Tech Note T23: Probabilistic error cancellation: paper notes

Summary for key notation of *Probabilistic error cancellation with sparse Pauli-Lindblad models on noisy quantum processors* [Ref. van den Berg et al. (2020)]

Symbol	Description	Value
	Circuit constants & indices	
n	number of qubits in circuit	$n = 1, 2, 3, \ldots$
l	number of layers in circuit	$l = 1, 2, 3, \ldots$
i	layer index within circuit	$i = 1, \ldots, l$
	Pauli-Lindblad model inputs	
k	model coefficient index, corresponds to a Pauli	$k = 1, 2, 3, \ldots$ or equiv. n-qubit Pauli string
b	fidelity vector indices	$b = 1, 2, \ldots$ or equiv. n-qubit Pauli string
$\mathcal{K}$	set of Pauli fidelity support indices for the sparse model $\mathcal{L}$ of Λ. Small set	$k \in \mathcal{K}$
$\mathcal{B}$	set of benchmark Paulis (Pauli fidelity support indices of Λ). Can be all of them, big set	$b \in \mathcal{B}$
N	number of error-mitigated circuit instances	$N = 1, 2, 3, \ldots$
	Pauli-Lindblad model variables	
γ	sampling overhead	$\gamma \geq 1$, $\gamma = \exp\left(\sum_{k\in\mathcal{K}} 2\lambda_k\right)$
$\gamma_i, \gamma(l)$	sampling overhead for the i-th layer and a total of l layers	$\gamma(l) = \Pi_{i=1}^{l} \gamma_i$
λ_k	k-th model coefficient	$\lambda_k \geq 0$
w_k	noise model weight in Λ factoring	$w_k := \frac{1}{2}\left(1 + e^{-2\lambda_k}\right)$
f_b	Pauli Λ fidelity of b-th Pauli index	$f_b := \frac{1}{2^n} \operatorname{Tr}\left(P_b^\dagger \Lambda\left(P_b\right)\right)$
f	vector of Pauli fidelities of Λ	$f = \{f_b\}_{b\in\mathcal{B}}$
$\hat{f}$	fidelity estimates for a set of	
$M(\mathcal{B},\mathcal{K})$	binary matrix with entries $M_{b,k} = \langle b, k\rangle_{sp}$ where the symplectic product is 0 is the Paulis commute and 1 if they do not	$\log(f) = -2M(\mathcal{B},\mathcal{K})\lambda$ (element-wise log) (used to fit with $\lambda \geq 0$)
	Quantum operators and superoperators	
P_k	Pauli operator, indexed by k	$P_k \in \{I, X, Y, Z\}^{\otimes n}$
$U, \mathcal{U}, \tilde{\mathcal{U}}_i$	unitary ideal gate; tilde: noisy i-th layer unitary	
Λ, Λ_i	noise channel	$\Lambda(\rho) = \exp[\mathcal{L}](\rho) = \Pi_{k\in\mathcal{K}}\left(w_k \cdot + (1-w_k) P_k \cdot P_k^\dagger\right)\rho$
Λ^{-1}	inverse noise map	$\Lambda^{-1}(\rho) = \exp[-\mathcal{L}](\rho) = \gamma \Pi_{k\in\mathcal{K}}\left(w_k \cdot - (1-w_k) P_k \cdot P_k^\dagger\right)\rho$
$\mathcal{L}(\rho)$	Lindblad generator	$\mathcal{L}(\rho) = \sum_{k\in\mathcal{K}} \lambda_k \left(P_k \rho P_k - \rho\right)$
$\left\langle \hat{A}_N \right\rangle$	average error mitigated estimate of $\left\langle \hat{A} \right\rangle$ for some operator $\hat{A}$ using N circuit instances	

23.1 Common questions

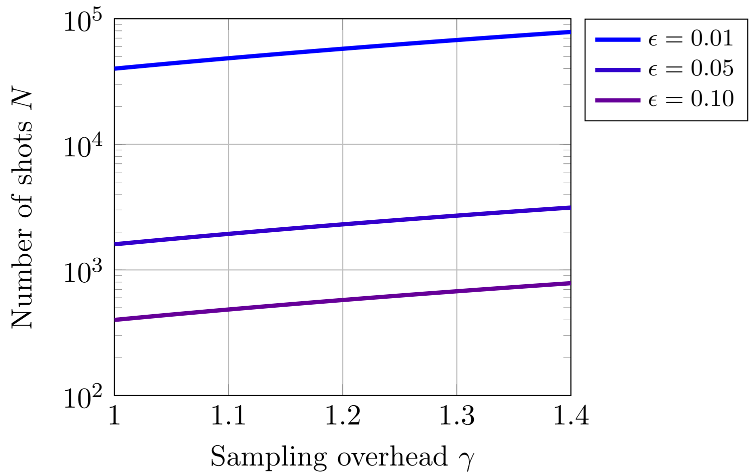
How many random circuits to run? For a maximum error bound ϵ between the the ideal and mitigated expectation values $\left|\left\langle\hat{A}\right\rangle_{\text{ideal}} - \left\langle\hat{A}_N\right\rangle\right| \leq \epsilon$ satisfied with a probability $1-\delta$ (assuming a weak enough noise, $C^{l\tau} \approx 1$), we can solve for the number of error-mitigated, random circuit instances N in van den Berg et al. (2020)

$$\epsilon = \gamma\frac{\sqrt{2\log\left(2/\delta\right)}}{\sqrt{N}} \ ,$$

$$\therefore \quad N = 2\log\left(2/\delta\right)\left(\frac{\gamma}{\epsilon}\right)^2 \ ,$$

$$N \approx 4\left(\frac{\gamma}{\epsilon}\right)^2 \quad \text{for } \delta \sim 0.01 \ .$$

For probability $\delta = 2\%$, $\log\left(2/\delta\right) = 2$, a small overhead; for $\delta = 0.01$, it is 2.3, and for $\delta = 0.001$, it is 3.3.



Bibliography

E. van den Berg, Z. K. Minev, and K. Temme, arXiv (2020), ISSN 23318422, 2012.09738, URL http://arxiv.org/abs/2012.09738. Nature Physics **16**, 233 (2020), ISSN 1745-2473, URL https://doi.org/10.1038/s41567-020-0847-3http://www.nature.com/articles/s41567-020-0847-3.

Caveat emptor These pages are a work in progress, inevitably imperfect, incomplete, and surely enriched with typos and unannounced inaccuracies. Sources credited in Bibliography to the best of my ability, though certain omissions certainly remain.

Notes by ©Zlatko K. Minev (zlatko-minev.com)