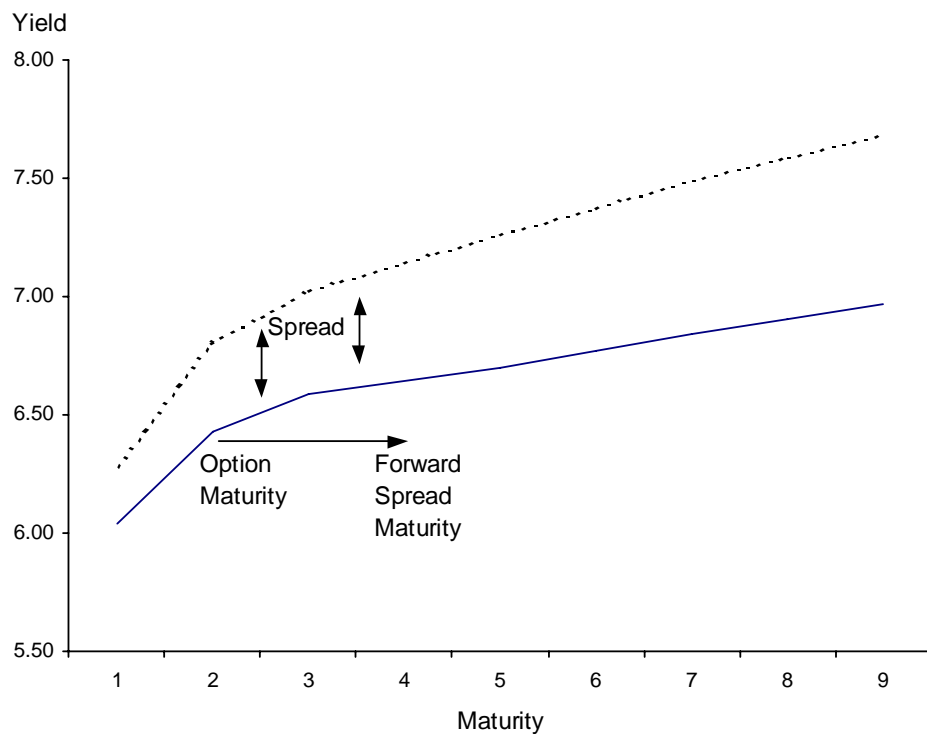

Figure 4: Illustration of a yield spread option on a BBB-rated bond.

4.2. Risk capital for a credit portfolio

The purpose of capital is to provide a cushion against losses for a financial institution. The amount of required economic capital is commensurate with the risk appetite of the financial institution. This boils down to choosing a confidence level in the loss (or value change) distribution of the institution with which senior management is comfortable. For instance, if the bank wishes to have an annual survival probability of 99%, this will require less capital than a survival probability of 99.9%, where the latter is the confidence level to which the New Basel Capital Accord is calibrated [4], and is typical for a regional bank (commensurate with a rating of about A-/BBB+, judging from Figure 3). The loss (or value change) distribution is arrived at through internal credit portfolio models.

One such credit portfolio model is CreditMetrics® [12], a portfolio application of the options-based model of firm default due to [19]. Given the credit rating distribution of exposures today, and inputs similar to the pricing of the yield spread option discussed in Section 4.1, namely the yield curve of default-free zero coupon bonds, the term structure of forward credit spreads, an estimate of the recovery rate in the event of default for each rating, and of course the credit migration matrix, the model generates a value distribution of the credit portfolio through stochastic simulation. An illustration is given in Figure 5 below. For instance, the 99% value-at-risk (VaR) is -14.24%, and the 99.9% VaR is -19.03%. Thus only for one year out of 1000 would the portfolio manager expect her portfolio to lose more that 19.03% of its value.
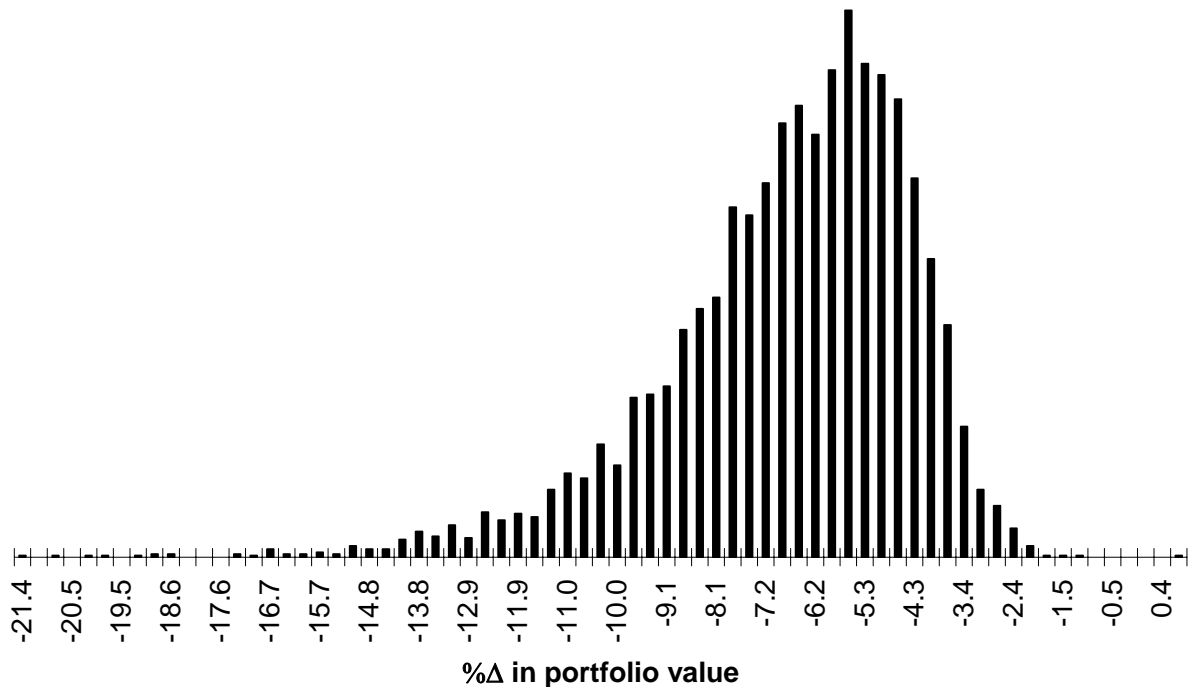
Figure 5: Illustrative example of a credit portfolio value change distribution using CreditMetrics®.

5. Summary

This chapter provided a brief overview of credit migration or transition matrices, which characterize past changes in credit quality of obligors (typically firms). They are cardinal inputs to many risk management applications, including portfolio risk assessment, the pricing of bonds and credit derivatives, and the assessment of regulatory capital as is the case for the New Basel Capital Accord. We addressed the question of how to estimate these matrices, how to make inference and compare them, and provided two examples of their use: the pricing of a derivative called a yield spread option, and the calculation of the value distribution for a portfolio of credit assets. The latter is especially useful for risk management of credit portfolios.

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