# Homework on optimization algorithms.

### P8160 Advanced Statistical Computing

# Problem 1:

Design an optmization algorithm to find the minimum of the continuously differentiable function

$$f(x) = -e^{-x}\sin(x)$$

on the closed interval [0, 1.5]. Write out your algorithm and implement it into  $\mathbf{R}$ .

# Answer: your answer starts here...

To find the minimum of a continuously function, we first make some changes to the function let

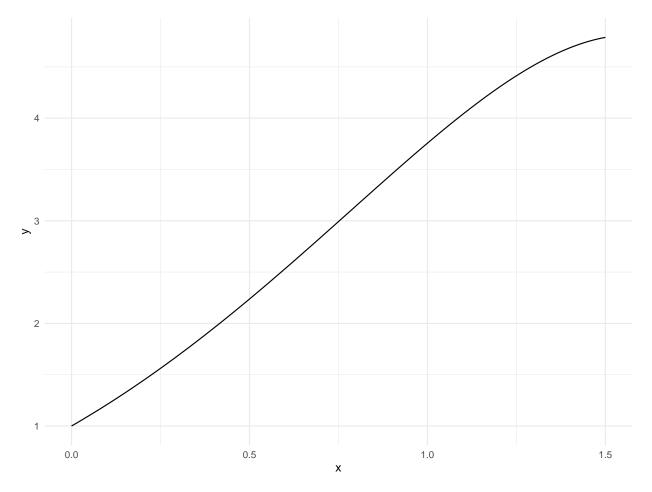
$$g(x) = e^x \sin(x)$$

and instead find the maximum of g(x).

The gradient of g(x) is:

$$\nabla g(x) = e^x(\sin(x) + \cos(x))$$

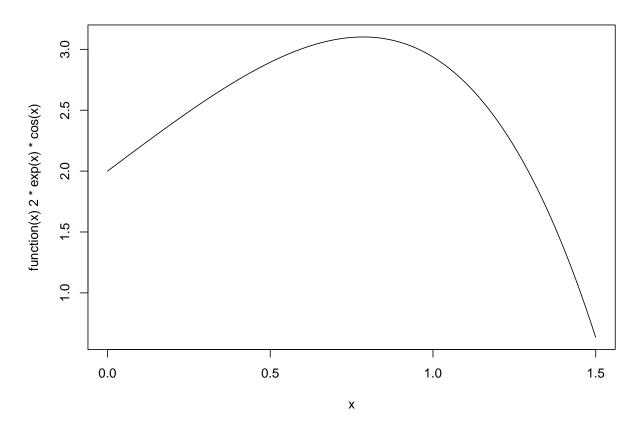
```
ggplot(tibble(x = seq(0,1.5,length = 10)),aes(x))+
geom_function(fun = function(x) exp(x)*(sin(x)+cos(x)))
```



and the Hessian is:

$$\nabla^2 g(x) = 2e^x \cos(x)$$

plot(function(x) 2\*exp(x)\*cos(x),xlim = c(0,1.5))

