

# El Bicho

DondeEstasCR7



07/11/2025

## Contents

<b>1 Algos</b>	<b>3</b>
1.1 Binary Search . . . . .	3
1.2 Fast Io . . . . .	3
1.3 Sliding Window . . . . .	4
1.4 Tablas Y Cotas . . . . .	5
1.5 Two Pointers . . . . .	7
<b>2 Bit Manipulation</b>	<b>8</b>
2.1 Bits . . . . .	8
<b>3 Combinatory</b>	<b>9</b>
3.1 Combi Brute Sin Mod . . . . .	9
3.2 Combinatory . . . . .	9
<b>4 Data Structures</b>	<b>10</b>
4.1 Fenwick Tree . . . . .	10
<b>5 Dp</b>	<b>11</b>
5.1 Digit Dp Pattern . . . . .	11
5.2 Digit Dp . . . . .	11
5.3 Edit Distance . . . . .	12
5.4 Knapsack . . . . .	12
5.5 Lcs . . . . .	13
<b>6 Geometry</b>	<b>13</b>
6.1 Convex Hull . . . . .	13
6.2 Point Operations . . . . .	14
<b>7 Graph</b>	<b>15</b>
7.1 Bellman Ford . . . . .	15
7.2 Bfs . . . . .	15
7.3 Bipartite . . . . .	16
7.4 Dfs . . . . .	16
7.5 Dfs 2D . . . . .	17
7.6 Dijkstra . . . . .	17

7.7 Disjoint Set Union Dsu . . . . .	18
7.8 Floyd Warshall . . . . .	18
7.9 Kruskal . . . . .	18
7.10 Lowest Common Ancestor Lca . . . . .	19
7.11 Prim . . . . .	20
7.12 Scc . . . . .	21
7.13 Topological Sort . . . . .	22
<b>8 Manhattan Distance</b>	<b>22</b>
8.1 Farthest Pair Of Points . . . . .	22
8.2 Nearest Neighbor In Each Octant . . . . .	22
<b>9 Number Theory</b>	<b>23</b>
9.1 Euler Toliente . . . . .	23
9.2 Gcd Lcm . . . . .	24
9.3 Number Theory . . . . .	24
9.4 Phi Euler . . . . .	25
9.5 Potenciacion Binaria . . . . .	25
9.6 Sieve . . . . .	25
9.7 Sieve Bitset . . . . .	26
9.8 Sum Of Divisors . . . . .	26
<b>10 Numerical Methods</b>	<b>27</b>
10.1 Calculating The Determinant Of A Matrix By Gauss . . . . .	27
10.2 Gauss Method For Solving System Of Linear Equations . . . . .	27
10.3 Gauss Solving Modular Slae . . . . .	28
10.4 Integration By Simpson'S Formula . . . . .	28
<b>11 Segment Tree</b>	<b>29</b>
11.1 Find Two Numbers . . . . .	29
11.2 Segment Tree . . . . .	29
11.3 Sparse Table . . . . .	30
<b>12 String</b>	<b>30</b>
12.1 Kmp . . . . .	30
12.2 Trie . . . . .	31
12.3 Z Algorithm . . . . .	32

---

## 1 Algos

### 1.1 Binary Search

```

1 // binary search en array ordenado
2 int n, target; cin >> n >> target;
3 vector<int> a(n);
4 for (int i = 0; i < n; i++) cin >> a[i];
5
6 // encontrar primera posicion >= target
7 int l = 0, r = n - 1, first_pos = n;
8 while (l <= r) {
9     int mid = (l + r) / 2;
10    if (a[mid] >= target) {
11        first_pos = mid;
12        r = mid - 1;
13    } else {
14        l = mid + 1;
15    }
16}
17
18 // encontrar ultima posicion <= target
19 l = 0, r = n - 1;
20 int last_pos = -1;
21 while (l <= r) {
22     int mid = (l + r) / 2;
23     if (a[mid] <= target) {
24         last_pos = mid;
25         l = mid + 1;
26     } else {
27         r = mid - 1;
28     }
29}
30
31 // binary search en funcion monotona
32 function<bool(int)> check = [&](int x) {
33     return true; // condicion
34};
35 l = 0, r = 1e9;
36 int ans = -1;
37 while (l <= r) {
38     int mid = (l + r) / 2;
39     if (check(mid)) {
40         ans = mid;
41         l = mid + 1; // o r = mid - 1 dependiendo del problema
42     } else {
43         r = mid - 1; // o l = mid + 1
44     }
45}

```

### 1.2 Fast Io

```

1 #include <bits/stdc++.h>
2 #include <ext/pb_ds/assoc_container.hpp>
3 #include <ext/pb_ds/tree_policy.hpp>
4
5 #define cpu() ios::sync_with_stdio(false);cin.tie(nullptr);
6
7 using namespace std;

```

```

8 using namespace __gnu_pbds;
9 template <class T>
10 using ordered_set = tree<T, null_type, less_equal<T>, rb_tree_tag,
11     tree_order_statistics_node_update>;
12
13 #define pb push_back
14 #define sz(a) ((int)(a).size())
15 #define ff first
16 #define ss second
17 #define all(a) (a).begin(), (a).end()
18 #define allr(a) (a).rbegin(), (a).rend()
19 #define approx(a) fixed << setprecision(a)
20
21 template <class T> void read(vector<T> &v);
22 template <class F, class S> void read(pair<F, S> &p);
23 template <class T, size_t Z> void read(array<T, Z> &a);
24 template <class T> void read(T &x) {cin >> x;}
25 template <class R, class... T> void read(R& r, T&... t){read(r); read(t...);}
26 template <class T> void read(vector<T> &v) {for(auto& x : v) read(x);}
27 template <class F, class S> void read(pair<F, S> &p) {read(p.ff, p.ss);}
28 template <class T, size_t Z> void read(array<T, Z> &a) { for(auto &x : a) read(x); }
29
30 template <class F, class S> void pr(const pair<F, S> &x);
31 template <class T> void pr(const T &x) {cout << x;}
32 template <class R, class... T> void pr(const R& r, const T&... t) {pr(r); pr(t...);}
33 template <class F, class S> void pr(const pair<F, S> &x) {pr("{", x.ff, ",", x.ss, "}")\n";}
34 void ps() {pr("\n");}
35 template <class T> void ps(const T &x) {pr(x); ps();}
36 template <class T> void ps(vector<T> &v) {for(auto& x : v) pr(x, '\n'); ps();}
37 template <class T, size_t Z> void ps(const array<T, Z> &a) { for(auto &x : a) pr(x, '\n'); ps();
38 }()
39
40 using ll = long long;
41 const double PI = 3.141592653589793;
42 const ll MX = 1e9 + 1;
43
44 void solve() {
45 }
46
47 int main() {
48     cpu();
49
50     int t = 1;
51     //cin >> t;
52     while (t--) {
53         solve();
54     }
55
56     return 0;
57 }

```

### 1.3 Sliding Window

```

1 int n, k; cin >> n >> k;
2 vector<int> a(n);
3 for (int i = 0; i < n; i++) cin >> a[i];

```

```

29     }
30     if (i + z_combined[i] - 1 > r_combined) {
31         l_combined = i;
32         r_combined = i + z_combined[i] - 1;
33     }
34 }
35 vector<int> occurrences;
36 for (int i = pattern.size() + 1; i < combined.size(); i++) {
37     if (z_combined[i] == pattern.size()) {
38         occurrences.push_back(i - pattern.size() - 1);
39     }
40 }
41 // z[i] = longitud del substring mas largo que empieza en i y es prefijo de s
42 // occurrences contiene las posiciones donde pattern aparece en text

```

```

36     int cur = 0;
37     for (char c : s) {
38         int idx = c - 'a';
39         if (nodes[cur].next[idx] == -1) return 0;
40         cur = nodes[cur].next[idx];
41     }
42     return nodes[cur].count;
43 }
44 };
45
46 // uso:
47 // Trie trie;
48 // int n; cin >> n;
49 // for (int i = 0; i < n; i++) {
50 //     string s; cin >> s;
51 //     trie.insert(s);
52 // }
53 // string query; cin >> query;
54 // bool exists = trie.search(query); // true si query existe en el trie
55 // int count = trie.count_prefix(query); // cantidad de strings que tienen query como
56 // prefijo
57
58 Trie trie;
59 trie.insert("hello");
60 trie.insert("hell");
61 bool found = trie.search("hello"); // true
62 int count = trie.count_prefix("hel"); // 2 (hello y hell)

