1) What do graph theory, many-body physics, the golden ratio, and Fibonacci anyons have in common?

and

2) Entanglement-enhanced learning of quantum processes at scale

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IBM Quantum

@CIFAR

Quantum advantage

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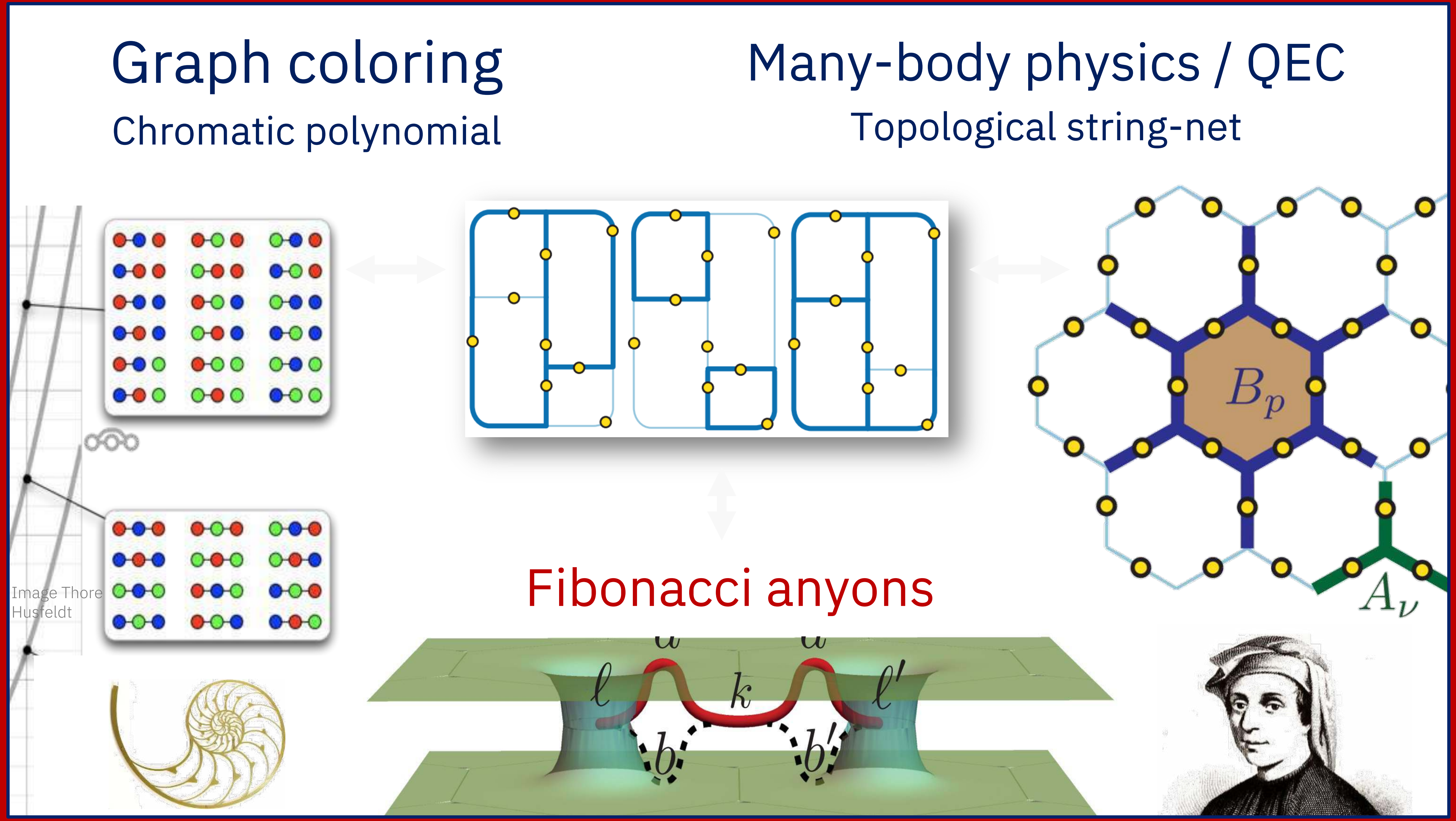
Quantum Physics

[Submitted on 18 Jun 2024]

Realizing string-net condensation: Fibonacci anyon braiding for universal gates and sampling chromatic polynomials

Zlatko K. Minev, Khadijeh Najafi, Swarnadeep Majumder, Juven Wang, Ady Stern, Eun-Ah Kim, Chao-Ming Jian, Guanyu Zhu

Aimed at a classically-hard problem and FTQC



Quantum Physics

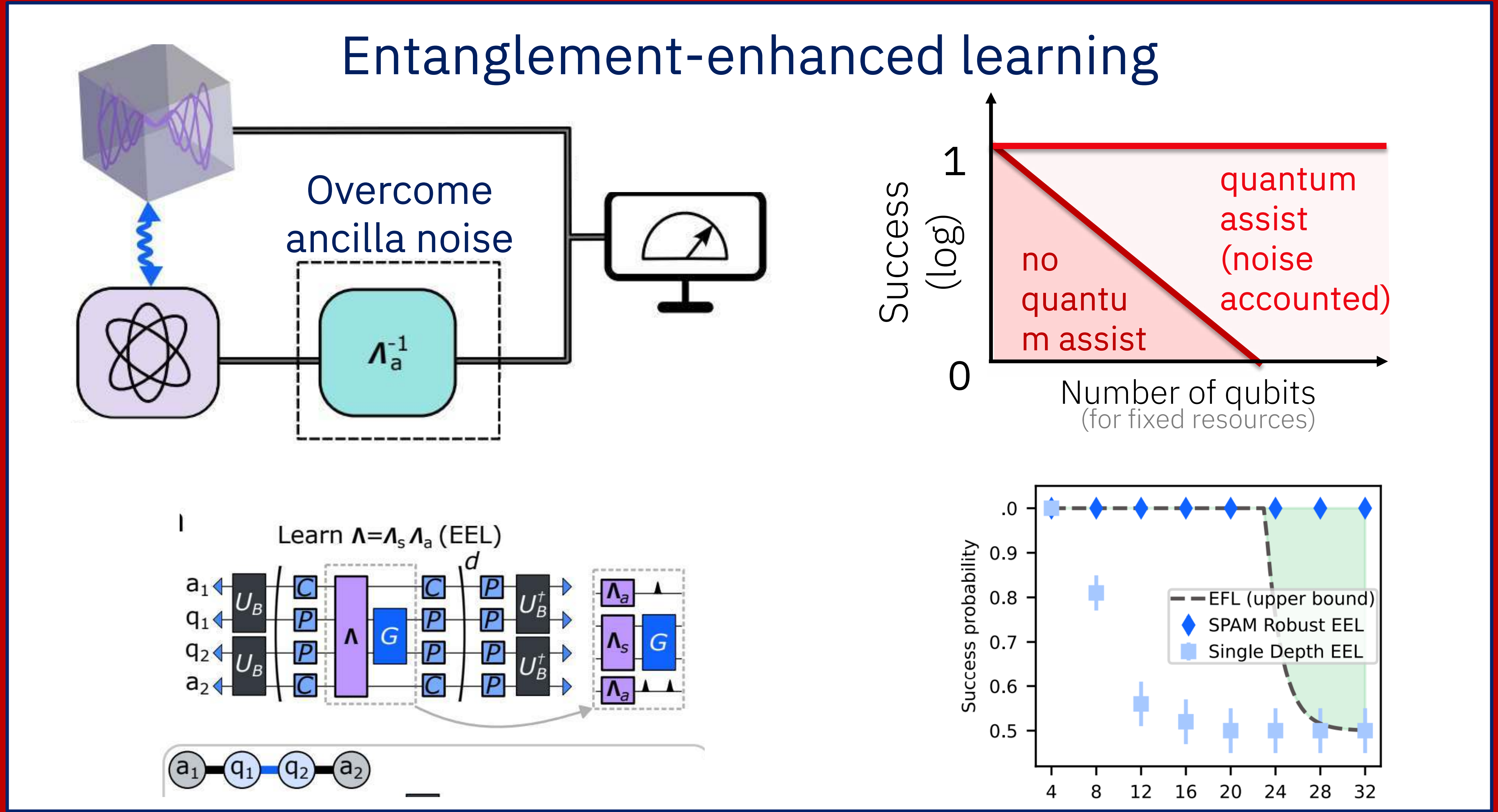
[Submitted on 6 Aug 2024]

Entanglement-enhanced learning of quantum processes at scale

Alireza Seif, Senrui Chen, Swarnadeep Majumder, Haoran Liao, Derek S. Wang, Moein Malekakhlagh, Ali Javadi-Abhari, Liang Jiang, Zlatko K. Minev

Aimed at sampling complexity

Project inspired by Robert Huang and John Preskill presentation & discussions at CIFAR QIS



arXiv > quant-ph > arXiv:2406.12820

Quantum Physics

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Realizing string-net condensation: Fibonacci anyon braiding for universal gates and sampling chromatic polynomials

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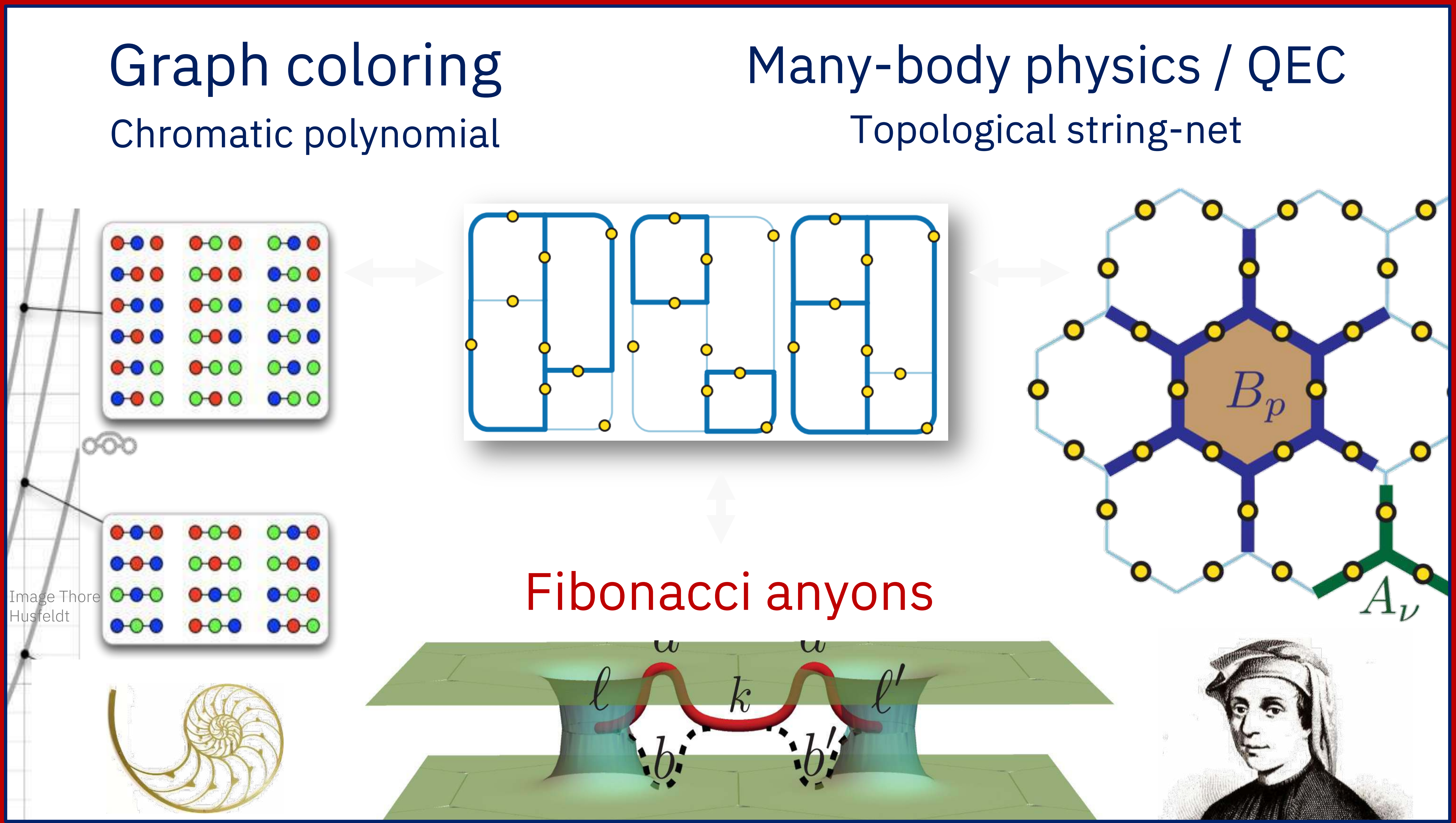

What do graph theory, many-body physics, the golden ratio, and Fibonacci anyons have in common?

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IBM Quantum

IBM Quantum
Guanyu Zhu
Swarnadeep Majumder
Sona Najafi

Cornell University
Prof. Eun-Ah Kim
Prof. Chao-Ming Jian

Harvard University
Juven Wang

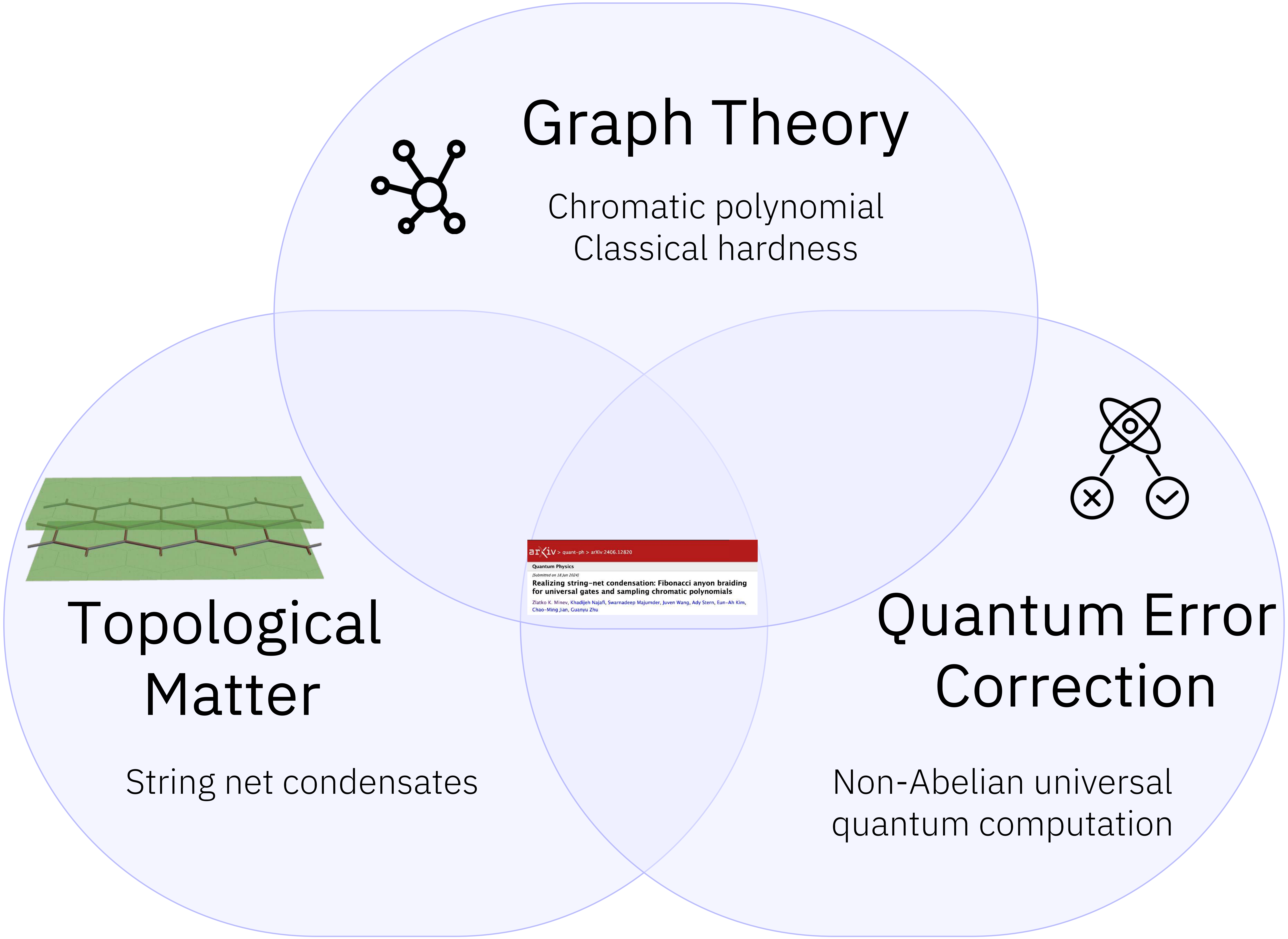
Weizmann Institute Science
Prof. Ady Stern







Acknowledgements: Sergey B. Bravyi, Vojtech Havlıcek, the IBM Quantum team, and many friends and colleagues



Icons: Freepik, Iconjam

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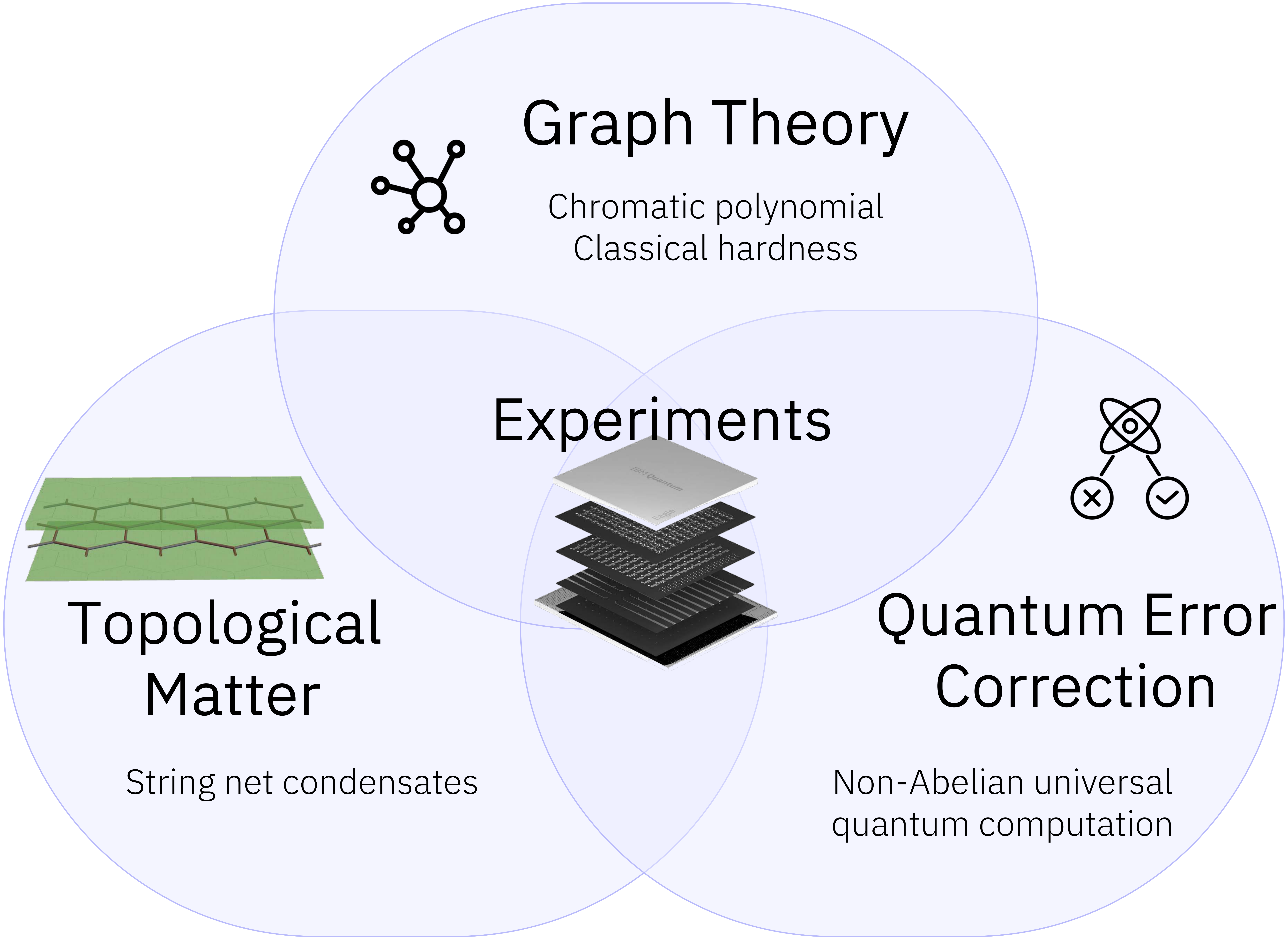

Icons: Freepik, Iconjam

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Graph Theory

Chromatic polynomial
Classical hardness

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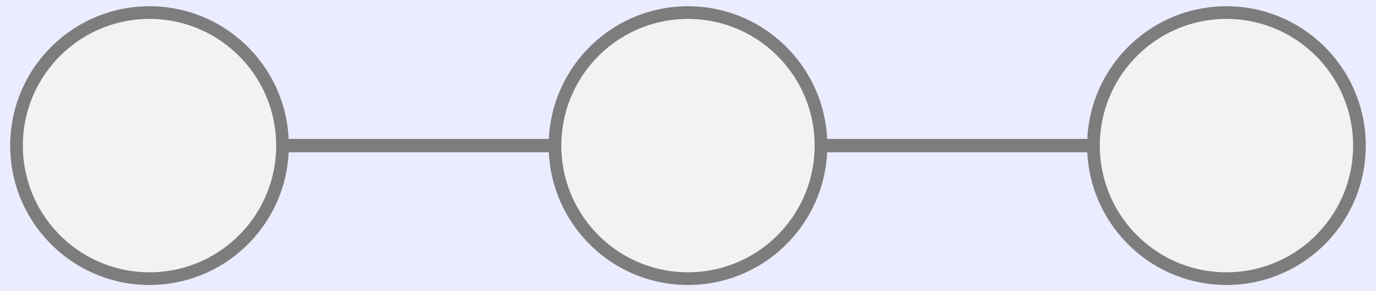
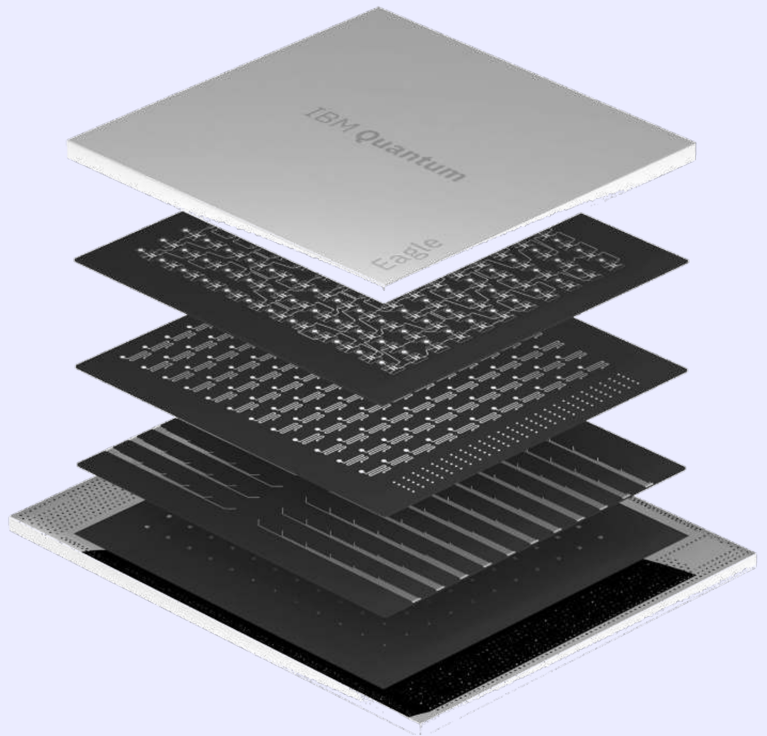
Graph coloring: Proper vertex k-coloring

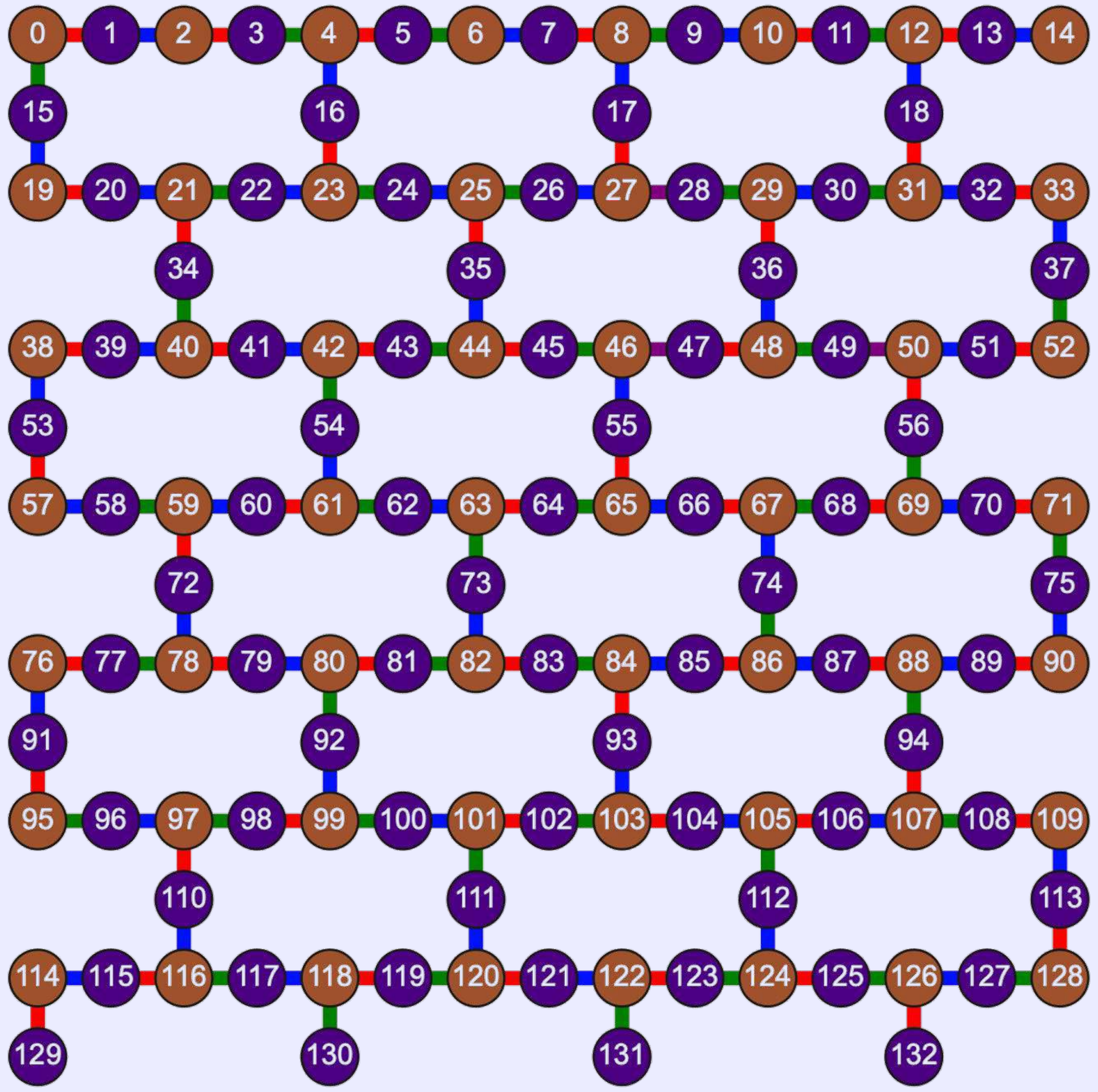
Task: Assign colors to the nodes such that *no two adjacent vertices* share the same color

Example: Graph G



We use graph coloring all the time

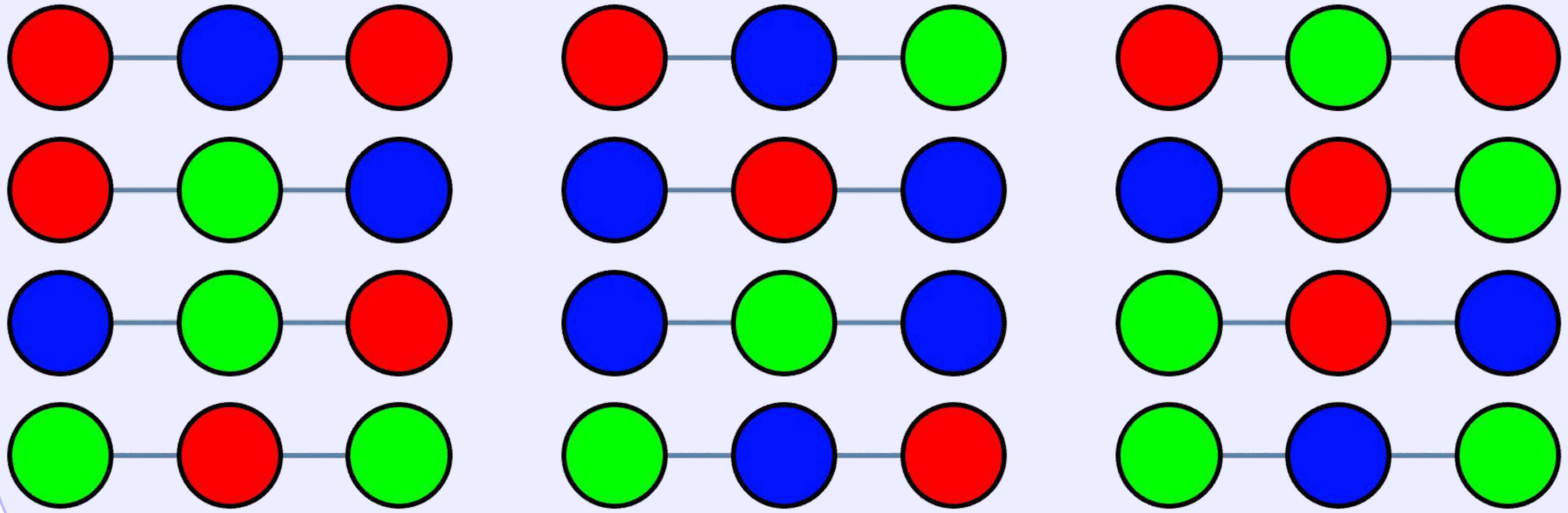




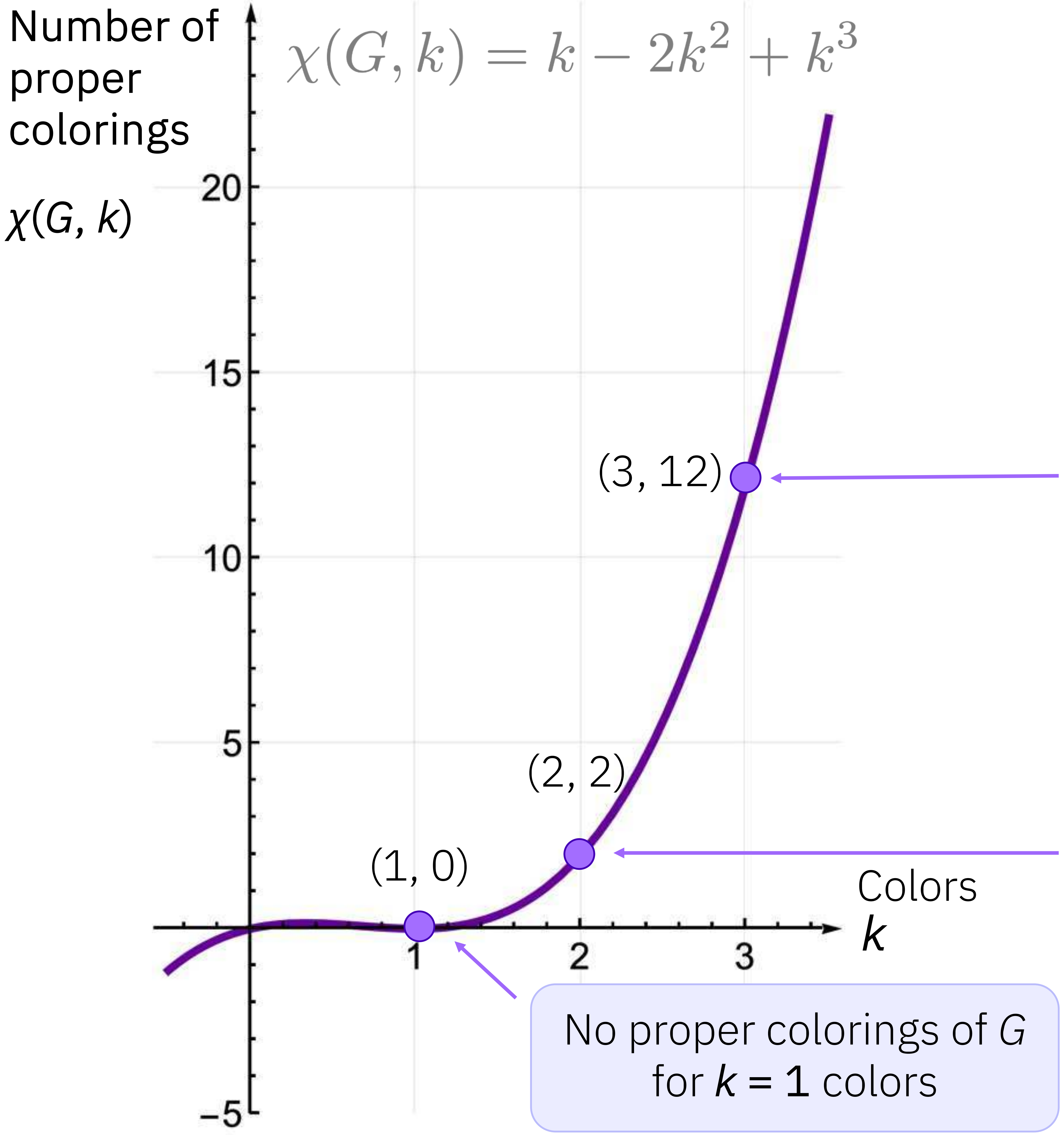
No proper colorings of G for k = 1 colors

All colorings of G for k = 2 colors: red, blue

All colorings of G for k = 3 colors: red, blue, green



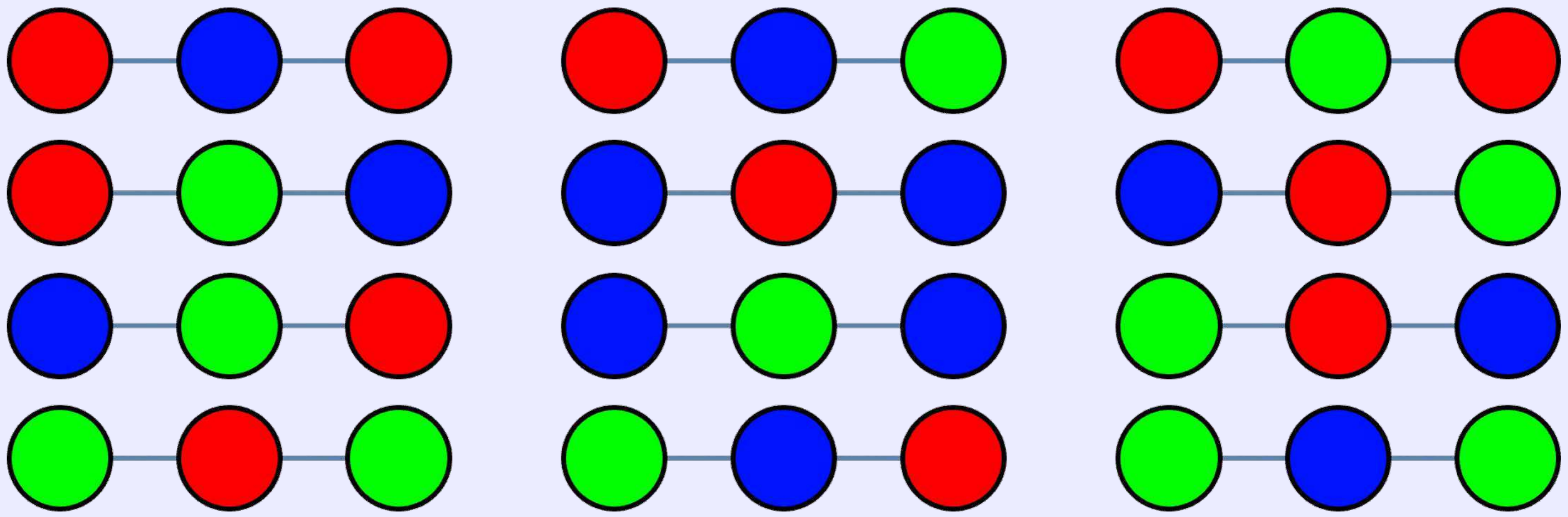
Chromatic polynomial: Proper vertex k-colorings of a graph



No proper colorings of G for $k = 1$ colors

The chromatic polynomial $\chi(G, k)$ of each graph G interpolates through the number of proper colorings for k colors.

