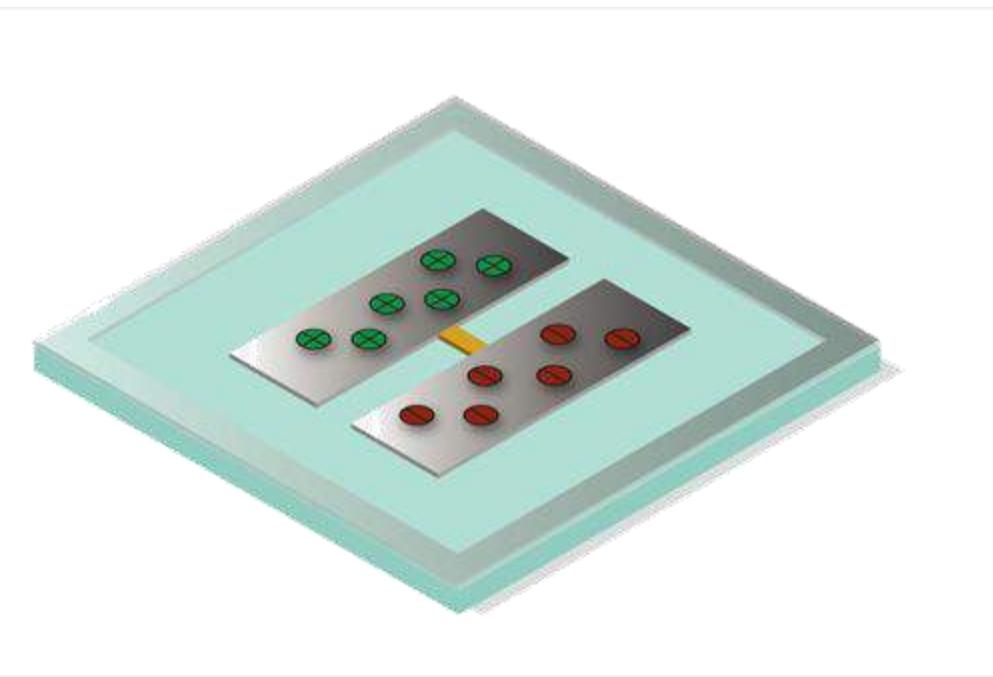


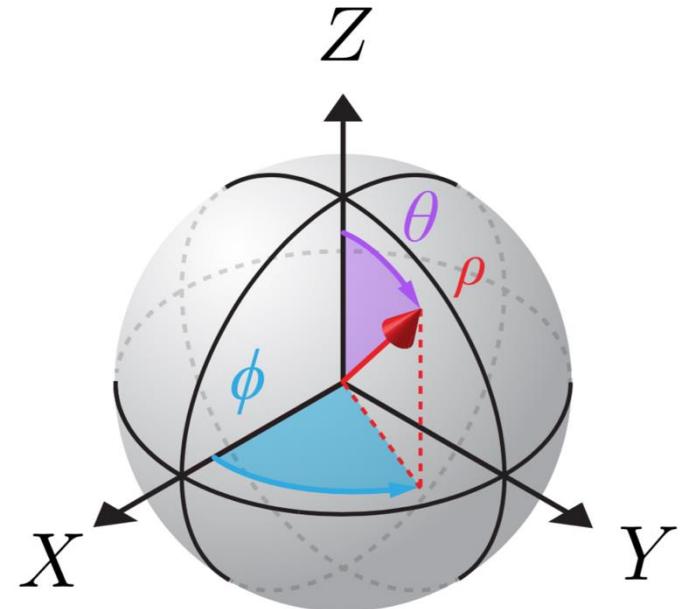
# Superconducting Qubits II

## Transmon & Measurements



Introduction

Zlatko K. Minev



IBM Quantum  
IBM T.J. Watson Research Center, Yorktown Heights, NY



@zlatko\_minev

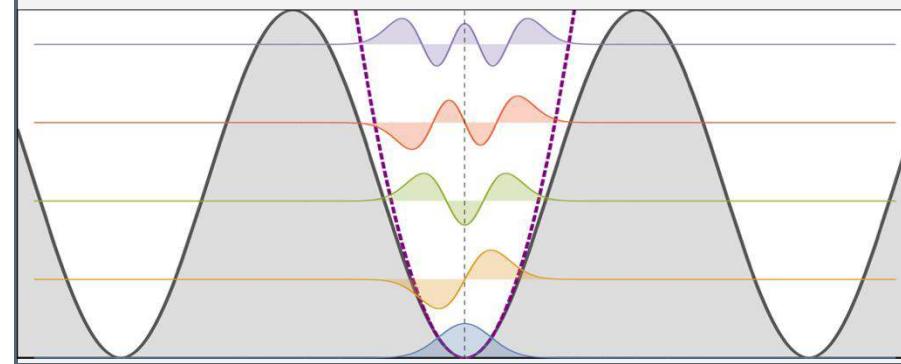


zlatko-minev.com

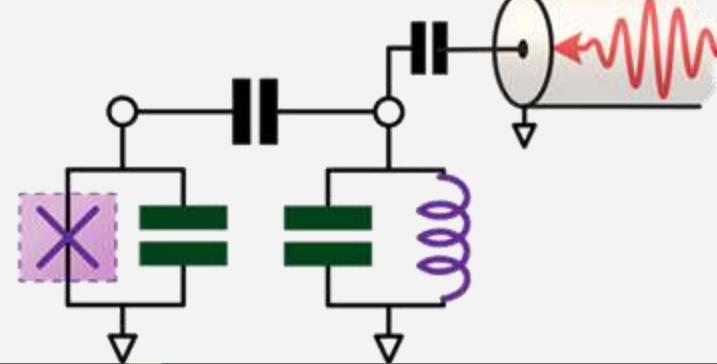
Image copyright:  
ZKM unless otherwise noted

# The road ahead

## Transmon qubit



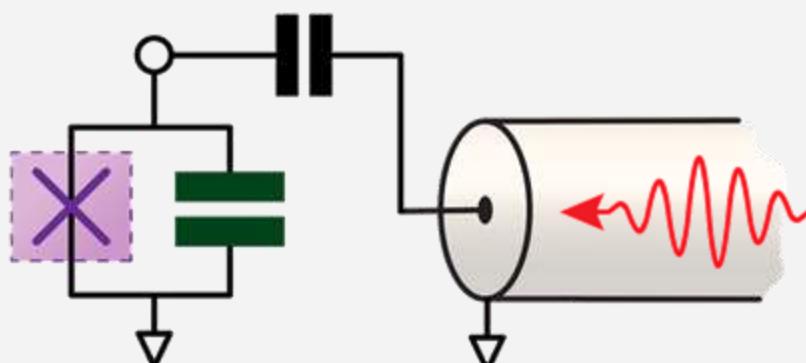
## Qubit and readout (/qubit)



