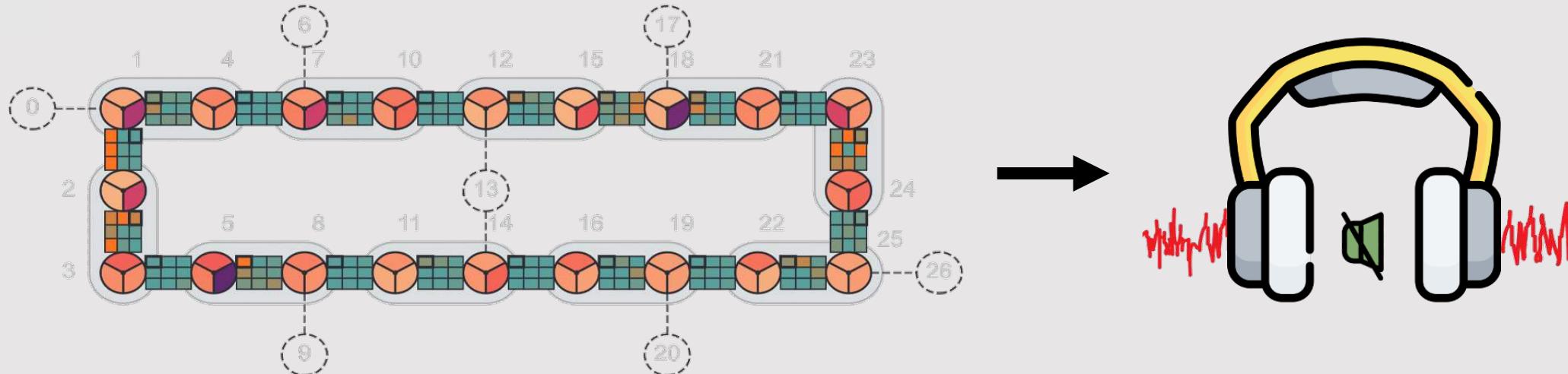


Got Slides?



# To learn and cancel quantum noise

Probabilistic error cancellation with  
sparse Pauli-Lindblad models on  
noisy quantum processors



Zlatko K. Minev

Ewout van den Berg, Zlatko K. Minev, Abhinav Kandala, Kristan Temme  
arXiv:2201.09866 (2022)



Acknowledgements: Broader IBM Quantum team



@zlatko\_minev



zlatko-minev.com

# Biggest challenge?

Please do share

# Biggest challenge?

hardware  
development

error correction  
overheads

scalability

engineering

need CS/EE  
talent

decoherence

high error rates

material  
quality

## Noise (Errors)

loss

heat

stability

algo  
development

importance of  
N in NISQ

gravity

modularization

hype

expectations

# Biggest challenge

---

Noise  
(Errors)



How to deal with noise?

# How to deal with errors due to noise?

Monitor

Error occurs  
Error detect



Quantum error correction

Shor, PRA (1995), ...

Monitor

Error anticipated  
Tell signal detected



Catch and reverse

Minev, Nature (2019), ...

No monitor

Error occurs  
Error undetected



Error mitigation

... subject of today

# Cancel quantum noise



# High-level message

## Learn

accurate, efficient, scalable



## Cancel

noise with noise,  
practical



## Cost

more noise more cost



# Outline



Idea

Probabilistic error cancelation (PEC) idea

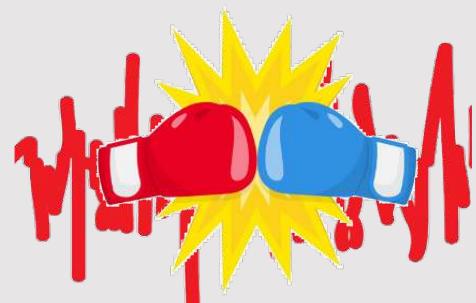
Why not possible at scale until now?



Learn

Challenge

Sparse Pauli-Lindblad model in experiment



Cancel  
(realization)

Mitigation in practice with Ising

Cost



# Outline

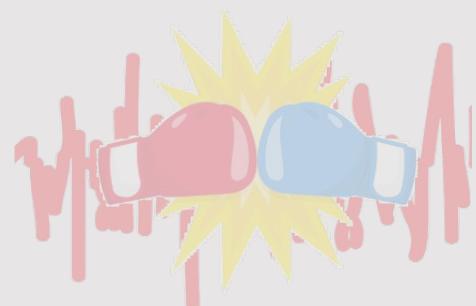
---



Idea

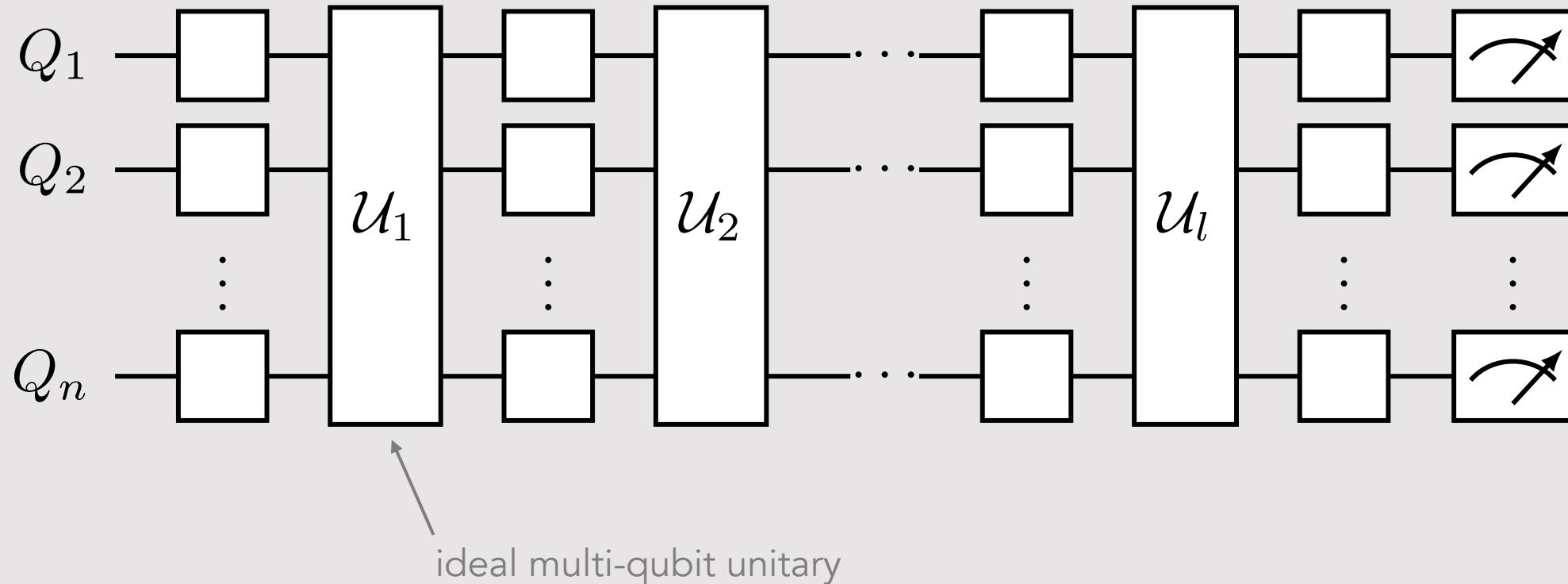


Learn



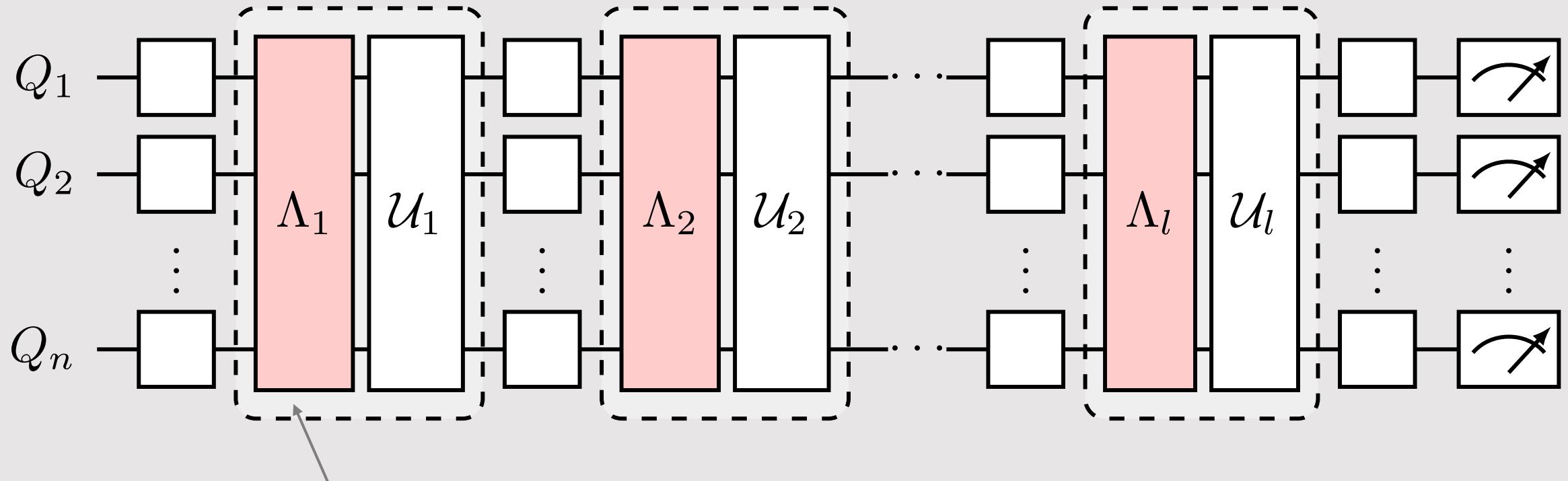
Cancel  
(realization)

# Ideal (noise-free) quantum circuit



A circuit can be decomposed into a layer construction  
Example: Trotterization of Ising model simulation

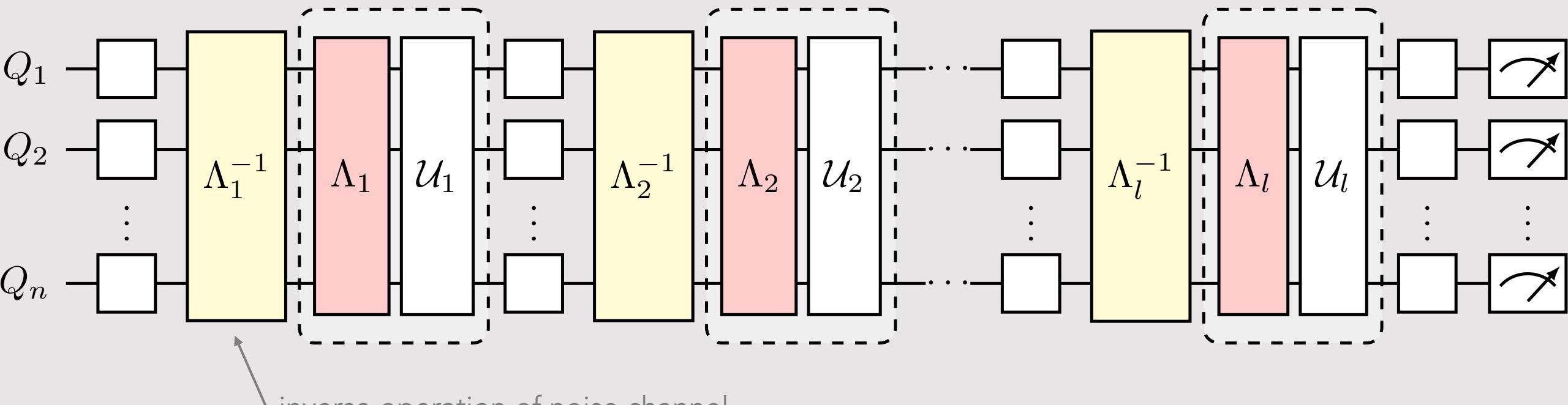
# Real (noisy) quantum circuit



multi-qubit noise channel  
inseparable from gate

completely positive and trace preserving (CPTP)  
representable by a  $4^n \times 4^n$  matrix

# Why not invert noise?

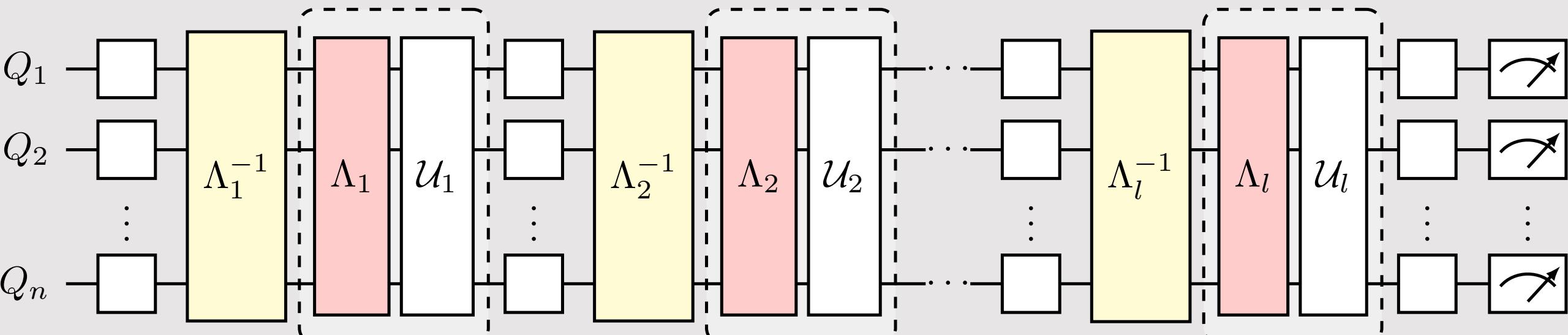


Not possible?

inverse operation of noise channel  
unphysical  
would need to know lost information due to noise  
non CPTP map  
has negative eigenvalues

...

# Probabilistic error cancellation

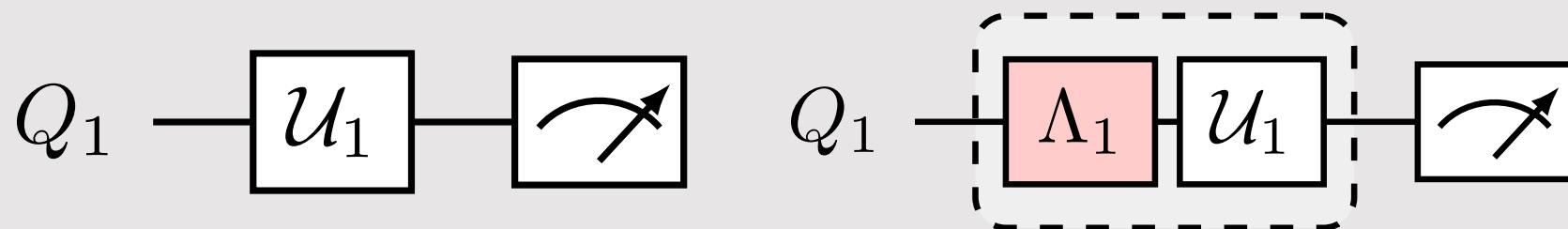


inverse operation of noise channel  
implement on average

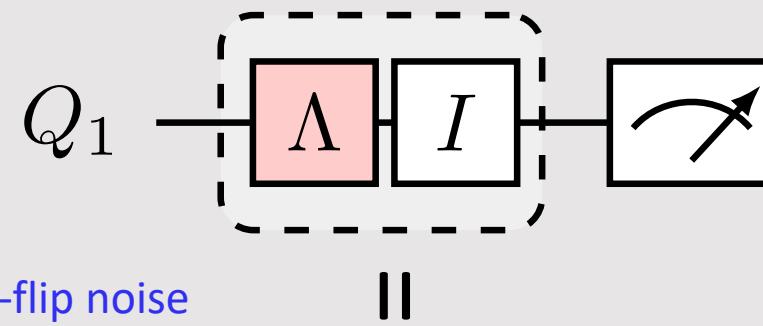
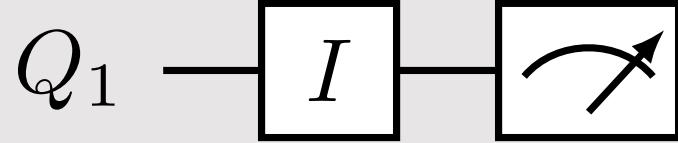
**K. Temme, S. Bravyi, and J. M. Gambetta**  
PRL 119, 180509 (2017)

See also S. Endo, S. Benjamin, and Y. Li  
Phys. Rev. X 8, 031027 (2018)

# Toy model

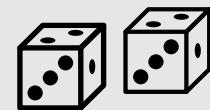


# Toy model: noise unraveling into quantum trajectories

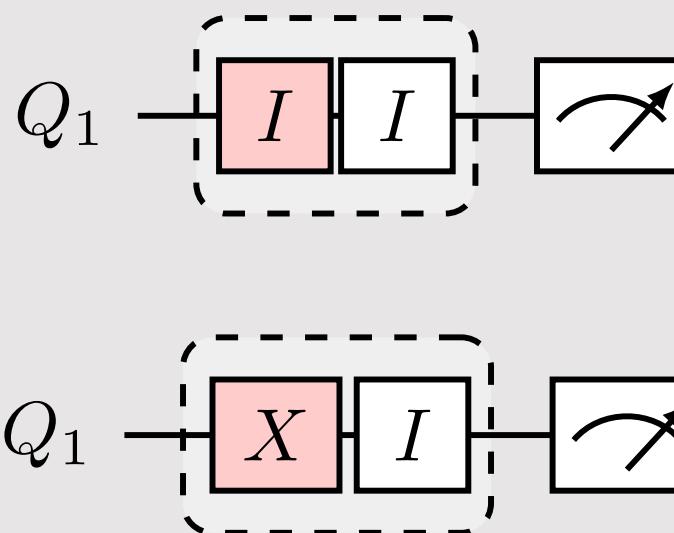


unraveling  
(quantum trajectories)

probability  $1-p$

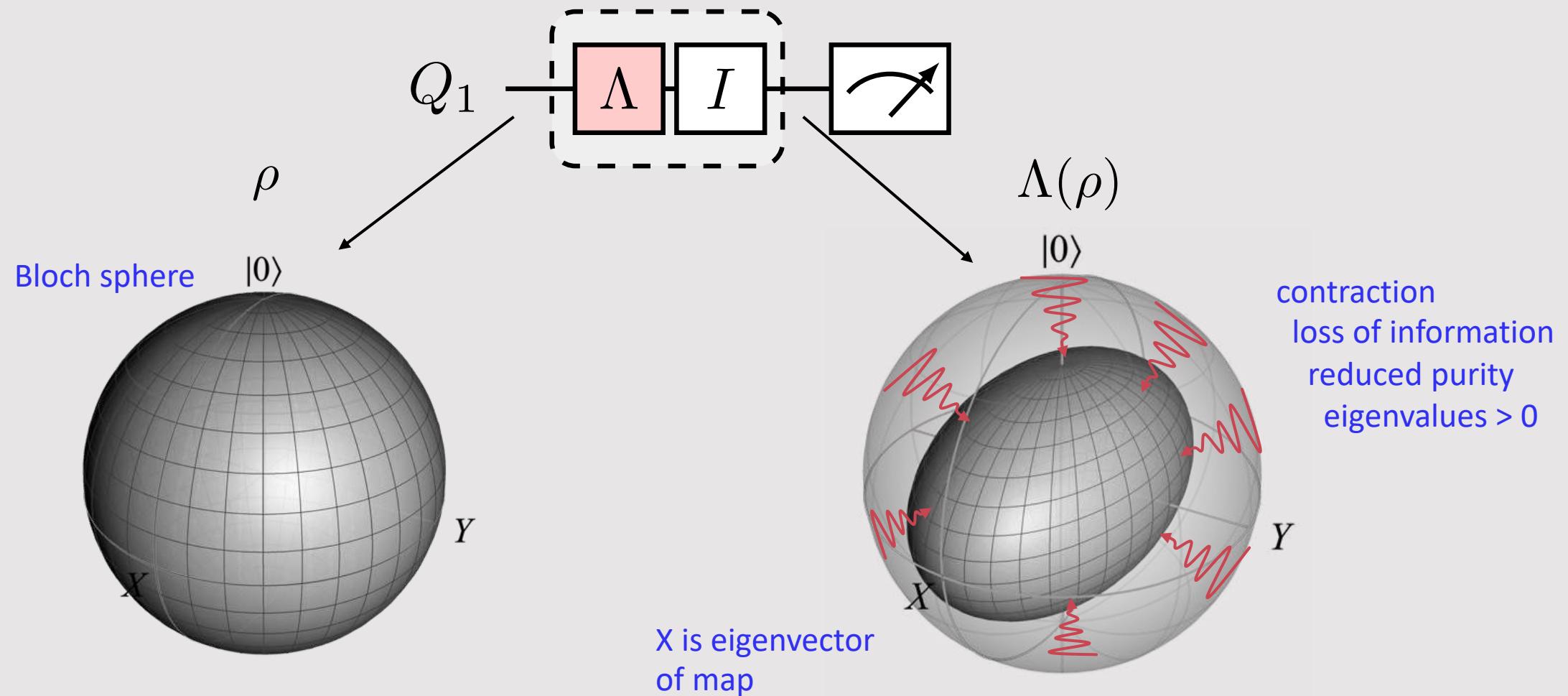


probability  $p$

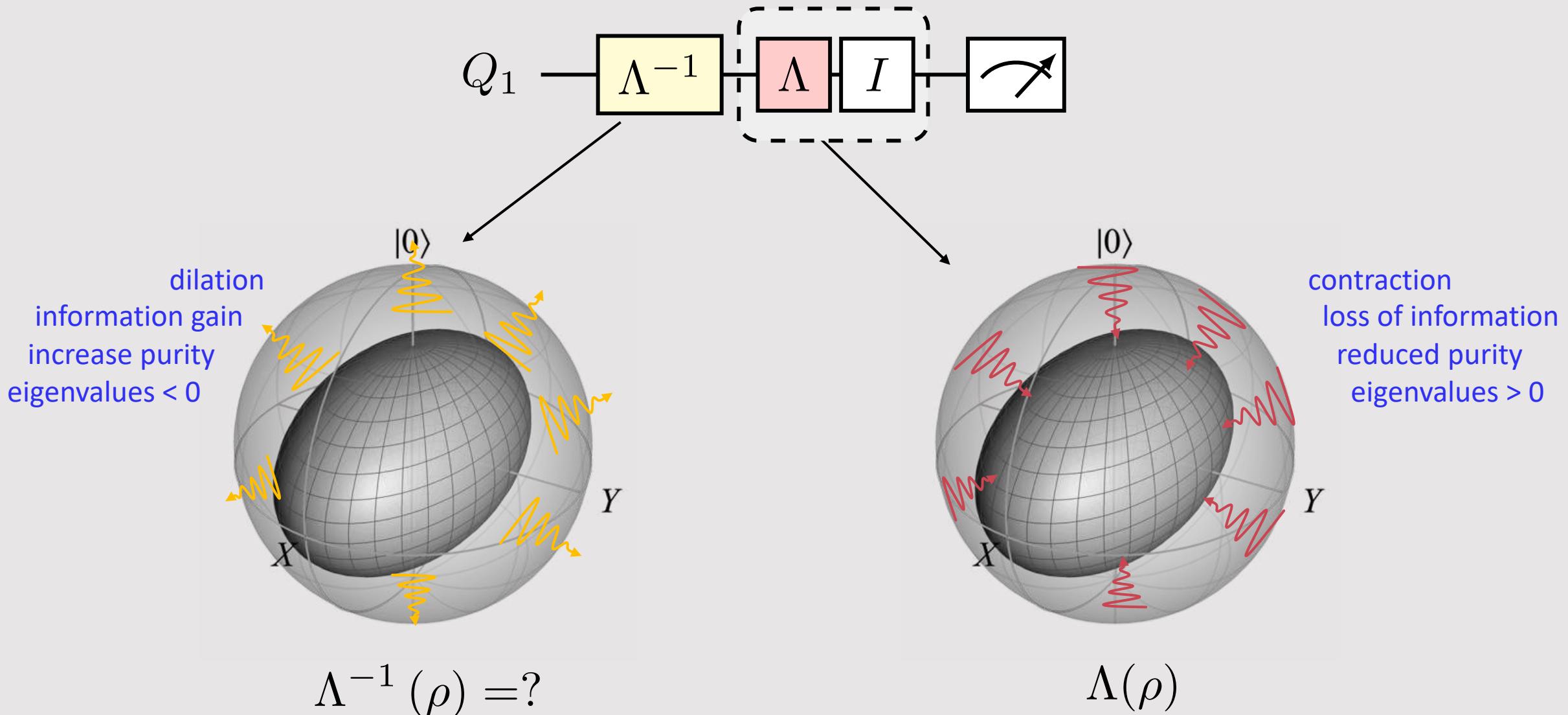


$$\Lambda(\rho) = (1 - p)I\rho I + pX\rho X$$

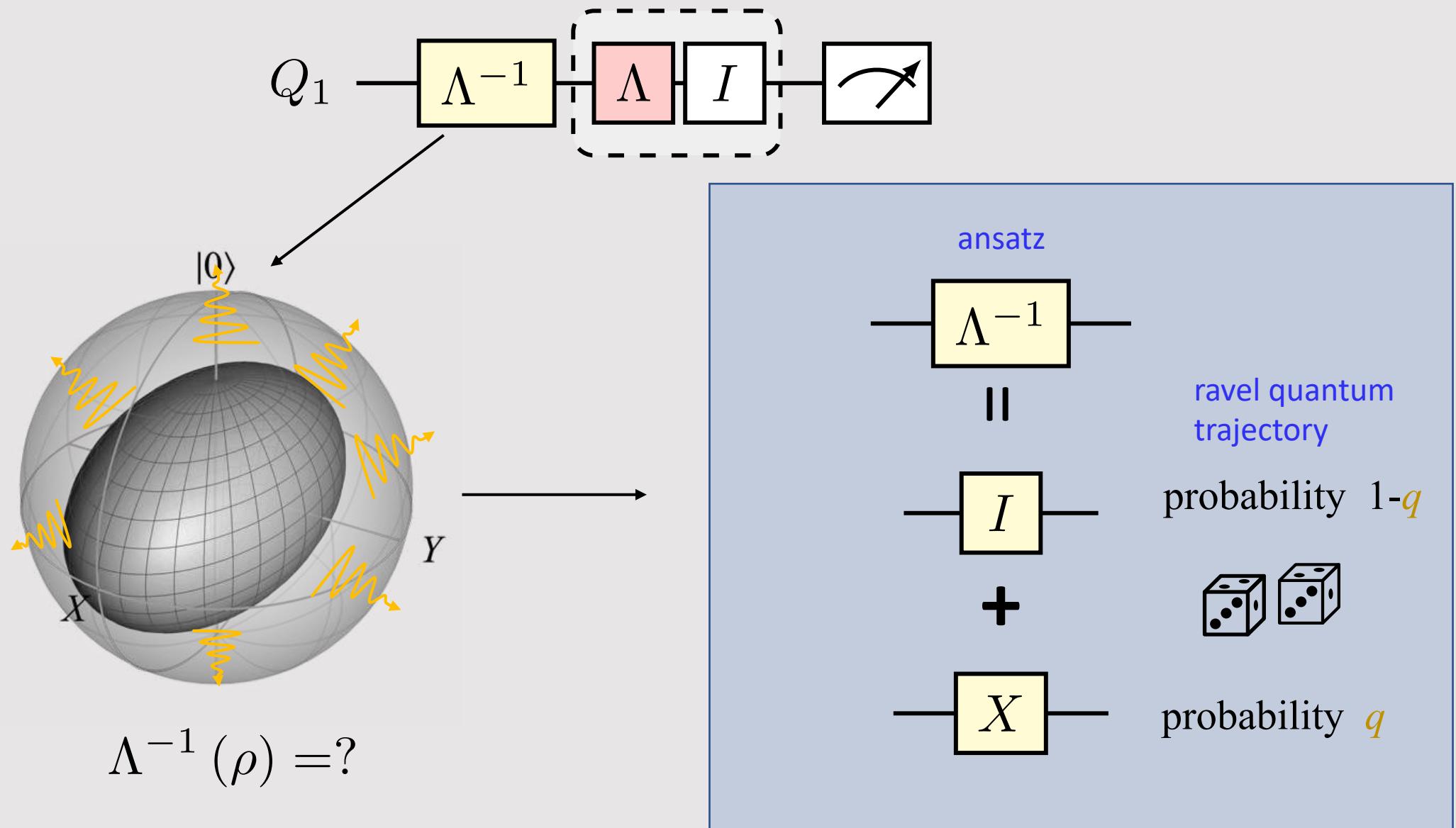
# Toy model: noise unraveling into quantum trajectories