All colorings of G for $k = 3$ colors: red, blue, green



All colorings of G for $k = 2$ colors: red, blue

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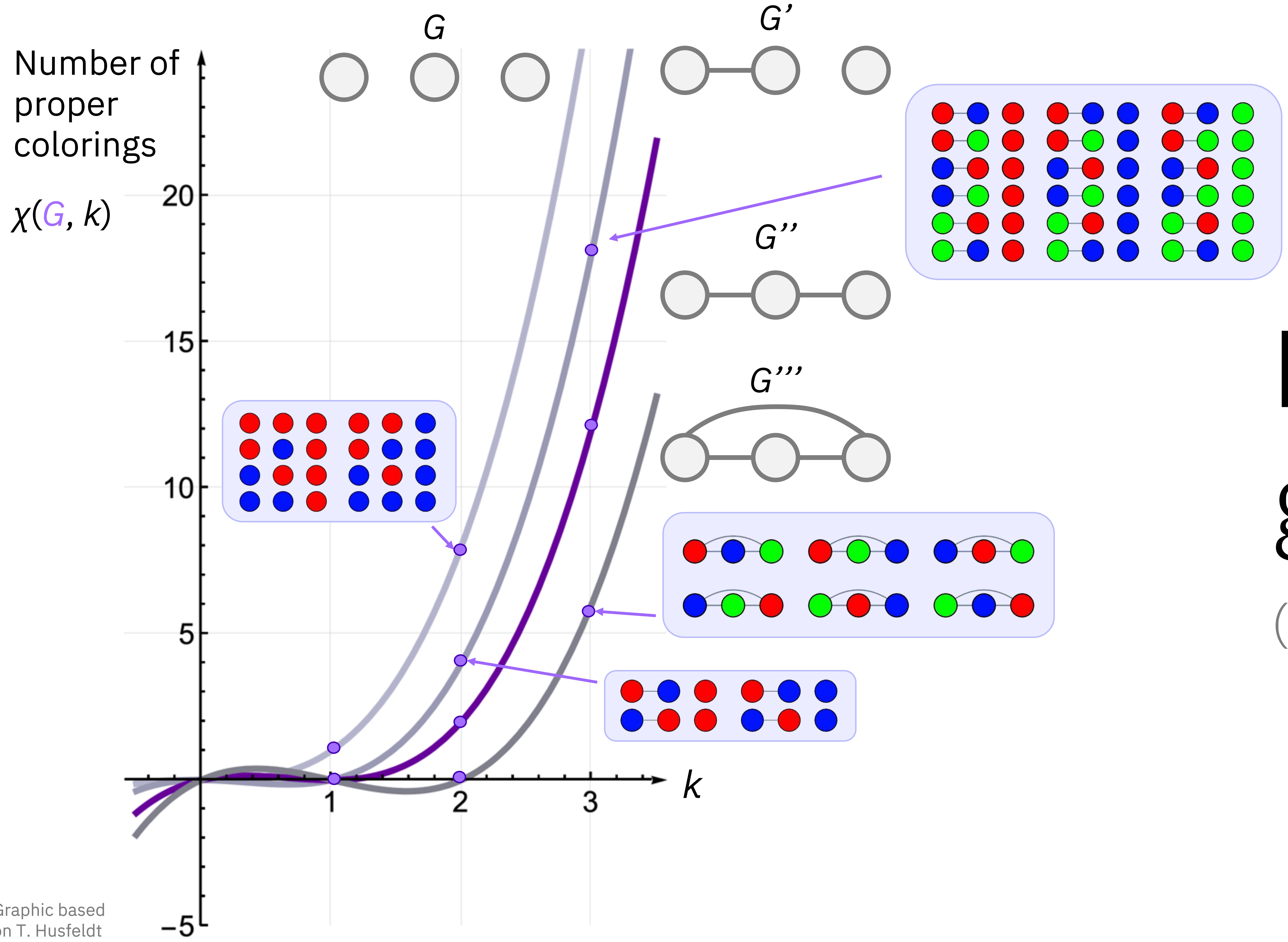
Chromatic polynomial: Proper vertex k-colorings of a graph



Different graphs

(non-isomorphic)

Graphic based on T. Husfeldt

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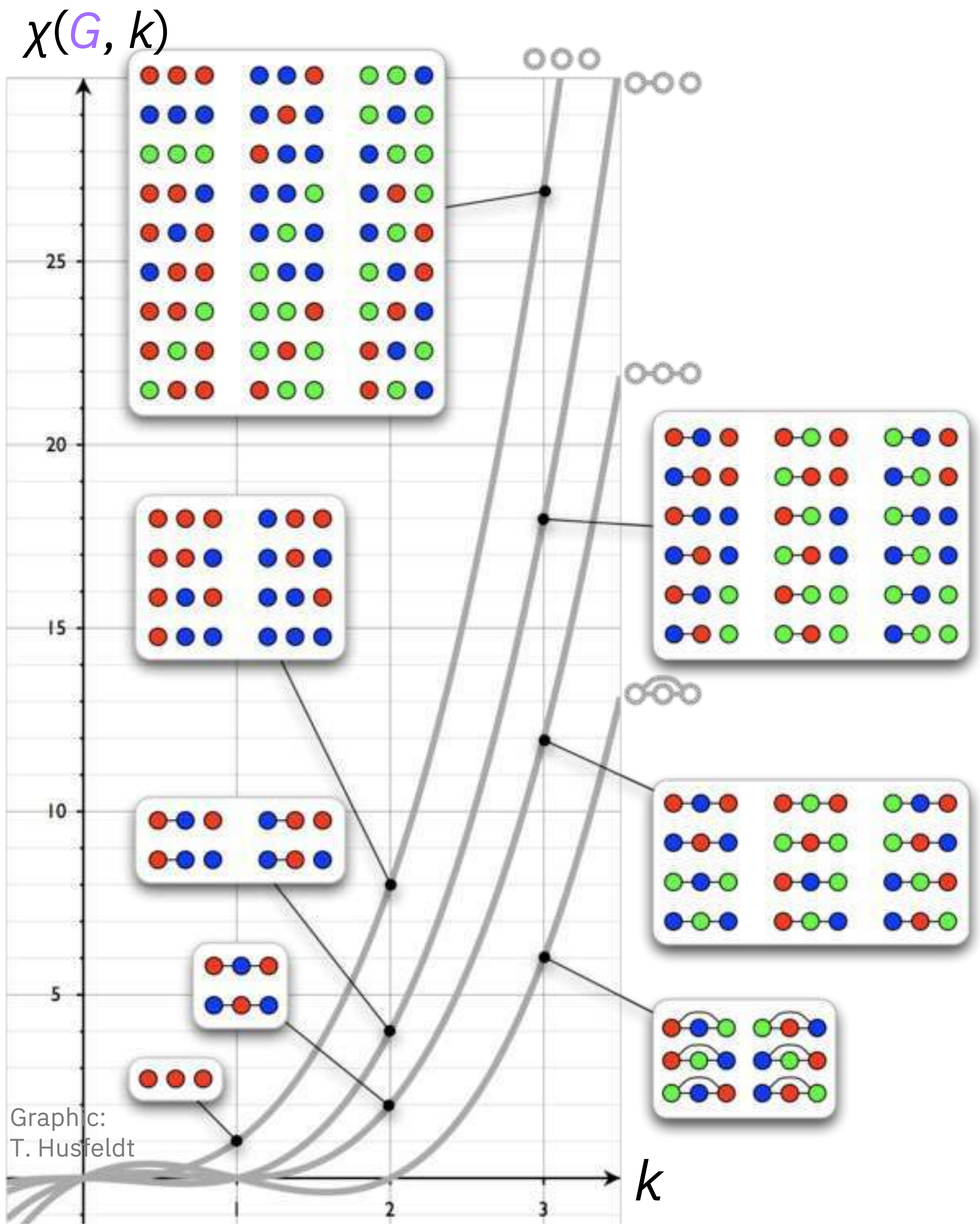
Classical hardness of chromatic polynomial



- Extremely well studied in complexity theory for 50+ years
  - Complexity of computing the chromatic polynomial is completely understood
- #P-hard, #P-complete
  - Calculating $P(G,k)$ exactly involves counting all possible valid colorings, a task that generally requires exponential time in the size of the graph for arbitrary G
  - Calculating the number of k=3 colorings of a graph $\chi(G,3)$ is the canonical example of a **#P-complete** problem
  - More generally, calculating the chromatic polynomial $\chi(G, k)$ is **#P-hard** (typically exponential or worse runtime) **for all k** (including negative integers and even all complex numbers)
  - Even **planar graphs** are #P-hard
- No efficient classical approximation algorithms for computing $\chi(G, k)$ are known for any k, except for the three easy points $k = 0, 1, 2$
  - And, no fully polynomial randomized approximation scheme (FPRAS) exists for rational k>2
- BQP-complete (decision version): additive approximation of the chromatic polynomial on a planar graph can be achieved with a polynomial-time quantum algorithm
- Possible path to quantum advantage?
- Relation **Fibonacci string-net condensate?**

Bencs, et al., Approximating the chromatic polynomial is as hard as computing it exactly. Comp. Complexity (2024)
Goldberg, et al., Inapproximability of the tutte polynomial, Information and Computation (2008)
Goldberg , et al., Inapproximability of the tutte polynomial of a planar graph, computational complexity (2012)
D. Vertigan, The Computational Complexity of Tutte Invariants for Planar Graphs, SIAM J. Comput. (2005)
Aharonov, et al. Polynomial Quantum Algorithms for Additive approximations of the Potts model and other Points of the Tutte Plane. (2007)

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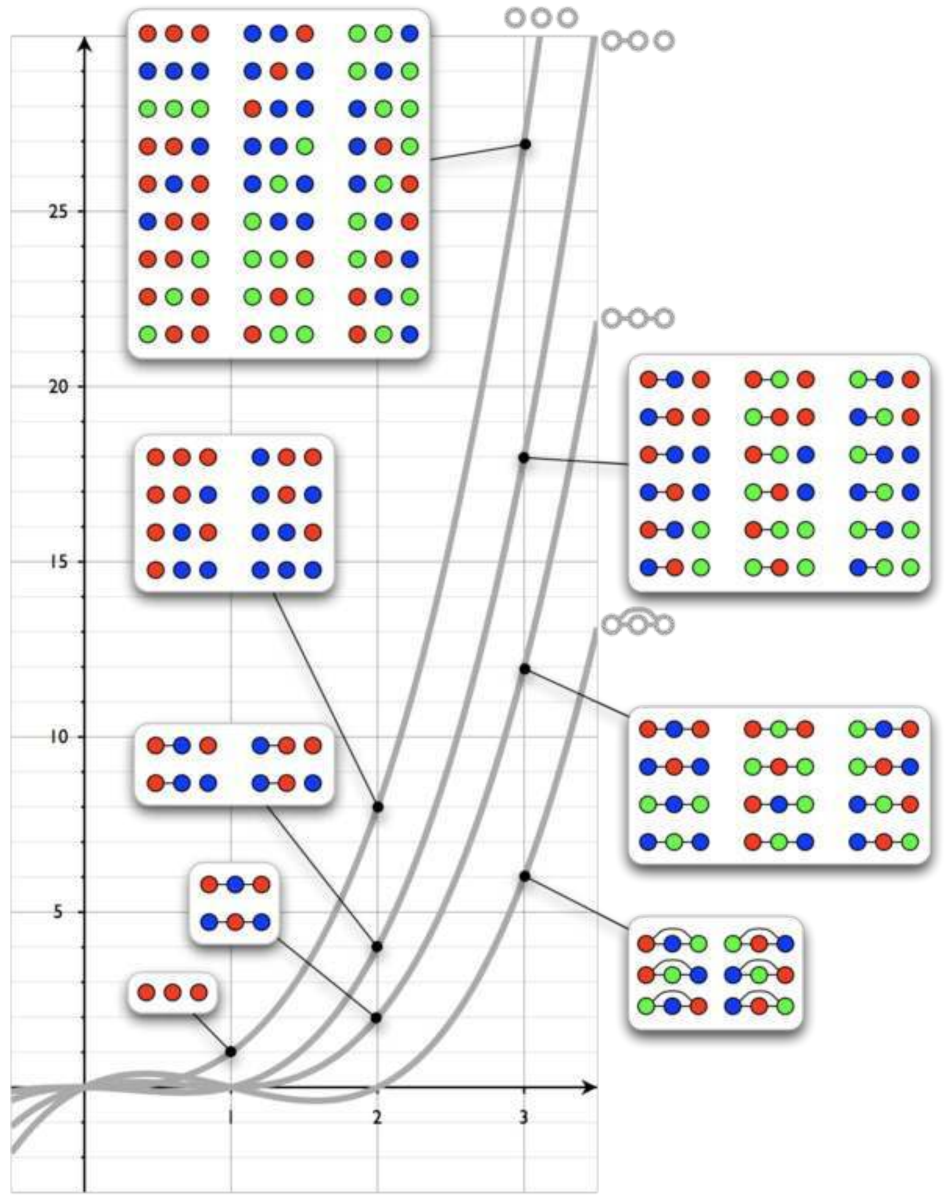
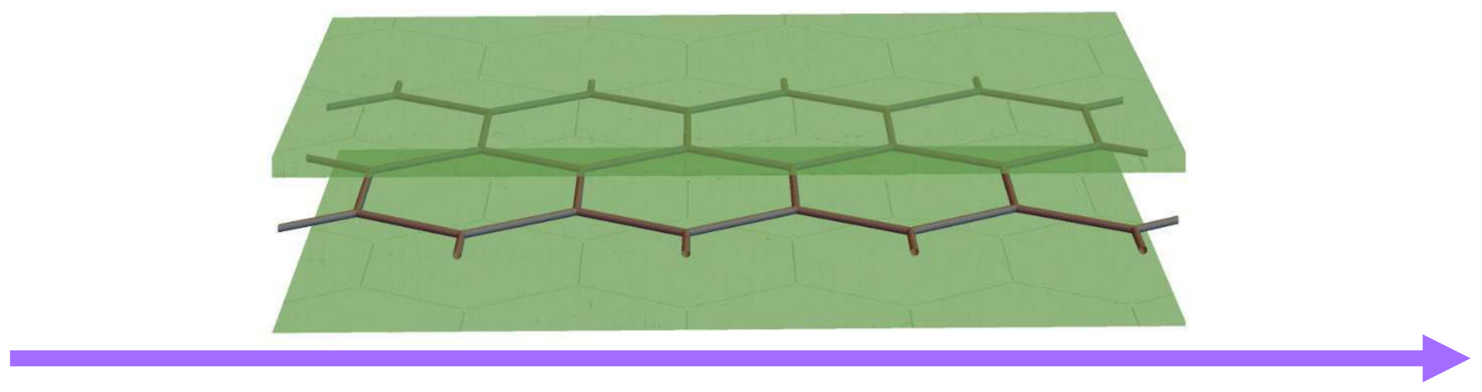
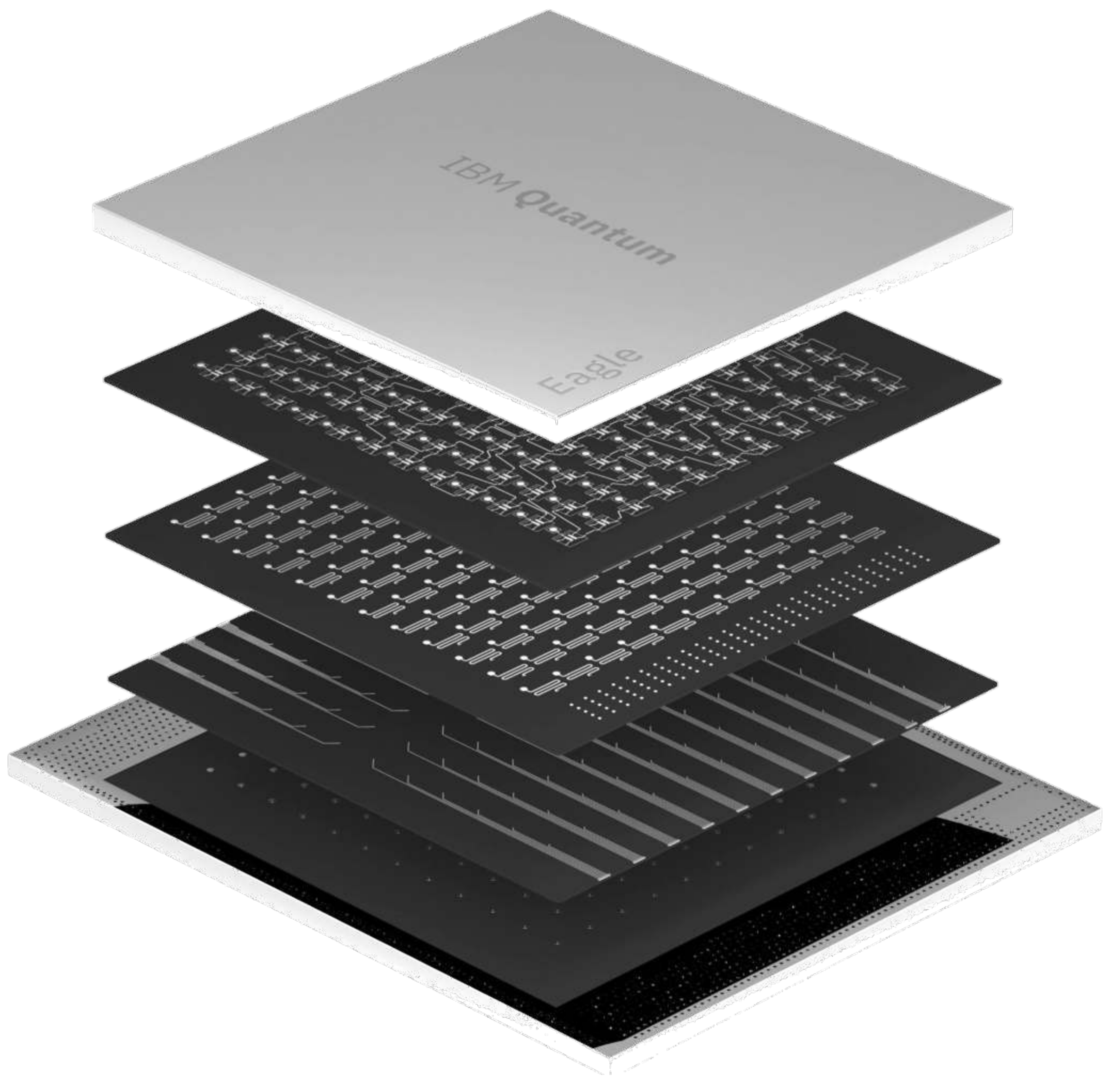
Can the chromatic polynomial be encoded in an efficiently-preparable many-body state and sampled on near-term hardware?

First proof-of-principle protocol and experiments







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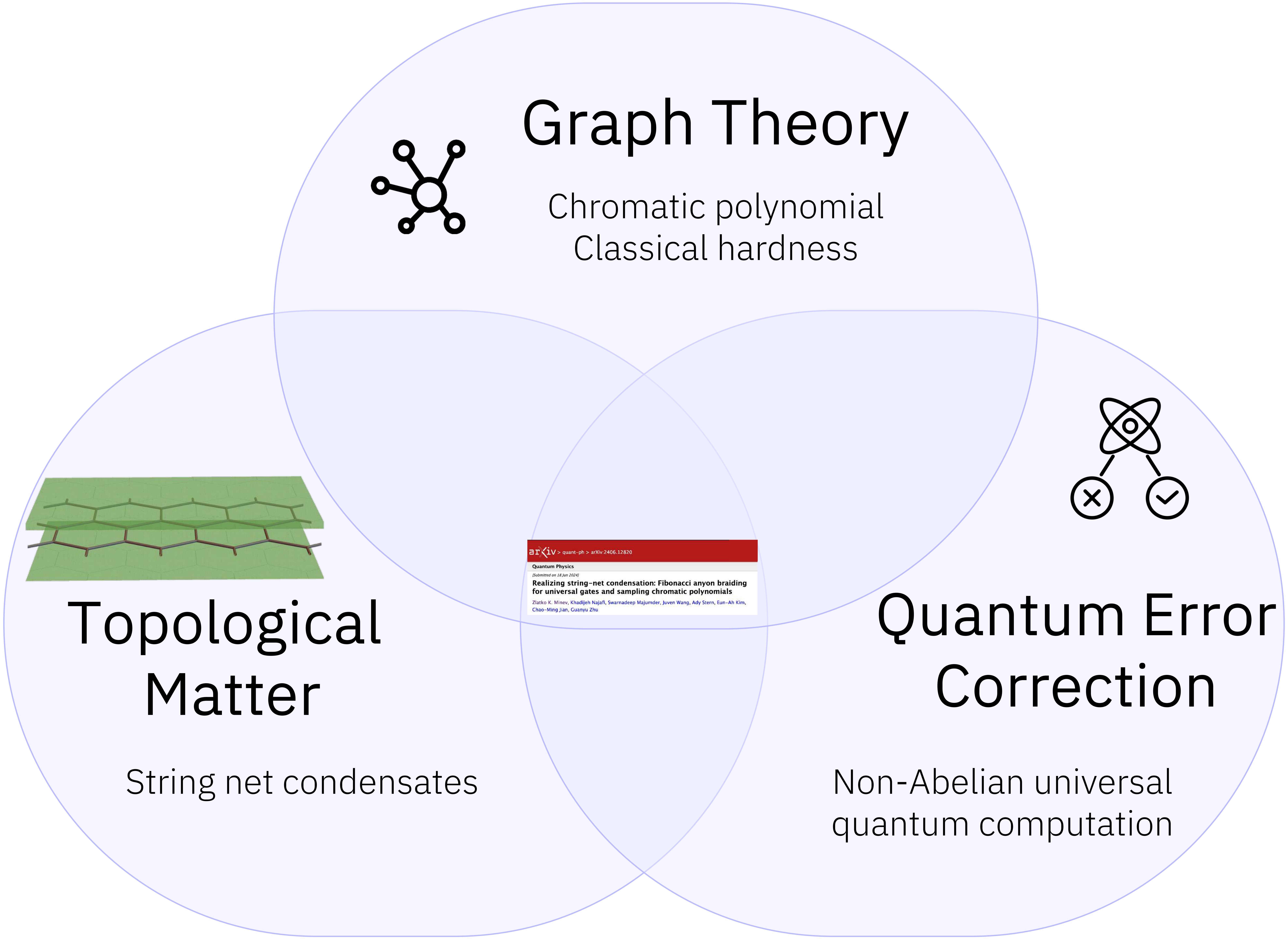

Icons: Freepik, Iconjam

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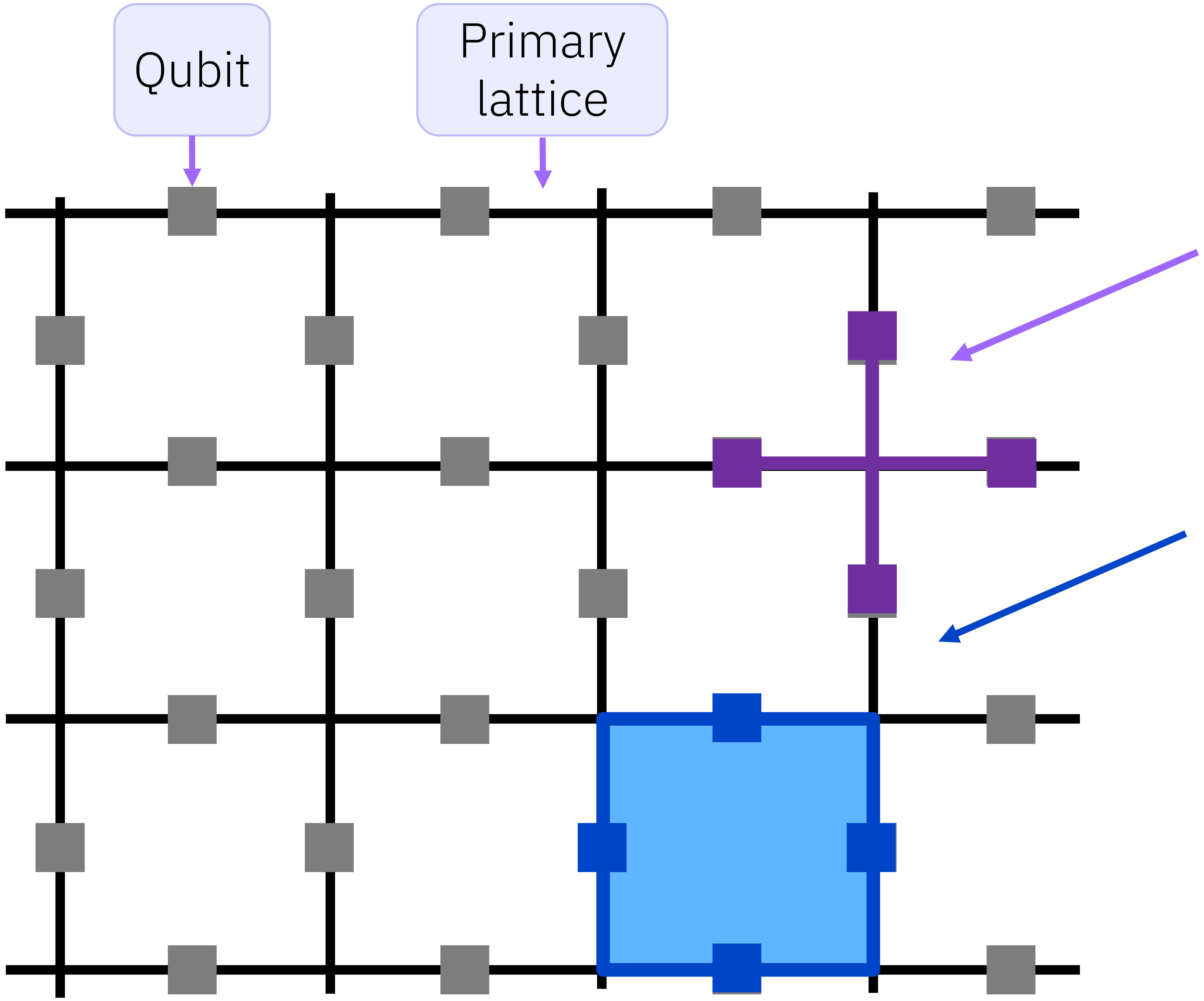
Topological matter and QEC

Start with something we know: Toric Code



Josiah Sinclair
Morning session

Vertex $$A_v = \prod_{j \in v} X_j$$

Plaquette $$B_p = \prod_{j \in p} Z_j$$

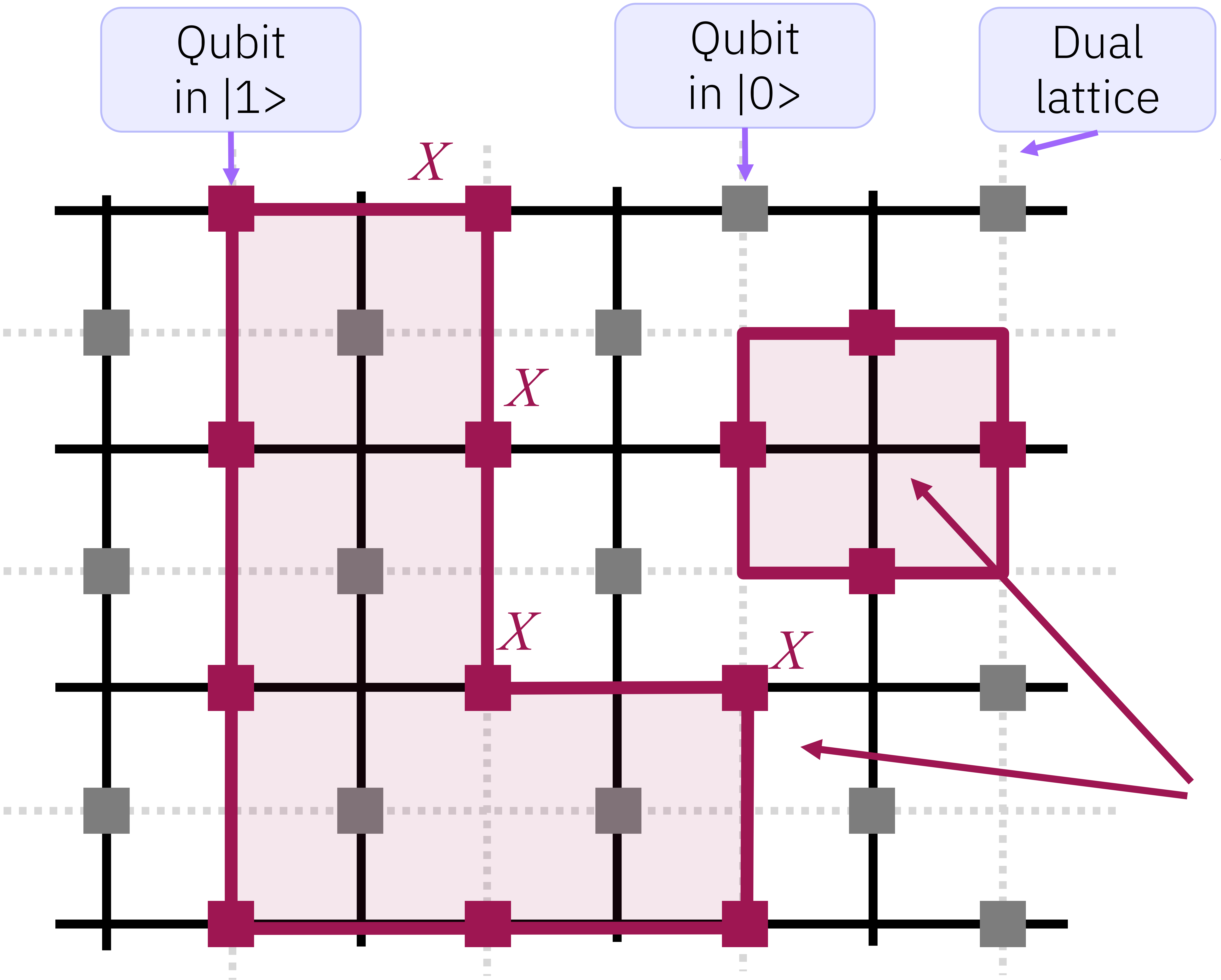
Hamiltonian $$H = -\sum_v A_v - \sum_p B_p$$

Commute $$[A_v, B_p] = 0$$

Toric code:
Kitaev, Proceedings of the 3rd Intl. Conf. of Quantum Comm. and Measurement (1997)
Kitaev, "Anyons in an exactly solved model and beyond". Annals of Physics. 321 (1): 2–111 (2006)

Topological matter and QEC

String nets in the Toric code



Vertex $A_v = \prod_{j \in v} X_j$

Plaquette $\boxed{B_p = \prod_{j \in p} Z_j}$

Hamiltonian $H = -\sum_v A_v \boxed{- \sum_p B_p}$

String loops Example of individual qubit state configuration (graph!) 010000111100

Example minimizes the B-plaquette term of Hamiltonian H.
All topologically-equivalent trivial loops on dual lattice have same energy, as the all zero state.

