

## 1 Nonlocal

Until now, you've been able to access names in parent frames, but you have not been able to modify them. The `nonlocal` keyword can be used to modify a binding in a parent frame. For example, consider `stepper`, which uses `nonlocal` to modify `num`:

```
def stepper(num):
    def step():
        nonlocal num # declares num as a nonlocal name
        num = num + 1 # modifies num in the stepper frame
        return num
    return step

>>> step1 = stepper(10)
>>> step1()           # Modifies and returns num
11
>>> step1()           # num is maintained across separate calls to step
12
>>> step2 = stepper(10) # Each returned step function keeps its own state
>>> step2()
11
```

As illustrated in this example, `nonlocal` is useful for maintaining state across different calls to the same function.

However, there are two important caveats with `nonlocal` names:

- **Global names** cannot be modified using the `nonlocal` keyword.
- **Names in the current frame** cannot be overridden using the `nonlocal` keyword. This means we cannot have both a local and nonlocal binding with the same name in a single frame.

Because `nonlocal` lets you modify bindings in parent frames, we call functions that use it **mutable functions**.

## Questions

- 1.1 Draw the environment diagram for the following code.

```
def stepper(num):  
    def step():  
        nonlocal num  
        num = num + 1  
        return num  
    return step
```

```
s = stepper(3)
```

```
s()
```

```
s()
```

- 1.2 Write a function that takes in a number  $n$  and returns a one-argument function. The returned function takes in a function that is used to update  $n$ . It should return the updated  $n$ .

```
def memory(n):  
    """  
    >>> f = memory(10)  
    >>> f(lambda x: x * 2)  
    20  
    >>> f(lambda x: x - 7)  
    13  
    >>> f(lambda x: x > 5)  
    True  
    """
```

## 2 Midterm Review

For any problems that may require it, the tree ADT is provided at the end of this packet for your convenience. This section is **far** longer than a typical discussion, and it is recommended that you also use it as a problem bank for your midterm studies! Best of luck, you got this!!

### Questions

- 2.1 Draw the environment diagram that results from executing the code below.

```
from operator import add
```

```
six = 1
```

```
def ty(one, a):  
    summer = one(a, six)  
    return summer
```

```
six = ty(add, 6)  
summer = ty(add, 6)
```

- 2.2 Write a function that takes in no arguments and returns two functions, **prepend** and **get**, which represent the “add to front of list” and “get the ith item” operations, respectively. Do not use any python built-in data structures like lists or dictionaries. You do not necessarily need to use all the lines.

```
def nonlocalist():
    """
    >>> prepend, get = nonlocalist()
    >>> prepend(2)
    >>> prepend(3)
    >>> prepend(4)
    >>> get(0)
    4
    >>> get(1)
    3
    >>> get(2)
    2
    >>> prepend(8)
    >>> get(2)
    3
    """
    get = lambda x: "Index out of range!"
    def prepend(value):
        nonlocal get
        -----
        f = _get-----
    def get(i):
        if i == 0:
            return value
        return ____ f ____ (____ i-1 ____ )
        -----
    return _prepend_____, _lambda x: get(x)_____
```

- 2.3 Fill in the definition of `f` below such that the interpreter prints as expected. **Your solution must be on one line.**

```
>>> f = _____
>>> f = f(10)
1
2
3
4
5
6
7
8
9
10
```

Then, given your definition of `f`, what will be printed below? (Assuming that the above lines have also been executed in the interpreter.)

```
>>> f

f = lambda x: [print(i) for i in range(1, x+1)]
1
...
n
[None, ...None]

f = lambda x: x and (f(x-1) or print(x))
```

- 2.4 (Spring 2015) Implement the `memory` function, which takes a number `x` and a single-argument function `f`. It returns a function with a peculiar behavior that you must discover from the doctests. You may only use names and call expressions in your solution. You may not write numbers or use features of Python not yet covered in the course.

```

square = lambda x: x * x
double = lambda x: 2 * x
def memory(x, f):
    """Return a higher-order function that prints its
    memories.
    >>> f = memory(3, lambda x: x)
    >>> f = f(square)
    3
    >>> f = f(double)
    9
    >>> f = f(print)
    6
    >>> f = f(square)
    3
    None
    """
    def g(h):
        f(x)
        print(-----)
        return memory(x, h)
    return g

```

- 2.5 It's Hog again! Write a commentary function `announce_losses` that takes in a player `who` and returns a commentary function that announces whenever that player loses points.

```
def announce_losses(who, last_score=0):
    """
    >>> f = announce_losses(0)
    >>> f1 = f(10, 0)
    >>> f2 = f1(1, 10) # Player 0 loses points due to swine swap
    Oh no! Player 0 just lost 9 point(s).
    >>> f3 = f2(7, 10)
    >>> f4 = f3(7, 11) # Should not announce when player 0's score does not change
    >>> f5 = f4(11, 12)
    """

    assert who == 0 or who == 1, 'The who argument should indicate a player.'
    def say(score0, score1):
        if who == 0:
            score = score0
        elif who == 1:
            score = score1
        if score < last_score:
            print("Oh no! Player", who, "just lost", last_score - score, "point(s).")
            return announce_losses(who, score)
    return say
```