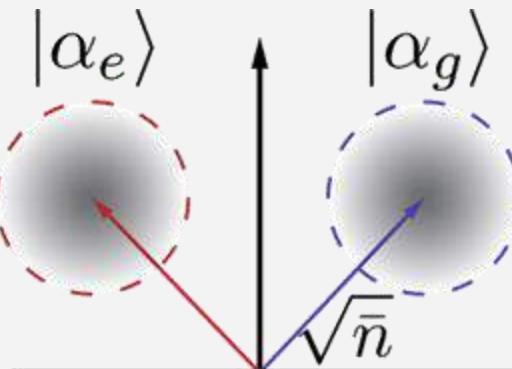
## Lab



## Qubit control

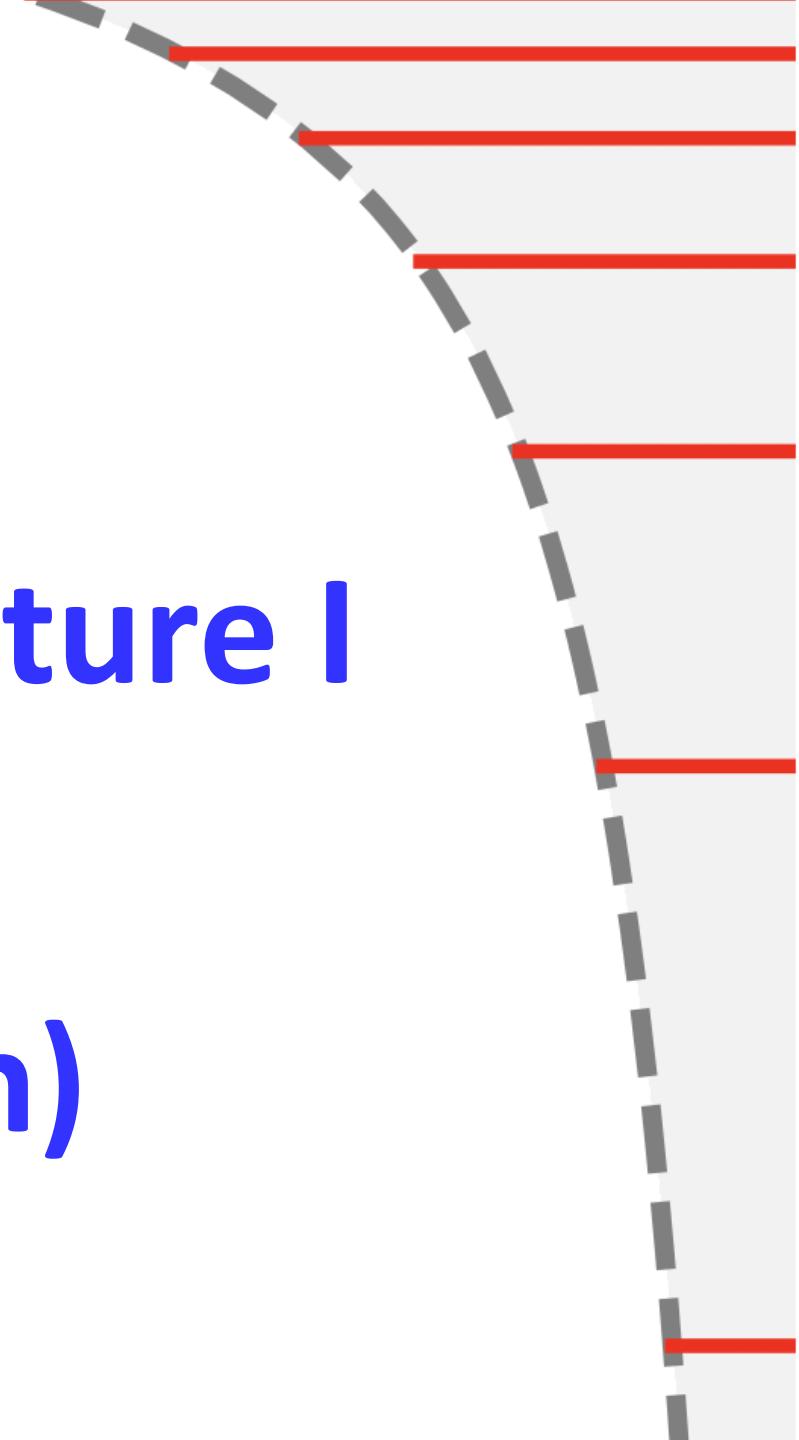


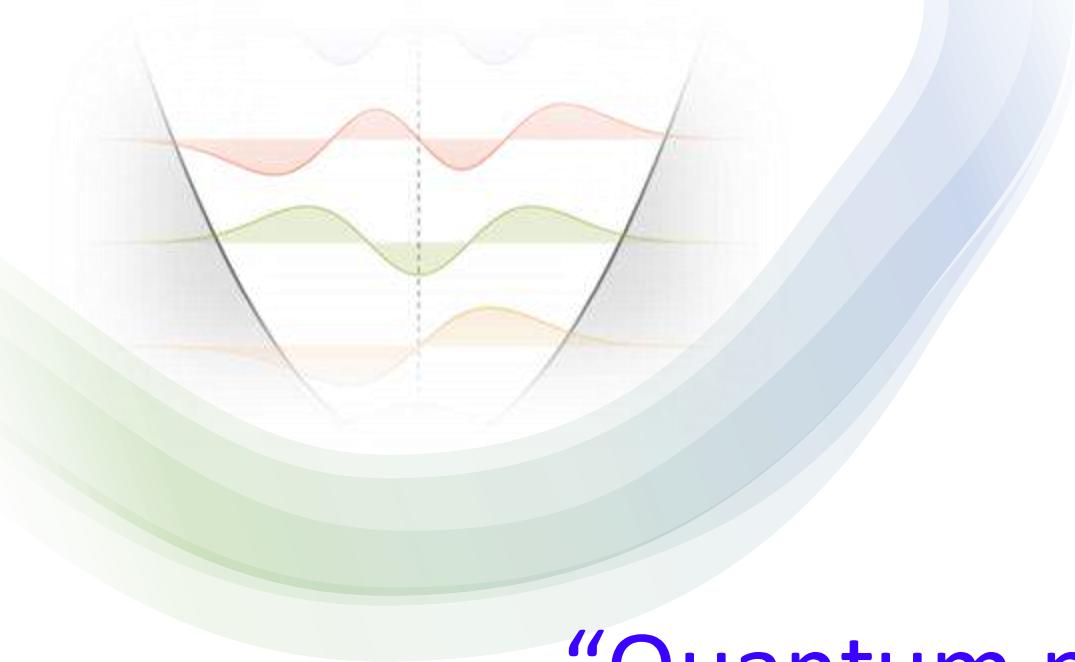
## On measurements



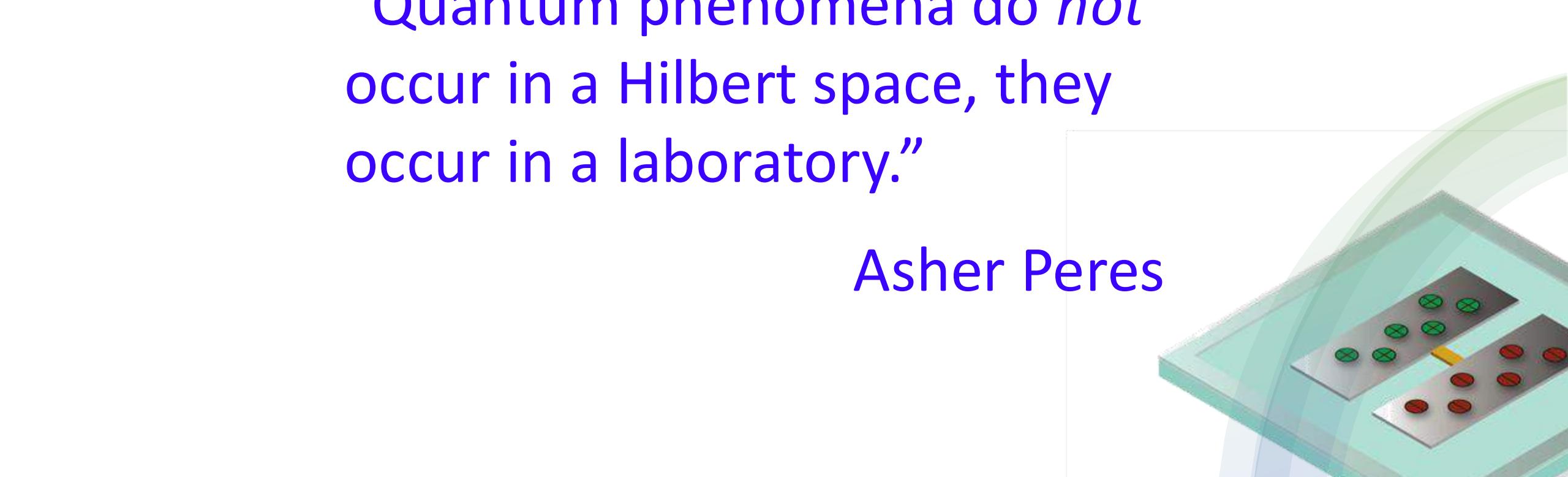
\* Road image: based on Freepik

# Takeaways from Lecture I (and building on)



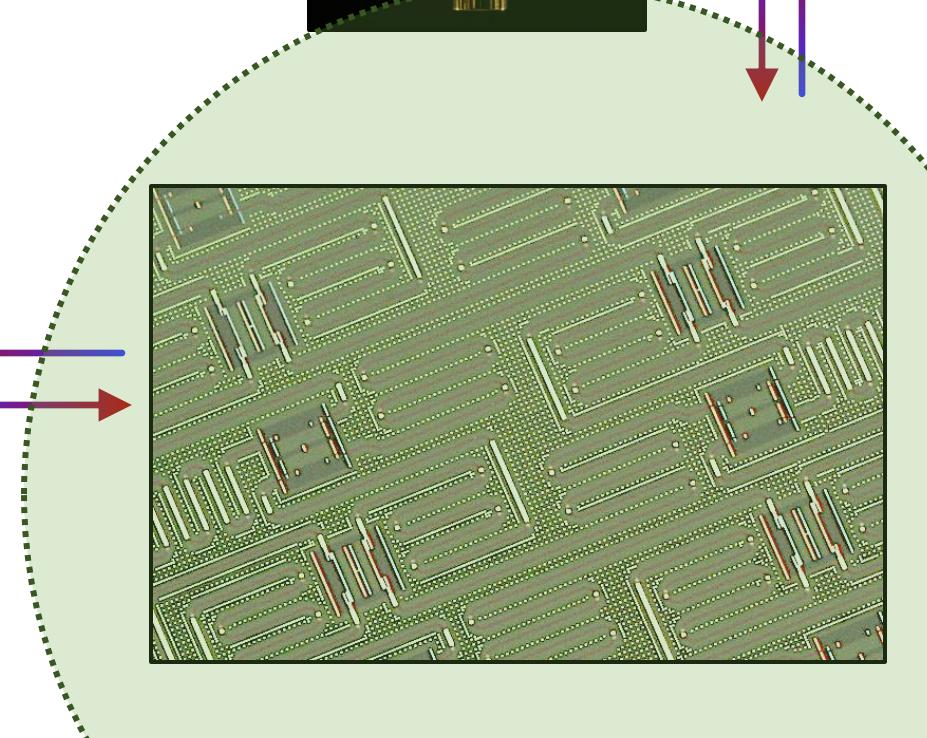
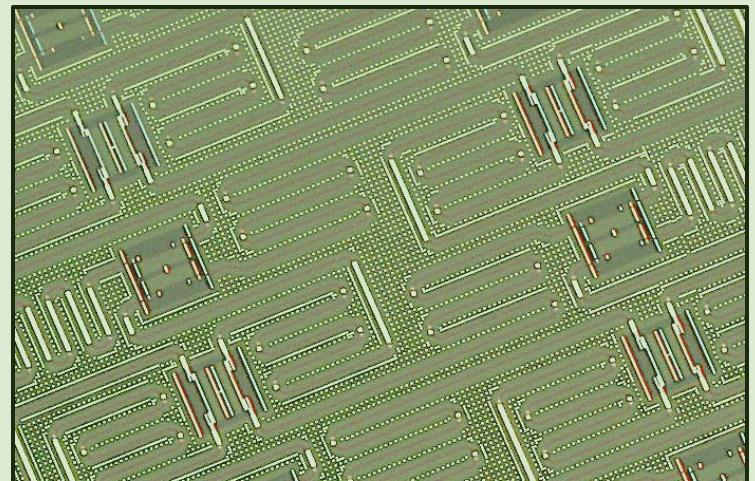
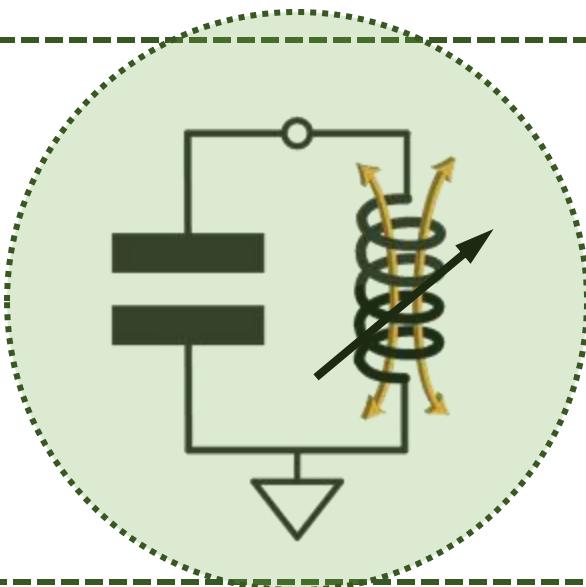
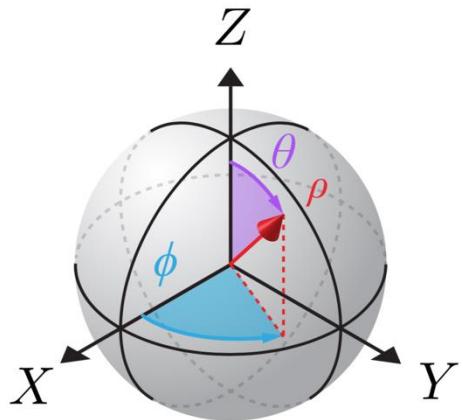
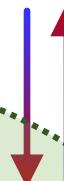
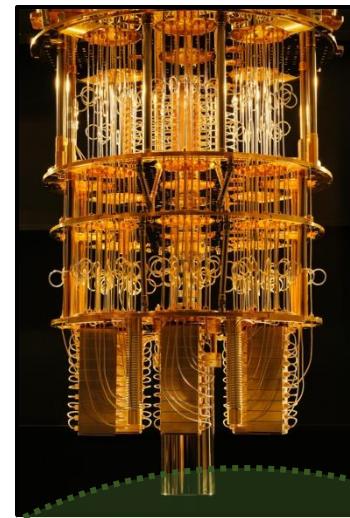


*“Quantum phenomena do not  
occur in a Hilbert space, they  
occur in a laboratory.”*

