QGSS22 Intro to projective noise

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Introduction to quantum noise
Measurement theory & projection noise

Qiskit Global Summer School on Quantum Simulations

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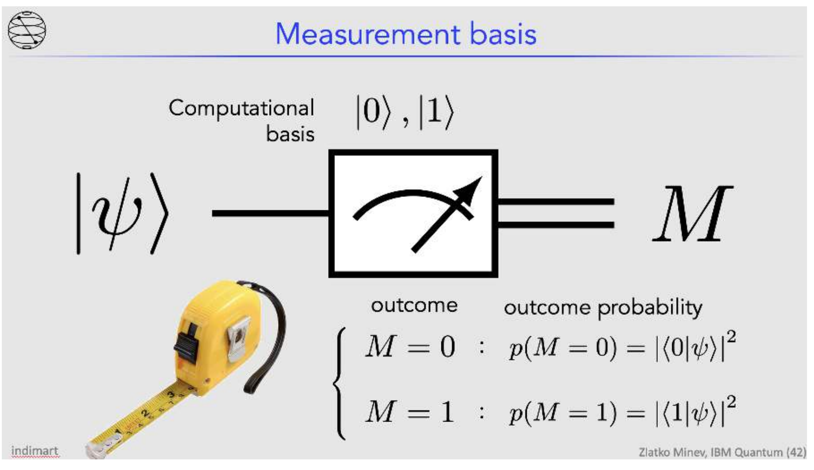
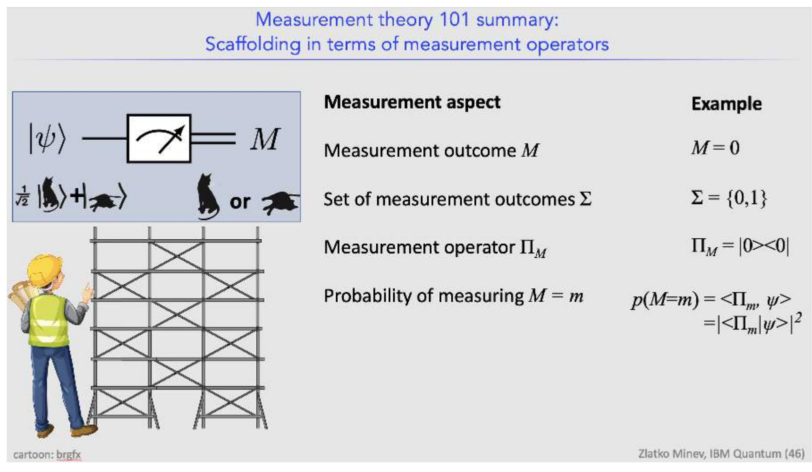
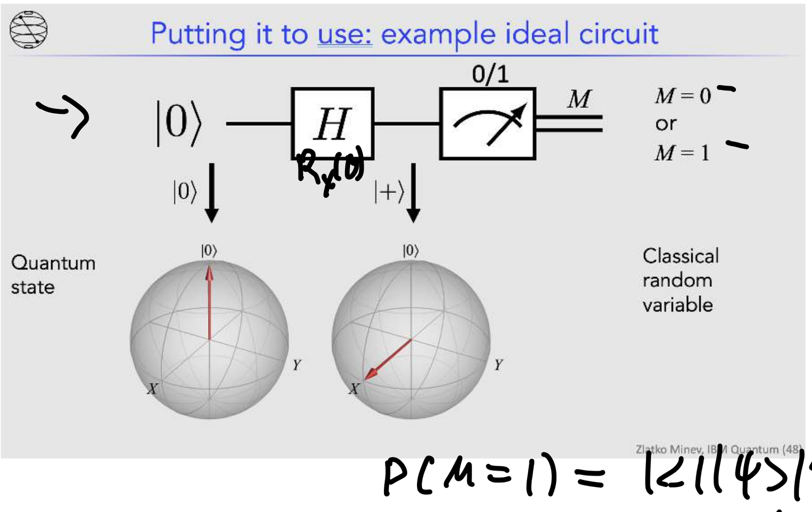






