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CSCI-630-02 Foundations of FIS
Homework3

1 (15 pts) Convert the following statements to **propositional** INF or CNF (you may find it easier to convert to non-normal-form sentences first, then use the rules of logic to create normal-form sentences). The capitalization is there to help you...If a particular student does not Attend class, they will Fail. If they do Attend, then they will Learn and not Fail. A student who Fails or does not Learn will be Grumpy. A student is Quiet if they are Grumpy. Use resolution to prove by contradiction (or show to be unprovable) whether the student is guaranteed to Attend class, whether they are guaranteed to be Grumpy, or guaranteed to be Quiet. (These are three separate proofs, but you can reuse portions from one to the next if you want to.)

A represents that students attend class, **F** represents students fail the exam, **L** represents students will learn, **G** represents students will be Grumpy, **Q** represents students are Quiet

According to description of the question 1, we can get INF expressions as followed:

If a particular student does not Attend class, they will Fail.

$$\neg A \rightarrow F$$

If they do Attend, then they will Learn and not Fail

$$A \rightarrow L \wedge \neg F$$

A student who Fails or does not Learn will be Grumpy.

$$F \vee \neg L \rightarrow G$$

A student is Quiet if they are Grumpy.

$$G \rightarrow Q$$

According to description of the question 1, we can get CNF expressions as followed:

$$A \vee F$$

$$\neg A \vee (L \wedge \neg F) = (\neg A \vee L) \wedge (\neg A \vee \neg F)$$

$$\neg (F \vee \neg L) \vee G = (\neg F \wedge L) \vee G = (\neg F \vee G) \wedge (L \vee G)$$

$$\neg G \vee Q$$

Note:
The last column of the Truth Table represents
$(A \vee F) \wedge ((\neg A \vee L) \wedge (\neg A \vee \neg F)) \wedge ((\neg F \vee G) \wedge (L \vee G)) \wedge (\neg G \vee Q)$

Truth Table

A	F	L	G	Q	$A \vee F$	$(\neg A \vee L) \wedge (\neg A \vee \neg F)$	$(\neg F \vee G) \wedge (L \vee G)$	$\neg G \vee Q$	
F	F	F	F	F	F	T	F	T	F
F	F	F	F	T	F	T	F	T	F
F	F	F	T	F	F	T	T	F	F
F	F	F	T	T	F	T	T	T	F
F	F	T	F	F	F	T	T	T	F
F	F	T	F	T	F	T	T	T	F
F	F	T	T	F	F	T	T	F	F
F	F	T	T	T	F	T	T	T	F
F	T	F	F	F	T	T	F	T	F
F	T	F	F	T	T	T	F	T	F
F	T	F	T	F	T	T	T	F	F
F	T	F	T	T	T	T	T	T	T
F	T	T	F	F	T	T	F	T	F
F	T	T	F	T	T	T	F	T	F
F	T	T	T	F	T	T	T	F	F
F	T	T	T	T	T	T	T	T	T
T	F	F	F	F	T	F	F	T	F
T	F	F	F	T	T	F	F	T	F
T	F	F	T	F	T	F	T	F	F

T	F	F	T	T	T	F	T	T	F
T	F	T	F	F	T	T	T	T	T
T	F	T	F	T	T	T	T	T	T
T	F	T	T	F	T	T	T	F	F
T	F	T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	F	T	F
T	T	F	F	T	T	F	F	T	F
T	T	F	T	F	T	F	T	F	F
T	T	F	T	T	T	F	T	T	F
T	T	T	F	F	T	F	F	T	F
T	T	T	F	T	T	F	F	T	F
T	T	T	T	F	T	F	T	F	F
T	T	T	T	T	T	F	T	T	F

whether the student is guaranteed to Attend class.