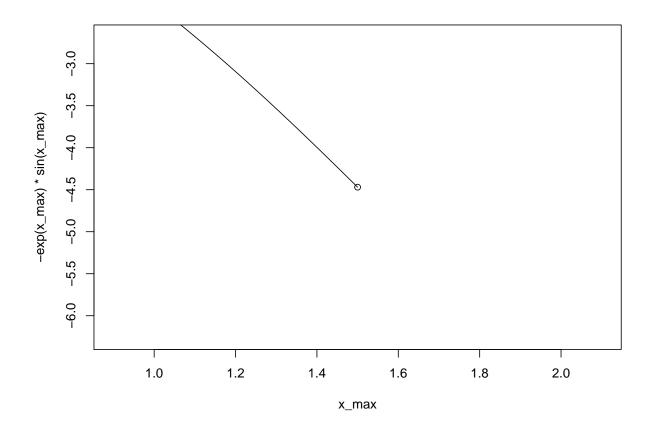


the hessian is greater than 0 everywhere in [0,1.5], so we can't use Newton method.

```
goose_egg =
  function(
    fun,
    left = NULL,
    right = NULL,
    range = NULL,
    ratio = 0.618,
    tol = 10e-4,
  ){
    if (!any(left,right)){
      left = range[1]
      right = range[2]
    }
    mid_1 = left + ratio*(right - left)
    f_mid_1 = fun(mid_1)
    mid_2 = mid_1 + ratio*(right-mid_1)
    f_{mid_2} = fun(mid_2)
```

```
f_left = fun(left)
    f_right = fun(right)
    i = 1
    while (abs(f_left - f_right)>tol && i<1000){</pre>
      i = i + 1
      if (f_mid_1 < f_mid_2) {</pre>
       f_left = f_mid_1
       left = mid_1
      } else {
        f_right = f_mid_2
        right = mid_2
      mid_1 = left + ratio * (right - left)
      f_mid_1 = fun(mid_1)
      mid_2 = mid_1 + ratio * (right - mid_1)
      f_mid_2 = fun(mid_2)
    }
    return(mean(mid_1,mid_2))
  }
x_{max} = goose_{egg}(function(x) exp(x)*sin(x), range = c(0,1.5))
print(x_max)
## [1] 1.5
plot(x_max,-exp(x_max)*sin(x_max))
```

 $plot(function(x) \{-exp(x)*sin(x)\}, xlim = c(0,1.5), add = T)$ 



### Problem 2:

The Poisson distribution, written as

$$P(Y = y) = \frac{\lambda^y e^{-\lambda}}{y!}$$

for  $\lambda > 0$ , is often used to model "count" data — e.g., the number of events in a given time period.

A Poisson regression model states that

$$Y_i \sim \text{Poisson}(\lambda_i),$$

where

$$\log \lambda_i = \alpha + \beta x_i$$

for some explanatory variable  $x_i$ . The question is how to estimate  $\alpha$  and  $\beta$  given a set of independent data  $(x_1, Y_1), (x_2, Y_2), \ldots, (x_n, Y_n)$ .

- 1. Generate a random sample  $(x_i, Y_i)$  with n = 500 from the Possion regression model above. You can choose the true parameters  $(\alpha, \beta)$  and the distribution of X.
- 2. Write out the likelihood of your simulated data, and its Gradient and Hessian functions.
- 3. Develop a modify Newton-Raphson algorithm that allows the step-halving and re-direction steps to ensure ascent directions and monotone-increasing properties.
- 4. Write down your algorithm and implement it in R to estimate  $\alpha$  and  $\beta$  from your simulated data.

### Answer: your answer starts here...

#### 2.1

```
print("hello world")

X = rbind(rep(1,500),rnorm(500))
Beta = runif(2)
lambda = exp(t(X)%*%Beta)
Y = map(lambda,~rpois(1,.x)) %>% unlist()
dat = list(y = Y, x=X)
ans = glm(Y~0+t(X),family = poisson())
```

#### 2.2

• The log-likelihood of Poisson distribution is

$$l(Y;\lambda) = \sum \{y * log(\lambda) - \lambda - log(y!)\}\$$

OR

$$l(Y; \alpha, \beta) = \sum \{y * (\alpha + x\beta) - exp(\alpha + x\beta) - log(y!)\}\$$

• The Score funtion is

$$\nabla(Y; \alpha, \beta) = \frac{\partial}{\partial \lambda} l(Y; \lambda) = (\sum \{y - exp(\alpha + x\beta)\}, \sum \{y * x - x * exp(\alpha + x\beta)\})$$

• The hessian is

$$\nabla^2(Y;\lambda) = \frac{\partial^2}{\partial \lambda^2} l(Y;\lambda)$$

$$= \begin{pmatrix} \sum -exp(\alpha + x\beta) & \sum -x * exp(\alpha + x\beta) \\ \sum -x * exp(\alpha + x\beta) & \sum -x^2 * exp(\alpha + x\beta) \end{pmatrix}$$

which is negative defined everywhere.

```
return(list(
    loglink = loglink,
    fisher = fisher,
    gradient = gradient,
    hessian = hessian
    ))
}
Poisson(Y,X,c(7,2))
```

```
## $loglink
## [1] -3974589
##
## $fisher
##
## [1,] 8.21e+08
##
## $gradient
## [1] -3980250 -7302954
##
## $hessian
##
                      [,2]
            [,1]
## [1,] -3981198 -7.3e+06
## [2,] -7303206 -1.6e+07
```

