

# CS5011 Introduction to Search

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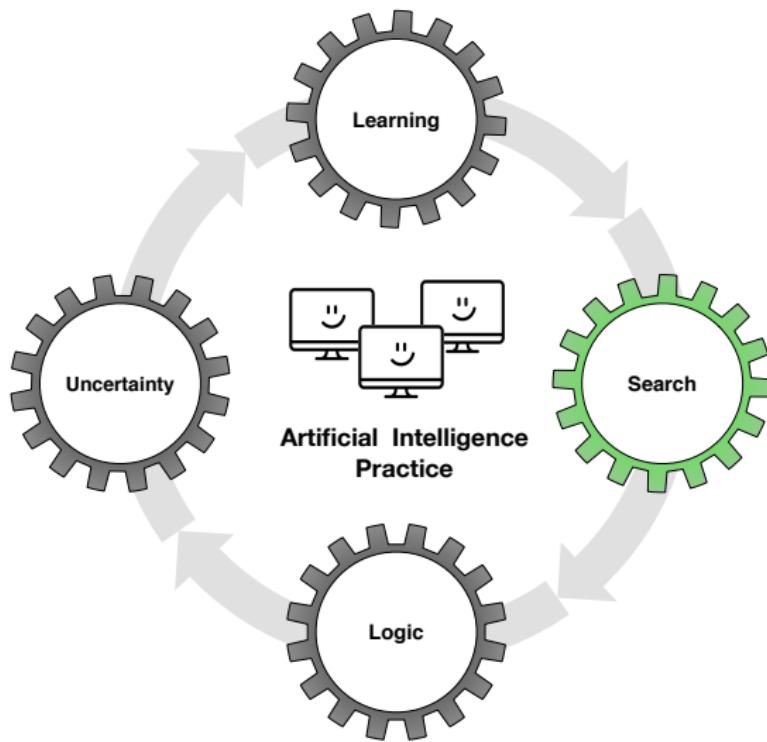
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09-10-2017

# Where are we?

How to search for a solution to solve a problem?



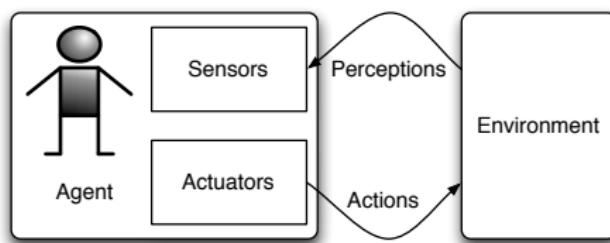
# Different definitions of AI

Thinking Humanly	Thinking Rationally	Acting Rationally
Thinking Humanly		
Cognitive modelling How the mind works CS+ Psychology	Right thinking Logic Rules to express thoughts	
Turing test Can machines be 'mistaken' for humans? Chat bots	Acting agents Autonomous action Achieve the best outcome	

# Agent

An agent is something that acts ... in AI:

- acts autonomously
- perceives the environment
- adapts to the environment
- pursues goals...



We need an agent **function** that 'maps' perceptions to actions

# So far?

- Learning: when the programmer does not know the agent ‘function’, we may use learning to improve performance from observations
- We now move to the case in which we know how to formulate solutions (part of the agent function), but we need to find a way to reach a goal among those solutions.
- How?

By constructing possible solutions and search for one that satisfies the goal

# Search in AI

Search is a central topic in Artificial Intelligence, many applications for example...

# Example: Route Planning

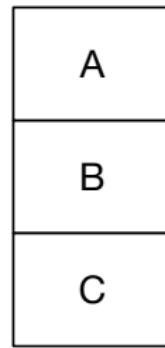
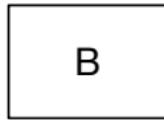
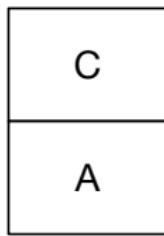


# Example: Route Planning

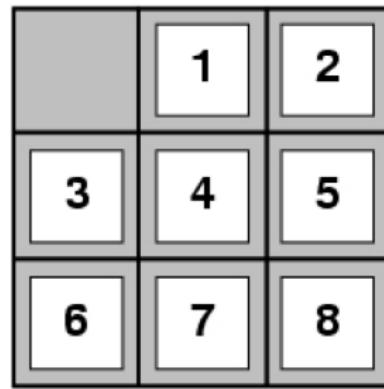
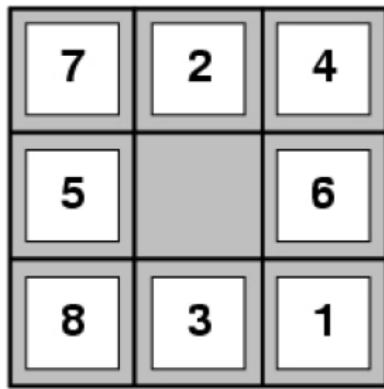


By A. Smith (Taken by A. Smith at RoboCup Rescue 2008 competition.)

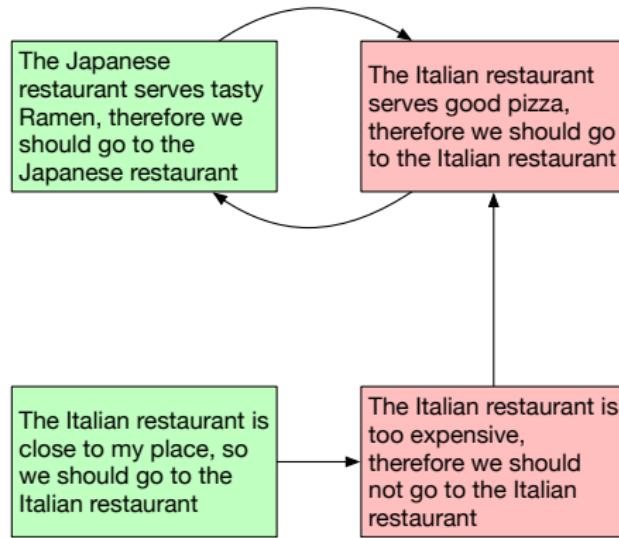
# Example: Planning



## Example: 8-Puzzle



# Example: Evaluate arguments



# Common components

- An agent is in an **initial situation** and wants to achieve a certain **goal**.
- At any point in time an agent has different simple **actions available**. Executing a particular sequence of such actions may or may not achieve the goal.
- Each action brings the agent from a **state** to another state.
- **Search** is the process of inspecting several such sequences and choosing one that achieves the goal.
- For some applications, each sequence of actions may be associated with a certain **cost**.
- A search problem where the agent aims at reaching the goal but also at doing so at minimal cost is an **optimisation problem**.