```

### 12.3 Z Algorithm

```
1 string s; cin >> s;
2 int n = s.size();
3 vector<int> z(n);
4 int l = 0, r = 0;
5 for (int i = 1; i < n; i++) {
6     if (i <= r) {
7         z[i] = min(r - i + 1, z[i - 1]);
8     }
9     while (i + z[i] < n && s[z[i]] == s[i + z[i]]) {
10         z[i]++;
11     }
12     if (i + z[i] - 1 > r) {
13         l = i;
14         r = i + z[i] - 1;
15     }
16 }
17
18 string pattern, text;
19 cin >> pattern >> text;
20 string combined = pattern + "#" + text;
21 vector<int> z_combined(combined.size());
22 int l_combined = 0, r_combined = 0;
23 for (int i = 1; i < combined.size(); i++) {
24     if (i <= r_combined) {
25         z_combined[i] = min(r_combined - i + 1, z_combined[i - l_combined]);
26     }
27     while (i + z_combined[i] < combined.size() && combined[z_combined[i]] == combined[i + z_combined[i]]) {
28         z_combined[i]++;
29     }
30 }
```

```

4 // ventana deslizante de tamano k
5 deque<int> dq;
6 for (int i = 0; i < n; i++) {
7     while (!dq.empty() && dq.front() <= i - k) dq.pop_front();
8     while (!dq.empty() && a[dq.back()] <= a[i]) dq.pop_back();
9     dq.push_back(i);
10    if (i >= k - 1) {
11        // a[dq.front()] es el maximo en ventana [i-k+1, i]
12    }
13}
14

15 // minimo en ventana de tamano k
16 dq.clear();
17 for (int i = 0; i < n; i++) {
18     while (!dq.empty() && dq.front() <= i - k) dq.pop_front();
19     while (!dq.empty() && a[dq.back()] >= a[i]) dq.pop_back();
20     dq.push_back(i);
21     if (i >= k - 1) {
22         // a[dq.front()] es el minimo en ventana [i-k+1, i]
23     }
24}
25

```

## 1.4 Tablas Y Cotas

```

1 // Primeros 180 Primos:
2 2 3 5 7 11 13 17 19 23 29 31 37 41 43 47 53 59 61 67 71 73 79 83 89
3 97 101 103 107 109 113 127 131 137 139 149 151 157 163 167 173 179
4 181 191 193 197 199 211 223 227 229 233 239 241 251 257 263 269 271
5 277 281 283 293 307 311 313 317 331 337 347 349 353 359 367 373 379
6 383 389 397 401 409 419 421 431 433 439 443 449 457 461 463 467 479
7 487 491 499 503 509 521 523 541 547 557 563 569 571 577 587 593 599
8 601 607 613 617 619 631 641 643 647 653 659 661 673 677 683 691 701
9 709 719 727 733 739 743 751 757 761 769 773 787 797 809 811 821 823
10 827 829 839 853 857 859 863 877 881 883 887 907 911 919 929 937 941
11 947 953 967 971 977 983 991 997 1009 1013 1019 1021 1031 1033 1039
12 1049 1051 1061 1063 1069

13
14 // Primos cercanos a 10^n
15 9941 9949 9967 9973 10007 10009 10037 10039 10061 10067 10069 10079
16 99961 99971 99989 99991 100003 100019 100043 100049 100057 100069
17 999959 999961 999979 999983 1000003 1000033 1000037 1000039 9999943
18 9999971 9999973 9999991 10000019 10000079 10000103 10000121 99999941
19 99999959 99999971 99999989 100000007 100000037 100000039 100000049
20 999999893 999999929 999999937 1000000007 1000000009 1000000021
21 1000000033

22
23 // Cantidad de primos menores que 10^n
24 pi(10^1) = 4 -> [2, 3, 5, 7]
25 pi(10^2) = 25 -> [2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67,
   71, 73, 79, 83, 89]
26 pi(10^3) = 168
27 pi(10^4) = 1.229
28 pi(10^5) = 9.592
29 pi(10^6) = 78.498
30 pi(10^7) = 664.579
31 pi(10^8) = 5.761.455
32 pi(10^9) = 50.847.534
33 pi(10^10) = 455.052.511

```

```

34 pi(10^11) = 4.118.054.813
35 pi(10^12) = 37.607.912.018
36
37 // Cantidad de divisores
38 sigma0(60) = 12 -> [1, 2, 3, 4, 6, 10, 12, 15, 20, 30, 60]
39 sigma0(120) = 16 -> [1, 2, 3, 4, 6, 8, 10, 12, 15, 20, 24, 30, 40, 60, 120]
40 sigma0(180) = 18 -> [1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 30, 36, 60, 90, 180]
41 sigma0(240) = 20 -> [1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 16, 20, 24, 30, 40, 60, 80, 120, 240]
42 sigma0(360) = 24
43 sigma0(720) = 30
44 sigma0(840) = 32
45 sigma0(1.260) = 36
46 sigma0(1.680) = 40
47 sigma0(10.080) = 72
48 sigma0(15.120) = 80
49 sigma0(50.400) = 108
50 sigma0(83.160) = 128
51 sigma0(110.880) = 144
52 sigma0(498.960) = 200
53 sigma0(554.400) = 216
54 sigma0(1.081.080) = 256
55 sigma0(1.441.440) = 288
56 sigma0(4.324.320) = 384
57 sigma0(8.648.640) = 448
58
59 // Suma de divisores
60 sigma1(96) = 252 -> [1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, 96]
61 sigma1(108) = 280 -> [1, 2, 3, 4, 6, 9, 12, 18, 27, 36, 54, 108]
62 sigma1(120) = 360 -> [1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 40, 60, 120]
63 sigma1(144) = 403 -> [1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36, 48, 72, 144]
64 sigma1(168) = 480
65 sigma1(960) = 3.048
66 sigma1(1.008) = 3.224
67 sigma1(1.080) = 3.600
68 sigma1(1.200) = 3.844
69 sigma1(4.620) = 16.128
70 sigma1(4.680) = 16.380
71 sigma1(5.040) = 19.344
72 sigma1(5.760) = 19.890
73 sigma1(8.820) = 31.122
74 sigma1(9.240) = 34.560
75 sigma1(10.080) = 39.312
76 sigma1(10.920) = 40.320
77 sigma1(32.760) = 131.040
78 sigma1(35.280) = 137.826
79 sigma1(36.960) = 145.152
80 sigma1(37.800) = 148.800
81 sigma1(60.480) = 243.840
82 sigma1(64.680) = 246.240
83 sigma1(65.520) = 270.816
84 sigma1(70.560) = 280.098
85 sigma1(95.760) = 386.880
86 sigma1(98.280) = 403.200
87 sigma1(100.800) = 409.448
88 sigma1(491.400) = 2.083.200
89 sigma1(498.960) = 2.160.576
90 sigma1(514.080) = 2.177.280
91 sigma1(982.800) = 4.305.280
92 sigma1(997.920) = 4.390.848

```

```

12
13 string pattern, text;
14 cin >> pattern >> text;
15 string combined = pattern + "#" + text;
16 vector<int> pi_combined(combined.size());
17 for (int i = 1; i < combined.size(); i++) {
18     int j = pi_combined[i - 1];
19     while (j > 0 && combined[i] != combined[j]) {
20         j = pi_combined[j - 1];
21     }
22     if (combined[i] == combined[j]) j++;
23     pi_combined[i] = j;
24 }
25 vector<int> occurrences;
26 for (int i = pattern.size() + 1; i < combined.size(); i++) {
27     if (pi_combined[i] == pattern.size()) {
28         occurrences.push_back(i - 2 * pattern.size());
29     }
30 }
31 // pi[i] = longitud del prefijo mas largo que es sufijo en s[0..i]
32 // occurrences contiene las posiciones donde pattern aparece en text

