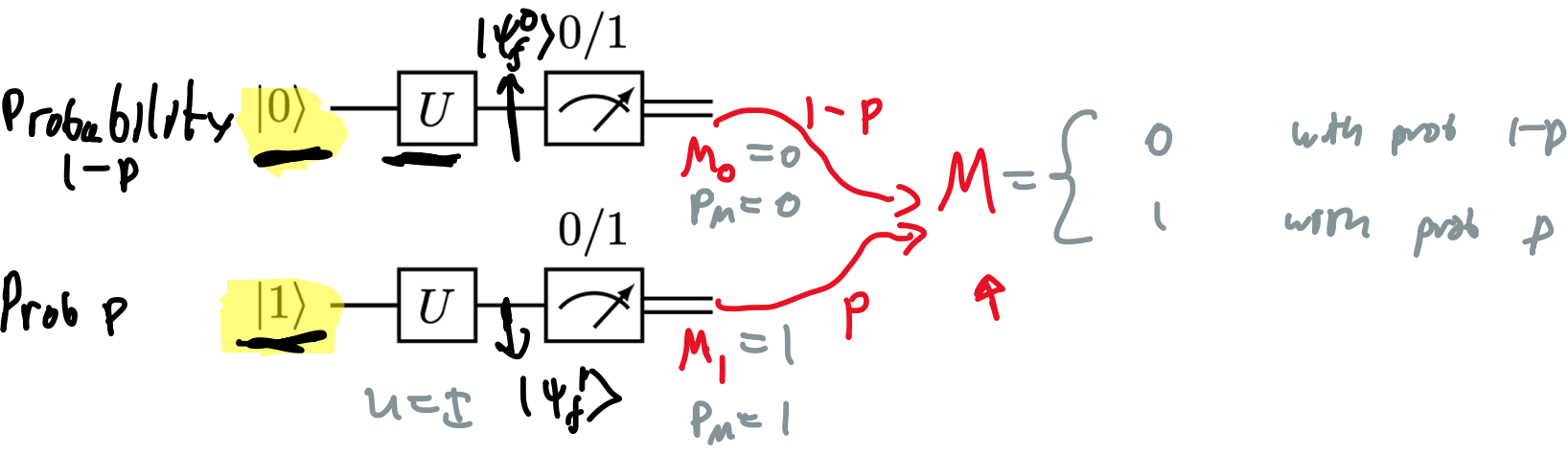


Introduction to quantum noise

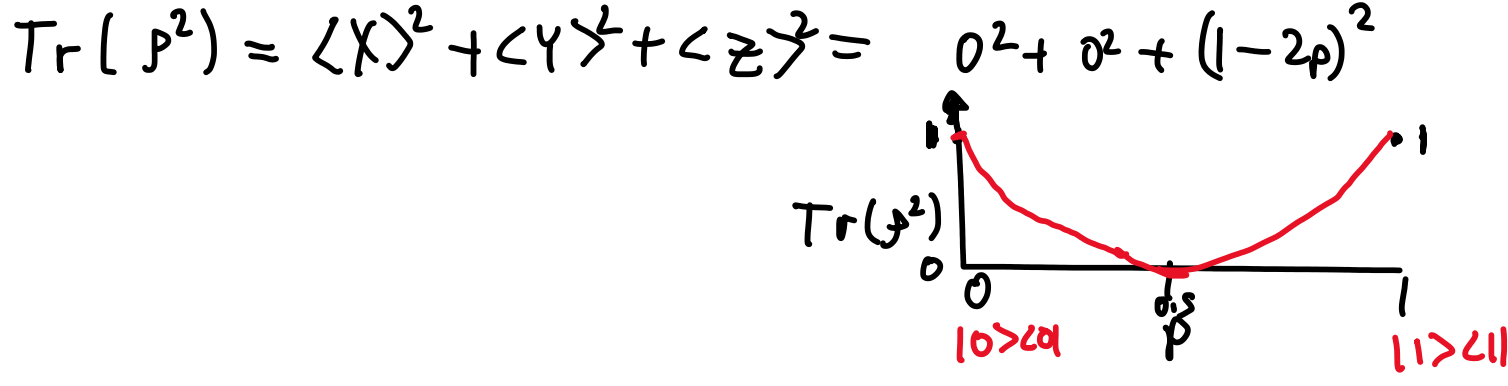
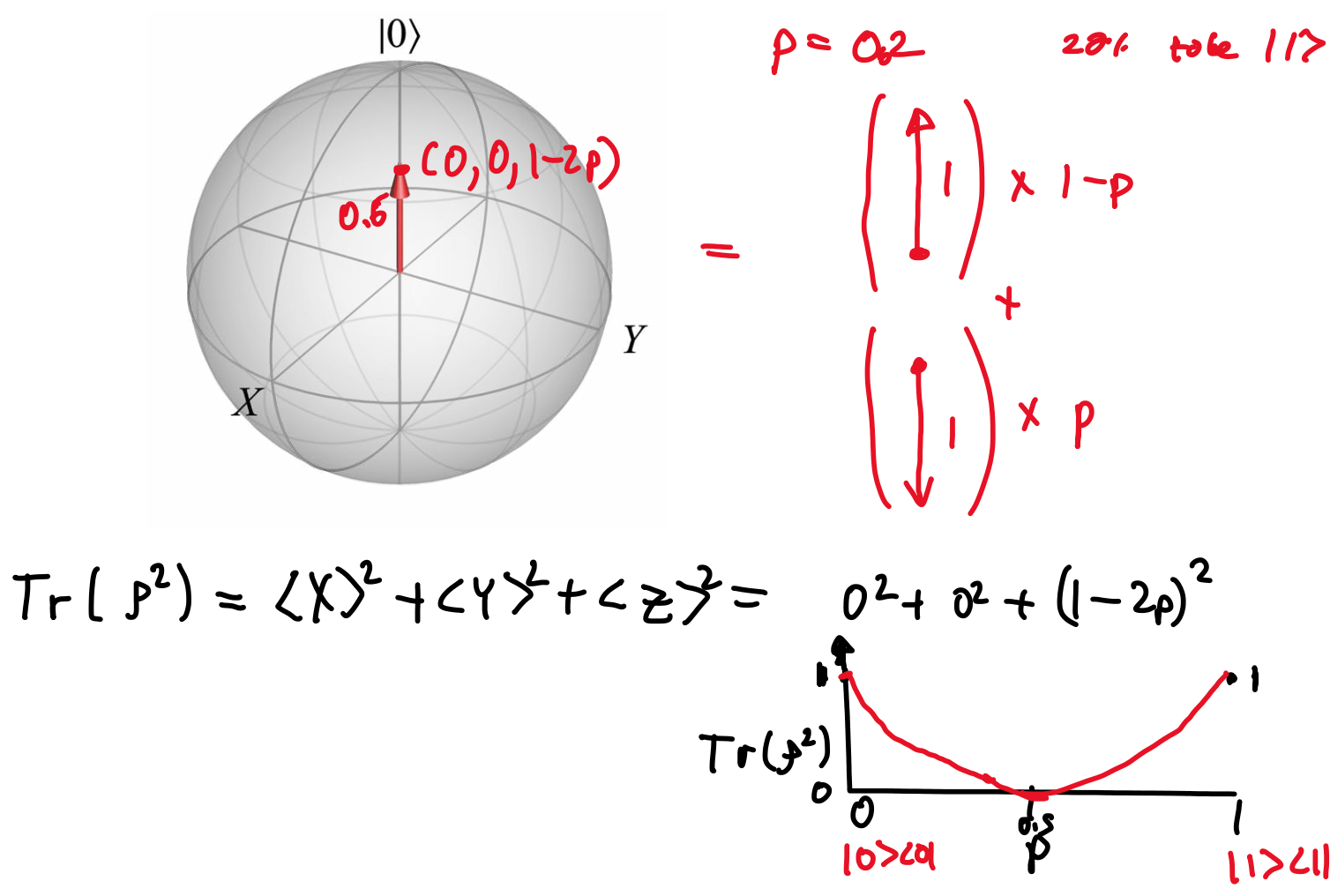
State preparation noise

