Projection noise

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Introduction to quantum noise

Measurement theory & projection noise

Qiskit Global Summer School on Quantum Machine Learning

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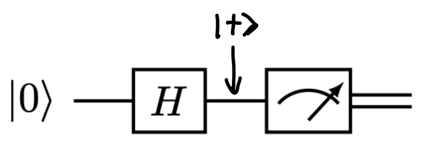
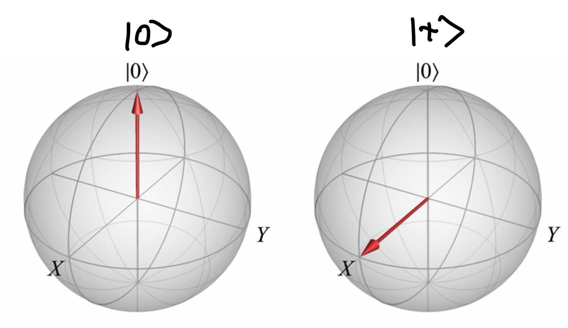


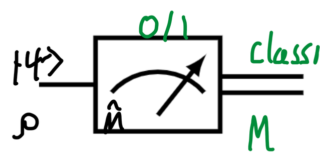


Measurement: theory 101

The standard (von Neumann) measurement of a quantum system.

von Neumann measurement is efficient, strong, and projective

Measurement operator (observable) $\hat{M}$



$m \leftarrow$ outcome

$m=0 \rightarrow |0>$

$m=1 \rightarrow |1>$

Is the qubit in $|1>$?

$$\hat{M} = |1><1| = \begin{array}{c|cc} & <0| & <1| \\ \hline |0> & 0 & 0 \\ |1> & 0 & 1 \end{array}$$

$$= 0|0><0| + 1|1><1|$$

$$= \sum_m m|m><m|$$

$m \in \{0,1\} \leftarrow$ possible outcomes

$|m> \in \{|0>, |1>\}$

$$\hat{M} = \sum_m m\hat{\Pi}_m$$

$$\hat{\Pi}_m = |m><m|$$

$$\hat{I} = \sum_m \hat{\Pi}_m \quad \text{(resolution)}$$

Probability of outcome

$$p(m=0) = <|0><0|>$$

$$p(m=1) = <|1><1|>$$

$$\hat{\Pi}_0 = |0><0|$$

$$\hat{\Pi}_1 = |1><1|$$

$$\hat{\Pi}_0 + \hat{\Pi}_1 = |0><0| + |1><1|$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \hat{I}$$

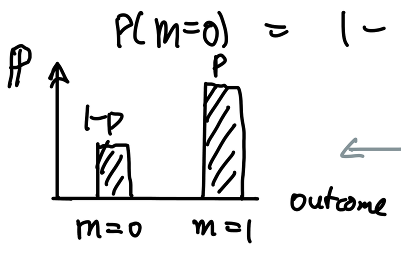
$$p(0) + p(1) = 1$$

$$P(m=0) = <\psi|0><0|\psi> = |<0|\psi>|^2$$

$$P(m=1) = <\psi|1><1|\psi> = |<1|\psi>|^2 = p$$

$$P(0) + P(1) = |<0|\psi>|^2 + |<1|\psi>|^2 = ||\psi>|^2 = |<\psi|\psi>| = 1$$

$$P(m=0) = 1 - P(1)$$



Bernoulli Distribution

$$\mathbb{P}(M) \sim \mathcal{B}(p)$$

probability ↑ ↖ classical random variable corresp: measurement outcome ↖ parameter

Statistics

$$\mathbb{E}[M] = \sum_m m\,\mathbb{P}(m) = 0\,\mathbb{P}(m=0) + 1\,\mathbb{P}(m=1)$$

classical ↑ classical ↑

$$= \mathbb{P}(m=1)$$

$$= p$$

$$= \sum_m m<\hat{\Pi}_m>$$

$$= \sum_m m<|m><m|>$$

$$= <\sum_m m|m><m|>$$

$$= <\hat{M}>$$

Quantum

$$\mathbb{V}[M] = \mathbb{E}[M^2] - \underline{\mathbb{E}[M]^2}_{\swarrow p^2} = <\hat{M}^2> - <\hat{M}>^2$$