# Route planning components

- State space: What are the possible states?

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  - ▶ Position on the map
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  - ▶ Dundee airport
- Goal: When have we found a solution?

# Route planning components

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  - ▶ Moves on the map
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  - ▶ School of Computer Science, St Andrews
- Search: How do we go from initial state to goal?

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  - ▶ Navigate through the (possibly huge) space of states (positions on map)
- Cost: How costly is a given move?

# Route planning components

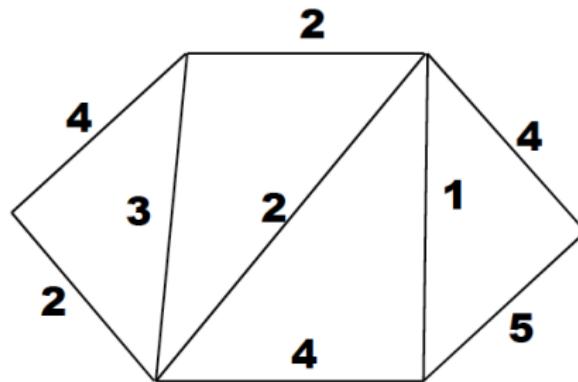
- State space: What are the possible states?
  - ▶ Position on the map
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  - ▶ School of Computer Science, St Andrews
- Search: How do we go from initial state to goal?
  - ▶ Navigate through the (possibly huge) space of states (positions on map)
- Cost: How costly is a given move?
  - ▶ Distance, fuel cost, etc...

## Another Example: Travel Salesperson 1

- Travel around a set of cities visiting each city exactly once and return home
- Traditionally, problem is to minimise total distance travelled
- Can also be cast as a decision problem: does there exist a tour with distance  $\leq$  limit

## Another Example: Travel Salesperson 2

- Problem: graph with a cost for each edge
- Solution: tour visiting all nodes exactly once, returning to base meeting some cost limit (or reaching minimum cost)
  - ▶ e.g. minimum cost is 21

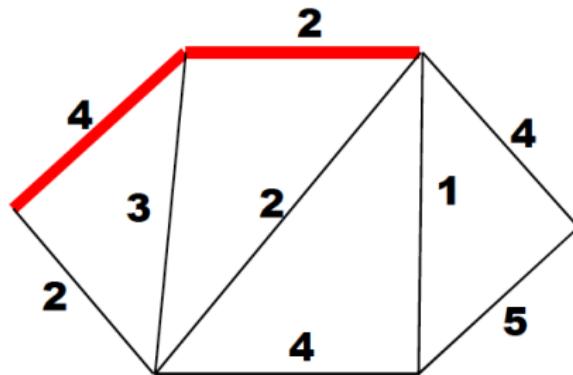


## Another Example: Travel Salesperson 3

- Easy to check that tour costs no more than limit
- Complete search might need to check every possible tour
  - ▶ If a poor algorithm is used
  - ▶ We can do much better than checking every tour

## Another Example: Travel Salesperson 4

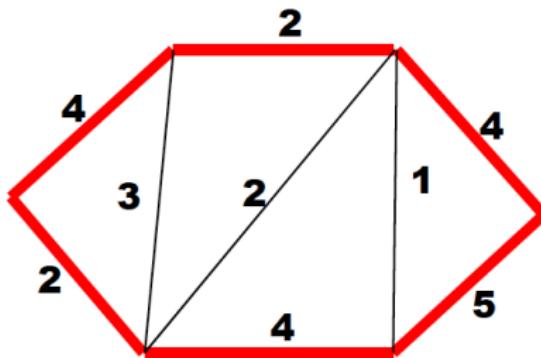
- A search state is a partial tour or a complete tour
- A partial tour:



- N.B. This is just one way of formulating TSP for search

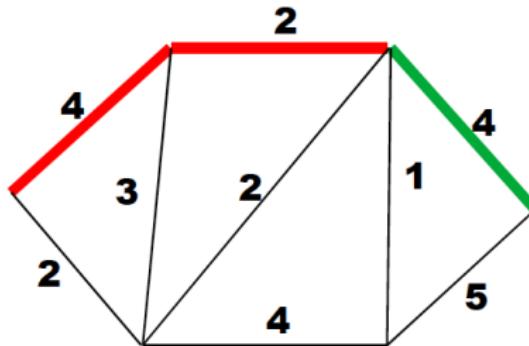
## Another Example: Travel Salesperson 5

- A solution is a complete tour (within the distance limit)
  - ▶ A special type of search state
- A complete tour:



## Another Example: Travel Salesperson 6

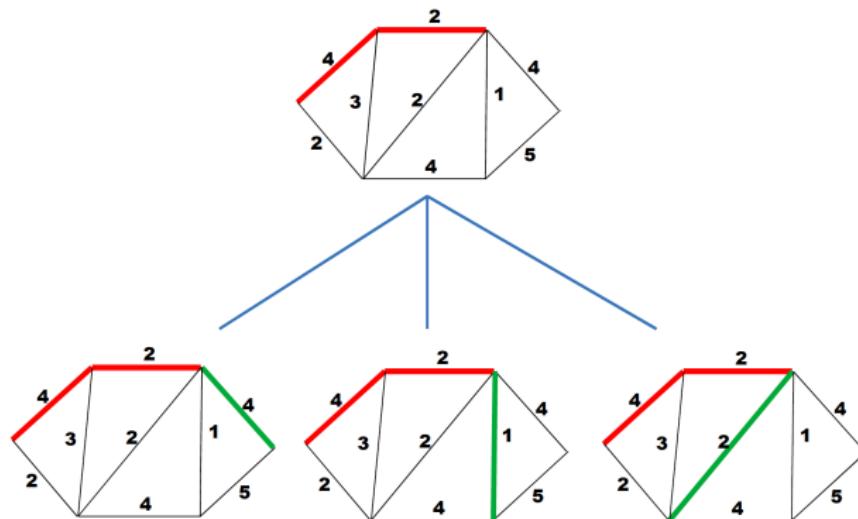
- We can construct a solution (if one exists) by repeatedly extending the empty tour?
- A partial tour (red) extended (in green):



