Homework 5 on MCMC

Leave your name and uni here

Due: 04/18/2020, by 11:59pm

Problem 1

Derive the posterior distributions in the following settings:

1. Suppose $X_1, ..., X_n$ iid sample from $N(\theta, \sigma^2)$ distribution, the prior distribution of θ is $N(\mu, \tau^2)$, derive the posterior distribtuion of θ given $\mathbf{X}$:

2.Suppose $X_1, ..., X_n$ iid sample from $U(0, \theta)$ distribution, the prior distribution of θ is Pareto distribution with pdf

$$\pi(\theta) = \frac{\alpha\beta^\alpha}{\theta^{\alpha+1}} I\{\theta \geq \beta\}$$

with known β and α

Answer: your answer starts here...

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#R codes:
```

Problem 2

Suppose there are three possible weathers in a day: rain, nice, cloudy. The transition probabilities are

rain nice cloudy

rain 0.5 0.5 0.25

nice 0.25 0 0.25

cloudy 0.25 0.5 0.5

where the columns represent the `origin" and the rows represent the`destination" of each step. The initial probabilities of the three states are given by (0.5,0, 0.5) for (rain, nice, cloudy). Answer the following questions

1. Compute the probabilities of the three states on the next step of the chain.
2. Find the stationary distribution of the chain
3. Write an R algorithm for the realization of the chain and illustrate the feature of the chain.

Answer: your answer starts here...

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#R codes:
```

problem 3 Consider the bivariate density

$$f(x,y) \propto \binom{n}{x} y^{x+a-1}(1-y)^{n-x+b-1}, x = 0, 1, \cdots, n, 0 \leq y \leq 1$$

Complete the following tasks:

1. Write the algorithm of the Gibbs sampler, implement it in R program, and generate a chain with target joint density $f(x,y)$
2. Use a Metropolis sampler to generate a chain with target joint density $f(x;y)$ and implement in R program.
3. Suppose $n = 30, a = 9, b = 14$, use simulations to compare the performance of the above two methods.

Answer: your answer starts here...

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#R codes:
```