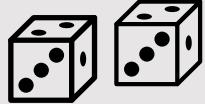
# Inverse of noise map is not physical

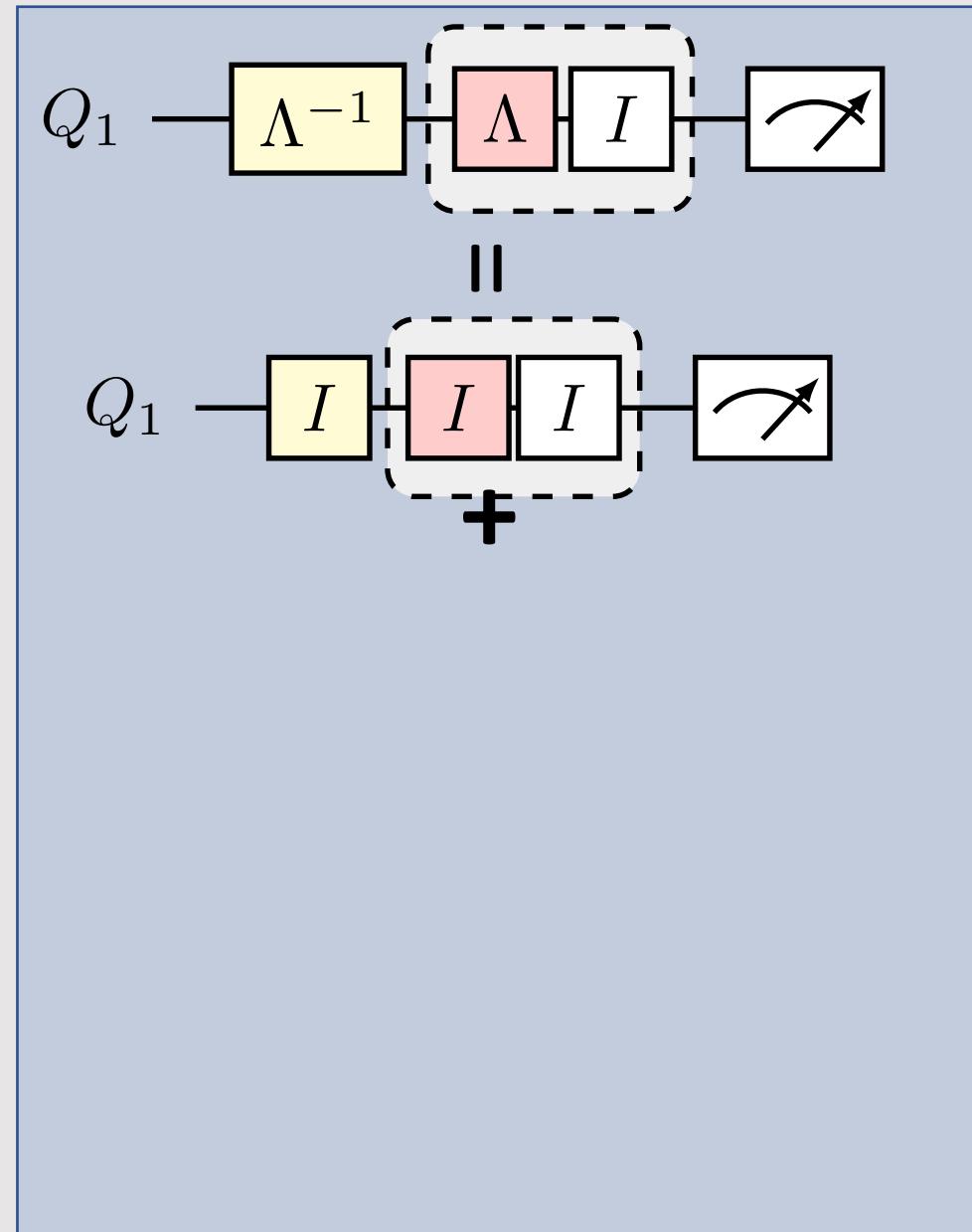


# Inverse of noise map is not physical



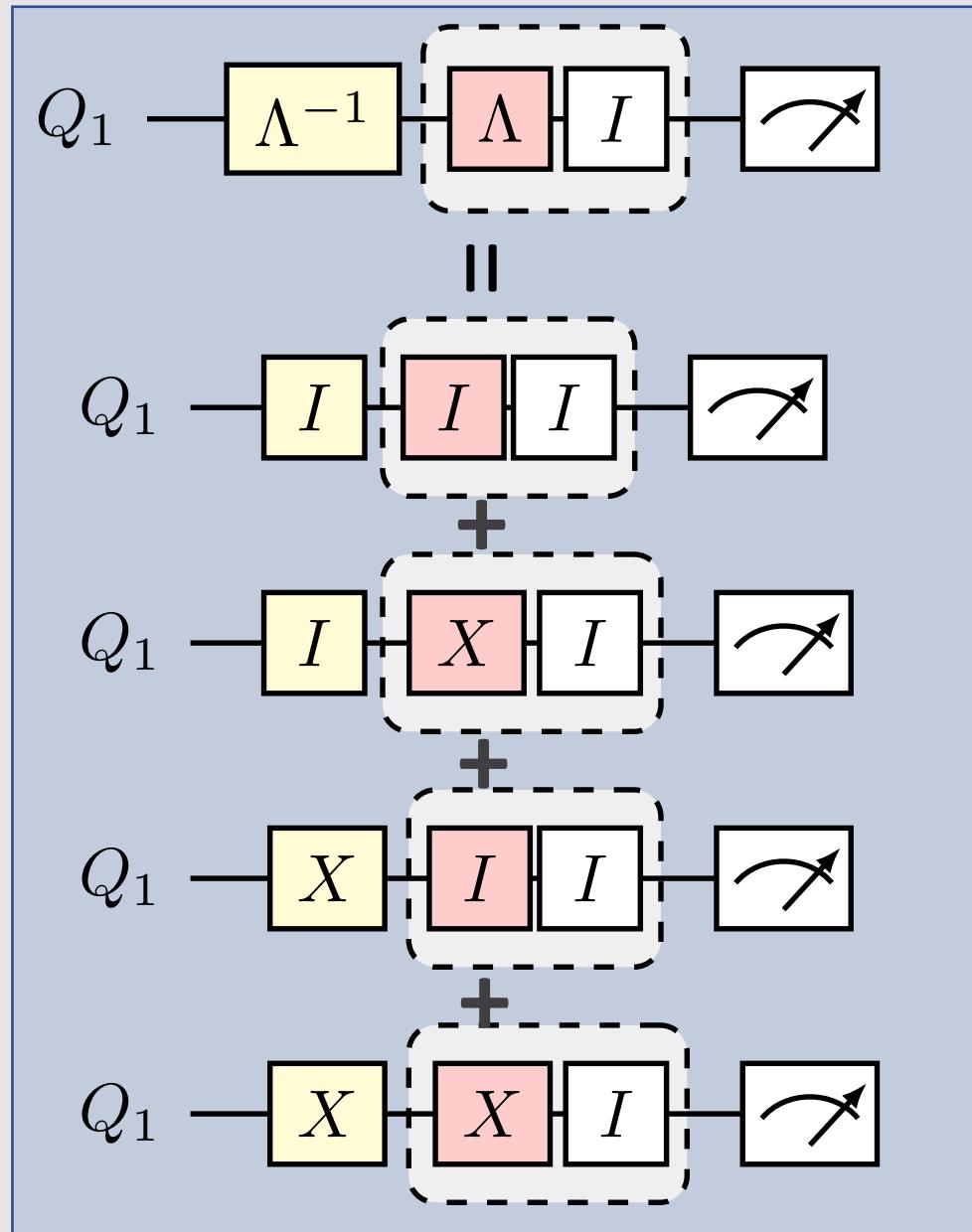
# Raveling quantum trajectories to undo noise

No error      probability  
 $(1-q)(1-p)$   




# Raveling quantum trajectories to undo noise

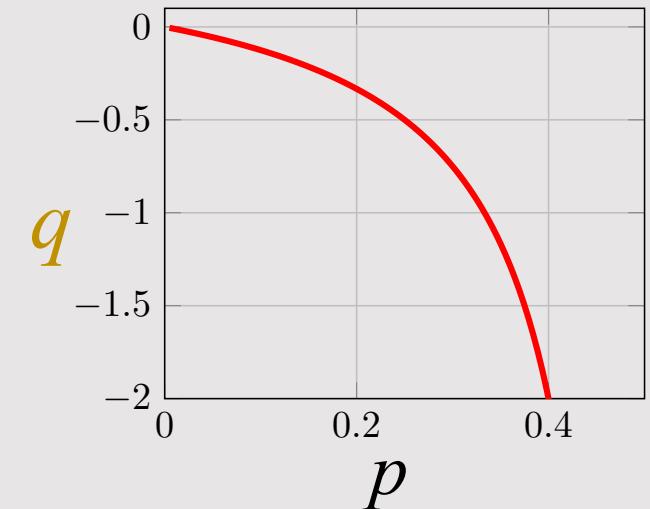
No error	probability $(1-q)(1-p)$	
ERROR!	$(1-q)p$	
ERROR!	$q(1-p)$	
Error CANCELED!	$qp$	



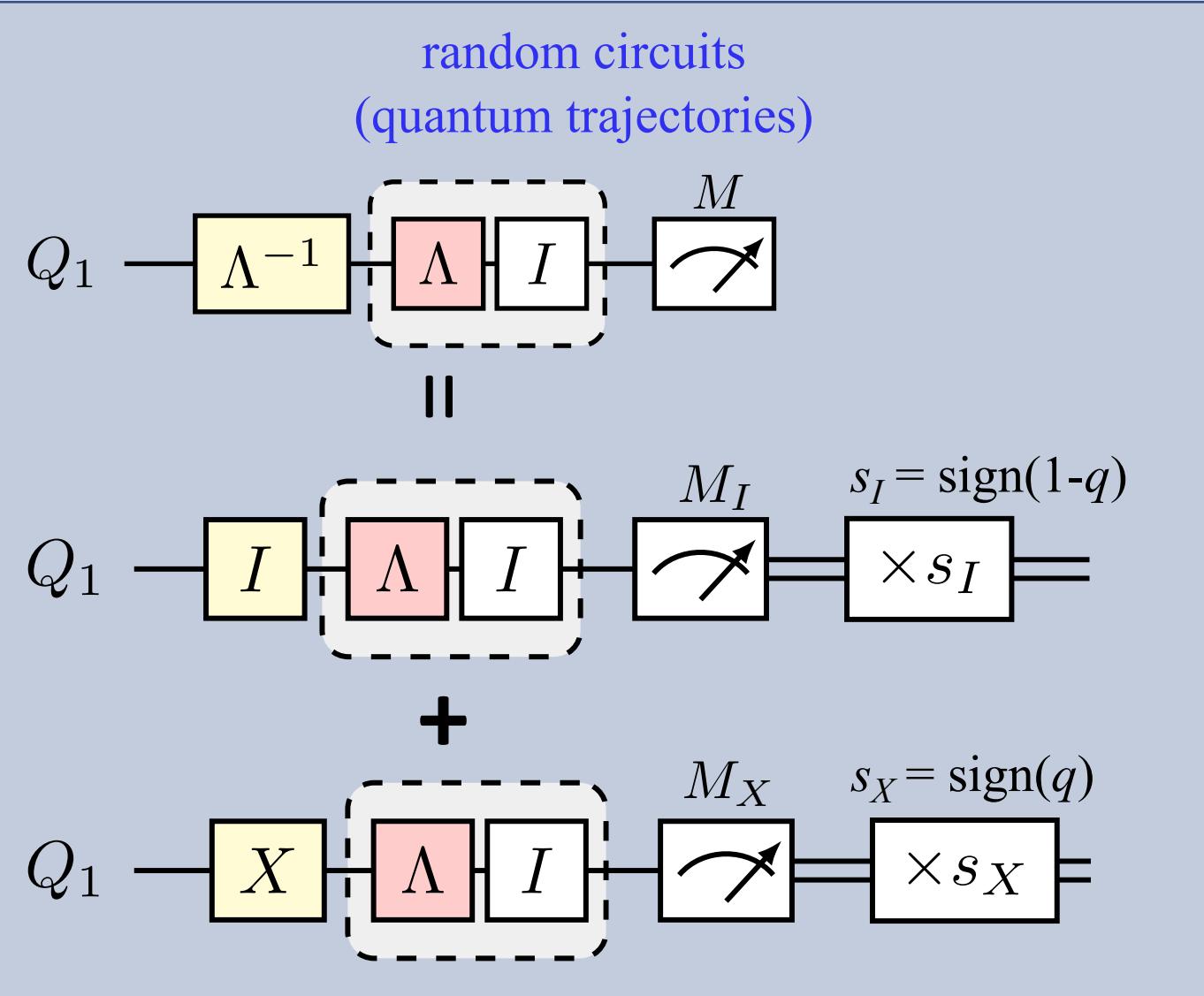
Solution to noise free!

$$q = \frac{-p}{1 - 2p}$$

Sign & scale:  
quasi-probability

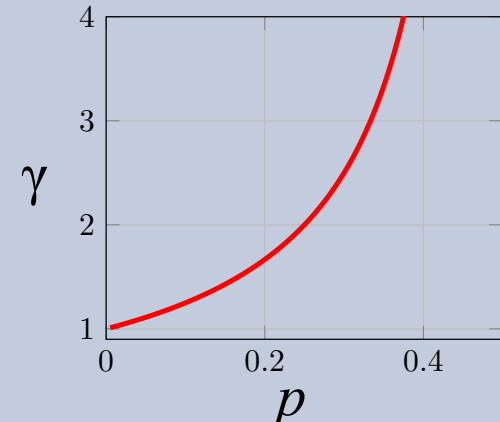


# How to implement?



sampling overhead

$$\gamma = |1-q| + |q|$$



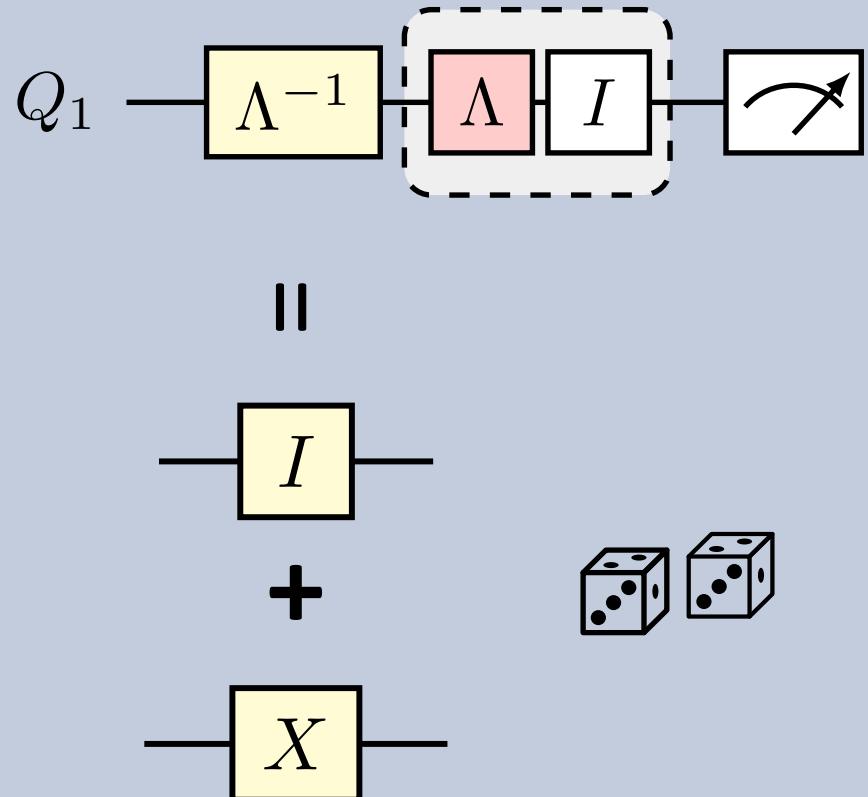
mitigated expectation

$$\langle M \rangle = \gamma(s_I P_I M_I + s_X P_X M_X)$$

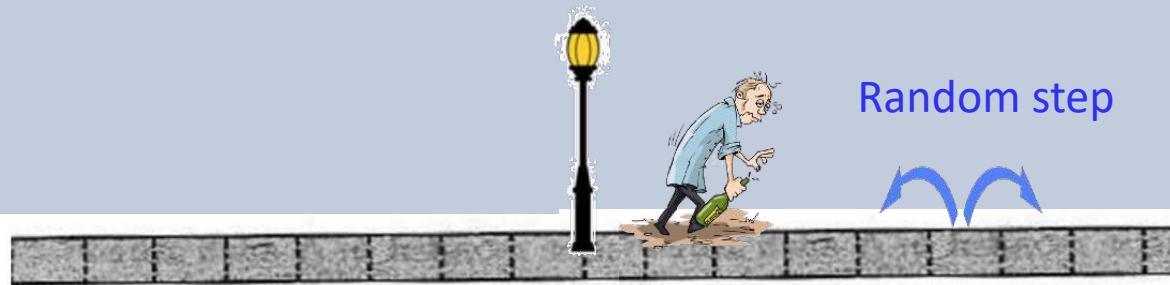
Gain: Bias-free estimate!

Cost: Variance

# Cancelling noise with noise



# Cancelling noise with noise: Drunkard's classical random walk analogy



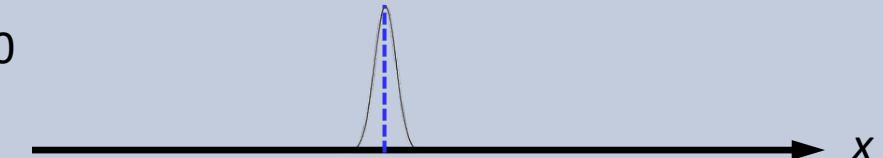
Random step

$$P(1 \text{ step left}) = \frac{1}{2} - p$$

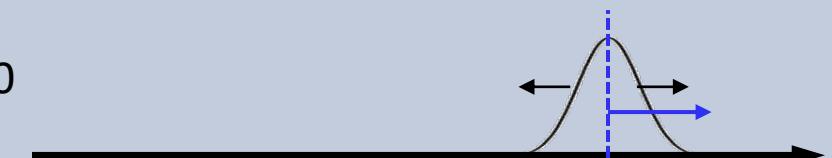
$$P(1 \text{ step right}) = \frac{1}{2} + p$$

Distribution of random walk

$t = 0$



$t > 0$



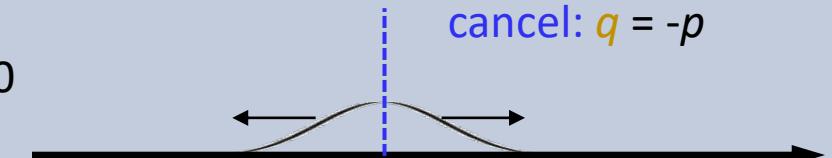
add 2<sup>nd</sup> random process  
wind blows

$$P(1 \text{ step left}) = \frac{1}{2} + q$$

$$P(1 \text{ step right}) = \frac{1}{2} - q$$

Distribution of random walk with wind

$t > 0$



Gain: Bias-free estimate!  
Cost: Variance

# Generalizing: Raveling trajectories with quasiprobabilities

Channel we *want* to implement

CPTP operation we *can* implement

$$\mathcal{C}(\cdot) = \sum_i a_i \mathcal{F}_i(\cdot)$$

Real coefficients, turn into *quasi-probability*

Putting the following techniques all on the same footing

## Technique

Prob. error cancelation (PEC)

Circuit cutting (knitting) of gates

Circuit cutting of wires

Classical sim. algorithms (QP)

## Channel $\mathcal{C}$

noise inverse

non-local gate

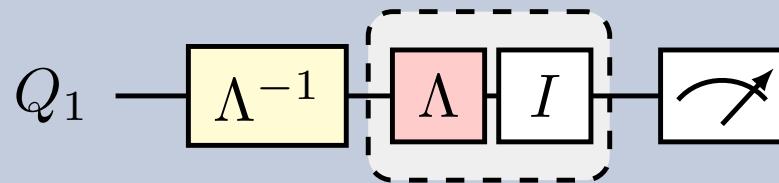
large unitary

unitary

# PEC: Nice, but why hasn't worked so far for experiments?

## Practical challenges

Small scale

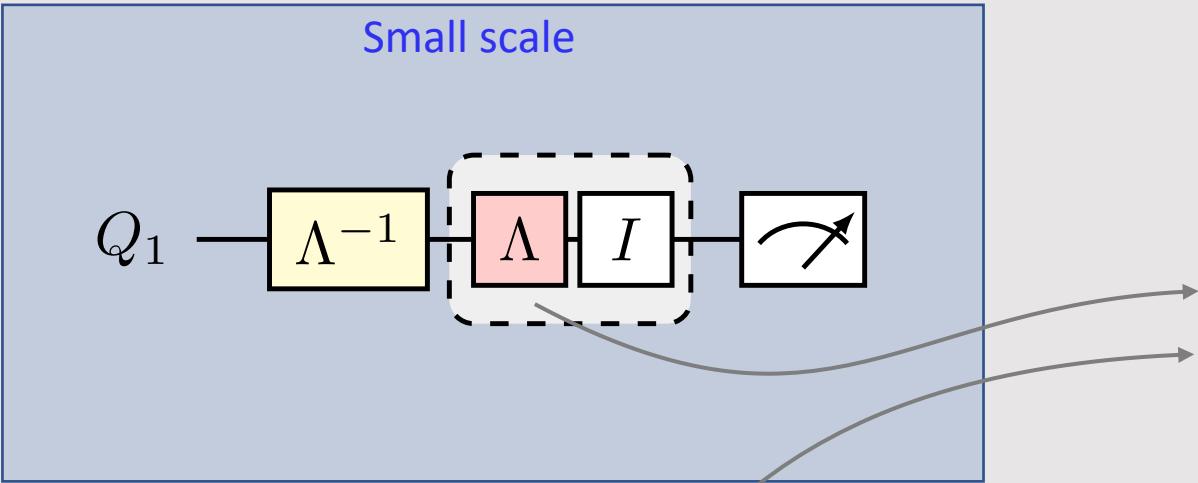


Critically hinges on knowing the full noise near perfectly

Despite the method's theoretical appeal (1-10), practical challenges have limited its demonstration to the one and two-qubit level (2, 3)

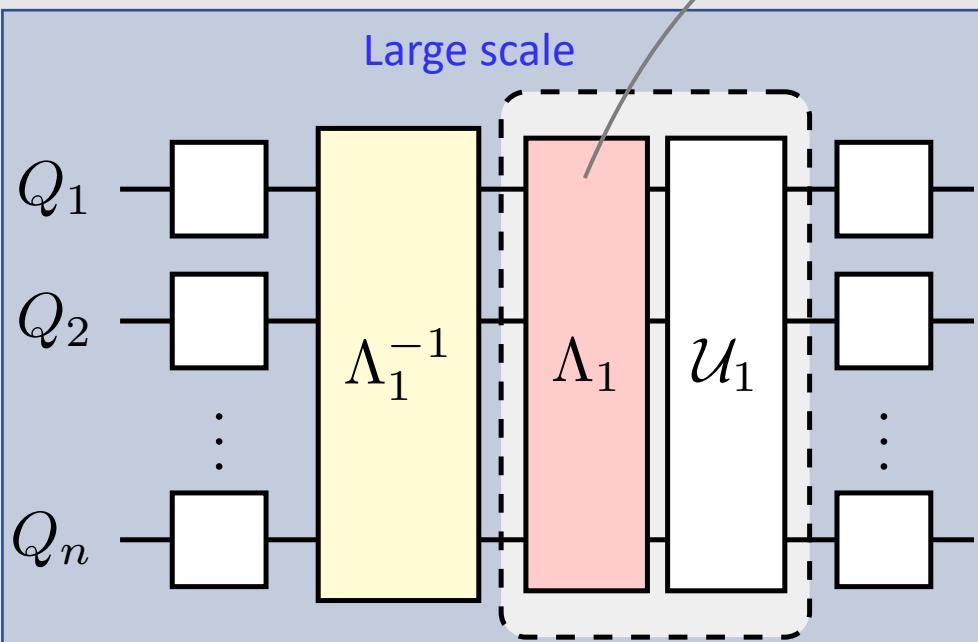
1. S. Endo, S. C. Benjamin, Y. Li, Physical Review X 8, 031027 (2018).
2. C. Song, et al., Science Advances 5, arXiv:2109.04457(2019).
3. S. Zhang, et al., Nature Communications 11, 587 (2020).
4. C. Piveteau, D. Sutter, S. Woerner, arXiv:2101.09290 (2021).
5. S. Endo, et al., J. Physi Soc. of Japan 90, 032001 (2021).
6. C. Piveteau, et al., arXiv:2103.04915 (2021).
7. R. Takagi, Phys. Rev. Research 3, 033178 (2021).
8. R. Takagi, S. Endo, S. Minagawa, M. Gu, arXiv:2109.04457 (2021).
9. Y. Guo, S. Yang, arXiv preprint arXiv:2201.00752 (2022).
10. ...