## Another Example: Travel Salesperson 7

- However, it might not be clear how to extend a search state
- There might be more than one option
- We need to explore a **search tree**
- Search trees are an abstraction, we never store them in memory

## Another Example: Travel Salesperson 8



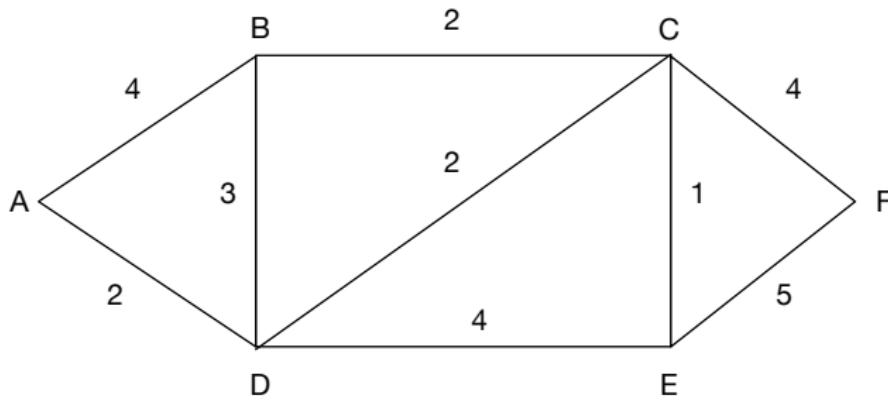
# Problem Formulation

What to define:

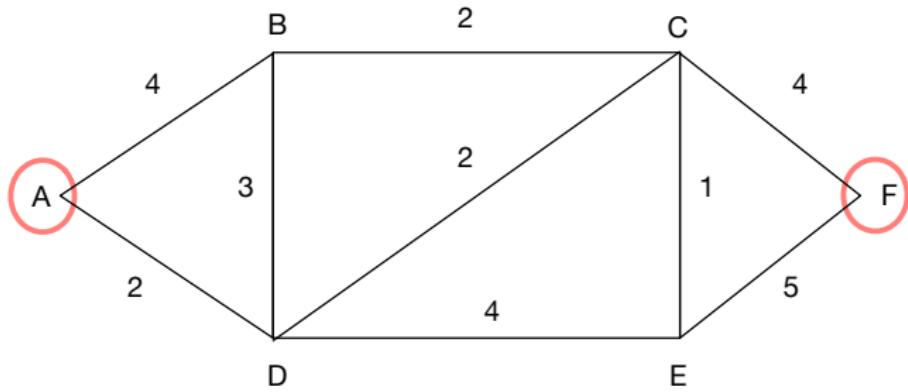
- State space
- Actions: moves between states
- Initial State
- Goal
- Search (successor function)
- Cost

Note: The type of problem formulation can have a big influence on the difficulty of finding a solution.

Let's focus on a slightly different problem...



Let's focus on a slightly different problem... find the route from A to F

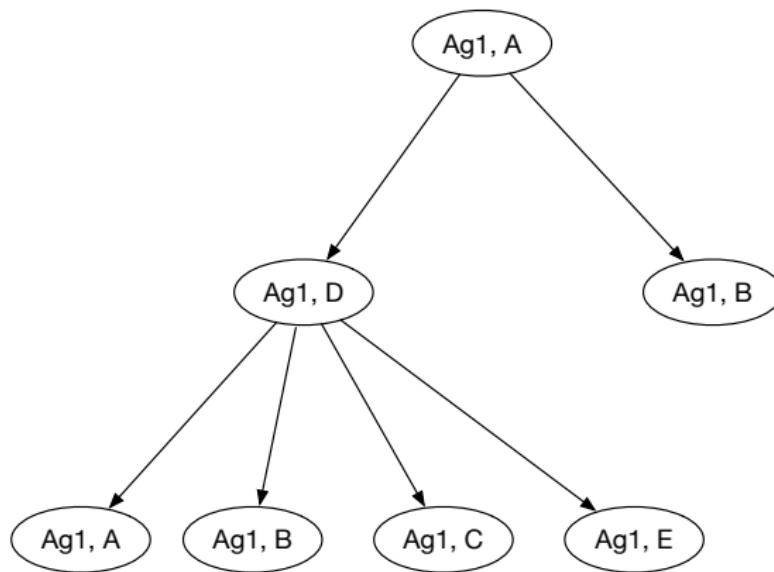


# Problem formulation

- State space: agent  $Ag1$  in a location  $locX$  (eg.  $\langle Ag1, C \rangle$ )
- Initial State:  $\langle Ag1, A \rangle$
- Successor function:  $Ag1$  moves from  $locA$  to an adjacent  $locB$   
Note: States may be redundant or illegal
- Goal State:  $\langle Ag1, F \rangle$
- Cost: **step cost** cost of one move from  $locA$  to  $locB$   
**path cost** sum of the step costs so far

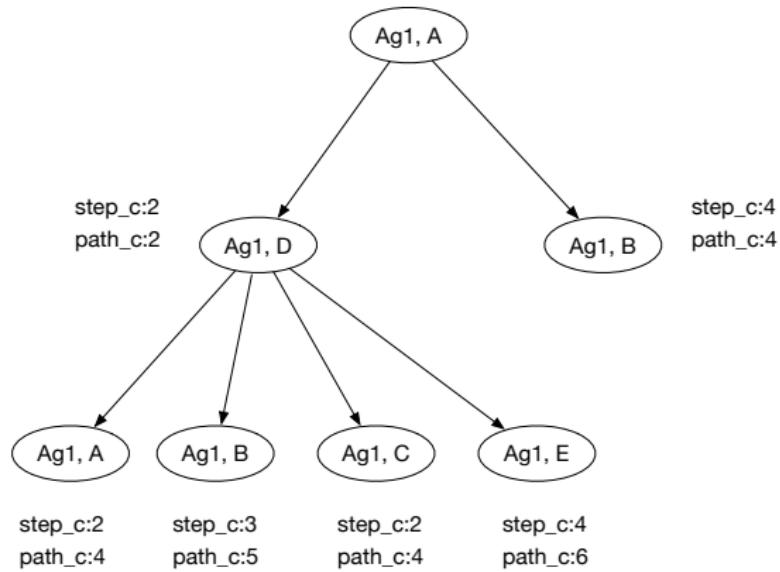
# General search

From the initial state, produce all successive states step by step → search tree.