A	F	L	G	Q	A v F	(¬A v L) ^ (¬A v ¬F)	(¬ F v G) ^ (L v G)	¬ G v Q	
F	T	F	T	T	T	T	T	T	T
T	F	T	T	T	T	T	T	T	T

According to the red column, the student is not guaranteed to Attend class.

whether they are guaranteed to be Grumpy

A	F	L	G	Q	A v F	(¬A v L) ^ (¬A v ¬F)	(¬ F v G) ^ (L v G)	¬ G v Q	
F	T	F	T	T	T	T	T	T	T
T	F	T	F	F	T	T	T	T	T

According to the red column, students are not guaranteed to be Grumpy

whether they are guaranteed to be Quiet.

A	F	L	G	Q	A v F	(¬A v L) ^ (¬A v ¬F)	(¬ F v G) ^ (L v G)	¬ G v Q	
F	T	F	T	T	T	T	T	T	T
T	F	T	F	F	T	T	T	T	T

According to the red column, students are not guaranteed to be Quiet

2 (15 pts) For each of the following sentences, if it is valid or unsatisfiable, prove it to be so, using truth tables or logical rules, or if it satisfiable but not valid, provide at least one satisfying and one non-satisfying assignment of variables.

1. Young -> Young
2. Young -> Happy

3. (Young -> Happy) -> (¬Young -> ¬Happy)
4. Young v Happy v (Young -> Happy)
5. (Young -> Happy) -> ((Young ^ Smart) -> Happy)
6. (Happy -> Smart) ^ (Smart -> ¬Young) ^ ¬(Young -> ¬Happy)

Truth Table

Young	Young -> Young
T	T
F	T

1 is valid and satisfiable because all the results in the last column are true

Young	Happy	Young -> Happy
F	F	T
F	T	T
T	F	F
T	T	T

2 is not valid because there is a row which result in the last column is false
2 is satisfiable because at least one result in the last column is true

Young	Happy	Young->Happy	¬Young->¬Happy	(Young->Happy)-> (¬Young->¬Happy)
F	F	T	T	T
F	T	T	F	F
T	F	F	T	T
T	T	T	T	T

3 is not valid because there is a row which result in the last column is false
3 is satisfiable because at least one result in the last column is true

Young	Happy	Young->Happy	Young v Happy v (Young->Happy)
F	F	T	T
F	T	T	T
T	F	F	T
T	T	T	T

4 is valid and satisfiable because all the results in the last column are true

Y	H	S	Y->H	Y^S	Y^S->H	(Y->H)-> Y^S->H
F	F	F	T	F	T	T
F	F	T	T	F	T	T
F	T	F	T	F	T	T
F	T	T	T	F	T	T
T	F	F	F	F	T	T
T	F	T	F	T	F	T
T	T	F	T	F	T	T
T	T	T	T	T	T	T

5 is valid and satisfiable because all the results in the last column are true

Y	H	S	H->S	S->¬Y	Y->¬H	¬(Y->¬H)	(H->S) ^ (S->¬Y) ^ (¬(Y->¬H))
F	F	F	T	T	T	F	F
F	F	T	T	T	T	F	F
F	T	F	F	T	T	F	F
F	T	T	T	T	T	F	F
T	F	F	T	T	T	F	F
T	F	T	T	F	T	F	F
T	T	F	F	T	F	T	F
T	T	T	T	F	F	T	F

6 is neither valid or satisfiable because all the results in the last column are false

3 (20 pts) First convert the following sentences to **first-order** INF or CNF (you may need more than one INF/CNF sentence for some of these):

- All ducks are either male or female.
- Any duck that lays an egg is female.
- A duck will quack if and only if it is male.*
- Thistle and Junior are ducks.
- Either Thistle or Junior has laid an egg.
- Junior quacks.

*Note: in real life, it is only the female ducks that quack. I changed it here to make the problem work a little differently.
Now, prove by contradiction, using resolution, that Thistle laid the egg.