# PEC: Nice, but why hasn't worked so far? Challenges



2 qubits  
10 qubits  
50 qubits  
noise param values  $10^{-2} - 10^{-5}$   
additive error sampling cost ( $>10^2 - 10^{10}$ )

255 parameters  
 $10^{12}$  parameters  
 **$10^{60}$  parameters**



## Challenges

### learning complexity

- efficient
- scalable
- accurate
- compact, tractable representation

### noise in full device

- cross-talk
- correlated errors
- parallel gates

# Outline

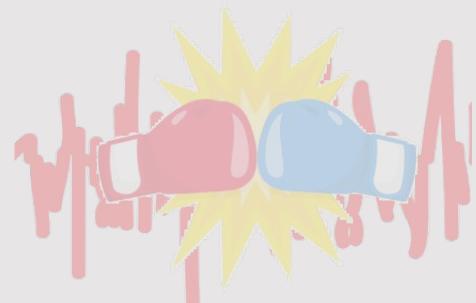
---



Idea

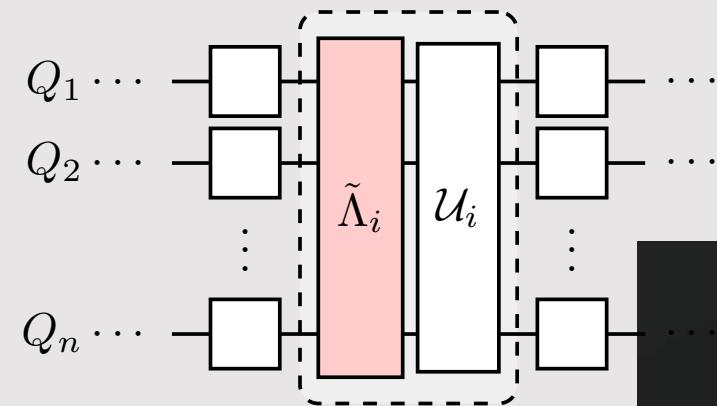


Learn

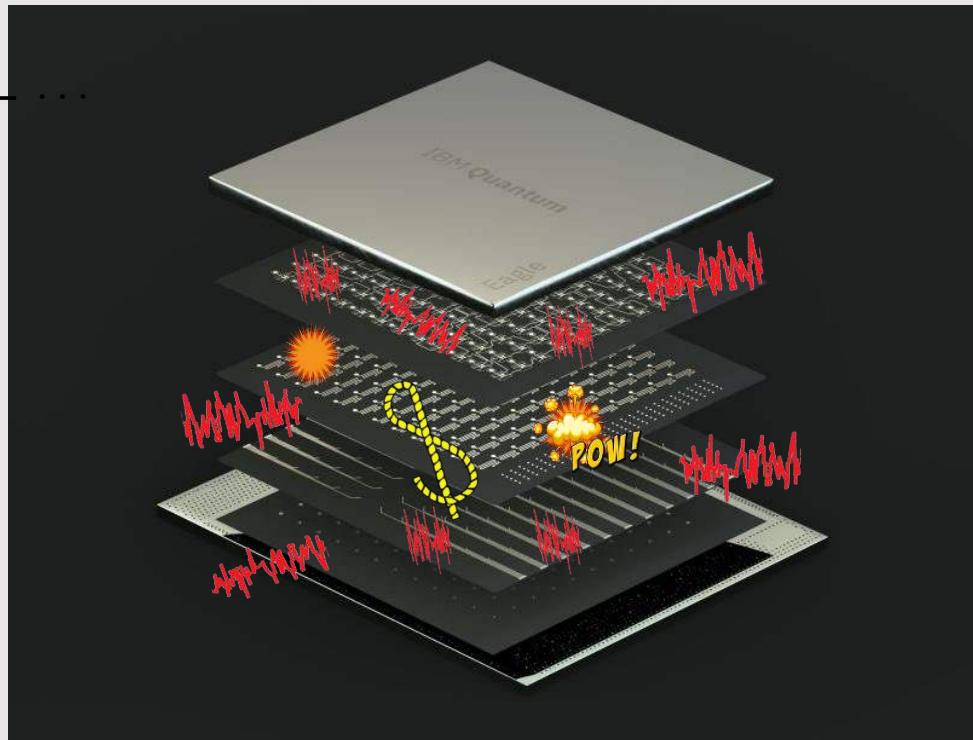


Cancel  
(realization)

# Is it possible to learn the noise with accuracy, efficiency, and scalability?

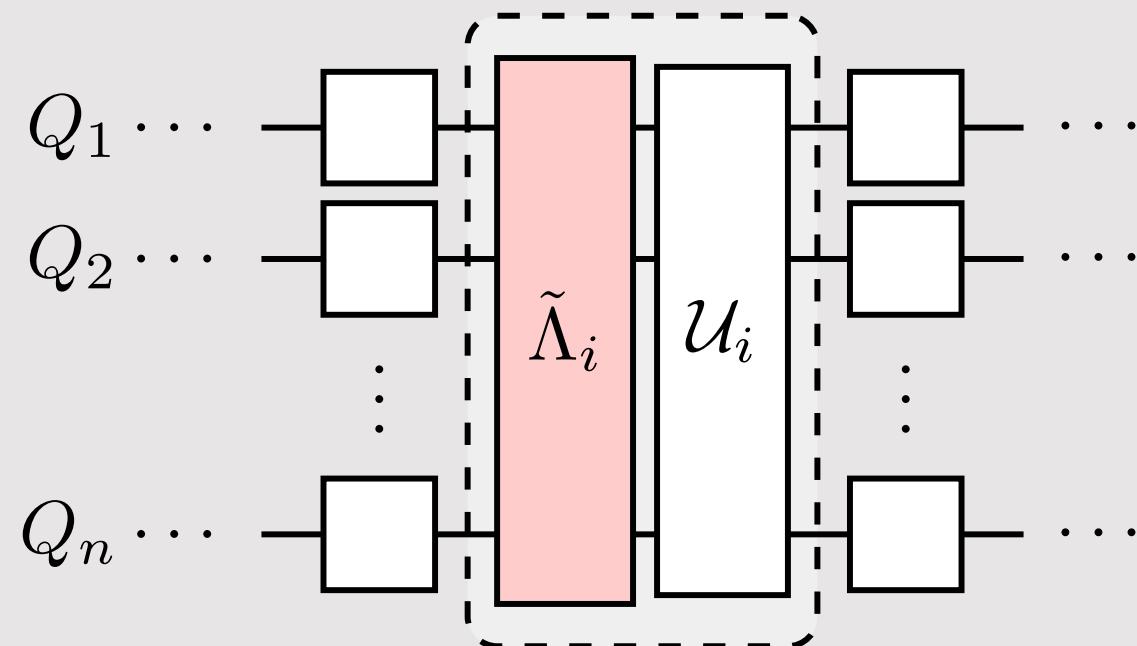


Energy relaxation  $T_1$   
Dephasing  $T_2$   
Coherent errors ZZ  
Classical crosstalk  
Quantum crosstalk  
State preparation error  
Measurement correlated errors  
...



Control errors  
Photon shot noise  
1/f charge noise  
1/f flux noise  
Nonequilibrium quasiparticles  
Leakage  
Cosmic rays  
...

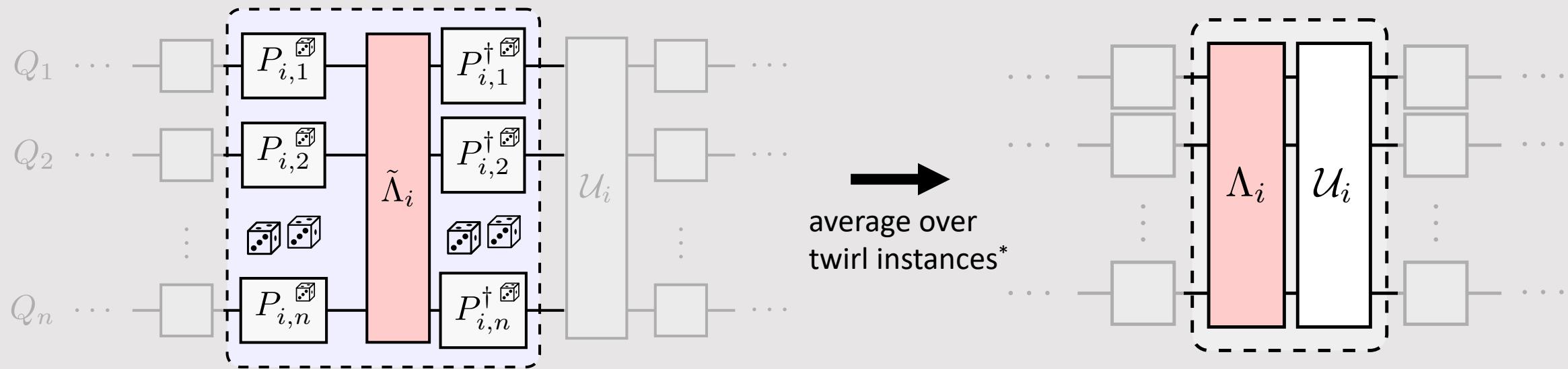
# Step 1: Simplify the noise



noise that includes cross-talk errors, etc.  
characterized by some  $4^n \times 4^n$  matrix

# Step 1: Simplify the noise: twirl

twirl reduces to noise  $4^n \times 4^n$  matrix to diagonal one with  $4^n$  entries in Pauli basis



## Twirling references

1. C. H. Bennett, et al., Phys. Rev. Lett. 76, 722 (1996).
2. E. Knill, arXiv:0404104 (2004).
3. O. Kern, G. Alber, D. L. Shepelyansky, EPJ D 32, 153 (2005).
4. M. R. Geller, Z. Zhou, Physical Review A 88, 012314 (2013).
5. J. J. Wallman, J. Emerson, Physical Review A 94, 052325 (2016)
6. Hashim *et al.*, Phys. Rev. X 11, 041039 (2021)
7. Tutorial: zlatko-minev.com/blog/twirling (2022)
8. ...

## Tutorial



SCAN ME

## Stochastic Pauli channel

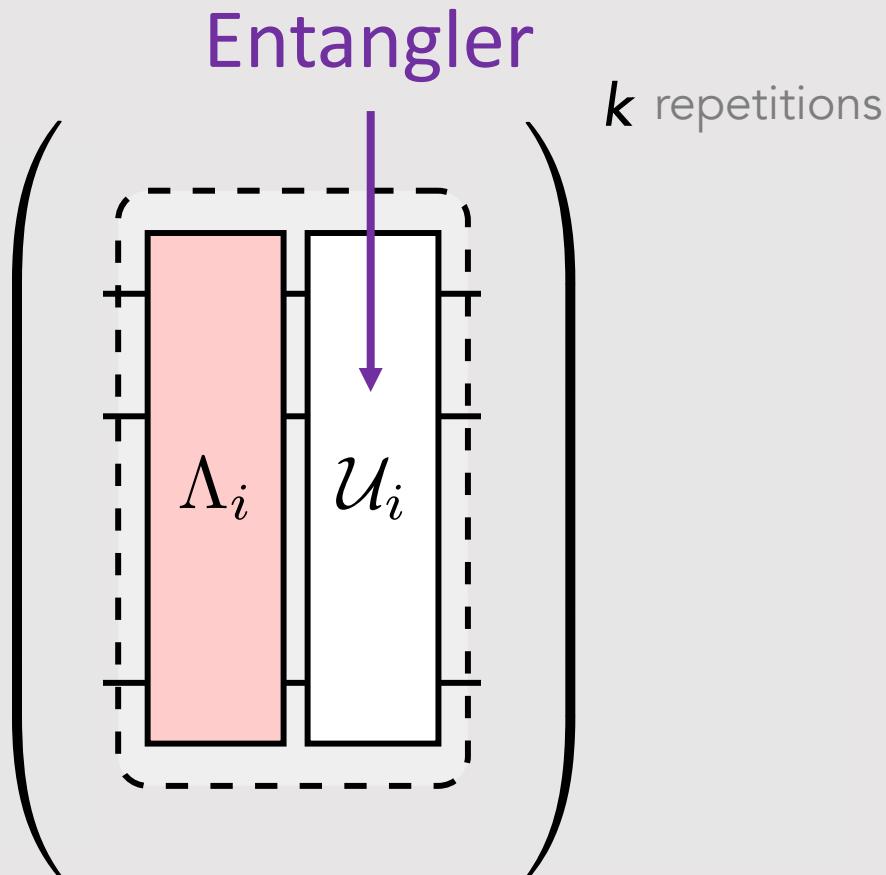
$$\Lambda_i(\rho) = \sum_{a=0}^{4^n - 1} c_{ia} P_a \rho P_a^\dagger$$

$$\Lambda(P_a) = f_a P_a$$

eigenvecs are Paulis

\* some sub-Clifford twirl group (use Paulis)

# Step 2: Ideally, amplify the noise and learn



Ideally wish

$$\cancel{\Lambda_i^k(P_a) - f_{ia}^k P_a}$$

Akin to:

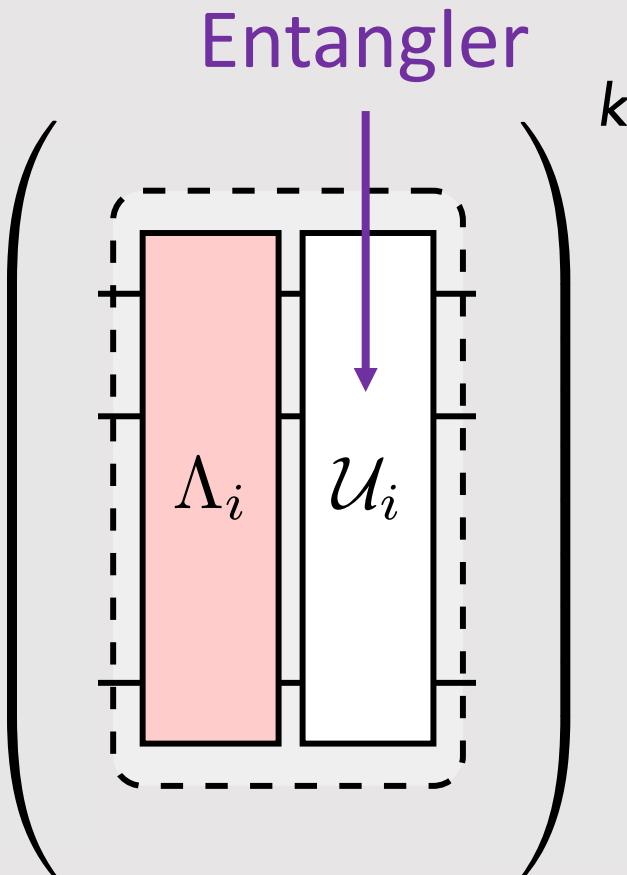
RB, Cycle RB, K-body noise reconstruction, ...

S.T. Flammia and J.J. Wallman ACM Trans QC 1, 3 (2020), ...

Erhard *et al.*, arXiv:1902.08543; Ferracin *et al.*, arXiv:2201.10672, ...

For something of a review of protocols, see Helsen, *et al.*, arXiv:2010.07974

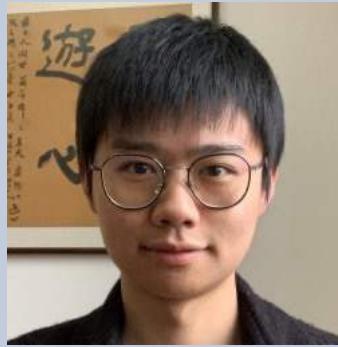
## Step 2: Ideally, amplify the noise and learn



Ideally wish

$$\cancel{\Lambda_i^k (P_a) - f_{ia}^k P_a}$$

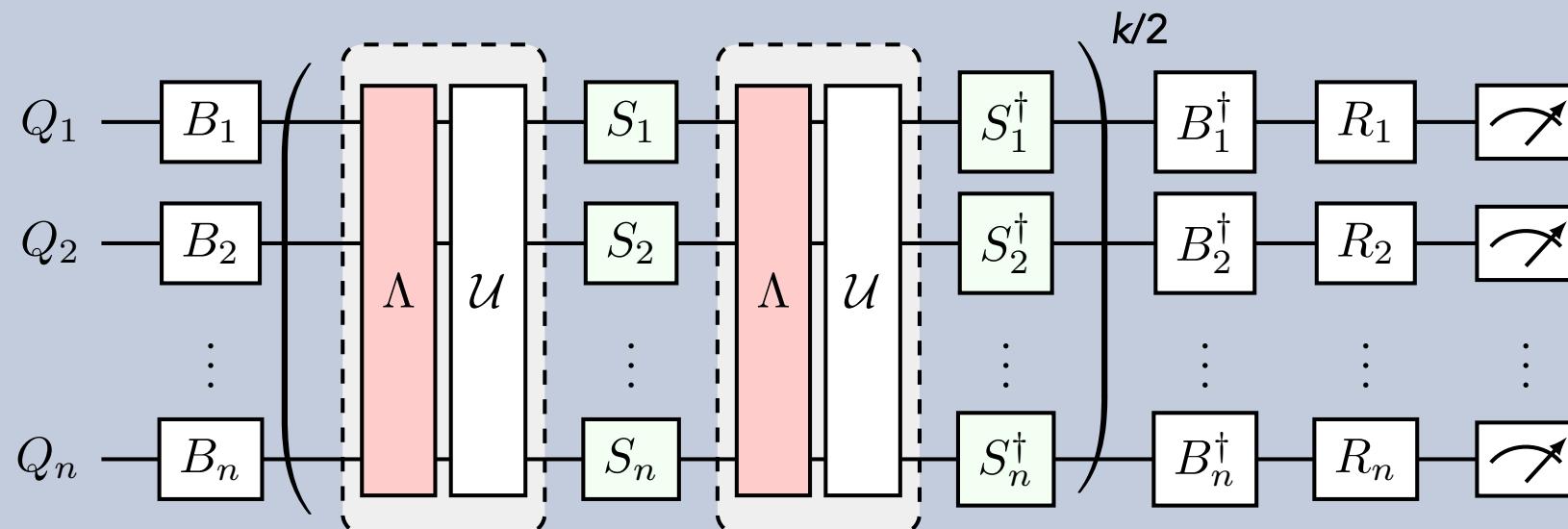
### Fundamental no-go theorem on learning



For general and in-depth  
Senrui Chen, Y. Liu, M. Otten, A. Seif, B. Fefferman, L. Jiang  
arXiv:2206.06362 (2022)  
or supplement of our paper for qubit version and work by  
S. Flammia, S Benjamin, and teams.

# Solution: Custom protocol + weak assumption

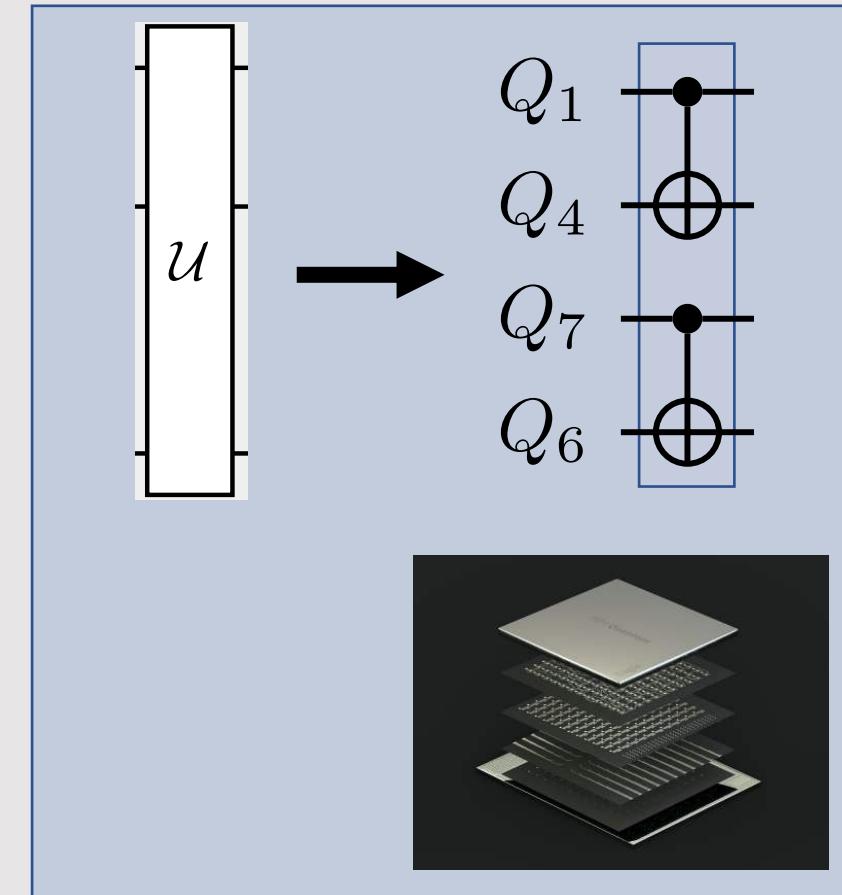
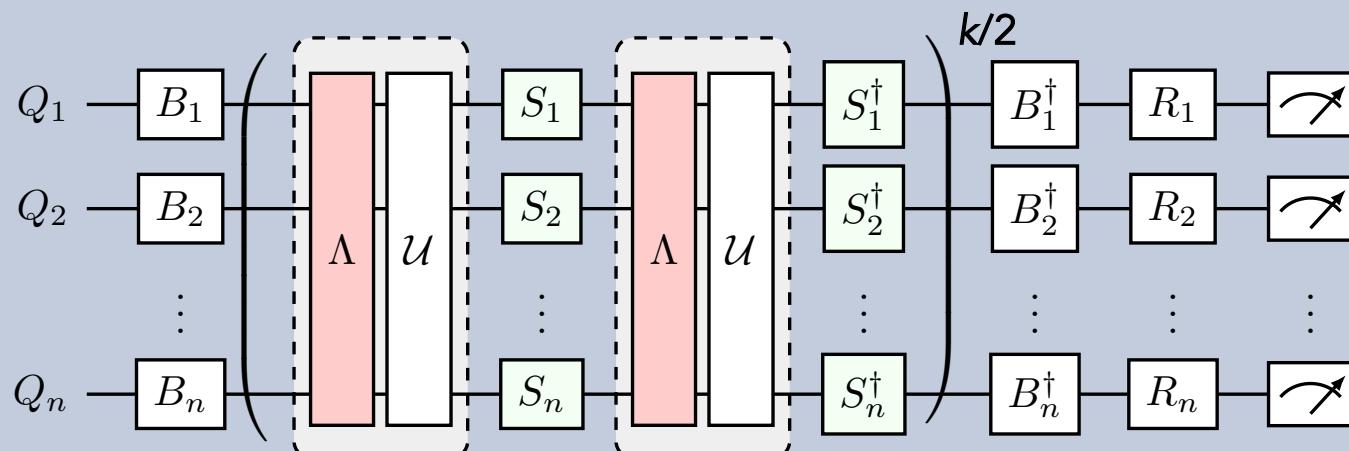
Learning circuits akin to cycle RB