#### 2.3

the Newton method updating is:

$$\nabla g(x_{k+1}) = \nabla g(x_k) + \eta * \nabla^2 g(x_k)(x_{k+1} - x_k)$$

where  $\eta$  is the step size that ensure  $\nabla g(x_{k+1}) > \nabla g(x_k)$ 

```
#Develop a modify Newton-Raphson algorithm that allows the
\#step-halving and
#re-direction steps
#to ensure ascent directions and monotone-increasing properties.
newton_update =
  function(fun,
           previous_theta,
           y,x,
           step_size = 1,
           optimizer = F,
           backtracking = T,
           tol = 1e-8) {
    #take previous gradient and a updated hessian, return update gradient with
    #backtracking
    #if (abs(fun(y,x,previous_theta)$loglink) == Inf) stop("Check your log-likelihood")
    trial = 0
    gradient = fun(y,x,previous_theta)$gradient
```

```
if (is.function(optimizer)) {
      hessian = optimizer(y,x, fun,) # get H
    } else{
      if (is.numeric(optimizer)) {
        H = optimizer # use H
      } else{
        hessian = fun(y,x, previous_theta)$hessian
        H = solve(hessian)
        while (all(eigen(H)$values > 0)) {# eigen decomposition
          P = eigen(hessian)
          lambda = max(P$values)
          hessian =
            P$vectors %*% (diag(P$values) - (lambda + tol) * diag(length(P$values))) %*%
            solve(P$vectors)
          H = solve(hessian)
     }
    }
    #updating
    cur_theta = previous_theta - step_size * H %*% gradient
    #backtracking
    while (backtracking & fun(y,x,cur_theta)$loglink < fun(y,x,previous_theta)$loglink & trial < 2000)
      step_size = step_size / 2
     trial = trial + 1 # avoild dead loops
      cur_theta = previous_theta - step_size * H %*% gradient
    }
    return(cur_theta)
newton_update(fun = Poisson,previous_theta = c(7,2), y=Y,x =X)
        [,1]
##
## [1,]
## [2,]
naive_newton =
  function(fun,
           init_theta = 1,
           y,x,
           tol = 1e-8,
           maxtiter = 2000,
           optimizer = F,
           ...) {
    f = fun(y,x,init_theta)
    if (any(is.null(f$loglink),
```

```
is.null(f$gradient),
        is.null(f$hessian))) {
  stop("fun input must return both gradient and hessian")
result = tibble()
i = 0
cur_theta = init_theta
prevlog = -Inf # \nabla g(x_{k})
while (any(abs(f$loglink)==Inf,abs(f$loglink - prevlog) > tol) && i < maxtiter) {
  i = i + 1
  prev_theta = cur_theta
  prevlog = f$loglink
  cur_theta = newton_update(fun, prev_theta,y,x)
  f = fun(y,x,cur_theta)
  result =
    rbind(result, tibble(
      iter = i,
      x_i = list(prev_theta),
      g(x_i) = prevlog
}
return(list(theta = cur_theta,result = result))
```

```
Beta_hat = naive_newton(Poisson,init_theta = c(7,2),Y,X)$theta

tibble(
  term = c("alpha", "beta"),
  theta = Beta,
  theta_hat = Beta_hat
) %>%
  knitr::kable()
```

| term          | theta            | theta_hat      |
|---------------|------------------|----------------|
| alpha<br>beta | $0.585 \\ 0.234$ | 0.606<br>0.247 |

#### Problem 3:

The data breast-cancer.csv have 569 row and 33 columns. The first column **ID** lables individual breast tissue images; The second column **Diagnonsis** indentifies if the image is coming from cancer tissue or benign cases (M=malignant, B = benign). There are 357 benign and 212 malignant cases. The other 30 columns correspond to mean, standard deviation and the largest values (points on the tails) of the distributions of the following 10 features computed for the cellnuclei;

• radius (mean of distances from center to points on the perimeter)

- texture (standard deviation of gray-scale values)
- perimeter
- area
- smoothness (local variation in radius lengths)
- compactness (perimeter2 / area 1.0)
- concavity (severity of concave portions of the contour)
- concave points (number of concave portions of the contour)
- symmetry
- fractal dimension ("coastline approximation" 1)

The goal is to build a predictive model based on logistic regression to facilitate cancer diagnosis;

- 1. Build a logistic model to classify the images into malignant/benign, and write down your likelihood function, its gradient and Hessian matrix.
- 2. Build a logistic-LASSO model to select features, and implement a path-wise coordinate-wise optimization algorithm to obtain a path of solutions with a sequence of descending  $\lambda$ 's.
- 3. Write a report to summarize your findings.

