Measurement noise

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Introduction to quantum noise

Measurement error

Qiskit Global Summer School on Quantum Machine Learning

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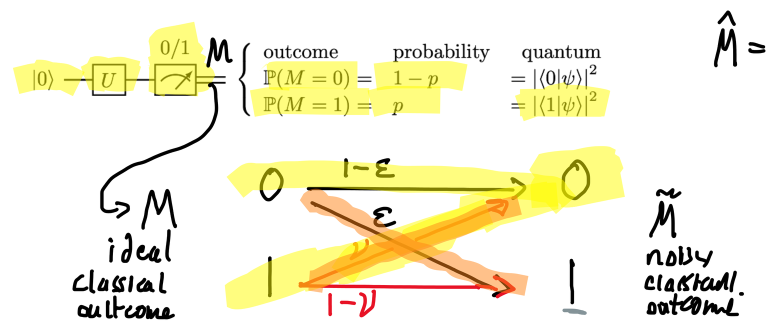


$$\hat{M} = |1\rangle\langle 1| = \tfrac{1}{2}(\hat{I} - \hat{Z})$$

$$P_M = \begin{pmatrix} \mathbb{P}(M=0) \\ \mathbb{P}(M=1) \end{pmatrix} = \begin{pmatrix} 1-p \\ p \end{pmatrix} \qquad P_{\tilde{M}} = \begin{pmatrix} \mathbb{P}(\tilde{M}=0) \\ \mathbb{P}(\tilde{M}=1) \end{pmatrix} = \begin{pmatrix} 1-\tilde{p} \\ \tilde{p} \end{pmatrix}$$

$$\begin{cases} \mathbb{P}(\tilde{M}=0) = \mathbb{P}(\tilde{M}=0|M=0)\,\mathbb{P}(M=0) + \overbrace{\mathbb{P}(\tilde{M}=0|M=1)}^{\text{error}}\,\mathbb{P}(M=1) \\ \mathbb{P}(\tilde{M}=1) = \mathbb{P}(\tilde{M}=1|M=0)\,\mathbb{P}(M=0) + \mathbb{P}(\tilde{M}=1|M=1)\,\mathbb{P}(M=1) = \varepsilon(1-p) + (1-\nu)p = \tilde{p} \end{cases}$$

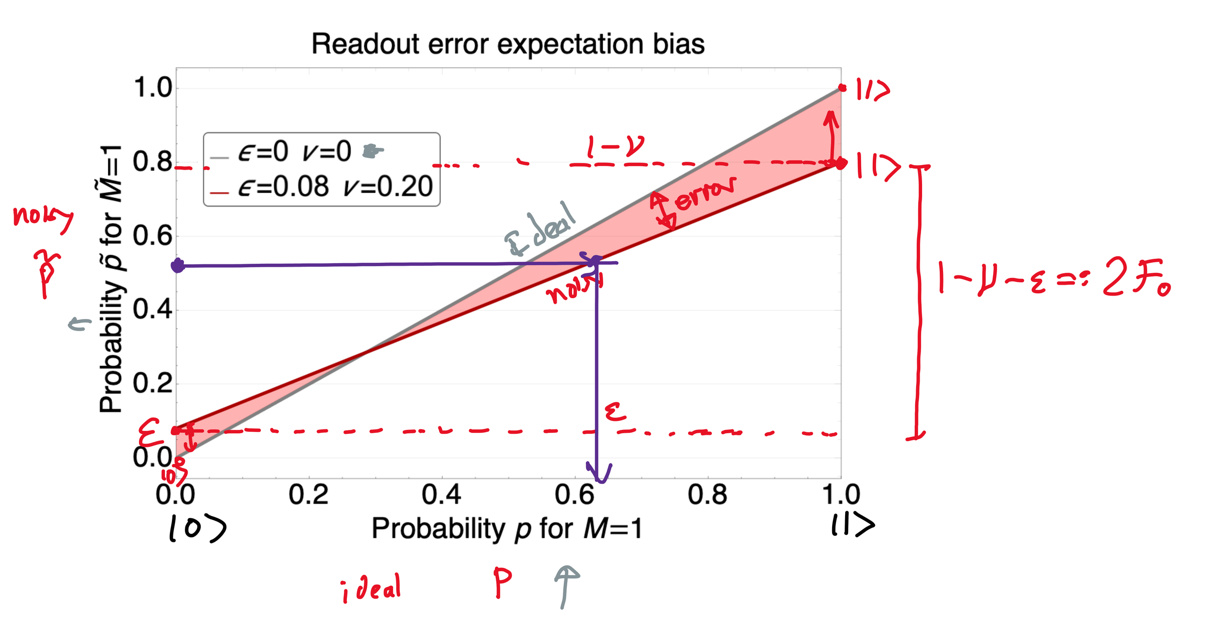
$$\underset{\text{noisy}}{P_{\tilde{M}}} = \underset{\text{readout error matrix}}{A}\ \underset{\text{ideal}}{P_M}$$

