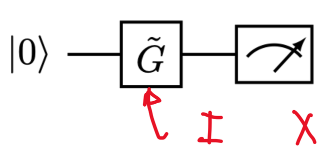
Monday, July 5, 2021 9:23 AM

Introduction to quantum noise

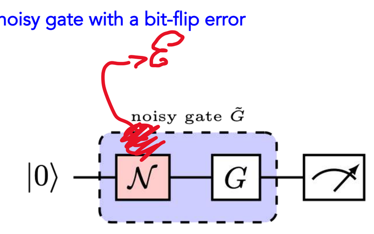
Incoherent noise & Quantum trajectories

Qiskit Global Summer School on Quantum Machine Learning
Zlatko K. Minev

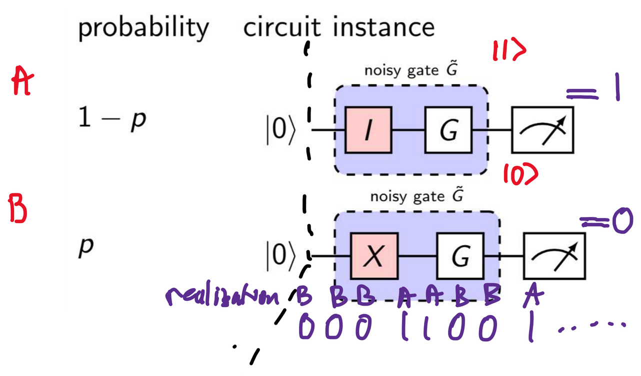
Quantum jumps: Unraveling of a quantum trajectory



Model of a noisy gate with a bit-flip error



$G = X$



$\mathcal{U}_A = GI = G = X$ $\quad 1-p$

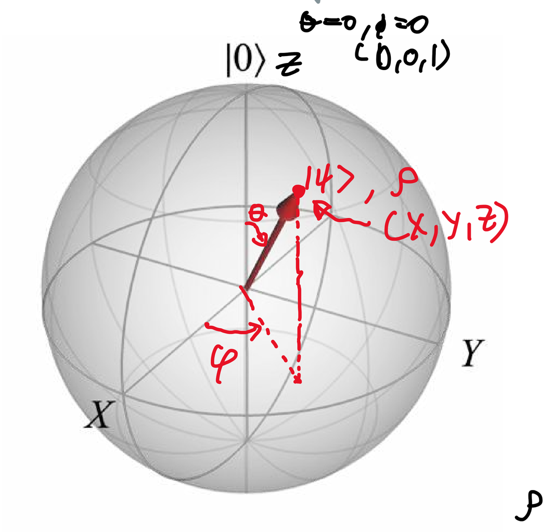
$\mathcal{U}_B = GX = XX = I$ $\quad p$

$\langle \hat{M} \rangle = \underbrace{(1-p)}_{\text{no error}} 1 + \underbrace{p}_{\text{error}} 0 = 1-p$

ideal $\langle \hat{M} \rangle = 1$

$\rho_0 = (1-p)\, |0\rangle\langle 0| + p\, |0\rangle\langle 0| = |0\rangle\langle 0|$ $\qquad \rho = |\psi\rangle\langle\psi| = \vec{\psi} \cdot \vec{\psi}^{+}$

$|0\rangle \rightleftarrows (0,0,1)$, $\theta = 0, \phi = 0$



$|\psi\rangle = \begin{pmatrix} \cos\theta/2 \\ \sin\theta/2\, e^{i\phi} \end{pmatrix}$

$\langle\psi|\psi\rangle = \left(\cos\theta/2 \quad \sin\frac{\theta}{2} e^{-i\phi}\right) \cdot \begin{pmatrix} \vdots \end{pmatrix}$

$= (\cos\theta/2)^2 + \left[\sin(\tfrac{\theta}{2})\, e^{-i\phi} e^{i\phi}\right]^2$

$= \cos^2 + \sin^2$

$= 1$

$0 \le \theta \le \pi \qquad 0 \le \phi \le 2\pi$

$\rho = |\psi\rangle\langle\psi| = \begin{pmatrix} \cos\theta/2 \\ \sin\theta/2\, e^{+i\phi} \end{pmatrix} \left(\cos\theta/2 \quad \sin\frac{\theta}{2} e^{-i\phi} \right)$

$= \begin{pmatrix} (\cos\theta/2)^2 & \cos\theta/2 \sin\theta/2\, e^{-i\phi} \\ \cos\theta/2 \sin\theta/2\, e^{+i\phi} & (\sin\theta/2)^2 \end{pmatrix}$

$(\cos\theta/2 \sin\theta/2 = \frac{1}{2}\sin\theta)$

$= \begin{pmatrix} \cos^2\theta/2 & \frac{1}{2}\sin\theta[\cos\phi - i\sin\phi] \\ \text{H.c.} & \sin^2\theta/2 \end{pmatrix}$