```

**12.2 Trie**

```

1 struct Trie {
2     struct Node {
3         vector<int> next;
4         bool is_end;
5         int count;
6         Node() : next(26, -1), is_end(false), count(0) {}
7     };
8     vector<Node> nodes;
9     Trie() { nodes.emplace_back(); }
10
11 void insert(string& s) {
12     int cur = 0;
13     for (char c : s) {
14         int idx = c - 'a';
15         if (nodes[cur].next[idx] == -1) {
16             nodes[cur].next[idx] = nodes.size();
17             nodes.emplace_back();
18         }
19         cur = nodes[cur].next[idx];
20         nodes[cur].count++;
21     }
22     nodes[cur].is_end = true;
23 }
24
25 bool search(string& s) {
26     int cur = 0;
27     for (char c : s) {
28         int idx = c - 'a';
29         if (nodes[cur].next[idx] == -1) return false;
30         cur = nodes[cur].next[idx];
31     }
32     return nodes[cur].is_end;
33 }
34
35 int count_prefix(string& s) {

```

```

39     int mid = (left + right) >> 1;
40     if(pos <= mid)
41         modify(pos, value, 2 * node, left, mid);
42     else
43         modify(pos, value, 2 * node + 1, mid + 1, right);
44     tree[node] = op(tree[2 * node], tree[2 * node + 1]);
45 }
46
47 T query(int l, int r, int node, int left, int right) {
48     if(r < left || l > right) return Z;
49     if(l <= left && right <= r) return tree[node];
50     int mid = (left + right) >> 1;
51     T leftSum = query(l, r, 2 * node, left, mid);
52     T rightSum = query(l, r, 2 * node + 1, mid + 1, right);
53     return op(leftSum, rightSum);
54 }
55
56 public:
57     void build(vector<T>& a) { build(a, 1, 0, N-1); }
58     void modify(int pos, T value) { modify(pos, value, 1, 0, N-1); }
59     T query(int l, int r) { return query(l, r, 1, 0, N-1); }
60 };

```

### 11.3 Sparse Table

```

1 int n; cin >> n;
2 vector<long long> a(n);
3 for (int i = 0; i < n; i++) cin >> a[i];
4 int k = log2(n) + 1;
5 vector<vector<long long>> st(n, vector<long long>(k));
6 for (int i = 0; i < n; i++) st[i][0] = a[i];
7 for (int j = 1; j < k; j++) {
8     for (int i = 0; i + (1 << j) <= n; i++) {
9         st[i][j] = min(st[i][j - 1], st[i + (1 << (j - 1))][j - 1]);
10    }
11 }
12 function<long long(int, int)> query = [&](int l, int r) {
13     int j = log2(r - l + 1);
14     return min(st[l][j], st[r - (1 << j) + 1][j]);
15 };
16 // query(l, r) = minimo en rango [l, r] en O(1)
17 // cambiar min por max para maximo
18 // cambiar min por gcd para GCD en rango

```

## 12 String

### 12.1 Kmp

```

1 string s; cin >> s;
2 int n = s.size();
3 vector<int> pi(n);
4 for (int i = 1; i < n; i++) {
5     int j = pi[i - 1];
6     while (j > 0 && s[i] != s[j]) {
7         j = pi[j - 1];
8     }
9     if (s[i] == s[j]) j++;
10    pi[i] = j;
11 }

```

```

93 sigma1(1.048.320) = 4.464.096
94 sigma1(4.979.520) = 22.189.440
95 sigma1(4.989.600) = 22.686.048
96 sigma1(5.045.040) = 23.154.768
97 sigma1(9.896.040) = 44.323.200
98 sigma1(9.959.040) = 44.553.600
99 sigma1(9.979.200) = 45.732.192
100
101 // Factoriales
102 0! = 1 (int)
103 1! = 1
104 2! = 2
105 3! = 6
106 4! = 24
107 5! = 120
108 6! = 720
109 7! = 5.040
110 8! = 40.320
111 9! = 362.880
112 10! = 3.628.800
113 11! = 39.916.800
114 12! = 479.001.600 (int)
115 13! = 6.227.020.800 (11)
116 14! = 87.178.291.200
117 15! = 1.307.674.368.000
118 16! = 20.922.789.888.000
119 17! = 355.687.428.096.000
120 18! = 6.402.373.705.728.000
121 19! = 121.645.100.408.832.000
122 20! = 2.432.902.008.176.640.000 (11)
123 21! = 51.090.942.171.709.400.000 (__int128_t)
124
125 // Limites de enteros
126 max signed char = 127
127 max unsigned char = 255
128 max signed int = 2.147.483.647
129 max unsigned int = 4.294.967.295
130 max signed long long = 9.223.372.036.854.775.807
131 max unsigned long long = 18.446.744.073.709.551.615
132 max signed __int128_t = 170.141.183.460.469.231.731.687.303.715.884.105.727
133 max unsigned __int128_t = 340.282.366.920.938.463.463.374.607.431.768.211.456

```

### 1.5 Two Pointers

```

1 int n, target; cin >> n >> target;
2 vector<int> a(n);
3 for (int i = 0; i < n; i++) cin >> a[i];
4
5 // encontrar subarray con suma = target
6 int l = 0, sum = 0;
7 for (int r = 0; r < n; r++) {
8     sum += a[r];
9     while (sum > target && l <= r) {
10         sum -= a[l++];
11    }
12    if (sum == target) {
13        // subarray [l, r] tiene suma = target
14    }
15 }

```

```

16 // encontrar numero de subarrays con suma <= target
17 l = 0, sum = 0;
18 long long count = 0;
19 for (int r = 0; r < n; r++) {
20     sum += a[r];
21     while (sum > target && l <= r) {
22         sum -= a[l++];
23     }
24     count += r - l + 1;
25 }
26 // count = numero de subarrays con suma <= target

```

## 2 Bit Manipulation

Técnicas para manipular bits individuales y operaciones a nivel de bit. Incluye macros útiles para competencias de programación.

### 2.1 Bits

Macros esenciales para manipulación de bits: verificar potencias de 2, establecer/limpiar bits, contar bits, y operaciones con LSB/MSB.

```

1 using ull = unsigned long long;
2 const ull UNSIGNED_LL_MAX = 18'446'744'073'709'551'615;
3 // Verifica si S es potencia de dos (y distinto de cero)
4 #define isPowerOfTwo(S) ((S) && !((S) & ((S) - 1)))
5 // Retorna la potencia de dos mas cercana a S
6 #define nearestPowerOfTwo(S) (1LL << lround(log2(S)))
7 // Calcula S % N cuando N es potencia de dos
8 #define modulo(S, N) ((S) & ((N) - 1))
9
10 // Verifica si el bit esta encendido (bit en 1)
11 #define isOn(S, i) ((S) & (1LL<<(i)))
12 // Enciende el bit (Lo pone en 1)
13 #define setBit(S, i) ((S) |= (1LL<<(i)))
14 // Apaga el bit (Lo pone en 0)
15 #define clearBit(S, i) ((S) &= ~(1LL<<(i)))
16 // Invierte el estado del bit (0 <-> 1)
17 #define toggleBit(S, i) ((S) ^= (1LL<<(i)))
18 // Enciende los primeros 'n' bits (idx-0)
19 #define setAll(S, n) ((S) = ((n)>=64 ? ~0LL : (1LL << (n))-1))
20
21 // Extrae el bit menos significativo 0100 (Least Significant Bit)
22 #define lsb(S) ((S) & -(S))
23 // Numero de ceros a la derecha (Posicion del LSB, idx-0)
24 #define idxLastBit(x) __builtin_ctzll(x)
25 // Extrae el bit mas significativo 0100 (Most Significant Bit)
26 #define msb(S) (1LL << (63 - __builtin_clzll(S)))
27 // Posicion del MSB (63 - ceros a la izquierda, idx-0)
28 #define idxFirstBit(x) (63 - __builtin_clzll(x))
29
30 #define countAllOnes(x) __builtin_popcountll(x)
31 // Apaga el ultimo bit encendido (el menos significativo)
32 #define turnOffLastBit(S) ((S) & ((S) - 1))
33 // Enciende el ultimo cero menos significativo
34 #define turnOnLastZero(S) ((S) | ((S) + 1))
35 // Apaga todos los bits encendidos mas a la derecha consecutivos
36 #define turnOffLastConsecutiveBits(S) ((S) & ((S) + 1))
37 // Enciende los ceros consecutivos mas a la derecha

