Homework 6

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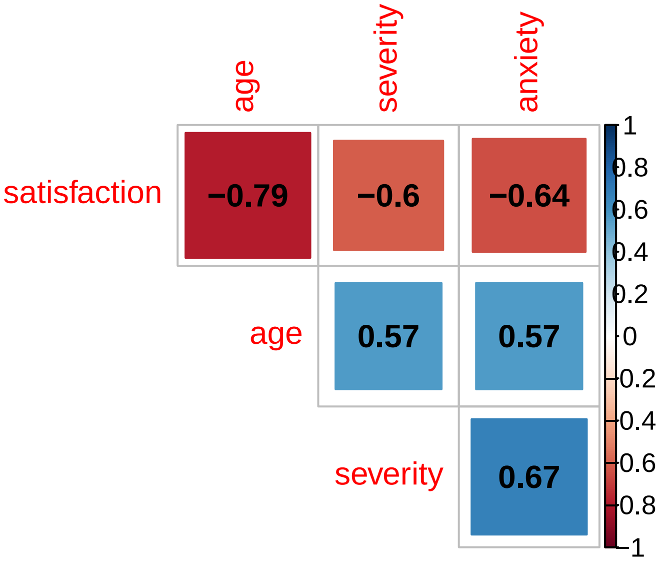
Problem 1(15p)

A hospital administrator wishes to test the relationship between 'patient's satisfactionscore' (Y) and three potential predictors:'age', 'severity of illness', and 'anxiety level' (see dataset'PatSatisfaction.csv'). The administrator randomly selected 46 patients, collected the data, and asked for your help with the analysis.

1. Create a correlation matrixfor all variablesand interpret your findings. Focus onthe correlationvalues between each predictor and the outcomeof interest. (2p)



2. Fit a multiple regression model including all three predictorsand test whether at least one of these variables is significant. State the hypotheses, test-statistic, decision rule and conclusion. (3p)

To test if at least one of these model is significant, we propose hypothesis:

$H_0 \ : \ \beta_1 = \beta_2 = \beta_3 = 0$

H_1 : at least one of the coefficient not equal to 0

The model we fit is:

$$statisfaction = 158.49 - 1.14 age - 0.44 severity - 13.47 anxiety$$

We campared with the model with only intercept, and with ANOVA, we have

$$F^* = \frac{MSR(0|X1X2X3)}{MSE(X1X2X3)} \sim F_{df_L - df_S, df_L}$$

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## Analysis of Variance Table
##
## Model 1: satisfaction ~ 1
## Model 2: satisfaction ~ age + severity + anxiety
##   Res.Df   RSS Df Sum of Sq  F  Pr(>F)
## 1     45 13369
## 2     42  4249  3      9120 30 1.5e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

With F statistics of 30, at 95% confidence level, we have critical value of 2.827, we reject the null hypothesis and conclude that at least one predictor have coefficent not equal to zero.

3. Show the regression results for all estimated slope coefficients with 95% CIs. Interpret the coefficient and 95% CI associated with 'severity of illness'. (5p)

term	conf.low	conf.high
(Intercept)	121.91	195.071
age	-1.57	-0.708
severity	-1.44	0.551
anxiety	-27.80	0.858

The CI for coefficient of disease severity is (-1.43, 0.55), we interpret as: the mean change of satisfaction give all the same except for disease severity per unit is somewhere within this interval, and 0 is included.