Qiskit Global Summer School on Quantum Machine Learning
Zlatko K. Mineev



Density operator

$$\rho = (1-p) |0\rangle\langle 0| + p |1\rangle\langle 1|$$
$$= \rho_{no error} \rho_{ideal} + \rho_{error} \rho_{error}$$
$$\rho_F = \rho_{no error} \rho_{ideal} + \rho_{error} \rho_{error}$$
$$= (1-p) |\psi_F\rangle\langle \psi_F| + p |\psi_F'\rangle\langle \psi_F'|$$
$$= (1-p) U |0\rangle\langle 0| U^\dagger + p U |1\rangle\langle 1| U^\dagger$$
$$= U \left[(1-p) |0\rangle\langle 0| + p |1\rangle\langle 1| \right] U^\dagger$$
$$= U \rho U^\dagger$$
$$\rho_U = U \rho U^\dagger$$
$$|0\rangle\langle 0| = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$
$$|1\rangle\langle 1| = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$
$$\rho = \frac{1}{2} \begin{pmatrix} 1-p & 0 \\ 0 & p \end{pmatrix} = \frac{1}{2} (\hat{I} + (1-2p)\hat{Z})$$
$$\begin{cases} \langle X \rangle = \text{Tr}[\rho \hat{X}] = 0 \\ \langle Y \rangle = \text{Tr}[\rho \hat{Y}] = 0 \\ \langle Z \rangle = \text{Tr}[\rho \hat{Z}] = 1-2p \end{cases}$$
$$\text{Tr}[\rho_A \rho_B] = 2\delta_{AB} \quad \text{for } A, B \in \{X, Y, Z\}$$



Scaling to larger number of qubits

$$[(1-p)|0\rangle\langle 0| + p|1\rangle\langle 1|]^{\otimes n} \equiv U \equiv \text{Measurement}$$
$$\rho_0 = [(1-p)|0\rangle\langle 0| + p|1\rangle\langle 1|]^{\otimes n}$$
$$= (1-p)^n |00\dots 0\rangle\langle 00\dots 0| + \dots + p^n |11\dots 1\rangle\langle 11\dots 1|$$
$$P(M=00\dots 0) = \begin{cases} 1 & \text{if } p=0 \text{ (ideal case)} \\ (1-p)^n \approx 1-np + O(p^2) & \text{for } p>0 \end{cases}$$
$$\langle Z_k \rangle = \langle 111\dots 111 \rangle = 1-2p$$
$$\langle Z Z Z \dots Z \rangle = \text{Tr}[Z^{\otimes n} \rho] = (1-2p)^n$$
$$\rho_0 = \prod_{k=1}^n \frac{1}{2} (\hat{I}_k + (1-2p)\hat{Z}_k)$$
$$= \frac{1}{2^n} (1+\underline{Z}) \otimes (1+\underline{Z}) \otimes \dots \otimes (1+\underline{Z})$$
$$= \frac{1}{2^n} [(1-2p)^n \underline{Z}^{\otimes n} + \dots]$$
$$\text{Tr}[\rho_0^2] = \prod_{k=1}^n \text{Tr}(\rho_{0,k}^2) = (1-2p)^{2n}$$