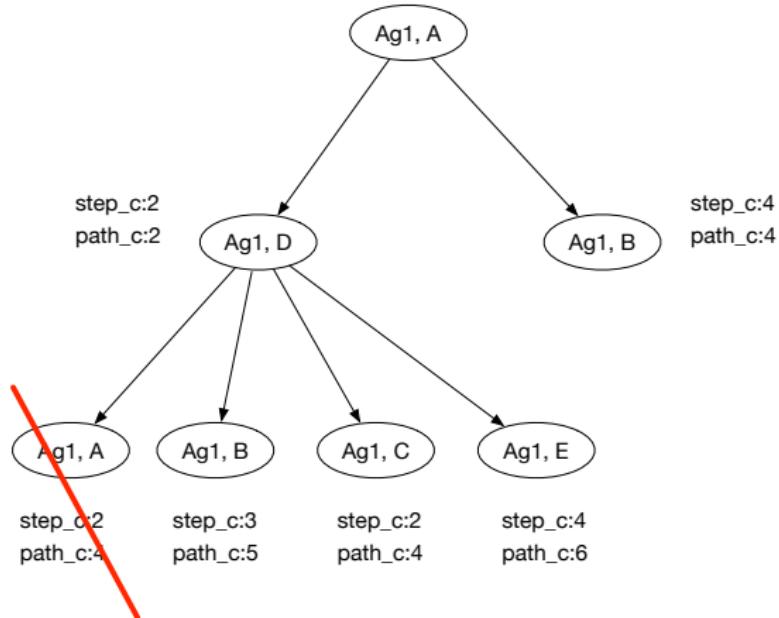
# Cost

From the initial state, produce all successive states step by step → search tree.



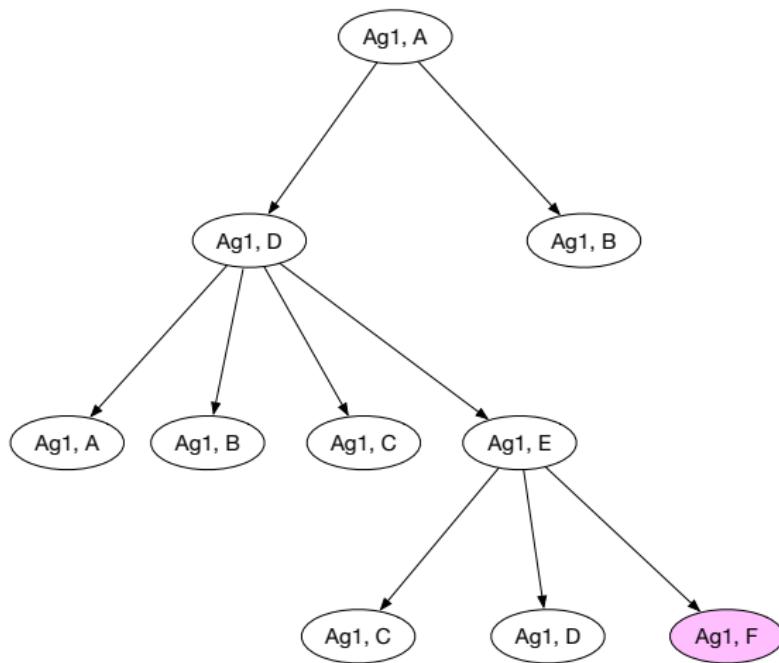
# Redundant state

From the initial state, produce all successive states step by step → search tree.



# Finding the goal

From the initial state, produce all successive states step by step → search tree.



# Implementing Search - Data Structures

Data structure for nodes in the search tree:

- **State:** State in the state space
- **Node:** Containing a state, pointer to predecessor, action, depth, and path cost
- **Depth:** number of steps along the path from the initial state
- **Path Cost:** Cost of the path from the initial state to the node
- **Frontier:** Memory for storing expanded nodes. Eg., a stack or a queue

# Implementing Search - Functions

General functions to implement:

- **Make-Node(Node node, State state)**: Creates a node from a state
- **Goal-Test(State state,State goal)**: Returns true if state is a goal state
- **Successor-Fn(State node, Problem problem)**: Implements the successor function, i.e. expands a set of new nodes given all actions applicable in the state
- **Cost(State state1,State state2)**: Returns the cost for executing action in state
- **Insert(Node node, Frontier frontier)**: Inserts a new node into the frontier
- **Remove(int index, Frontier frontier)**: Returns the first node from the frontier

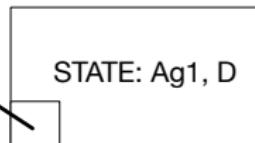
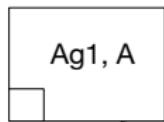
# General Search Algorithm

```
function MAKE-NODE(node,state) returns a node
    n ← a new Node
    STATE[n] ← state
    PARENT-NODE[n] ← node
    ACTION[n] ← move from STATE[node] to state
    PATH-COST[n] ← PATH-COST[node] + COST(STATE[node],state)
    DEPTH[n] ← DEPTH[node] + 1
    return n

end
```

# Node

PARENT-NODE:



ACTION: move  
PATH-COST: 2  
DEPTH: 1

# General Search Algorithm

```
function TREE-SEARCH(problem,frontier) returns a solution, or failure
  initial_node  $\leftarrow$  MAKE-NODE(null,initial_state)
  frontier  $\leftarrow$  INSERT( initial_node, frontier)
  loop do
    if frontier is empty return failure
    nd  $\leftarrow$  REMOVE(index, frontier)
    if GOAL-TEST(STATE[nd], goal)
      return nd
    else
      frontier  $\leftarrow$  INSERT-ALL (EXPAND(nd, problem, frontier))
    end loop
  end
```

# General Search Algorithm

```
function EXPAND(node, problem, frontier) returns a set of nodes
  next_states  $\leftarrow$  SUCCESSOR-FN(STATE[node],problem)
  successors  $\leftarrow$  empty set
    for each state in next_states
      nd  $\leftarrow$  MAKE-NODE(node,state)
      add nd to successors
    end for
  return successors
end
```