4. Obtain an interval estimate for a new patient's satisfaction with the following characteristics:Age=35, Severity=42, Anxiety=2.1. Interpret the interval. (2p)

The 95% confidence prediciton interval for Age=35, Severity=42, Anxiety=2.1 is (50.06, 93.3)

5. a) Test whether 'anxiety level' can be dropped from the regression model, given the other two covariates are retained. State the hypotheses, test-statistic, decision rule and conclusion. (1.5p)

We campared the full model with model without anxiety, and with ANOVA, we have

$H_0 \ : \ \beta_{anxiety} = 0$

$H_1 \ : \ \beta_{anxiety} \neq 0$

$$F^* = \frac{MSR(X3|X1X2)}{MSE(X1X2X3)} \sim F_{df_L - df_S, df_L}$$

```
## Analysis of Variance Table
##
## Model 1: satisfaction ~ (age + severity + anxiety) - anxiety
## Model 2: satisfaction ~ age + severity + anxiety
##   Res.Df  RSS Df Sum of Sq   F Pr(>F)
## 1     43 4613
## 2     42 4249  1       364 3.6  0.065 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

At 95% confidence level, we can't reject the null hypothesis and conclude that the model include *anxiety* is not superior or different from model without *anxiety*.

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b) How areR2/R2-adjusted impacted by the action that youtook in part 5-a)? (1.5p)
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model	r_square	adj_r_square
full model	0.682	0.659
model withour anxiety	0.655	0.639

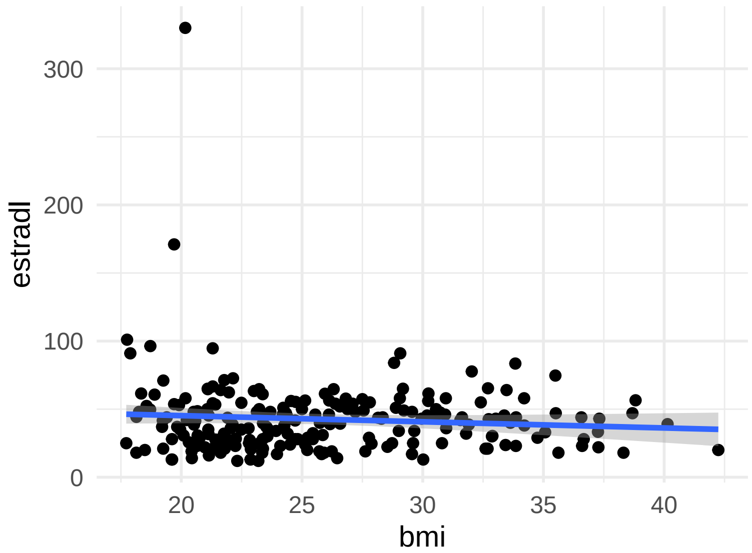
Problem 2(15p)

Obesity is very common in American society and is a risk factor for breast cancer in postmenopausal women. One mechanism explaining why obesity is a risk factor is that it may raise estrogen levels in women. In particular, one biomarker of estrogen, serum estradiol, is a strong risk factor for breast cancer. To better assess these relationships, researchers studied a group of 210 premenopausal women and recorded the following information('Estradl.csv'):

- Estradiol hormonal serum levels(Estradl);
- BMI = weight (kg)/height2(m2); measure of overall adiposity(used to indicate obesity, e.g., BMI>30; note that for this analysis we will use the continuous measurements);
- Ethnicity (Ethnic= 1 if African American, = 0 if Caucasian)
- Age (Entage);
- Number of children (Numchild);
- Age at menarche (Agemenar= age when menstrual periods began).

1. Is there a crude association between BMI and serum estradiol?

   a) Generate a scatter plot with the overlaid regression line. Comment.(2.5p)



b) Provide the summary regression output and comment on the nature of the relationship (i.e., sign,

term	estimate	std.error	statistic	p.value
(Intercept)	54.310	9.51	5.71	0.00
bmi	-0.453	0.36	-1.26	0.21

As shown, the coefficient of BMI is -0.453, with negative sign but also with a p.value of 0.21. The intercept is 54.31 , with p.value of 3.796×10^{-8}.