3

### 3.1

the data is a binomial outcome response, which follows a Bernoulli distribution, Using logit link, which

$$log(\frac{p}{1-p}) = X^T \beta$$

s.t

$$p = \frac{exp(X\beta)}{1 + exp(X\beta)}$$

the log-likelihood of Bernoulli is

$$l(Y;\beta) = \sum \{y*log(\frac{p}{1-p}) + log(1-p)\} = \sum \{y*X^T\beta - log(1 + exp(X^T\beta))\}$$

The gradient is

$$\nabla l(Y;\beta) = \left(\frac{\partial}{\partial \beta_i} l(Y;\beta)\right) = \left(\sum \left(y * x_i - \frac{x_i exp(X^T \beta)}{1 + exp(X^T \beta)}\right)\right)$$

and the hessian is

$$\nabla^2 l(Y;\beta) = (\sum -\frac{x_i * x_j * exp(X^T\beta)}{(1 + exp(X^T\beta)^2)})$$

```
Bernoulli =
  function(y,x,
           theta vec){
   if (length(theta_vec)!=ncol(x)) stop("length of theta_vec must match dim of x")
   if (is.factor(y)) y = y %>% as.numeric()-1
   X = x
   loglink = sum(Y * X%*%theta_vec - log(1+exp(X%*%theta_vec)))
   if (abs(loglink) == Inf){
    stop("Choose a better starting value")
   fisher = var(Y * X%*%theta_vec - log(1+exp(X%*%theta_vec)))
   X = x*1e-0
   gradient = map(1:length(theta_vec),
                   ~ sum(Y * X[, .x] - X[, .x] * exp(X %*% theta_vec) /
                           (1 + exp(X %*% theta_vec)))) %>% unlist()
   hessian =
      expand.grid(i = seq(1,length(theta_vec)),
                  j = seq(1,length(theta_vec))) %>%
      summarise(beta = map2(i,j,~sum(-X[,.x]*X[,.y]*exp(X%*%theta_vec)/(1+exp(X%*%theta_vec))^2))) %%
     unnest(beta) %>%
     pull(beta)
   hessian = matrix(hessian,ncol = length(theta_vec))
   return(
      list(loglink = loglink,
           fisher = fisher,
           gradient = gradient*1e+0,
           hessian = hessian*1e+0)
  }
Bernoulli(Y,X,rep(0,10))
## $loglink
## [1] -394
##
## $fisher
##
        [,1]
## [1,]
##
## $gradient
                              1707.73 -21099.85
                                                     5.60
## [1]
           317.09
                     907.67
                                                              -1.09
                                                                        -8.82
## [8]
            -4.74
                      10.64
                                 4.58
##
## $hessian
                      [,2] [,3]
                                                                        [,7]
##
             [,1]
                                         [,4]
                                                   [,5]
                                                             [,6]
```

```
[1,]
          -30154
                   -39461 -196955 -1489947 -194.847 -2.23e+02
                                                                    -205.45
##
   [2,]
          -39461
                   -55557 -257249 -1865996 -264.207 -2.94e+02
                                                                    -258.39
##
   [3,] -196955 -257249 -1287036 -9765529 -1270.701 -1.47e+03 -1358.71
   [4,] -1489947 -1865996 -9765529 -78593927 -9101.146 -1.10e+04 -11005.14
##
##
   [5,]
            -195
                     -264
                              -1271
                                       -9101
                                                -1.349 -1.50e+00
                                                                      -1.30
                     -294
##
  [6,]
            -223
                              -1467
                                      -11036
                                                -1.500 -1.94e+00
                                                                     -1.85
##
  [7,]
            -205
                     -258
                              -1359
                                                -1.300 -1.85e+00
                                                                     -2.02
                                      -11005
            -114
## [8,]
                     -141
                              -754
                                       -6154
                                                -0.713 -9.68e-01
                                                                     -1.02
                                                -2.514 -2.81e+00
##
   [9,]
            -366
                     -498
                              -2387
                                       -17084
                                                                      -2.44
                               -815
                                        -5750
                                                -0.869 -9.62e-01
## [10,]
            -125
                     -172
                                                                     -0.82
##
              [,8]
                        [,9]
                                 [,10]
##
   [1,]
         -114.280
                    -366.09
                             -125.098
         -141.187
                             -171.984
##
   [2,]
                    -498.30
##
  [3,] -753.925 -2387.38 -815.186
## [4,] -6153.567 -17083.93 -5750.212
##
   [5,]
           -0.713
                      -2.51
                               -0.869
##
  [6,]
           -0.968
                      -2.81
                               -0.962
## [7,]
           -1.023
                      -2.44
                               -0.820
## [8,]
           -0.554
                      -1.33
                               -0.443
## [9,]
           -1.331
                      -4.78
                               -1.632
## [10,]
           -0.443
                      -1.63
                               -0.568
```