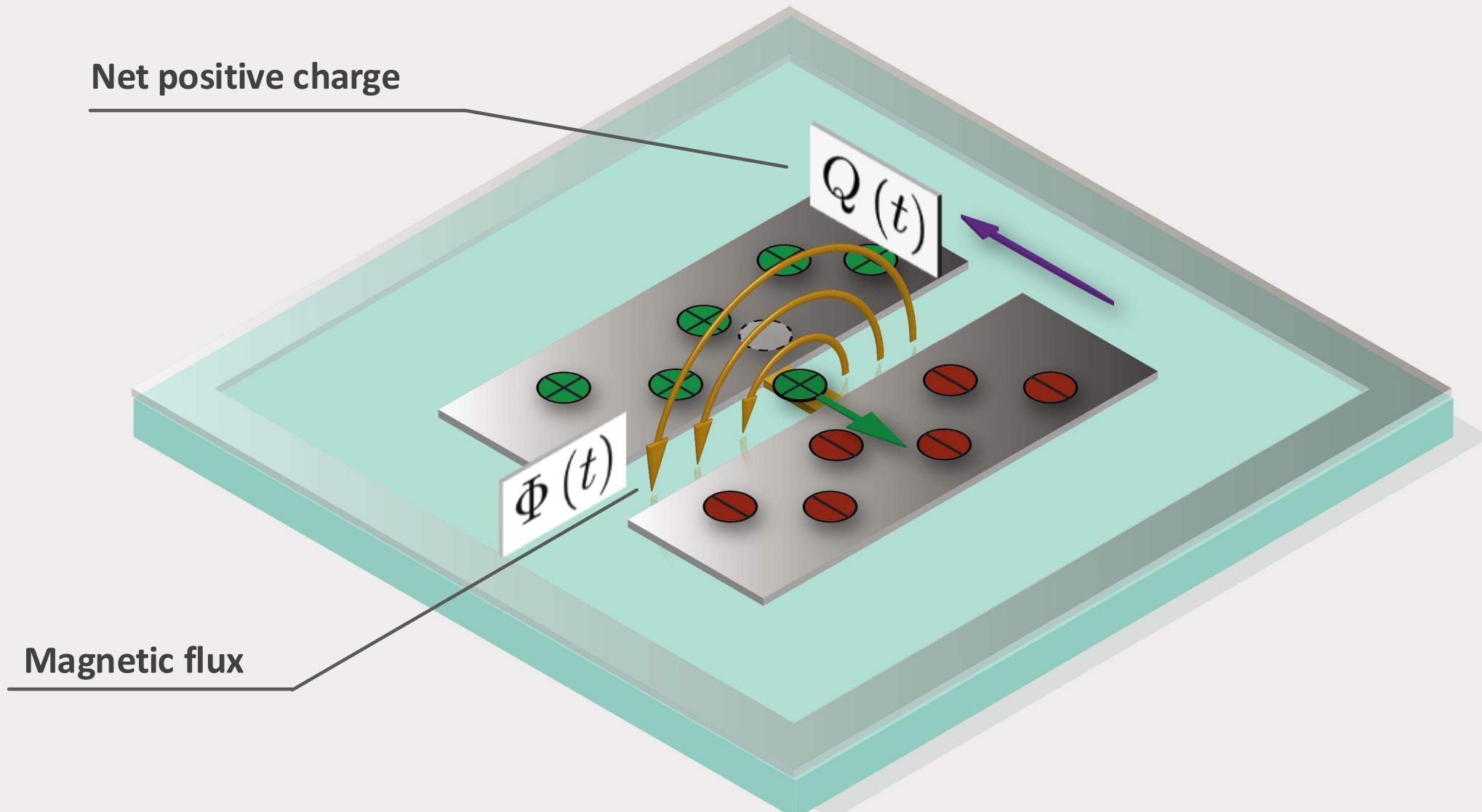


Asher Peres

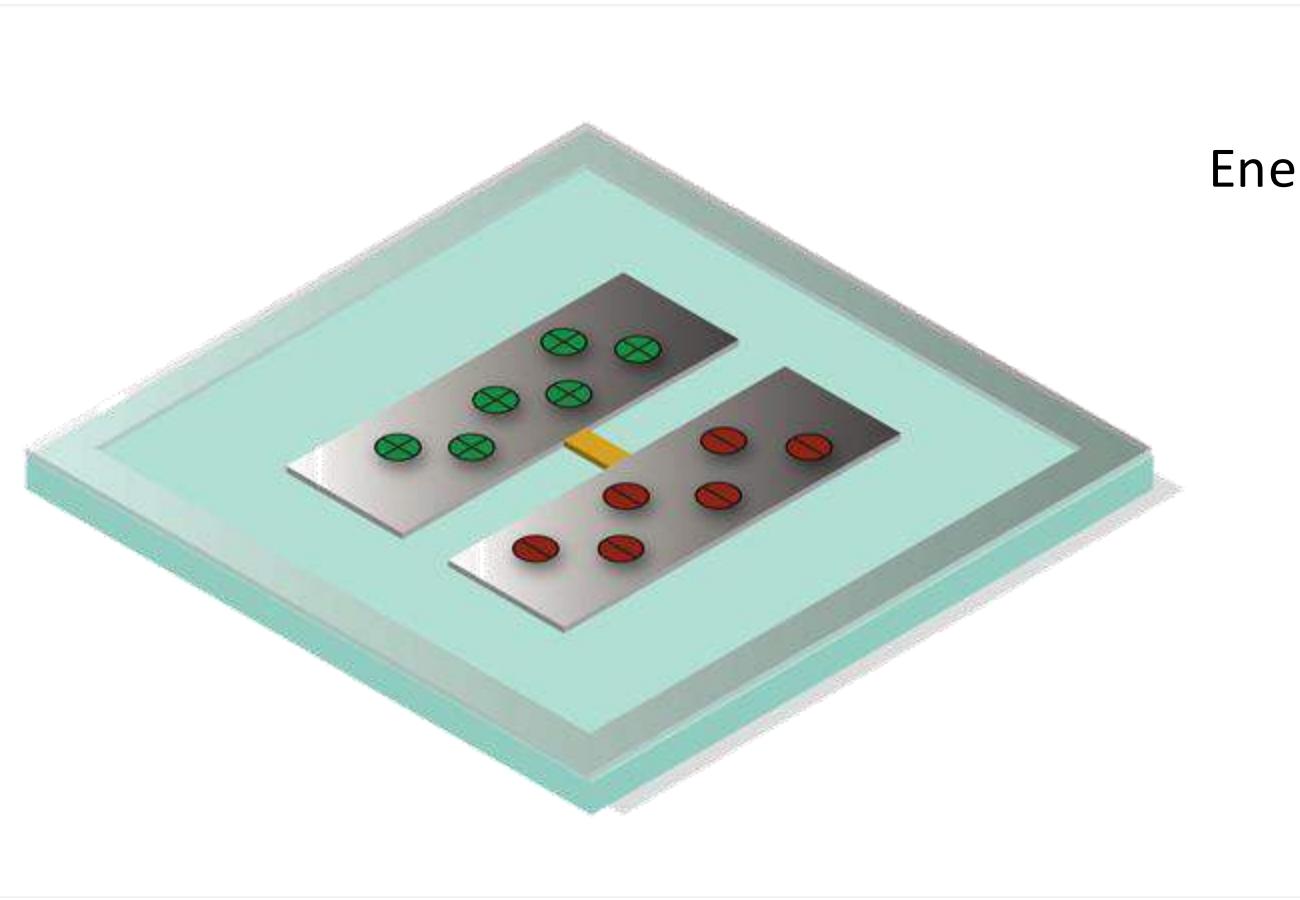
# Qubit in the cloud



# Transmon qubit: physical picture



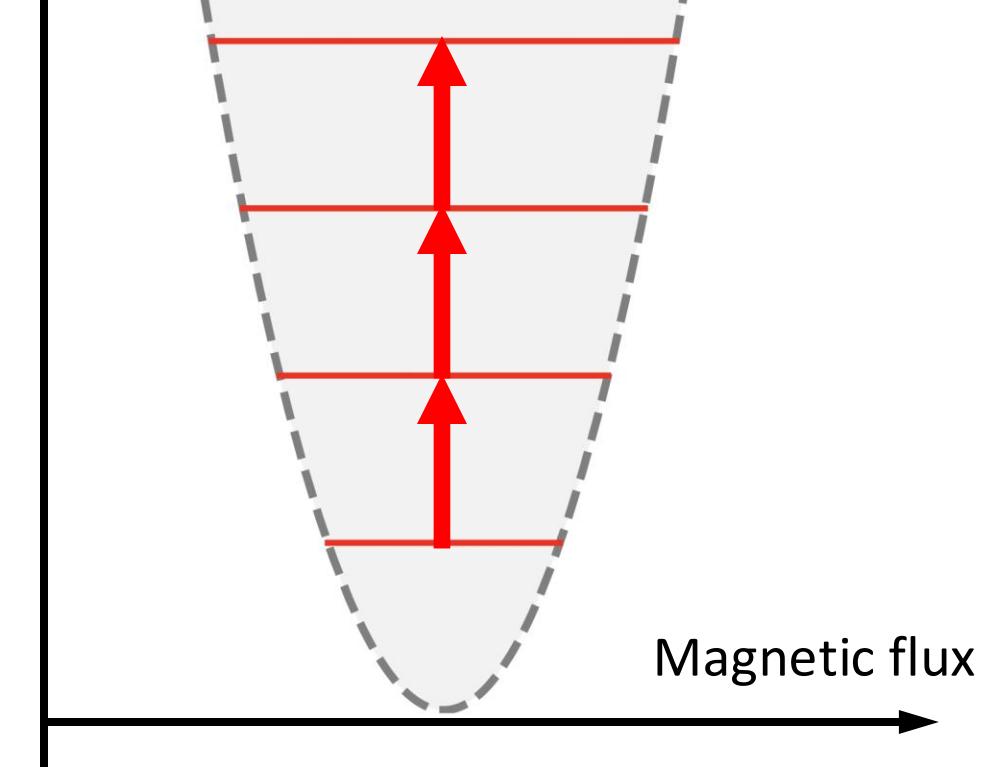
# A first approximation



Energy

## Harmonic oscillator

$$\hat{H} |n\rangle = \hbar\omega_0 \left( n + \frac{1}{2} \right) |n\rangle$$



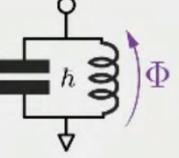
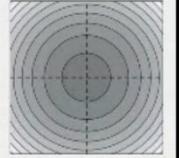
# Classical to quantum bridge



Bruno @brunormzg · 15h

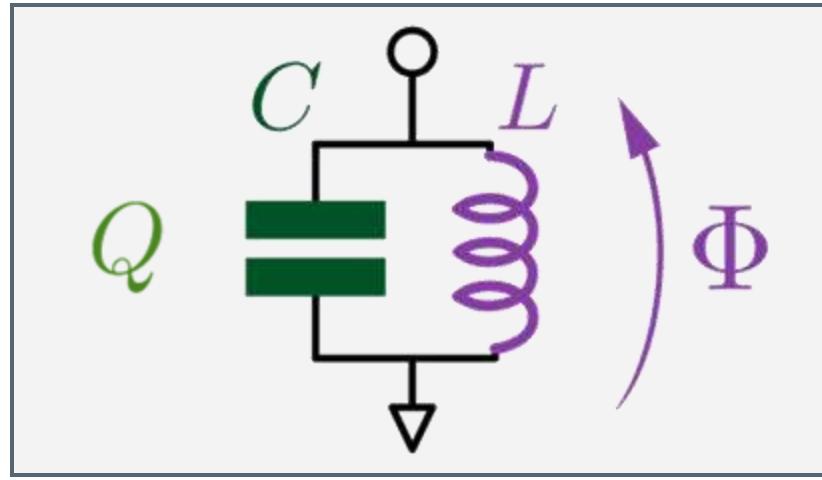
This is perhaps my favorite of many "aha!" moments that I've had during the #qgss. I didn't quite grasp the origin of the creation/annihilation operators, but that is no more. Thanks a lot, looking forward to tomorrow's lecture!!

@zlatko\_minev @qiskit @IBM

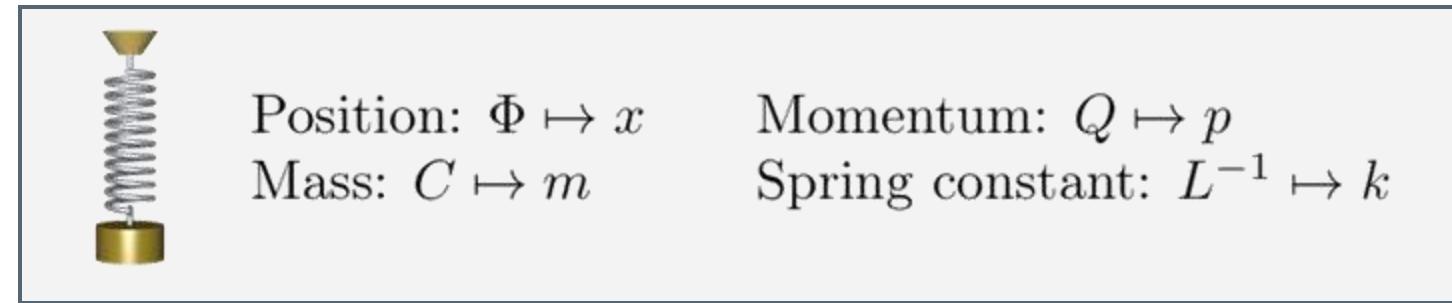
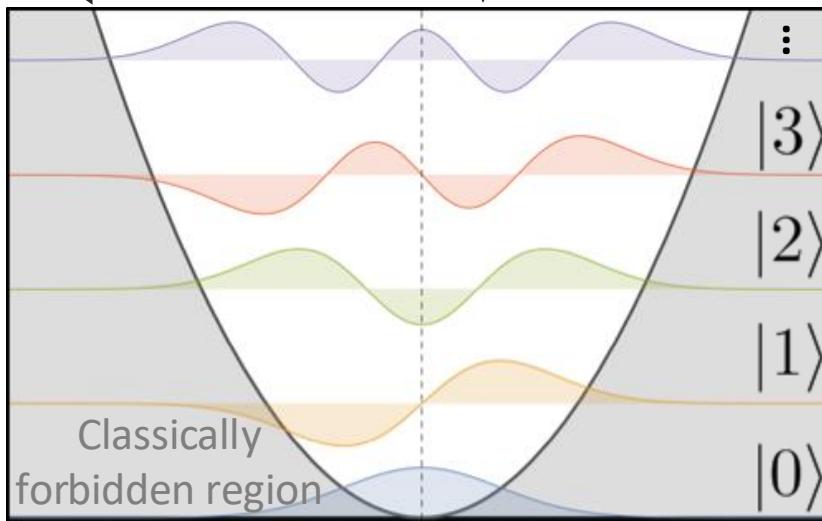
 **The classical and quantum oscillator** 