```

## 11 Segment Tree

### 11.1 Find Two Numbers

```

1 // "find two number where the sum is x, and gcd(a, b) > 1" b
2 auto find = [&](ll x){
3     for(int d = 2; d <= x / 2; d++){
4         if(x % d == 0){
5             ll m = 1, n = (x / d) - 1;
6             ll a = d * m, b = d * n;
7             if(__gcd(a, b) > 1){
8                 cout<< a << ' ' << b;
9                 ps();
10                return;
11            }
12        }
13    }
14};

```

### 11.2 Segment Tree

```

1 // "This segment_tree I understand better how it works"
2 template<typename T>
3 struct seg_tree {
4     int N;
5     T Z = 0;
6     vector<T> tree;
7
8     seg_tree(int N) : N(N) {
9         tree.resize(4 * N);
10    }
11
12    seg_tree(vector<T>& A) {
13        N = (int)A.size();
14        tree.resize(4 * N);
15        build(A, 1, 0, N-1);
16    }
17
18    private:
19    T op(T a, T b) {
20        return a + b;
21    }
22
23    void build(vector<T>& a, int node, int left, int right) {
24        if(left == right) {
25            tree[node] = a[left];
26            return;
27        }
28        int mid = (left + right) >> 1;
29        build(a, 2 * node, left, mid);
30        build(a, 2 * node + 1, mid + 1, right);
31        tree[node] = op(tree[2 * node], tree[2 * node + 1]);
32    }
33
34    void modify(int pos, T value, int node, int left, int right) {
35        if(left == right) {
36            tree[node] = value;
37            return;
38        }

```

```

26     ++row;
27 }
28
29 ans.assign (m, 0);
30 for (int i=0; i<m; ++i)
31     if (where[i] != -1)
32         ans[i] = a[where[i]][m] / a[where[i]][i];
33 for (int i=0; i<n; ++i) {
34     double sum = 0;
35     for (int j=0; j<m; ++j)
36         sum += ans[j] * a[i][j];
37     if (abs (sum - a[i][m]) > EPS)
38         return 0;
39 }
40
41 for (int i=0; i<m; ++i)
42     if (where[i] == -1)
43         return INF;
44
45 } 
```

### 10.3 Gauss Solving Modular Slae

```

1 int gauss (vector < bitset<N> > a, int n, int m, bitset<N> & ans) {
2     vector<int> where (m, -1);
3     for (int col=0, row=0; col<m && row<n; ++col) {
4         for (int i=row; i<n; ++i)
5             if (a[i][col]) {
6                 swap (a[i], a[row]);
7                 break;
8             }
9             if (! a[row][col])
10                continue;
11             where[col] = row;
12
13             for (int i=0; i<n; ++i)
14                 if (i != row && a[i][col])
15                     a[i] ^= a[row];
16             ++row;
17     }
18     // The rest of implementation is the same as above
19 } 
```

### 10.4 Integration By Simpson'S Formula

```

1 const int N = 1000 * 1000; // number of steps (already multiplied by 2)
2
3 double simpson_integration(double a, double b){
4     double h = (b - a) / N;
5     double s = f(a) + f(b); // a = x_0 and b = x_2n
6     for (int i = 1; i <= N - 1; ++i) { // Refer to final Simpson's formula
7         double x = a + h * i;
8         s += f(x) * ((i & 1) ? 4 : 2);
9     }
10    s *= h / 3;
11    return s;
12 } 
```

```

38 #define turnOnLastConsecutiveZeroes(S) ((S) | ((S) - 1))
39
40 // mascara de bits (mask -> subconjunto) 0(2^N)
41 for (int mask = 0; mask < (1 << N); mask++)
42
43 // Recorrer subconjuntos de un superconjunto (menos el vacio)
44 int b = 0b1011; // Representacion binaria de un decimal en int
45 for (int i = b; i; i = (i - 1) & b) {
46     cout << bitset<4>(i) << "\n";
47 }
48
49 void printBin(ll x) {
50     // 63 -> unsigned ll, 62 -> ll, 31 -> unsigned int, 30 -> int
51     for (ll i = 63; i >= 0; i--)
52         cout << ((x >> i) & 1);
53     cout << '\n';
54 } 
```

## 3 Combinatory

### 3.1 Combi Brute Sin Mod

```

1 // nCk brute force sin MOD n <= 20
2 long long nCk_bruteforce(long long n, long long k) {
3     if (k < 0 || k > n) return 0;
4     long long res = 1;
5     for (long long i = 1; i <= k; i++) {
6         res = res * (n - i + 1) / i; // aqui la division es exacta
7     }
8     return res;
9 }
10
11 // nPk brute force sin MOD n <= 20
12 long long nPk_bruteforce(long long n, long long k) {
13     if (k < 0 || k > n) return 0;
14     long long res = 1;
15     for (long long i = 0; i < k; i++) {
16         res *= (n - i);
17     }
18     return res;
19 } 
```

### 3.2 Combinatory

OJO: Es necesario usar binpow con MOD primo

```

1 // Devuelve el inverso modular de a mod MOD
2 // Usa el Teorema Pequeno de Fermat: a^(MOD-2) === a^(-1) (mod MOD)
3 // (valido solo si MOD es primo)
4 ll inv(ll a, ll p = MOD) {
5     return binpow(a, p - 2, p);
6 }
7
8 // Factoriales e inversos factoriales precomputados
9 // fact[n] = n! mod MOD
10 // invf[n] = (n!)^(-1) mod MOD
11 // Precomputa en O(n)
12 vector<ll> fact(MAXN + 1), invf(MAXN + 1);
13 
```

```

14 void precompute_factorials() {
15     fact[0] = 1;
16     for (int i = 1; i <= MAXN; i++) {
17         fact[i] = fact[i - 1] * i % MOD;
18     }
19     invf[MAXN] = inv(fact[MAXN]);
20     for (int i = MAXN; i > 0; i--) {
21         invf[i - 1] = invf[i] * i % MOD;
22     }
23 }
24
25 // Combinatoria de n en k: nCk(n, k) para n <= 10^6
26 // "n choose k" = n! / (k! * (n-k)!) mod MOD
27 // Retorna 0 si k > n
28 ll nCk(ll n, ll k) {
29     if (k < 0 || k > n) return 0;
30     return fact[n] * invf[k] % MOD * invf[n - k] % MOD;
31 }
32
33 // Permutacion de n en k: nPk(n, k) para n <= 10^6
34 // Calcula permutaciones: "n permute k" = n! / (n-k)! mod MOD
35 // Retorna 0 si k > n
36 ll nPk(ll n, ll k) {
37     if (k < 0 || k > n) return 0;
38     return fact[n] * invf[n - k] % MOD;
39 }

```

## 4 Data Structures

### 4.1 Fenwick Tree

```

1 struct FenwickTree {
2     vector<long long> tree;
3     int n;
4
5     FenwickTree(int size) : n(size + 1) {
6         tree.assign(n, 0);
7     }
8
9     void update(int idx, long long delta) {
10        for (idx++; idx < n; idx += idx & -idx) {
11            tree[idx] += delta;
12        }
13    }
14
15    long long query(int idx) {
16        long long sum = 0;
17        for (idx++; idx > 0; idx -= idx & -idx) {
18            sum += tree[idx];
19        }
20        return sum;
21    }
22
23    long long range_query(int l, int r) {
24        return query(r) - query(l - 1);
25    }
26};
27

```

## 10 Numerical Methods

### 10.1 Calculating The Determinant Of A Matrix By Gauss

```

1 const double EPS = 1E-9;
2 int n;
3 vector < vector<double> > a (n, vector<double> (n));
4
5 double det = 1;
6 for (int i=0; i<n; ++i) {
7     int k = i;
8     for (int j=i+1; j<n; ++j)
9         if (abs (a[j][i]) > abs (a[k][i]))
10             k = j;
11     if (abs (a[k][i]) < EPS) {
12         det = 0;
13         break;
14     }
15     swap (a[i], a[k]);
16     if (i != k)
17         det = -det;
18     det *= a[i][i];
19     for (int j=i+1; j<n; ++j)
20         a[i][j] /= a[i][i];
21     for (int j=0; j<n; ++j)
22         if (j != i && abs (a[j][i]) > EPS)
23             for (int k=i+1; k<n; ++k)
24                 a[j][k] -= a[i][k] * a[j][i];
25 }
26
27 cout << det;