Toric code:
Kitaev, Proceedings of the 3rd Intl. Conf. of Quantum Comm. and Measurement (1997)
Kitaev, "Anyons in an exactly solved model and beyond". Annals of Physics. 321 (1): 2–111 (2006)

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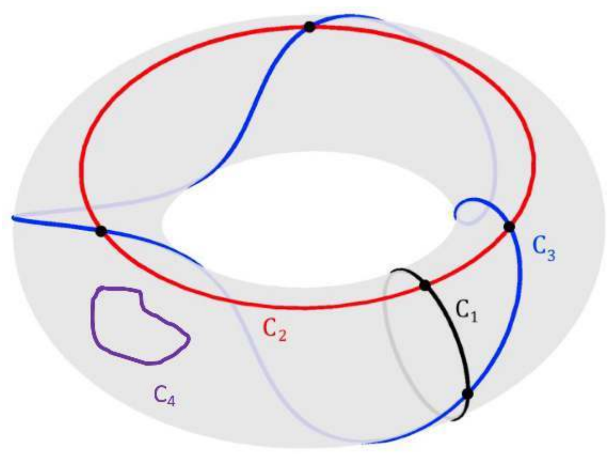
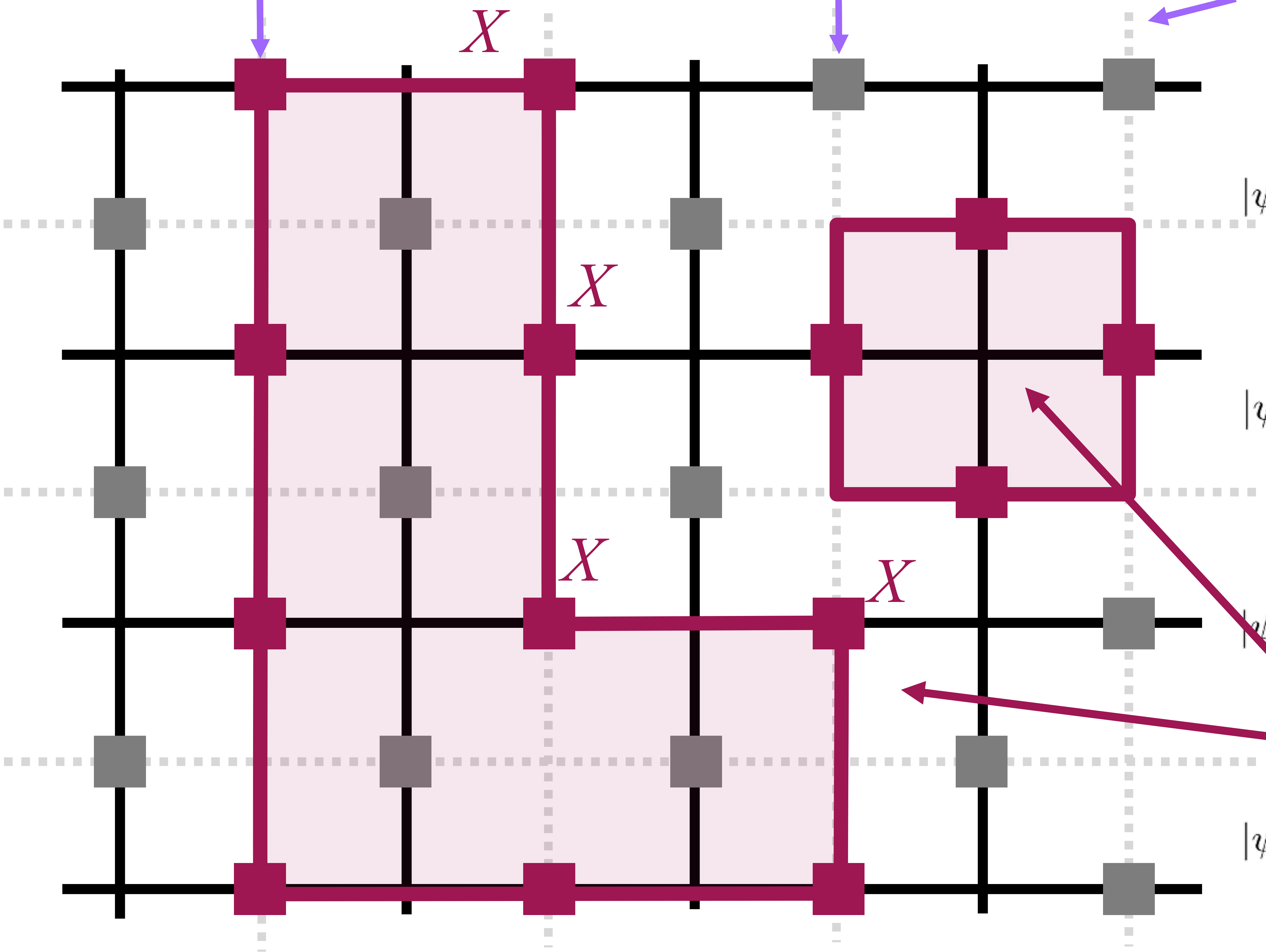
Topological matter and QEC



Ground state



Topological ground (vacuum) states
of Toric code with periodic b.c. on torus.

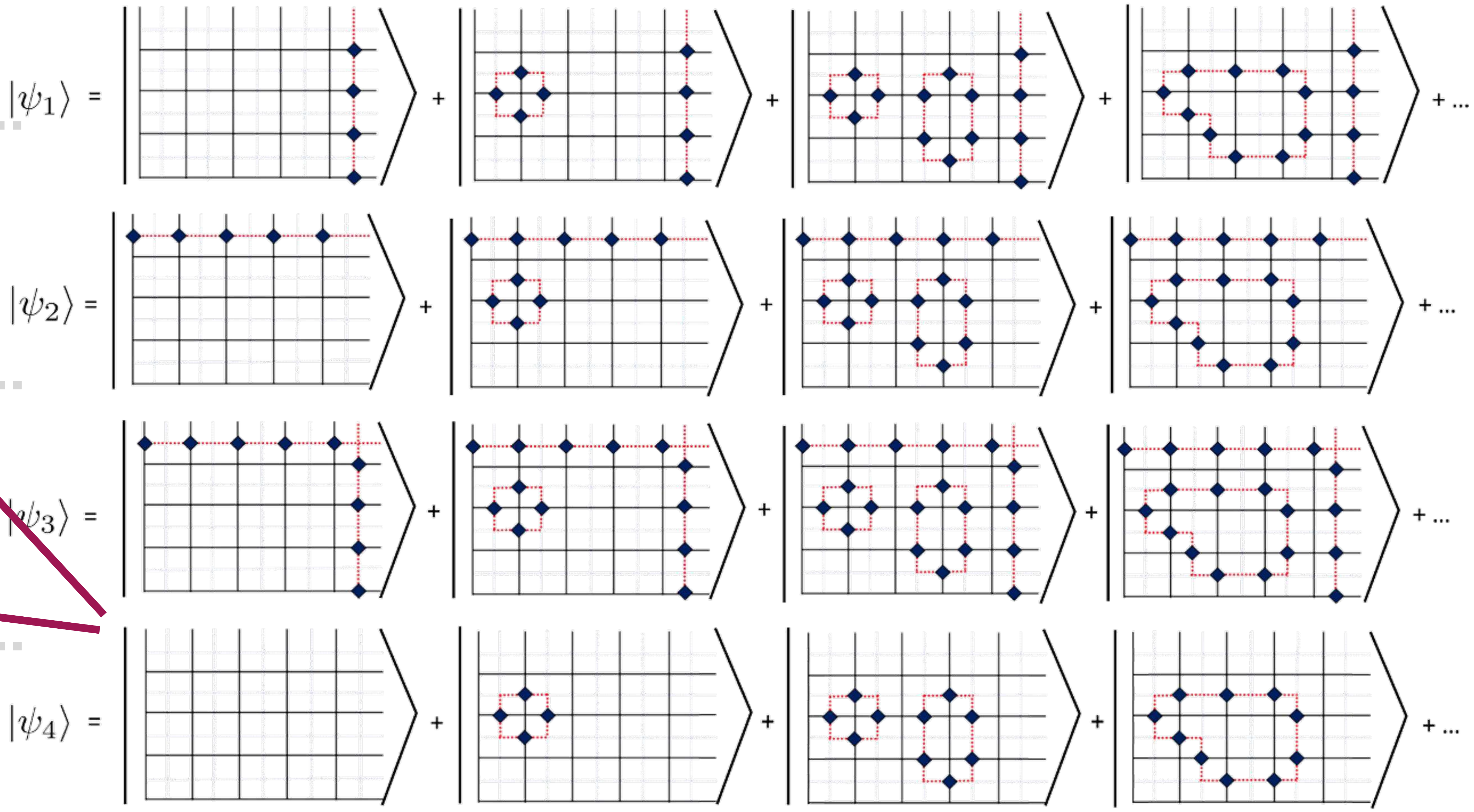


Image: D. Kufel

Toric code:
Kitaev, Proceedings of the 3rd Intl. Conf. of Quantum Comm. and Measurement (1997)
Kitaev, "Anyons in an exactly solved model and beyond". Annals of Physics. 321 (1): 2–111 (2006)

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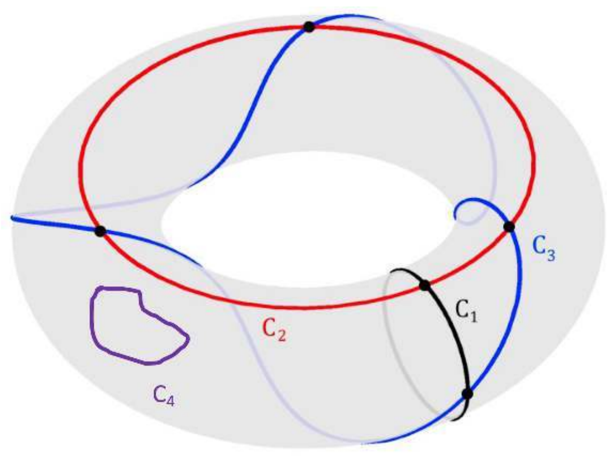
Topological matter and QEC



Ground state

Qubit in |1>

Qubit in |0>

Dual lattice

Topological: Robust to local errors/perturbations

Ground state is a **condensate** of closed strings

Each string graph has **same phase and amplitude!**

Topological ground (vacuum) states
of Toric code with periodic b.c. on torus.

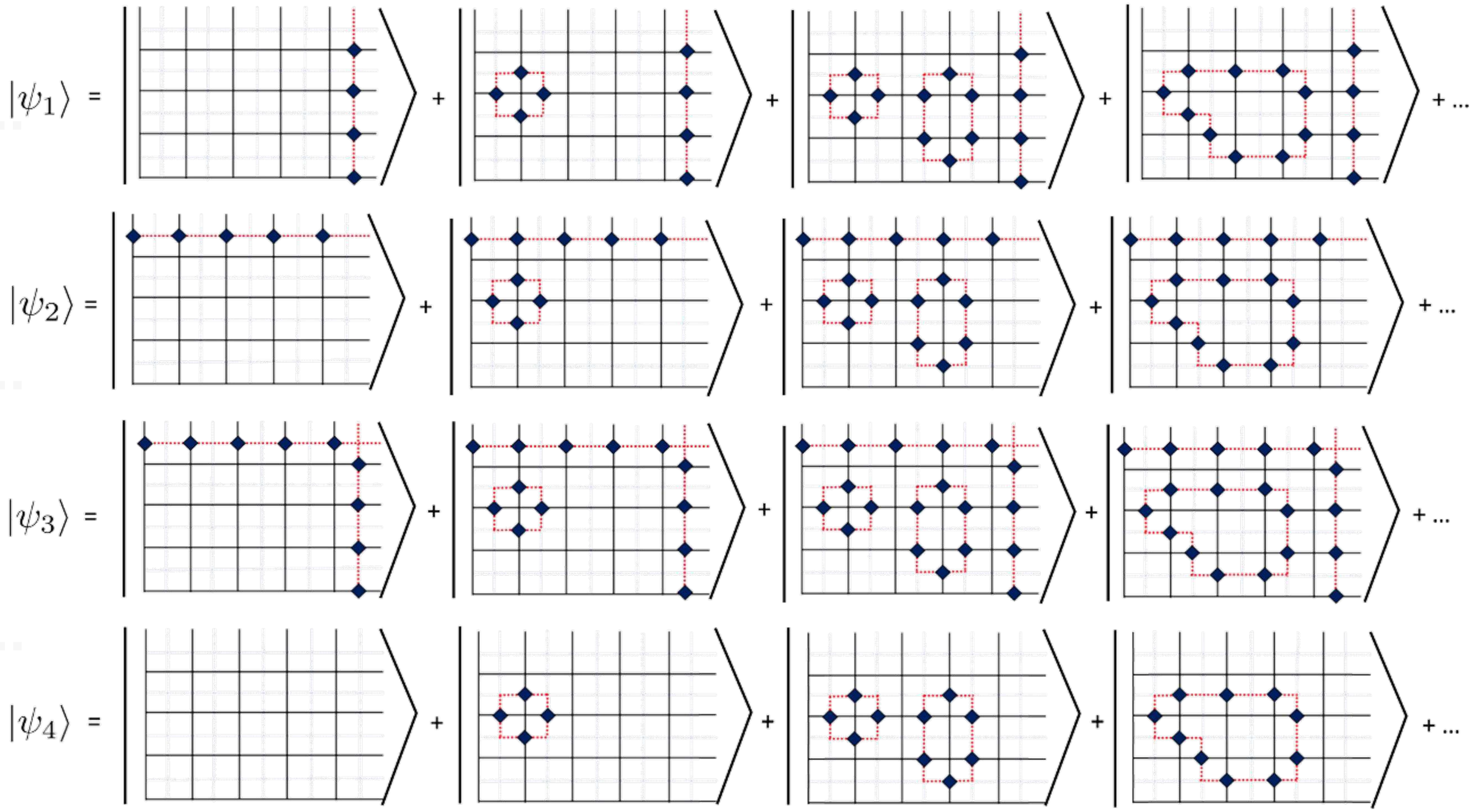


Image: D. Kufel

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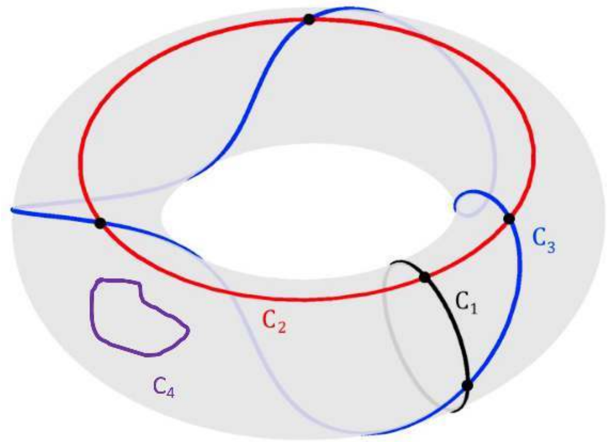
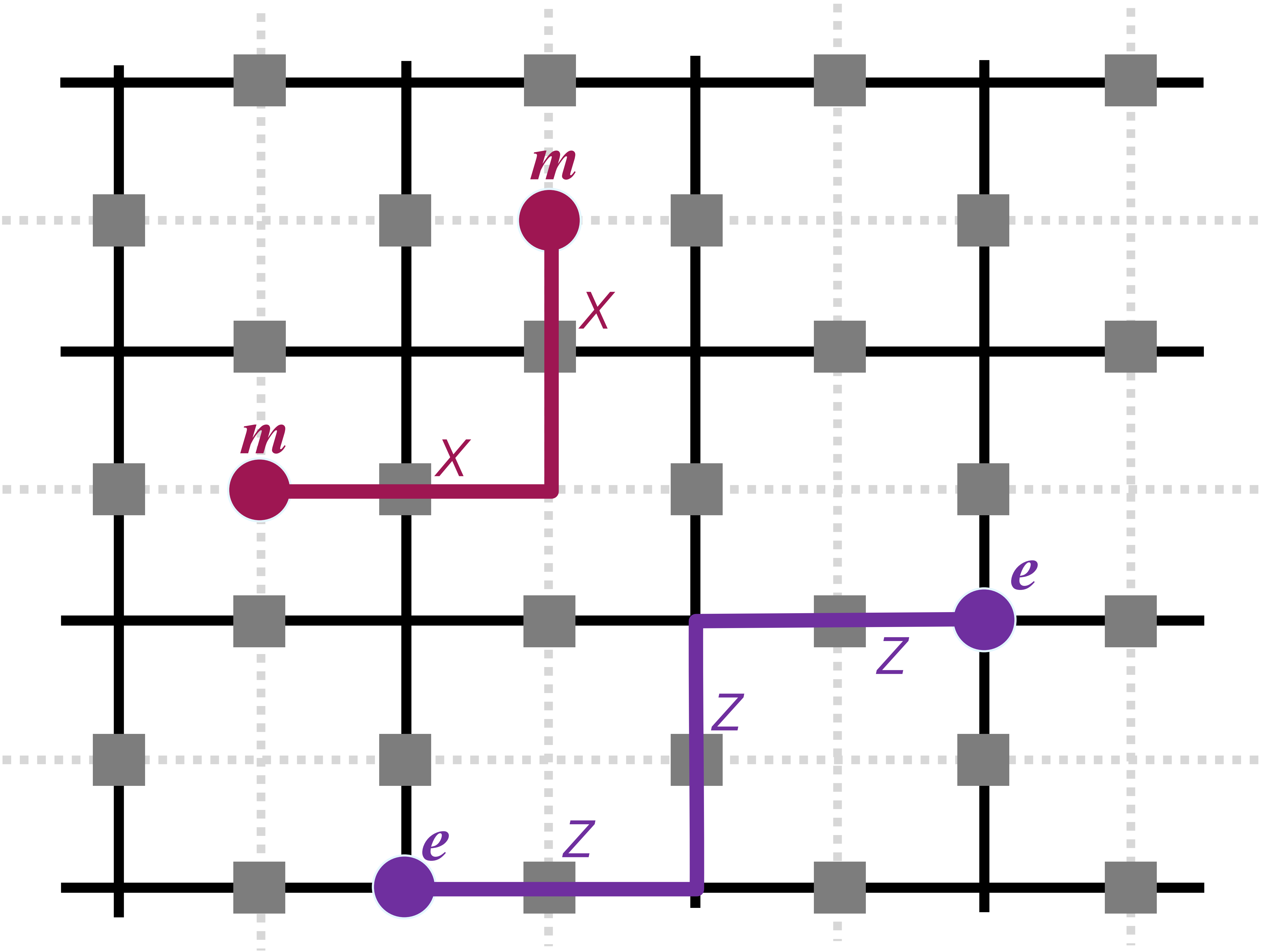
Topological matter and QEC



Quasiparticle excitations: **Abelian anyons**



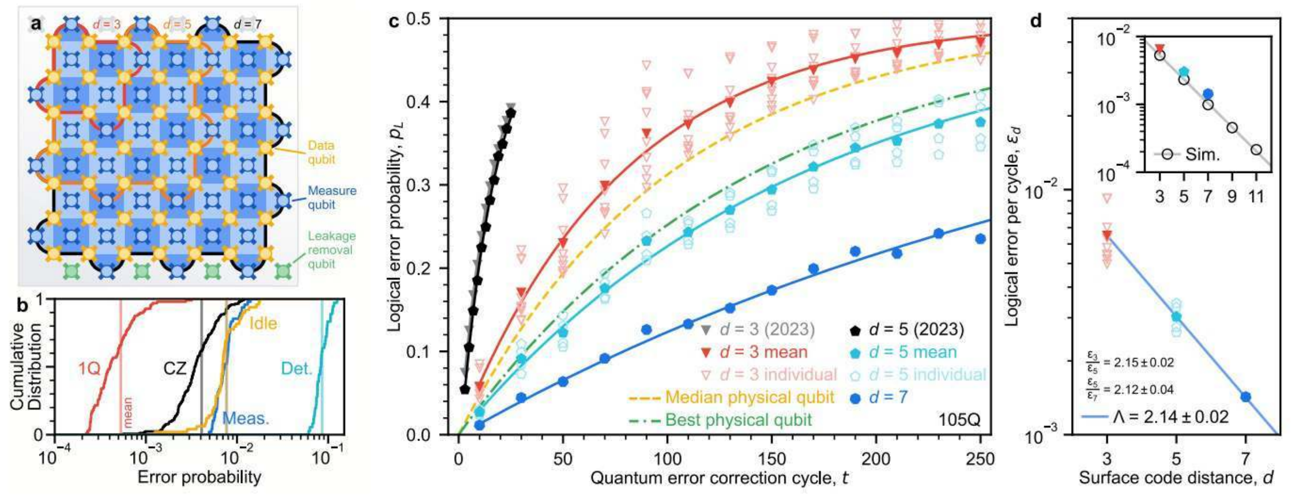
Anyons at the ends of broken strings

Properties topological, several flavors

Braiding → logical Clifford gates

No native universal gates 😿

See recent advances "*Quantum error correction below the surface code threshold*" by Google team



Toric code:
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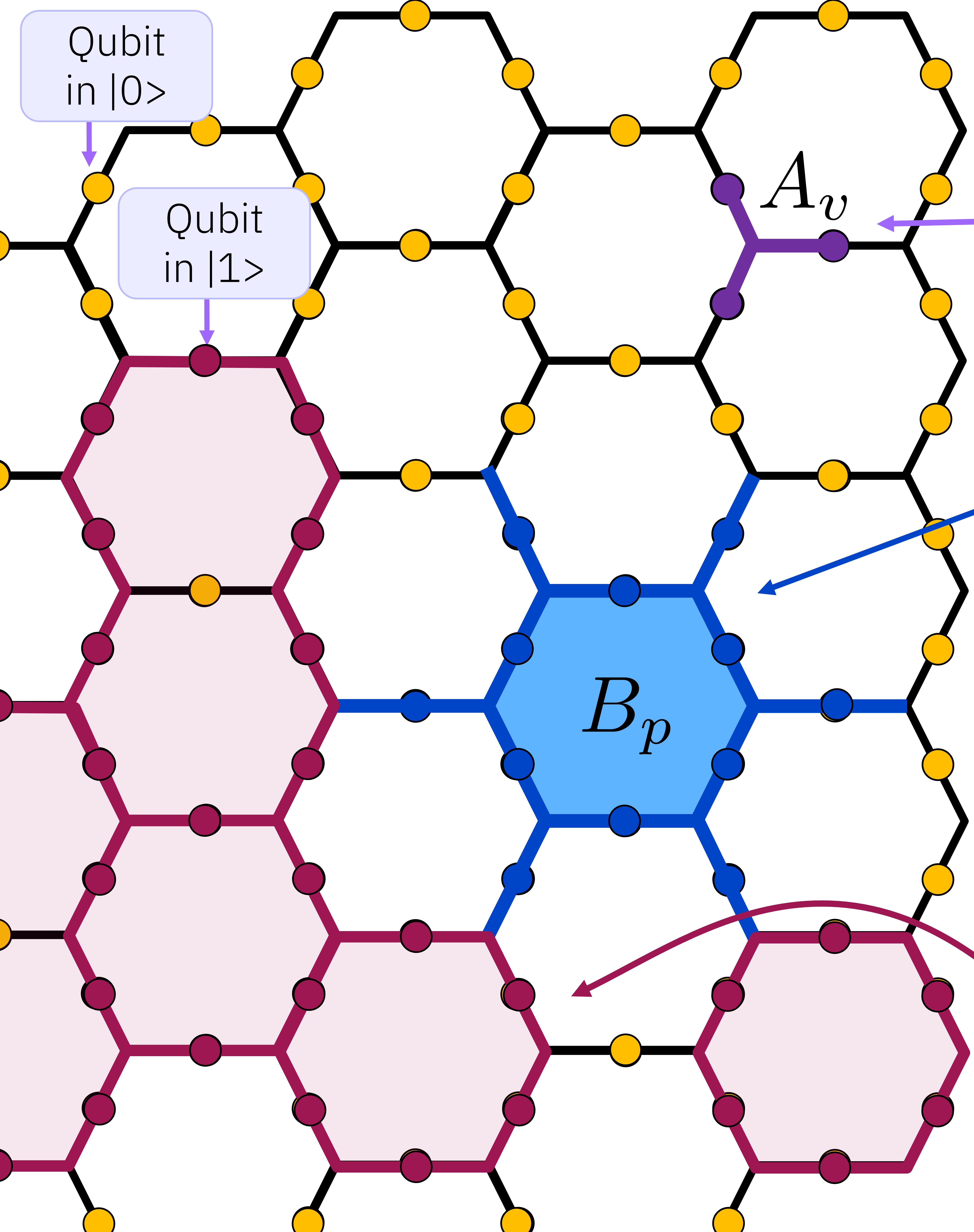

Levin-Wen model

Vertex $$A_v \left| {}_{i}\!\curlywedge^{j}_{k} \right\rangle = \delta_{ijk} \left| {}_{i}\!\curlywedge^{j}_{k} \right\rangle$$

Projection $$\delta_{ijk} = \begin{cases} 1 & \text{if } ijk = 000, 011, 101, 110, 111, \\ 0 & \text{otherwise.} \end{cases}$$

Plaquette B_p

B_p is a twelve-qubit interaction – hard! 😱

Golden ratio hidden inside

Hamiltonian $$\cancel{H =} - \sum_v A_v - \sum_p B_p$$

String nets

M. A. Levin and X.-G. Wen, Phys. Rev. B 71, 045110 (2005).
M. Levin and X.-G. Wen, Rev. Mod. Phys. 77, 871 (2005).

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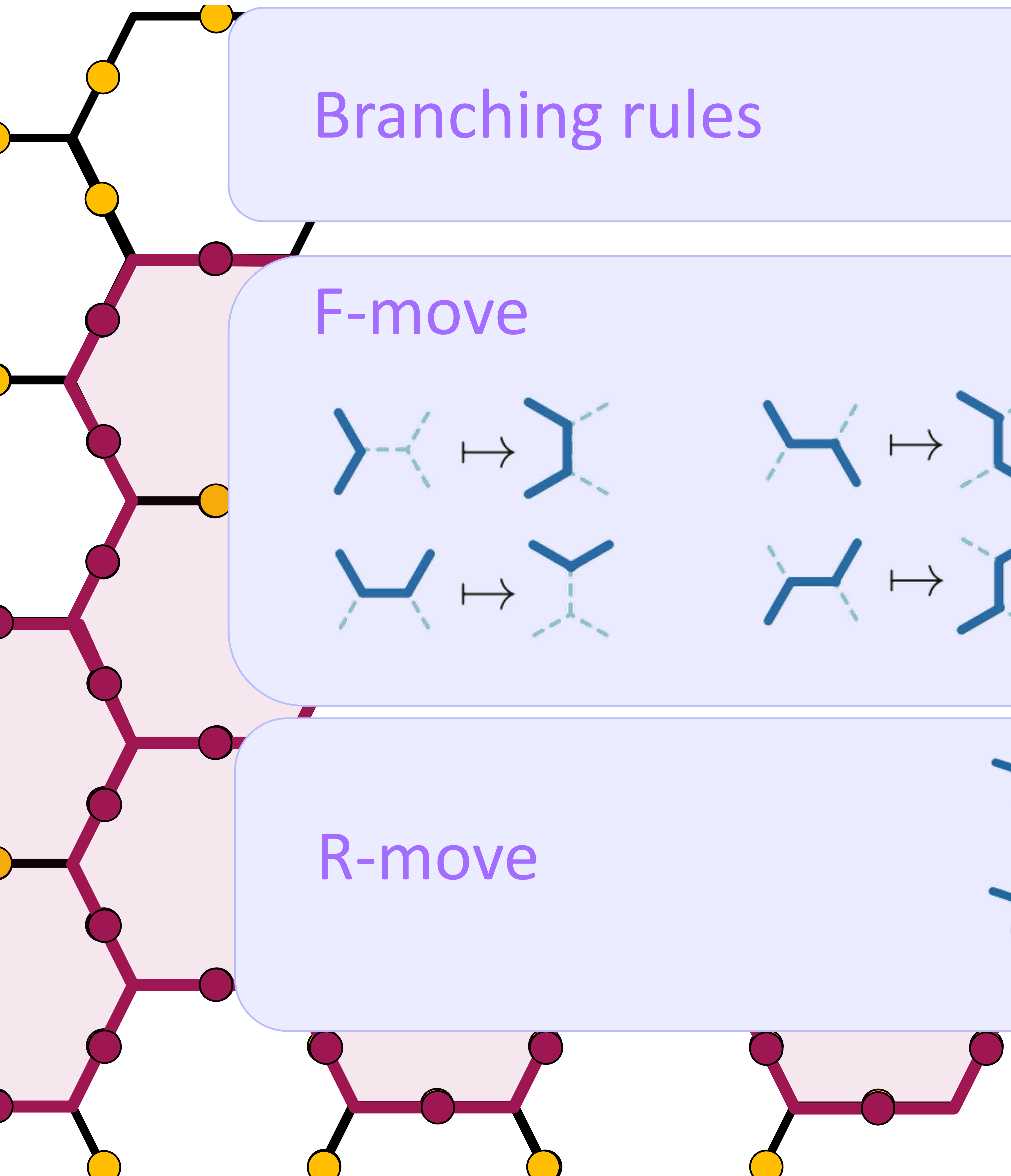
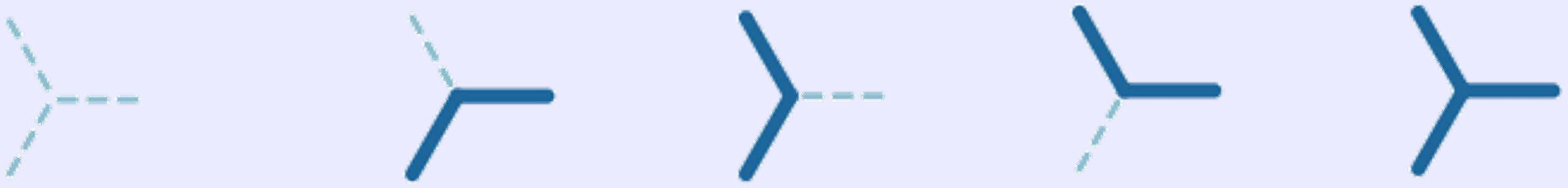
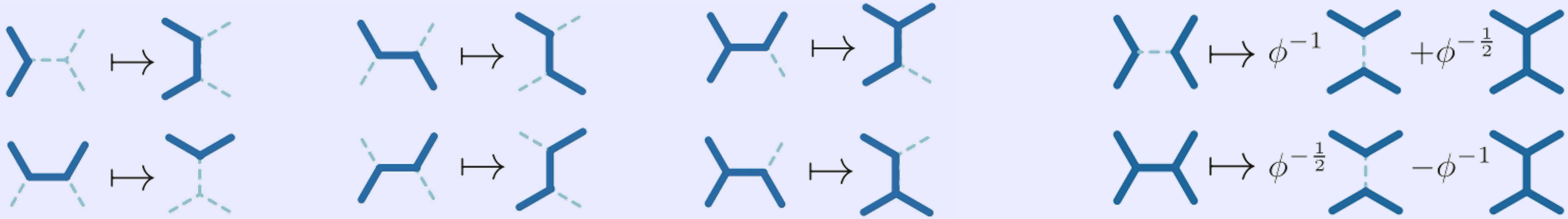
Geometry rules and deforming graphs



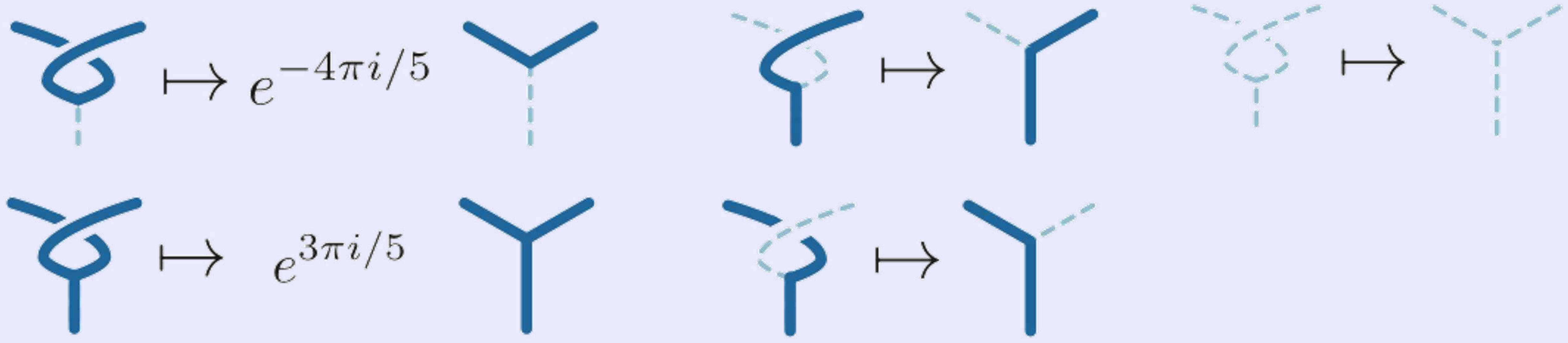
Branching rules



F-move



R-move



String-net condensation offers a general mechanism to study party- and time-reversal invariant topological phases

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Fibonacci string-net condensate (Fib-SNC) vacuum

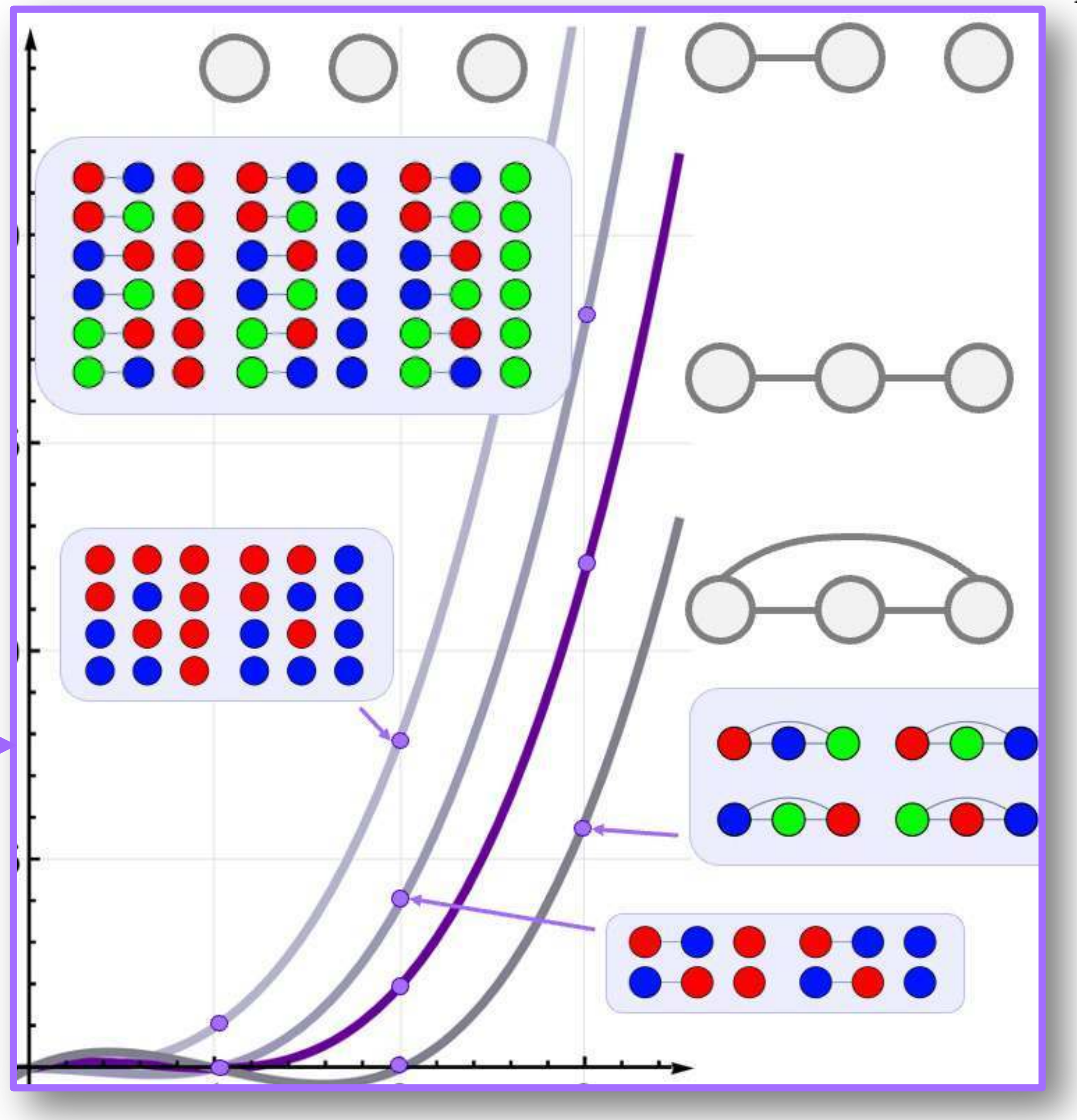
The Fib-SNC vacuum can be expressed as a condensation (superposition) of all closed-string network configurations (string diagrams) satisfying the branching (fusion) rules.

