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CSCI-630-02 Foundations of FIS
Homework5

1 (25 pts) Consider this slightly modified version of the example Markov model and HMM for Working, Surfing, Meeting, as discussed in class. (It is also available in MyCourses.) First, on paper, write the transition matrix that represents this system. Then, write a little piece of code in the language of your choice to compute the following:

$$A = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{3} \\ \frac{1}{4} & \frac{1}{4} & \frac{2}{3} \\ \frac{1}{4} & \frac{1}{4} & 0 \end{bmatrix}$$

$$Working = \begin{bmatrix} 0.3 \\ 0.6 \\ 0.1 \end{bmatrix} \; Surfing = \begin{bmatrix} 0.5 \\ 0.2 \\ 0.3 \end{bmatrix} \; Meeting = \begin{bmatrix} 0.1 \\ 0.8 \\ 0.1 \end{bmatrix}$$

a. The probability of being in each of the three states at each of the first 12 time steps.

[Working, Surfing, Meeting] is the format, shown as followed:

Step1: [0.5, 0.25, 0.25]

Step2: [0.45833333, 0.35416667, 0.1875]

Step3: [0.46875, 0.328125, 0.203125]

Step4: [0.46614583, 0.33463542, 0.19921875]

Step5: [0.46679688, 0.33300781, 0.20019531]

Step6: [0.46663411, 0.33341471, 0.19995117]

Step7: [0.4666748, 0.33331299, 0.20001221]

Step8: [0.46666463, 0.33333842 , 0.19999695]

Step9: [0.46666718, 0.33333206, 0.20000076]

Step10: [0.46666654, 0.33333365 , 0.19999981]

Step11: [0.4666667, 0.33333325, 0.20000005]

Step12: [0.4666667, 0.33333325 0.20000005]

b. The same as the previous, but in the case where I appear unhappy, then meh, then happy, then the same sequence repeated three more times.

[Working, Surfing, Meeting] is the format, shown as followed

Step1: 0.33333333333333337, 0.5, 0.16666666666666669

Step2: 0.5513513513513513, 0.12432432432432433, 0.32432432432432434

Step3: 0.38976377952755914, 0.5610236220472441, 0.04921259842519685

Step4: 0.31913987651692577, 0.5266127315307644, 0.15424739195231

Step5: 0.5508835196538046, 0.12167327803822575, 0.32744320230796975

Step6: 0.3888383644058903, 0.5622354751827626, 0.048926160411347015

Step7: 0.31922030233290677, 0.5264619331257998, 0.1543177645412934

Step8: 0.550886156845035, 0.12168822212186434, 0.3274256210331008

Step9: 0.3888435747414012, 0.5622286521243554, 0.04892777313424326

Step10: 0.3192198494442207 , 0.5264627822920861, 0.1543173682636931

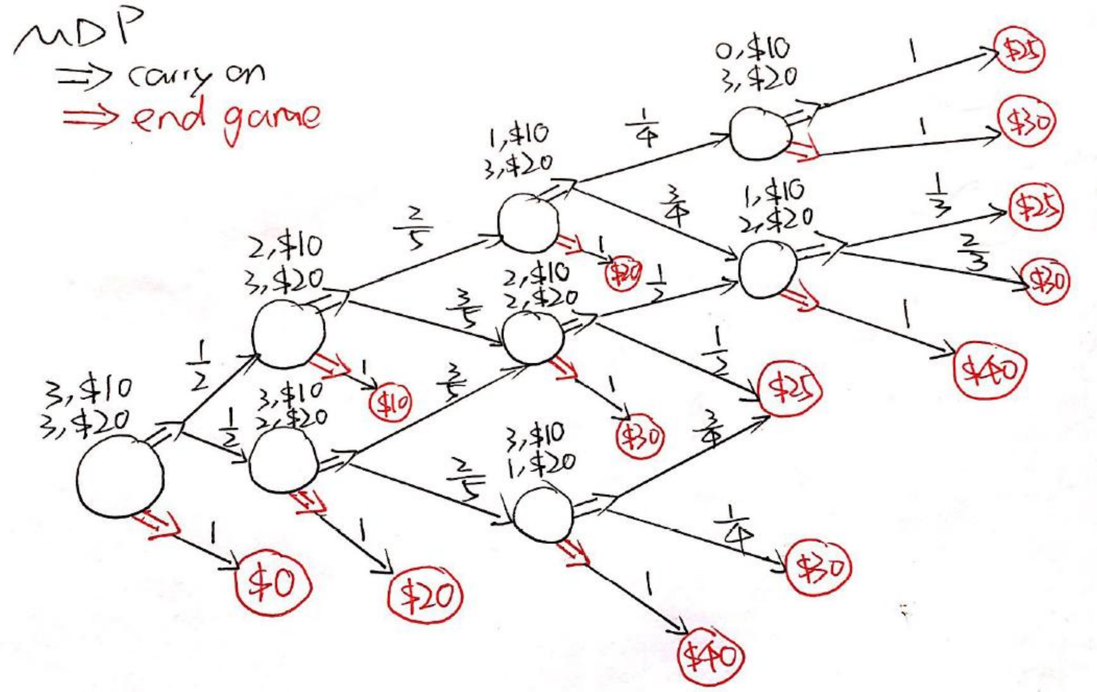
Step11: 0.5508861419942171, 0.12168813796723106, 0.3274257200385517