	Classical	Quantum
Hamiltonian	$\mathcal{H} = \frac{Q^2}{2C} + \frac{\Phi^2}{2L}$ $= \frac{1}{2} \hbar \omega_0 (\alpha^* \alpha + \alpha \alpha^*)$	$\hat{H} = \frac{\hat{\Phi}^2}{2L} + \frac{\hat{Q}^2}{2C}$ $= \hbar \omega_0 (\hat{a}^\dagger \hat{a} + \hat{a} \hat{a}^\dagger)$
Phase space	$\alpha(t) = \sqrt{\frac{1}{2\hbar Z}} [\Phi(t) + iZQ(t)]$ $\alpha(t) = \alpha(0) e^{-i\omega_0 t}$ $\Phi(t) = \sqrt{\frac{\hbar Z}{2}} (\alpha^*(t) + \alpha(t))$ $Q(t) = i\sqrt{\frac{\hbar}{2Z}} (\alpha^*(t) - \alpha(t))$	$\hat{a} = \sqrt{\frac{1}{2\hbar Z}} (\hat{\Phi} + iZ\hat{Q})$ $\hat{a}(t) = \hat{a}(0) e^{-i\omega_0 t}$ <p style="text-align: right;">(Heisenberg picture)</p> $\hat{\Phi} = \Phi_{\text{ZPF}} (\hat{a}^\dagger + \hat{a})$ $\hat{Q} = iQ_{\text{ZPF}} (\hat{a}^\dagger - \hat{a})$

# The LC quantum harmonic oscillator



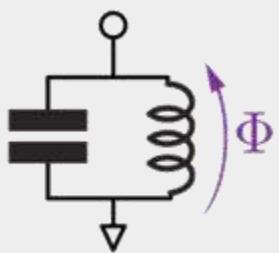
$$\mathcal{E}_{\text{ind}}(\Phi) = \frac{\Phi^2}{2L} \quad \psi_n(\Phi) \equiv \langle \Phi | n \rangle$$



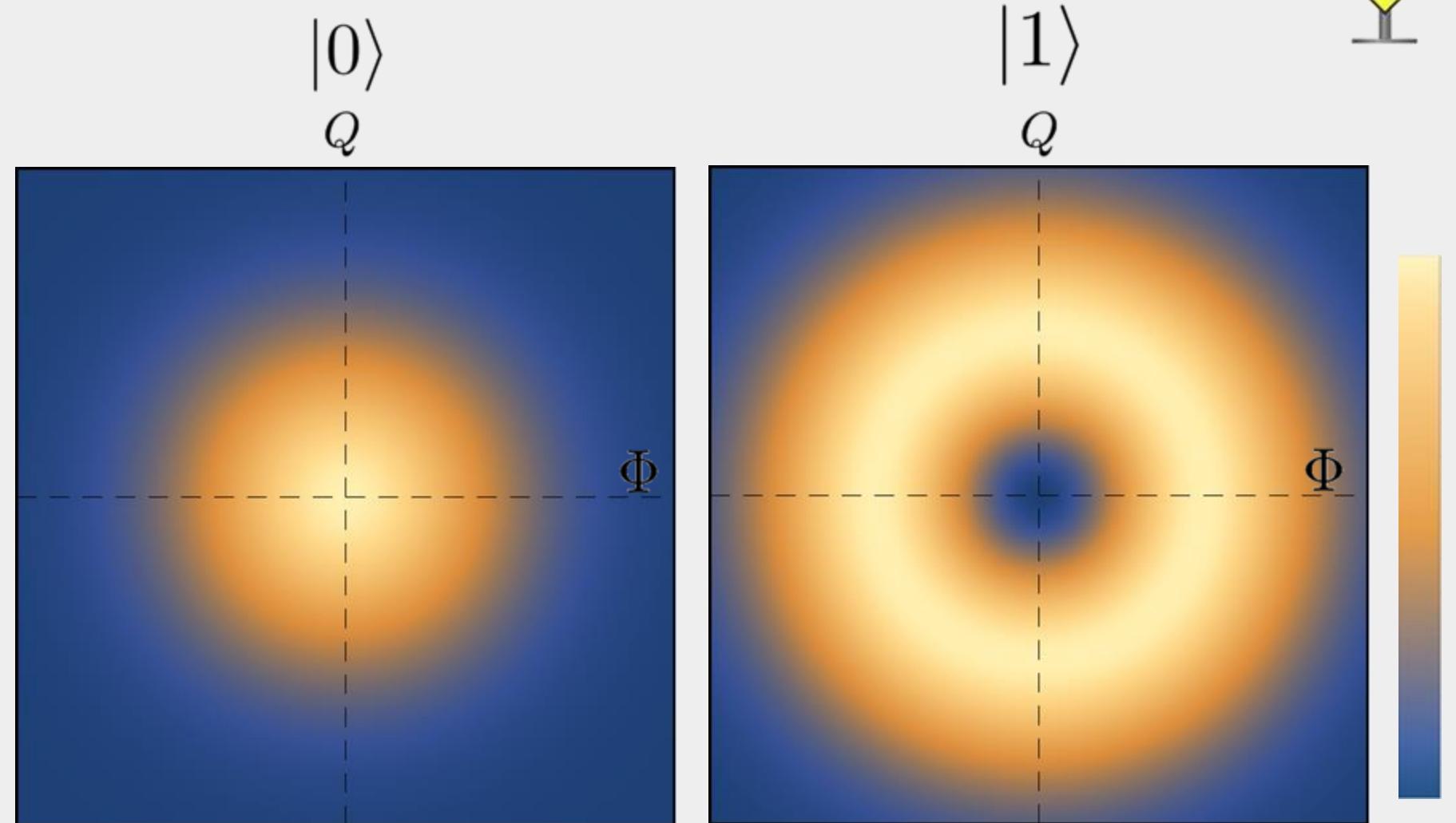
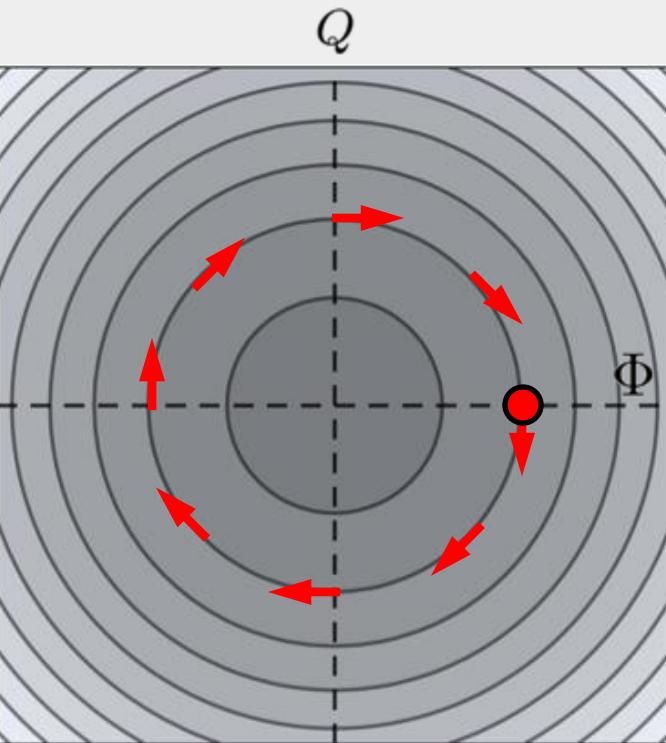
$$\hat{H}(\hat{\Phi}, \hat{Q}) = \frac{\hat{\Phi}^2}{2L} + \frac{\hat{Q}^2}{2C} = \hbar\omega_0 \left( \hat{a}^\dagger \hat{a} + \frac{1}{2} \right) \quad \omega_0^2 = \frac{1}{LC}$$

$$\hat{H}|n\rangle = \hbar\omega_0 \left( n + \frac{1}{2} \right) |n\rangle \quad Z_0 = \frac{L}{C}$$

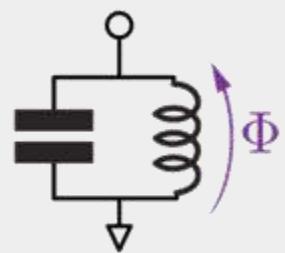
$$\begin{aligned} \hat{\Phi} &= \Phi_{\text{ZPF}} (\hat{a}^\dagger + \hat{a}) & \Phi_{\text{ZPF}} &= \sqrt{\frac{\hbar}{2} Z_0} & \langle 0 | \hat{\Phi}^2 | 0 \rangle &= \Phi_{\text{ZPF}}^2 \\ \hat{Q} &= iQ_{\text{ZPF}} (\hat{a}^\dagger - \hat{a}) & Q_{\text{ZPF}} &= \sqrt{\frac{\hbar}{2} Z_0^{-1}} & \langle 0 | \hat{Q}^2 | 0 \rangle &= Q_{\text{ZPF}}^2 \end{aligned}$$



# Phase-space (Husimi Q function)



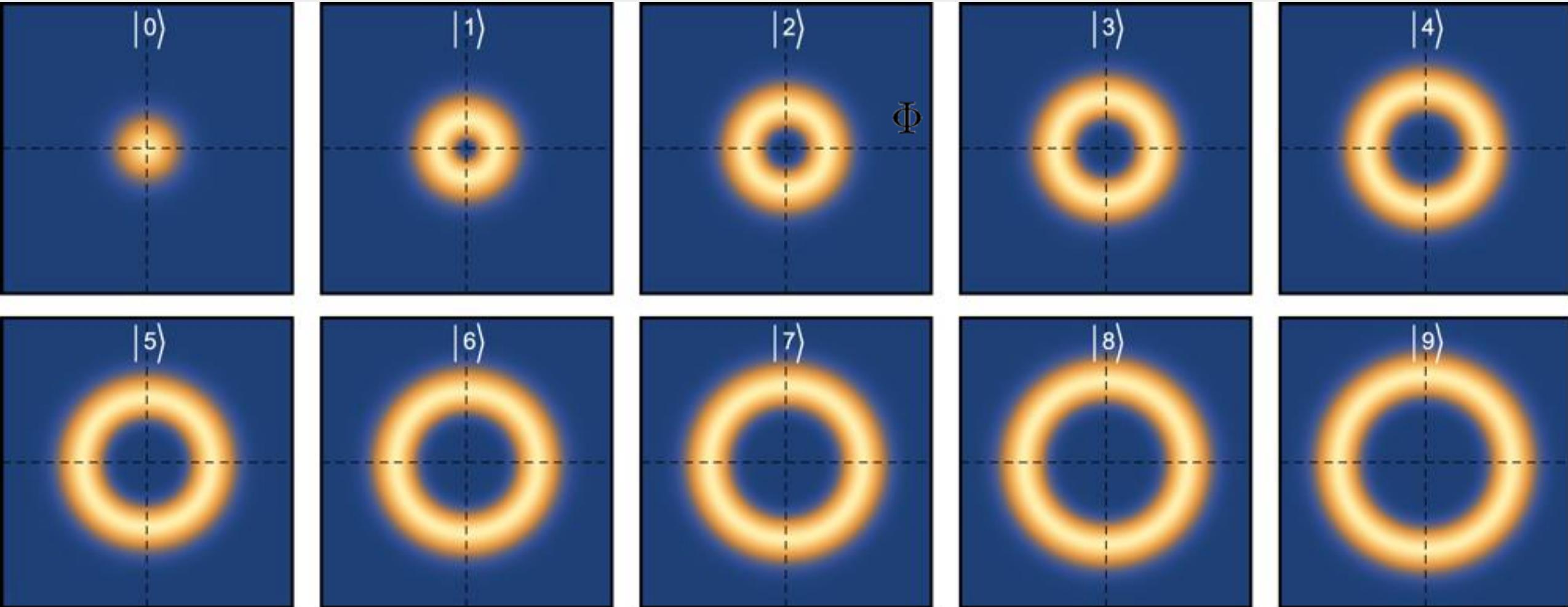
$$Q(\alpha) = \frac{1}{\pi} \langle \alpha | \hat{\rho} | \alpha \rangle$$



