

zl9901 Assignment 2

Question 1

a Estimated covariance matrix for all variables except for invoice_no

	fan	radiator	engine	temp	starts	low_oil	oil_light
fan	0.1626	-0.002189	-0.09832	0.04806	0.02046	-0.00111	0.002804
radiator	-0.002189	0.1586	-0.0510	0.0226	0.01164	0.0006	0.001942
engine	-0.09832	-0.0510	0.2115	-0.1043	-0.04145	0.001455	-0.002266
temp	0.04806	0.0226	-0.1043	0.2474	0.01976	0.000022	-0.00047
starts	0.02046	0.01164	-0.04145	0.01976	0.1339	0.001327	0.001266
low_oil	-0.00111	0.0006	0.001455	0.000022	0.001327	0.2126	-0.1496
oil_light	0.002804	0.001942	-0.002266	-0.00047	0.001266	-0.1496	0.2432

$$\text{Cov}(X, X) = \text{Var}(X) = E(X^2) - [E(X)]^2$$

$$\text{Cov}(X, Y) = E(XY) - E(X)E(Y)$$

b Complete joint probability mass function table

fan	radiator	engine	temp	starts	Low_oil	Oil_light	prob
F	F	F	F	F	F	F	0.0001
F	F	F	F	F	F	T	0.0004
F	F	F	F	F	T	F	0.0001
F	F	F	F	F	T	T	0
F	F	F	F	T	F	F	0.0004
F	F	F	F	T	F	T	0.0014
F	F	F	F	T	T	F	0.0009
F	F	F	F	T	T	T	0.0002
F	F	F	T	F	F	F	0
F	F	F	T	F	F	T	0.0004
F	F	F	T	F	T	F	0.0003
F	F	F	T	F	T	T	0
F	F	F	T	T	F	F	0.0007
F	F	F	T	T	F	T	0.0026
F	F	F	T	T	T	F	0.0020
F	F	F	T	T	T	T	0.0001
F	F	T	F	F	F	F	0.0016
F	F	T	F	F	F	T	0.0038
F	F	T	F	F	T	F	0.0022
F	F	T	F	F	T	T	0
F	F	T	F	T	F	F	0.0025
F	F	T	F	T	F	T	0.0089

F	F	T	F	T	T	F	0.0051
F	F	T	F	T	T	T	0.0006
F	F	T	T	F	F	F	0
F	F	T	T	F	F	T	0.0005
F	F	T	T	F	T	F	0.0009
F	F	T	T	F	T	T	0
F	F	T	T	T	F	F	0
F	F	T	T	T	F	T	0.0017
F	F	T	T	T	T	F	0.0008
F	F	T	T	T	T	T	0
F	T	F	F	F	F	F	0.0001
F	T	F	F	F	F	T	0.0006
F	T	F	F	F	T	F	0.0004
F	T	F	F	F	T	T	0.0001
F	T	F	F	T	F	F	0.0022
F	T	F	F	T	F	T	0.0072
F	T	F	F	T	T	F	0.0031
F	T	F	F	T	T	T	0.0006
F	T	F	T	F	F	F	0.0003
F	T	F	T	F	F	T	0.0008
F	T	F	T	F	T	F	0.0008
F	T	F	T	F	T	T	0
F	T	F	T	T	F	F	0.0027
F	T	F	T	T	F	T	0.0101
F	T	F	T	T	T	F	0.0051
F	T	F	T	T	T	T	0.0003
F	T	T	F	F	F	F	0.0059
F	T	T	F	F	F	T	0.0200
F	T	T	F	F	T	F	0.0092
F	T	T	F	F	T	T	0.0007
F	T	T	F	T	F	F	0.0123
F	T	T	F	T	F	T	0.0443
F	T	T	F	T	T	F	0.0230
F	T	T	F	T	T	T	0.0028
F	T	T	T	F	F	F	0.0003
F	T	T	T	F	F	T	0.0021
F	T	T	T	F	T	F	0.0013
F	T	T	T	F	T	T	0.0001
F	T	T	T	T	F	F	0.0011
F	T	T	T	T	F	T	0.0056