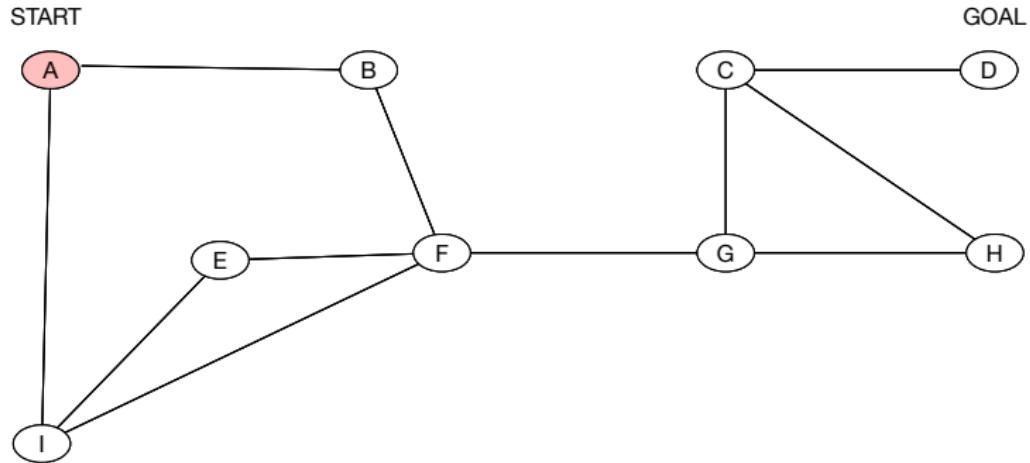
# Search Strategies

- Uninformed or blind searches:
  - ▶ No information on the length or cost of a path to the solution
- Informed search:
  - ▶ Uses information on the length or cost of a path to choose the next step (heuristics)

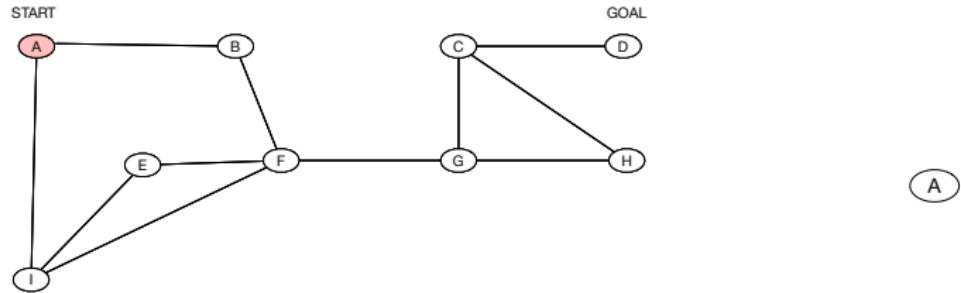
# Search Strategies

- Uninformed or blind searches:
  - ▶ breadth-first search
  - ▶ depth-first search
  - ▶ uniform-cost search
  - ▶ iterative deepening
- Informed search (strategy+heuristics)
  - ▶ Greedy Best first
  - ▶ A\*

# Breadth-first search

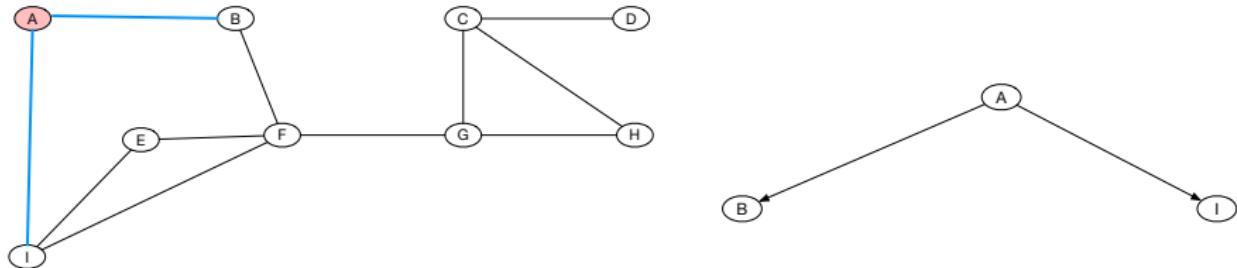


# Breadth-first search



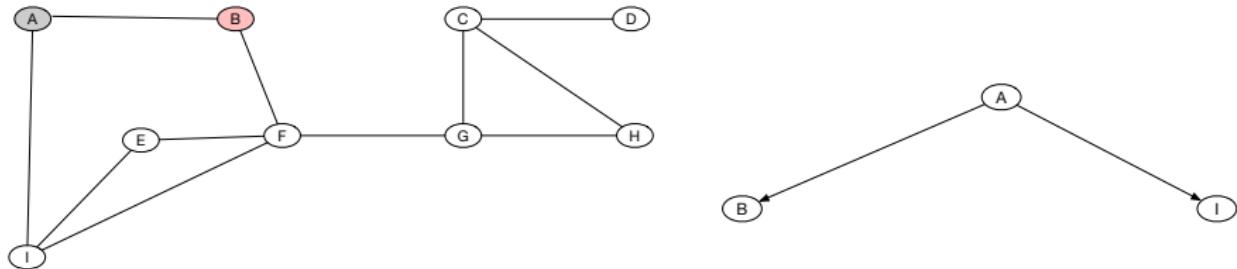
- Current node:
- Frontier: {A}
- Explored: { }

# Breadth-first search



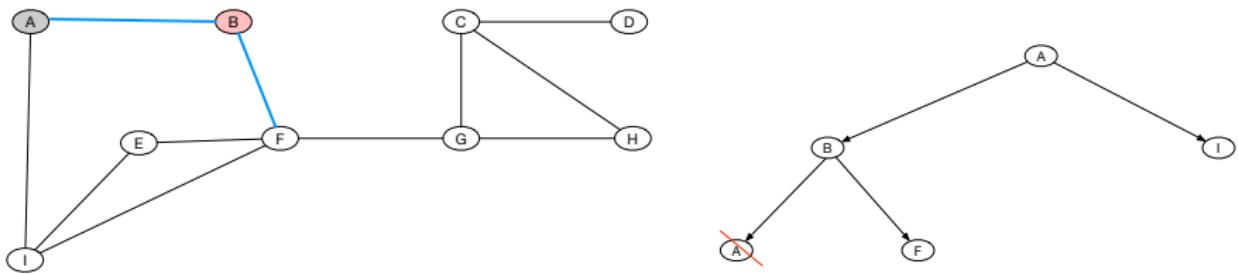
- Current node: A
- Frontier: {B,I}
- Explored: { A }