$$\widetilde{\Psi}(G)^2 = \frac{\Psi(G)^2}{\Psi(\mathrm{vac})^2} = \frac{1}{\phi+2}\chi(\hat{G}, \phi+2)$$

Relative sampling probability of vacuum bitstrings

Golden ratio

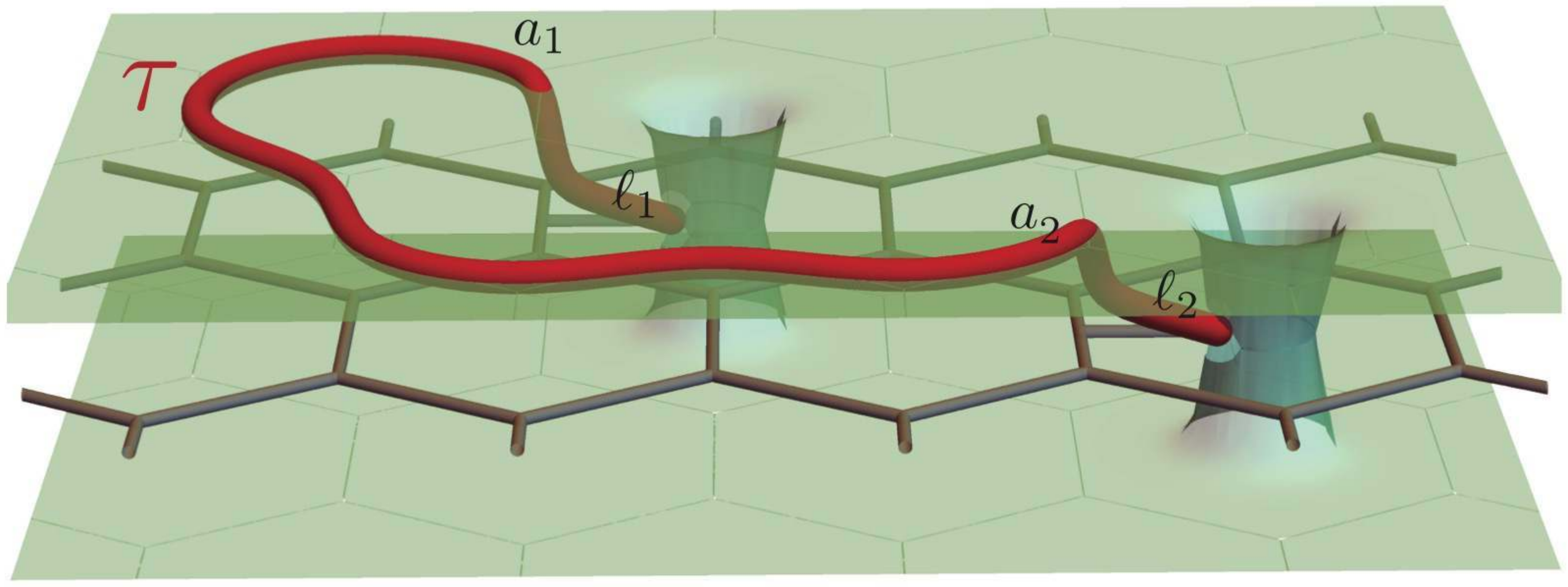
Chromatic polynomial of the dual graph



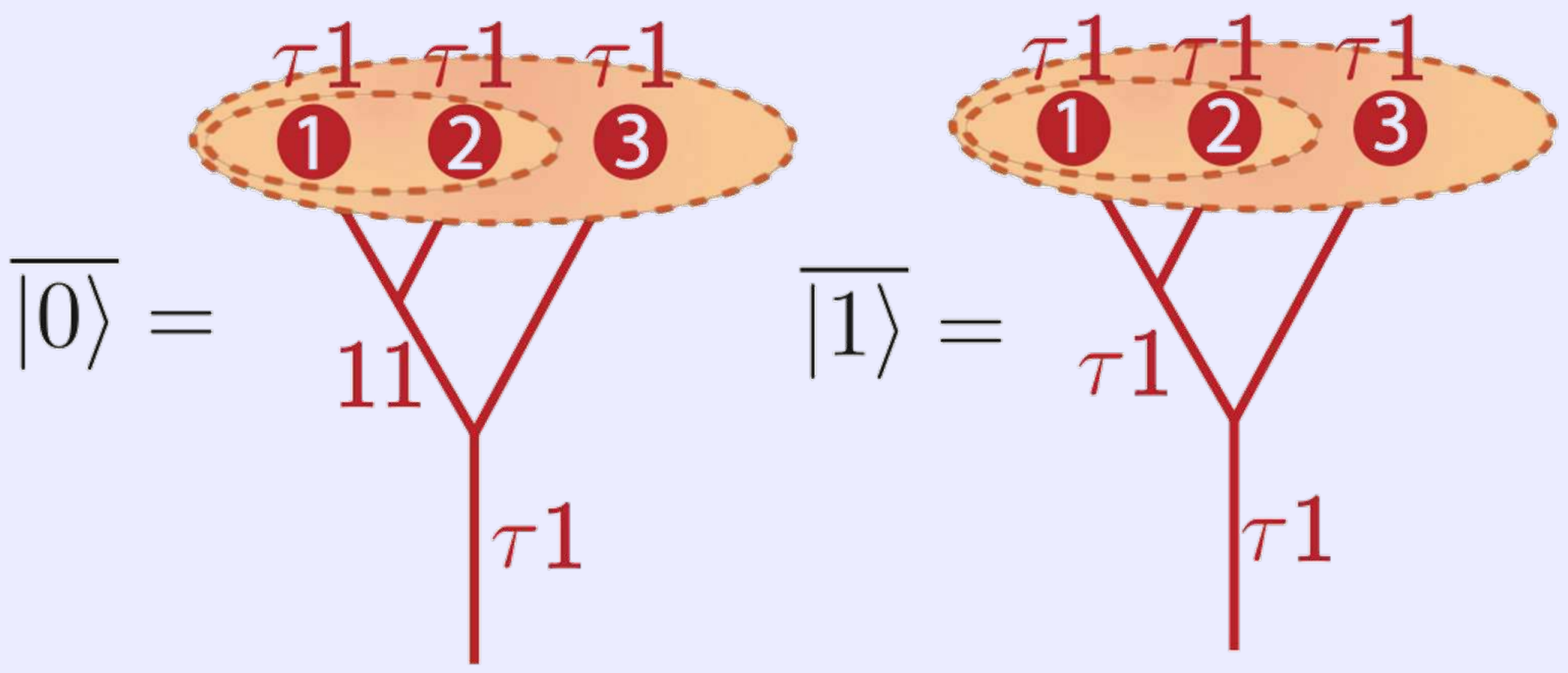
The exact evaluation of the wavefunction amplitude $\Psi(G)$ is #P-hard.

L. Fidkowski, et al., From String Nets to Nonabelions, Commun. Math. Phys. (2009).
P. Fendley and E. Fradkin, Realizing non-Abelian statistics in time-reversal-invariant systems, Phys. Rev. B (2005).
P. Fendley and V. Krushkal, Tutte chromatic identities from the Temperley–Lieb algebra, Geometry & Topology (2009).
P. Fendley and V. Krushkal, Link invariants, the chromatic polynomial and the Potts model, Adv. in The. and Math. Phys. (2010).

Excitations: Fibonacci anyons as a universal FQTC primitive

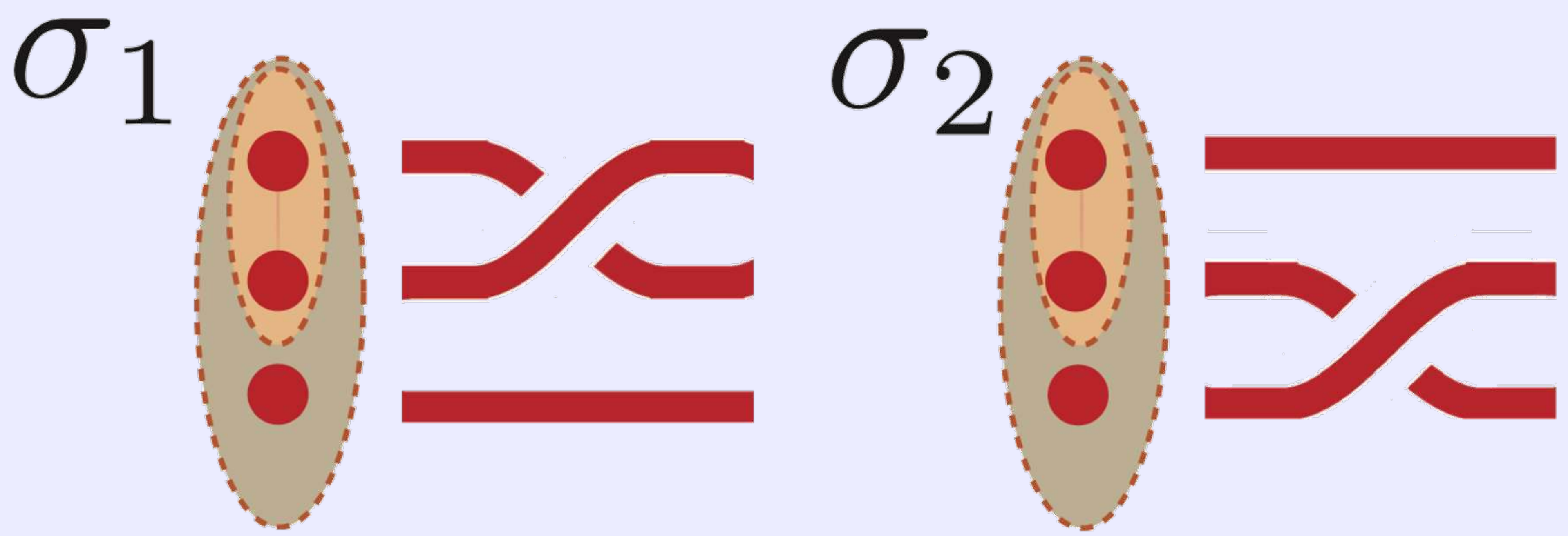


Logical qubit encoding



- **Universal gate set** for fault-tolerant quantum computation
  - No need for any additional gates for fault-tolerant quantum computation (FQTC) (unlike Toric, surface, LDPC, etc.)
  - Highly-entangled states, hard to simulate classically
  - Rooted in Fibonacci sequence and golden ratio
- **Non-abelian topological order and QEC**
  - Beyond stabilizer codes (Toric, surface, ...)

Logical non-Clifford gates



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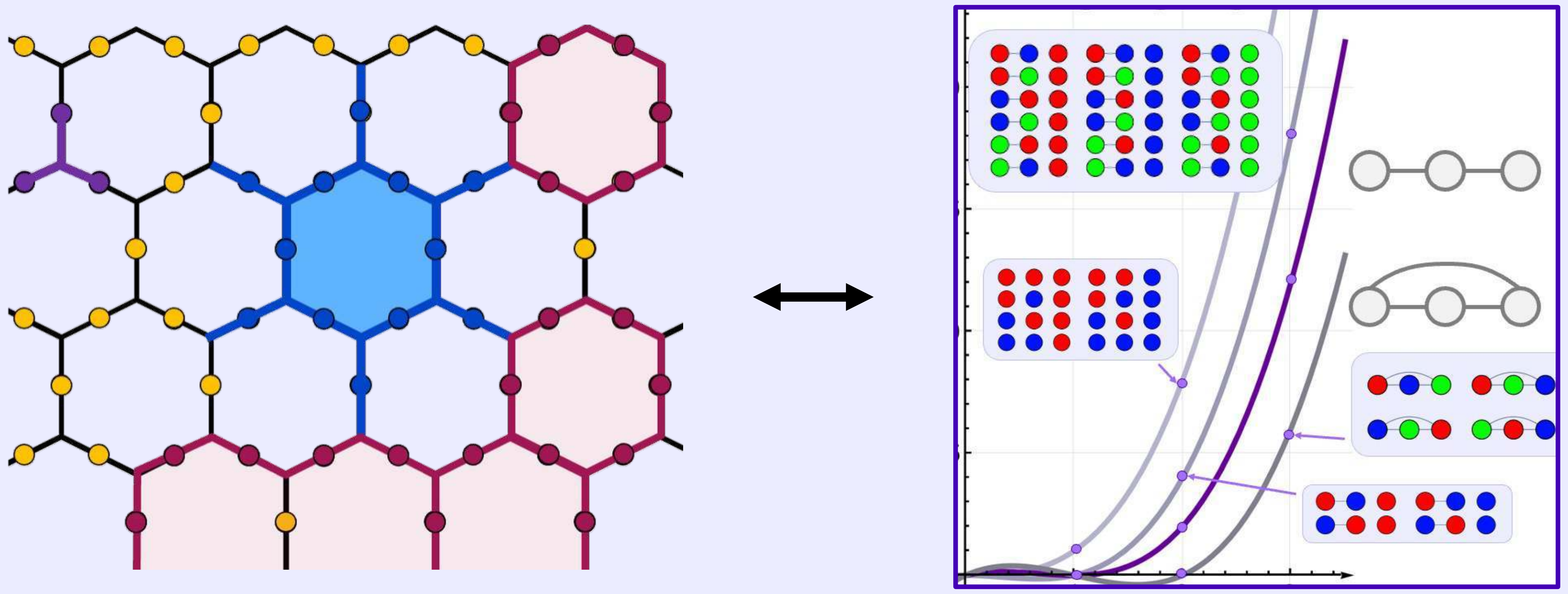
Fibonacci string-net condensate and anyons for universal FQTC

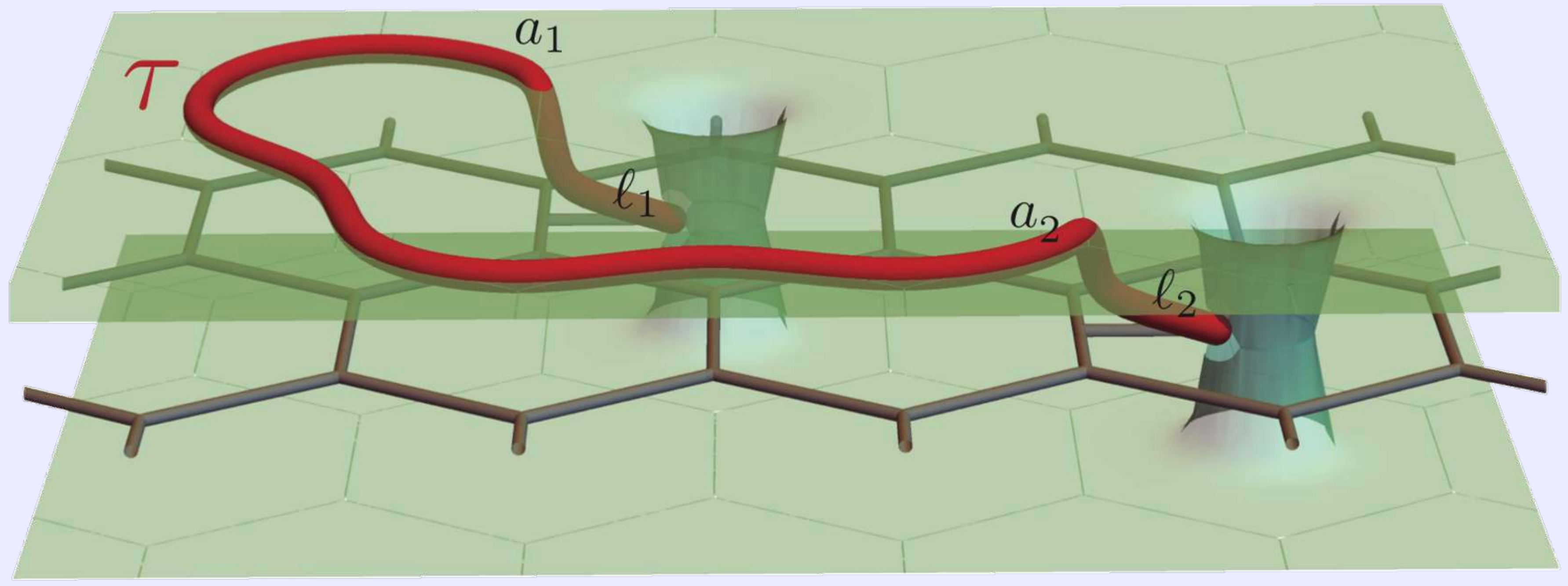
Goal 1

Realize faithful Fibonacci string-net condensate to sample chromatic polynomials (classically hard)



Goal 2

Braid non-Abelian Fibonacci anyons to perform logical gate that is non-Clifford (universal)



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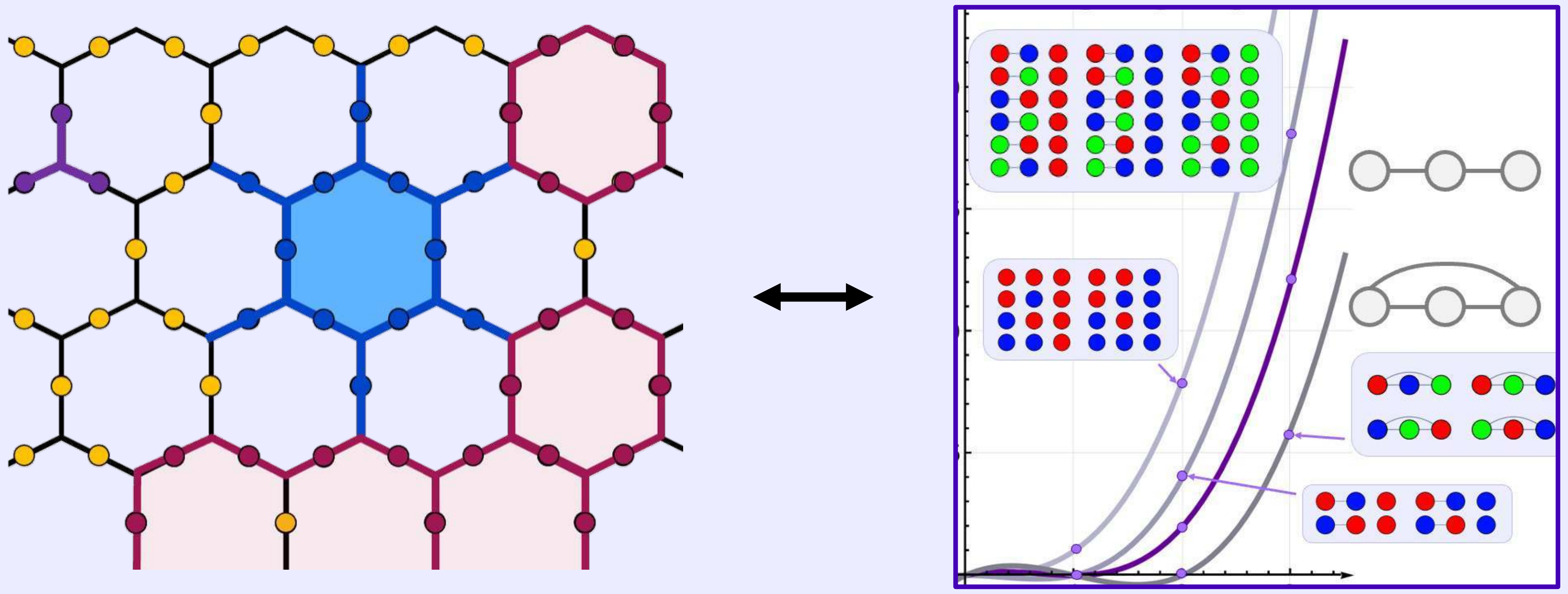
The challenges

Goal 1

Realize faithful Fibonacci string-net condensate to sample chromatic polynomials (classically hard)



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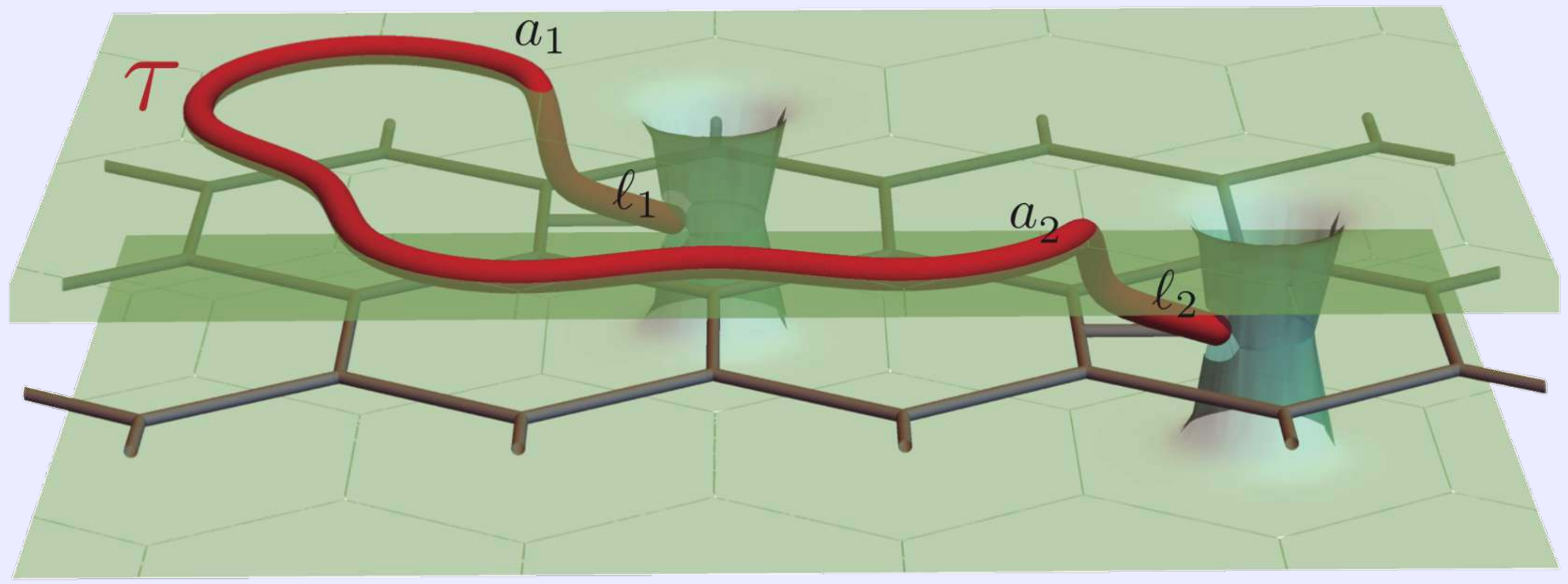

Unreachable so far. Why?

- B_p is a 12-qubit operator
  Circuit depths are incredibly large
- Operations require many F-moves, requiring high-weight controlled operations

Despite successful implementation of topological states with Abelian anyons [1, 2] and even non-Abelian Ising [3, 4] and D_4 anyons [5], whose braiding is restricted to Clifford gates at best. Recent attempts [6] showed promise, but 12-qubit operators forced the use of approximations, making graph condensation exploration practically infeasible.

[1] Science, 374 1242 (2021); [2] Science 374, 1237 (2021); [3] Nature 618, 264 (2023);
[4] Chinese Phys. Lett. 40, 060301 (2023); [5] Nature 626, 505 (2024); [6] Nature Phys. 20, 1469 (2024)

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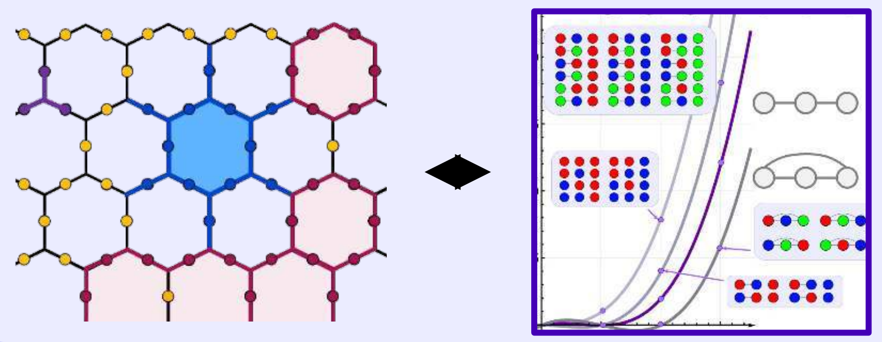
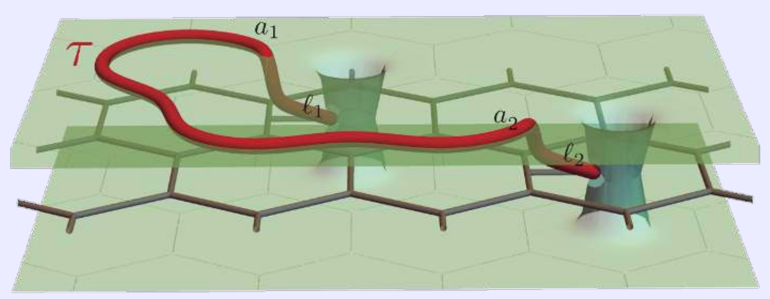
Solution! Overcoming the challenges

Realize faithful Fibonacci string-net condensate
to sample chromatic polynomials (classically hard)



Braid non-Abelian Fibonacci anyons
to perform logical gate that is non-Clifford (universal)



Dynamical string-net preparation (DSNP)

- **Scalable, suited** for hardware
- **Efficient** in qubits / gates - few qubits can represent full plaquette
- **General** for any string nets. See paper for explanation
- **Exact**, no approximations!

How?
- Based on **reconfigurable graphs** + some theory team magic :)
- Build graphs in virtual not physical space (see manuscript)

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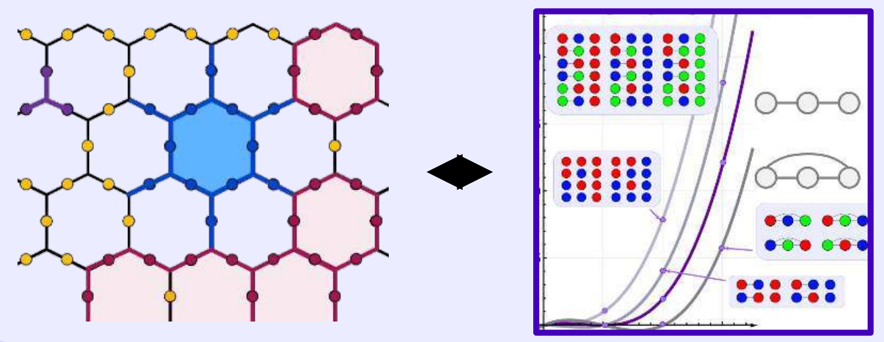
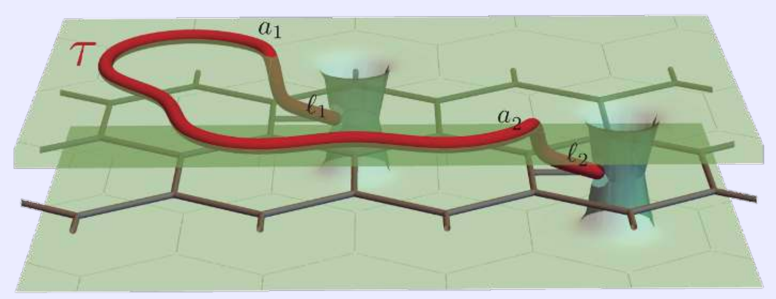
Solution! Overcoming the challenges

Realize faithful Fibonacci string-net condensate
to sample chromatic polynomials (classically hard)



Braid non-Abelian Fibonacci anyons
to perform logical gate that is non-Clifford (universal)



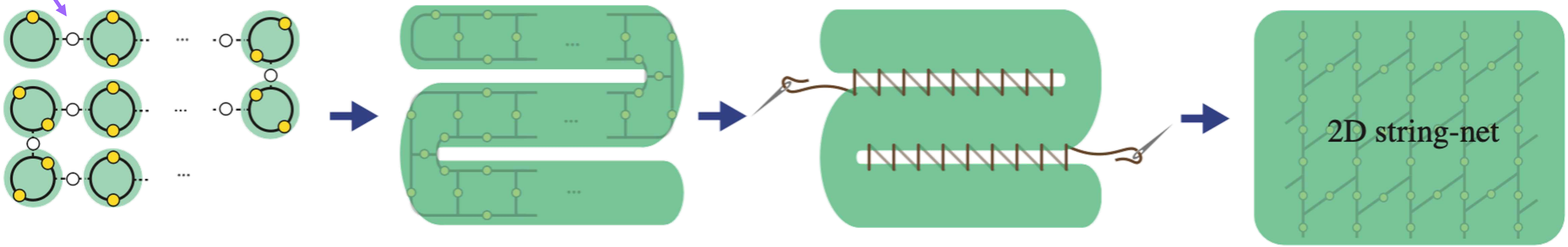
Dynamical string-net preparation (DSNP)

- **Exact**, no approximations!
- **General** for any string nets. See paper for explanation
- **Efficient** in qubits / gates - few qubits can represent full plaquette
- **Scalable, suited** for hardware

+ Many execution improvements

- new SOTA hardware with low-crosstalk
- optimized compiling and error aware-layout
- composite error-mitigation
- see supplemental information
- ...

Qubit



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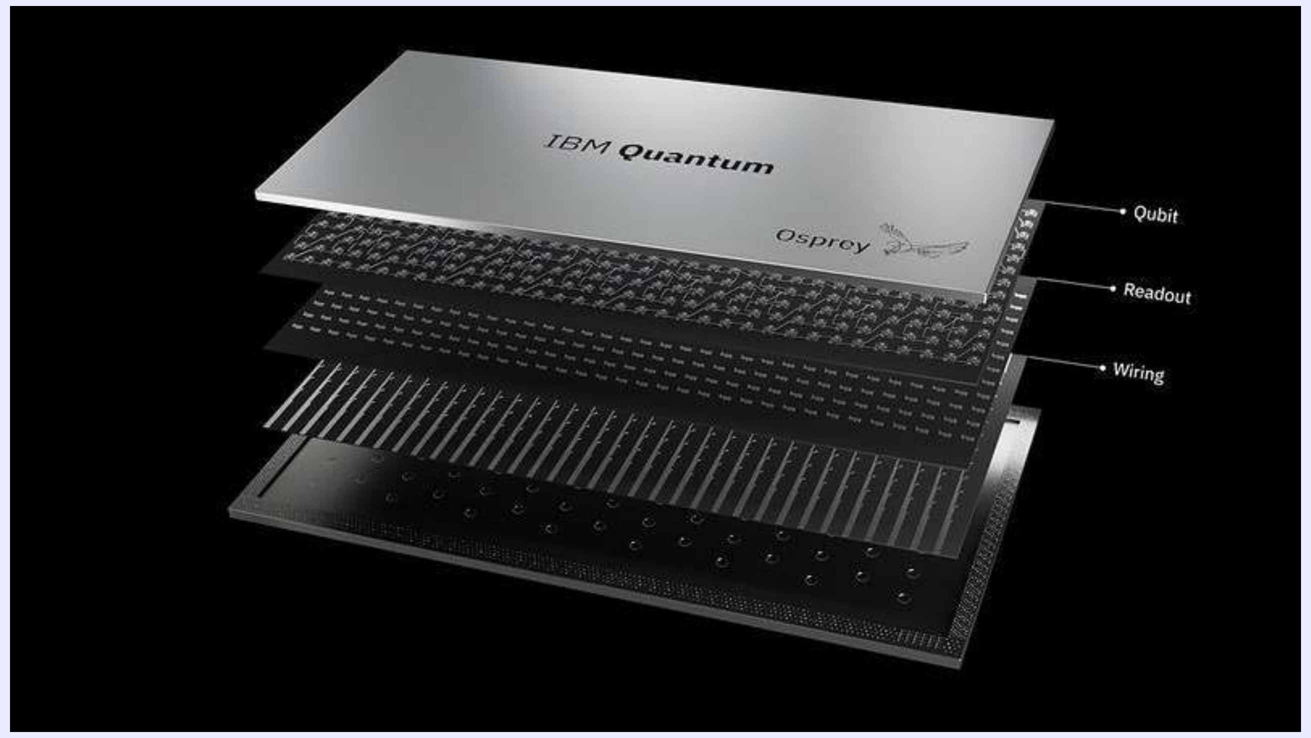
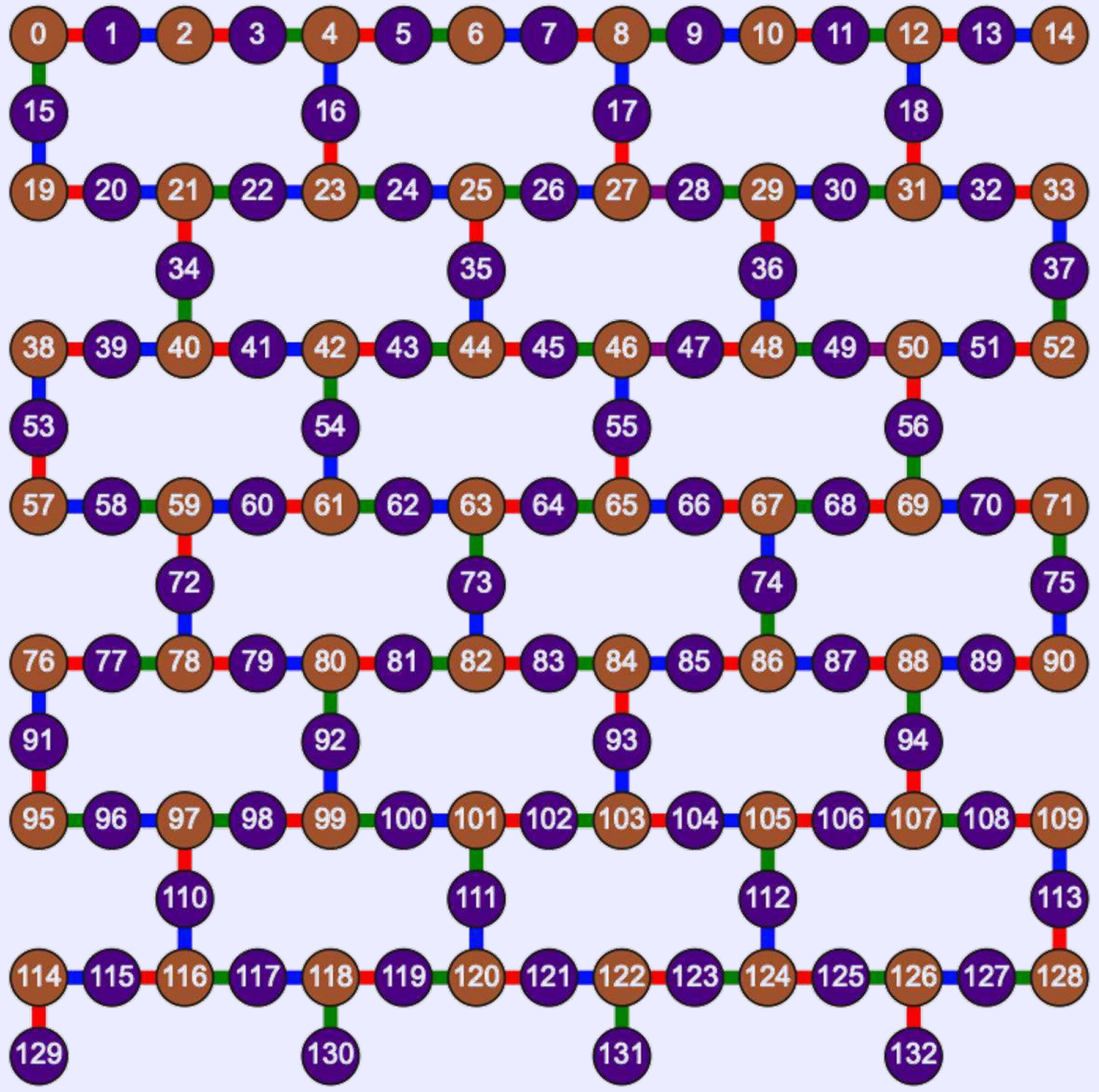
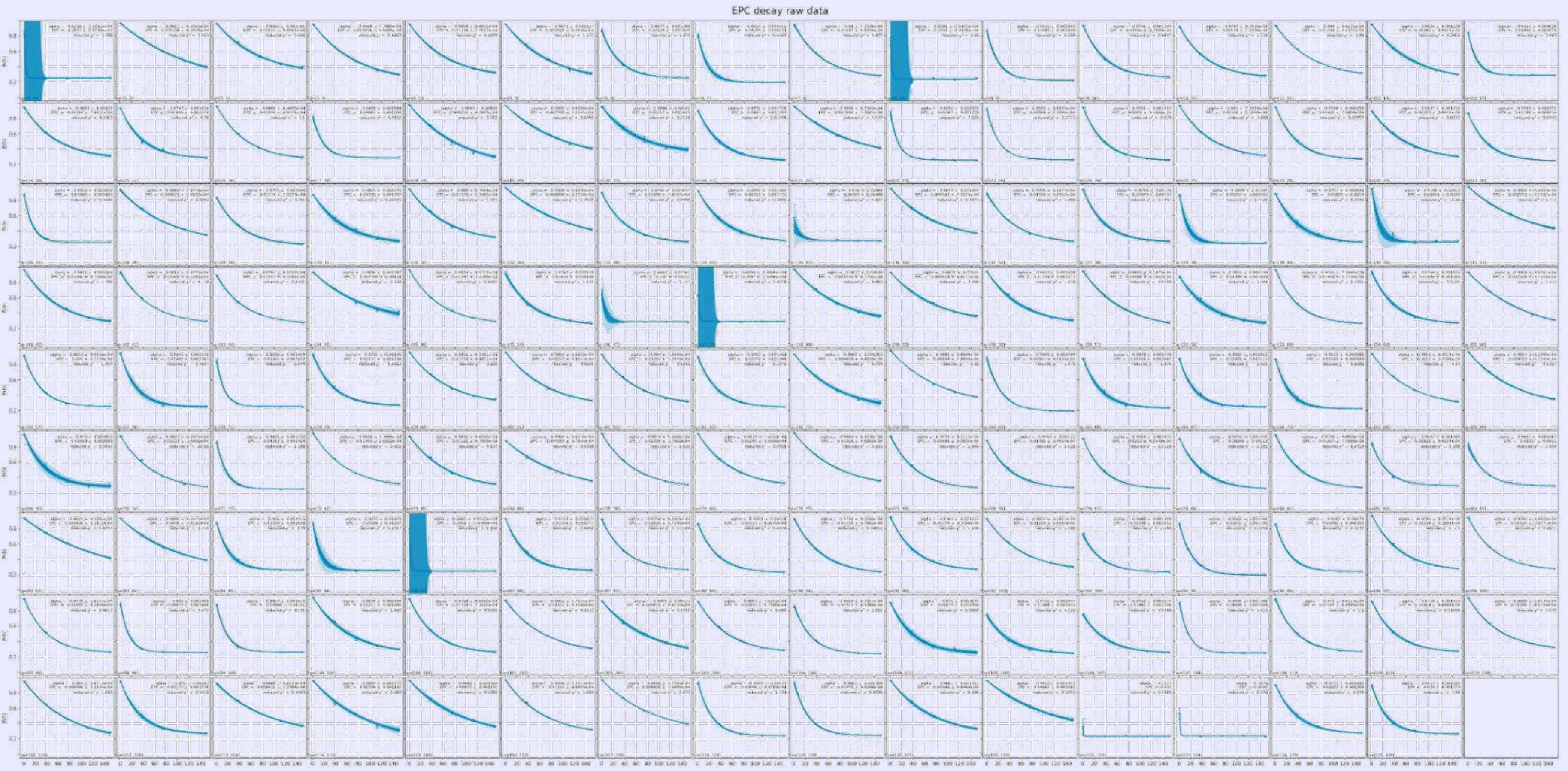
Experiment