$= \frac{1}{2}(\hat{I} + x\hat{X} + y\hat{Y} + z\hat{Z}) \leftarrow$

$\langle \hat{X} \rangle = x = \sin\theta\cos\phi = \mathrm{Tr}[\hat{X}\rho]$ $\quad \hat{X} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$\langle \hat{Y} \rangle = y = \sin\theta\sin\phi = \mathrm{Tr}[\hat{Y}\rho]$ $\quad \mathrm{Tr}[I] = 2$

$\langle \hat{Z} \rangle = z = \cos\theta = \mathrm{Tr}[\hat{Z}\rho]$ $\quad \mathrm{Tr}[X^2] = \mathrm{Tr}[I] = 2$

Case A: $|\psi_A\rangle = \mathcal{U}_A|0\rangle = |1\rangle \Rightarrow \rho_A = |\psi_A\rangle\langle\psi_A| = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$

Case B: $|\psi_B\rangle = \mathcal{U}_B|0\rangle = |0\rangle \Rightarrow \rho_B = |\psi_B\rangle\langle\psi_B| = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

Ensemble: $\rho = P_A\rho_A + \overset{\text{error}}{P_B}\rho_B$ $\quad = P_A U_A|0\rangle\langle 0|U_A^{+} + P_B U_B|0\rangle\langle 0|U_B^{+}$ $\quad |\psi_B\rangle = U_B|0\rangle$, $|\psi_A\rangle = U_A|0\rangle$

$= (1-p)\, |\psi_A\rangle\langle\psi_A| + p\, |\psi_B\rangle\langle\psi_B|$ $\quad = (1-p)\, U_A|0\rangle\langle 0|U_A^{+} + p\, U_B|0\rangle\langle 0|U_B^{+}$

$= (1-p)\, |1\rangle\langle 1| + p\, |0\rangle\langle 0|$ $\qquad |\psi_A\rangle^{+} = \langle\psi_A|$

$= 1-p \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} + p \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ $\qquad (U|0\rangle)^{+} = |0\rangle^{+}U^{+} = \langle 0|U^{+} =$

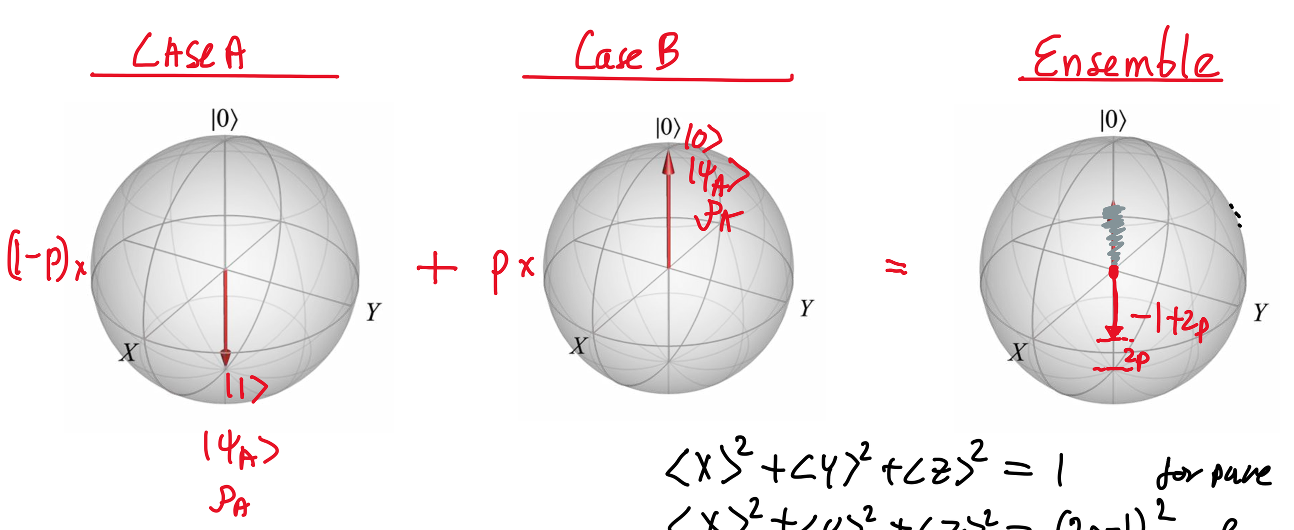
$= \begin{array}{c|cc} & \langle 0| & \langle 1| \\ \hline |0\rangle & p & 0 \\ |1\rangle & 0 & 1-p \end{array}$ $\qquad (AB)^{+} = B^{+}A^{+}$

Bloch $\quad x = \mathrm{Tr}(\hat{X}\rho) = 0$ $\qquad \mathrm{Tr}[P_\alpha P_\beta] = \delta_{\alpha\beta} 2$