Step12: 0.3888435454002347, 0.5622286905473118, 0.0489277640524536

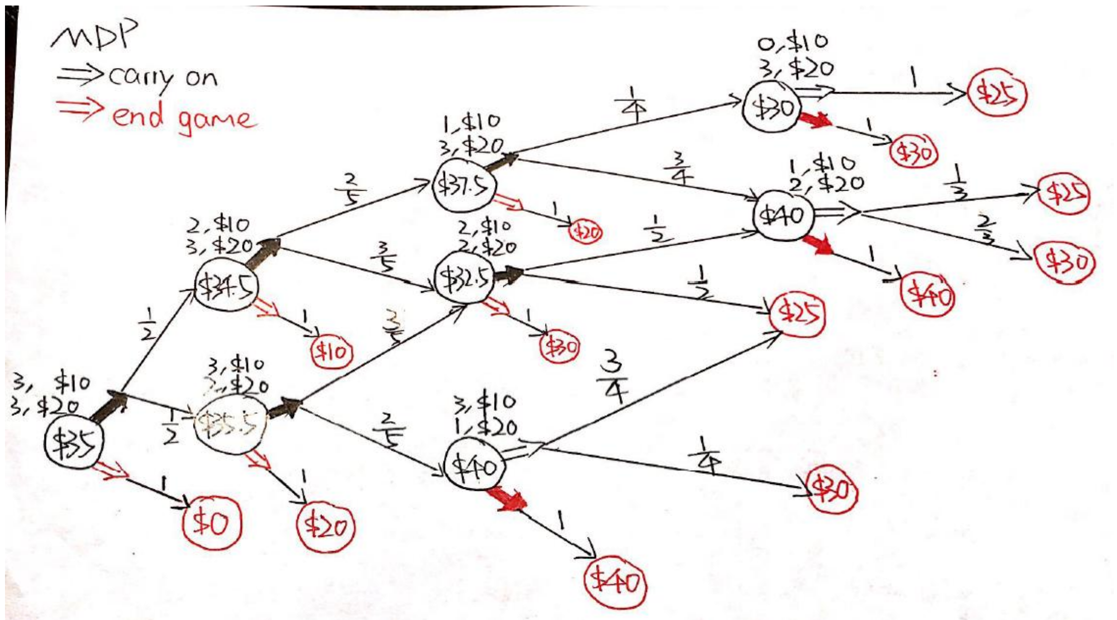
Submit the code and include the output for each question in your writeup.

2 (25 pts) Consider the following game, which can be modeled as an MDP. When the game starts, I am holding an envelope with three $10 bills and three $20 bills. You will draw (and hold on to) random bills from the envelope, one at a time. At any point, if you have less than $50 in hand, you can stop and take the money you are holding, or you can keep playing. If at any point you draw a bill that puts you at $50 or more, the game is over and you win only half of what you are holding. (So, for example, if you have $30 in hand and draw a 20, you would end up with $50/2 = $25.)

a. Draw the MDP that represents this game. Reuse any states that would appear more than once, but make sure that your states obey the Markov property. (Hint: consider the contents of the envelope as the game progresses.)



b. Compute the optimal policy and show the value of each state of the game. How much would you expect to win on average by playing this game?



The optimal policy is marked as filled
The value expected to win by average is $35