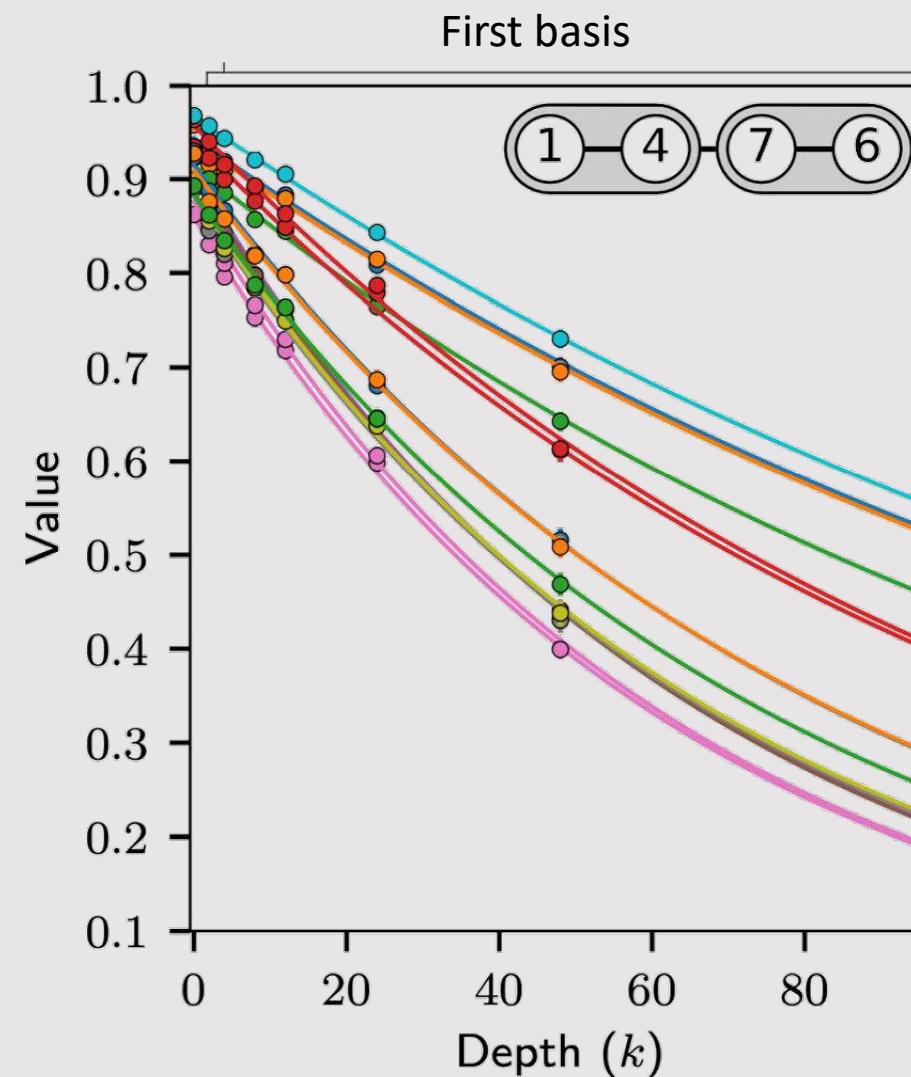
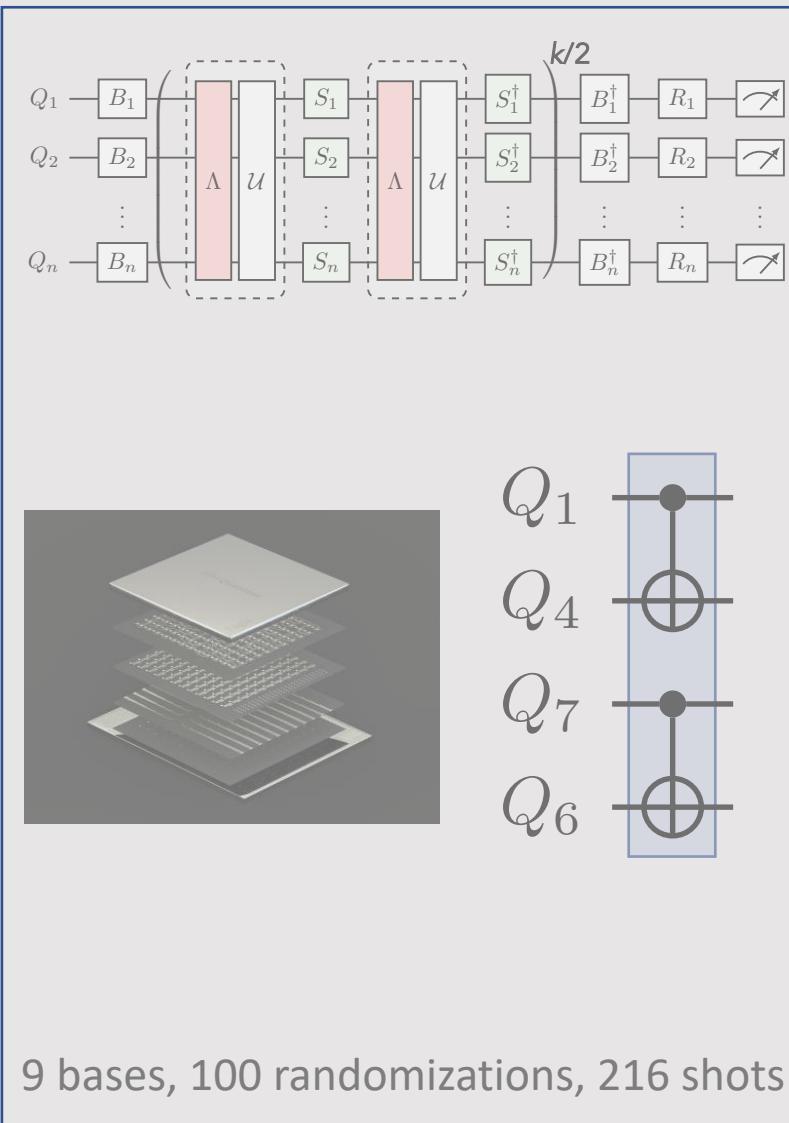
(ask me in questions for how it works)

# In a simple experiment?

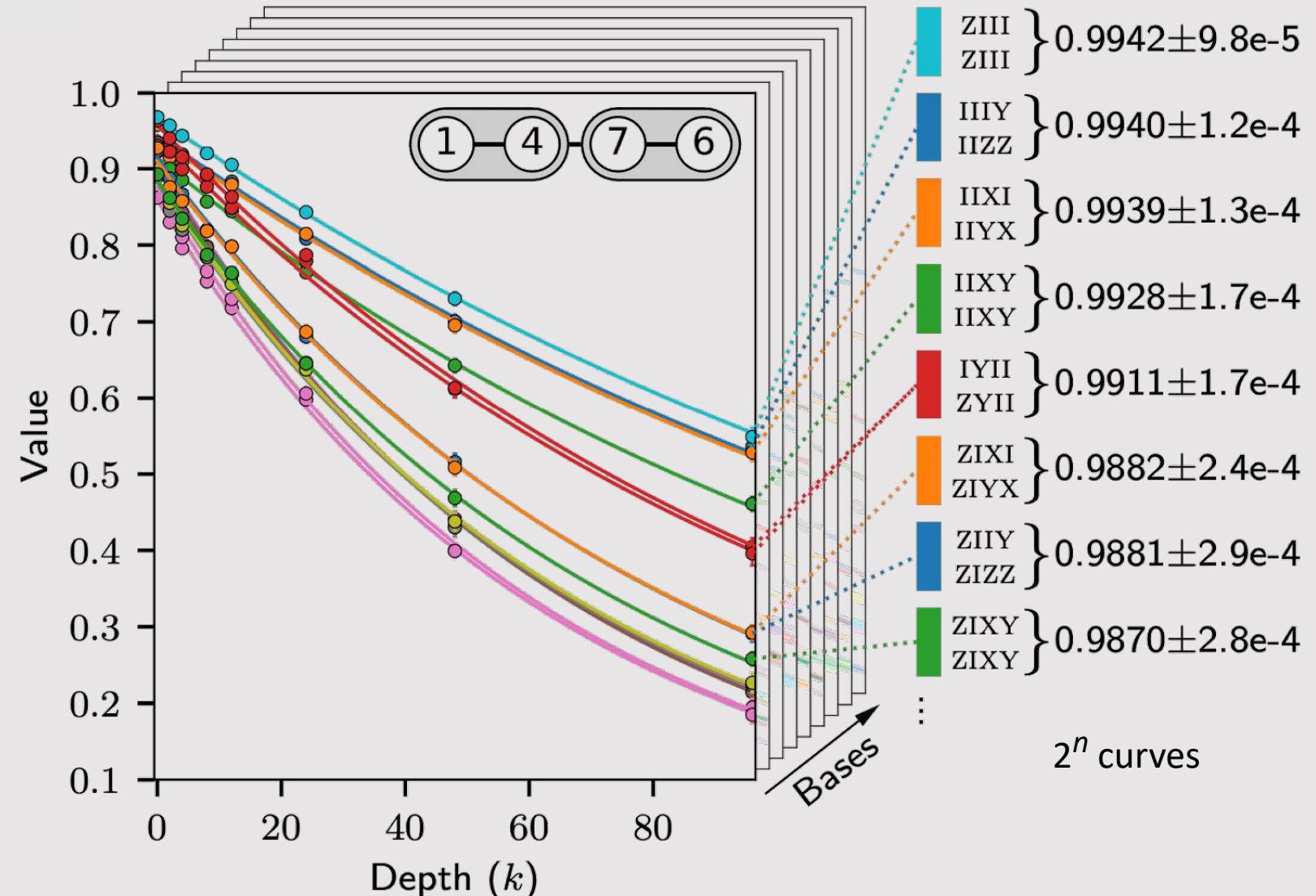
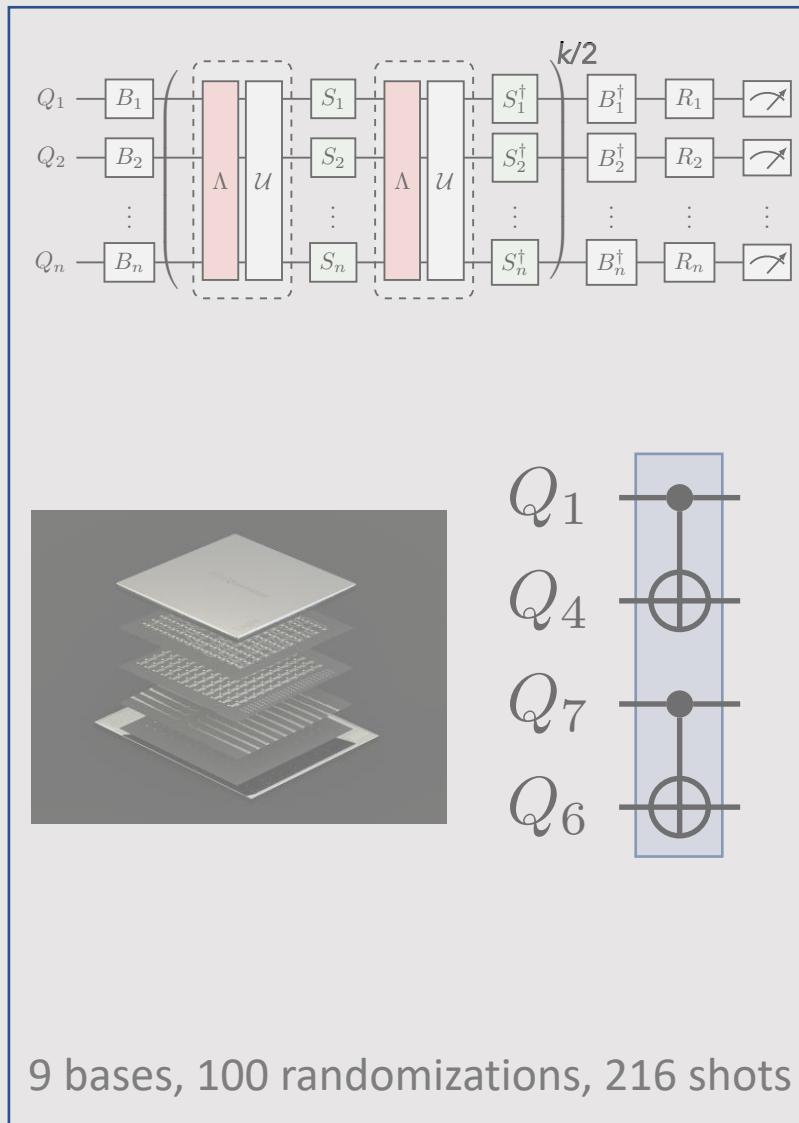
## Learning circuits



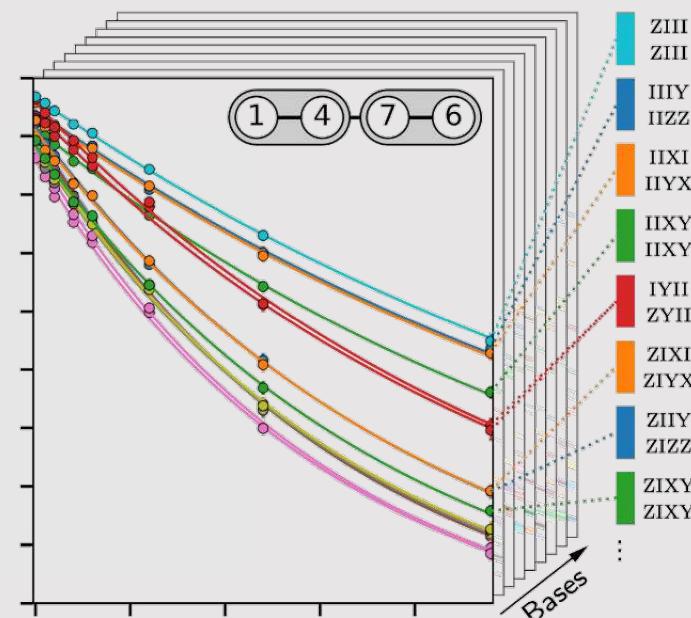
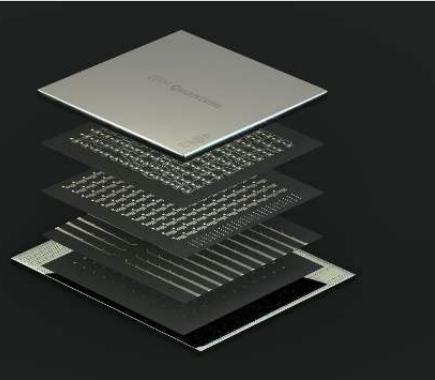
# Learning the noise: raw data



# Learning the noise: raw data



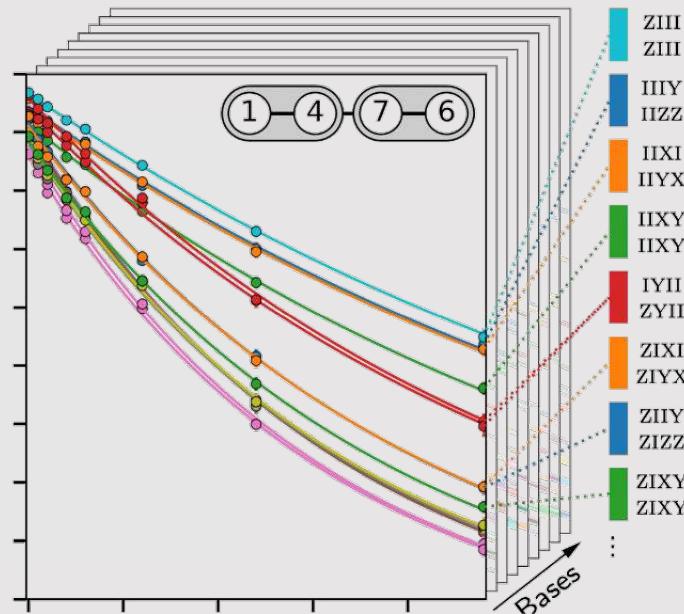
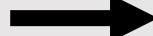
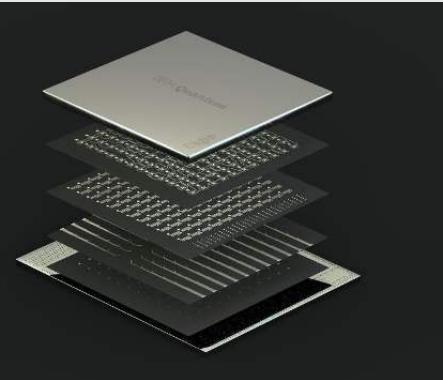
# Reconstructing quantum channel from measurement data



$$\Lambda(\rho) = \sum_{a=0}^{4^n - 1} c_a P_a \rho P_a^\dagger$$

Still  $4^n$

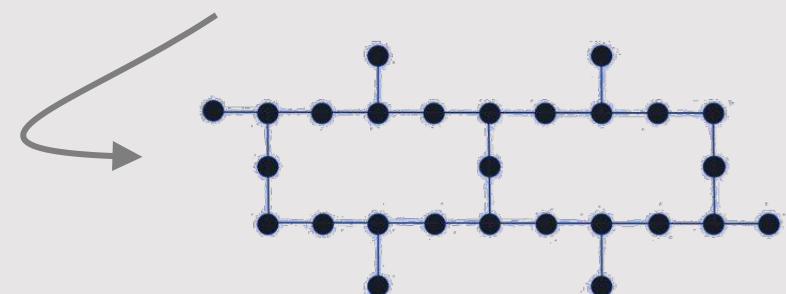
# Sparse Pauli-Lindblad model



$$\Lambda(\rho) = \sum_{a=0}^{4^n - 1} c_a P_a \rho P_a^\dagger$$

$$\Lambda(\rho) = \exp[\mathcal{L}](\rho)$$

$$\mathcal{L}(\rho) = \sum_{k \in \mathcal{K}} \lambda_k (P_k \rho P_k - \rho)$$



Magic



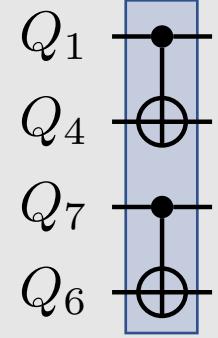
icon: Eucalyp

Highlight: Ewout van den Berg

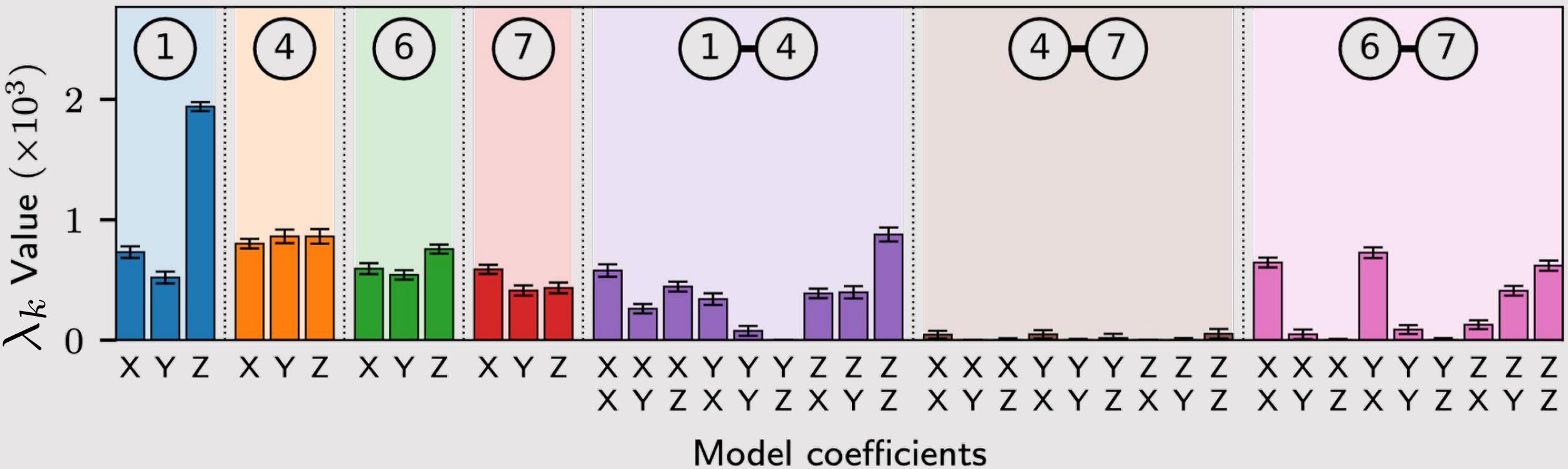
See Peter Zoller talk tomorrow for Hamiltonian learning

Zlatko Minev, IBM Quantum (40)

# Sparse Lindblad tomogram

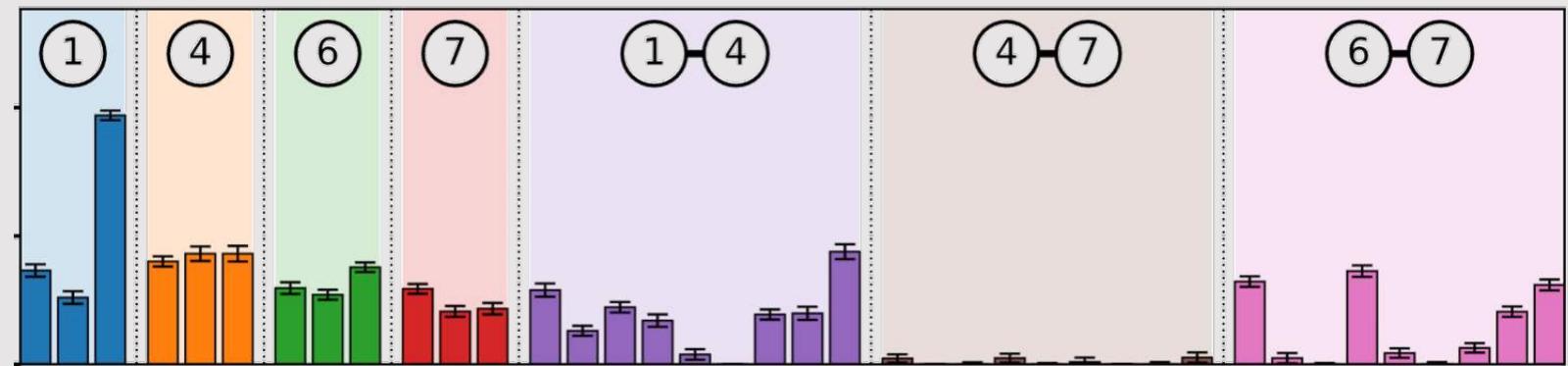


$$\mathcal{L}(\rho) = \sum_{k \in \mathcal{K}} \lambda_k \left( P_k \rho P_k^\dagger - \rho \right)$$



Model coefficients

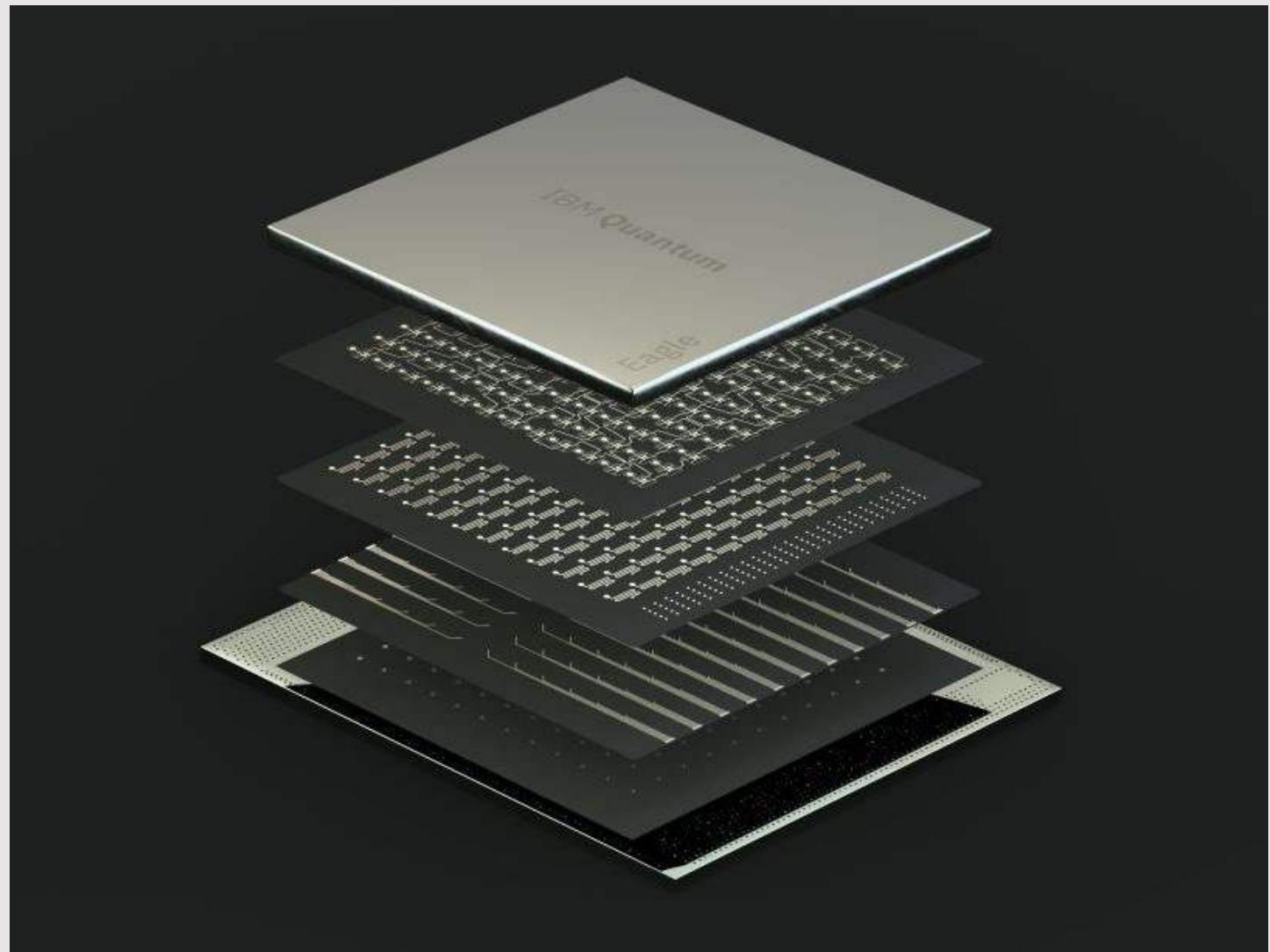
# Fingerprint



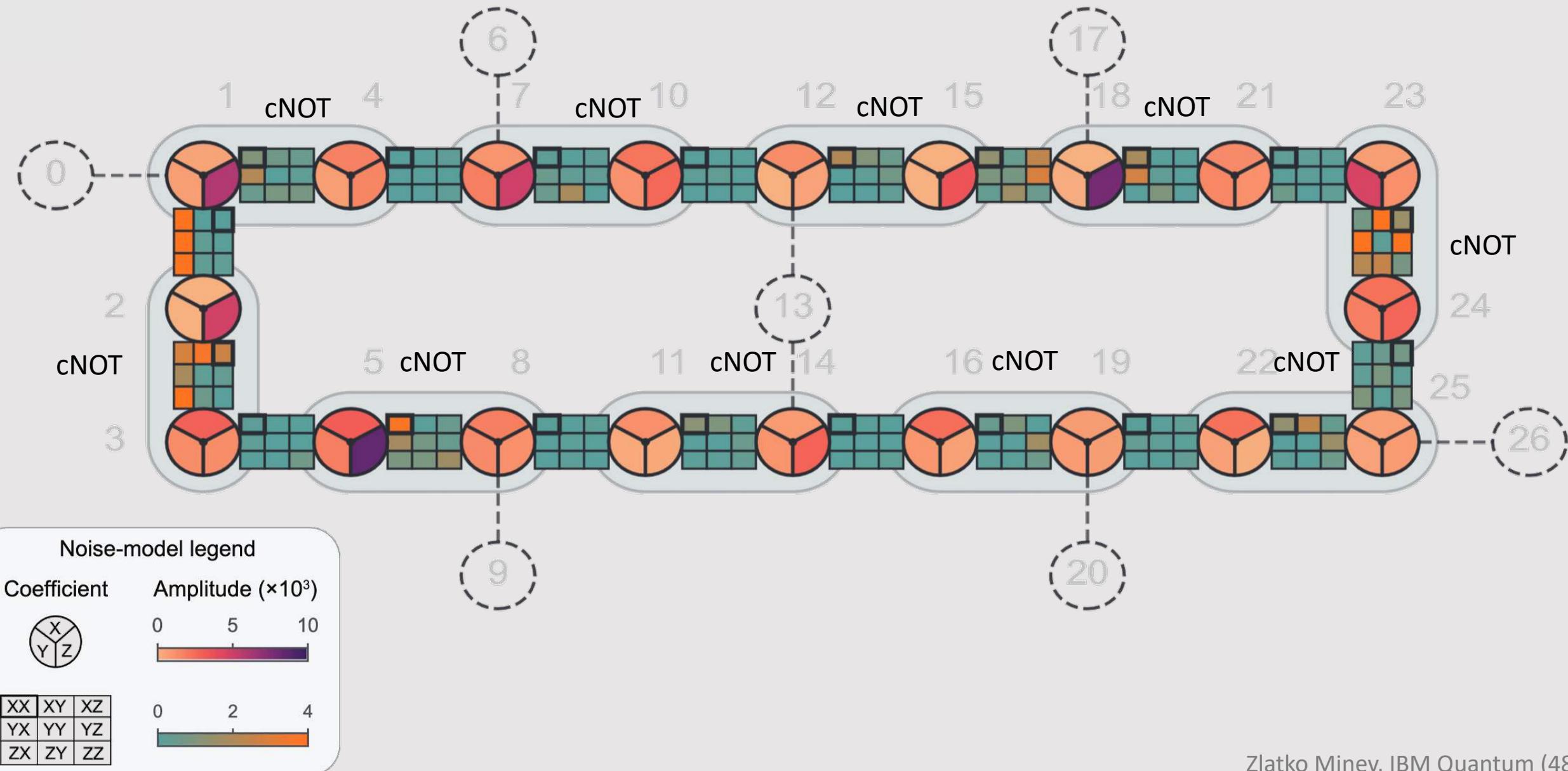
Application: Device inspection/optimization