```

### 10.2 Gauss Method For Solving System Of Linear Equations

```

1 const double EPS = 1e-9;
2 const int INF = 2; // it doesn't actually have to be infinity or a big number
3
4 int gauss (vector < vector<double> > a, vector<double> & ans) {
5     int n = (int) a.size();
6     int m = (int) a[0].size() - 1;
7
8     vector<int> where (m, -1);
9     for (int col=0, row=0; col<m && row<n; ++col) {
10        int sel = row;
11        for (int i=row; i<n; ++i)
12            if (abs (a[i][col]) > abs (a[sel][col]))
13                sel = i;
14        if (abs (a[sel][col]) < EPS)
15            continue;
16        for (int i=col; i<=m; ++i)
17            swap (a[sel][i], a[row][i]);
18        where[col] = row;
19
20        for (int i=0; i<n; ++i)
21            if (i != row) {
22                double c = a[i][col] / a[row][col];
23                for (int j=col; j<=m; ++j)
24                    a[i][j] -= a[row][j] * c;
25            }

```

```

4 if (!is_prime[0]) is_prime.assign(N+1, true);
5 is_prime[0] = is_prime[1] = false;
6 for (int p = 2; p * p <= N; p++) {
7     if (is_prime[p]) {
8         for (int i = p * p; i <= N; i += p) {
9             is_prime[i] = false;
10    }
11 }
12 }
13 }
```

## 9.7 Sieve Bitset

```

1 // Hasta N = 10^8 aprox en 1s
2 const int MAX_V = 1e7 + 5;
3 bitset<MAX_V> composite;
4 void sieve() {
5     composite[0] = composite[1] = true;
6     for (int i = 2; i * i < MAX_V; i++) {
7         if (composite[i]) continue;
8         for (int j = i * i; j < MAX_V; j += i) {
9             composite[j] = true;
10        }
11    }
12 }
13
14 int main() {
15     sieve();
16     for (int i = 2; i < 100; i++) {
17         cout << i << " is_prime:" << !composite[i] << '\n';
18     }
19 }
```

## 9.8 Sum Of Divisors

```

1 /* Sum of divs
2 long long SumOfDivisors(long long num) {
3     long long total = 1;
4
5     for (int i = 2; (long long)i * i <= num; i++) {
6         if (num % i == 0) {
7             int e = 0;
8             do {
9                 e++;
10                num /= i;
11            } while (num % i == 0);
12
13            long long sum = 0, pow = 1;
14            do {
15                sum += pow;
16                pow *= i;
17            } while (e-- > 0);
18            total *= sum;
19        }
20    }
21    if (num > 1) {
22        total *= (1 + num);
23    }
24    return total;
25 }
```

```

28 // uso:
29 // int n; cin >> n;
30 // FenwickTree ft(n);
31 // for (int i = 0; i < n; i++) {
32 //     long long x; cin >> x;
33 //     ft.update(i, x);
34 // }
35 // ft.update(idx, delta); // actualizar elemento en idx
36 // long long sum = ft.range_query(l, r); // suma en rango [l, r]
```

## 5 Dp

### 5.1 Digit Dp Pattern

```

1 string pattern; cin >> pattern; // ejemplo: "xxxxx3xxxx" donde x = digito libre
2 int n = pattern.size();
3 long long k; cin >> k; // modulo
4
5 vector<vector<vector<long long>>> dp(n, vector<vector<long long>>(k, vector<long long>(2,
6 -1)));
7
8 function<long long(int, int, bool, bool)> solve = [&](int pos, int rem, bool tight, bool
9 started) {
10     if (pos == n) {
11         return (started && rem == 0) ? 1LL : 0LL;
12     }
13     if (started && !tight && dp[pos][rem][tight ? 1 : 0] != -1) {
14         return dp[pos][rem][tight ? 1 : 0];
15     }
16     long long res = 0;
17     if (pattern[pos] != 'x' && pattern[pos] != 'X') {
18         int fixed_digit = pattern[pos] - '0';
19         bool new_tight = tight && (fixed_digit == 9);
20         bool new_started = started || (fixed_digit > 0);
21         int new_rem = (rem * 10 + fixed_digit) % k;
22         res += solve(pos + 1, new_rem, new_tight, new_started);
23     } else {
24         int limit = tight ? 9 : 9;
25         int start_digit = (pos == 0) ? 1 : 0; // primer digito no puede ser 0
26         for (int d = start_digit; d <= limit; d++) {
27             bool new_tight = tight && (d == limit);
28             bool new_started = started || (d > 0);
29             int new_rem = (rem * 10 + d) % k;
30             res += solve(pos + 1, new_rem, new_tight, new_started);
31         }
32     }
33     if (started && !tight) {
34         dp[pos][rem][tight ? 1 : 0] = res;
35     }
36     return res;
37 }
38
39 long long result = solve(0, 0, true, false);
40 // result = cantidad de numeros que siguen el patron y son divisibles por k
41 // ejemplo: pattern = "xxxxx3xxxx", k = 7
42 // cuenta numeros tipo 1234534567 que son divisibles por 7
43 // x o X = digito libre, cualquier otro caracter = digito fijo
```

### 5.2 Digit Dp

```

1 string s; cin >> s; // numero como string (puede ser muy grande, tipo 10^100)
2 int n = s.size();
3 long long k; cin >> k; // modulo
4
5 vector<vector<vector<long long>>> dp(n, vector<vector<long long>>(k, vector<long long>(2,
6   -1)));
7
8 function<long long(int, int, bool, bool)> solve = [&](int pos, int rem, bool tight, bool
9   started) {
10  if (pos == n) {
11    return (started && rem == 0) ? 1LL : 0LL;
12  }
13  if (started && !tight && dp[pos][rem][tight ? 1 : 0] != -1) {
14    return dp[pos][rem][tight ? 1 : 0];
15  }
16  long long res = 0;
17  int limit = tight ? (s[pos] - '0') : 9;
18  for (int d = 0; d <= limit; d++) {
19    bool new_tight = tight && (d == limit);
20    bool new_started = started || (d > 0);
21    int new_rem = (rem * 10 + d) % k;
22    res += solve(pos + 1, new_rem, new_tight, new_started);
23  }
24  if (started && !tight) {
25    dp[pos][rem][tight ? 1 : 0] = res;
26  }
27  return res;
28}
29 long long result = solve(0, 0, true, false);
30 // result = cantidad de numeros <= s que son divisibles por k
31 // para contar en rango [a, b]: result_b - result_a-1
32 // ejemplo: s = "1000000", k = 7 -> contar numeros de 0 a 1000000 divisibles por 7

```

### 5.3 Edit Distance

```

1 string s1, s2; cin >> s1 >> s2;
2 int n = s1.size(), m = s2.size();
3 vector<vector<int>> dp(n + 1, vector<int>(m + 1));
4 for (int i = 0; i <= n; i++) dp[i][0] = i;
5 for (int j = 0; j <= m; j++) dp[0][j] = j;
6 for (int i = 1; i <= n; i++) {
7  for (int j = 1; j <= m; j++) {
8    if (s1[i - 1] == s2[j - 1]) {
9      dp[i][j] = dp[i - 1][j - 1];
10   } else {
11     dp[i][j] = 1 + min({dp[i - 1][j], dp[i][j - 1], dp[i - 1][j - 1]});
12   }
13 }
14 }
15 // dp[n][m] = edit distance (minimo numero de operaciones: insertar, eliminar, reemplazar)
16 // para convertir s1 en s2

```

### 5.4 Knapsack

```

1 int n, capacity; cin >> n >> capacity;
2 vector<int> weight(n), value(n);
3 for (int i = 0; i < n; i++) {
4  cin >> weight[i] >> value[i];
5 }

```

37 }

### 9.4 Phi Euler

$\Phi(n)$  = contar la cantidad de numeros coprimos entre 1 a n

```

1 int phi(int n) {
2  int ans = n;
3  for(int i = 2; i * i <= n; i++) {
4    if(n % i == 0) {
5      while (n % i == 0) {
6        n /= i;
7      }
8      ans -= ans / i;
9    }
10 }
11 if(n > 1) {
12   ans -= ans / n;
13 }
14 return ans;
15 }

16 /* phi(n) -> complejo: O(log(log(n)))
17 void phi_1_to_n(int n) {
18  vector<int> phi(n + 1);
19  for (int i = 0; i <= n; i++)
20    phi[i] = i;
21
22  for (int i = 2; i <= n; i++) {
23    if (phi[i] == i) {
24      for (int j = i; j <= n; j += i)
25        phi[j] -= phi[j] / i;
26    }
27  }
28 }
29