$$\mathbb{E}[M^2] = \sum_m m^2\,\mathbb{P}(m)$$

$$= \sum_m m^2<|m><m|>$$

$$= <\sum_m m^2|m><m|>$$

$$= <\hat{M}^2>$$

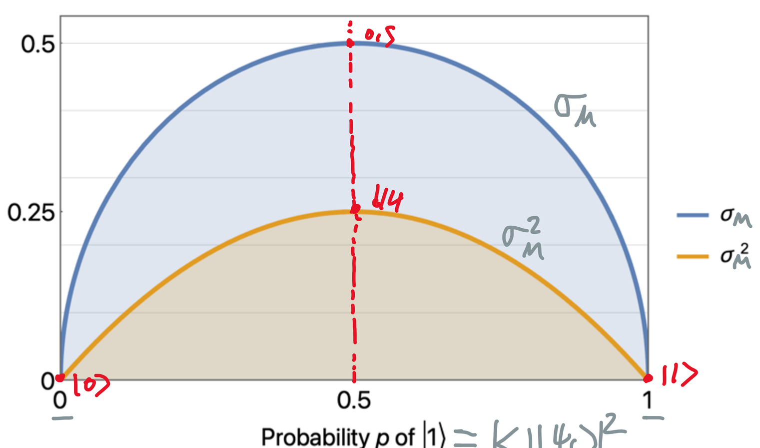
$$= \cancel{0^2(1-p)} + 1^2 p$$

$$= p$$

$$\mathbb{V}[M] = p - p^2$$

$$= p(1-p) \qquad = 0 \text{ if } p=0$$

$$= \sigma_M^2 \qquad = 0 \text{ if } p=1$$



$= |<1|\psi>|^2$

$= |<1|U_{alg}|0>|^2$

↑ outcome ↑ algo ↑ initial

Random coin toss

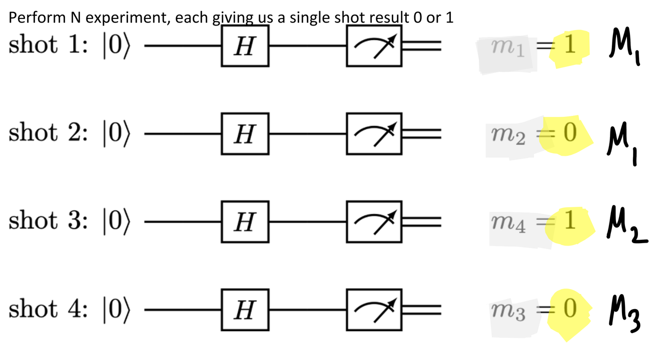
Bernoulli distribution

Parameters determined by quantum algorithm

Projection noise and sampling error

Let's turn to the example of finite number of shots we execute for our experiment.

Perform N experiment, each giving us a single shot result 0 or 1



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For 3 samples, there are 2^3 possible outcome sequences. i.d.d. $M \sim M_n$

m_1 m_2 m_3

000
001
010
011
100
101
110
111

Sample mean

$$S := \frac{1}{N}\sum_{n=1}^{N} M_n = \frac{1}{3}(0+0+1) = \frac{1}{3}$$

↖ random variable ↖ #shots

001 →

Calculation worked out in bonus section

$$= \frac{1}{3}(1+0+1) = 2/3$$

$$\mathbb{E}[S] = \mathbb{E}[\frac{1}{N}\sum_{n=1}^{N} M_n]$$

$$= 1/N \sum_{n=1}^{N} \mathbb{E}[M_n] \quad \text{i.i.d.}$$

$$= \cancel{1/N\,N}\ \mathbb{E}[M]$$

$$= \mathbb{E}[M]$$

$$= <\hat{M}>$$

$$\mathbb{E}[aM] = a\,\mathbb{E}[M]$$

$$\mathbb{E}[aM_1 + bM_2] = a\,\mathbb{E}[M_1] + b\,\mathbb{E}[M_2]$$

$= p$ for a qubit

$$\mathbb{V}[S] = \mathbb{V}\left[\tfrac{1}{N}\sum_{n=1}^{N} M_n\right]$$

$\mathrm{cov}[M_n, M_{n'}] = 0$ for $n \neq n'$

$$= \frac{1}{N^2}\sum_{n=1}^{N} \underbrace{\mathbb{V}[M_n]}_{\mathbb{V}[M]}$$

$$= \frac{p(1-p)}{N} \simeq \frac{\mathbb{V}[M]}{N}$$

Recall:

$\mathbb{V}[aX] = a^2\,\mathbb{V}[X]$

$\mathbb{V}[aX + bY] = a^2\,\mathbb{V}[X] + b^2\,\mathbb{V}[Y] + 2ab\,\mathrm{Cov}[X,Y]$ ← zero

$\mathrm{Cov}[X,Y] = \mathbb{E}[XY] - \mathbb{E}[X]\,\mathbb{E}[Y]$

error bar on expectation value $\sigma_S = \sqrt{\frac{p(1-p)}{N}}$ ← quantum projection noise

← #shots finite sampling noise