$\Sigma = \{0,1\}$

$\hat{\Pi}_0 = |0\rangle\langle 0|$

$\hat{\Pi}_1 = |1\rangle\langle 1|$

$|\psi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$

$|\psi\rangle = \begin{pmatrix}1\\0\end{pmatrix}$

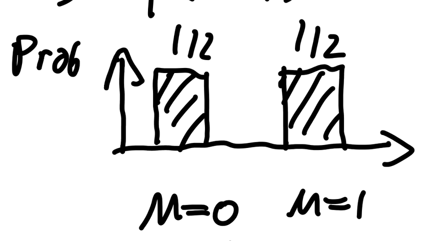
$|\psi\rangle^\dagger = \begin{pmatrix}1\\0\end{pmatrix}^\dagger = (1 \;\; 0)$

$$\begin{aligned}
p(M=1) &= |\langle 1|\psi\rangle|^2 \\
&= \langle\psi|\hat{\Pi}_1|\psi\rangle \\
&= \tfrac{1}{2}(1\;1)\begin{pmatrix}0&0\\0&1\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix} \\
&= \tfrac{1}{2}(1\;1)\begin{pmatrix}0\\1\end{pmatrix} \\
&= \tfrac{1}{2}\cdot 1 \\
&= \tfrac{1}{2}
\end{aligned}$$

$$\begin{aligned}
p(M=0) &= |\langle 0|\psi\rangle|^2 \\
&= \hat{\Pi}_0|\psi\rangle \\
&= \tfrac{1}{2}(1\;1)\begin{pmatrix}1&0\\0&0\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix} \\
&= \tfrac{1}{2}(1\;1)\cdot\begin{pmatrix}1\\0\end{pmatrix} \\
&= \tfrac{1}{2}\cdot 1 \\
&= \tfrac{1}{2}
\end{aligned}$$

$p(M=1) + p(M=0) = 1 \Rightarrow p(M=1) = 1 - p(M=0)$

$M : \begin{cases} 1 & p = 1/2 \\ 0 & p = 1/2 \end{cases}$



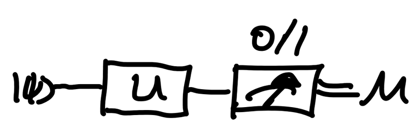
Statistics: Mean

Random variable M drawn from distribution

$$\mathbb{E}[M] = \sum_{m\in\Sigma} m\, p(M=m)$$

classical expectation value

$$= 0\cdot p(M=0) + 1\, p(M=1) = 0\tfrac{1}{2} + 1\tfrac{1}{2} = \tfrac{1}{2}$$



more general case

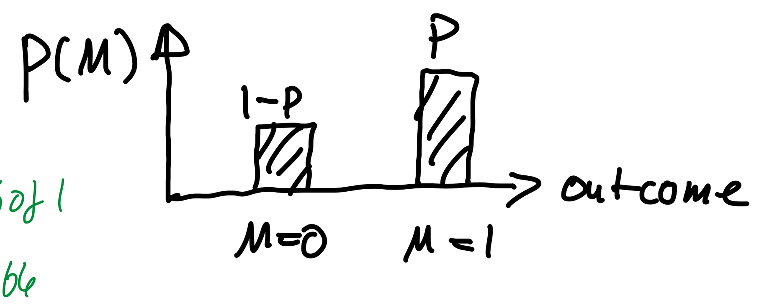
Prob to find $M=1$: $p(M=1) = p$ $\quad p \in [0,1]$

every possible qubit case

Bernoulli Distribution

$p(M) \sim B(p)$ ← prob of 1

classical probability ← classical rand variable



$$\begin{aligned}
\mathbb{E}[M] &= \sum_{m\in\Sigma} m\, p(M=m) \\
&= 0\, p(M=0) + 1\, p(M=1) \\
&= 0(1-p) + 1\cdot p \\
&= p
\end{aligned}$$

classical

$$\begin{aligned}
\mathbb{E}[M] &= \sum_{m\in\Sigma} m\, p(M=m) \\
&= \sum_{m\in\Sigma} m \langle\hat{\Pi}_m\rangle \\
&= \sum_{m\in\Sigma} m \langle\psi|\hat{\Pi}_m|\psi\rangle \\
&= \langle\psi|\boxed{\sum_{m\in\Sigma} m\hat{\Pi}_m}|\psi\rangle \\
&= \langle\psi|\hat{M}|\psi\rangle \\
&= \langle\hat{M}\rangle
\end{aligned}$$