```

### 9.5 Potenciacion Binaria

```

1 using ll = long long;
2 const int MAXN = 1e6; // limite superior de n
3 const ll MOD = 1e9 + 7; // primo grande
4
5 // Potenciacion binaria modular a^b mod p
6 ll binpow(ll a, ll b, ll m = MOD) {
7  a %= m;
8  ll res = 1;
9  while (b > 0) {
10   if (b & 1)
11     res = res * a % m;
12   a = a * a % m;
13   b >>= 1;
14 }
15 return res;
16 }

```

### 9.6 Sieve

```

1 // Criba de Eratostenes: Hasta N = 10^6
2 void sieve(vector<bool>& is_prime) {
3  int N = (int) is_prime.size();

```

```

30     }
31 }
32 for(int i = 1; i <= n; i++) {
33     cout << i << ' ' << phi[i] << '\n';
34 }
35 }
36 };

```

## 9.2 Gcd Lcm

```

1 // Maximo comun divisor (GCD): Algoritmo de Euclides
2 int gcd(int a, int b) {
3     if (a > b) swap(a, b);
4     if (a == 0) return b;
5     return gcd(b % a, a);
6 }
7
8 // Minimo comun multiplo (LCM): Calculado con GCD
9 int lcm(int a, int b) {
10    return (a * b) / gcd(a, b);
11 }

```

## 9.3 Number Theory

```

1 // Divisores de N: Hasta N = 10^6
2 vector<int> divisores(int N) {
3     vector<int> divs;
4     for (int d = 1; d * d <= N; d++) {
5         if (N % d == 0) {
6             divs.push_back(d);
7             if (N / d != d) divs.push_back(N / d);
8         }
9     }
10    return divs;
11 }
12
13 // Factorizacion de N: Hasta N = 10^6
14 vector<pair<int, int>> factorizar(int N) {
15     vector<pair<int, int>> facts;
16     for (int p = 2; p * p <= N; p++) {
17         if (N % p == 0) {
18             int exp = 0;
19             while (N % p == 0) {
20                 exp++;
21                 N /= p;
22             }
23             facts.push_back({ p, exp });
24         }
25     }
26     if (N > 1) facts.push_back({ N, 1 });
27     return facts;
28 }
29
30 // Primalidad: Hasta N = 10^6 - 0(sqrt(N))
31 bool isPrime(int N) {
32     if (N < 2) return false;
33     for (int d = 2; d * d <= N; d++) {
34         if (N % d == 0) return false;
35     }
36     return true;

```

```

6 vector<long long> dp(capacity + 1, 0);
7 for (int i = 0; i < n; i++) {
8     for (int w = capacity; w >= weight[i]; w--) {
9         dp[w] = max(dp[w], dp[w - weight[i]] + value[i]);
10    }
11 }
12 // dp[capacity] = valor maximo que se puede obtener con capacidad maxima
13 // para version con items ilimitados, cambiar el loop: for (int w = weight[i]; w <= capacity
14 ; w++)
15

```

## 5.5 Lcs

```

1 string s1, s2; cin >> s1 >> s2;
2 int n = s1.size(), m = s2.size();
3 vector<vector<int>> dp(n + 1, vector<int>(m + 1, 0));
4 for (int i = 1; i <= n; i++) {
5     for (int j = 1; j <= m; j++) {
6         if (s1[i - 1] == s2[j - 1]) {
7             dp[i][j] = dp[i - 1][j - 1] + 1;
8         } else {
9             dp[i][j] = max(dp[i - 1][j], dp[i][j - 1]);
10        }
11    }
12
13 // dp[n][m] = longitud de LCS (Longest Common Subsequence)
14
15 string reconstruct_lcs() {
16     string lcs = "";
17     int i = n, j = m;
18     while (i > 0 && j > 0) {
19         if (s1[i - 1] == s2[j - 1]) {
20             lcs += s1[i - 1];
21             i--;
22         } else if (dp[i - 1][j] > dp[i][j - 1]) {
23             i--;
24         } else {
25             j--;
26         }
27     }
28     reverse(lcs.begin(), lcs.end());
29     return lcs;
30 }
31 // lcs = string de la LCS

```

## 6 Geometry

### 6.1 Convex Hull

```

1 struct Point {
2     long long x, y;
3     Point(long long x = 0, long long y = 0) : x(x), y(y) {}
4     Point operator-(const Point& p) const { return Point(x - p.x, y - p.y); }
5     long long cross(const Point& p) const { return x * p.y - y * p.x; }
6     long long cross(const Point& a, const Point& b) const { return (a - *this).cross(b - *this
7         ); }
8     bool operator<(const Point& p) const { return x < p.x || (x == p.x && y < p.y); }
9 }
10 vector<Point> convex_hull(vector<Point>& points) {

```

```

11 int n = points.size();
12 if (n <= 1) return points;
13 sort(points.begin(), points.end());
14 vector<Point> hull;
15 for (int i = 0; i < n; i++) {
16     while (hull.size() >= 2 && hull[hull.size() - 2].cross(hull.back(), points[i]) <= 0) {
17         hull.pop_back();
18     }
19     hull.push_back(points[i]);
20 }
21 int lower = hull.size();
22 for (int i = n - 2; i >= 0; i--) {
23     while (hull.size() > lower && hull[hull.size() - 2].cross(hull.back(), points[i]) <= 0)
24     {
25         hull.pop_back();
26     }
27     hull.push_back(points[i]);
28 }
29 hull.pop_back();
30 return hull;
31 }
32 // uso:
33 // int n; cin >> n;
34 // vector<Point> points(n);
35 // for (int i = 0; i < n; i++) {
36 //     cin >> points[i].x >> points[i].y;
37 // }
38 // vector<Point> hull = convex_hull(points);
39 // hull contiene los puntos del convex hull en orden counter-clockwise

```

## 6.2 Point Operations

```

1 struct Point {
2     long long x, y;
3     Point(long long x = 0, long long y = 0) : x(x), y(y) {}
4     Point operator+(const Point& p) const { return Point(x + p.x, y + p.y); }
5     Point operator-(const Point& p) const { return Point(x - p.x, y - p.y); }
6     Point operator*(long long k) const { return Point(x * k, y * k); }
7     long long dot(const Point& p) const { return x * p.x + y * p.y; }
8     long long cross(const Point& p) const { return x * p.y - y * p.x; }
9     long long cross(const Point& a, const Point& b) const { return (a - *this).cross(b - *this)
10    ); }
11    long long norm2() const { return x * x + y * y; }
12    double norm() const { return sqrt(norm2()); }
13    bool operator<(const Point& p) const { return x < p.x || (x == p.x && y < p.y); }
14    bool operator==(const Point& p) const { return x == p.x && y == p.y; }
15 };
16
17 long long orientation(Point a, Point b, Point c) {
18     return (b - a).cross(c - a);
19 }
20
21 bool collinear(Point a, Point b, Point c) {
22     return orientation(a, b, c) == 0;
23 }
24
25 bool cw(Point a, Point b, Point c) {
26     return orientation(a, b, c) < 0;
27 }

```

```

9     vector<int> ids(ps.size());
10    iota(ids.begin(), ids.end(), 0);
11    vector<tuple<long long, int, int>> edges;
12    for (int rot = 0; rot < 4; rot++) { // for every rotation
13        sort(ids.begin(), ids.end(), [&](int i, int j){
14            return (ps[i].x + ps[i].y) < (ps[j].x + ps[j].y);
15        });
16        map<int, int, greater<int>> active; // (xs, id)
17        for (auto i : ids) {
18            for (auto it = active.lower_bound(ps[i].x); it != active.end();
19                 active.erase(it++)) {
20                int j = it->second;
21                if (ps[i].x - ps[i].y > ps[j].x - ps[j].y) break;
22                assert(ps[i].x >= ps[j].x && ps[i].y >= ps[j].y);
23                edges.push_back({(ps[i].x - ps[j].x) + (ps[i].y - ps[j].y), i, j});
24            }
25            active[ps[i].x] = i;
26        }
27        for (auto &p : ps) { // rotate
28            if (rot & 1) p.x *= -1;
29            else swap(p.x, p.y);
30        }
31    }
32    return edges;
33 }

