

# Coherent error: ZZ [bonus]

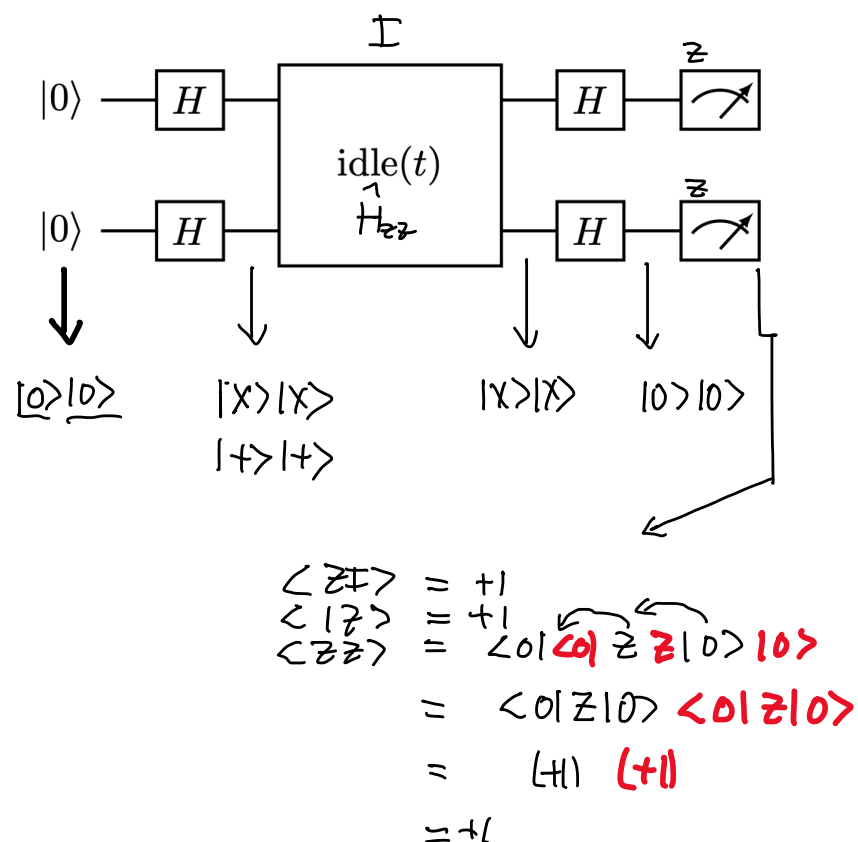
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## Introduction to quantum noise

### Coherent errors

Qiskit Global Summer School on Quantum Machine Learning

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Hadamard gate

$$H = \begin{bmatrix} |0\rangle & |1\rangle \end{bmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{pmatrix}$$

$$\begin{cases} H|0\rangle = |+\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \\ H|1\rangle = |-\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \end{cases}$$

$$X|+\rangle = +|+\rangle$$

$$X|-\rangle = -|-\rangle$$

$$|+\rangle := \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$|-\rangle := \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

$$\begin{cases} Z|0\rangle = +|0\rangle \\ Z|1\rangle = -|1\rangle \end{cases}$$

Noisy

ZZ Interaction

$$\hat{H} = \frac{1}{2} \hbar \omega \hat{Z} \hat{Z}$$

$$\hat{U}(t) = \exp(-i \hat{H} t / \hbar)$$

$$= \exp(-i \frac{\omega t}{2} \hat{Z} \hat{Z})$$

$$= \cos(\frac{\omega t}{2}) \hat{I} - i \sin(\frac{\omega t}{2}) \hat{Z} \hat{Z}$$

$$= \hat{R}_{ZZ}(\theta = \omega t)$$

$$|0\rangle|0\rangle \xrightarrow{HH} |+\rangle|+\rangle \xrightarrow{\text{idle}} R_{ZZ}(\theta)|+\rangle|+\rangle = \cos \frac{\theta}{2} |+\rangle|+\rangle - i \sin \frac{\theta}{2} |-\rangle|-\rangle \xrightarrow{HH} \cos \frac{\theta}{2} |0\rangle|0\rangle - i \sin \frac{\theta}{2} |1\rangle|1\rangle$$

$$R_{ZZ}(\theta)|+\rangle|+\rangle = \cos \frac{\theta}{2} |+\rangle|+\rangle - i \sin \frac{\theta}{2} (Z|+\rangle)_1 (Z|+\rangle)_2$$

$$Z|+\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = |-\rangle$$

$$Z|+\rangle = |-\rangle$$

$$Z|-\rangle = |+\rangle$$

$$\langle ZI \rangle = \cos \omega t$$

$$\langle IZ \rangle = \cos \omega t$$

$$\langle ZZ \rangle = 1$$