Quantum

Observable

$\langle\hat{\Pi}_m\rangle = \langle\psi|\hat{\Pi}_m|\psi\rangle = \langle\psi|m\rangle\langle m|\psi\rangle$

linear

$\langle a\hat{B}\rangle = a\langle\hat{B}\rangle$

$\langle\hat{A}+\hat{B}\rangle = \langle\hat{A}\rangle + \langle\hat{B}\rangle$

define

$\hat{M} := \sum_{m\in\Sigma} m\hat{\Pi}_m$

defn

Different observables

For example $\hat{M} = 0|0\rangle\langle 0| + 1|1\rangle\langle 1| = \frac{1}{2}(\hat{I}+\hat{Z}) = \begin{pmatrix}0&0\\0&1\end{pmatrix}$

$\hat{M} = (+1)|0\rangle\langle 0| - 1|1\rangle\langle 1| = \hat{Z} = \begin{pmatrix}1&0\\0&-1\end{pmatrix}$

For $\hat{M} = \hat{Z}$ $\quad \Sigma = \{-1, +1\}$ $\quad$ Pauli Observables

$\hat{M} = \hat{X}$

$$\mathcal{S} = \{+, -\}$$
$$\hat{\Pi}_+ = |+\rangle\langle +|$$
$$\hat{\Pi}_- = |-\rangle\langle -|$$
$$\hat{X} = \sum_{m \in \mathcal{S}} m |m\rangle\langle m|$$
$$= +1 |+\rangle\langle +| - |-\rangle\langle -|$$

$$\hat{X}|+\rangle = +1|x\rangle$$
$$\hat{X}|-\rangle = -1|x\rangle$$
$$|+\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$$
$$|-\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$$

Statistic: Variance

$$\mathbb{V}[M] = \mathbb{E}[M^2] - \mathbb{E}[M]^2$$
$$= \underline{\langle \hat{M}^2 \rangle} - \langle \hat{M} \rangle^2$$

$$\langle \hat{M} \rangle = \langle \psi | \hat{M} | \psi \rangle$$
$$\langle \hat{M}^2 \rangle = \langle \psi | \hat{M} \cdot \hat{M} \cdot | \psi \rangle$$

Return to $\mathcal{S} = \{0, 1\}$
$$\hat{\Pi}_0 = |0\rangle\langle 0|$$
$$\hat{\Pi}_1 = |1\rangle\langle 1|$$
$$p(M=1) = p$$