# Breadth-first search



- Current node: B
- Frontier: {I}
- Explored: { A }

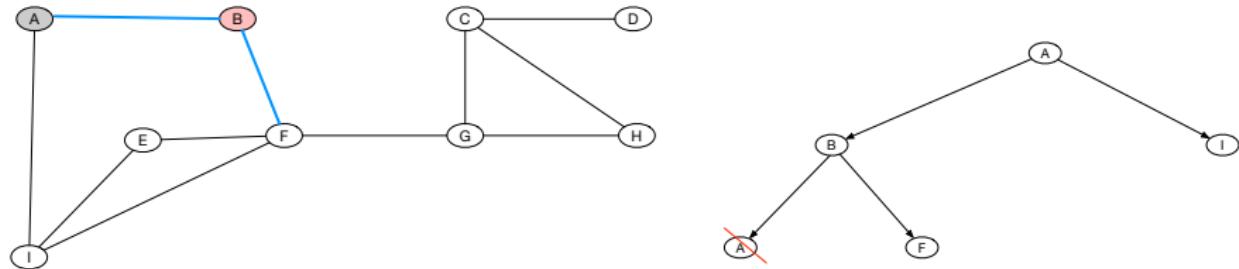
# Breadth-first search



- Current node: B
- Frontier: {I, A, F}
- Explored: { A ,B }

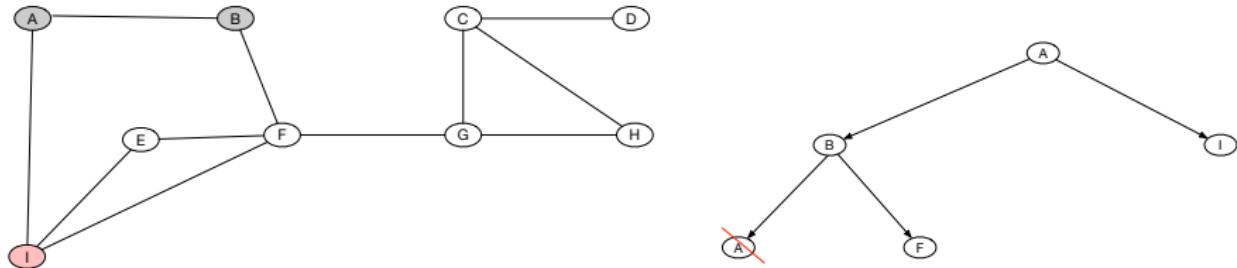
But A has been explored already

# Breadth-first search



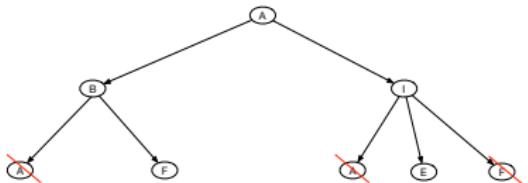
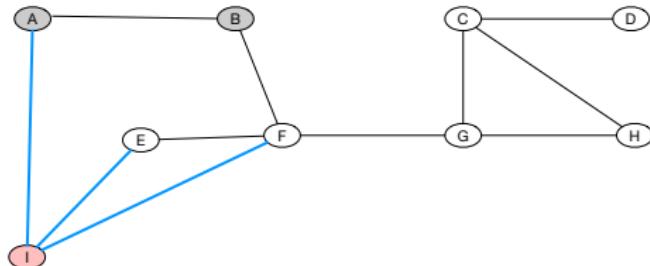
- Current node: B
- Frontier: {I,F}
- Explored: { A ,B }

# Breadth-first search



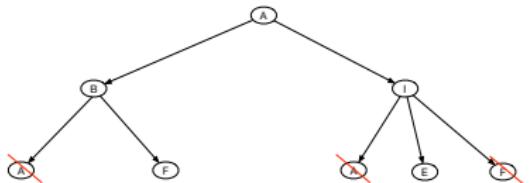
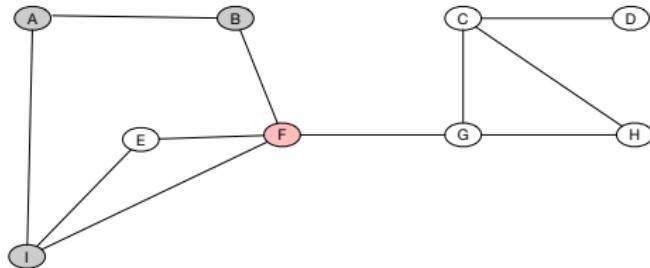
- Current node: I
- Frontier: {F}
- Explored: { A ,B }

# Breadth-first search



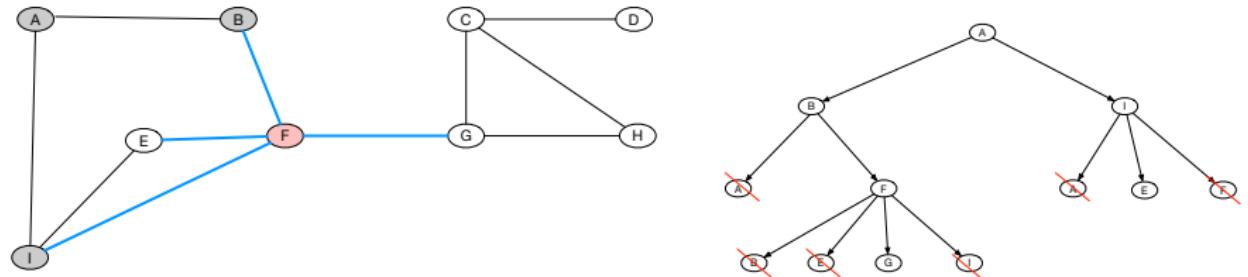
- Current node: I
- Frontier: {F,E}
- Explored: { A ,B ,I }

# Breadth-first search



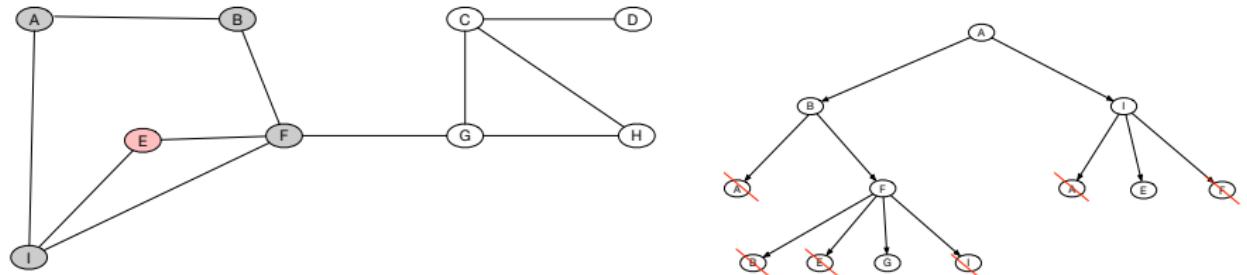
- Current node: F
- Frontier: {E}
- Explored: { A ,B ,I }