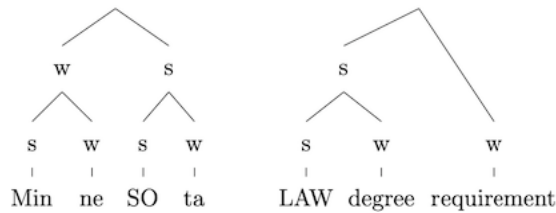
- 2.6 (Fall 2013) The CS 61A staff has developed a formula for determining what a fox might say. Given three strings—a start, a middle, and an end—a fox will say the start string, followed by the middle string repeated a number of times, followed by the end string. These parts are all separated by single hyphens.

Complete the definition of `fox_says`, which takes the three string parts of the fox's statement (start, middle, and end) and a positive integer `num` indicating how many times to repeat middle. It returns a string. You cannot use any `for` or `while` statements. Use recursion in `repeat`. Moreover, you cannot use string operations other than the `+` operator to concatenate strings together.

```
def fox_says(start, middle, end, num):
    """
    >>> fox_says('wa', 'pa', 'pow', 3)
    'wa-pa-pa-pa-pow'
    >>> fox_says('fraka', 'kaka', 'kow', 4)
    'fraka-kaka-kaka-kaka-kaka-kow'
    """
    def repeat(k):
        if k == 1:
            return middle
        return middle + '-' + repeat(k-1)

    return start + '-' + repeat(num) + '-' + end
```

- 2.7 The study of stress is still an open field of inquiry in linguistics—why do we say “alaBama,” but “aLABama” and “alabaMA” make us cringe? Or how did it come to be that “AMERICAN history professor” and “american HISTORY professor” mean two different things? One model that we use to understand stress actually employs the tree data structure!



In the above diagrams, every node has a “strong” child and a “weak” child, and primary stress is placed on the leaf that has the greatest number of strong parents. In the spirit of computational linguistics, let’s write a function that, given one of these tree structures, identifies the stressed part of a word or phrase.<sup>1</sup>

```
def primary_stress(t):
    """
    >>> word = tree("", [
        tree("w", [tree("s", [tree("min")]), tree("w", [tree("ne")])]),
        tree("s", [tree("s", [tree("so")]), tree("w", [tree("ta")])])])
    >>> primary_stress(word)
    'so'
    >>> phrase = tree("", [
        tree("s", [tree("s", [tree("law")]), tree("w", [tree("degree")])]),
        tree("w", [tree("requirement")])])
    >>> primary_stress(phrase)
    'law'
    """

    def helper(t, num_s):
        if is_leaf(t):
            return [label(t), num_s]
        if label(t) == "s":
            num_s = num_s + 1
        return max([helper(b, num_s) for b in branches(t)],
                    key = lambda a: a[1])
    return helper(t, 0)[0]
```

<sup>1</sup>Inspiration for this problem comes from Liberman, Mark and Alan Prince. 1977. *On stress and linguistic rhythm*. *Linguistic Inquiry*. 8:249-336., and from the course Linguistics 111 (Phonology).

- 2.8 Consider the subset sum problem: you are given a list of integers and a number  $k$ . Is there a subset of the list that adds up to  $k$ ? For example:

```
>>> subset_sum([2, 4, 7, 3], 5)      # 2 + 3 = 5
True
>>> subset_sum([1, 9, 5, 7, 3], 2)
False
>>> subset_sum([1, 1, 5, -1], 3)
False
```

*Note:* You can use the `in` operator to determine if an element belongs to a list:

```
>>> 3 in [1, 2, 3]
True
>>> 4 in [1, 2, 3]
False
```

```
def subset_sum(seq, k):
```

```
    if not seq:
        return False
    if k in seq:
        return True
    for i in seq:
        s = seq
        s.remove(i)
        if subset_sum(s, k-i):
            return True
    return False
```

### 3 Tree ADT

# Constructor

```
def tree(label, branches=[]):
    """Construct a tree with the given label value and a list of branches."""
    for branch in branches:
        assert is_tree(branch)
    return [label] + list(branches)
```

# Selector

```
def label(tree):
    """Return the label value of a tree."""
    return tree[0]
```

# Selector

```
def branches(tree):
    """Return the list of branches of the given tree."""
    return tree[1:]
```

```
def is_tree(tree):
    """Returns True if the given tree is a tree, and False otherwise."""
    if type(tree) != list or len(tree) < 1:
        return False
    for branch in branches(tree):
        if not is_tree(branch):
            return False
    return True
```

# For convenience

```
def is_leaf(tree):
    """Returns True if the given tree's list of branches is empty, and False
    otherwise."""
    return not branches(tree)
```