#### # Bernoulli(dat,ans\$coefficients[-1] %>% as.vector())

#### 3.2

```
soft_threshold =
  function(beta, lambda) {
    if (abs(beta)>lambda) {
        if (beta > 0) return(beta - lambda)
        return(beta + lambda)
    }
    return(0)
    }
```

```
bfun =
  function(y, x, theta_vec) {
    if (is.factor(y))
        y = y %>% as.numeric() - 1
    p_prev = exp(x %*% theta_vec) / (1 + exp(x %*% theta_vec)) # row matrix

w_prev = p_prev * (1 - p_prev) # row matrix

z_prev = x %*% theta_vec + (y - p_prev) / w_prev # row * 1 matrix

loglink =
    #sum(w_prev * (z_prev - x %*% theta_vec) ^ 2) / (2 * length(y))
    sum(w_prev * (z_prev - x %*% theta_vec) ^ 2) / (2)

return(list(
    loglink = loglink,
```

```
p = p_prev,
w = w_prev,
z = z_prev
))

gfun = function(y,x,theta_vec){
z = y
w = rep(1/length(y),length(y))
loglink = sum(w *(y - x %*% theta_vec) ^ 2) / 2
return(
list(z=z,w=w,loglink=loglink)
)
}
```

```
Qlasso =
  function(y,
           х,
           family_fun,
           lambda = 0,
           theta_vec = NA,
           maxiter = 500,
           tol = 1e-6,
           intercept = T,
           ...) {
    # data preparation
   if (is.factor(y))
      y = y \%  as.numeric() - 1
   y = y
   x = scale(x)
   xsd = attr(x, "scaled:scale") %>% as.vector()
   if (intercept) {
     x = cbind(rep(1, length(y)), x) # add alpha
      xsd = append(1, xsd)
    # Manually choosing starting value
   if (!is.na(theta_vec)) {
      if (length(theta_vec) != ncol(x))
       theta_vec = append(rep(0, ncol(x) - length(theta_vec)), theta_vec)
   } else
      theta_vec = rep(0, ncol(x))
    #update part
   iter = 0
   cur_result = family_fun(y, x, theta_vec)$loglink
   if (abs(cur_result) == Inf|is.na(cur_result))
      stop("Diverge at starting value")
   prev_result = -Inf
   while (abs(cur_result - prev_result) > tol
```

```
iter = iter + 1
     prev_result = cur_result
      for (i in 1:length(theta_vec)) {
        #weight
        fun_prev = family_fun(y, x, theta_vec)
        z_prev = fun_prev$z # working response
        w_prev = fun_prev$w # weight
        #update
        cur_theta =
            sum((z_prev - x[, -i] %*% theta_vec[-i]) * x[, i] * w_prev)
         #soft-threshold
        if (i > 1) {
          cur_theta =
            soft_threshold(cur_theta,
                           lambda) / sum(w_prev * (x[, i] ^ 2))
        } else {
          cur_theta = cur_theta / sum(w_prev * (x[, i] ^ 2))
        #update theta
       theta_vec[[i]] = cur_theta
      cur_result = family_fun(y, x, theta_vec)$loglink + lambda * sum(abs(theta_vec))
   return(list(coefficient = theta_vec / xsd,
                loglikelihood = cur_result))
 }
cv_Qlasso =
  function(y,x,family_fun,lambda=NA,number = 5,intercept = T){
   result = tibble()
   lambda = as.vector(lambda)
   if (all(is.na(lambda))) {
     Y = y
     if (is.factor(y)) Y = y %>% as.numeric()-1
     Y = as.vector(Y)
     max_lambda = length(Y)*max(colSums(diag(Y) %*% scale(x)))
     lambda = exp(seq(log(max_lambda),-10,length=20))
   block_length = length(y)/number
   for (lam in lambda) {
     for (i in 1:number) {
        #create training partition
       test_index = (1 + (number-1)*block_length):(number*block_length)
       trainX = x[-test_index,]
       testX = x[test_index,]
       trainY = y[-test_index]
       testY = y[test_index]
```