- Ask me about dynamical decoupling (DD) application

# Learning noise across large device?



# Noise tomogram for 20Q Ising-ring Trotter layer



# Outline

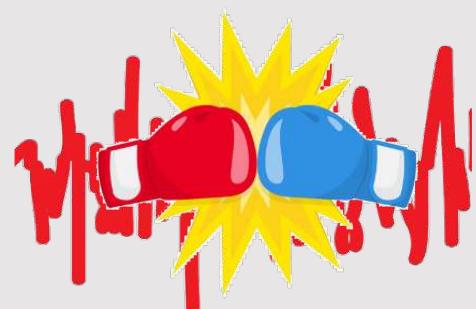
---



Idea



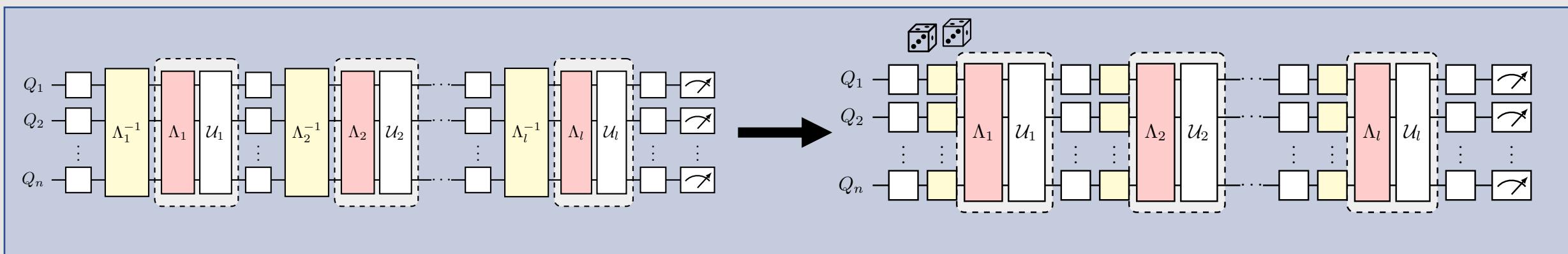
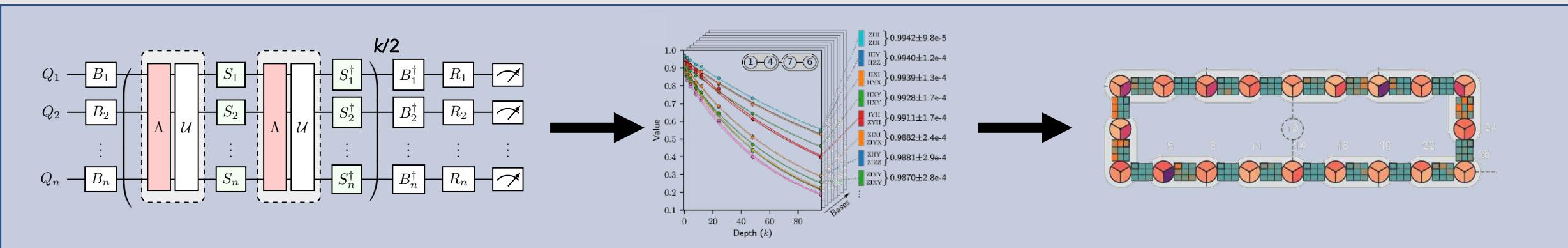
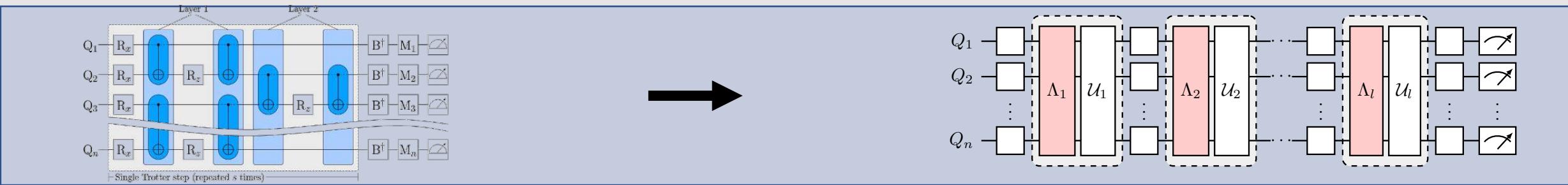
Learn



Cancel  
(realization)

PEC mitigation:  
inverting noise to  
obtain expectation  
values

# Protocol overview



# PEC + Simulating transverse-field Ising model time evolution



$$H = -J \sum_j Z_j Z_{j+1} + h \sum_j X_j$$

Study global magnetization

$$\vec{M} := \sum_n (\langle X \rangle_n, \langle Y \rangle_n, \langle Z \rangle_n) / N$$

$$h = 1, J = -0.15, \delta t = 1/4$$

# Trotterized time evolution

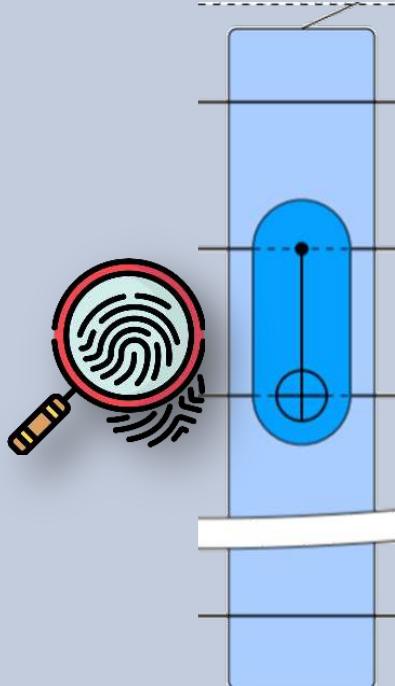


## Evolution Trotter layers

Layer 1



Layer 2



$$\gamma_1 = 1.0309 \pm 8.40 \cdot 10^{-5} \quad \gamma_2 = 1.0384 \pm 2.20 \cdot 10^{-4}$$

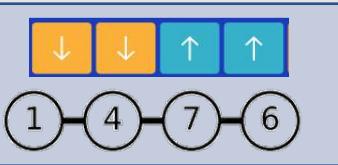
## Observables

$$\vec{M} := \sum_n (\langle X \rangle_n, \langle Y \rangle_n, \langle Z \rangle_n) / N$$

Example: Observables for  $N = 4$

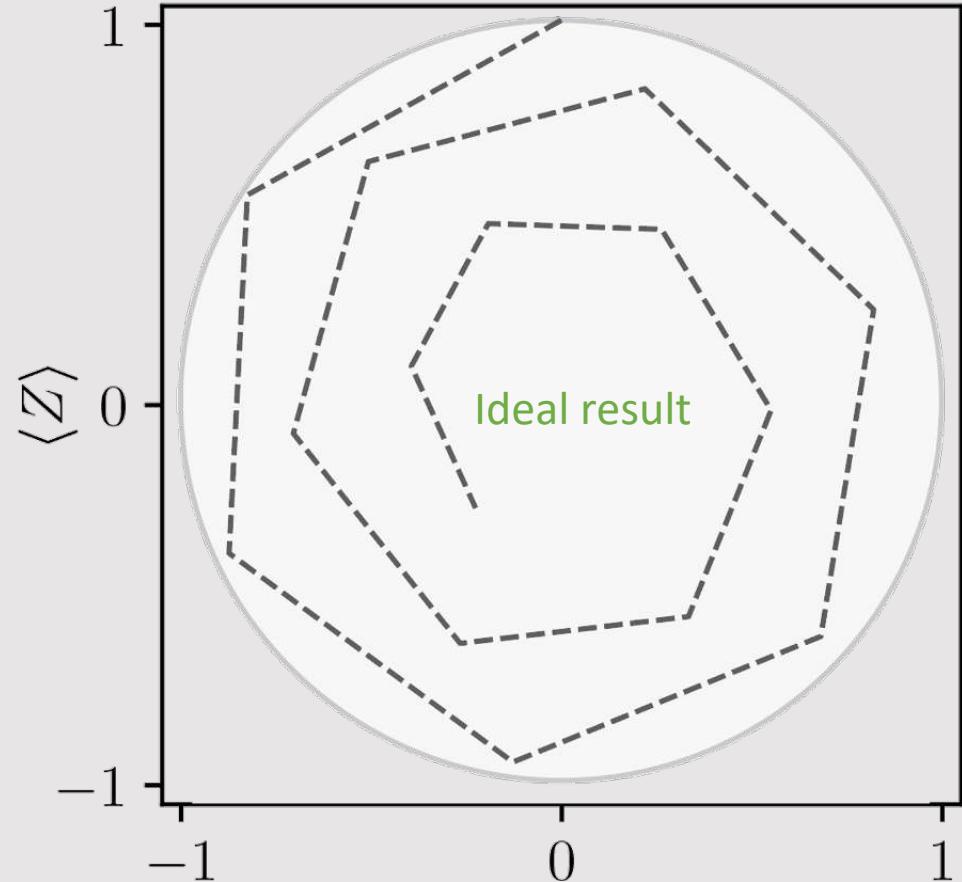
XIII	YIII	ZIII
IXII	IYII	IZII
IIXI	IIYI	IIZI
IIIX	IIIY	IIIZ

# Ideal Ising model evolution

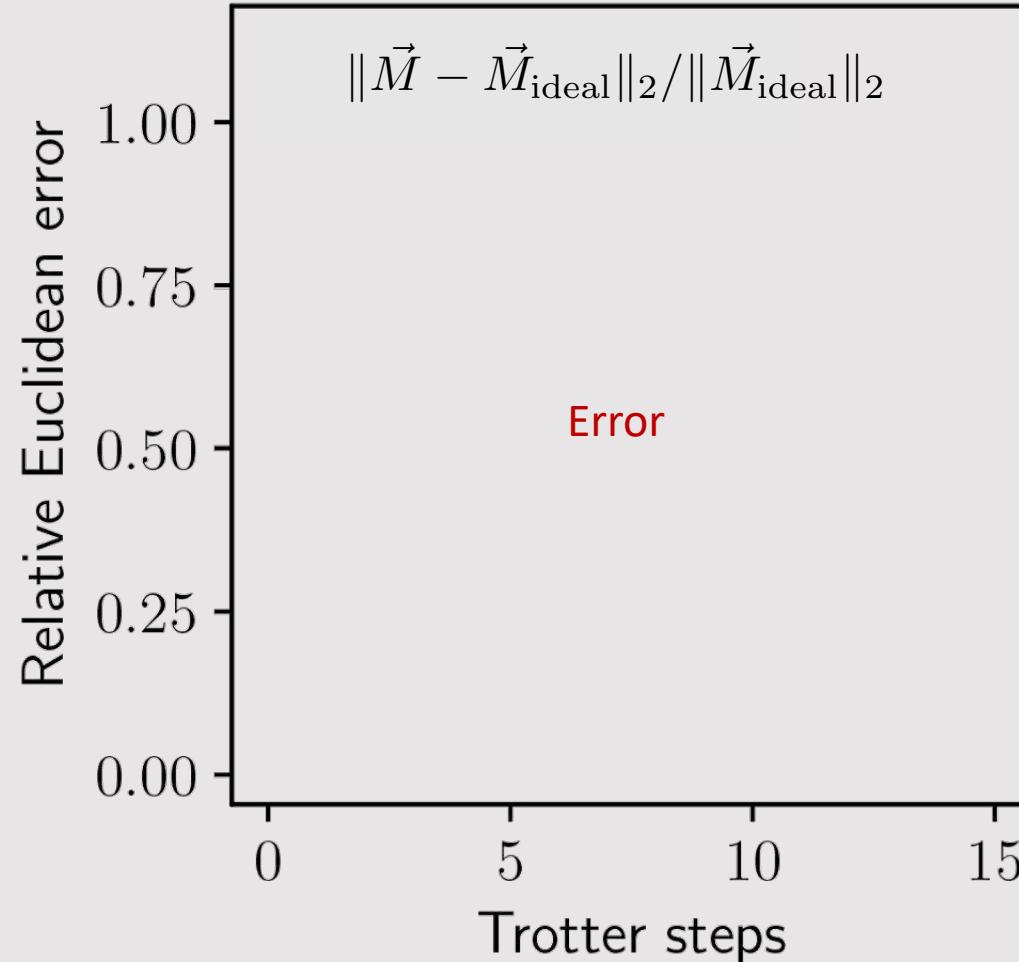


----- Ideal

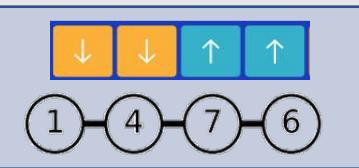
$$\vec{M} := \sum_n (\langle X \rangle_n, \langle Y \rangle_n, \langle Z \rangle_n) / N$$



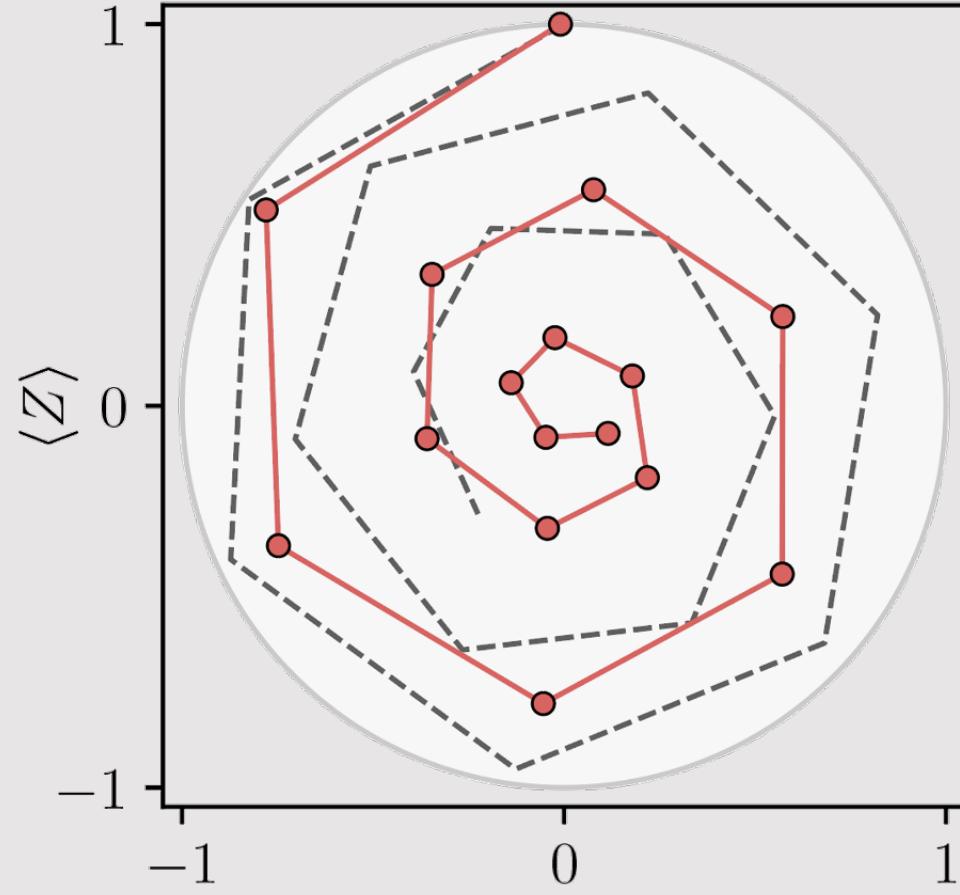
$$h = 1, J = -0.15, \delta t = 1/4$$



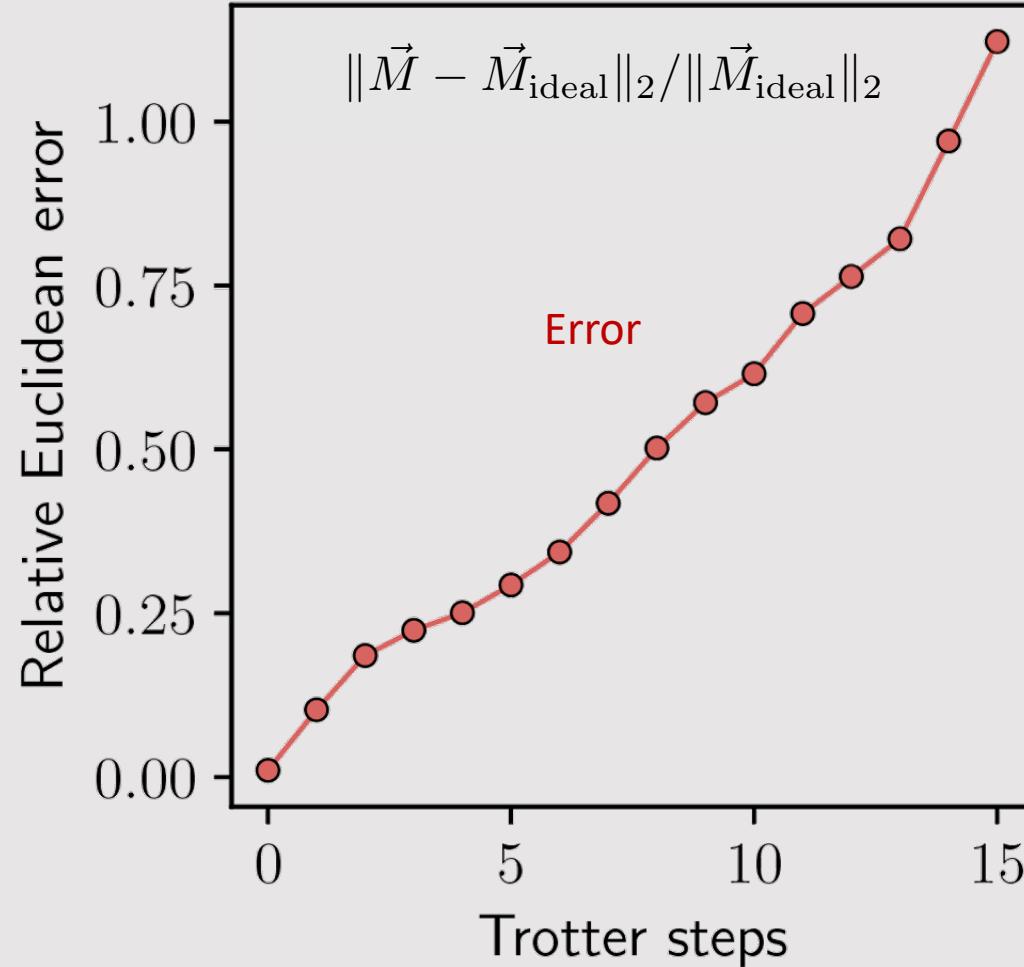
# Without PEC: but with DD & twirl readout mitigation