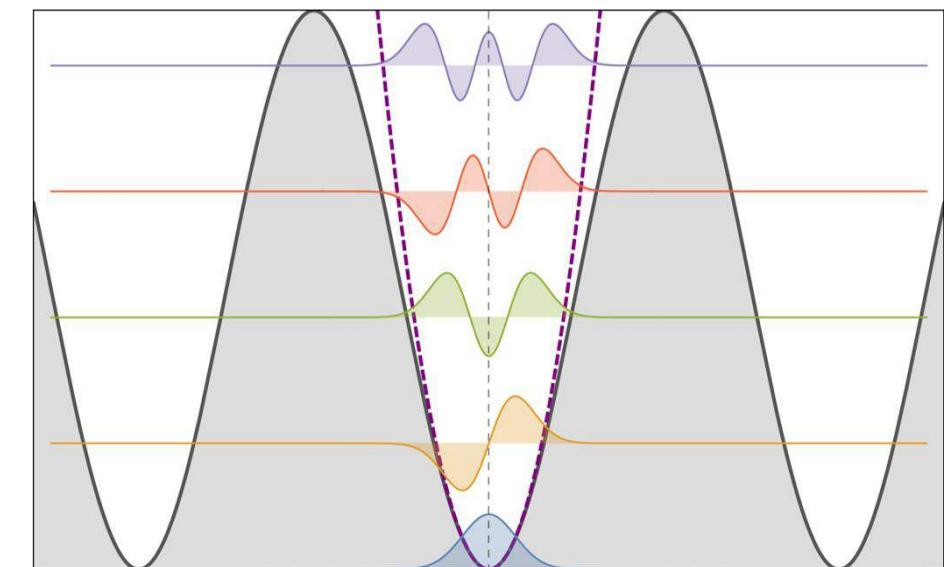
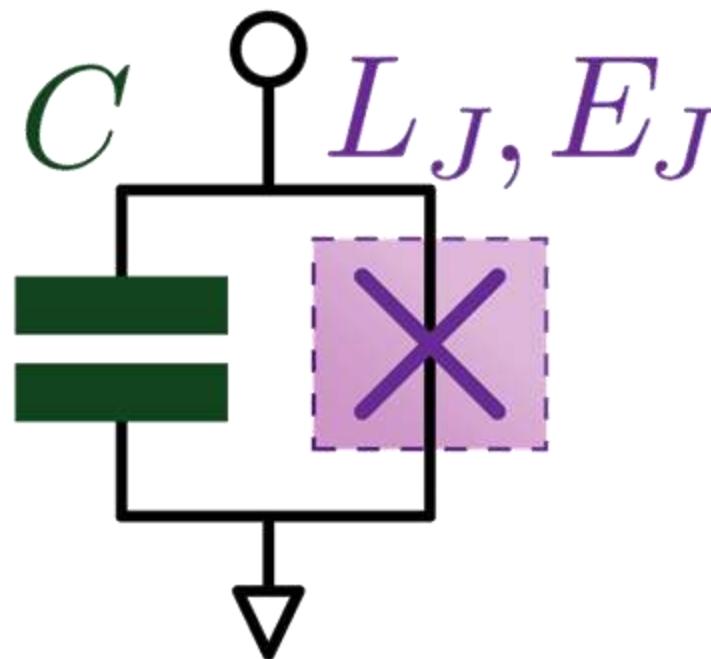
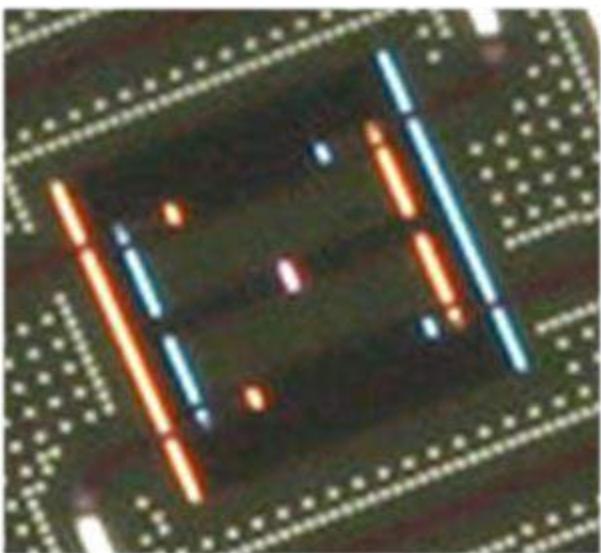
# The cloud of quantum uncertainty



