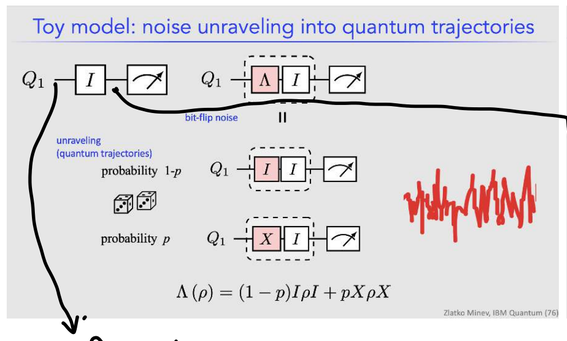
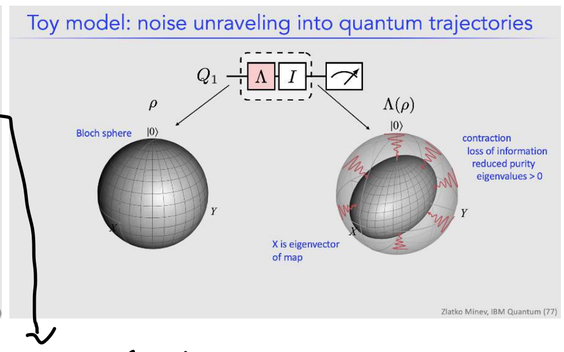
QGSS22 Incoherent noise

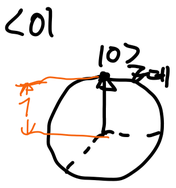
Wednesday, June 1, 2022 12:39 PM



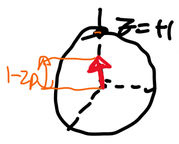


$\rho_0 = |0\rangle\langle 0|$

$\langle X\rangle = 0$
$\langle Y\rangle = 0$
$\langle Z\rangle = 1$



$\rho = (1-p)I|0\rangle\langle 0|I + pX\rho X$



$= (1-p)|0\rangle\langle 0| + p|1\rangle\langle 1|$

$= \begin{pmatrix} 1-p & 0 \\ 0 & p \end{pmatrix} = (\cdot\cdot)I + (\cdot\cdot)Z$

$\rho_0 = |0\rangle\langle 0| = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \frac{I+Z}{2}$

$\langle X\rangle = \mathrm{Tr}(X\rho_0)$
$= \frac{1}{2}\mathrm{Tr}(X(I+Z))$
$= \frac{1}{2}\mathrm{Tr}(X + X\cdot Z)$

$\mathrm{Tr}(X) = \mathrm{Tr}(Y) = \mathrm{Tr}(Z) = 0$
$\mathrm{Tr}(I) = 2$

$\langle X\rangle = \frac{1}{2}\{\mathrm{Tr}(X) + \mathrm{Tr}(X\cdot Z)\}$
$X\cdot Z = Y$
$= 0$

$\langle Y\rangle = 0$

$\langle Z\rangle = \mathrm{Tr}(Z\rho_0)$
$= \frac{1}{2}\mathrm{Tr}(Z + Z\cdot Z)$
$= \frac{1}{2}[\mathrm{Tr}(Z) + \mathrm{Tr}(I)]$
$= 2/2 = 1$

$\mathrm{Tr}(\rho) = 1$

$\mathrm{Tr}(\rho^2) = \mathrm{Tr}\begin{pmatrix} (1-p)^2 & \\ & p^2 \end{pmatrix} = (1-p)^2 + p^2$
$= 1 - 2p + p^2 + p^2$
$= 1 - 2p + 2p^2$
$= 1 + 2p(p-1)$
$= \begin{cases} 1 & \text{if } p = 0 \\ 1 & \text{if } p = 1 \end{cases}$

$\langle X\rangle = ? = 0$
$\langle Y\rangle = ? = 0$
$\langle Z\rangle = \mathrm{Tr}(Z\cdot\begin{pmatrix} 1-p & 0 \\ 0 & p \end{pmatrix})$
$= \mathrm{Tr}\left(\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\cdot\begin{pmatrix} 1-p & 0 \\ 0 & p \end{pmatrix}\right)$
$= \mathrm{Tr}\begin{pmatrix} 1-p & \\ & -p \end{pmatrix}$
$= 1 - 2p$