&& iter < maxiter) {</pre>

```
trainF = Qlasso(trainY,trainX,family_fun,lambda = lam,intercept = intercept)
        if (intercept) {prev_coef = trainF$coefficient[-1]
        }else {prev_coef = trainF$coefficient}
        #test
        testF = family_fun(testY,testX,prev_coef)
        result =
         result %>%
          rbind(expand.grid(coefficient = trainF$coefficient,
                            lambda = lam,
                            loglikelihood = testF$logli+ lam*sum(abs(trainF$coefficient))) %>%
                  cbind(tibble(term = (1:length(trainF$coefficient))-1) %>%
                          mutate_all(as.character))
                )
      }
   }
    # take the mean of everything
   result = result %>%
      group_by(lambda,term) %>%
      summarise(across(c(coefficient,loglikelihood),mean))%>%
      arrange((loglikelihood)) %>%
      nest(coef = c(coefficient,term))
    coef = result[1,] %>%
      unnest() %>%
      pull(coefficient)
   lambda = result[1,"lambda"] %>% as.numeric()
   loglikelihood = result[1,"loglikelihood"] %>% as.numeric()
   return(list(
      coefficient = coef,
      loglikelihood = loglikelihood,
      lambda = lambda,
      cvtable = result
   ))
 }
tibble(glm = glm(Y_g~X_g)$coefficient,
       glmnet_0lambda = coef(glmnet(X_g,Y_g,lambda = 0)) %% as.vector(),
       Qlasso_Olambda = Qlasso(Y_g,X_g,gfun)$coefficient,
       Qlasso_01lambda = Qlasso(Y_g, X_g, gfun, 5)$coefficient,
       glmnet_01lambda = coef(glmnet(X_g,Y_g,lambda = 5)) %>% as.vector()
       ) %>%
```

Table 2: Function comparison at Gaussian Family

knitr::kable(caption = "Function comparison at Gaussian Family")

| glm   | glmnet_0lambda | Qlasso_0lambda | Qlasso_01lambda | glmnet_01lambda |
|-------|----------------|----------------|-----------------|-----------------|
| 0.076 | 0.076          | 3.306          | 3.306           | 2.480           |
| 6.042 | 6.042          | 6.042          | 2.061           | 2.082           |

| glm       | glmnet_0lambda | Qlasso_0lambda | Qlasso_01lambda | glmnet_01lambda |
|-----------|----------------|----------------|-----------------|-----------------|
| ${6.986}$ | 6.986          | 6.986          | 2.533           | 2.556           |
| 3.413     | 3.413          | 3.413          | 0.000           | 0.000           |
| 6.519     | 6.519          | 6.519          | 3.248           | 3.265           |
| 0.504     | 0.504          | 0.504          | 0.000           | 0.000           |
| 8.601     | 8.601          | 8.601          | 4.482           | 4.503           |
| 6.227     | 6.227          | 6.227          | 0.524           | 0.554           |
| 0.305     | 0.305          | 0.305          | 0.000           | 0.000           |
| 7.785     | 7.785          | 7.785          | 3.410           | 3.431           |
| 0.469     | 0.469          | 0.469          | 0.000           | 0.000           |