$Q$

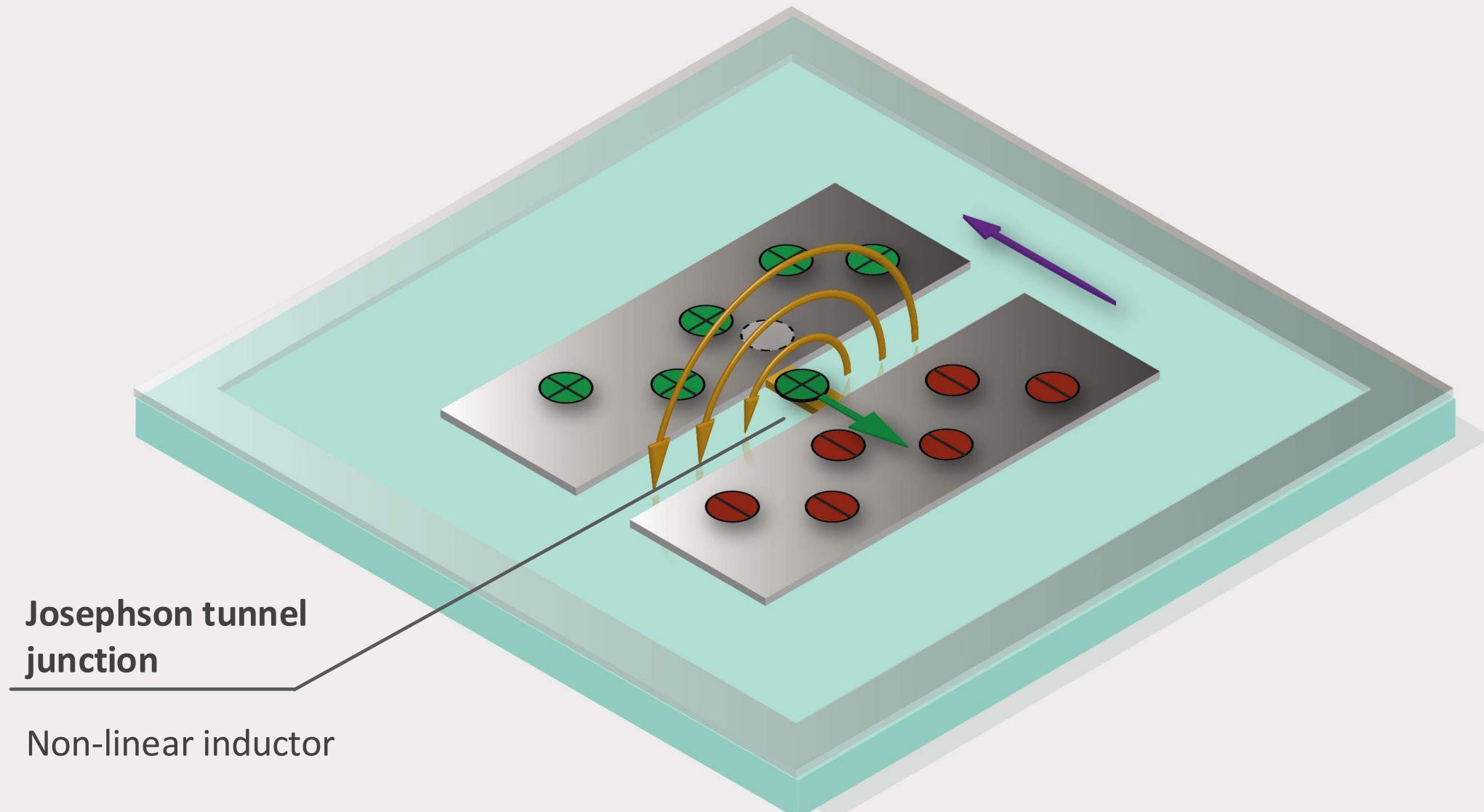


# Transmon Qubit

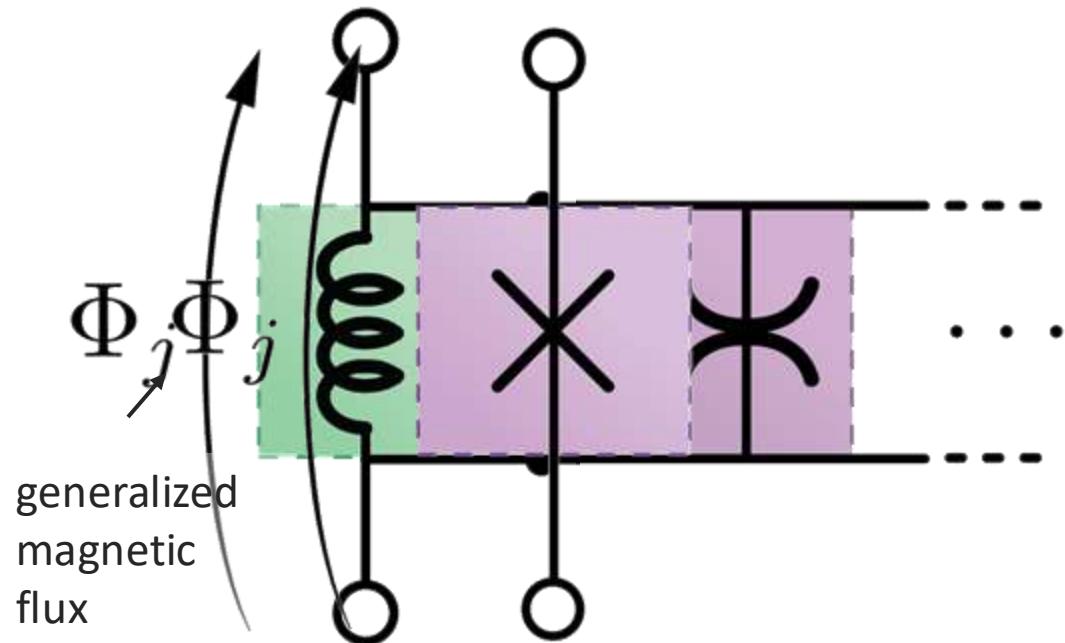


Introduction

# The transmon as a non-linear oscillator



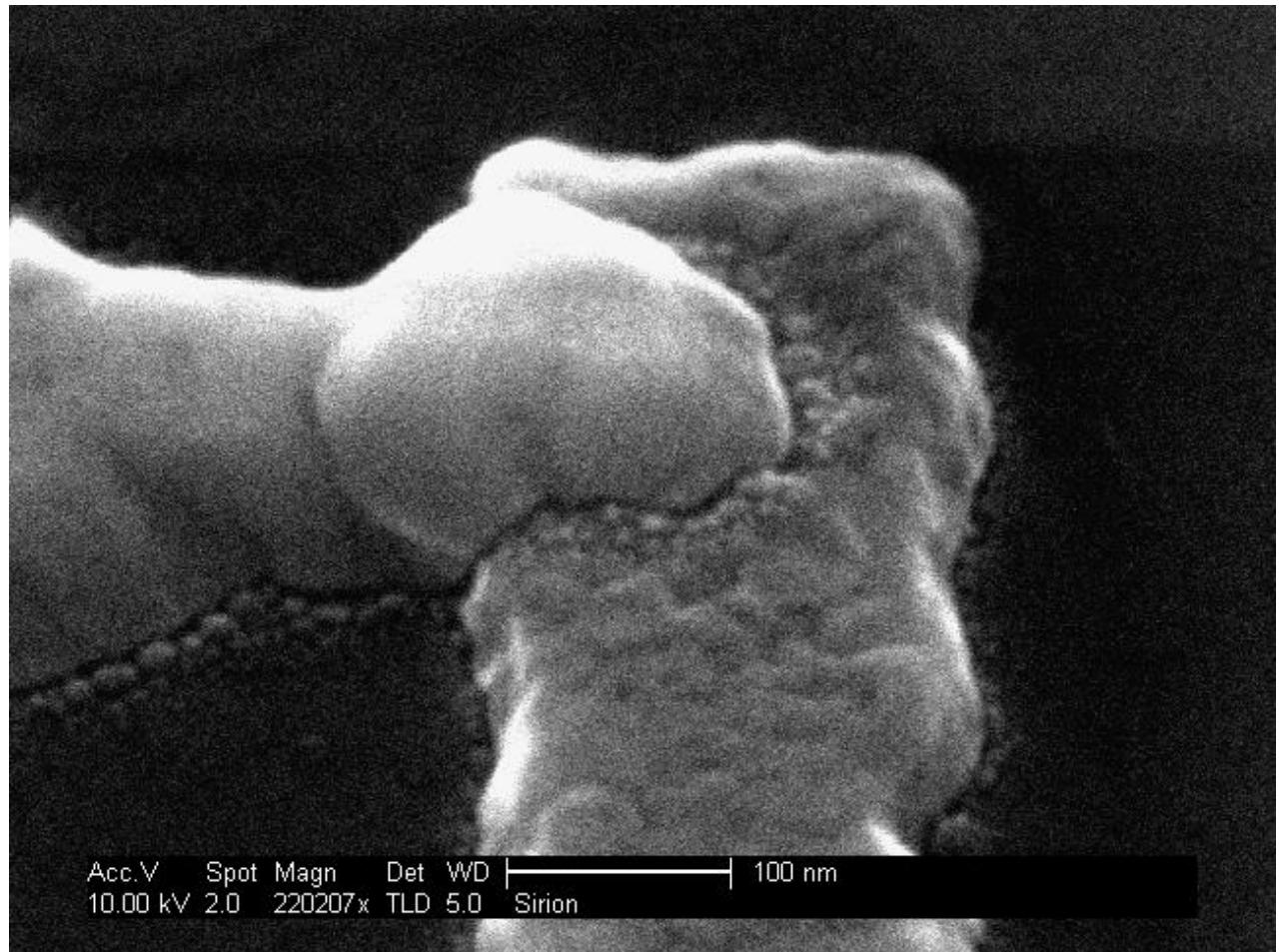
# Josephson tunnel junction



$$\mathcal{E}_j(\Phi_j) = E_j(1 - \cos(\Phi_j/\phi_0))$$

$$\phi_0 \equiv \hbar/2e = \mathcal{E}_j^{\text{lin}}(\Phi_j) + \mathcal{E}_j^{\text{nl}}(\Phi_j)$$

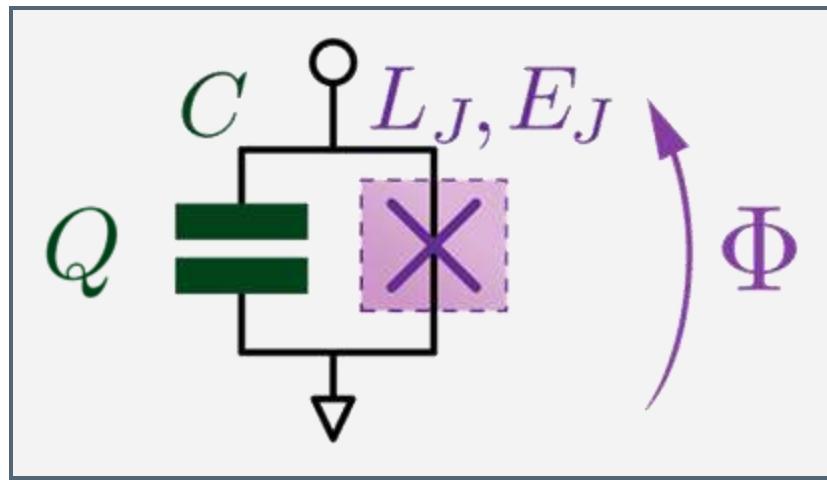
$$= \frac{E_j}{2} \left( \frac{\Phi_j}{\phi_0} \right)^2 - \frac{E_j}{4!} \left( \frac{\Phi_j}{\phi_0} \right)^4 + \mathcal{O}(\Phi_j^6)$$



SEM image: L. Frunzio

Circuit image: Minev et al., EPR to appear (2020)

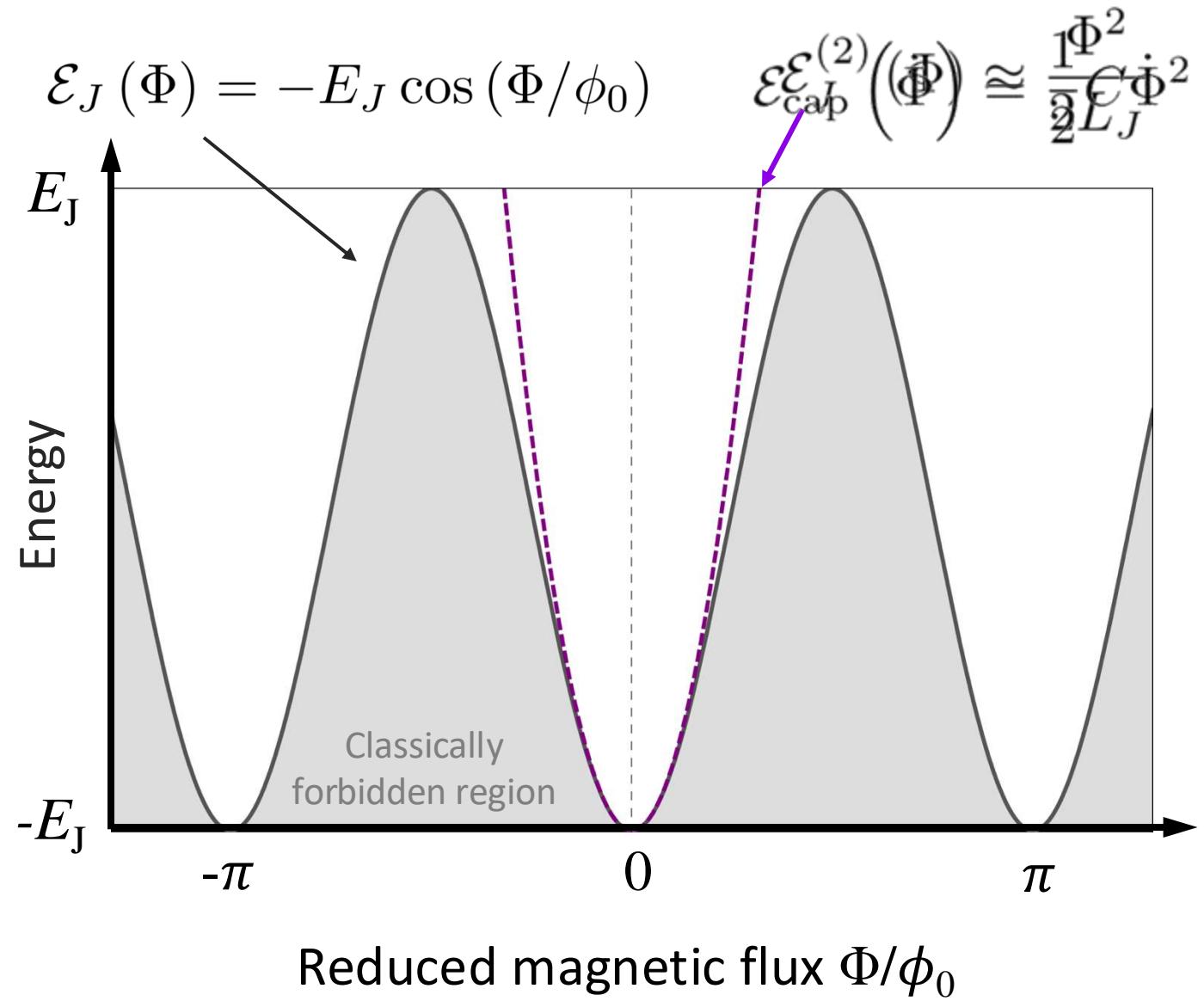
# The Transmon qubit



$$E_J = \frac{\phi_0^2}{L_J}$$

$$\phi_0 = \frac{\hbar}{2e}$$

$$\approx 3.3 \times 10^{-16} \text{ Wb}$$

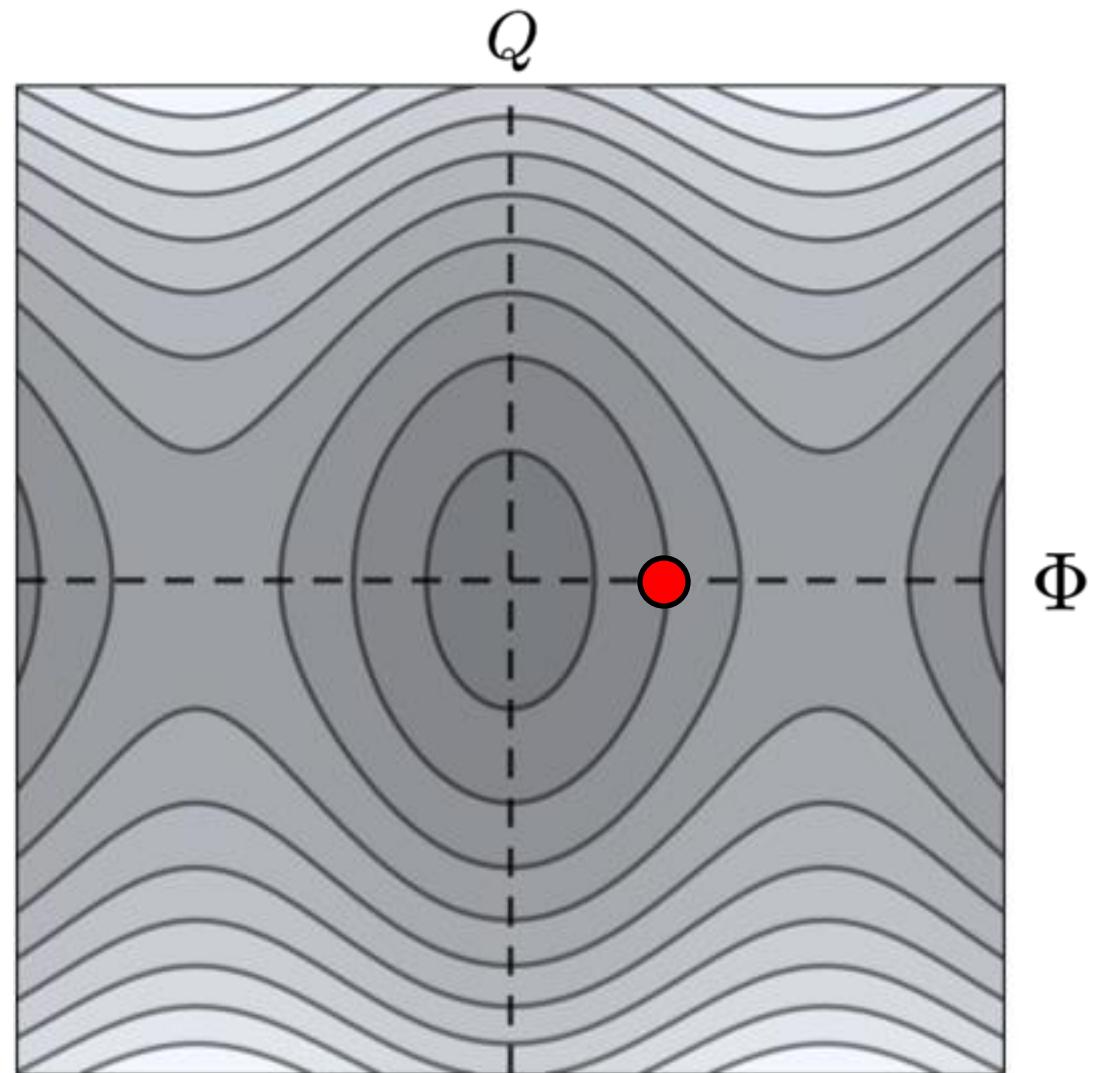


# Semi-classical intuition: phase space picture

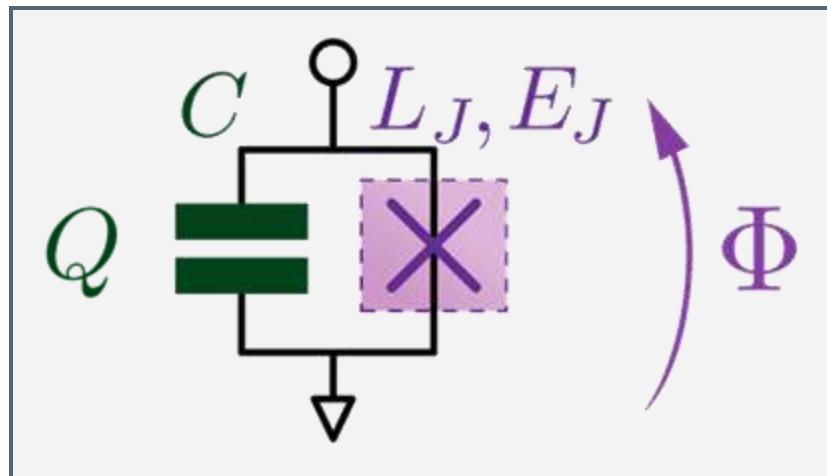
$$\mathcal{H}(\Phi, Q) = \frac{Q^2}{2C} - E_J \cos(\Phi/\phi_0)$$