1) Male(x) ^ Female(x) -> False
2) Duck(x) ^ Lay(x) -> Female(x)
3) Duck(x) ^ Quack(x) -> Male(x)
4) True -> Duck(T)
5) True -> Duck(J)
6) True -> Lay(T) v Lay(J)
7) True -> Quack(J)
8) Lay(T) -> False

9) Duck(J) -> Male(J)
10) True -> Male(J)
11) Duck(J) ^ Lay(J) ^ Male(J) -> False
12) Lay(J) ^ Male(J) -> False
13) Lay(J) -> False
14) True -> Lay(T)

3)+7): Duck(J) -> Male(J)
Thus we can get a new expression as 9)
9) Duck(J) -> Male(J)

5)+9): True -> Male(J)
Thus we can get a new expression as 10)
10) True -> Male(J)

1)+2): Duck(J) ^ Lay(J) ^ Male(J) -> False
Thus we can get a new expression as 11)

11) Duck(J) ^ Lay(J) ^ Male(J) -> False

5)+11): Lay(J) ^ Male(J) -> False
Thus we can get a new expression as 12)
12) Lay(J) ^ Male(J) -> False

10)+12): Lay(J) -> False
Thus we can get a new expression as 13)
13) Lay(J) -> False

6)+13): True -> Lay(T)
Thus we can get a new expression as 14)
14) True -> Lay(T)

14)+8): True -> False
Contradiction

4(25 pts) R&N 8.14 - write a Prolog program `family.p` to solve this, including all the family tree data, but skip the "m-th cousin" part. Use the simpler definition of in-laws such as here. Hand in all your code and the results of the questions posed to your program. Make sure that you have allowed Prolog to show you **all** of the answers to each question!

Elizabeth's grandchildren

```
?- grand_child(Children,elizabeth).
Children = william ;
Children = harry ;
Children = peter ;
Children = zara ;
Children = beatrice ;
Children = eugenie ;
Children = louise ;
Children = james.
```

Diana's brother-in-law

```
?- brother_in_law(Mybrothers,diana).
Mybrothers = andrew ;
Mybrothers = andrew ;
Mybrothers = edward ;
Mybrothers = edward ;
Mybrothers = mark ;
Mybrothers = mark ;
```

Zara's great-grandparents

```
?- great_grand_parent(GreatGrandParent,zara).
GreatGrandParent = george ;
GreatGrandParent = mum ;
```

Eugenie's ancestors

```
?- ancestor(Myancestor,eugenie).
Myancestor = andrew ;
Myancestor = sarah ;
Myancestor = elizabeth ;
Myancestor = philip ;
Myancestor = george ;
Myancestor = mum ;
```

5 (10 pts) Examine the totally fictional data set here. Presume that this data was collected at Midnight Oil - each line represents one drink that was ordered, and whether it was hot, sweet, and/or caffeinated.

a. Based on this data set, what is your best estimate of P(Caff)?
b. What is your best estimate of P(Sweet | Hot)?
c. Based on this data, do any two of these variables appear to be independent? If so, which two? If not, why not?

a $P(Caff) = \frac{13}{30}$

b P(Sweet | Hot)= $\frac{P(Sweet \wedge Hot)}{P(Hot)} = \frac{4}{15}$

c $P(Sweet) = \frac{13}{30}$

$P(Hot) = \frac{15}{30}$

$P(Caff \mid Hot) = \frac{P(Caff \wedge Hot)}{P(Hot)} = \frac{9}{15}$

$P(Caff \mid Sweet) = \frac{P(Caff \wedge Sweet)}{P(Sweet)} = \frac{2}{13}$

Because P(Caff | Hot) is not equal to P(Caff), Caff and Hot are dependent variables
Because P(Caff | Sweet) is not equal to P(Caff), Caff and Sweet are dependent variables

$P(Hot \mid Sweet) = \frac{P(Hot \wedge Sweet)}{P(Sweet)} = \frac{4}{13}$

$P(Hot \mid Caff) = \frac{P(Hot \wedge Caff)}{P(Caff)} = \frac{9}{13}$

Because P(Hot | Sweet) is not equal to P(Hot), Hot and Sweet are dependent variables
Because P(Hot | Caff) is not equal to P(Hot), Hot and Caff are dependent variables