$y = \mathrm{Tr}(\hat{Y}\rho) = 0$ $\qquad \alpha, \beta \in \{X, Y, Z\}$

$z = \mathrm{Tr}(\hat{Z}\rho) = \mathrm{Tr}\left(\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} p & 0 \\ 0 & 1-p \end{pmatrix}\right) = \mathrm{Tr}\begin{pmatrix} p & 0 \\ 0 & p-1 \end{pmatrix} = 2p-1$

$\rho_{final} = \mathcal{E}(\rho_{in}) = \sum_K P_K U_K \rho_{in} U_K^{+}$ $\qquad K \in \{A, B\}$



$\langle X \rangle^2 + \langle Y \rangle^2 + \langle Z \rangle^2 = 1$ for pure

$\langle X \rangle^2 + \langle Y \rangle^2 + \langle Z \rangle^2 = (2p-1)^2$ for our impure state

$\left(-\boxed{\tilde{G}}- \right)^d -\boxed{\nearrow}-$

$\rho_{in} -\boxed{\tilde{G}}- = \rho_{in} -\boxed{\mathcal{E}}-\boxed{X}-$

$$\boxed{\begin{aligned} \rho &= \frac{1}{2}(I + x\hat{X} + y\hat{Y} + z\hat{Z}) \\ &= \frac{1}{2}\left(\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & x \\ x & 0 \end{pmatrix} + \begin{pmatrix} 0 & -iy \\ iy & 0 \end{pmatrix} + \begin{pmatrix} z & 0 \\ 0 & -z \end{pmatrix}\right) \\ &= \frac{1}{2}\begin{pmatrix} 1+z & x-iy \\ x+iy & 1-z \end{pmatrix} \qquad x, y, z \in [-1, 1],\ x^2+y^2+z^2 \le 1 \end{aligned}}$$

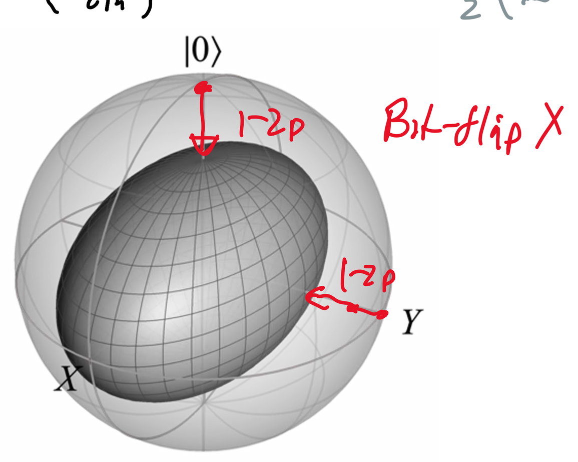
$\rho_{in} = \frac{1}{2}(I + x_{in}\hat{X} + y_{in}\hat{Y} + z_{in}\hat{Z})$

$\rho_{out} = \mathcal{E}(\rho_{in}) = (1-p)\, I\rho_{in}I + p\, X\rho_{in}X$ $\qquad X\rho X = X\frac{1}{2}(\hat{I} + x\hat{X} + \cdots)X$

$= (1-p)\begin{pmatrix} x_{in} \\ y_{in} \\ z_{in} \end{pmatrix} + p\begin{pmatrix} +x_{in} \\ -y_{in} \\ -z_{in} \end{pmatrix}$ $\qquad = \frac{1}{2}(\hat{X}\hat{I}\hat{X} + x\hat{X}\hat{X}\hat{X} + \cdots)$

$\qquad = \frac{1}{2}(\hat{I} + x\hat{X} - y\hat{Y} - z\hat{Z})$

$= \begin{pmatrix} x_{in} \\ (1-2p)\, y_{in} \\ (1-2p)\, z_{in} \end{pmatrix}$ error



$X\rho_{in}X = \begin{pmatrix} x_{in} \\ -y_{in} \\ -z_{in} \end{pmatrix}$

$\tilde{G}^d(\rho) = \cdots X\mathcal{E}(X\mathcal{E}(\rho)X)X\cdots = \begin{pmatrix} x_{in} \\ (-(1-2p))^d y_{in} \\ (-(1-2p))^d z_{in} \end{pmatrix} = \rho_f(d)$

$$\rho_{in} = |0\rangle\langle 0| = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \xrightarrow{\tilde{G}^d} \begin{pmatrix} 0 \\ 0 \\ (2p-1)^d \end{pmatrix}$$

$$\langle \tilde{Z} \rangle = \mathrm{Tr}(Z \rho_{out}) = z_{out} = (2p-1)^d \qquad \text{if } p=0 \Rightarrow \langle Z \rangle_{ideal} = (-1)^d$$

$$0 \le p \le 1$$

$$\langle \tilde{Z} \rangle = (2p-1)^d \simeq 1 - 2dp$$

error

Linear in p