$$E = \hbar\omega_0 \left( n + \frac{1}{2} \right)$$

$$= \frac{Q^2}{2C} - E_J \cos(\Phi/\phi_0)$$



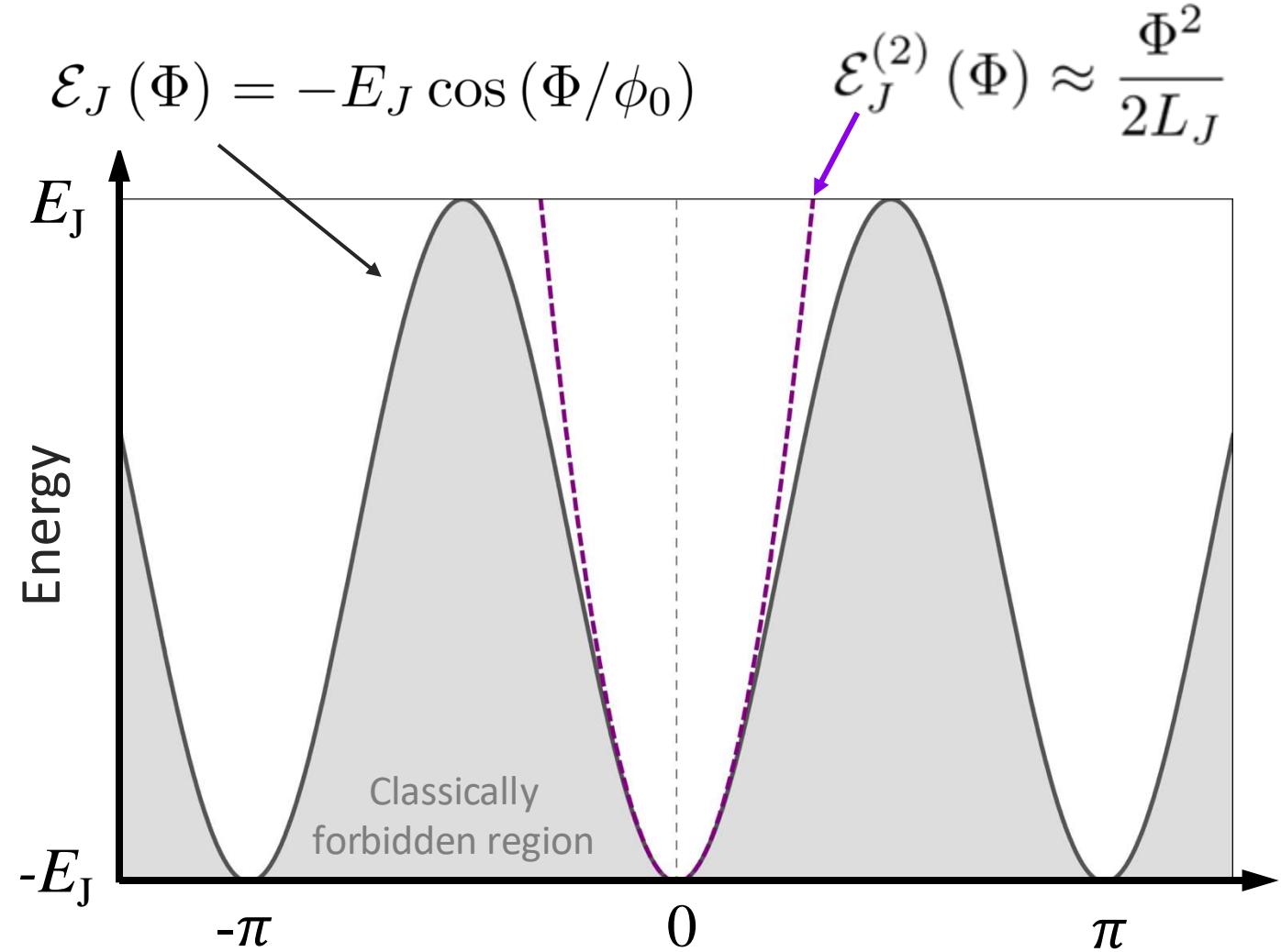
# The Transmon qubit



$$\hat{H} = \frac{\hat{Q}^2}{2C} - E_J \cos\left(\hat{\Phi}/\phi_0\right)$$

$$\approx \frac{\hat{Q}^2}{2C} + \frac{\hat{\Phi}^2}{2L_J} - \frac{E_J}{4!} \left( \frac{\hat{\Phi}}{\phi_0} \right)^4$$

$$= \hbar\omega_0 \hat{a}^\dagger \hat{a} - \frac{E_J \phi_{\text{ZPF}}^4}{4!} (\hat{a} + \hat{a}^\dagger)^4$$



# Hand-written notes

Day 2

$$[-E_S \phi_{ZPF}^4] (\hat{a} + \hat{a}^\dagger)^4$$

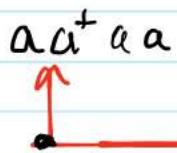
Drop huts

$$(a + a^\dagger)(a + a^\dagger)(a + a^\dagger)(a + a^\dagger)$$

$$\begin{aligned} &aaaa + aaaa^\dagger + aa a^\dagger a + \dots \\ &\quad \dots \end{aligned}$$



<sup>4</sup>wave mixing

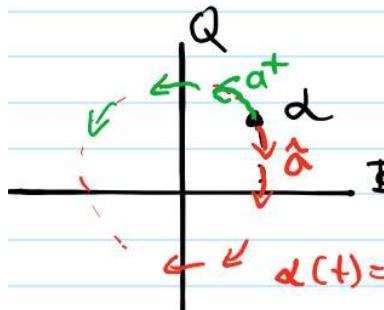


$$\begin{aligned} [a, a^\dagger] &= 1 \\ a a^\dagger - a^\dagger a &= 1 \\ a a^\dagger &= 1 + a^\dagger a \end{aligned}$$

$$(a^\dagger a + 1) a a$$

$$a^\dagger a a + a a$$

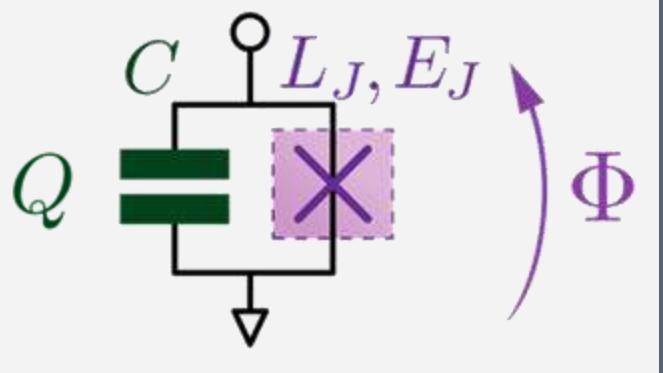
# Hand-written notes


$$a^+ a^3 + a^2 \xrightarrow{\text{non-classical term}} 4\text{-wave mixing}$$
$$\alpha(t) = \alpha(0) e^{-i\omega_0 t}$$
$$\hat{\alpha}(t) = \hat{\alpha}(0) e^{-i\omega_0 t}$$
$$\hat{\alpha}^+(t) = \hat{\alpha}(0) e^{+i\omega_0 t}$$
$$\begin{aligned} a^+ a^3 &= a^+ e^{+i\omega_0 t} a^3 e^{-i\omega_0 t} \\ &= a^+ a^3 e^{-2i\omega_0 t} \xrightarrow{\text{rotating}} \\ \hat{\alpha}^2 \hat{\alpha}^2 &= (\hat{\alpha}^+ e^{+i\omega_0 t})^2 (\hat{\alpha}^- e^{-i\omega_0 t})^2 \\ &= (\hat{\alpha}^+)^2 (\hat{\alpha})^2 e^{+2i\omega_0 t} e^{-2i\omega_0 t} \\ &= \hat{\alpha}^+ \hat{\alpha}^- \xrightarrow{\text{non-rotating}} \end{aligned}$$

$$\hat{H}_0 = \hat{H}_0 + \hat{H}_R(t)$$

$$\text{1st appr., } \hat{H} \approx \hat{H}_0$$

# The Transmon qubit



$$= \hbar\omega_0 \hat{a}^\dagger \hat{a} - \frac{E_J \phi_{\text{ZPF}}^4}{4!} (3 + 6\cancel{\hat{a}^2} + 12\hat{a}^\dagger \hat{a} + 6\cancel{\hat{a}^{\dagger 2}} + \cancel{\hat{a}^4} + 4\cancel{\hat{a}^\dagger \hat{a}^3} + 6\cancel{\hat{a}^{\dagger 2} \hat{a}^2} + 4\cancel{\hat{a}^{\dagger 3} \hat{a}} + \cancel{\hat{a}^{\dagger 4}})$$

RWA or 1<sup>st</sup>  
order PT

$$\begin{aligned} \hat{H}_4^{\text{RWA}} &= \hbar\omega_0 \hat{a}^\dagger \hat{a} - \frac{E_J}{4!} \phi_a^4 (12\hat{a}^\dagger \hat{a} + 6\hat{a}^{\dagger 2} \hat{a}^2) \\ &= \boxed{\hbar(\omega_0 - \Delta_q) \hat{a}^\dagger \hat{a} - \frac{\hbar\alpha}{2} \hat{a}^{\dagger 2} \hat{a}^2} \end{aligned}$$

Due to nl & commutator

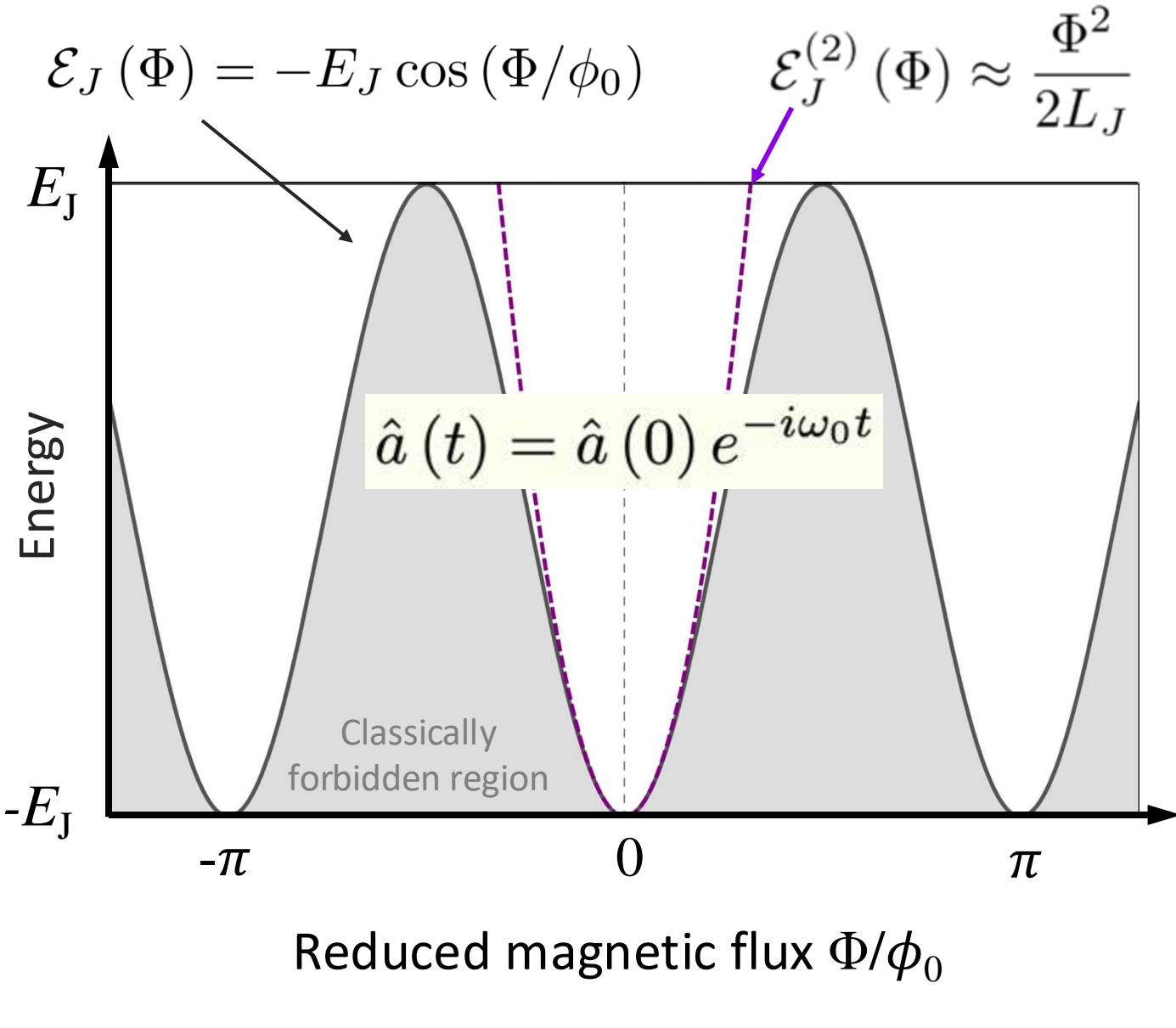
Due to non-linearity (nl)

$$\text{where } \hbar\Delta_q = \hbar\alpha \equiv \frac{1}{2} E_J \phi_{\text{ZPF}}^4$$