F	T	T	T	T	T	F	0.0027
F	T	T	T	T	T	T	0.0004
T	F	F	F	F	F	F	0.0005
T	F	F	F	F	F	T	0.0015
T	F	F	F	F	T	F	0.0009
T	F	F	F	F	T	T	0
T	F	F	F	T	F	F	0.0043
T	F	F	F	T	F	T	0.0145
T	F	F	F	T	T	F	0.0059
T	F	F	F	T	T	T	0.0012
T	F	F	T	F	F	F	0.0009
T	F	F	T	F	F	T	0.0031
T	F	F	T	F	T	F	0.0013
T	F	F	T	F	T	T	0.0002
T	F	F	T	T	F	F	0.0079
T	F	F	T	T	F	T	0.0228
T	F	F	T	T	T	F	0.0108
T	F	F	T	T	T	T	0.0011
T	F	T	F	F	F	F	0.0030
T	F	T	F	F	F	T	0.0131
T	F	T	F	F	T	F	0.0055
T	F	T	F	F	T	T	0.0006
T	F	T	F	T	F	F	0.0067
T	F	T	F	T	F	T	0.0283
T	F	T	F	T	T	F	0.0154
T	F	T	F	T	T	T	0.0019
T	F	T	T	F	F	F	0.0006
T	F	T	T	F	F	T	0.0013
T	F	T	T	F	T	F	0.0003
T	F	T	T	F	T	T	0
T	F	T	T	T	F	F	0.0014
T	F	T	T	T	F	T	0.0027
T	F	T	T	T	T	F	0.0014
T	F	T	T	T	T	T	0.0003
T	T	F	F	F	F	F	0.0033
T	T	F	F	F	F	T	0.0124
T	T	F	F	F	T	F	0.0059
T	T	F	F	F	T	T	0.0006
T	T	F	F	T	F	F	0.0276
T	T	F	F	T	F	T	0.1181

T	T	F	F	T	T	F	0.0586
T	T	F	F	T	T	T	0.0063
T	T	F	T	F	F	F	0.0047
T	T	F	T	F	F	T	0.0188
T	T	F	T	F	T	F	0.0103
T	T	F	T	F	T	T	0.0006
T	T	F	T	T	F	F	0.0408
T	T	F	T	T	F	T	0.1730
T	T	F	T	T	T	F	0.0854
T	T	F	T	T	T	T	0.0090
T	T	T	F	F	F	F	0.0020
T	T	T	F	F	F	T	0.0084
T	T	T	F	F	T	F	0.0043
T	T	T	F	F	T	T	0.0003
T	T	T	F	T	F	F	0.0050
T	T	T	F	T	F	T	0.0217
T	T	T	F	T	T	F	0.0115
T	T	T	F	T	T	T	0.0011
T	T	T	T	F	F	F	0.0001
T	T	T	T	F	F	T	0.0011
T	T	T	T	F	T	F	0.0006
T	T	T	T	F	T	T	0
T	T	T	T	T	F	F	0.0007
T	T	T	T	T	F	T	0.0024
T	T	T	T	T	T	F	0.0016
T	T	T	T	T	T	T	0

c Contingency table of fan belt

b	P(b)
T	0.7956
F	0.2044

Contingency table of radiator leak

r	P(r)
T	0.8024
F	0.1976

Contingency table of fan belt, radiator leak and engine overheat

f	r	e	P(e;f,r)
T	T	T	0.0955
T	T	F	0.9045
T	F	T	0.5176
T	F	F	0.4824
F	T	T	0.7930
F	T	F	0.2070
F	F	T	0.7487
F	F	F	0.2513

Contingency table of engine and temp light

e	t	P(t;e)
T	T	0.1054
T	F	0.8946
F	T	0.5987
F	F	0.4013

Contingency table of engine and starts

e	s	P(s;e)
T	T	0.7043
T	F	0.2957
F	T	0.9003
F	F	0.0997

Contingency table of low oil

l	P(l)
T	0.3067
F	0.6933

Contingency table of low oil and oil light

l	o	P(o;l)
T	T	0.0949
T	F	0.9051
F	T	0.7985
F	F	0.2015

- d Compute $P(\text{fan}=1 \mid \text{low_oil}=1, \text{starts}=0)$ using table from (b)

$$P(\text{fan} = 1 \mid \text{low_oil} = 1, \text{starts} = 0) = \frac{P(\text{fan}=1, \text{low_oil}=1, \text{starts}=0)}{P(\text{low_oil}=1, \text{starts}=0)} = \frac{0.4432}{0.5536} = 0.661$$

The process of calculation can be found in graphical.py

- e Compute $P(\text{fan}=1 \mid \text{low_oil}=1, \text{starts}=0)$ using table from (c)

Since low_oil and starts can't influence fan belt, they are independent from each other according to graphical model

$$P(\text{fan}=1 \mid \text{low_oil}=1, \text{starts}=0) = P(\text{fan}=1) = 0.7956$$

Question 2

a)