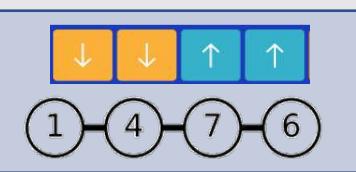
----- Ideal      —●— without PEC



$$h = 1, J = -0.15, \delta t = 1/4$$

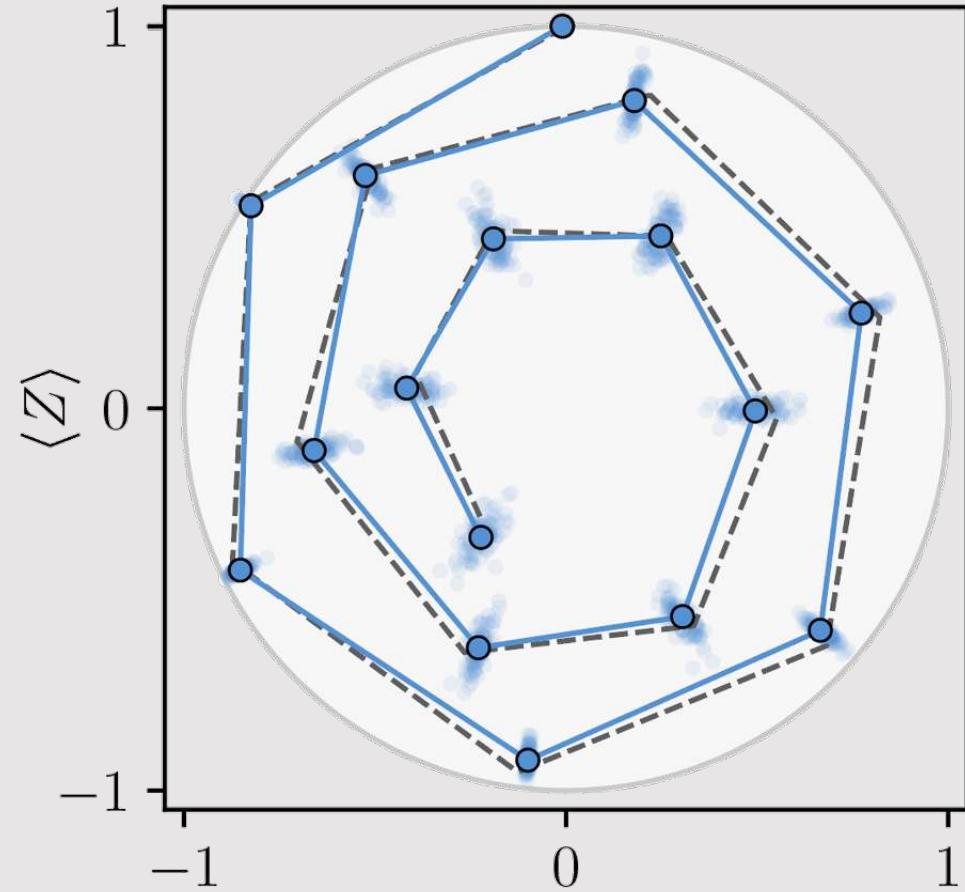


# With PEC



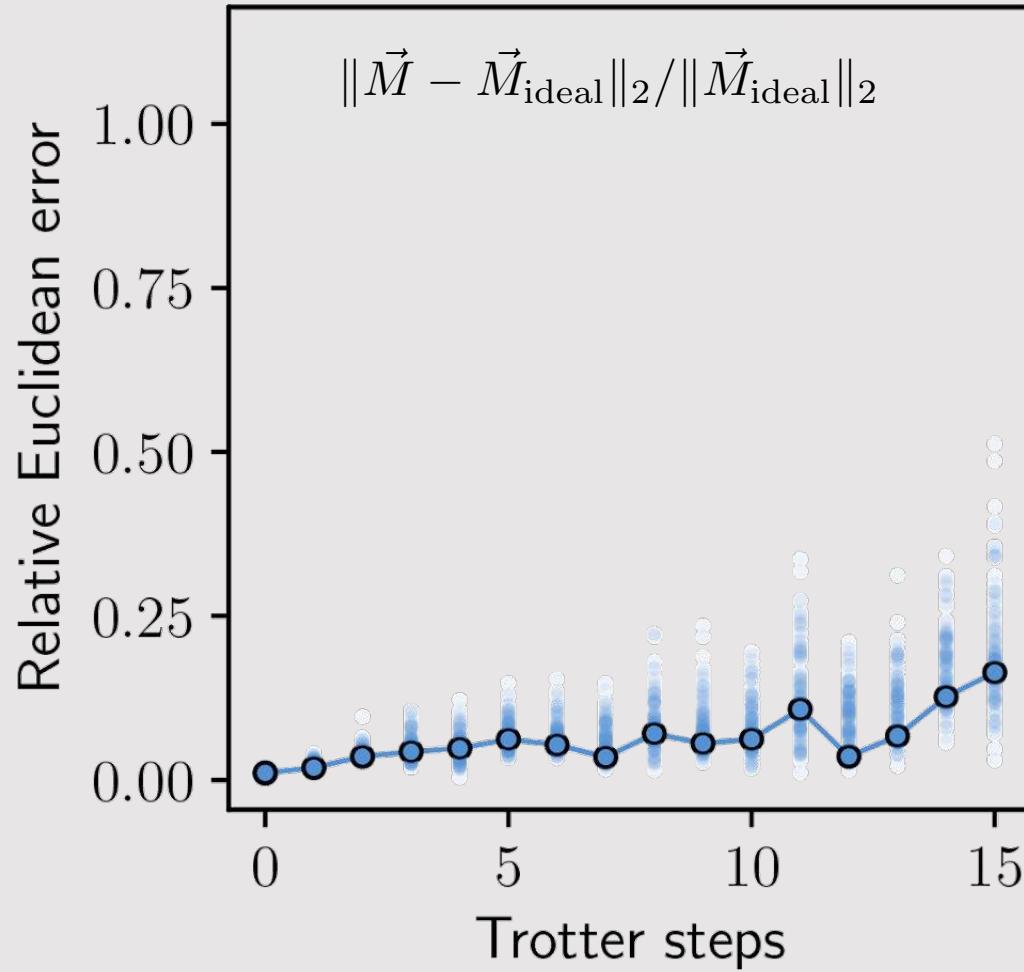
----- Ideal

—●— with PEC

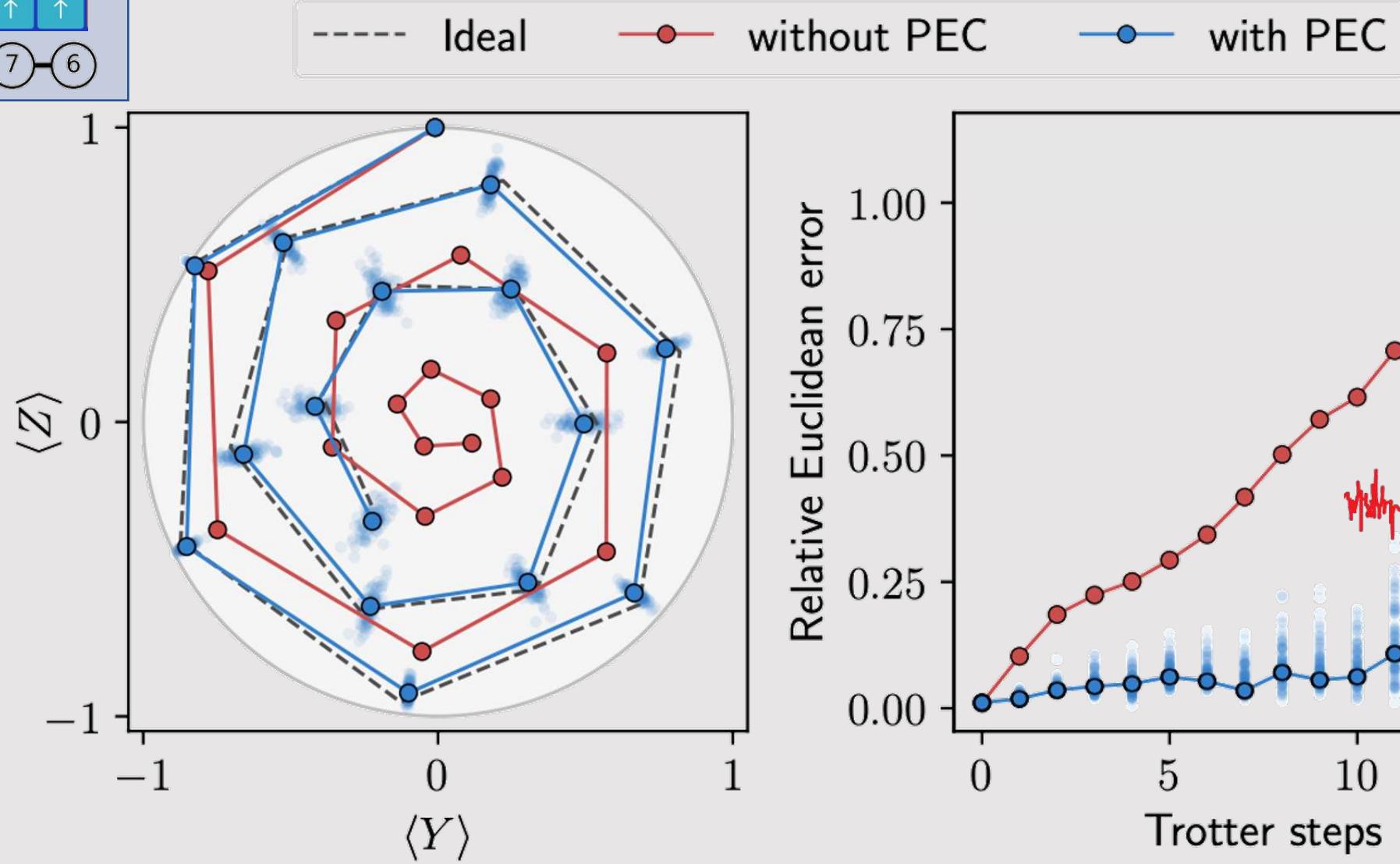
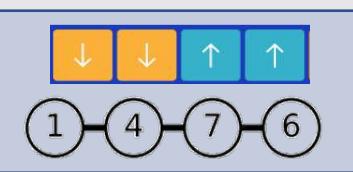


15 steps, depth 60

$h = 1, J = -0.15, \delta t = 1/4$

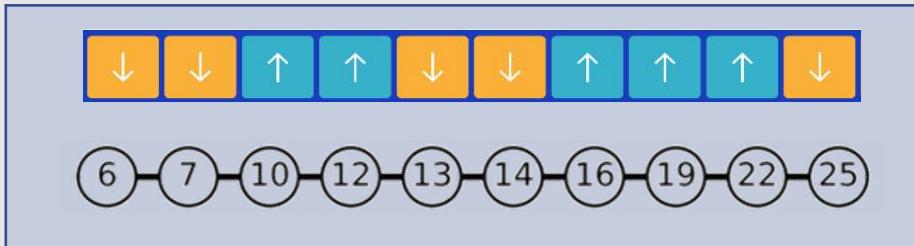


# With vs. without PEC

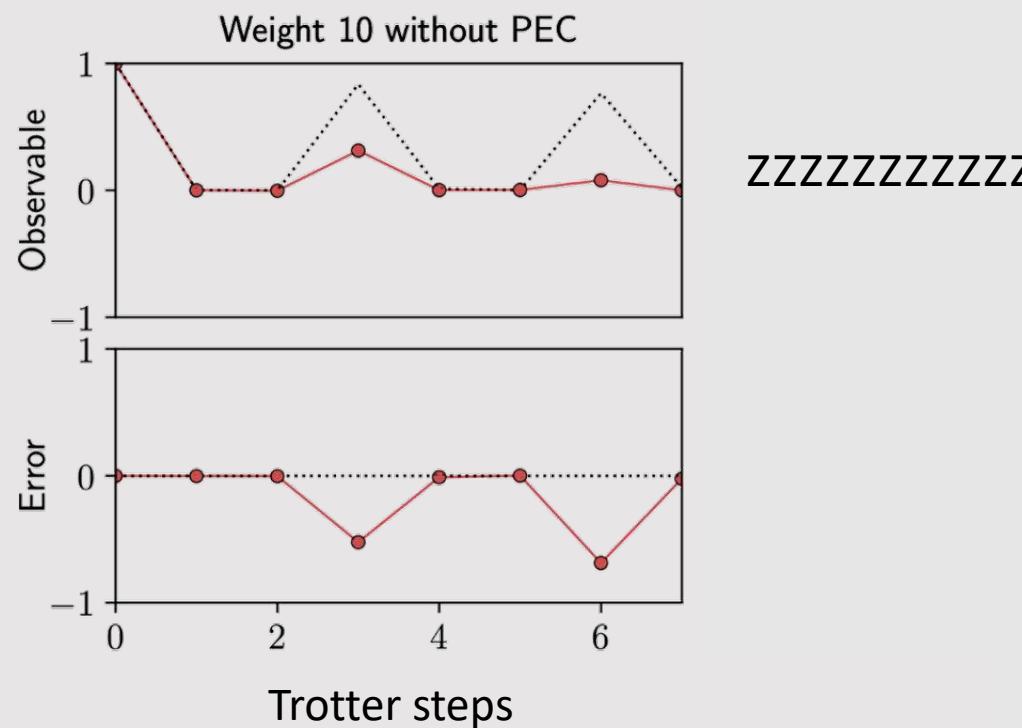


$$h = 1, J = -0.15, \delta t = 1/4$$

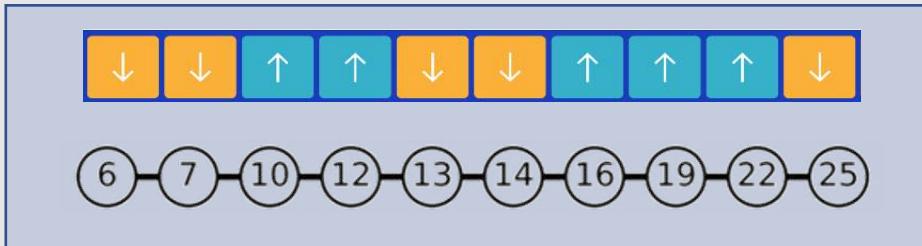
# With vs without PEC: 10 qubit high-weight observables



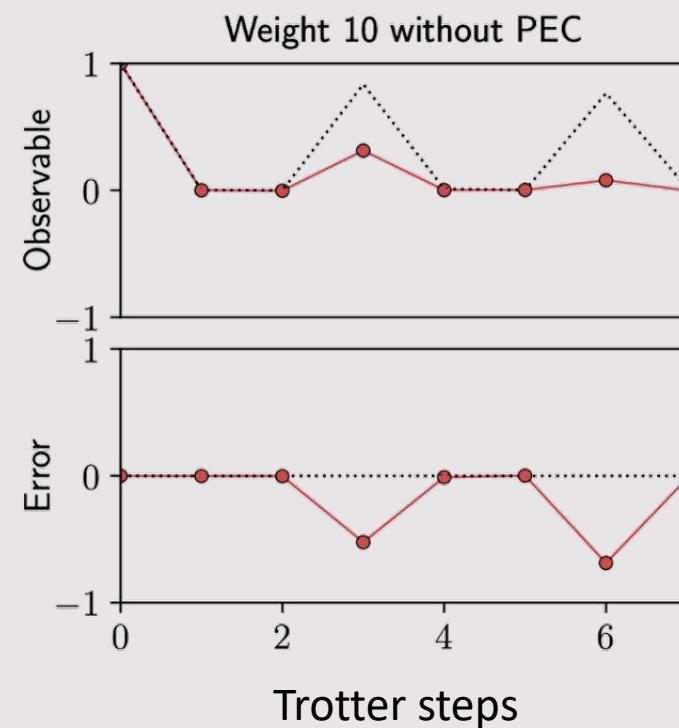
----- Ideal    —●— without PEC    —●— with PEC



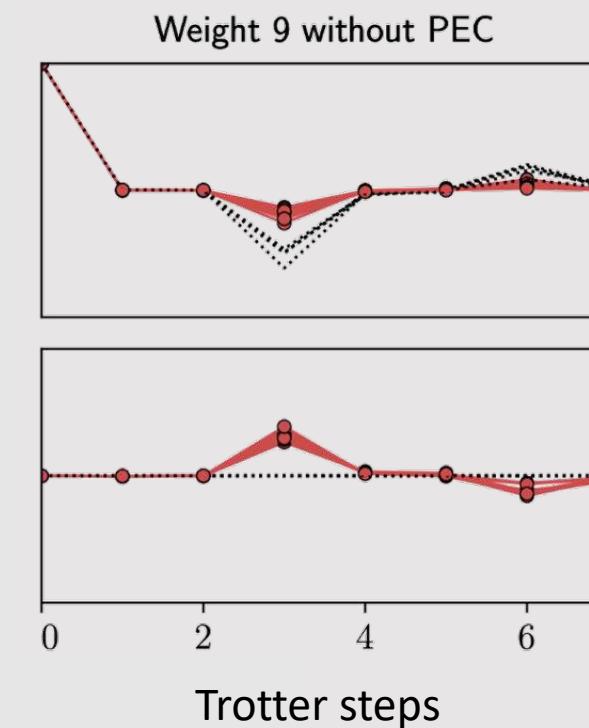
# With vs without PEC: 10 qubit high-weight observables



----- Ideal    -●- without PEC    -●- with PEC

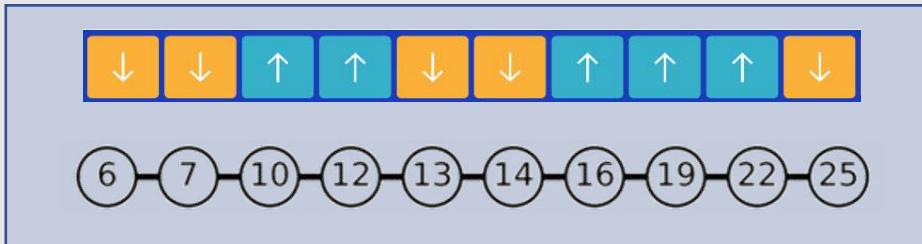


ZZZZZZZZZZZZ

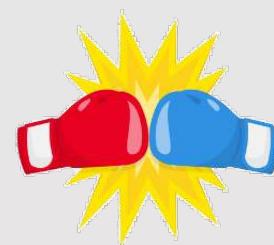
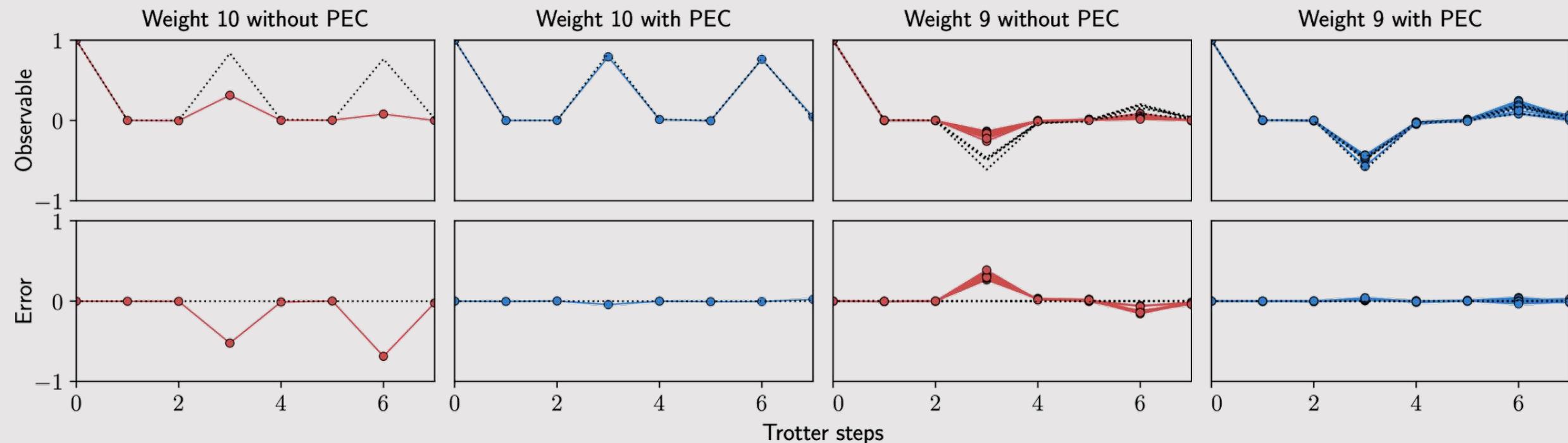


I Z Z Z Z Z Z Z Z Z Z  
Z I Z Z Z Z Z Z Z Z Z Z  
Z Z I Z Z Z Z Z Z Z Z Z Z  
...  
Z Z Z Z Z Z Z Z Z Z I

# With vs without PEC: 10 qubit high-weight observables

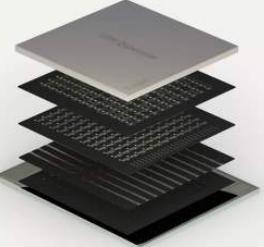


----- Ideal    -●- without PEC    -●- with PEC



# Scaling and error budget

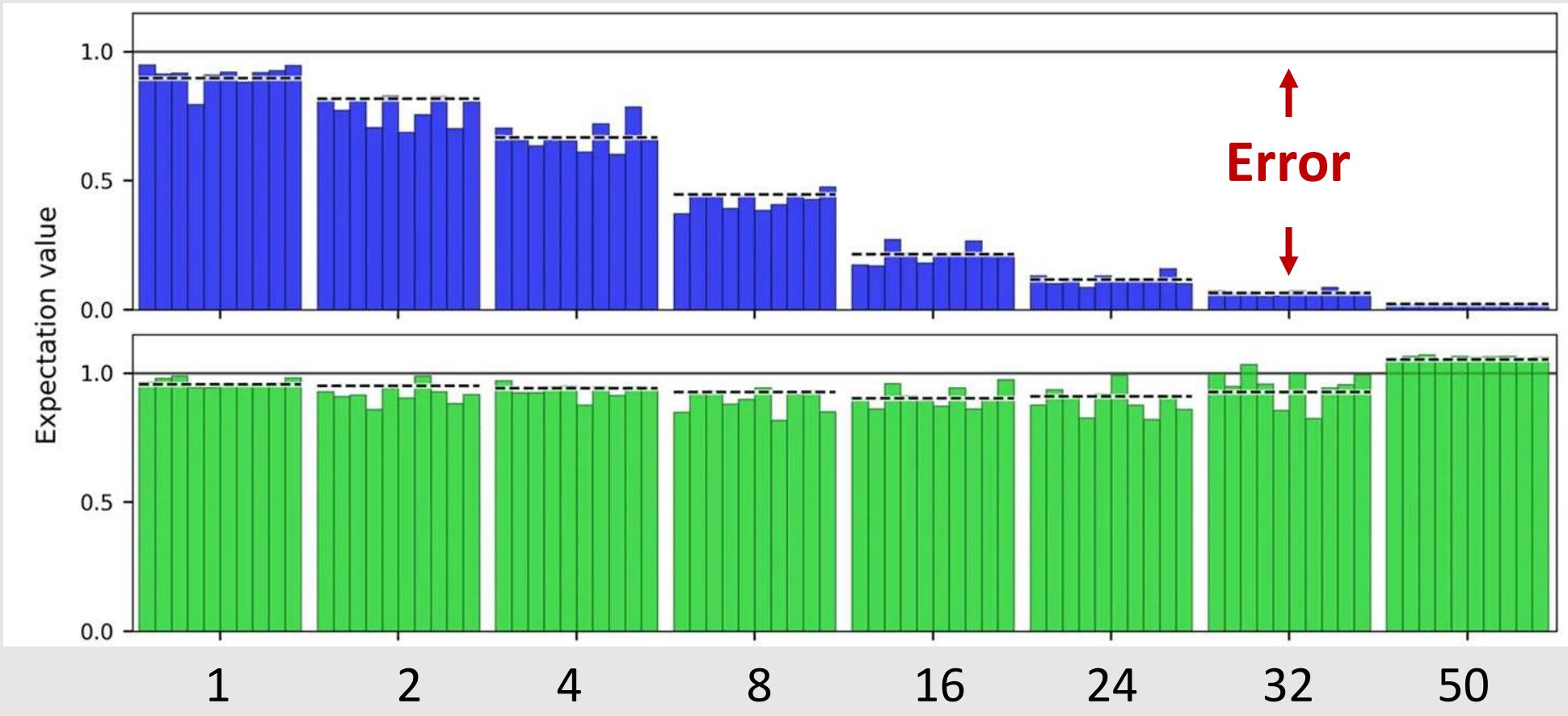




# 50 qubits

Z stabilizers of increasing weight

Without PEC

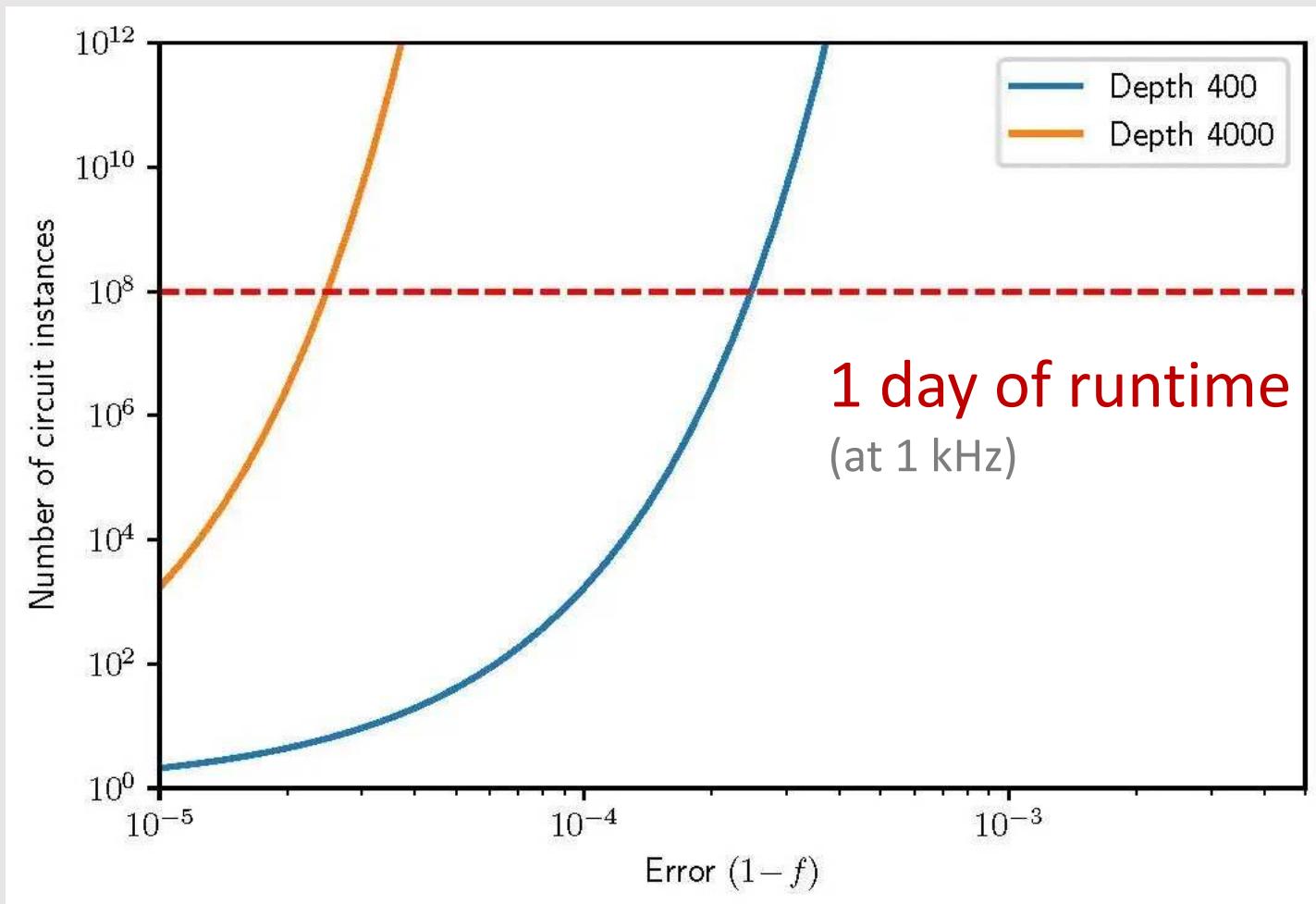


50Q, 2 layers of cNOT gates  
(preliminary data)

Weight of observable

# Path to 100+ qubits?

Estimating PEC overhead for Trotter circuits comprising 100 qubits



See also on speed: A. Wack, et al., Quality, speed, and scale: three key attributes to measure the performance of near-term quantum computers (2021).

*Q: Is it useful to build a quantum computer with a few percent error?*

- John Martinis



# Path to quantum computing

---

Noise-free estimators can be obtained from noisy quantum computers TODAY, at a runtime cost that is exponential in number of qubits  $n$  and circuit depth  $d$

$$\text{Runtime} = \beta d (\bar{\gamma})^{n^d} \text{ seconds}$$

$d$  is the depth of the quantum circuit

$\beta$  is a measure of the time per circuit layer operation (CLOPS)  
(increase by pushing **speed**)

$\bar{\gamma}$  is a measure of the collective quantum noise  
(increasing **quality** brings it closer to 1)

$n$  is the number of operational qubits  
(increase by pushing **scale**)

# Thank you!

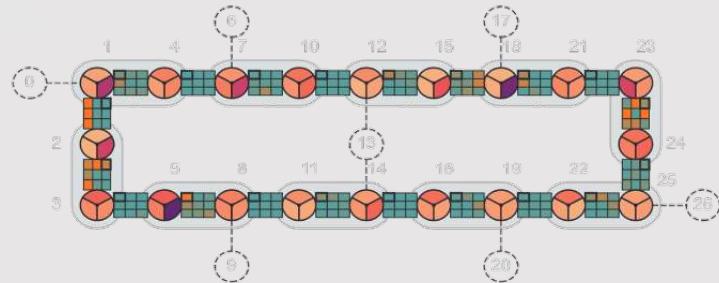
Lindbladian learning is accurate, efficient, and scalable

Powerful characterization and benchmarking tool

Enable the study and mitigation of noise in quantum processors at a new scale

Ewout van den Berg, Zlatko K. Minev, Abhinav Kandala, Kristan Temme  
arXiv:2201.09866 (2022)

Thanks to discussion with many folks on the broader IBM team.