$$\mathbb{E}[M^2] = \sum_{m \in \mathcal{S}} m^2 \, p(M=m)$$

$$p(M=m) = \langle |m\rangle\langle m| \rangle$$

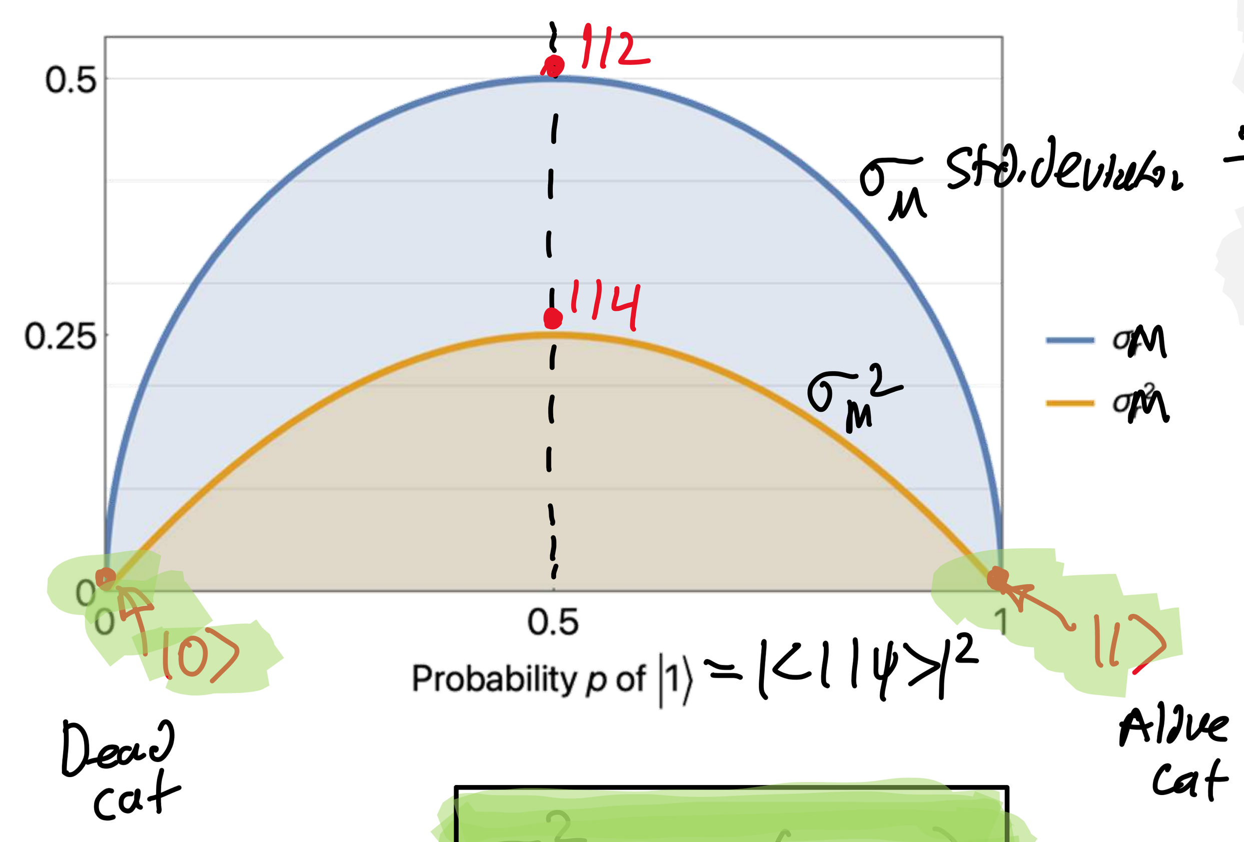
$$= \sum m^2 \langle |m\rangle\langle m| \rangle$$
$$= \left\langle \sum m^2 |m\rangle\langle m| \right\rangle$$
$$= \langle \hat{M}^2 \rangle$$
$$= \langle |1\rangle\langle 1| \rangle$$
$$= \langle \hat{\Pi}_1 \rangle$$
$$= p = \mathbb{E}[M]$$

$$\hat{M} = 0|0\rangle\langle 0| + 1|1\rangle\langle 1|$$
$$\hat{M}^2 = |1\rangle\langle 1| = M$$

$$\mathbb{V}[M] = p - p^2$$
$$= p(1-p)$$
$$= \sigma_M^2$$

0 iff $p = 0$ or $p = 1$

Variance of the random classical variable vs. probability to obtain 1



$$\boxed{\sigma_M^2 = p(1-p)}$$

Quantum system prepared by Quantum algo

M

Random coin toss
Governed by Bernoulli distribution
Parameters determined by quantum algorithm

Projection noise and sampling error

Let's turn to the example of finite number of shots we execute for our experiment.
Perform N experiment, each giving us a single shot result 0 or 1

shot 1: 10) 10) shot 2: 10) shot 3: 10) shot 4:

M_1

M_2

M_3

M_4

$\vdots$

M_N Total # of shots will be N

For 3 samples, there are 2^3 possible outcome sequences $(N=3)$

M_1	M_2	M_3
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

2^N possible combination

Sample mean?

$$S := \frac{1}{N}\sum_{n=1}^{N} M_n$$

S: random variable

N: # shots

n: index of the shot #

M_n: indep. identically distributed (i.i.d)

$M_n \sim M$

examp $S = \frac{1}{3}(1+0+1) = 2/3$

$$\mathbb{E}[S] = \mathbb{E}\left[\frac{1}{N}\sum_{n=1}^{N} M_n\right]$$

$$= \frac{1}{N}\sum_{n=1}^{N}\mathbb{E}[M] \qquad \mathbb{E}[M_n] = \mathbb{E}[M_{n'}] = \mathbb{E}[M]$$

$$= \frac{1}{\cancel{N}}\cancel{N}\,\mathbb{E}[M]$$

$$= \mathbb{E}[M]$$

$$\mathbb{E}[S] = \mathbb{E}[M] = \langle\hat{M}\rangle \quad = p \text{ for qubit } \hat{M} = |1\rangle\langle 1|$$

$$\mathbb{V}[S] = \frac{\mathbb{V}[M]}{N} = \frac{p(1-p)}{N}$$

$$\sigma_S = \sqrt{\frac{p(1-p)}{N}}$$

N: # shots

σ_S: error bar (std dev) on sample mean estimator for our expectation value.