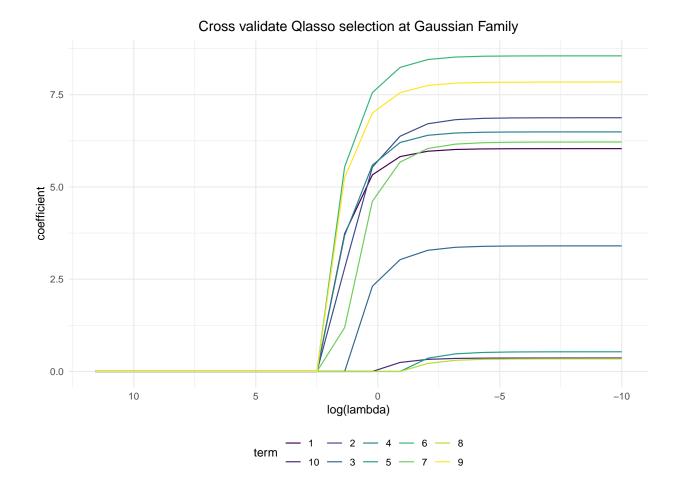
Table 3: Function comparision at binomial family

| glm     | $glmnet\_0lambda$ | Qlasso_0lambda | Qlasso_56.9lambda | ${\rm glmnet\_01lambda}$ |
|---------|-------------------|----------------|-------------------|--------------------------|
| 7.360   | 8.317             | -0.244         | 0.587             | 4.282                    |
| 2.049   | 1.608             | 1.814          | 0.000             | 0.000                    |
| -0.385  | -0.384            | -0.381         | -0.009            | -0.009                   |
| 0.072   | 0.120             | -0.010         | -0.022            | -0.022                   |
| -0.040  | -0.038            | -0.031         | 0.000             | 0.000                    |
| -76.432 | -75.580           | -78.093        | 0.000             | 0.000                    |
| 1.462   | 0.452             | 6.939          | 0.000             | 0.000                    |
| -8.469  | -8.730            | -9.772         | 0.000             | 0.000                    |
| -66.822 | -66.866           | -62.523        | -29.644           | -29.656                  |
| -16.278 | -16.226           | -16.658        | 0.000             | 0.000                    |
| 68.337  | 67.619            | 52.700         | 0.000             | 0.000                    |

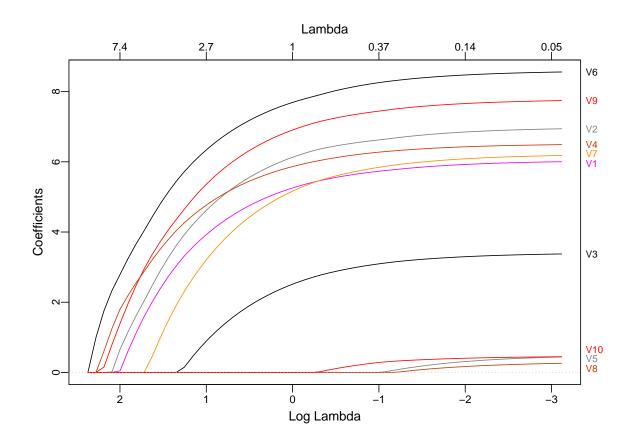
From above tables we can see that the Qlasso function of mine works reasonably close to glmnet except for the intercept terms in the case of Gaussian family. However, the result of binomial family behave rather different with L1 penalties. The function produce similar result with lambda = 0 except for intercept and once adding lamdba, the effect of lambda is rather different to glmnet.

```
gaussian_Qlasso = cv_Qlasso(Y_g,X_g,gfun)

gaussian_Qlasso$cvtable %>%
  unnest(coef) %>%
  filter(term != 0) %>%
  ggplot(aes(x = log(lambda), y = coefficient, color = term)) +
  geom_line() +
  scale_x_reverse() +
  labs(title = "Cross validate Qlasso selection at Gaussian Family")
```



```
gaussian_glmnet = cv.glmnet(X_g,Y_g)
plotmo::plot_glmnet(gaussian_glmnet$glmnet.fit)
```



```
tibble(term = c("intercept",colnames(X_g %>% as_tibble())),
    Qlasso = gaussian_Qlasso$coef,
    glmnet = coef(gaussian_glmnet) %>% as.vector()) %>%
knitr::kable(caption = "Gaussian family result")
```