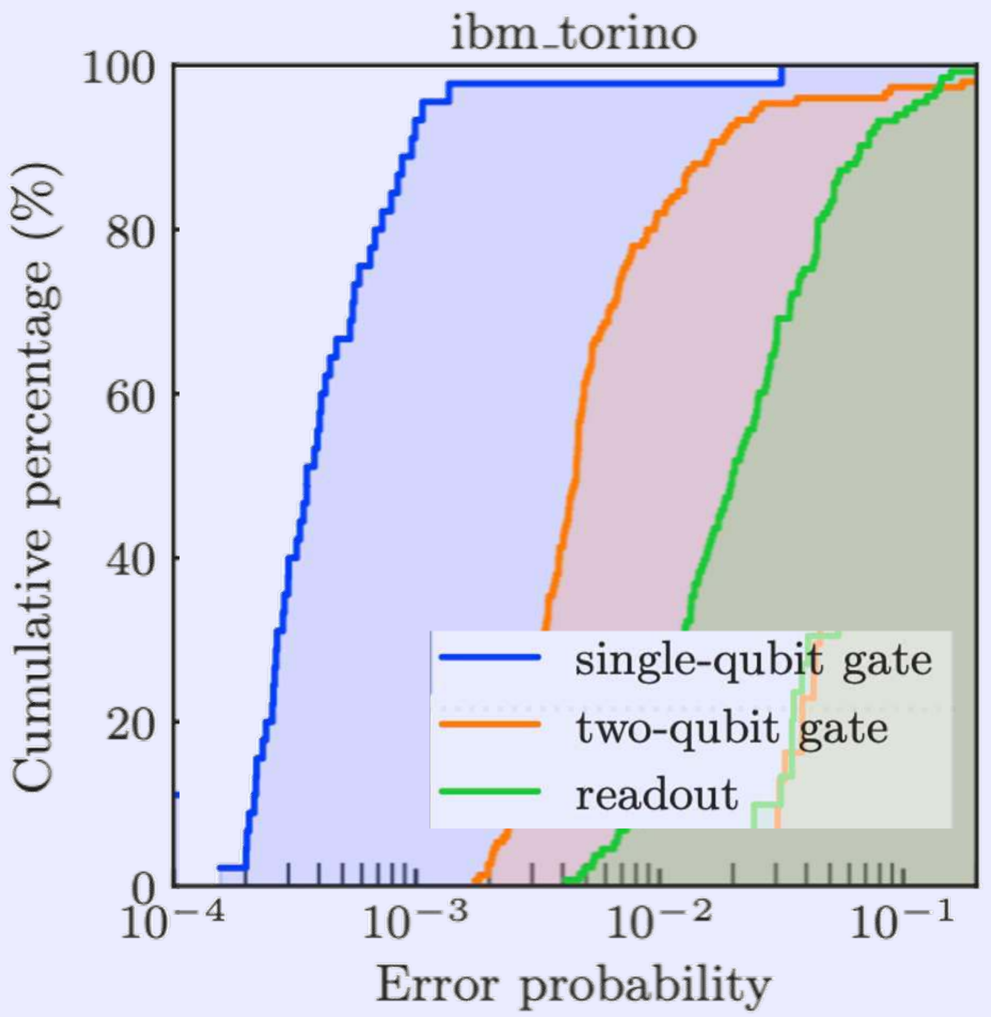




Heron device
Fast flux gates, low x-talk

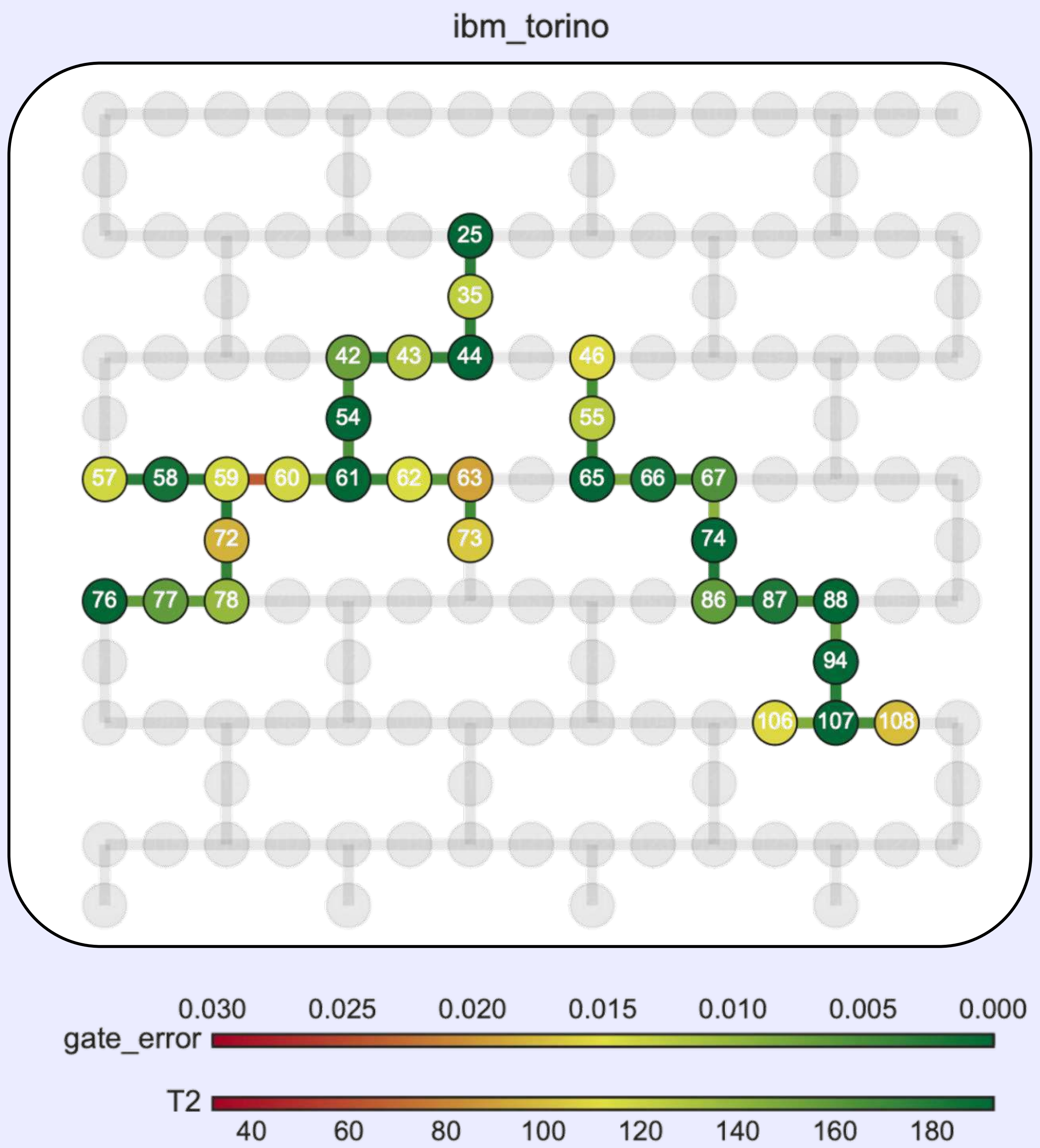
Real-time benchmarking

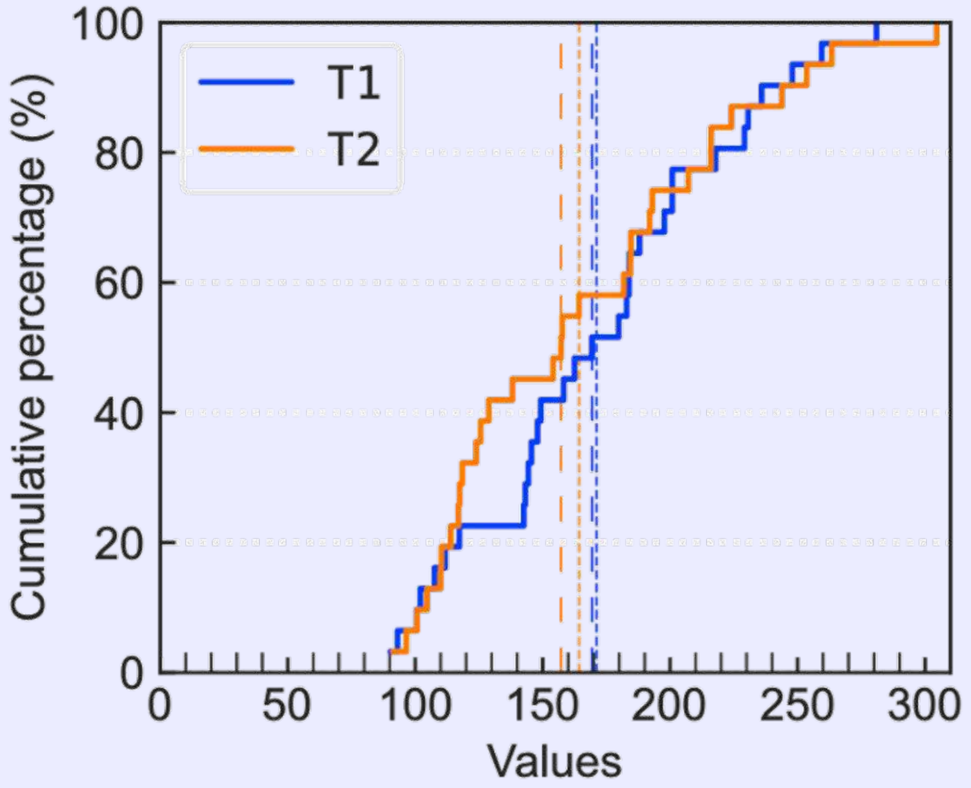


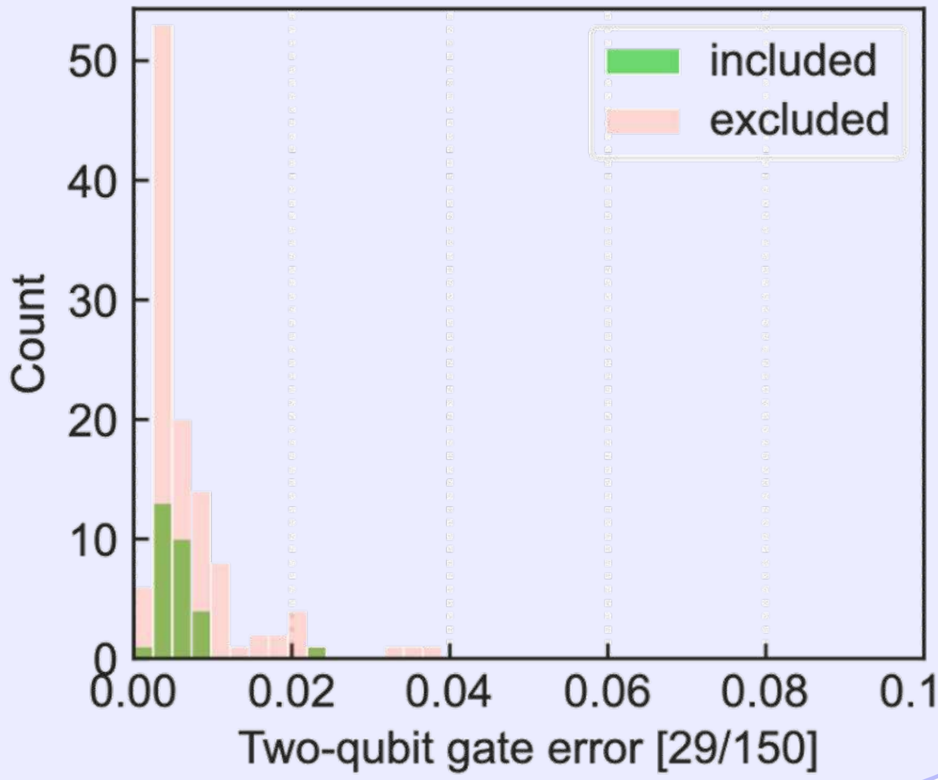


Optimized layout using real-time benchmarking data

(see appendix)

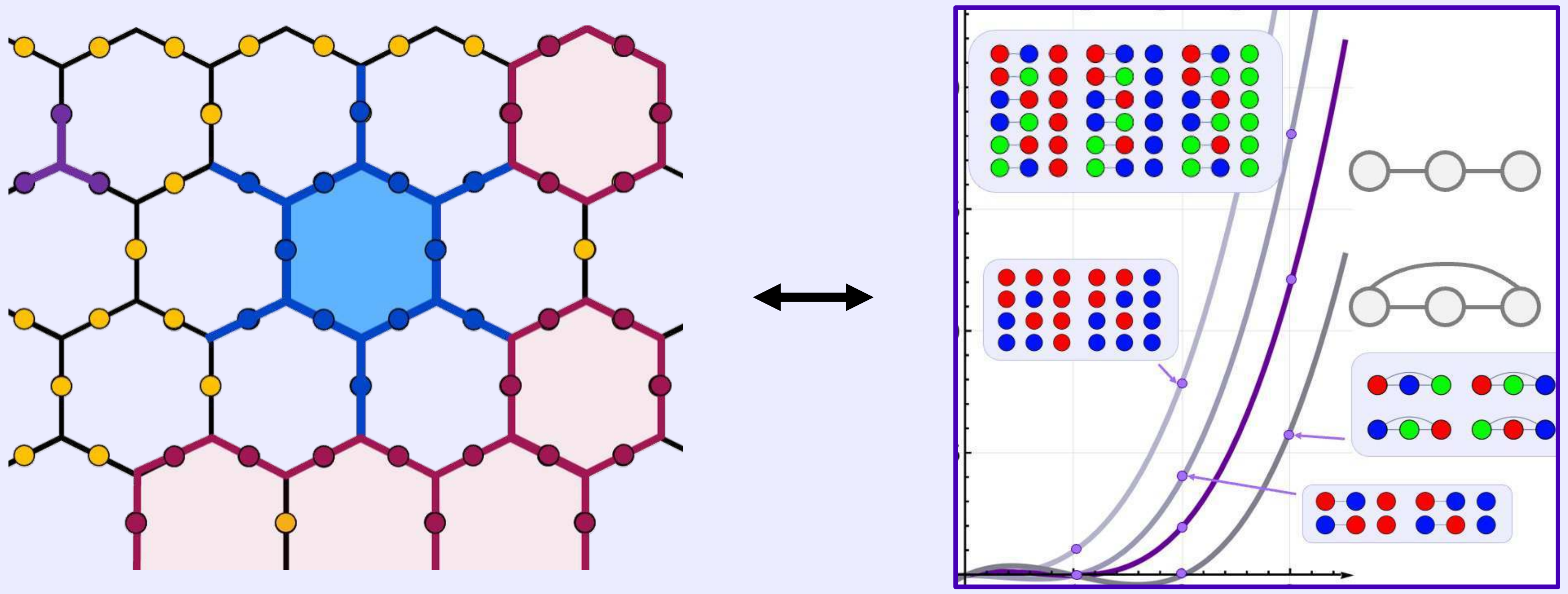




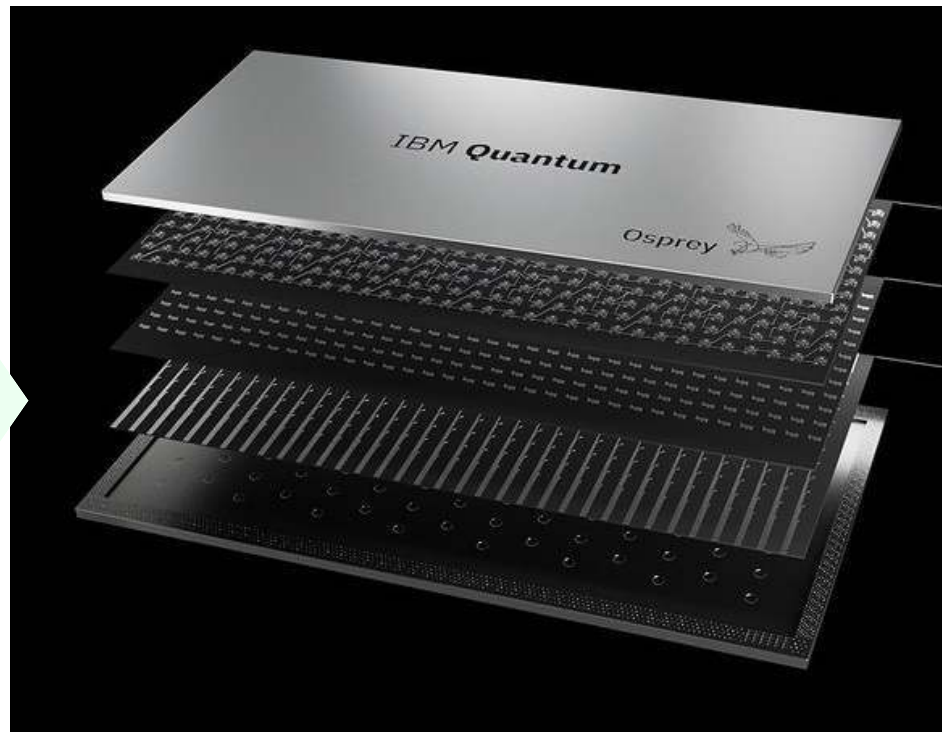


Preparing a 2D Fib-SNC with dynamic string-net preparation (DSNP)

Realize faithful Fibonacci string-net condensate to sample chromatic polynomials (classically hard)

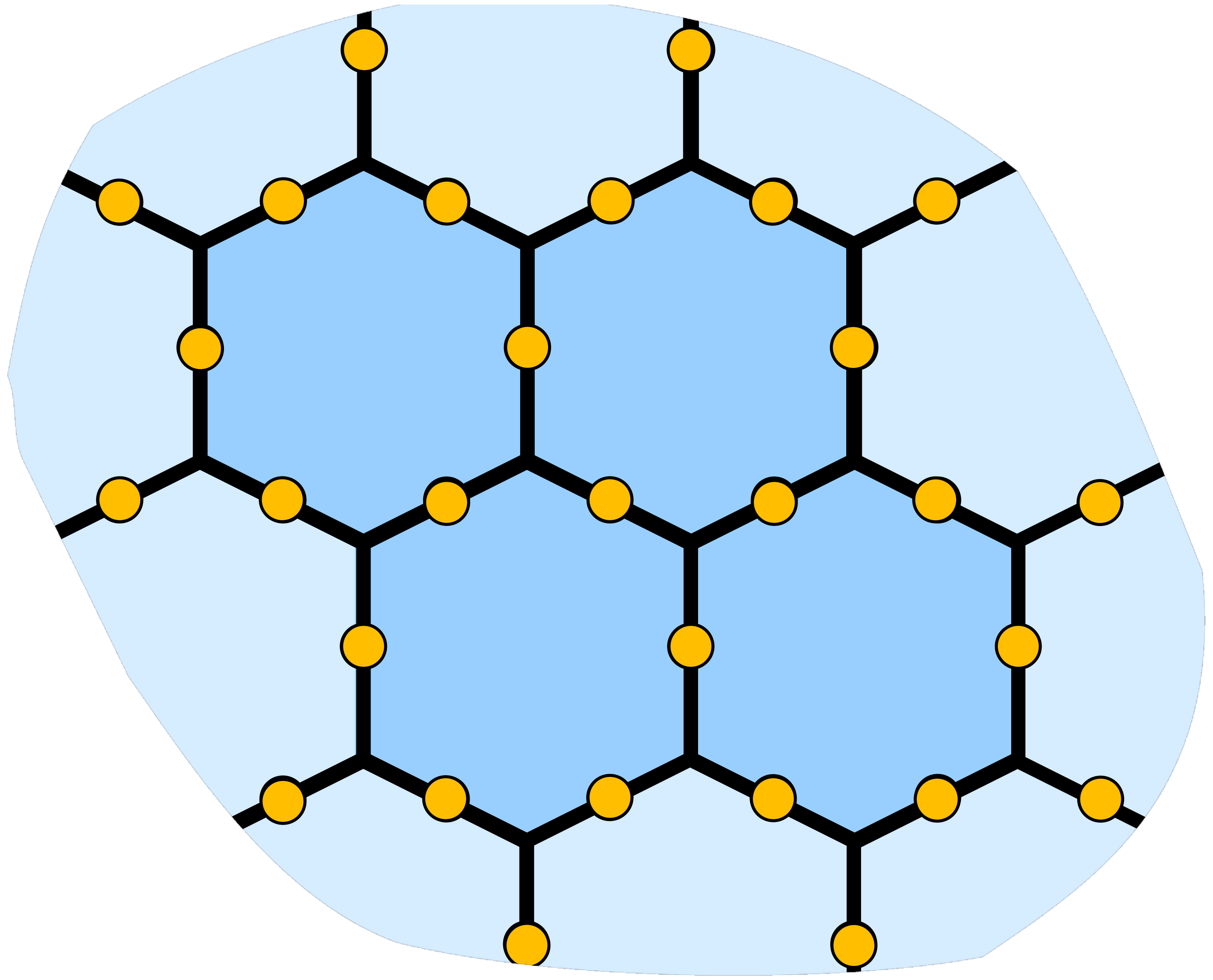


Dynamical string-net preparation (DSNP)

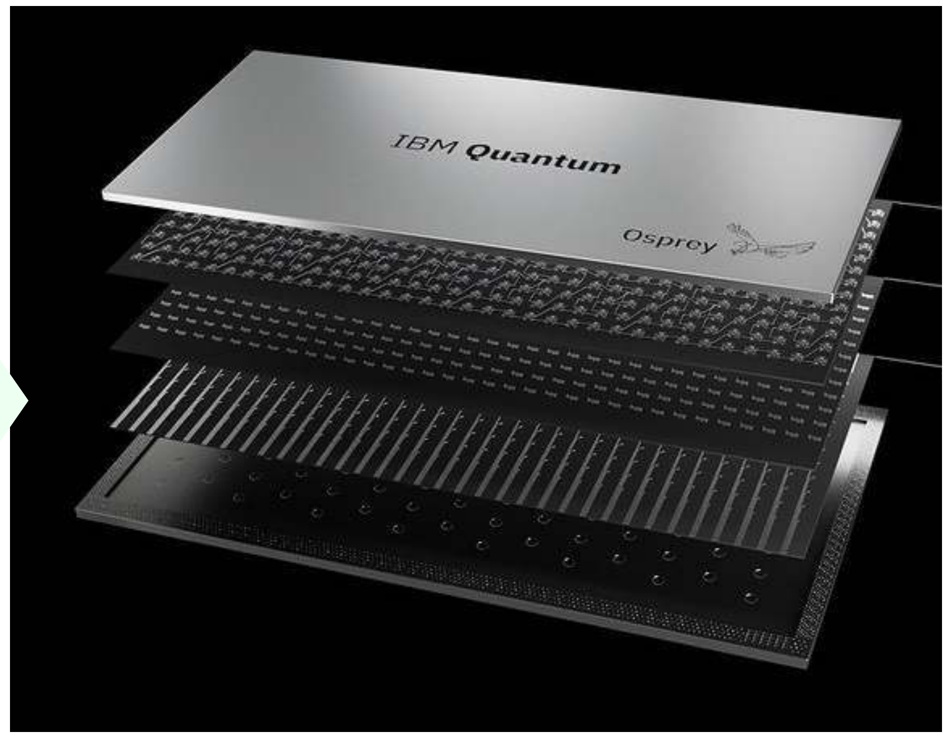


Two-dimensional four-plaquette Fib-SNC vacuum

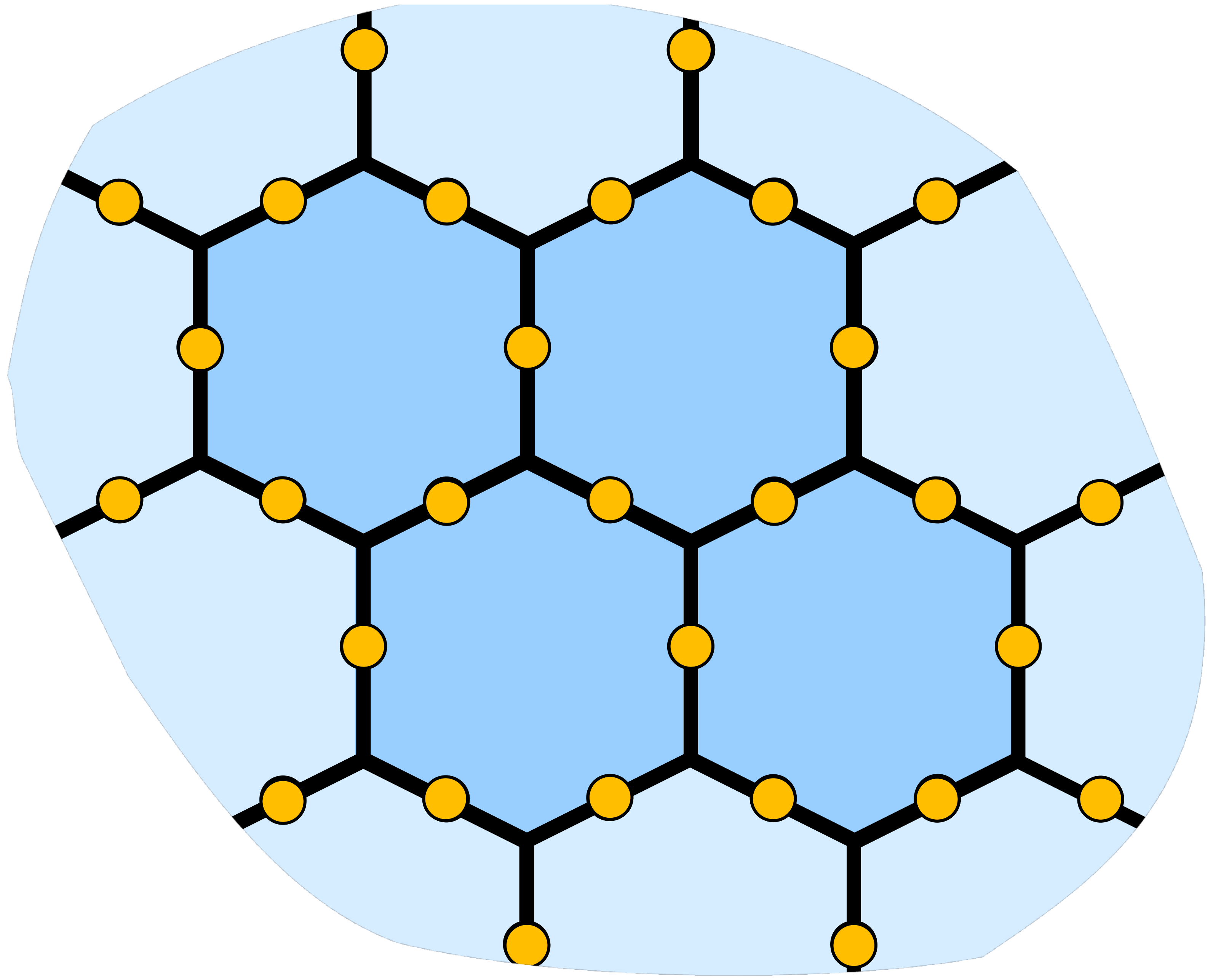
Start small
and simple



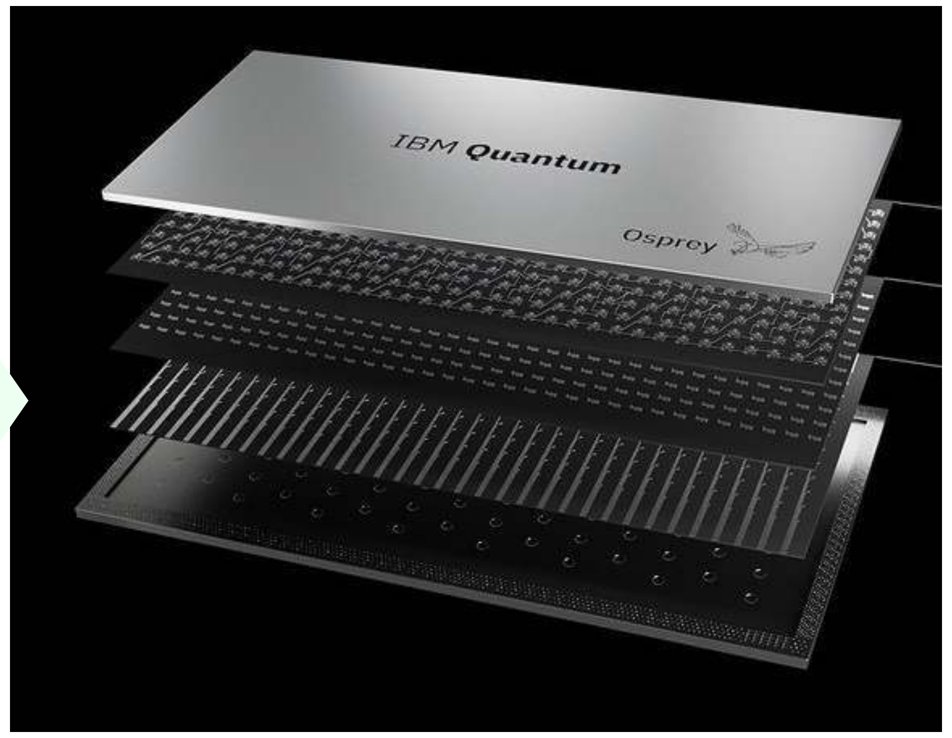
Dynamical string-net
preparation (DSNP)