# Breadth-first search



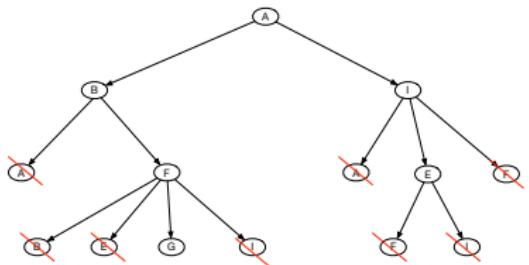
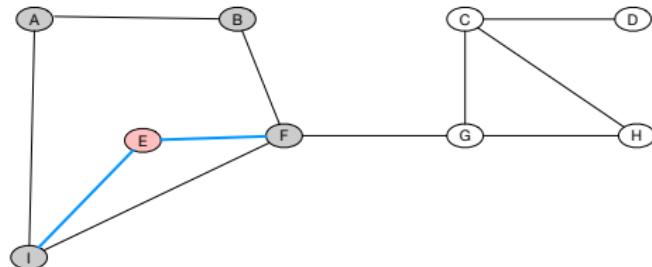
- Current node: F
- Frontier: {E,G}
- Explored: { A ,B ,I ,F }

# Breadth-first search



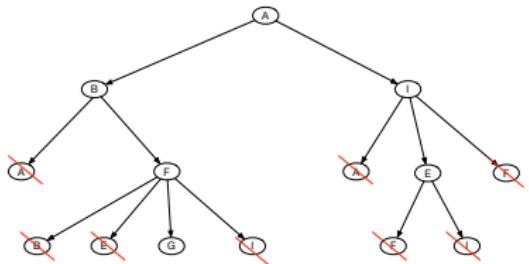
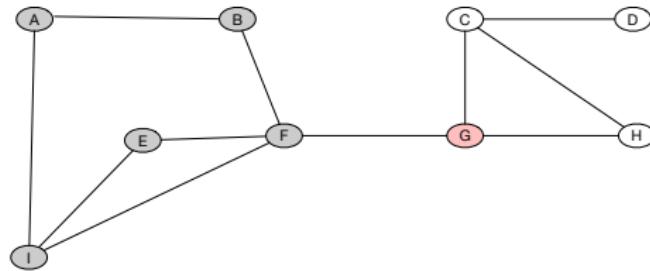
- Current node: E
- Frontier: {G}
- Explored: { A ,B ,I ,F }

# Breadth-first search



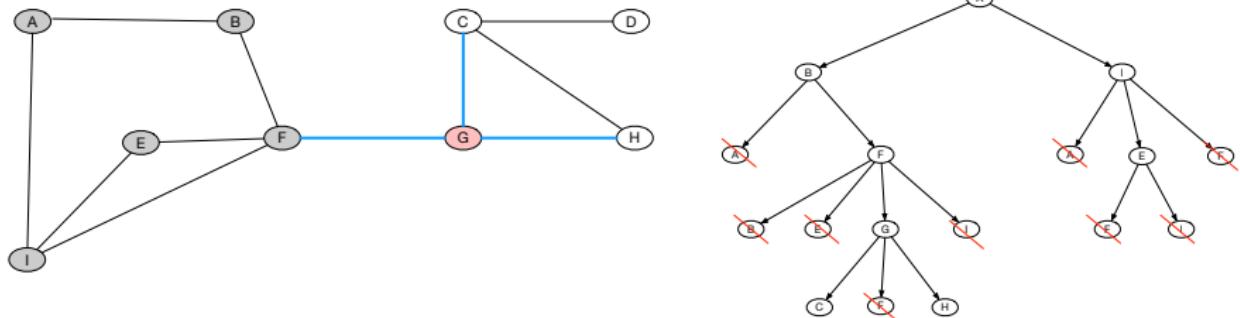
- Current node: E
- Frontier: {G}
- Explored: { A ,B ,I ,F ,E }

# Breadth-first search



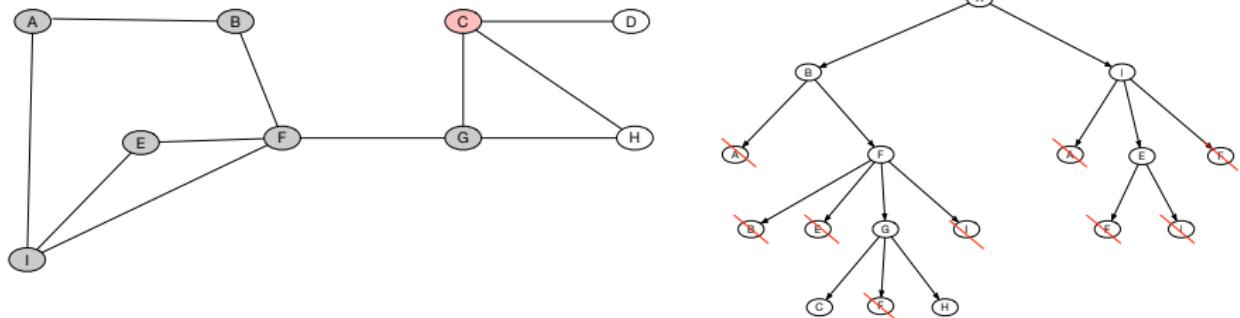
- Current node: G
- Frontier: {}
- Explored: { A ,B ,I ,F ,E }