Got Slides?

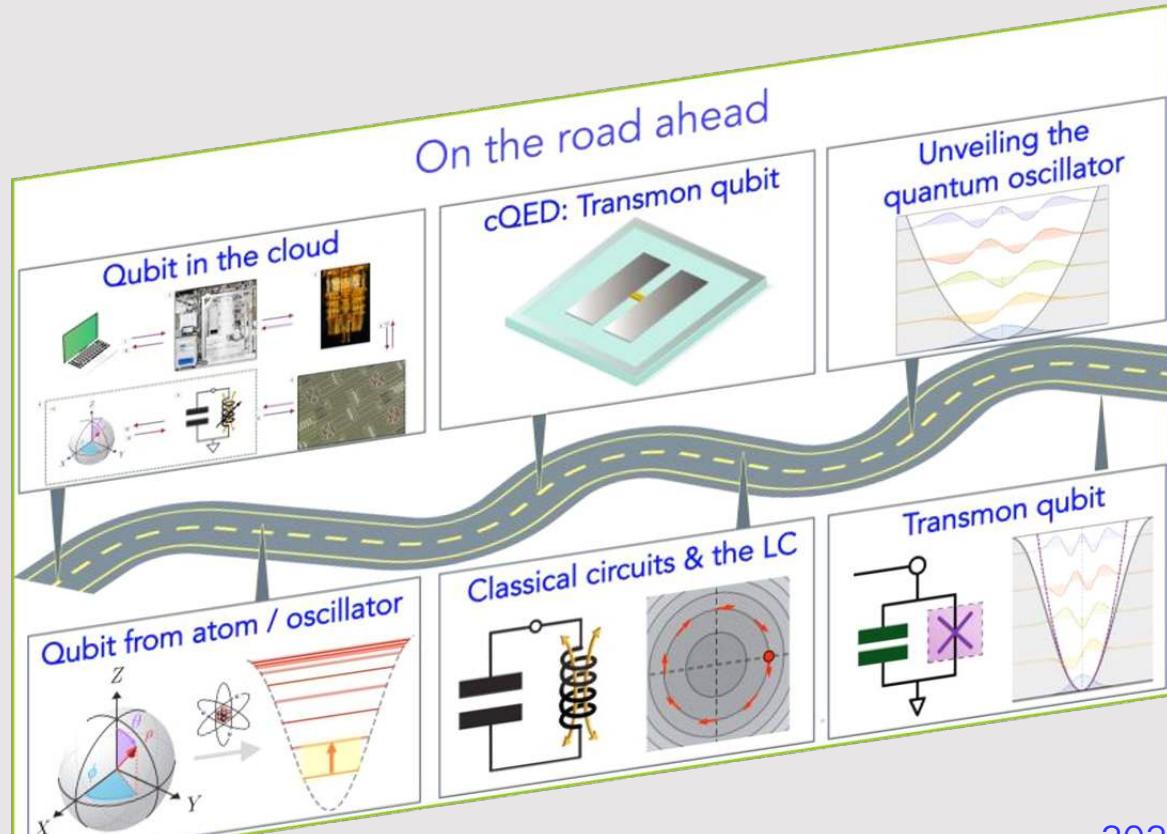


[zlatko-minev.com](http://zlatko-minev.com)

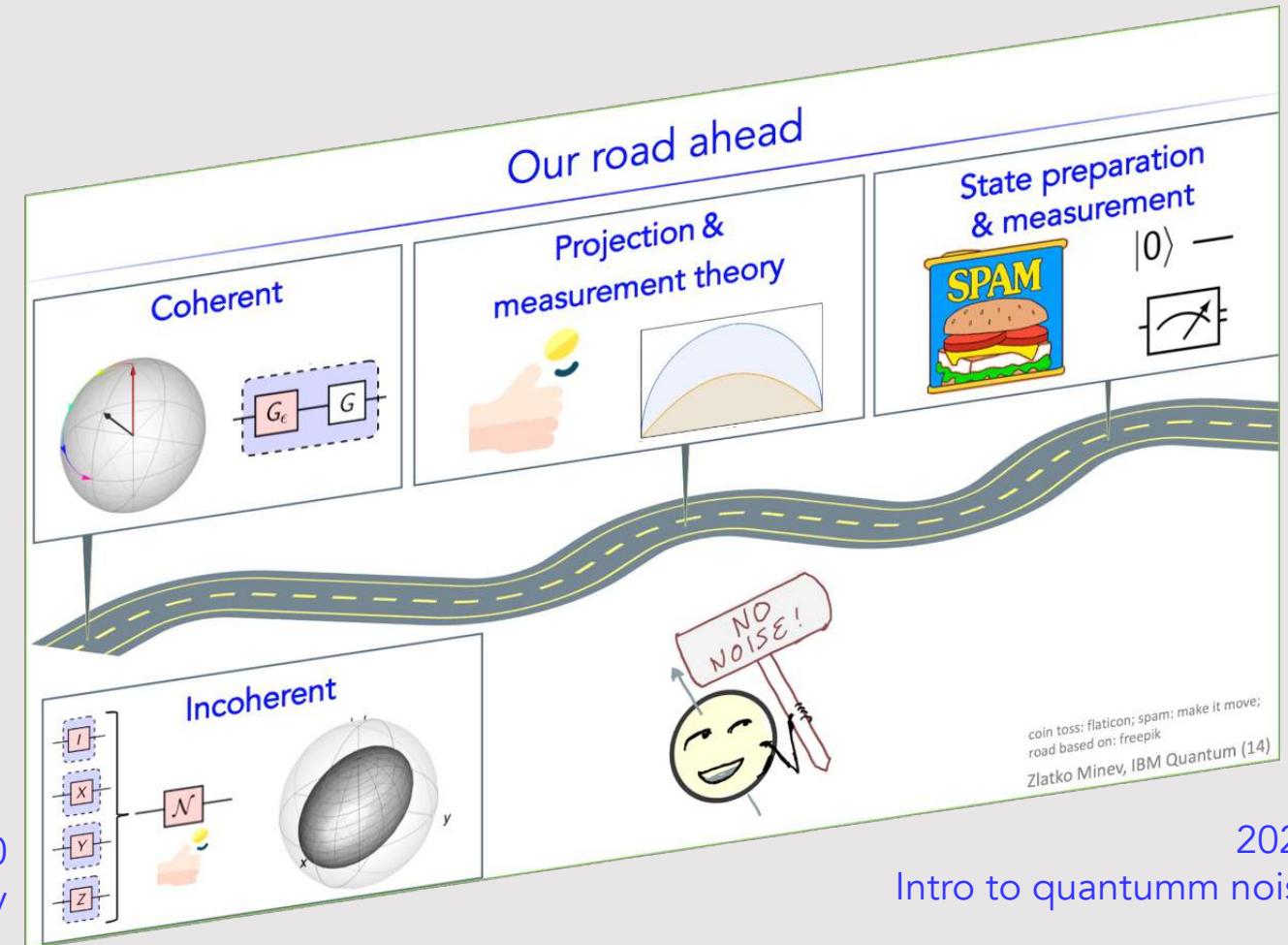


# Global Summer Schools QGSS

On the road ahead



Our road ahead



2022 coming up soon!

## Tech Note T23: Probabilistic error cancellation: paper notes

Summary for key notation of *Probabilistic error cancellation with sparse Pauli-Lindblad models on noisy quantum processors* [Ref. van den Berg et al. (2020)]

Symbol	Description	Value
Circuit constants & indices		
$n$	number of qubits in circuit	$n = 1, 2, 3, \dots$
$l$	number of layers in circuit	$l = 1, 2, 3, \dots$
$i$	layer index within circuit	$i = 1, \dots, l$
Pauli-Lindblad model inputs		
$k$	model coefficient index, corresponds to a Pauli	$k = 1, 2, 3, \dots$ or equiv. $n$ -qubit Pauli string
$b$	fidelity vector indices	$b = 1, 2, \dots$ or equiv. $n$ -qubit Pauli string
$\mathcal{K}$	set of Pauli fidelity support indices for the sparse model $\mathcal{L}$ of $\Lambda$ . Small set	$k \in \mathcal{K}$
$\mathcal{B}$	set of benchmark Paulis (Pauli fidelity support indices of $\Lambda$ ). Can be all of them, big set	$b \in \mathcal{B}$
$N$	number of error-mitigated circuit instances	$N = 1, 2, 3, \dots$
Pauli-Lindblad model variables		
$\gamma$	sampling overhead	$\gamma \geq 1$ , $\gamma = \exp(\sum_{k \in \mathcal{K}} 2\lambda_k)$
$\gamma_i, \gamma(l)$	sampling overhead for the $i$ -th layer and a total of $l$ layers	$\gamma(l) = \prod_{i=1}^l \gamma_i$
$\lambda_k$	$k$ -th model coefficient	$\lambda_k \geq 0$
$w_k$	noise model weight in $\Lambda$ factoring	$w_k := \frac{1}{2}(1 + e^{-2\lambda_k})$
$f_b$	Pauli $\Lambda$ fidelity of $b$ -th Pauli index	$f_b := \frac{1}{2^n} \text{Tr}(P_b^\dagger \Lambda(P_b))$
$f$	vector of Pauli fidelities of $\Lambda$	$f = \{f_b\}_{b \in \mathcal{B}}$
$\hat{f}$	fidelity estimates for a set of	
$M(\mathcal{B}, \mathcal{K})$	binary matrix with entries $M_{b,k} = \langle b, k \rangle_{sp}$ where the symplectic product is 0 if the Paulis commute and 1 if they do not	$\log(f) = -2M(\mathcal{B}, \mathcal{K})\lambda$ (element-wise log) (used to fit with $\lambda \geq 0$ )
Quantum operators and superoperators		
$P_k$	Pauli operator, indexed by $k$	$P_k \in \{I, X, Y, Z\}^{\otimes n}$
$U, \mathcal{U}, \tilde{\mathcal{U}}$	unitary ideal gate; tilde: noisy $i$ -th layer unitary	
$\Lambda, \Lambda_i$	noise channel	$\Lambda(\rho) = \exp[\mathcal{L}](\rho) = \prod_{k \in \mathcal{K}} (w_k \cdot + (1 - w_k)P_k \cdot P_k^\dagger) \rho$
$\Lambda^{-1}$	inverse noise map	$\Lambda^{-1}(\rho) = \exp[-\mathcal{L}](\rho) = \prod_{k \in \mathcal{K}} (w_k \cdot - (1 - w_k)P_k \cdot P_k^\dagger) \rho$
$\mathcal{L}(\rho)$	Lindblad generator	$\mathcal{L}(\rho) = \sum_{k \in \mathcal{K}} \lambda_k (P_k \rho P_k - \rho)$
$\langle \hat{A}_N \rangle$	average error mitigated estimate of $\langle \hat{A} \rangle$ for some operator $\hat{A}$ using $N$ circuit instances	

### 23.1 Common questions

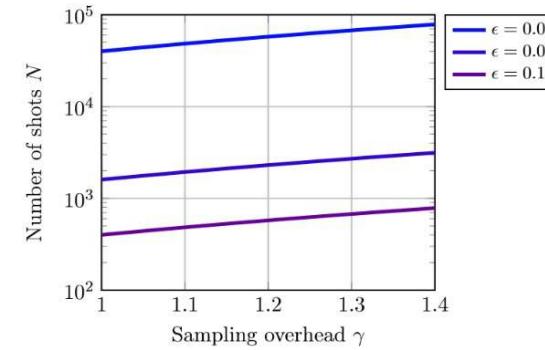
**How many random circuits to run?** For a maximum error bound  $\epsilon$  between the the ideal and mitigated expectation values  $|\langle \hat{A} \rangle_{\text{ideal}} - \langle \hat{A}_N \rangle| \leq \epsilon$  satisfied with a probability  $1 - \delta$  (assuming a weak enough noise,  $C^{lr} \approx 1$ ), we can solve for the number of error-mitigated, random circuit instances  $N$  in van den Berg et al. (2020)

$$\epsilon = \gamma \frac{\sqrt{2 \log(2/\delta)}}{\sqrt{N}},$$

$$\therefore N = 2 \log(2/\delta) \left(\frac{\gamma}{\epsilon}\right)^2,$$

$$N \approx 4 \left(\frac{\gamma}{\epsilon}\right)^2 \text{ for } \delta \sim 0.01.$$

For probability  $\delta = 2\%$ ,  $\log(2/\delta) = 2$ , a small overhead; for  $\delta = 0.01$ , it is 2.3, and for  $\delta = 0.001$ , it is 3.3.



### Bibliography

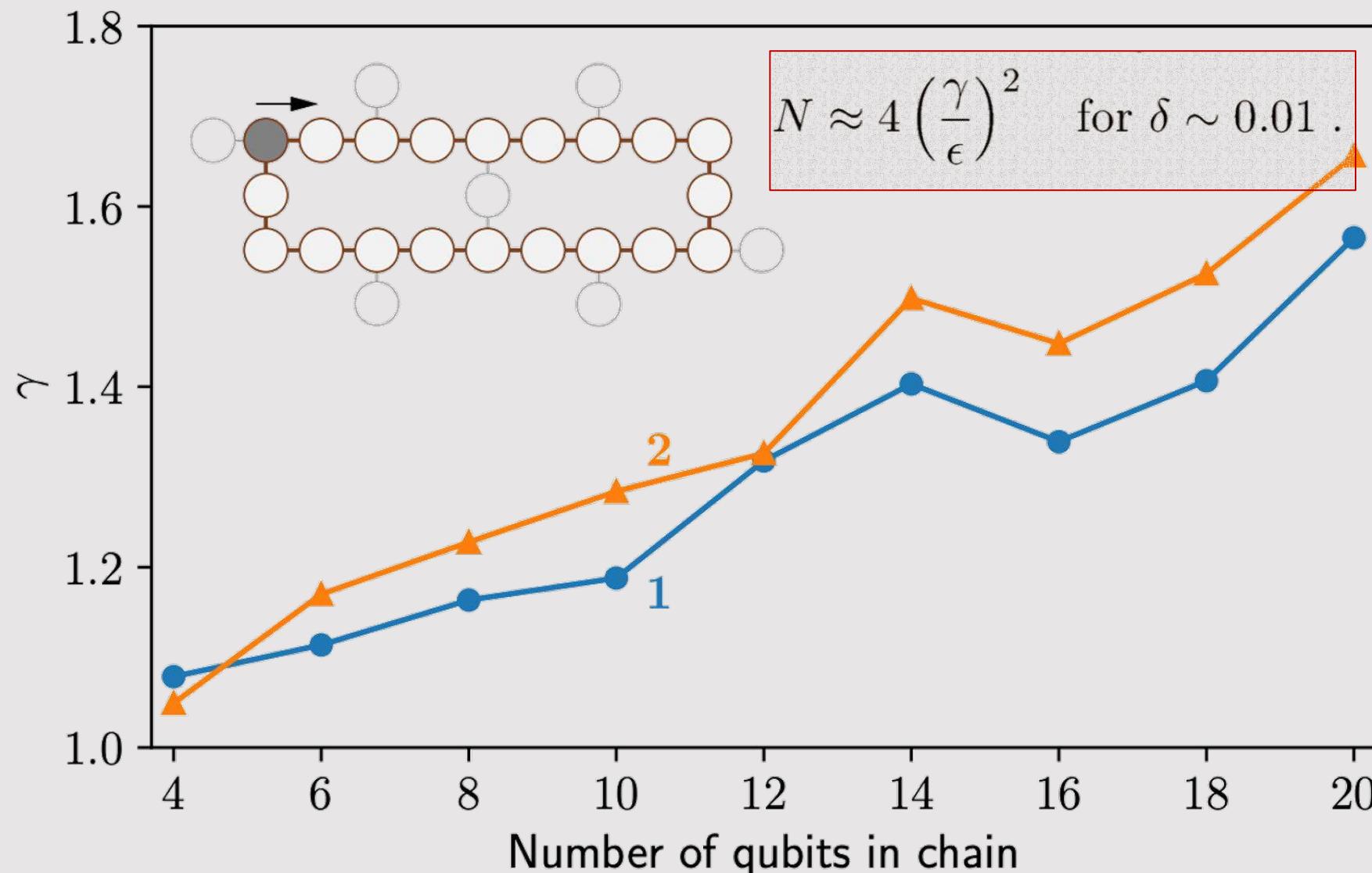
E. van den Berg, Z. K. Minev, and K. Temme, arXiv (2020), ISSN 23318422, 2012.09738, URL <http://arxiv.org/abs/2012.09738>.  
Nature Physics **16**, 233 (2020), ISSN 1745-2473, URL <https://doi.org/10.1038/s41567-020-0847-3>.

**Caveat emptor** These pages are a work in progress, inevitably imperfect, incomplete, and surely enriched with typos and unannounced inaccuracies. Sources credited in Bibliography to the best of my ability, though certain omissions certainly remain.

**Notes by** ©Zlatko K. Minev (zlatko-minev.com)

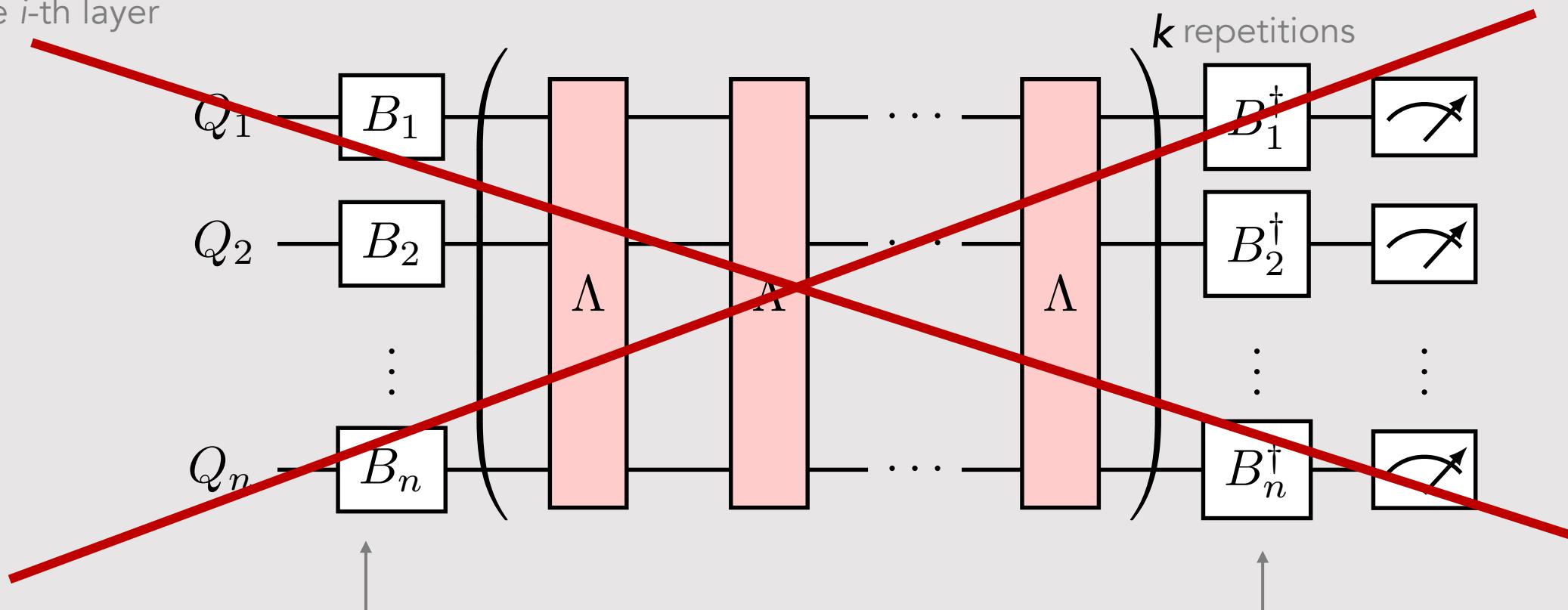
**Paper and talk notation summary:**  
<https://www.zlatko-minev.com/blog/pec-notation>

# Mitigation sampling overhead



# Step 2 wish: amplify noise

for the  $i$ -th layer



Since diagonal channel  
will amplify eigenvalues  
learn with multiplicative precision

Akin to:

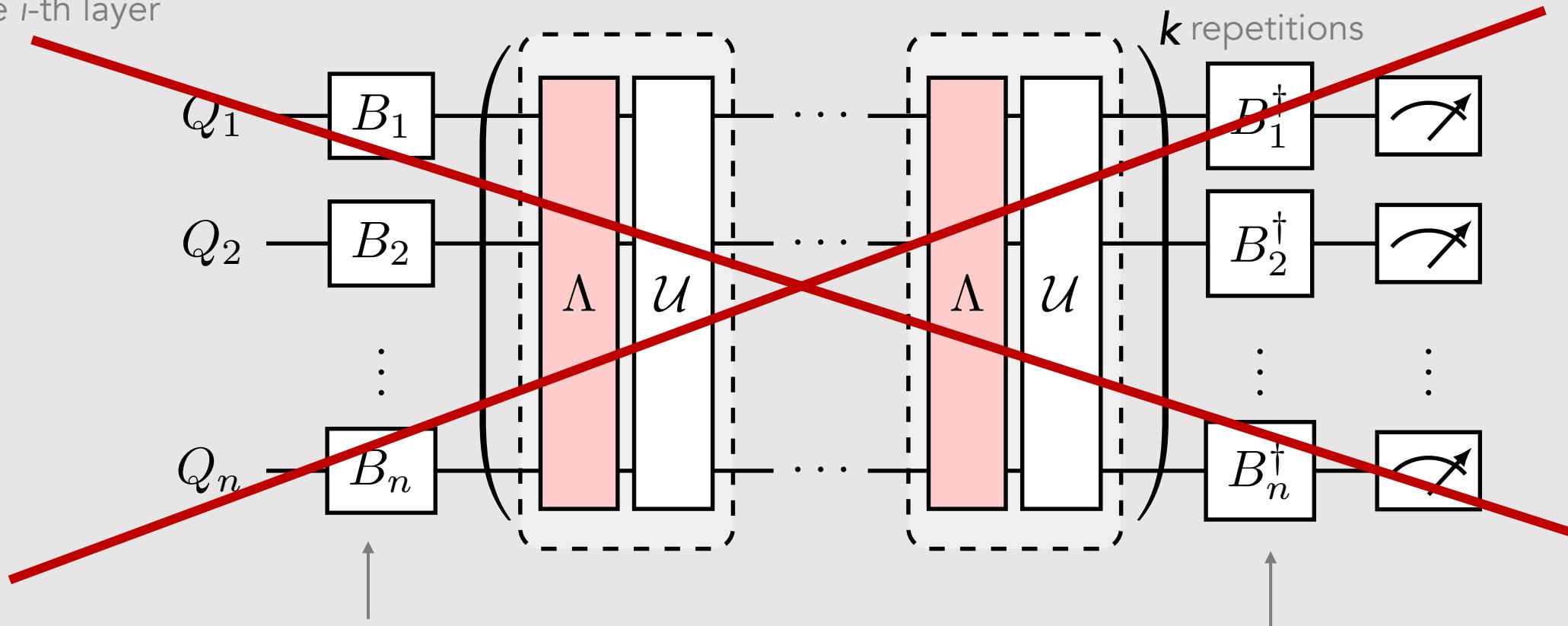
RB, Cycle RB, K-body noise reconstruction, ...

S.T. Flammia and J.J. Wallman ACM Trans QC 1, 3 (2020), ...

For something of a review of protocols, see Helsen, et al., *A general framework for randomized benchmarking* (arXiv:2010.07974)

# Step 2 wish: amplify noise with gates?

for the  $i$ -th layer



prepare circuit in pre-determined Pauli basis

Since diagonal channel will amplify eigenvalues learn with multiplicative precision

measure circuit in same pre-determined Pauli basis

Akin to:

RB, Cycle RB, K-body noise reconstruction, ...

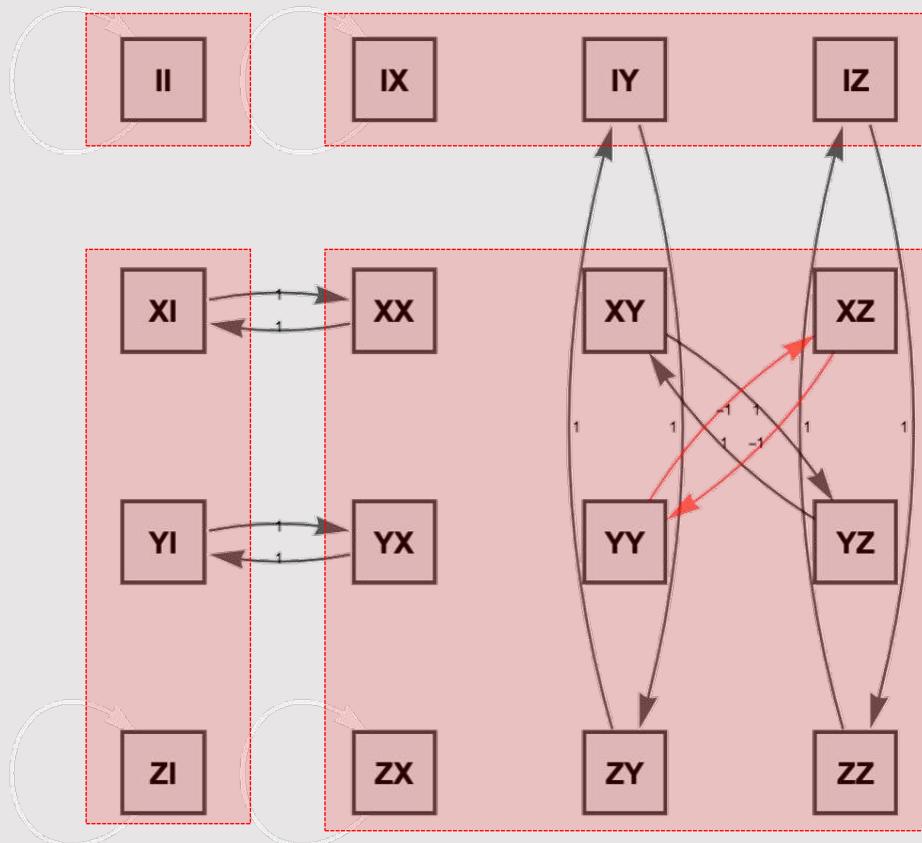
S.T. Flammia and J.J. Wallman ACM Trans QC 1, 3 (2020), ...