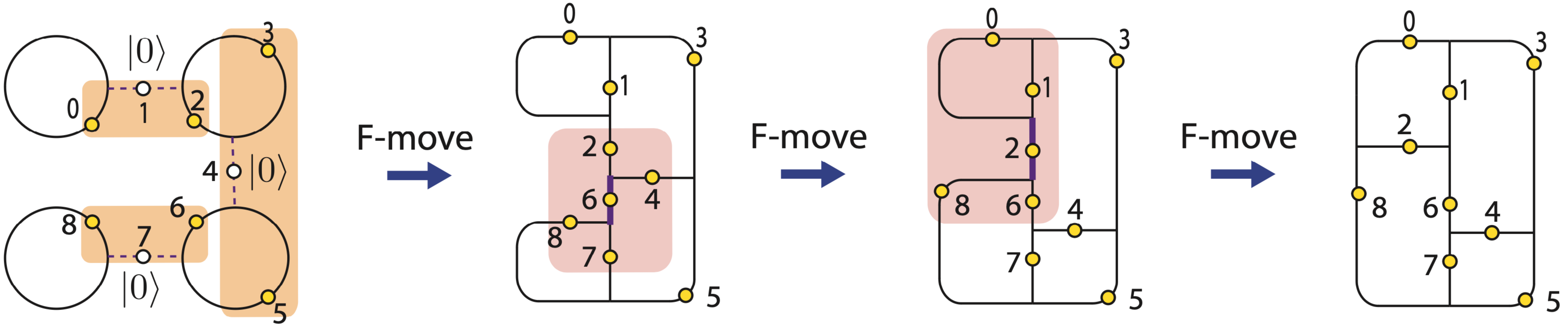


Two-dimensional four-plaquette Fib-SNC vacuum



Dynamical string-net preparation (DSNP)





+2 ancillas to simplify multi-control Toffolis -> 11 qubit

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Transpiled to device topology and native gates: Deep Circuits

After highest optimization level with more than 20 retries (see appendix for optimization details)
(Circuit for t1 braiding, but representative of idea)

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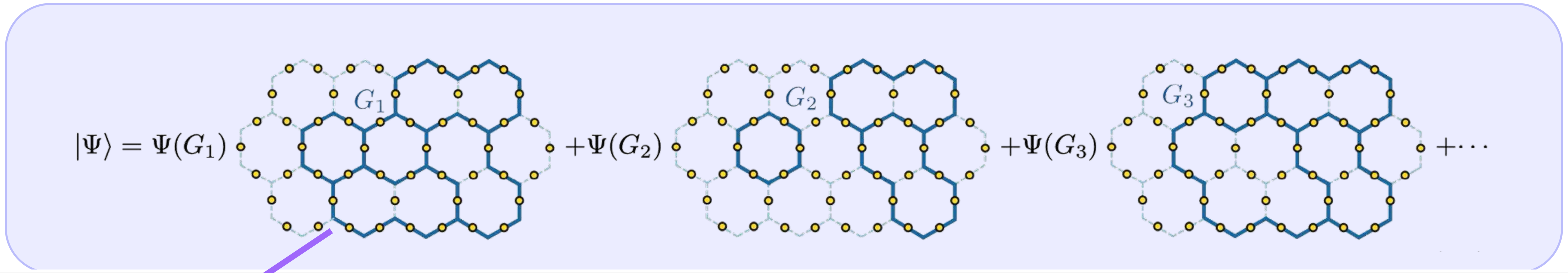
Raw experiment data: Sampling the Fib-SNC



Bulk EM: None; Readout EM: M3; DD: XY-4; Twirl: 1200; Shots: 30 x 10^6;

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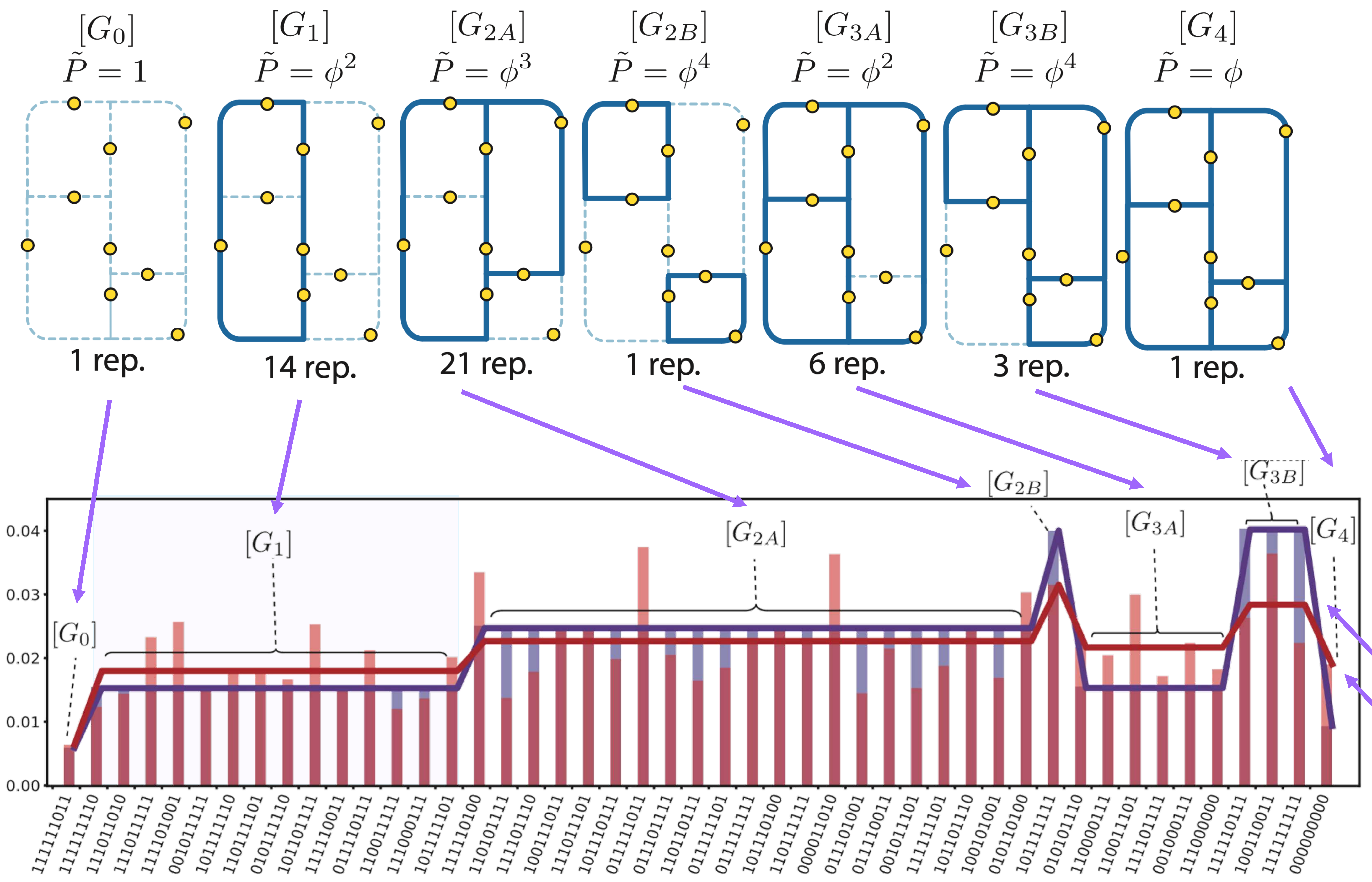
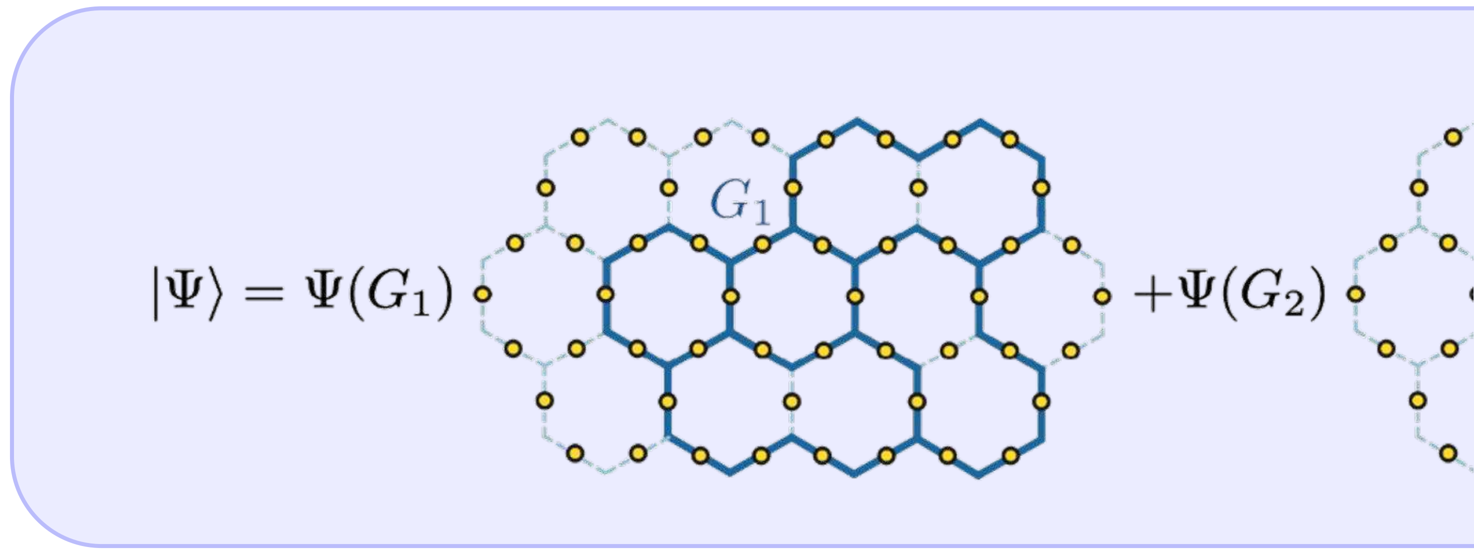
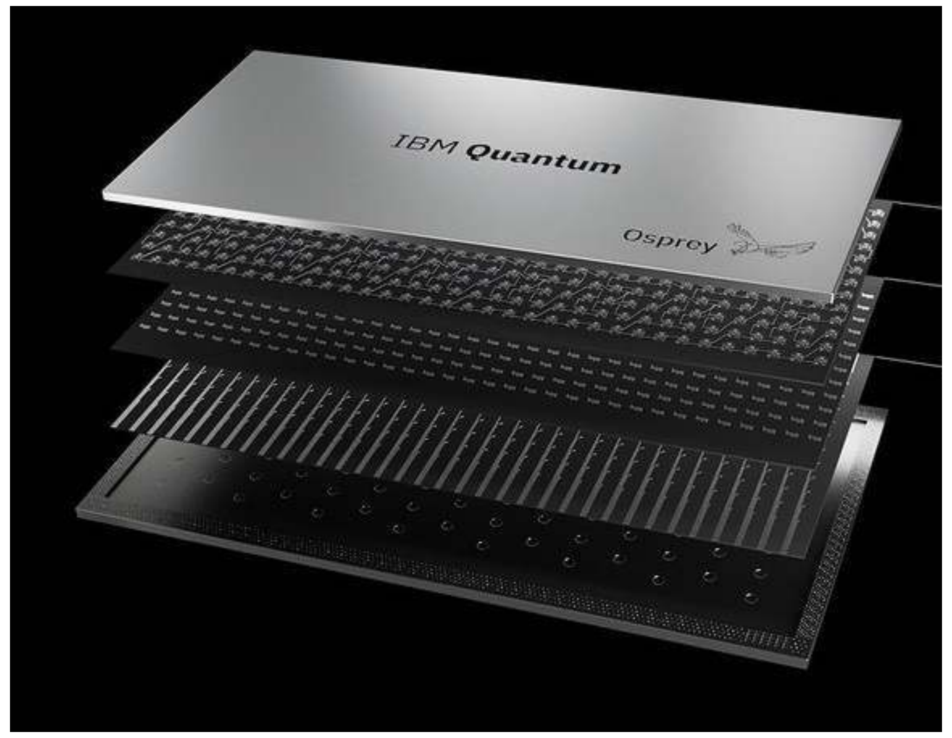
Data: Sampling Fib-SNC







Ideal

Experiment averaged over graph class

Bulk EM: None; Readout EM: M3; DD: XY-4; Twirl: 1200; Shots: 30 x 10^6;

$2^{11} = 2048$

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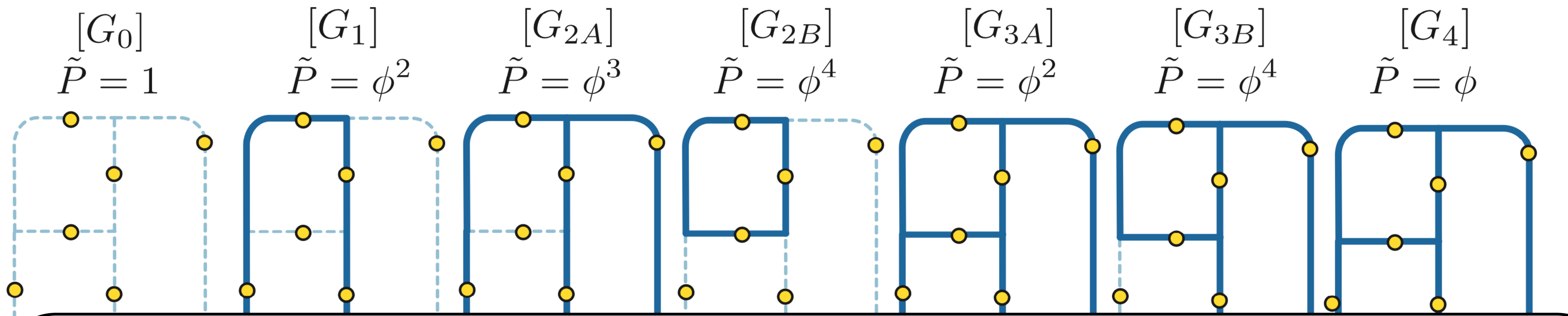
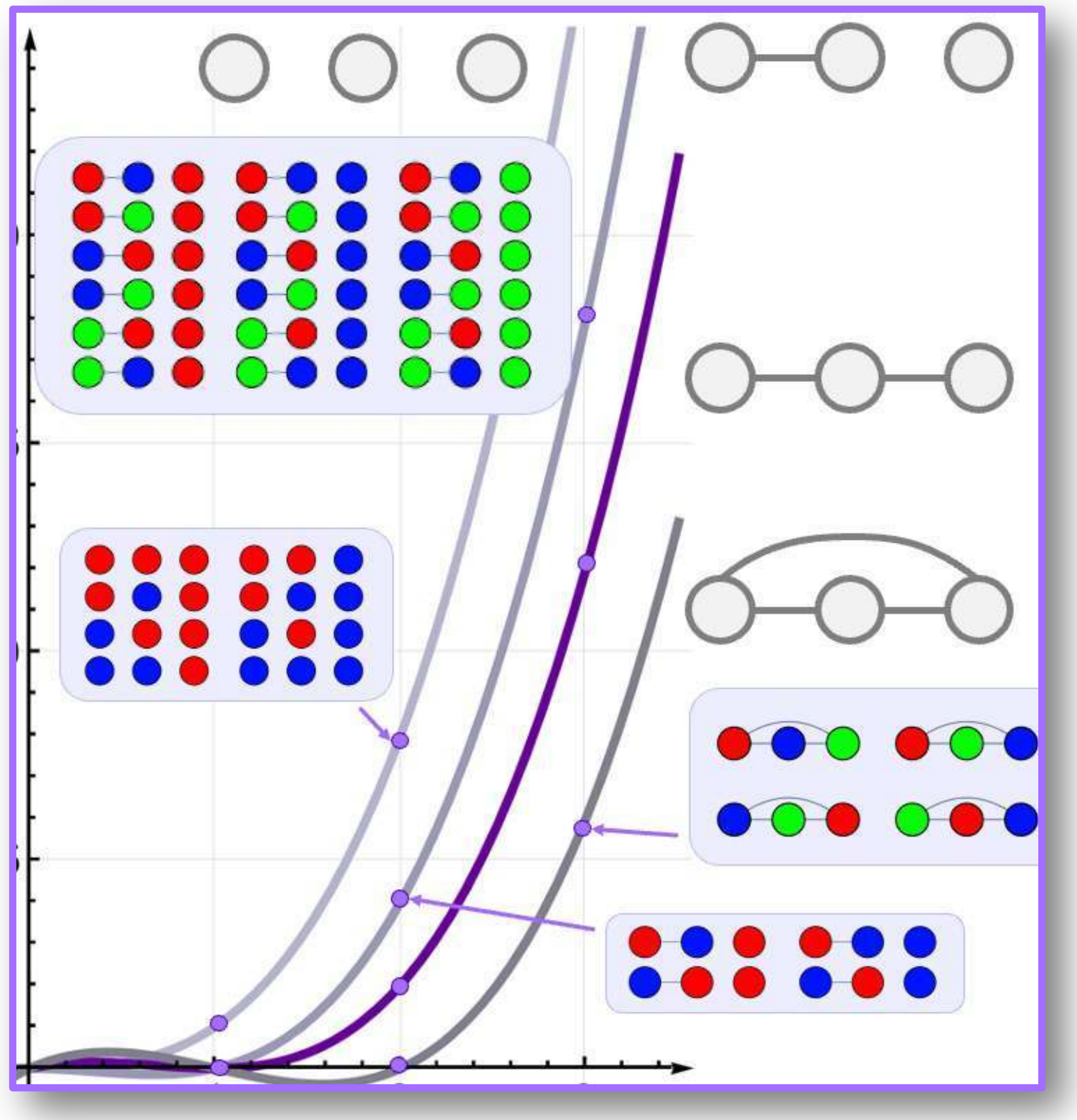


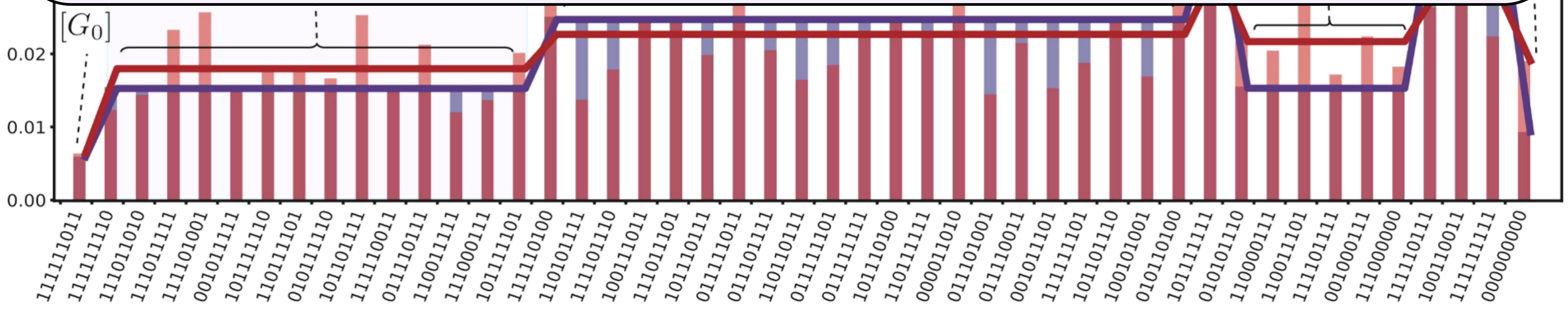


$$\widetilde{\Psi}(G)^2 = \frac{\Psi(G)^2}{\Psi(\mathrm{vac})^2} = \frac{1}{\phi + 2}\chi(\hat{G}, \phi + 2)$$

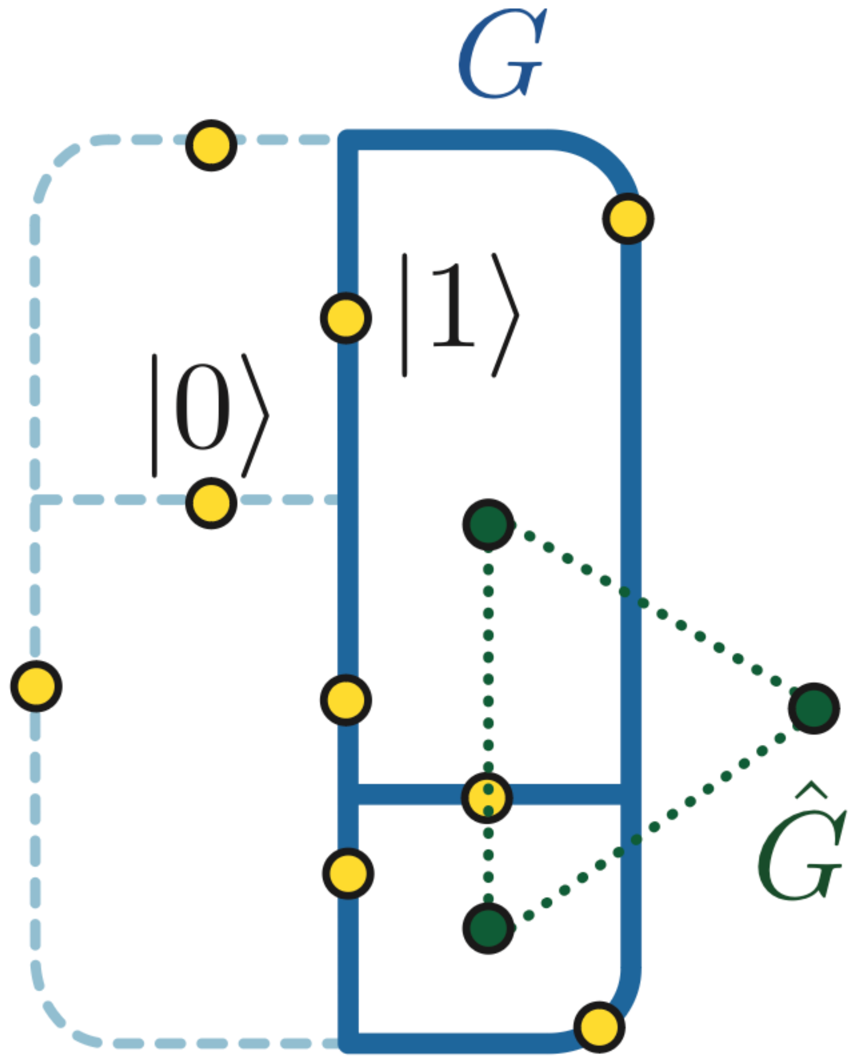
Relative sampling probability of vacuum bitstrings

Golden ratio

Chromatic polynomial of the dual graph



Bitstring configuration



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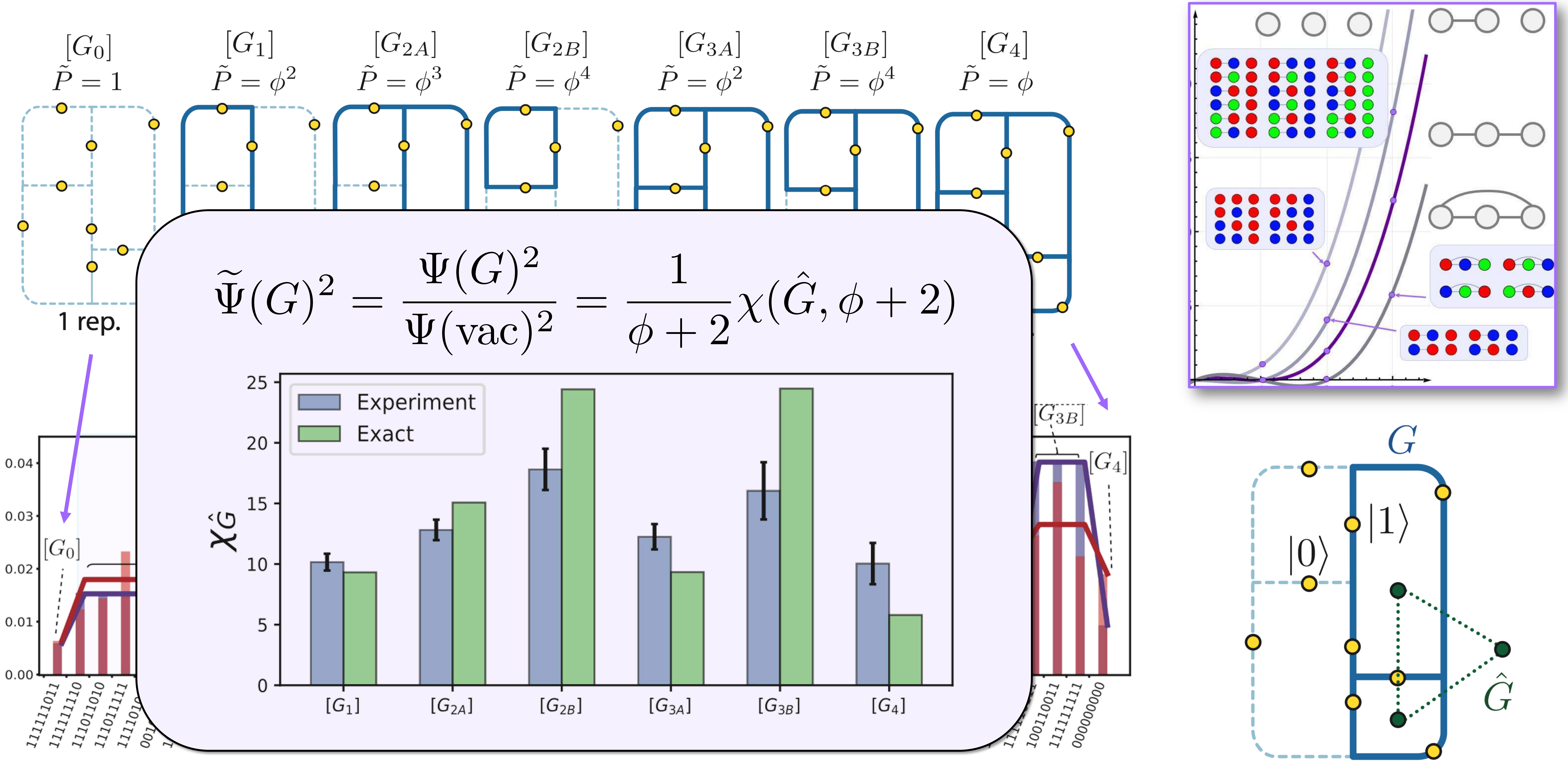

$$\widetilde{\Psi}(G)^2 = \frac{\Psi(G)^2}{\Psi(\mathrm{vac})^2} = \frac{1}{\phi+2}\chi(\hat{G},\phi+2)$$

Bitstring configuration

No error mitigation in bulk!

Bulk EM: None; Readout EM: M3; DD: XY-4; Twirl: 1200; Shots: 30 x 10⁶;

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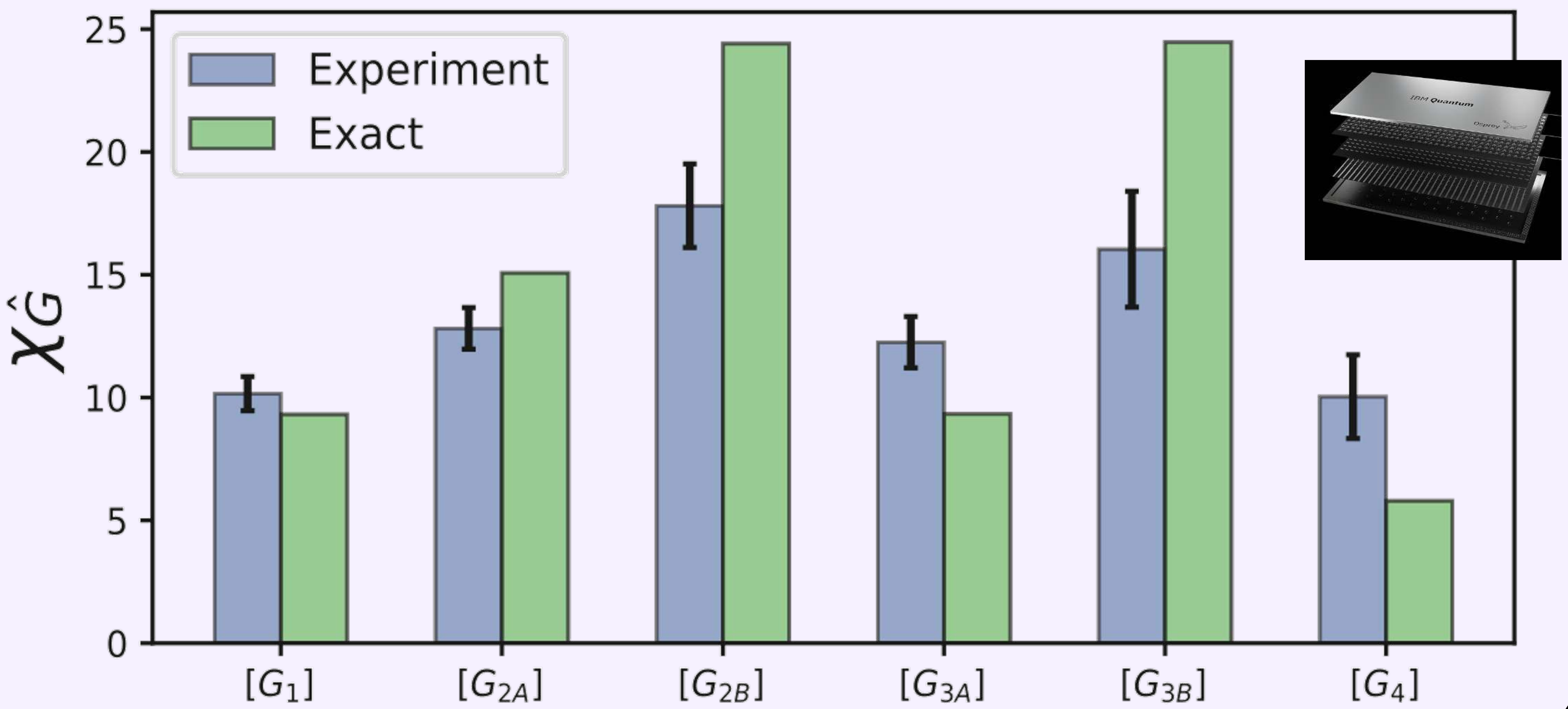
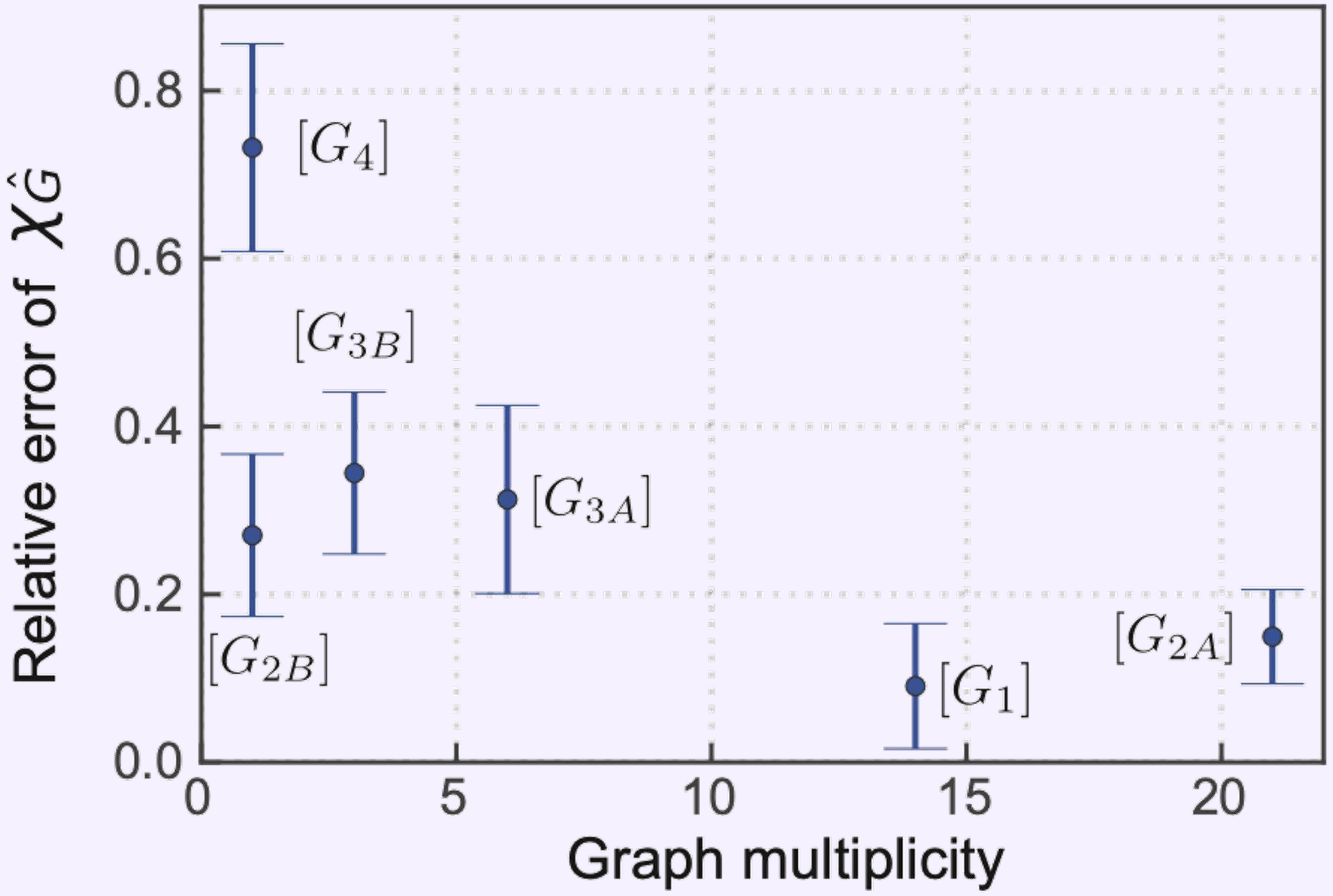
$$\widetilde{\Psi}(G)^2 = \frac{\Psi(G)^2}{\Psi(\mathrm{vac})^2} = \frac{1}{\phi+2}\chi(\hat{G}, \phi+2)$$





Bulk EM: None; Readout EM: M3; DD: XY-4; Twirl: 1200; Shots: 30 x 10^6;