$P(Sweet \mid Caff) = \frac{P(Sweet \wedge Caff)}{P(Caff)} = \frac{2}{13}$

$P(Sweet \mid Hot) = \frac{P(Sweet \wedge Hot)}{P(Hot)} = \frac{4}{15}$

Because P(Sweet | Caff) is not equal to P(Sweet), Sweet and Caff are dependent variables
Because P(Sweet | Hot) is not equal to P(Sweet), Sweet and Hot are dependent variables

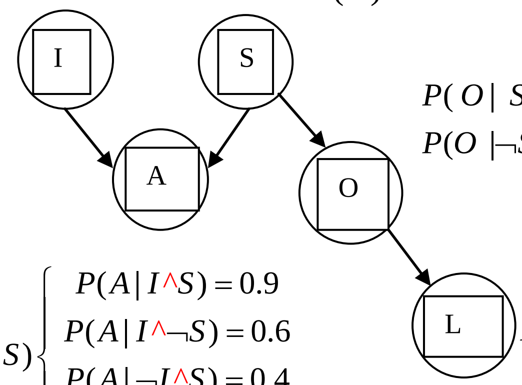
6 (15 pts) Consider the following set of boolean variables regarding a similar situation to problem 1, but probabilistic and about one randomly-chosen day:

- S : it is a pleasant sunny day
- A : Student attends class
- I : Class is interesting
- L : Student learns something about the subject (can happen outside of class!)
- O : people are sitting outside to have coffee

a. Draw a Bayes net that reasonably represents the relationships between those variables.
b. Annotate the Bayes net with all necessary probabilities so that any possible question about the variables can be answered For the values, choose any reasonable number other than 0 or 1 (and don't just use the same value over and over!).
c. Compute the following probabilities based on your network. You may reuse any computations as needed.

- P(O)
- P(I^S)
- P(¬A)
- P(I | L)

$P(I) = 0.5$ $\quad$ $P(S) = 0.$



$P(O \mid S) = 0.$
$P(O \mid \neg S) = 0$

$P(L \mid O) = 0.1$
$P(L \mid \neg O) = 0$

$$P(A \mid I, S)\begin{cases} P(A \mid I \wedge S) = 0.9 \\ P(A \mid I \wedge \neg S) = 0.6 \\ P(A \mid \neg I \wedge S) = 0.4 \\ P(A \mid \neg I \wedge \neg S) = 0.1 \end{cases}$$

$$P(O) = P(O \wedge S) + P(O \wedge \neg S) = P(O \mid S)P(S) + P(O \mid \neg S)P(\neg S)$$
$$= 0.7 * 0.3 + 0.1 * (1 - 0.3) = 0.28$$

$$P(I \wedge S) = P(I) * P(S) = 0.5 * 0.3 = 0.15$$

$$P(\neg A) = 1 - P(A)$$
$$= 1 - (P(A \wedge I \wedge S) + P(A \wedge I \wedge \neg S) + P(A \wedge \neg I \wedge S) + P(A \wedge \neg I \wedge \neg S))$$
$$= 1 - P(A \mid I \wedge S)P(I \wedge S) - P(A \mid I \wedge \neg S)P(I \wedge \neg S)$$
$$- P(A \mid \neg I \wedge S)P(\neg I \wedge S) - P(A \mid \neg I \wedge \neg S)P(\neg I \wedge \neg S)$$
$$= 1 - 0.9 * 0.5 * 0.3 - 0.6 * 0.5 * 0.7 - 0.4 * 0.5 * 0.3 - 0.1 * 0.5 * 0.7$$
$$= 0.56$$

$$P(I \mid L) = P(I) = 0.5$$

Because I and L are independent from my example