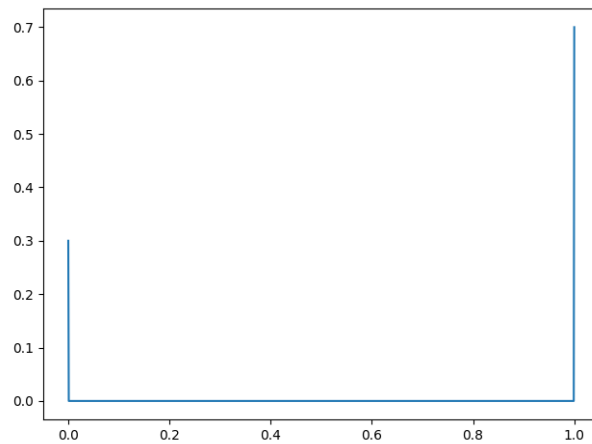


Fig 1 Bernoulli Distribution

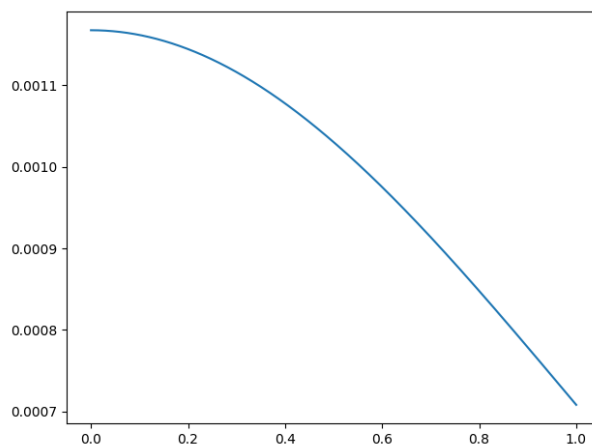


Fig 2 Gaussian Distribution

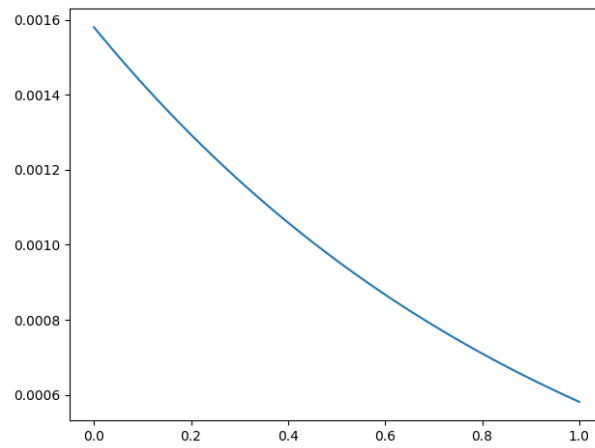


Fig 3 Laplace Distribution

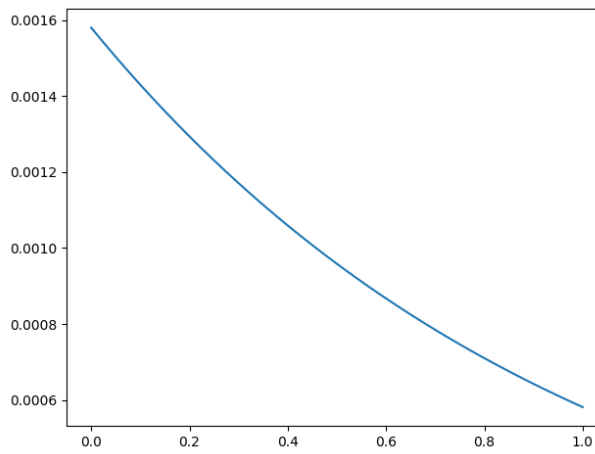


Fig 4 Exponential Distribution

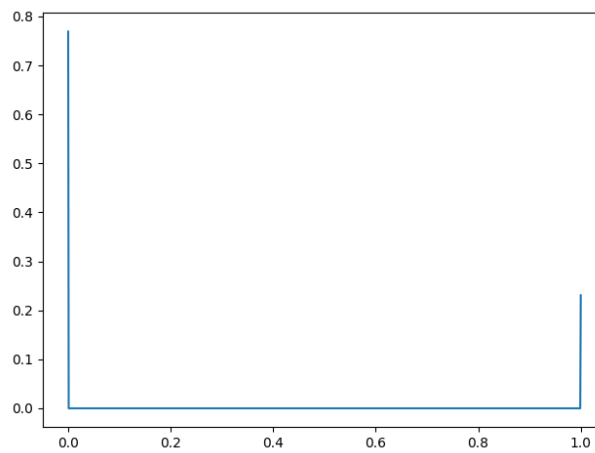


Fig 5 Poisson Distribution

b)

	Bernoulli(.7)	norm	laplace	expon	Poisson(.3)
Z	1.0	341.66518	316.40227	632.80455	0.96306368
KL(u x)	-0.0122446	0.0107650	0.0414068	0.0414068	-0.01207673
KL(x u)	6.29789047	0.0103818	0.0407311	0.0407311	6.368550636