2. How does the relationship between BMI and serum estradiol change after controlling for all the other

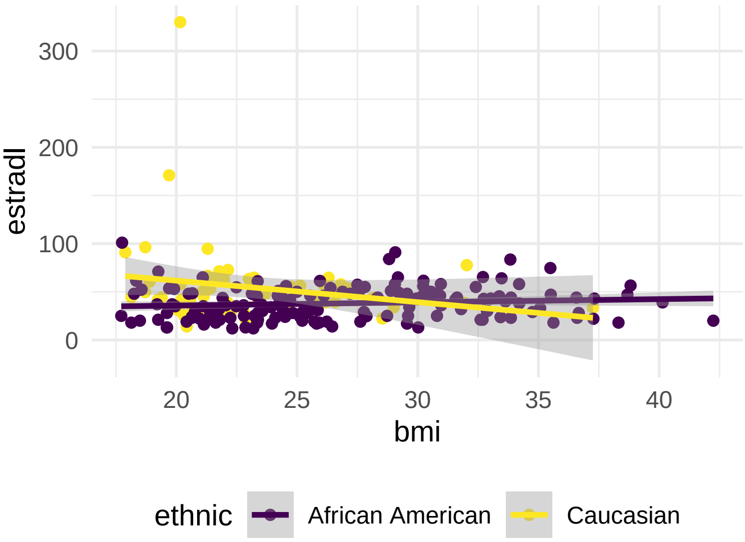
risk factors listed above? Provide the summary regression output and comment on the relationships observed for each of the predictors.(5p)

term	estimate	std.error	statistic	p.value
(Intercept)	26.157	13.072	2.001	0.047
ethnicCaucasian	16.058	4.449	3.609	0.000
entage	0.518	0.359	1.444	0.150
numchild	-0.491	1.244	-0.394	0.694
agemenar	0.107	0.169	0.635	0.526
bmi	-0.107	0.370	-0.288	0.774

[1] “After controlling other factors, the mean change per unit of ethnicCaucasian is 16.0579 with p.value of 4e-04;
After controlling other factors, the mean change per unit of entage is 0.518 with p.value of 0.1503;
After controlling other factors, the mean change per unit of numchild is -0.4906 with p.value of 0.6938;
After controlling other factors, the mean change per unit of agemenar is 0.1073 with p.value of 0.5264;
After controlling other factors, the mean change per unit of bmi is -0.1066 with p.value of 0.7737”

3. Now focus only the relationship between BMI and serum estradiol by ethnicity. Is there any evidence that these relationships vary for African American and Caucasian women?

   a) Use graphical displays and numerical summaries to sustain your conclusion.(2.5p)



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## Analysis of Variance Table
##
## Model 1: estradl ~ bmi
## Model 2: estradl ~ bmi + ethnic
##   Res.Df    RSS Df Sum of Sq    F  Pr(>F)
## 1    208 165281
## 2    207 155050  1     10231 13.7 0.00028 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

b) Based on your response in part 3-a), take additional steps to quantify the relationship between BMI a

ethnic	term	estimate	p_value	conf_low	conf_high
Caucasian	(Intercept)	106.285	0.004	34.78	177.785
Caucasian	bmi	-2.235	0.147	-5.28	0.809
African American	(Intercept)	29.075	0.000	15.56	42.589
African American	bmi	0.333	0.184	-0.16	0.826

```
## Model filterred possible outlier
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term	estimate	std.error	statistic	p.value
(Intercept)	63.021	15.217	4.14	0.000
bmi	-0.663	0.645	-1.03	0.308

We've seen that Ethnic plays a role in predicting Estradol, and through the fig we learn there's intercetion of two smooth line of different ethnics as an indicator of interaction. So we fitted a model with interaction.

term	estimate	std.error	statistic	p.value
(Intercept)	29.075	10.757	2.703	0.007
bmi	0.333	0.392	0.848	0.398
ethnicCaucasian	77.210	24.784	3.115	0.002
bmi:ethnicCaucasian	-2.568	1.029	-2.497	0.013

```
## Analysis of Variance Table
##
## Model 1: estradl ~ bmi
## Model 2: estradl ~ bmi * ethnic
##   Res.Df    RSS Df Sum of Sq    F  Pr(>F)
## 1    208 165281
## 2    206 150497  2     14784 10.1 6.4e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```