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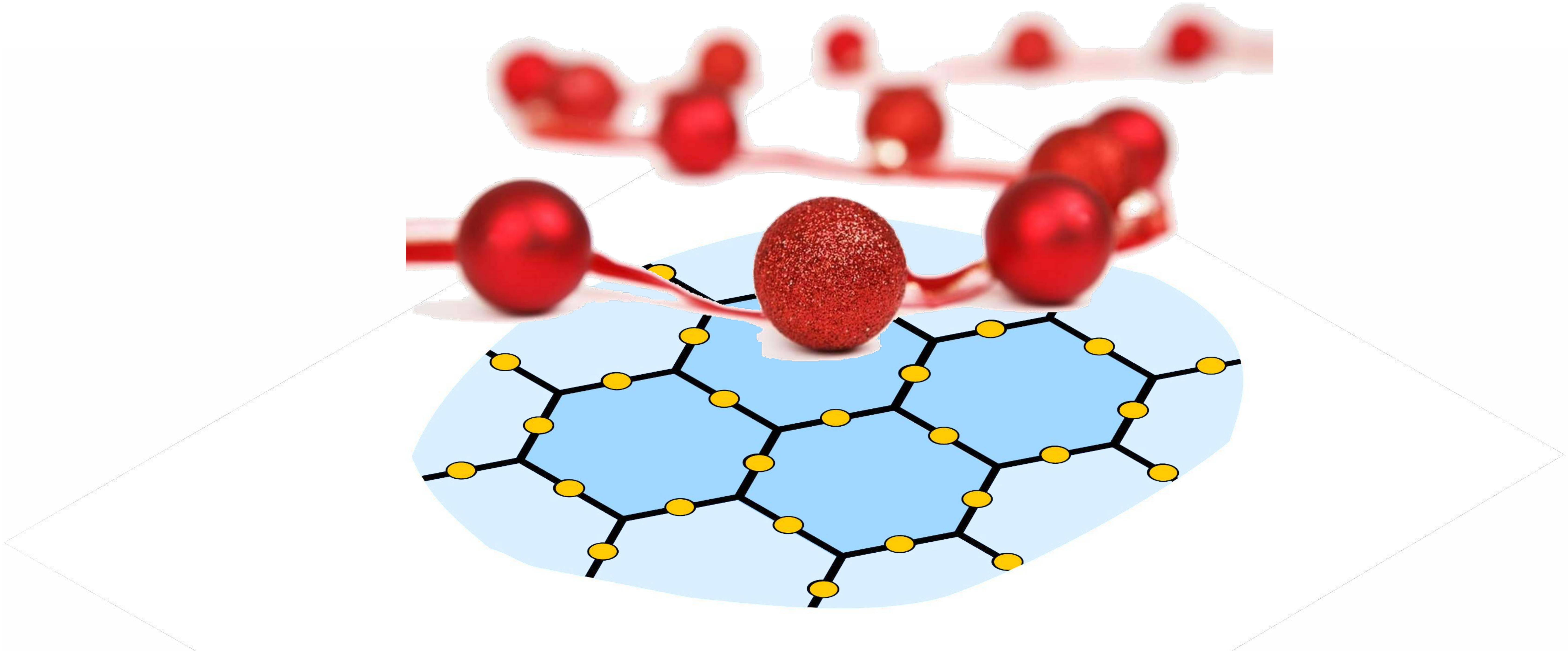
Fibonacci anyons



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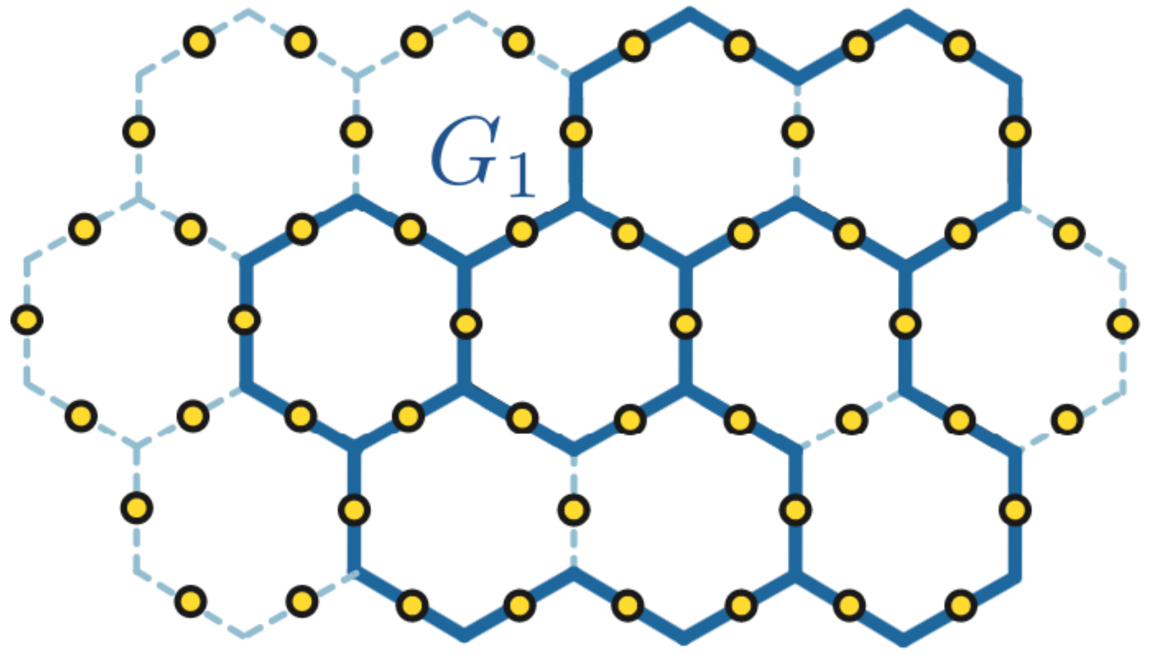
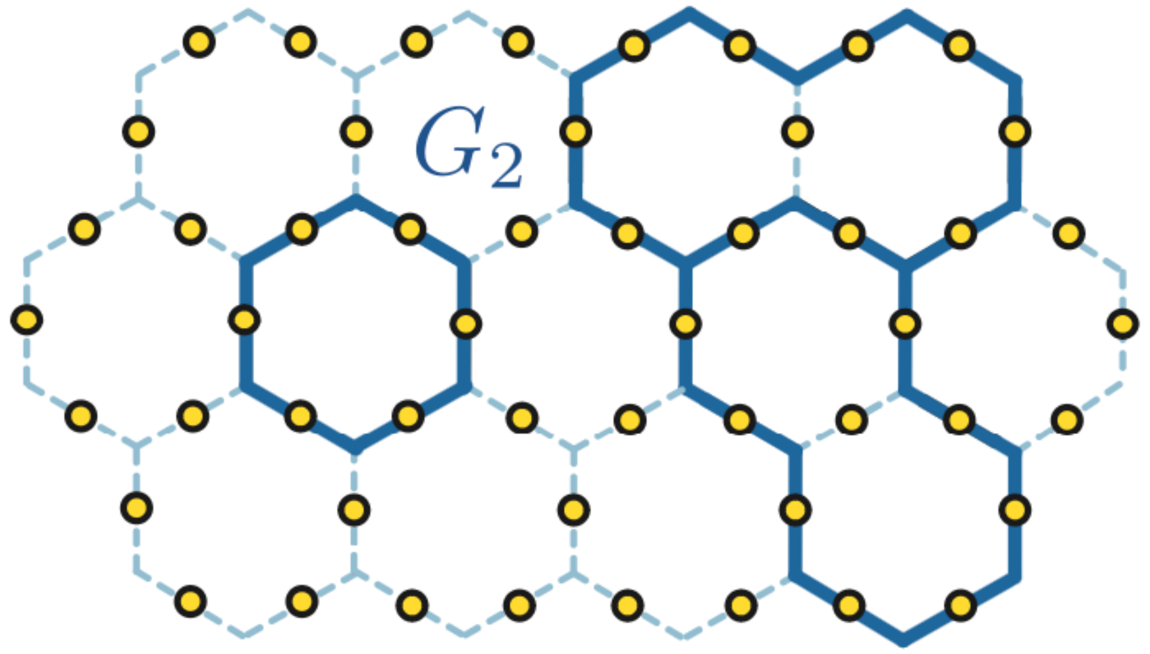
Fibonacci Anyons: Excitations of Fib-SNC

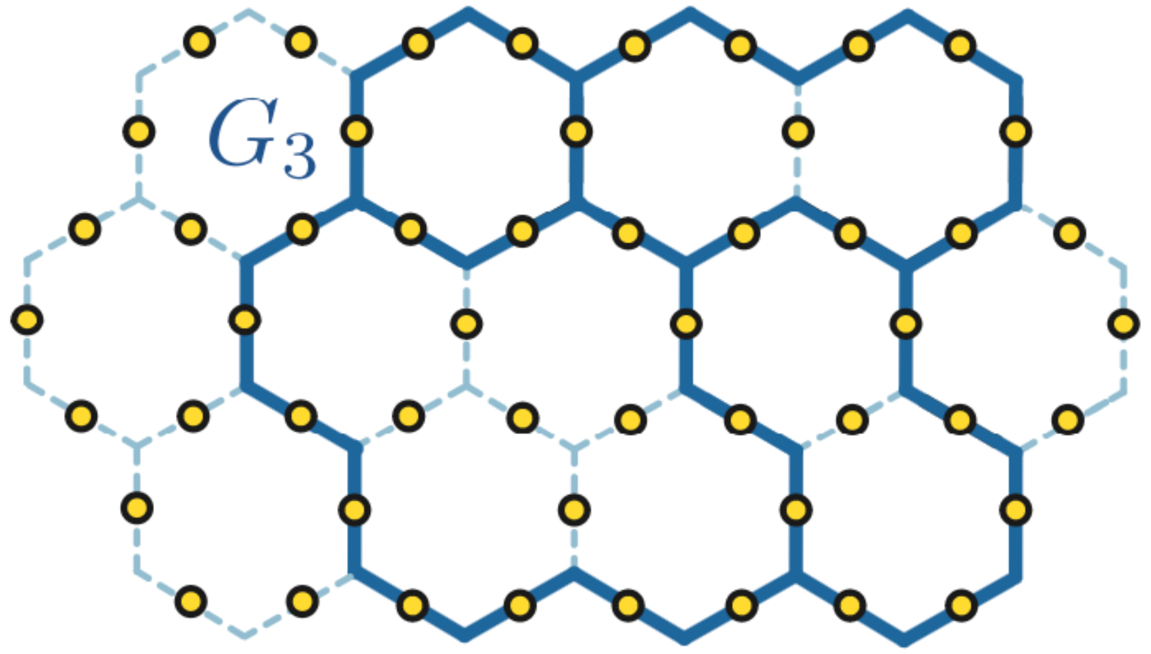
$|\Psi\rangle = \Psi(G_1)$



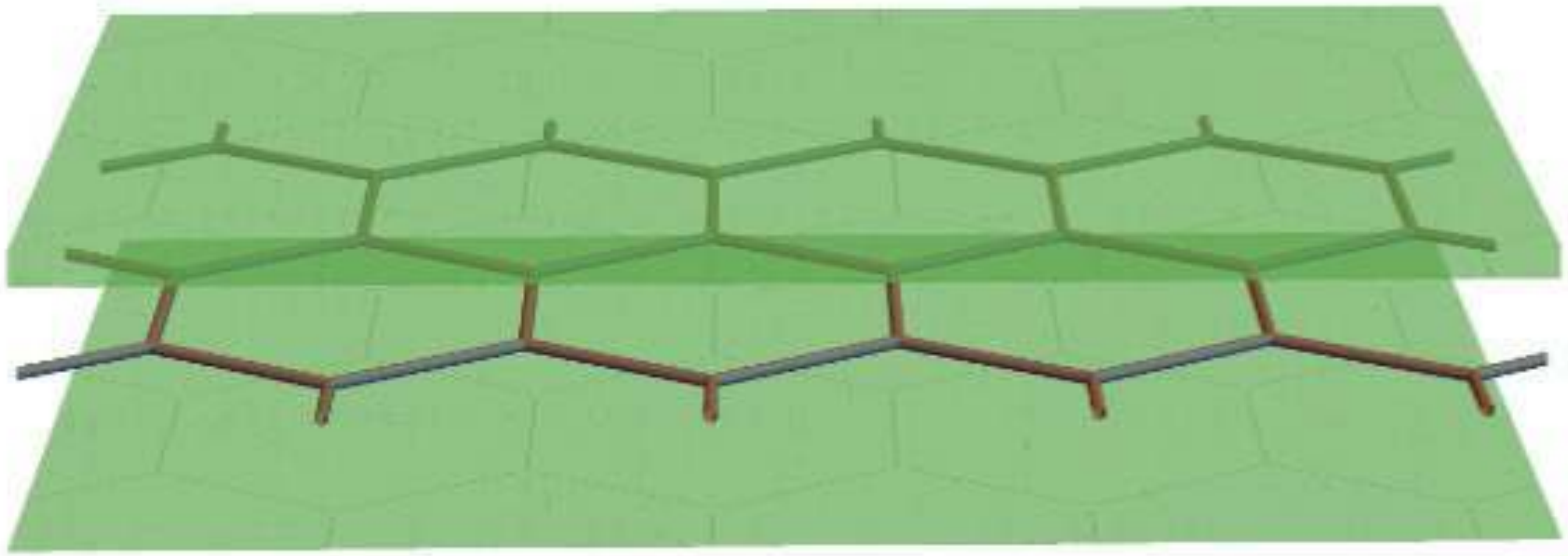
$+\Psi(G_2)$

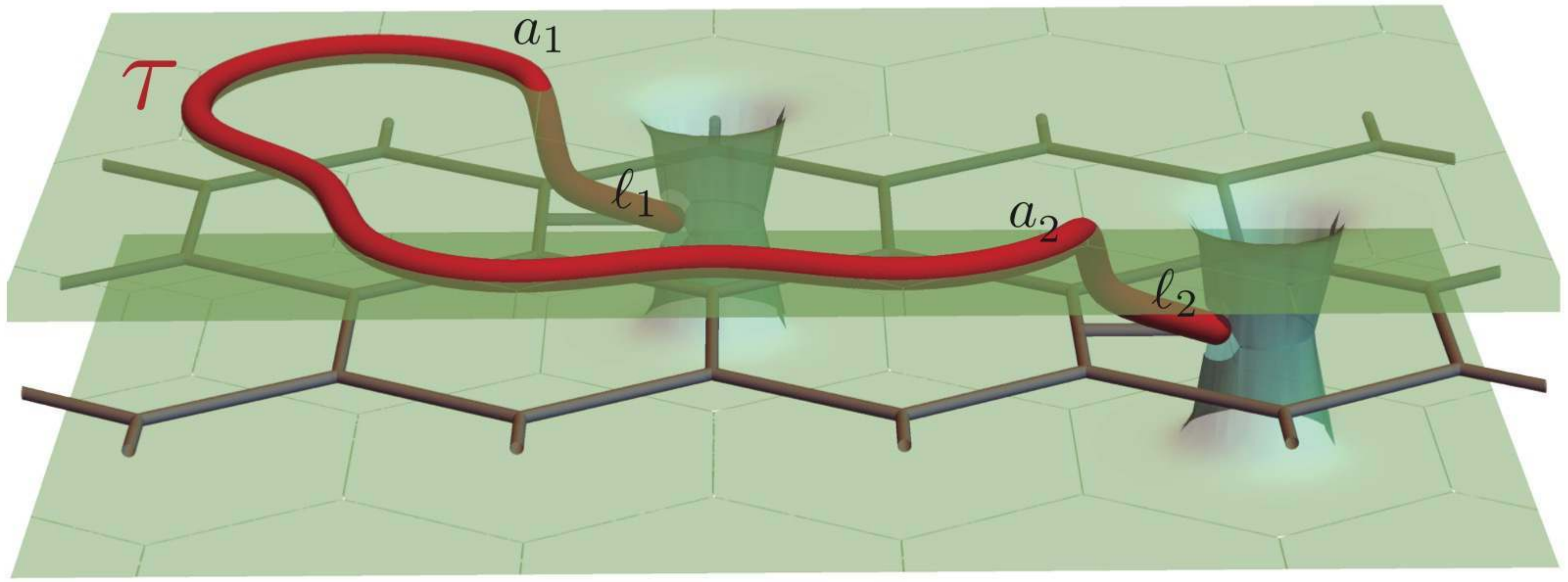


$+\Psi(G_3)$



$+\cdots$





The Fib-SNC is equivalent to the combined vacuum of a time-reversed pair of topological quantum field theories (TQFTs). Each copy of the TQFTs is represented as a green sheet (each resembles TQFT for filling-factor 12/5 quantum Hall state). Each wormhole connects the two copies. When encircled by the vacuum loop, these wormholes are effectively closed.

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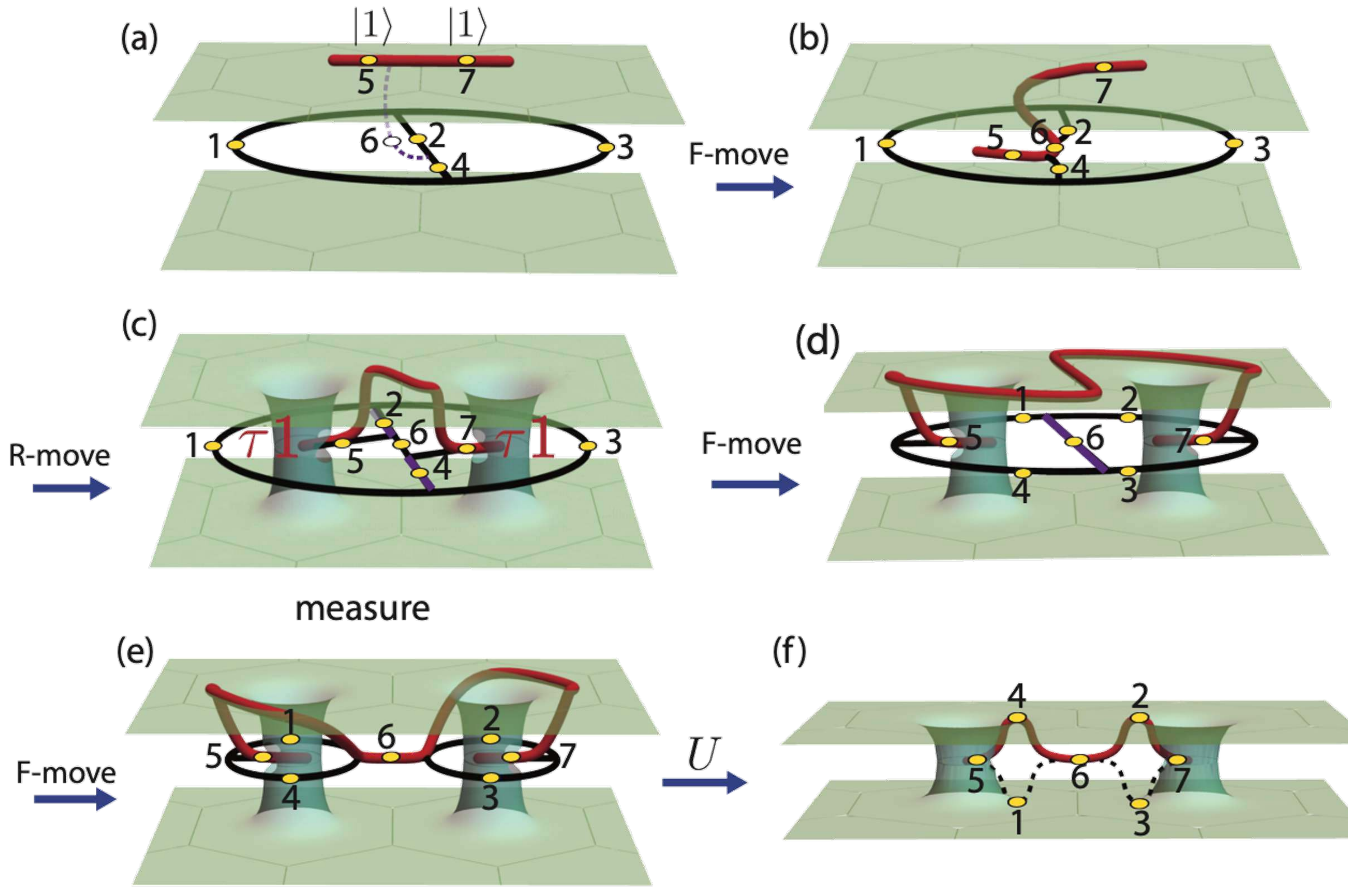
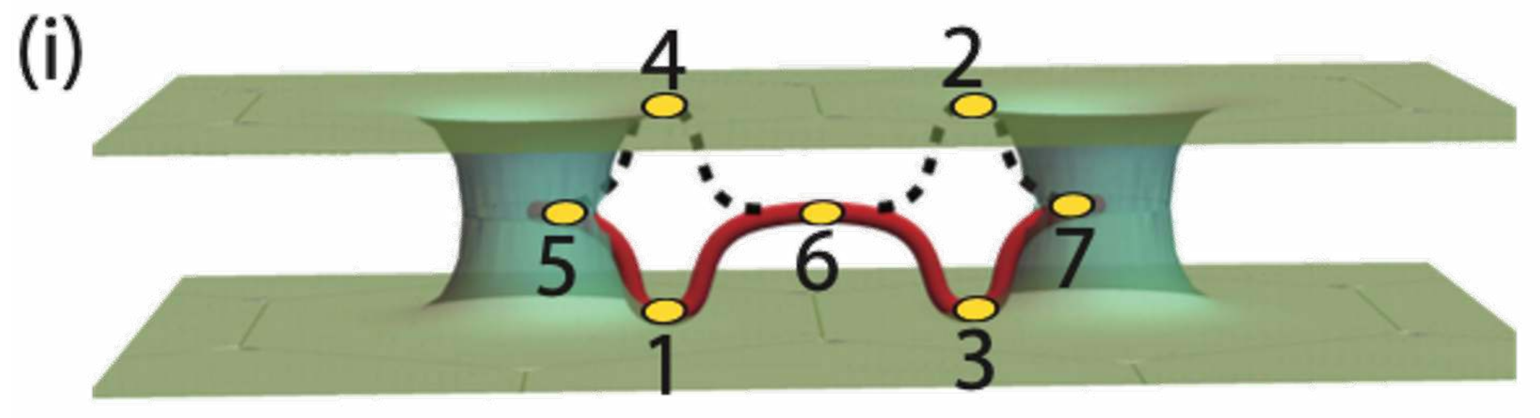
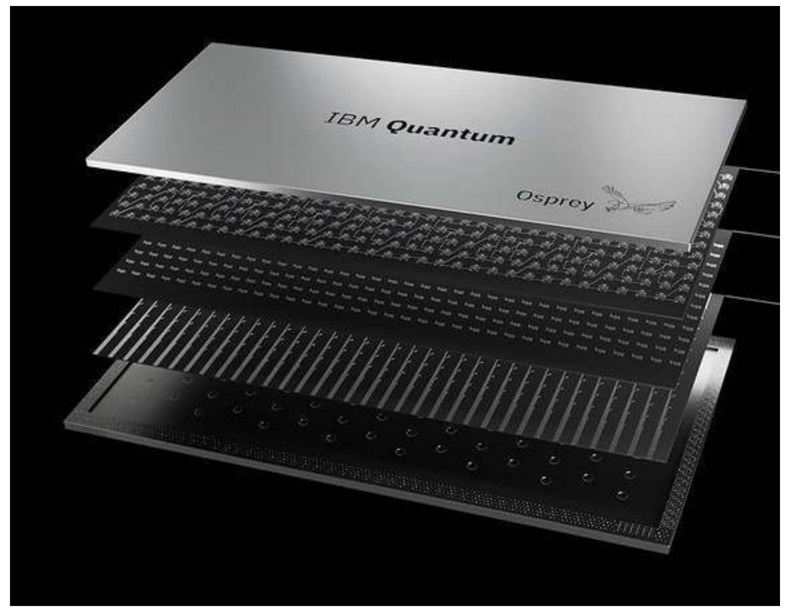
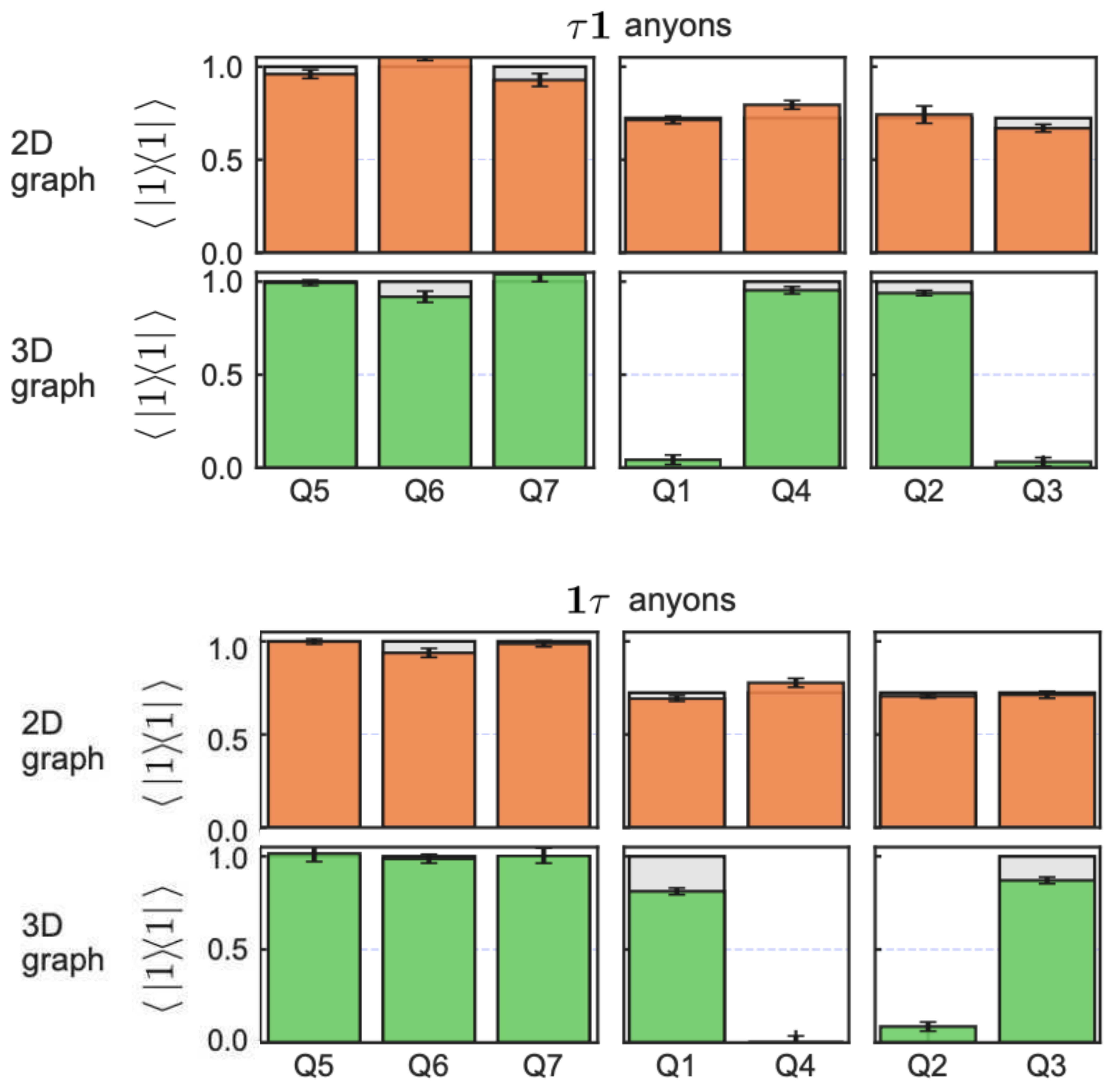
Creating Fibonaci anyon pairs and certifying their anyon charges







Theory (grey bars) **vs. experiment** (colored bars)



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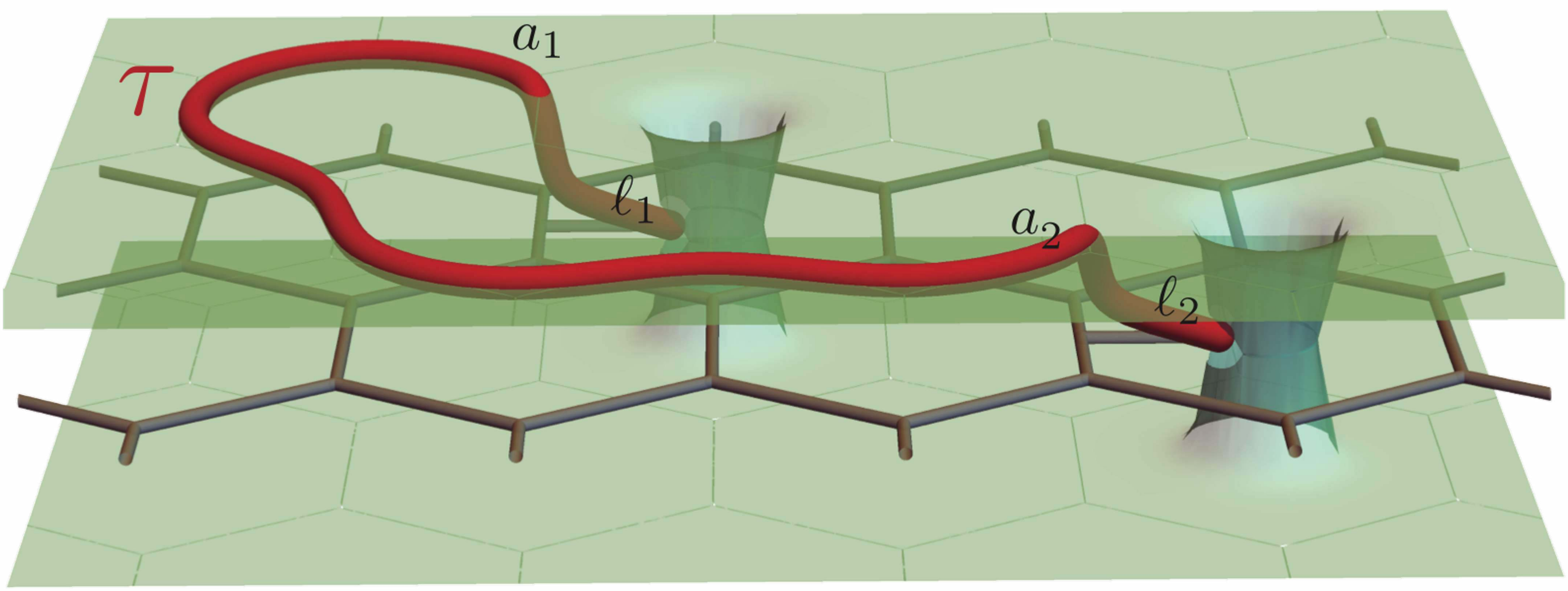
Braid non-Abelian Fibonacci anyons to perform logical gate that is non-Clifford (universal)



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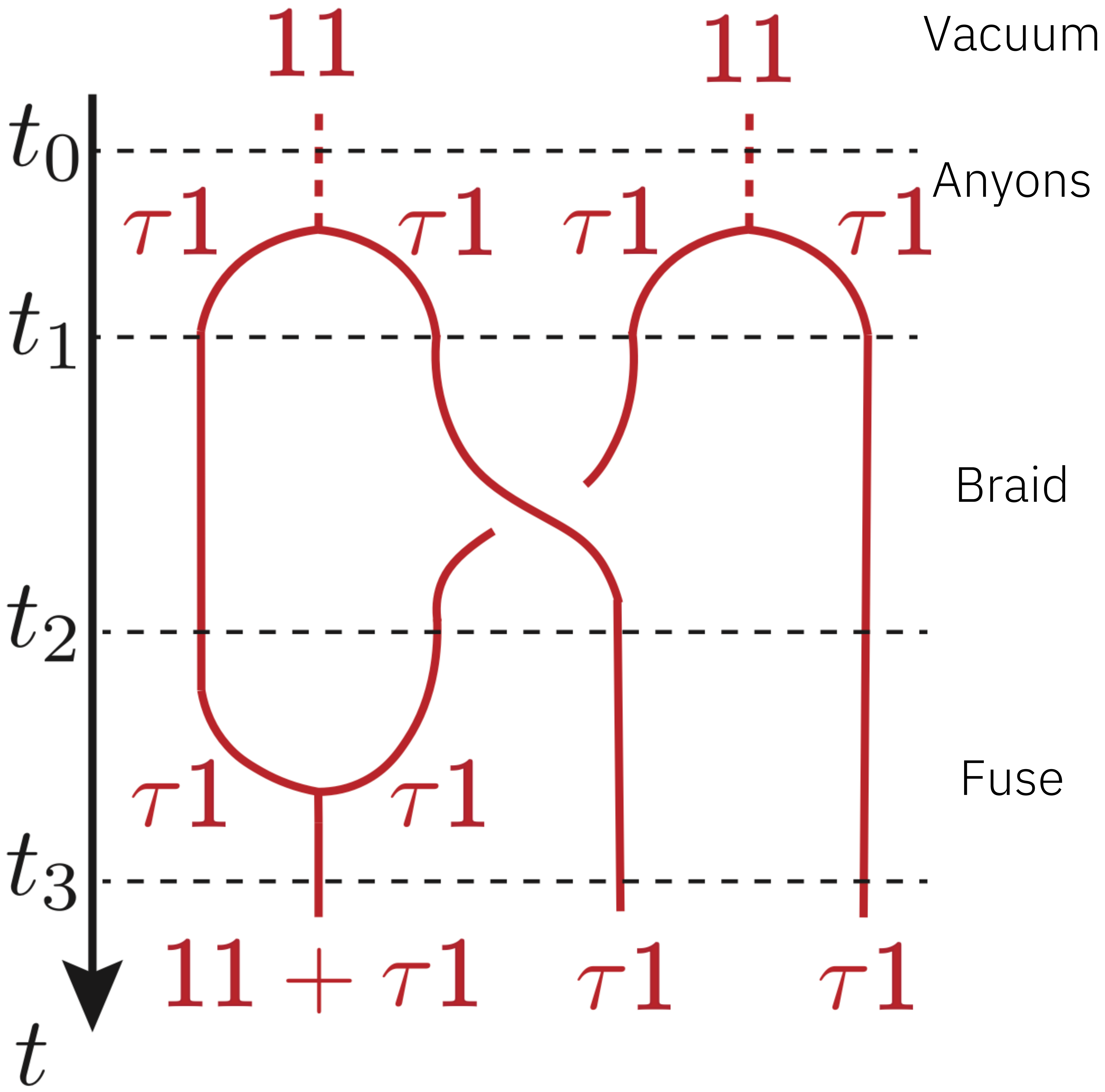
Non-Clifford logical gate: τ1 anyon braiding experiment



$\overline{|0\rangle}$

Non-Clifford logical gate

$$\sigma_2\overline{|0\rangle} = \phi^{-1}e^{4\pi i/5}\overline{|0\rangle} + \phi^{-1/2}e^{-3\pi i/5}\overline{|1\rangle}$$