Erhard *et al.*, arXiv:1902.08543; Ferracin *et al.*, arXiv:2201.10672, ...

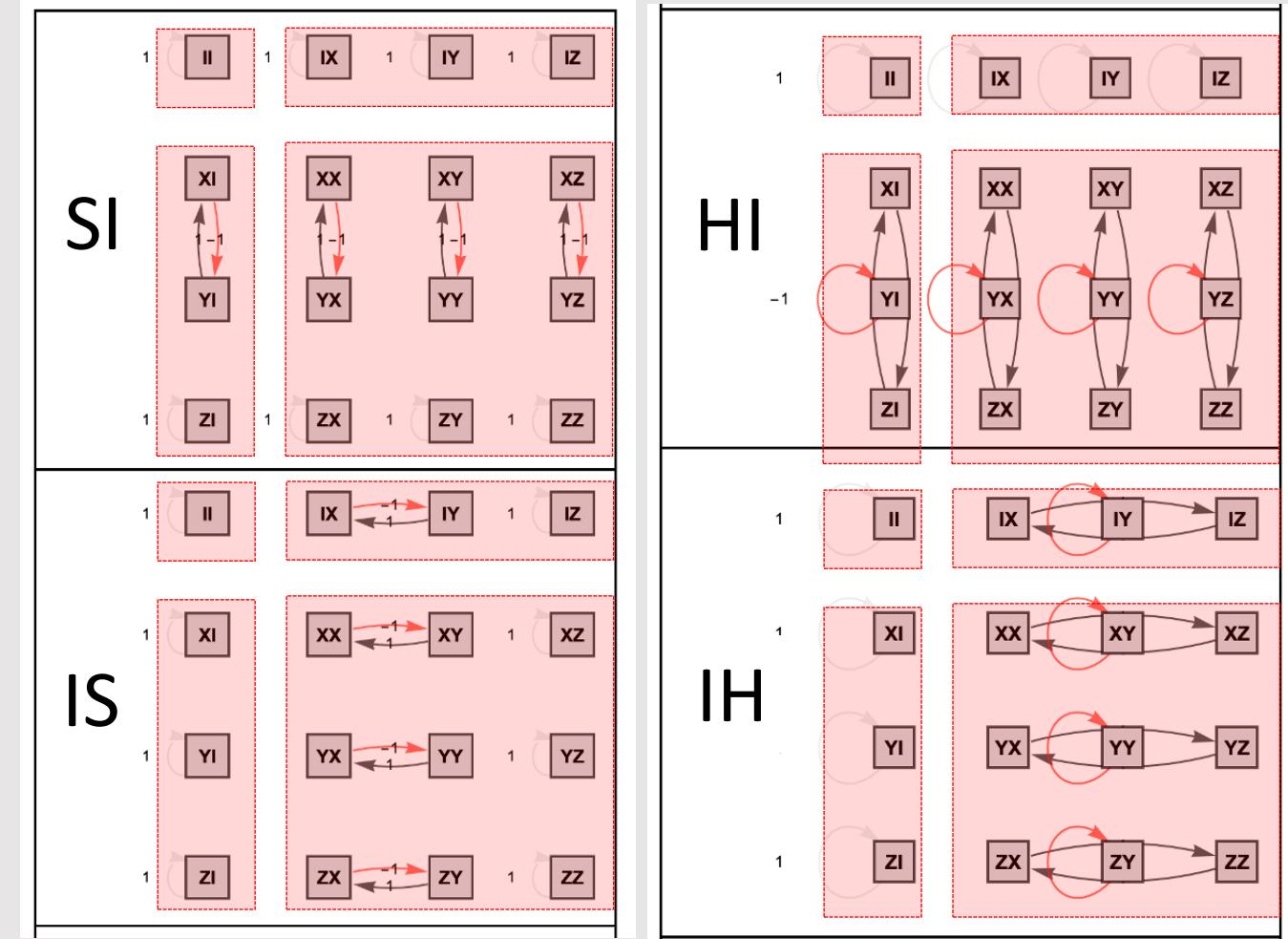
For something of a review of protocols, see Helsen, *et al.*, arXiv:2010.07974

# How to gates move state Paulis around?

cNOT



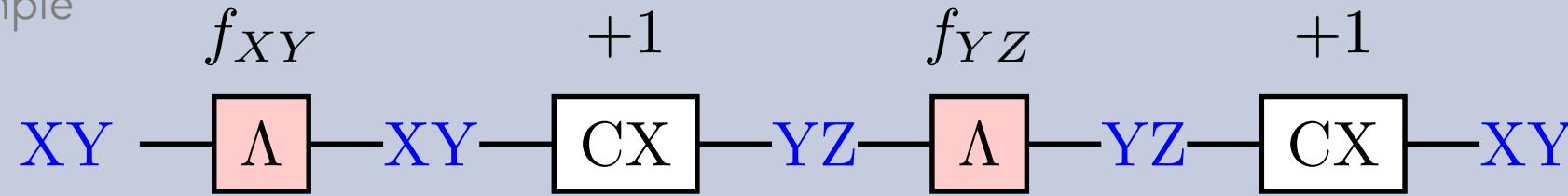
Example single qubit gates



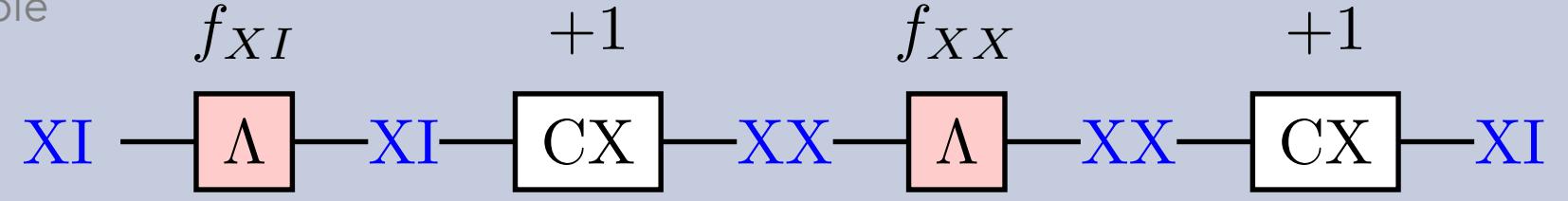
# Let's see how the amplification works with gates: no-go theorem

$$\Lambda(P_a) = f_a P_a$$

2Q example

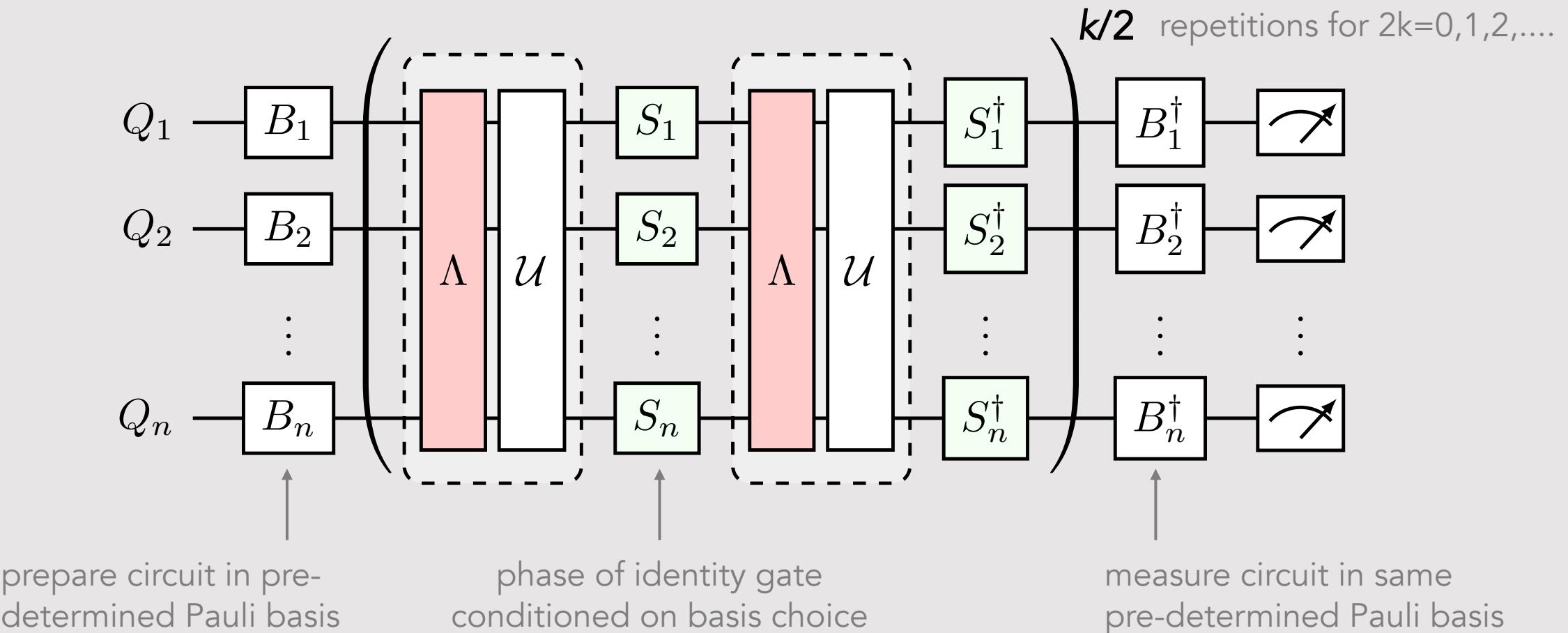


2Q example



fundamental degeneracy – can not undo some non-local – need entangling operation

## Step 2: Amplify & learn noise per layer



Akin to:

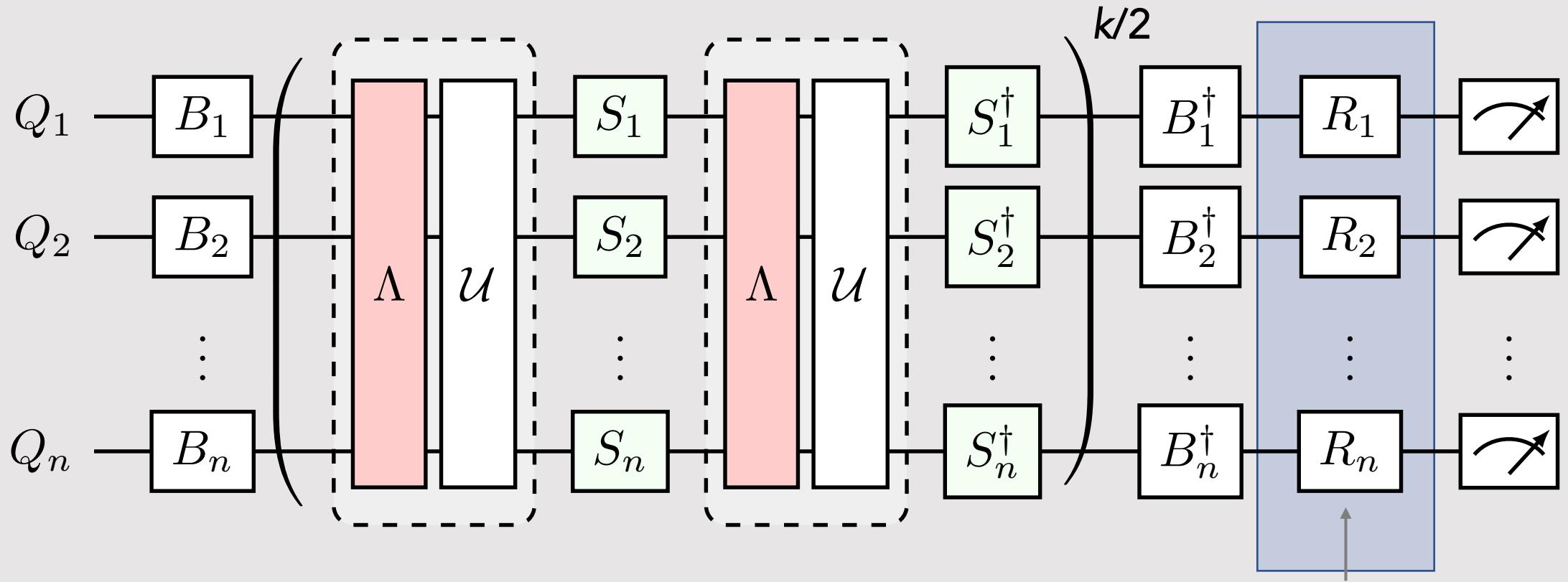
RB, Cycle RB, K-body noise reconstruction, ...

S.T. Flammia and J.J. Wallman ACM Trans QC 1, 3 (2020), ...

Erhard *et al.*, arXiv:1902.08543; Ferracin *et al.*, arXiv:2201.10672, ...

...

# Step 3: Twirl readout-error mitigation



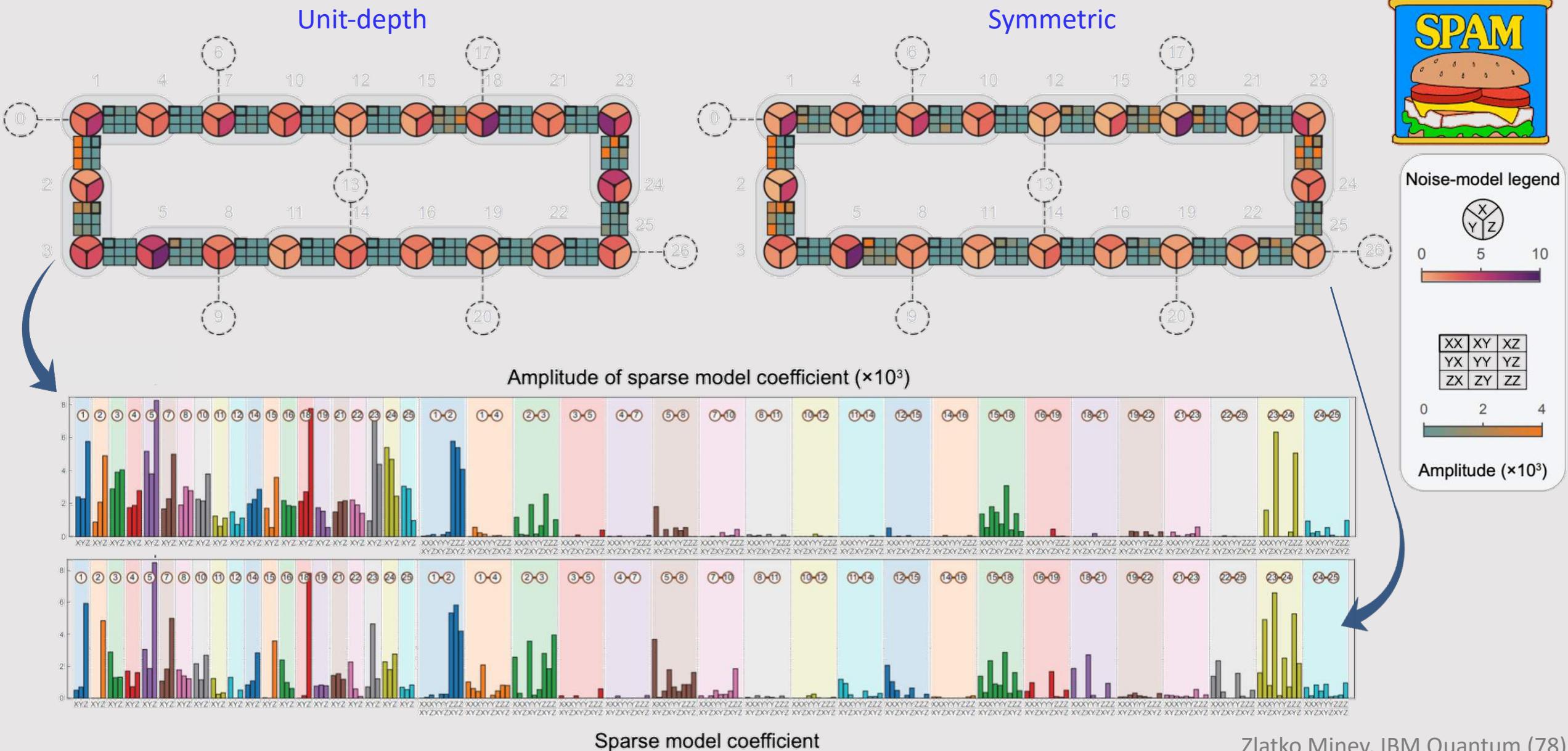
twirl readout to do readout  
error mitigation in-situ\*

\* Model-free readout-error mitigation for quantum expectation values

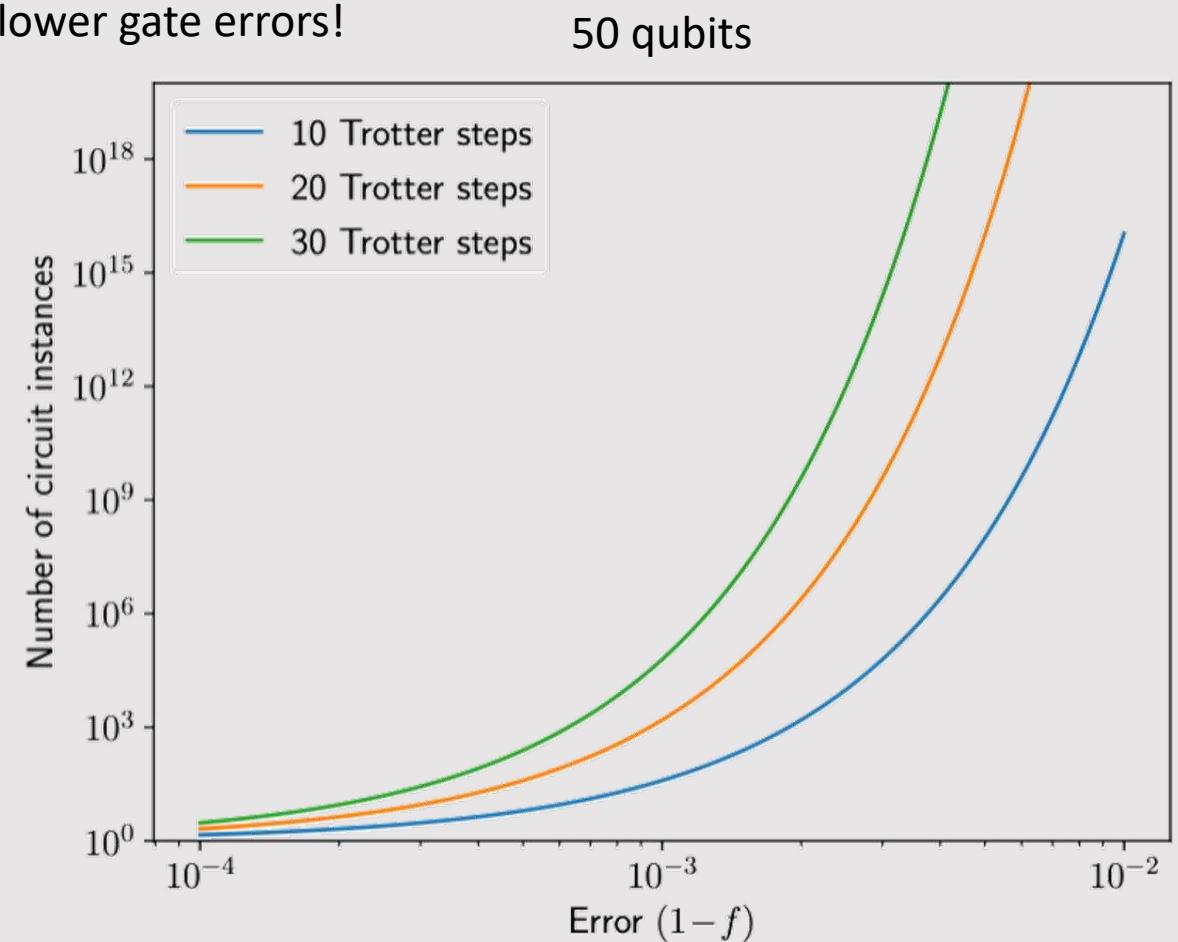
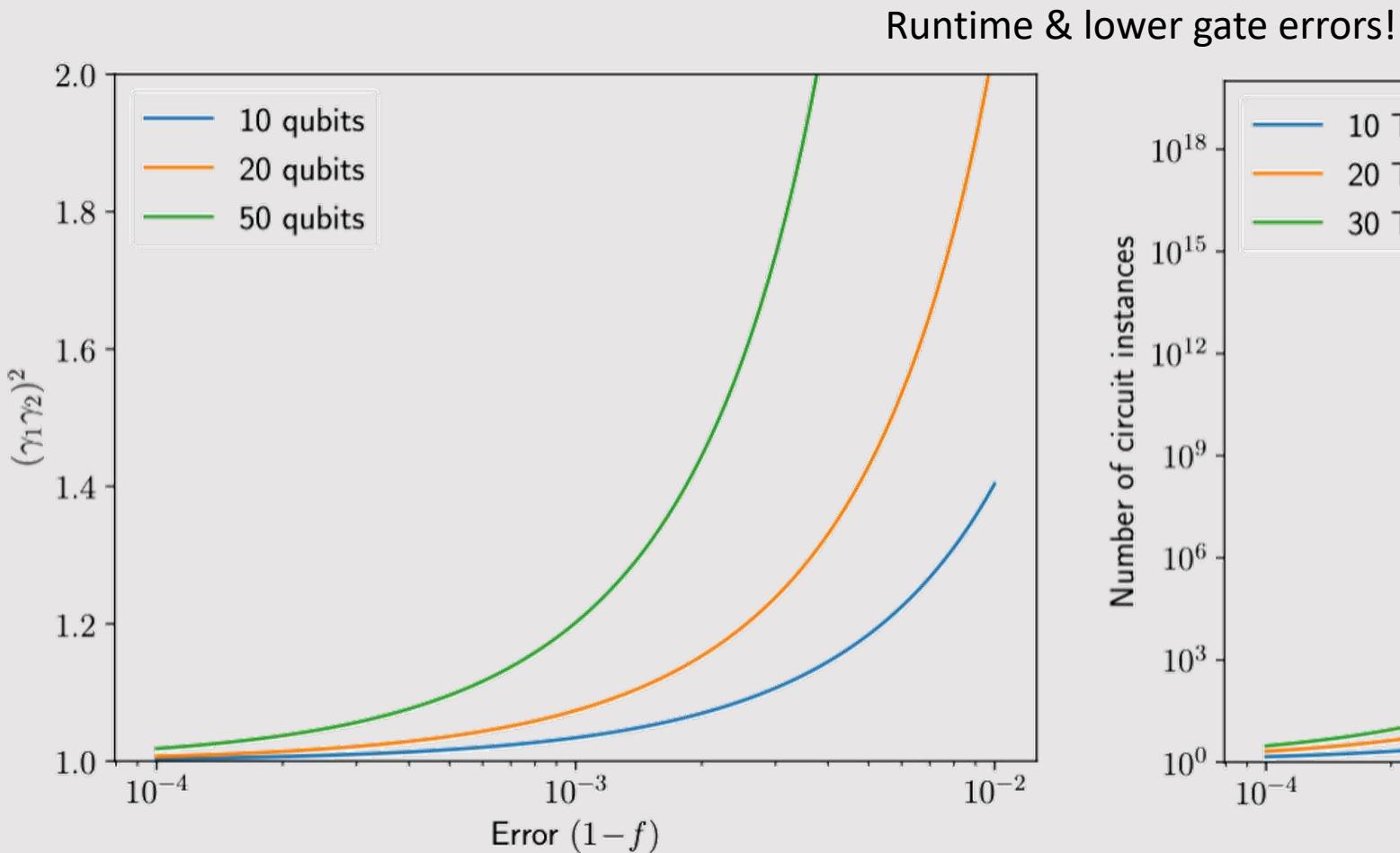
Ewout van den Berg, Zlatko K. Minev, and Kristan Temme

Phys. Rev. A 105, 032620 (2022)

# Complete (SPAM-sensitive) vs symmetry (SPAM-free) learning



# Outlook



# Bit-string probability distributions

From  $2^n$  Pauli expectation values to  $2^n$  bitstring probabilities  
Walsh-Hadamard (WH) transform

$$\begin{array}{lll} p(00\dots 00) & \xrightarrow{\text{WH}} & p(II\dots II) \\ p(00\dots 01) & & p(II\dots IZ) \\ p(00\dots 10) & & p(II\dots ZI) \\ \dots & & \dots \\ p(11\dots 11) & \xrightarrow{\text{WH}^{-1}} & p(ZZ\dots ZZ) \end{array}$$

Example for 2 qubits

	00	01	10	11
11	1	1	(	)
1Z	1	-1	(	-1
Z1	1	1	-1	-1
ZZ	1	-1	-1	1