Table 4: Gaussian family result

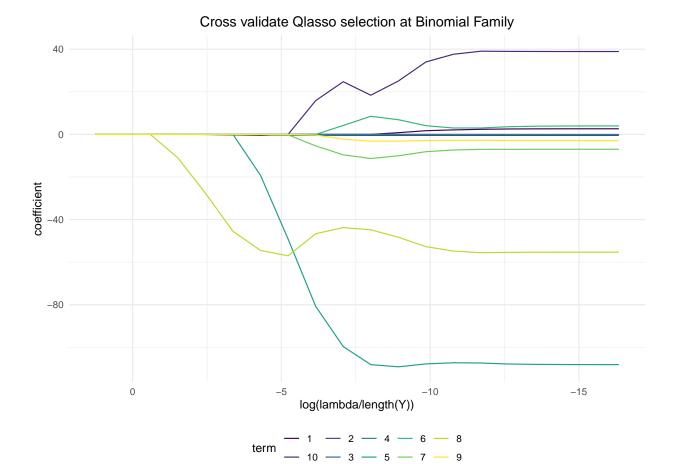
| term      | Qlasso | glmnet |
|-----------|--------|--------|
| intercept | 4.056  | 0.123  |
| V1        | 6.038  | 5.961  |
| V2        | 0.365  | 6.892  |
| V3        | 6.876  | 3.333  |
| V4        | 3.403  | 6.457  |
| V5        | 6.492  | 0.370  |
| V6        | 0.533  | 8.514  |
| V7        | 8.553  | 6.130  |
| V8        | 6.218  | 0.211  |
| V9        | 0.342  | 7.699  |
| V10       | 7.843  | 0.424  |

Using cross validation we can see that for gaussian, the penalty in the same scale as the glmnet, but because

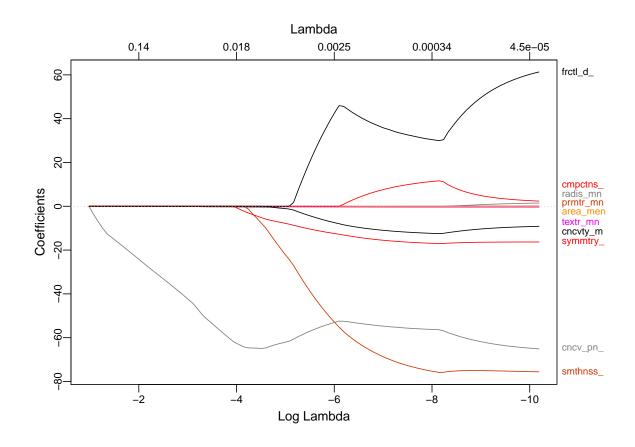
of the intercept is different, the optimum model chosen is different. same result(bug) can be observed in the binomial family, but rather deteriorate. The penalty although work similar to glmnet in terms of trends, but on a rather different scale(sample size). Also, the intercept produce by the function is not even the  $log(\frac{\bar{Y}}{1-\bar{Y}})|X=0$ , so the objective function failed to aid the function to choose the optimum model.

```
binomial_Qlasso = cv_Qlasso(Y,X,bfun)

binomial_Qlasso$cvtable %>%
   unnest(coef) %>%
   filter(term != 0) %>%
   ggplot(aes(x = log(lambda/length(Y)), y = coefficient, color = term)) +
   geom_line() +
   scale_x_reverse() +
   labs(title = "Cross validate Qlasso selection at Binomial Family")
```



```
binomial_glmnet = cv.glmnet(X,Y,family = "binomial")
plotmo::plot_glmnet(binomial_glmnet$glmnet.fit)
```



```
tibble(term = c("intercept",colnames(X)),
    Qlasso = binomial_Qlasso$coef,
    glmnet = coef(binomial_glmnet) %>% as.vector()) %>%
knitr::kable(caption = "Binomial family result")
```

Table 5: Binomial family result

| term                   | Qlasso | glmnet  |
|------------------------|--------|---------|
| intercept              | 0.373  | 14.867  |
| radius_mean            | 0.000  | -0.438  |
| texture_mean           | 0.000  | -0.201  |
| perimeter_mean         | 0.000  | 0.000   |
| area_mean              | 0.000  | 0.000   |
| smoothness_mean        | 0.000  | -3.431  |
| compactness_mean       | 0.000  | 0.000   |
| concavity_mean         | 0.000  | 0.000   |
| concave_points_mean    | 0.000  | -64.722 |
| symmetry_mean          | 0.000  | -3.772  |
| fractal_dimension_mean | 0.000  | 0.000   |

# 3.3

The result from