$$A = \begin{array}{c|cc} & M=0 & M=1 \\ \hline \tilde{M}=0 & \mathbb{P}(\tilde{M}=0|M=0) & \mathbb{P}(\tilde{M}=0|M=1) \\ \tilde{M}=1 & \mathbb{P}(\tilde{M}=1|M=0) & \mathbb{P}(\tilde{M}=1|M=1) \end{array}$$

$$= \begin{pmatrix} 1-\varepsilon & \nu \\ \varepsilon & 1-\nu \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \text{ for an ideal measurement}$$

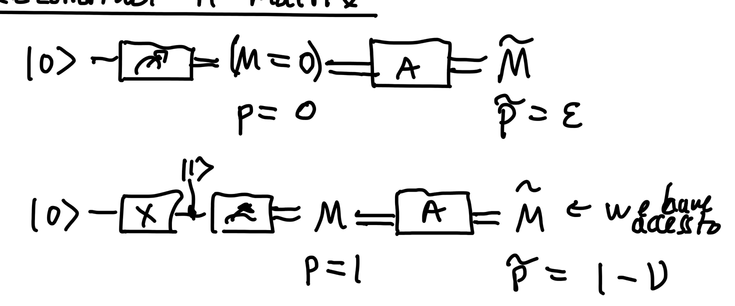
$$\sum_m A_{mn} = 1 \text{ for any } n \qquad \text{Stochastic matrix}$$

$$\begin{aligned} \tilde{p} &= \varepsilon(1-p) + (1-\nu)p \\ &= \varepsilon - p\varepsilon + p - \nu p \\ &= p + \varepsilon - (\nu+\varepsilon)p \end{aligned}$$



Bonus section content:

Reconstruct A matrix



Noise mitigation

we know A $\qquad \tilde{P}_M = A P_M$

measure $\tilde{p}$, $\tilde{P}_M$ noisy $\qquad P_M = A^{-1}\tilde{P}_M$

find p, P_M ideal $\qquad p =$

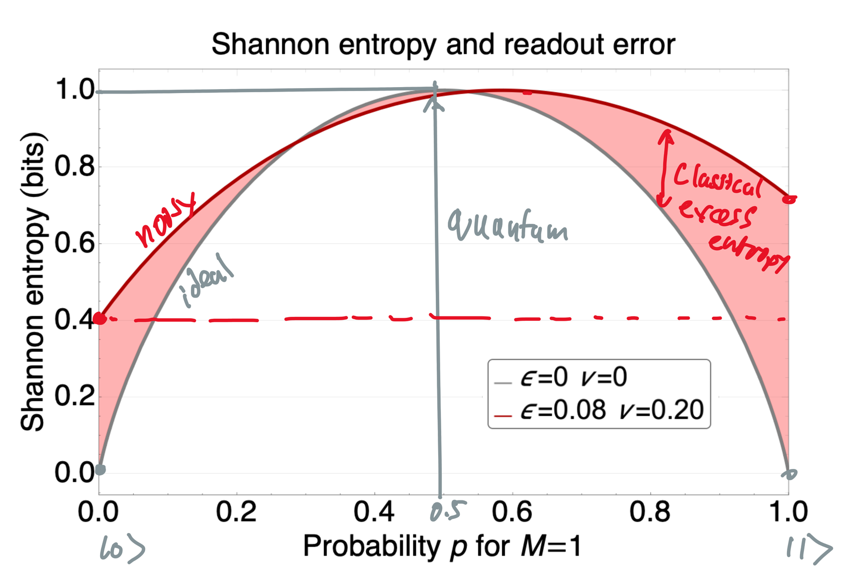
$\dim A = 2^n \times 2^n$ $\quad n = \#\text{qubits}$

Assignment Fidelity

$$\begin{aligned} \mathcal{F}_0 &= 1 - \tfrac{1}{2}\left[P(\tilde{M}=1|M=0) + P(\tilde{M}=0|M=1)\right] \\ &= \tfrac{1}{d}\,\mathrm{Tr}(A) \qquad d = 2^n,\ n = \#\text{qubits} \\ &= 1 - \tfrac{1}{2}(\mu+\nu) \end{aligned}$$

Shannon Entropy

$$\mathcal{H}(M) = \mathcal{H}(P_M) = -\sum_m p_m \log_2 p_m = \underbrace{-(1-p)\log_2(1-p) - p\log_2 p}_{\text{Binary entropy}}$$



Larger System

n qubits $\qquad \dim A = 2^n \times 2^n$