# Breadth-first search



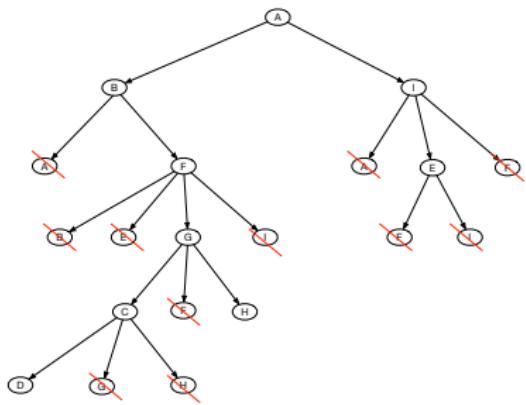
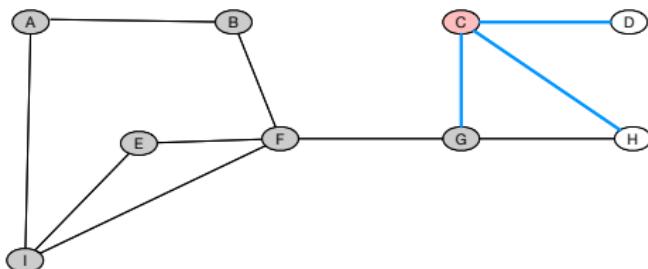
- Current node: G
- Frontier: {C,H}
- Explored: { A ,B ,I ,F ,E ,G }

# Breadth-first search



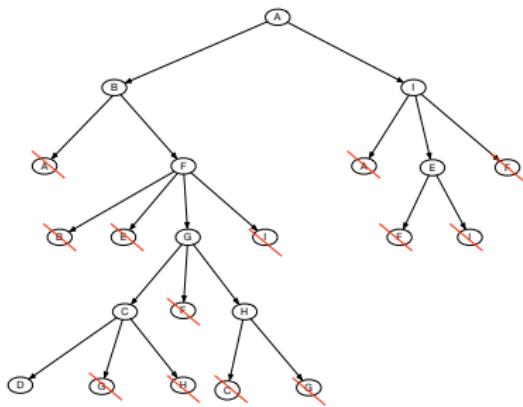
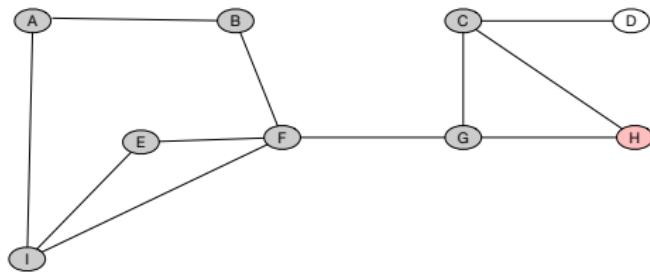
- Current node: C
- Frontier: {H}
- Explored: { A ,B ,I ,F ,E ,G }

# Breadth-first search



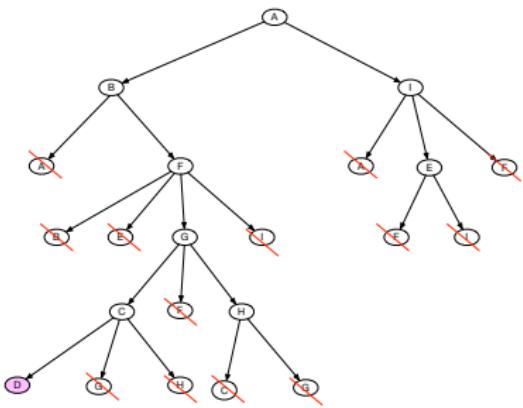
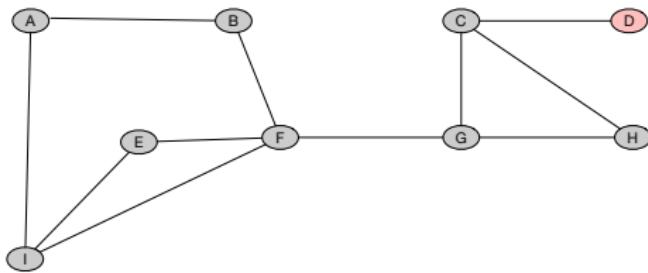
- Current node: C
- Frontier: {H,D}
- Explored: { A ,B ,I ,F ,E ,G ,C }

# Breadth-first search



- Current node: H
- Frontier: {D}
- Explored: { A ,B ,I ,F ,E ,G ,C ,H }

# Breadth-first search

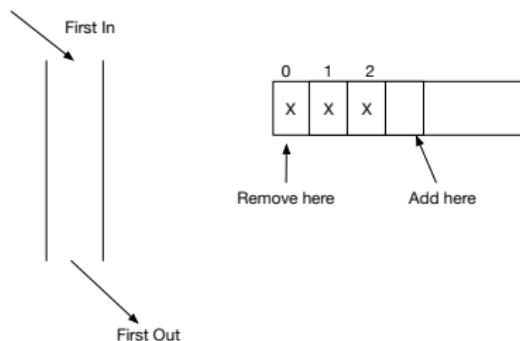


- Current node: D
- Frontier: {}
- Explored: { A ,B ,I ,F ,E ,G ,C ,H ,D }

# Breadth-first search

Expand all nodes at the same level & use:

- First-in-first-out (FIFO) → Queue structure for Frontier



- to avoid loops: check that explored nodes and frontier do not include the new node
- for this problem: SUCCESSOR-FN → connected node in alphabetical order

# Search Algorithm Example

```
function TREE-SEARCH(problem,frontier) returns a solution, or failure
initial_node  $\leftarrow$  MAKE-NODE(null,initial_state)
frontier  $\leftarrow$  INSERT-END( initial_node, frontier)
explored  $\leftarrow$  empty set
    loop do
        if frontier is empty return failure
        nd  $\leftarrow$  REMOVE-FRONT(frontier)
        Add nd to explored
        if GOAL-TEST(STATE[nd], goal)
            return nd
        else
            frontier  $\leftarrow$  INSERT-ALL (EXPAND(nd, problem, frontier, explored ))
        end loop
    end
```

# General Search Algorithm

```
function EXPAND(node, problem, frontier, explored) returns a set of nodes
    next_states  $\leftarrow$  SUCCESSOR-FN(STATE[node],problem)
    successors  $\leftarrow$  empty set
        for each state in next_states
            nd  $\leftarrow$  MAKE-NODE(node, state)
            if nd is not in explored or frontier then
                add nd to successors in alphabetical order
            end for
        return successors
    end
```

# Depth-first search

For next time try and compare the two approaches

# Summary

Search in AI:

- Search problems
- Search components
- Search trees
- BFS implementation

## Next time

No class tomorrow 10th due to DLS

Assignment 2 and informed search

# Reading Material

- S.J. Russell and P. Norvig. *Artificial Intelligence: A Modern Approach.* 3rd ed. Pearson Education, 2010 [Chapter 3]