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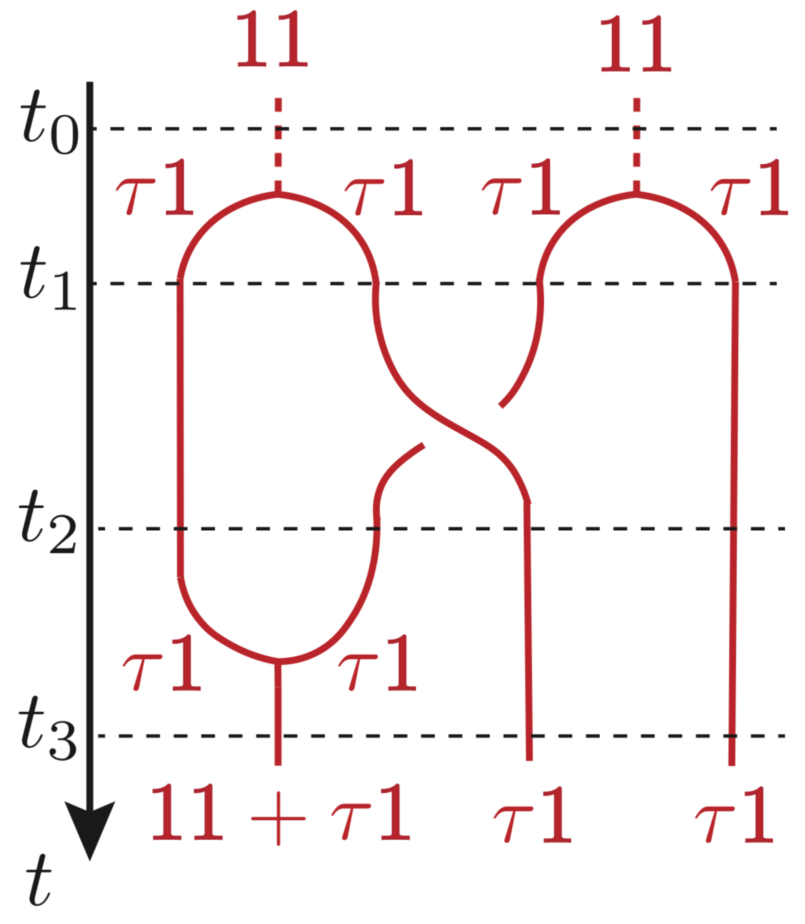
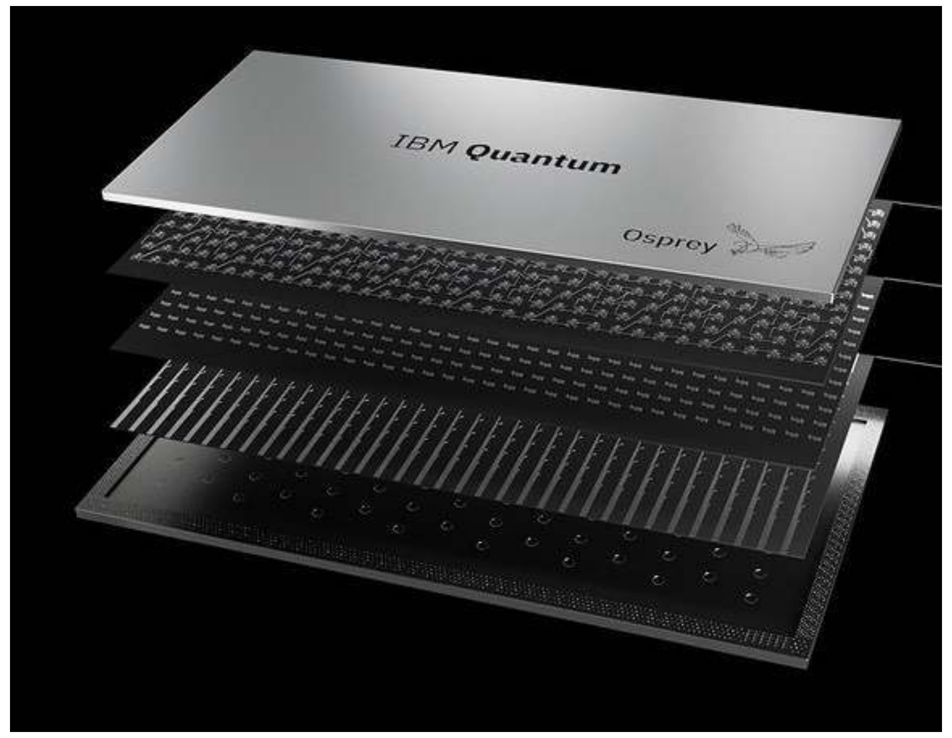
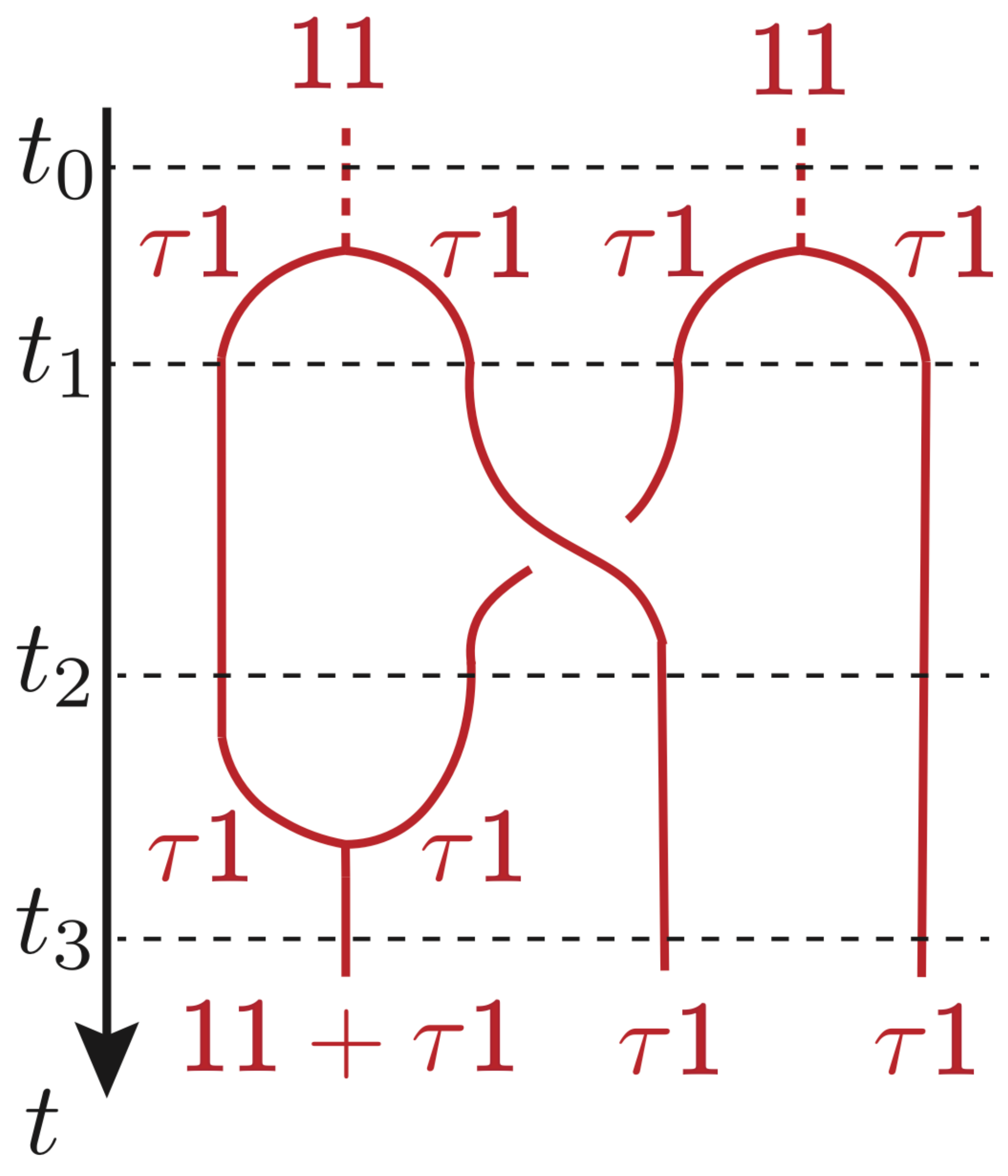
Non-Clifford logical gate: τ1 anyon braiding experiment



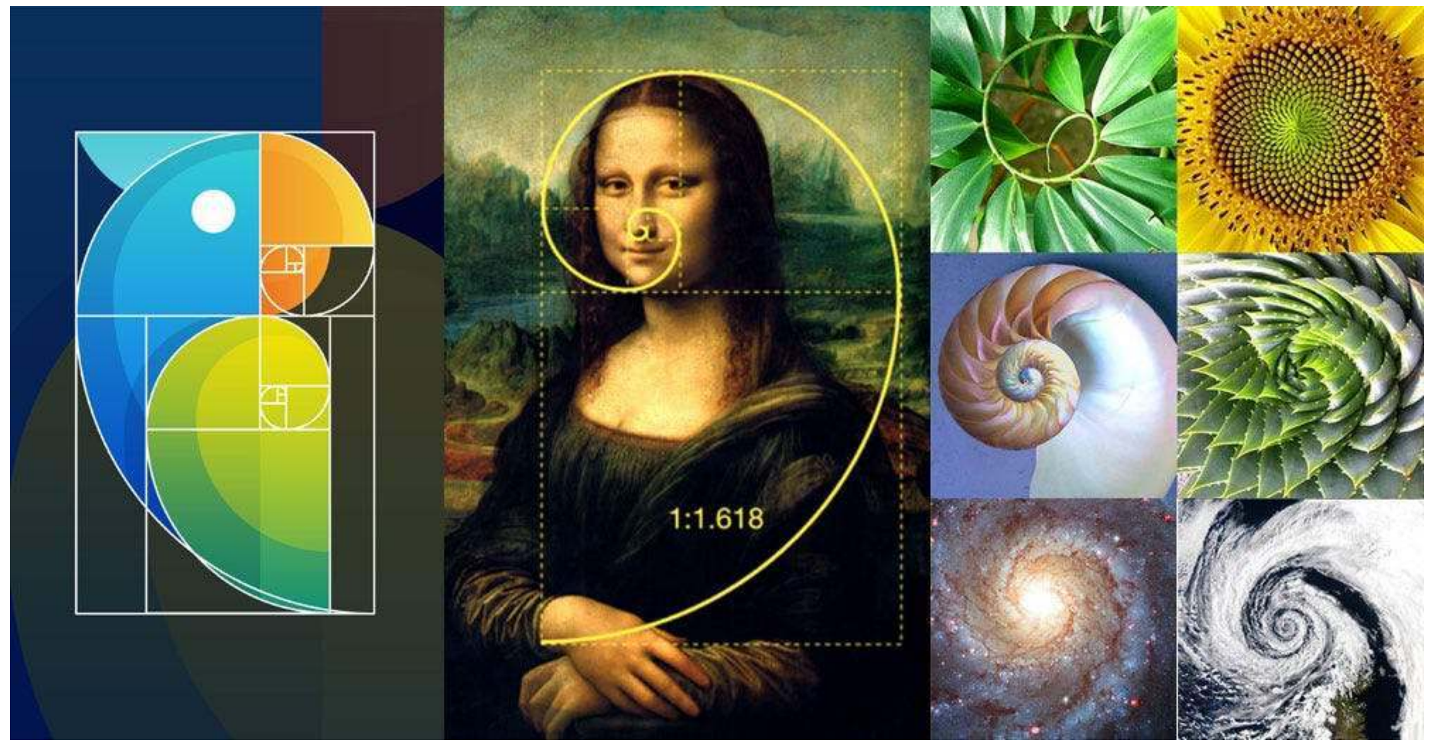
$$\sigma_2\overline{|0\rangle} = \phi^{-1}e^{4\pi\mathrm{i}/5}\overline{|0\rangle} + \phi^{-1/2}e^{-3\pi\mathrm{i}/5}\overline{|1\rangle}$$



Non-Clifford logical gate: τ1 anyon braiding experiment



$$\sigma_2 \overline{|0\rangle} = \phi^{-1} e^{4\pi i/5} \overline{|0\rangle} + \phi^{-1/2} e^{-3\pi i/5} \overline{|1\rangle}$$



$$\frac{|\langle 1|q_{[9]}\rangle|^2}{|\langle 0|q_{[9]}\rangle|^2} = \frac{1+\sqrt{5}}{2} \approx 1.61803.$$

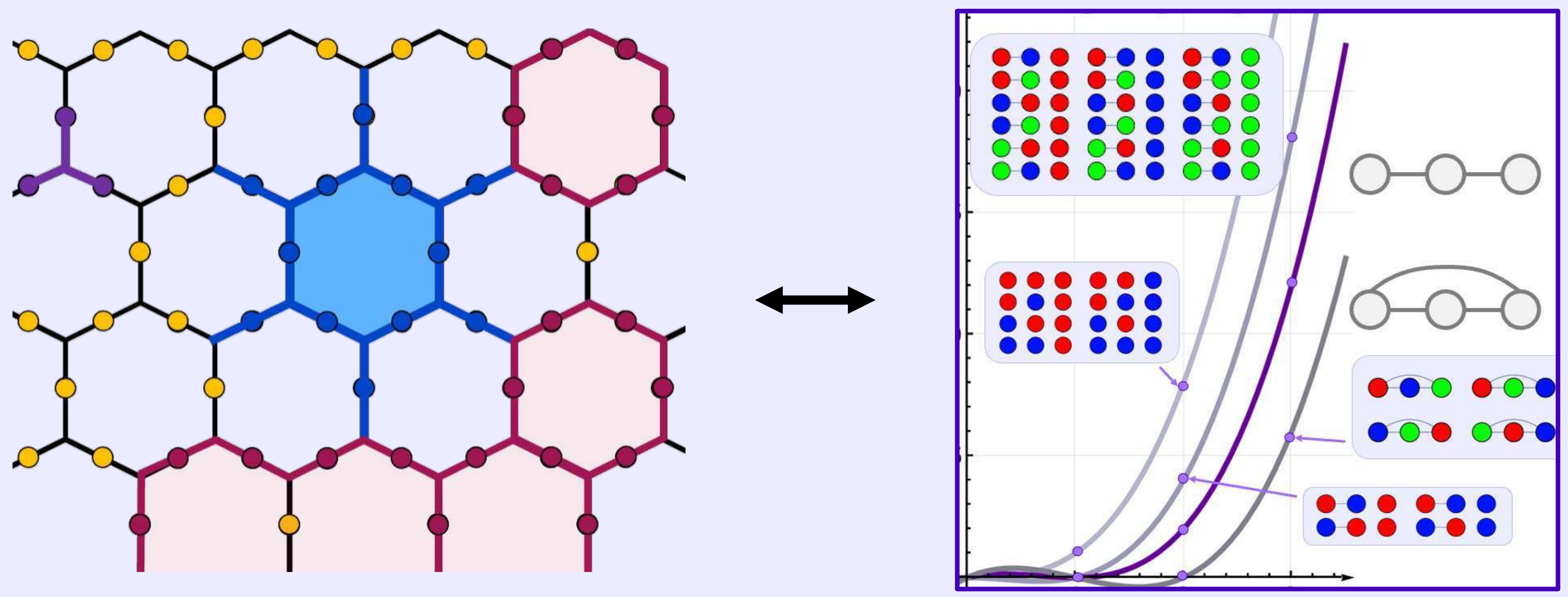
Experiment: $|1\rangle\langle 1| \,/\, |0\rangle\langle 0| = 1.65 \pm 0.14$

Bootstrap resampled experiment distributions

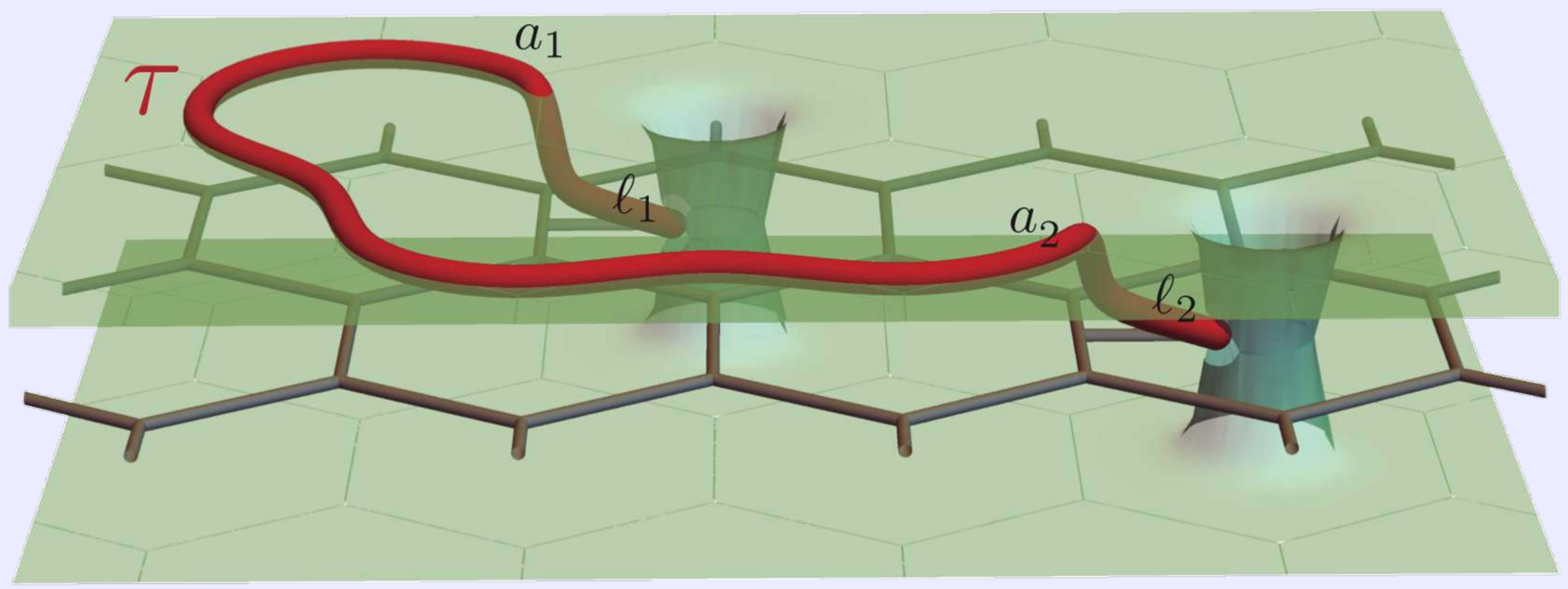
Bulk EM: dZNE; Readout EM: TREX; DD: XY-4;
Twirl: 200; Shots: 4 x 10^5;

Summary & Outlook

Realize faithful Fibonacci string-net condensate to sample chromatic polynomials (classically hard)



Braid non-Abelian Fibonacci anyons to perform logical gate that is non-Clifford (universal)



- Scalable approach to generating and manipulating Fib-SNC
  - Not static, but *dynamic* graph preparation
  - New pathways for the realization of complex quantum states
- Create, measure, and braid Fibonacci anyons
- Experimentally viable to sample the Fib-SNC to estimate the chromatic polynomial
  - New path to quantum advantage?
  - Sampling complexity? Bounds? Evaluate polynomial at other points? Optimize prep? Hybrid quantum-classical acceleration?
- DSNP: exciting avenues for the exploration of topological phases of matter and their application to FTQC

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Some future directions

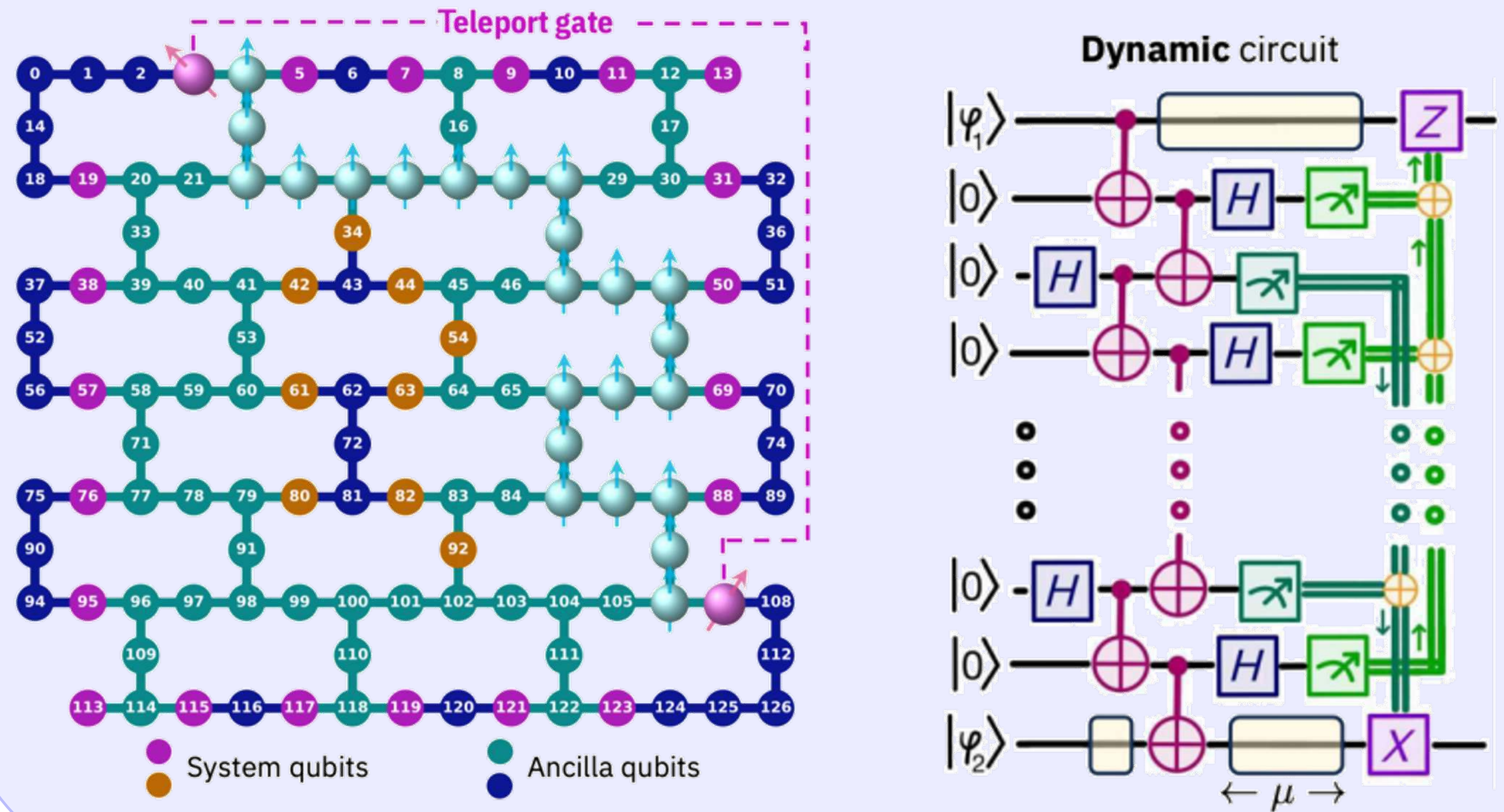
Dynamic circuits

PRX QUANTUM
a Physical Review journal

Efficient Long-Range Entanglement Using Dynamic Circuits

Elisa Bäumer, Vinay Tripathi, Derek S. Wang, Patrick Rall, Edward H. Chen, Swarnadeep Majumder, Alireza Seif, and Zlatko K. Minev

PRX Quantum **5**, 030339 – Published 22 August 2024



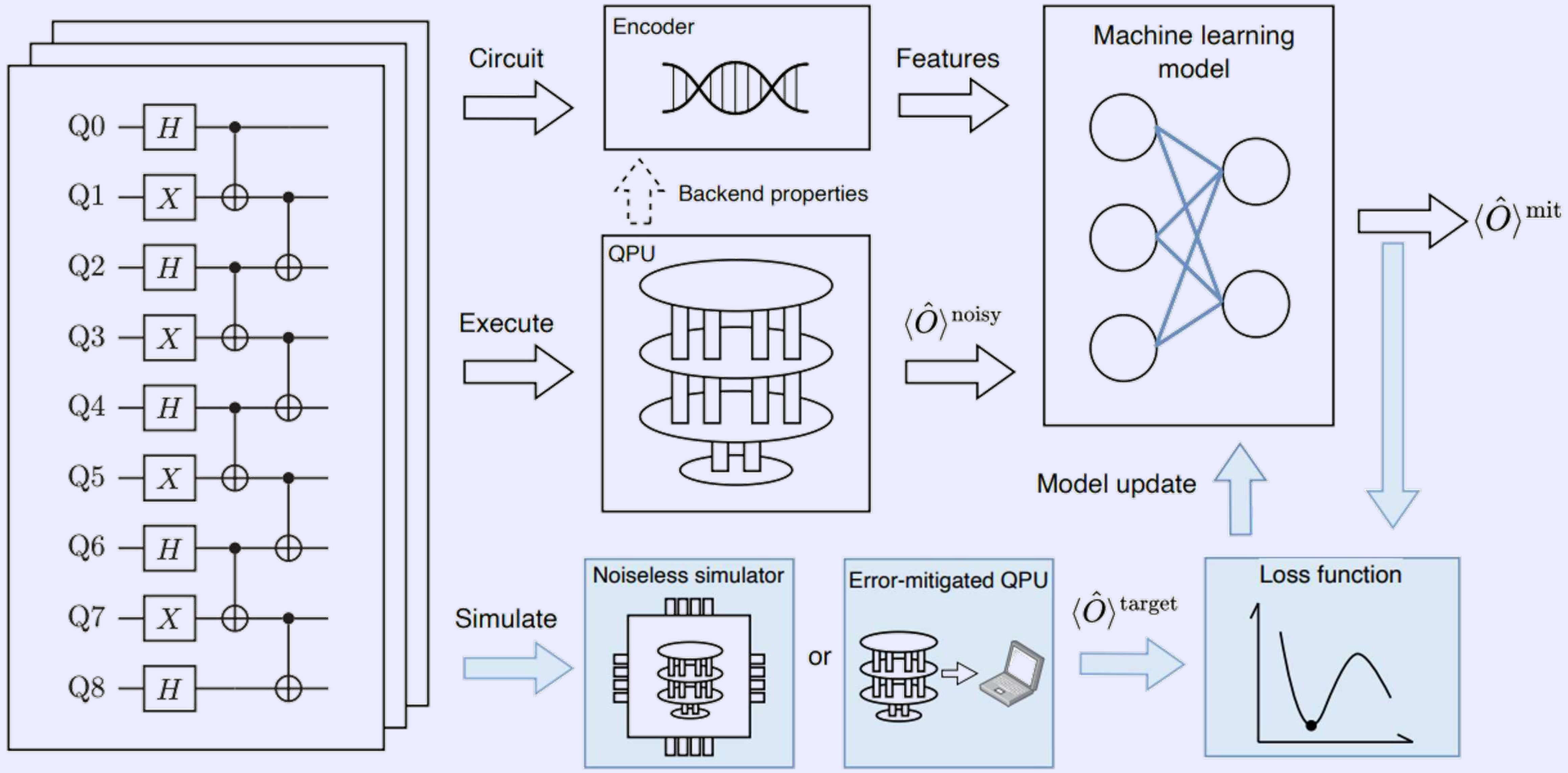
Improved EM

Quantum Physics

[Submitted on 29 Sep 2023]

Machine Learning for Practical Quantum Error Mitigation

Haoran Liao, Derek S. Wang, Iskandar Sitdikov, Ciro Salcedo, Alireza Seif, Zlatko K. Minev



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Quantum

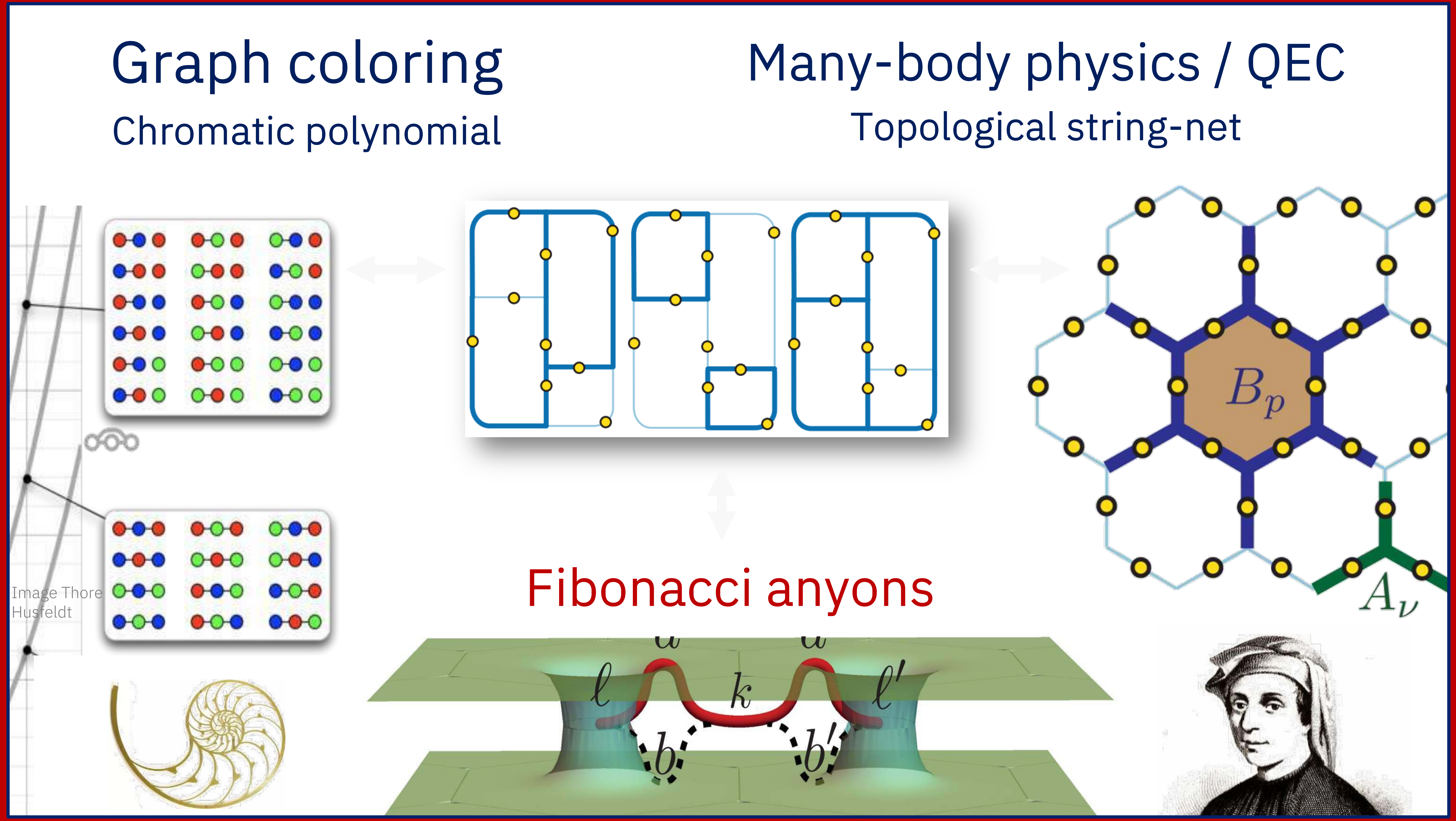
arXiv > quant-ph > arXiv:2406.12820

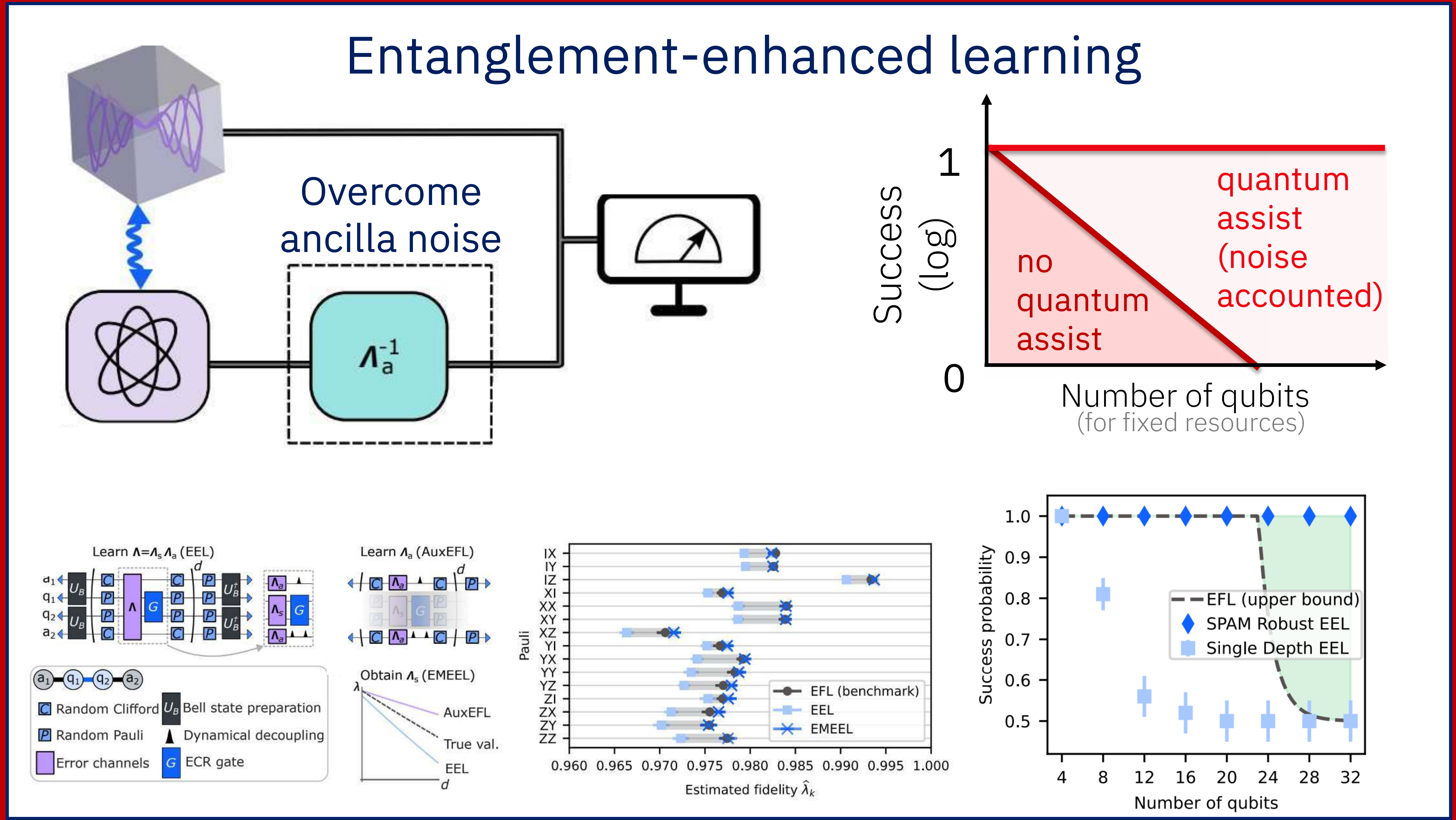
Quantum Physics

[Submitted on 18 Jun 2024]

Realizing string-net condensation: Fibonacci anyon braiding for universal gates and sampling chromatic polynomials

Zlatko K. Minev, Khadijeh Najafi, Swarnadeep Majumder, Juven Wang, Ady Stern, Eun-Ah Kim, Chao-Ming Jian, Guanyu Zhu





arXiv > quant-ph > arXiv:2408.03376

Quantum Physics

[Submitted on 6 Aug 2024]

Entanglement-enhanced learning of quantum processes at scale

Alireza Seif, Senrui Chen, Swarnadeep Majumder, Haoran Liao, Derek S. Wang, Moein Malekakhlagh, Ali Javadi-Abhari, Liang Jiang, Zlatko K. Minev

Project inspired by Robert Huang and John Preskill presentation at CIFAR QIS meeting

Thank you!

IBM Quantum