```

## 9 Number Theory

### 9.1 Euler Toliente

```

1 class EulerTotiente {
2     public:
3     /* metodo en O(sqrt(n))
4     template <typename T>
5     T euler_classic(T n) {
6         T result = n;
7         for(T i = 2; i * i <= n; i++) {
8             if(n % i == 0) {
9                 while(n % i == 0) n /= i;
10                result -= result / i;
11            }
12        }
13        if(n > 1) {
14            result -= result / n;
15        }
16        return result;
17    }
18
19    /* metodo en O(nlog(log(n)))
20    void euler_faster(int n) {
21        vector<int> phi(n + 1);
22        for(int i = 0; i <= n; i++) {
23            phi[i] = i;
24        }
25        for(int i = 2; i <= n; i++) {
26            if(phi[i] == i) {
27                for(int j = i; j <= n; j += i) {
28                    phi[j] -= phi[j] / i;
29                }
30            }
31        }
32    }
33 }

```

```

54     // reverse(components.begin(), components.end()); return components; // SCC in
55     // topological order
56     return components; // SCC in reverse topological order
57 }

```

### 7.13 Topological Sort

```

1 vector<int> top_sort(vector<vector<int>>& adj){
2     int n = adj.size();
3     bool cycle = false;
4     vector<int> sorted, color(n);
5     function<void(int)> dfs = [&](int u){
6         color[u] = 1;
7         for(int v : adj[u]){
8             if(color[v] == 0 && !cycle) dfs(v);
9             else if(color[v] == 1) cycle = true;
10        }
11        color[u] = 2;
12        sorted.push_back(u);
13    };
14    for(int i = 1; i < n; i++){
15        if(color[i] == 0 && !cycle) dfs(i);
16    }
17    if(cycle){return {};}
18    reverse(sorted.begin(), sorted.end());
19    return sorted;
20 }

```

## 8 Manhattan Distance

### 8.1 Farthest Pair Of Points

```

1 long long ans = 0;
2 for (int msk = 0; msk < (1 << d); msk++) {
3     long long mx = LLONG_MIN, mn = LLONG_MAX;
4     for (int i = 0; i < n; i++) {
5         long long cur = 0;
6         for (int j = 0; j < d; j++) {
7             if (msk & (1 << j)) cur += p[i][j];
8             else cur -= p[i][j];
9         }
10        mx = max(mx, cur);
11        mn = min(mn, cur);
12    }
13    ans = max(ans, mx - mn);
14 }

```

### 8.2 Nearest Neighbor In Each Octant

```

1 // Nearest Neighbor in each Octant in O(n log n)
2 struct point {
3     long long x, y;
4 };
5
6 // Returns a list of edges in the format (weight, u, v).
7 // Passing this list to Kruskal algorithm will give the Manhattan MST.
8 vector<tuple<long long, int, int>> manhattan_mst_edges(vector<point> ps) {

```

```

26 }
27
28 bool ccw(Point a, Point b, Point c) {
29     return orientation(a, b, c) > 0;
30 }
31
32 // uso:
33 // Point p1(1, 2), p2(3, 4), p3(5, 6);
34 // long long area = abs((p2 - p1).cross(p3 - p1)) / 2; // area del triangulo
35 // bool clockwise = cw(p1, p2, p3); // true si p1->p2->p3 es clockwise
36 // bool counter_clockwise = ccw(p1, p2, p3); // true si p1->p2->p3 es counter-clockwise

```

## 7 Graph

Algoritmos de grafos: DFS, BFS, componentes fuertemente conexas, y otras estructuras de datos para problemas de grafos.

### 7.1 Bellman Ford

```

1 int n, m; cin >> n >> m;
2 vector<tuple<int, int, long long>> edges;
3 for (int i = 0; i < m; i++) {
4     int u, v; cin >> u >> v;
5     long long w; cin >> w;
6     edges.emplace_back(u, v, w);
7 }
8 vector<long long> dist(n + 1, inf);
9 dist[1] = 0;
10 for (int i = 0; i < n - 1; i++) {
11     for (auto [u, v, w] : edges) {
12         if (dist[u] != inf) {
13             dist[v] = min(dist[v], dist[u] + w);
14         }
15     }
16 }
17 bool has_negative_cycle = false;
18 for (auto [u, v, w] : edges) {
19     if (dist[u] != inf && dist[u] + w < dist[v]) {
20         has_negative_cycle = true;
21         break;
22     }
23 }
24 // dist[u] = distancia minima desde nodo 1 hasta u
25 // has_negative_cycle = true si hay ciclo negativo alcanzable desde nodo 1

```

### 7.2 Bfs

```

1 vector<bool> vis(n+1);
2 queue<int> q;
3 function<void(int)> bfs = [&](int start) {
4     vis[start] = true;
5     q.push(start);
6     int levels = 1;
7     while (!q.empty()) {
8         int sz = q.size();
9         levels++;
10        while (sz--) {
11            int u = q.front();
12            q.pop();

```

```

13     for (int& v : adj[u]) {
14         if (vis[v]) continue;
15         vis[v] = true;
16         q.push(v);
17     }
18 }
19 }
20 };
21
22 for (int u = 1; u <= n; u++) {
23     if (vis[u]) continue;
24     bfs(u);
25 }

```

### 7.3 Bipartite

```

1 int N, M; cin >> N >> M;
2 vector<vector<int>> adj(N + 1);
3 while (M--) {
4     int u, v; cin >> u >> v;
5     adj[u].push_back(v);
6     adj[v].push_back(u);
7 }
8
9 vector<bool> vis(N + 1);
10 vector<int> col(N + 1, 0);
// bipartite graph
11 function<bool(int, int)> dfs = [&](int u, int c) {
12     vis[u] = 1;
13     col[u] = c;
14
15     for (auto v : adj[u]) {
16         if (vis[v] && col[u] == col[v]) return false;
17         else if (!vis[v] && !dfs(v, c ^ 1)) return false;
18     }
19     return true;
20 };
21
22 for (int i = 1; i <= N; i++) {
23     if (vis[i]) continue;
24     if (dfs(i, 1) == false) {
25         cout << "IMPOSSIBLE";
26         return;
27     }
28 }
29
30 for (int i = 1; i <= N; i++) cout << (col[i] ? 1 : 2) << ' ';

```

### 7.4 Dfs

```

1 vector<bool> vis(n+1);
2 function<void(int)> dfs = [&](int u) {
3     vis[u] = true;
4     for (int& v : adj[u]) {
5         if (vis[v]) continue;
6         dfs(v);
7     }
8 };
9
10 for (int u = 1; u <= n; u++) {

```

```

23 }
24 // mst_cost = costo del MST (Minimum Spanning Tree)

```

### 7.12 Scc

Algoritmo de Tarjan para encontrar componentes fuertemente conexas (SCC) en un grafo dirigido.

```

1 // "These works to find a componente fuertemente conexa that it's in directed graph"
2 struct SCC {
3     int N = 0, id;
4     vector<vector<int>> adj;
5     vector<int> ind, low;
6     stack<int> s;
7     vector<bool> in_stack;
8     vector<vector<int>> components;
9     vector<int> component_id;
10
11 //1-indexed
12 SCC(int n = 0){ N = n + 1, adj.assign(N, {}); }
13 SCC(const vector<vector<int>> & _adj){ adj = _adj, N = adj.size(); }
14
15 void add_edge(int from, int to){
16     adj[from].push_back(to);
17 }
18
19 void dfs(int u){
20     low[u] = ind[u] = id++;
21     s.push(u);
22     in_stack[u] = true;
23     for(int v : adj[u]){
24         if(ind[v] == -1){
25             dfs(v);
26             low[u] = min(low[u], low[v]);
27         }else if(in_stack[v]){
28             low[u] = min(low[u], ind[v]);
29         }
30     }
31     if(low[u] == ind[u]){
32         components.emplace_back();
33         vector<int> & comp = components.back();
34         while(true){
35             assert(!s.empty());
36             int x = s.top(); s.pop();
37             in_stack[x] = false;
38             component_id[x] = components.size() - 1;
39             comp.push_back(x);
40             if(x == u) break;
41         }
42     }
43 }
44
45 vector<vector<int>> get(){
46     ind.assign(N, -1); low.assign(N, -1); component_id.assign(N, -1);
47     s = stack<int>();
48     in_stack.assign(N, false);
49     id = 0;
50     components = {};
51     for(int i = 1; i < N; i++)
52         if(ind[i] == -1) dfs(i);
53 }

```

```

28     for(int v : adj[u]){
29         if(v == p) continue;
30         dfs(v, u);
31     }
32     out[u] = ++timer;
33 }
34
35 bool is_ancestor(int p, int u){
36     return in[p] <= in[u] && out[p] >= out[u];
37 }
38
39 int query(int u, int v){
40     if(is_ancestor(u, v)) return u;
41     if(is_ancestor(v, u)) return v;
42
43     for(int bit = l; bit >= 0; bit--){
44         if(is_ancestor(up[u][bit], v)) continue;
45         u = up[u][bit];
46     }
47     return up[u][0];
48 }
49
50 int ancestor(int u, int k){
51     if(depth[u] <= k) return -1;
52     for(int bit = 0; bit <= l; bit++){
53         if(k >> bit & 1) u = up[u][bit];
54     }
55     return u;
56 }
57
58 int distance(int u, int v){
59     return depth[u] + depth[v] - 2 * depth[query(u, v)];
60 }
61 };

```

## 7.11 Prim

```

1 int n, m; cin >> n >> m;
2 vector<vector<pair<int, long long>>> adj(n + 1);
3 for (int i = 0; i < m; i++) {
4     int u, v; cin >> u >> v;
5     long long w; cin >> w;
6     adj[u].emplace_back(v, w);
7     adj[v].emplace_back(u, w);
8 }
9 vector<bool> vis(n + 1);
10 pqg<pair<long long, int>> pq;
11 pq.push({OLL, 1});
12 long long mst_cost = 0;
13 while (!pq.empty()) {
14     auto [w, u] = pq.top(); pq.pop();
15     if (vis[u]) continue;
16     vis[u] = true;
17     mst_cost += w;
18     for (auto [v, weight] : adj[u]) {
19         if (!vis[v]) {
20             pq.push({weight, v});
21         }
22     }
23 }

```

```

11     if (vis[u]) continue;
12     dfs(u);
13 }

```

## 7.5 Dfs 2D

```

1 int N, M; cin >> N >> M;
2 vector<vector<char>> grid(N, vector<char>(M));
3 for (int i = 0; i < N; i++) {
4     for (int j = 0; j < M; j++) {
5         cin >> grid[i][j];
6     }
7 }
8
9 vector<vector<bool>> vis(N, vector<bool>(M));
10 int dx[4] = {-1, 1, 0, 0}, dy[4] = {0, 0, -1, 1};
11 function<void(int, int)> dfs = [&](int x, int y) {
12     vis[x][y] = 1;
13
14     for (int d = 0; d < 4; d++) {
15         int nx = x + dx[d], ny = y + dy[d];
16         if (0 <= nx && 0 <= ny && nx < N && ny < M && grid[nx][ny] == '.' && !vis[nx][ny]) {
17             dfs(nx, ny);
18         }
19     }
20 };
21
22 int comp = 0;
23 for (int i = 0; i < N; i++) {
24     for (int j = 0; j < M; j++) {
25         if (vis[i][j] || grid[i][j] == '#') continue;
26         dfs(i, j);
27         comp++;
28     }
29 }
30
31 cout << comp;

```

## 7.6 Dijkstra

```

1 int N, M; cin >> N >> M;
2 vector<vector<pair<int, long long>>> adj(N + 1);
3 for (int i = 0; i < M; i++) {
4     int u, v; cin >> u >> v;
5     long long w; cin >> w;
6     adj[u].emplace_back(v, w);
7 }
8 vector<long long> dis(N + 1, inf);
9 pqg<pair<long long, int>> pq;
10 dis[1] = 0;
11 pq.push({OLL, 1});
12 while (!pq.empty()) {
13     auto [d, node] = pq.top(); pq.pop();
14     if (dis[node] != d) continue;
15     for (auto [v, w] : adj[node]) {
16         if (d + w < dis[v]) {
17             dis[v] = d + w;
18             pq.push({dis[v], v});
19         }
20     }
21 }

```

```

21 }
22 for (int u = 1; u <= N; u++) {
23     cout << dis[u] << " ";
24 }

```

## 7.7 Disjoint Set Union Dsu

```

1 struct DSU {
2     vector<int> p, size;
3     DSU(int n){
4         p.resize(n + 1), size.resize(n + 1, 1);
5         for(int i = 1; i <= n; i++) p[i] = i;
6     }
7
8     int find(int x){
9         if(p[x] != x) p[x] = find(p[x]);
10        return p[x];
11    }
12
13    void merge(int x, int y){
14        x = find(x), y = find(y);
15        if(x == y) return;
16        if(size[x] < size[y]) swap(x, y);
17        size[x] += size[y];
18        p[y] = x;
19    }
20};

```

## 7.8 Floyd Warshall

```

1 int n; cin >> n;
2 vector<vector<long long>> dist(n + 1, vector<long long>(n + 1, inf));
3 for (int i = 1; i <= n; i++) dist[i][i] = 0;
4 int m; cin >> m;
5 for (int i = 0; i < m; i++) {
6     int u, v; cin >> u >> v;
7     long long w; cin >> w;
8     dist[u][v] = min(dist[u][v], w);
9     dist[v][u] = min(dist[v][u], w);
10}
11 for (int k = 1; k <= n; k++) {
12     for (int i = 1; i <= n; i++) {
13         for (int j = 1; j <= n; j++) {
14             if (dist[i][k] != inf && dist[k][j] != inf) {
15                 dist[i][j] = min(dist[i][j], dist[i][k] + dist[k][j]);
16             }
17         }
18     }
19 }
20 // dist[i][j] = distancia minima entre nodo i y nodo j

```

## 7.9 Kruskal

```

1 struct DSU {
2     vector<int> p, size;
3     DSU(int n) {
4         p.resize(n + 1), size.resize(n + 1, 1);
5         for (int i = 1; i <= n; i++) p[i] = i;
6     }
7     int find(int x) {

```

```

8         if (p[x] != x) p[x] = find(p[x]);
9         return p[x];
10    }
11    void merge(int x, int y) {
12        x = find(x), y = find(y);
13        if (x == y) return;
14        if (size[x] < size[y]) swap(x, y);
15        size[x] += size[y];
16        p[y] = x;
17    }
18 }
19
20 int n, m; cin >> n >> m;
21 vector<tuple<long long, int, int>> edges;
22 for (int i = 0; i < m; i++) {
23     int u, v; cin >> u >> v;
24     long long w; cin >> w;
25     edges.emplace_back(w, u, v);
26 }
27 sort(edges.begin(), edges.end());
28 DSU dsu(n);
29 long long mst_cost = 0;
30 for (auto [w, u, v] : edges) {
31     if (dsu.find(u) != dsu.find(v)) {
32         dsu.merge(u, v);
33         mst_cost += w;
34     }
35 }
36 // mst_cost = costo del MST (Minimum Spanning Tree)

```

## 7.10 Lowest Common Ancestor Lca

```

1 struct LCA{
2     int n, l, timer = 0;
3     vector<vector<int>> up, adj;
4     vector<int> depth, in, out;
5
6     LCA(int _n) {
7         n = _n + 1;
8         l = ceil(log2(n));
9         up.resize(n, vector<int>(l + 1));
10        adj.resize(n);
11        depth.resize(n);
12        in.resize(n);
13        out.resize(n);
14    }
15
16    void add_edge(int p, int u){
17        adj[p].push_back(u);
18        adj[u].push_back(p);
19    }
20
21    void dfs(int u = 1, int p = 1){
22        up[u][0] = p;
23        depth[u] = depth[p] + 1;
24        in[u] = ++timer;
25        for(int level = 1; level <= l; level++){
26            up[u][level] = up[up[u][level - 1]][level - 1];
27        }

```