“Lamb shift”  
due to ZPF

Anharmonicity

Zero-point  
fluctuations



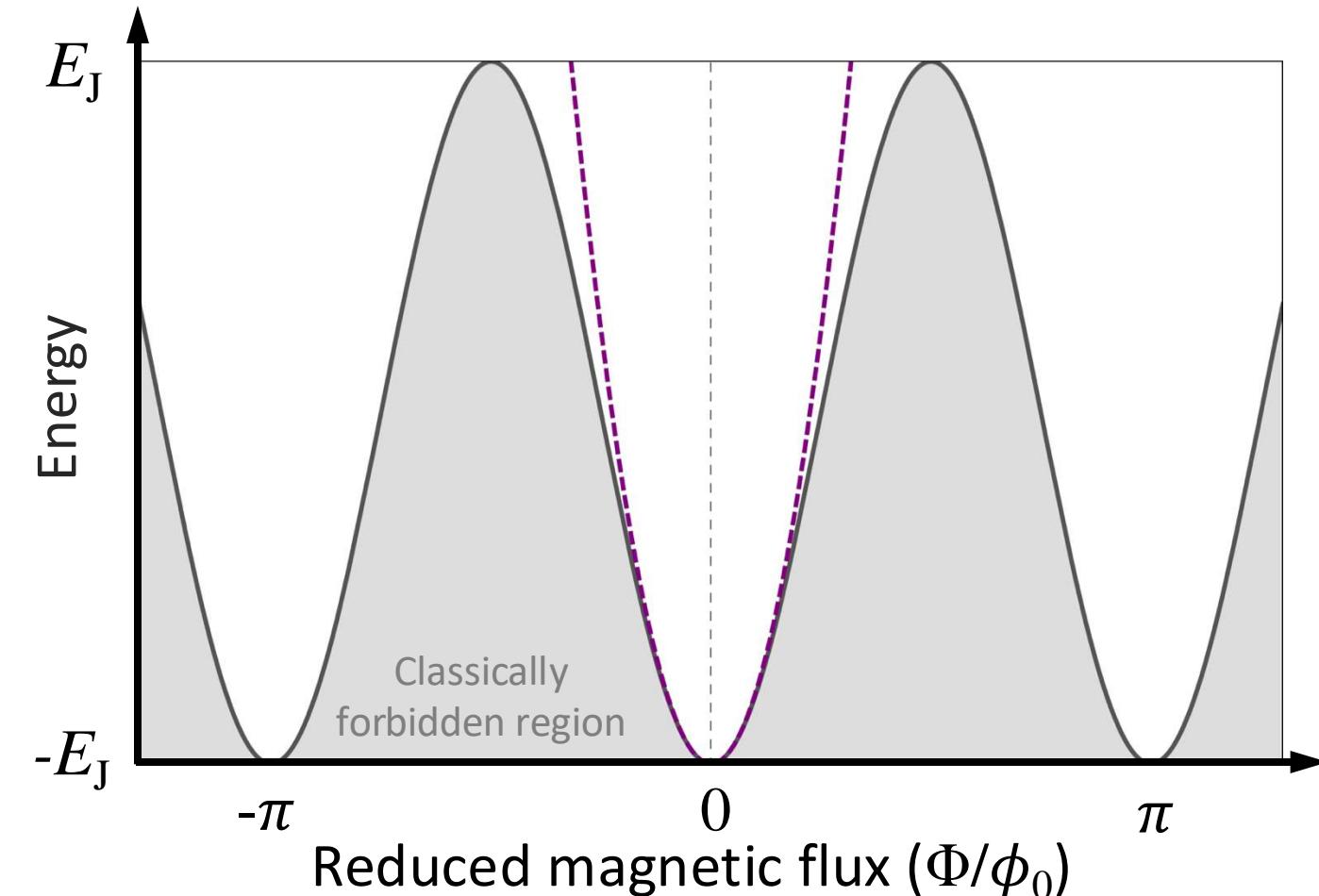
# The Kerr Hamiltonian of the transmon

$$\hat{N} \equiv \hat{a}^\dagger \hat{a}$$

$$\hat{H}_4^{\text{RWA}} \approx \hbar\omega_q \hat{N} - \frac{\hbar\alpha}{2} \hat{N} (\hat{N} - 1)$$

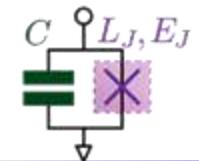
$\hat{H}_{\text{lin}}$  → Solution known (SHO)  
 $\hat{H}_{\text{nl}}$  → New from nonlinearity

$$\hat{H}_4^{\text{RWA}} |n\rangle = \hbar\omega_q n \left( 1 - \frac{\hbar\alpha_a}{2} (n - 1) \right) |n\rangle$$



1<sup>st</sup> order correction to energy:  $E_n^{(1)} = \langle n^{(0)} | \hat{H}_{\text{nl}} | n^{(0)} \rangle$  and to eigenstates:  $|n^{(1)}\rangle = \sum_{k \neq n} \frac{\langle k^{(0)} | \hat{H}_{\text{nl}} | n^{(0)} \rangle}{E_n^{(0)} - E_k^{(0)}} |k^{(0)}\rangle$ .

To first order perturbation theory the eigenstates do not change! Only the energy changes. Dispersive.

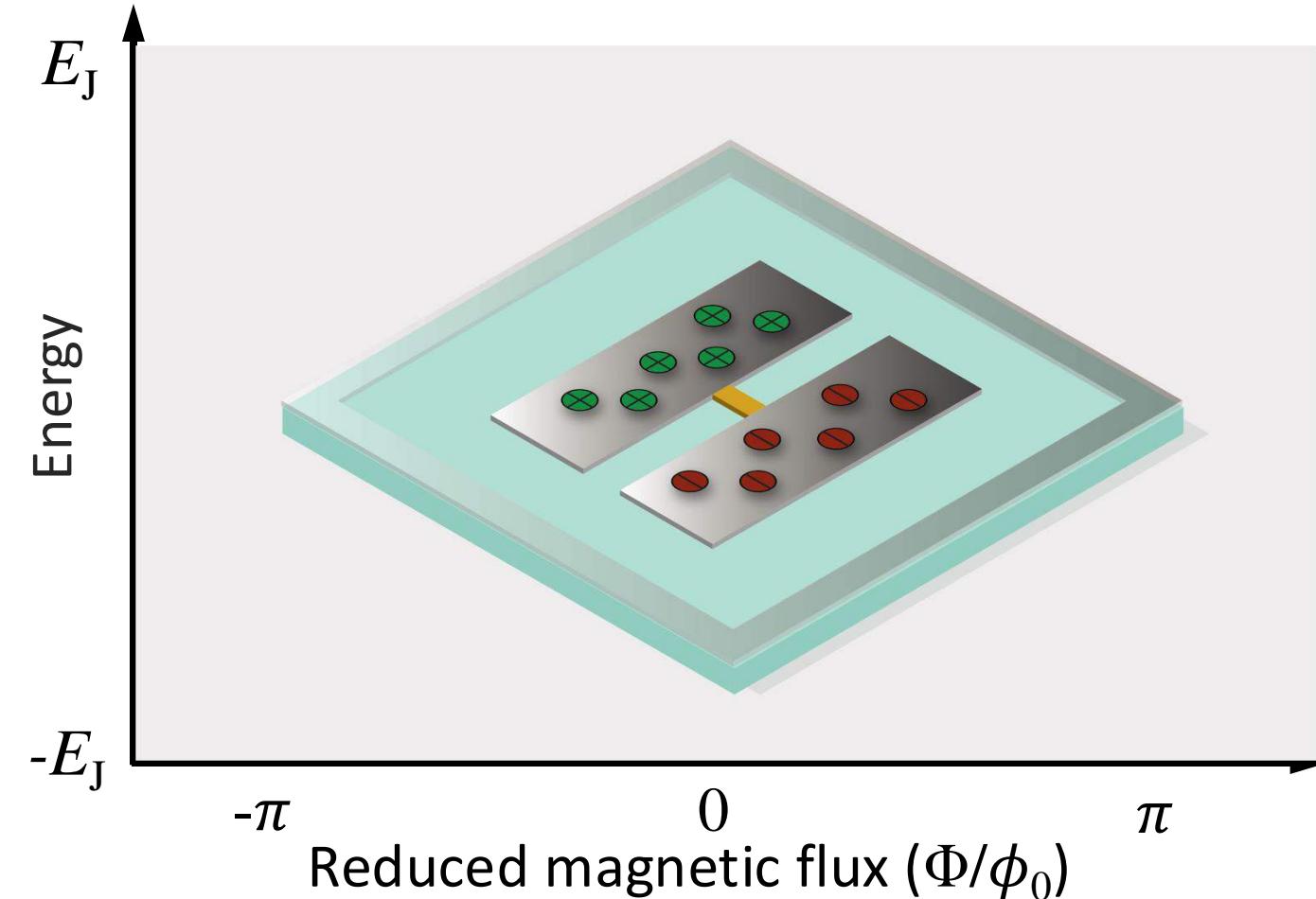


# Exploring a real transmon qubit

$$\hat{N} \equiv \hat{a}^\dagger \hat{a}$$

$$\hat{H}_4^{\text{RWA}} \approx \hbar\omega_q \hat{N} - \frac{\hbar\alpha}{2} \hat{N} (\hat{N} - 1)$$

$\hat{H}_{\text{lin}}$  Solution known (SHO)       $\hat{H}_{\text{nl}}$  New from nonlinearity



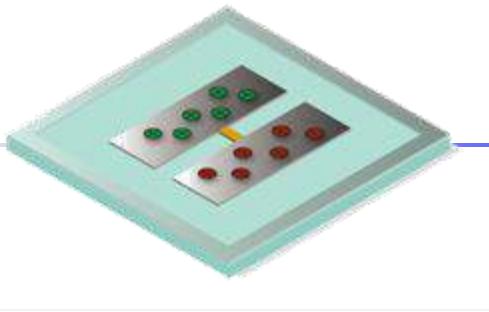
## Experimental parameters

$$L_J = 14 \text{ nH}$$

$$E_J = \frac{\phi_0^2}{L_J} = 12 \text{ GHz}$$

$$C_J = 65 \text{ fF}$$

$$E_C = \frac{e^2}{2C} = 0.3 \text{ GHz}$$



# Exploring a real transmon qubit

$$\hat{H}_4^{\text{RWA}} \approx \hbar\omega_q \hat{N} - \frac{\hbar\alpha}{2} \hat{N} (\hat{N} - 1)$$

Dispersive, states didn't change to 1<sup>st</sup> order  
“Lamb shift” due to ZPF

Parameters used in figure (of a measured qubit)

$$L_J = 14 \text{ nH}$$

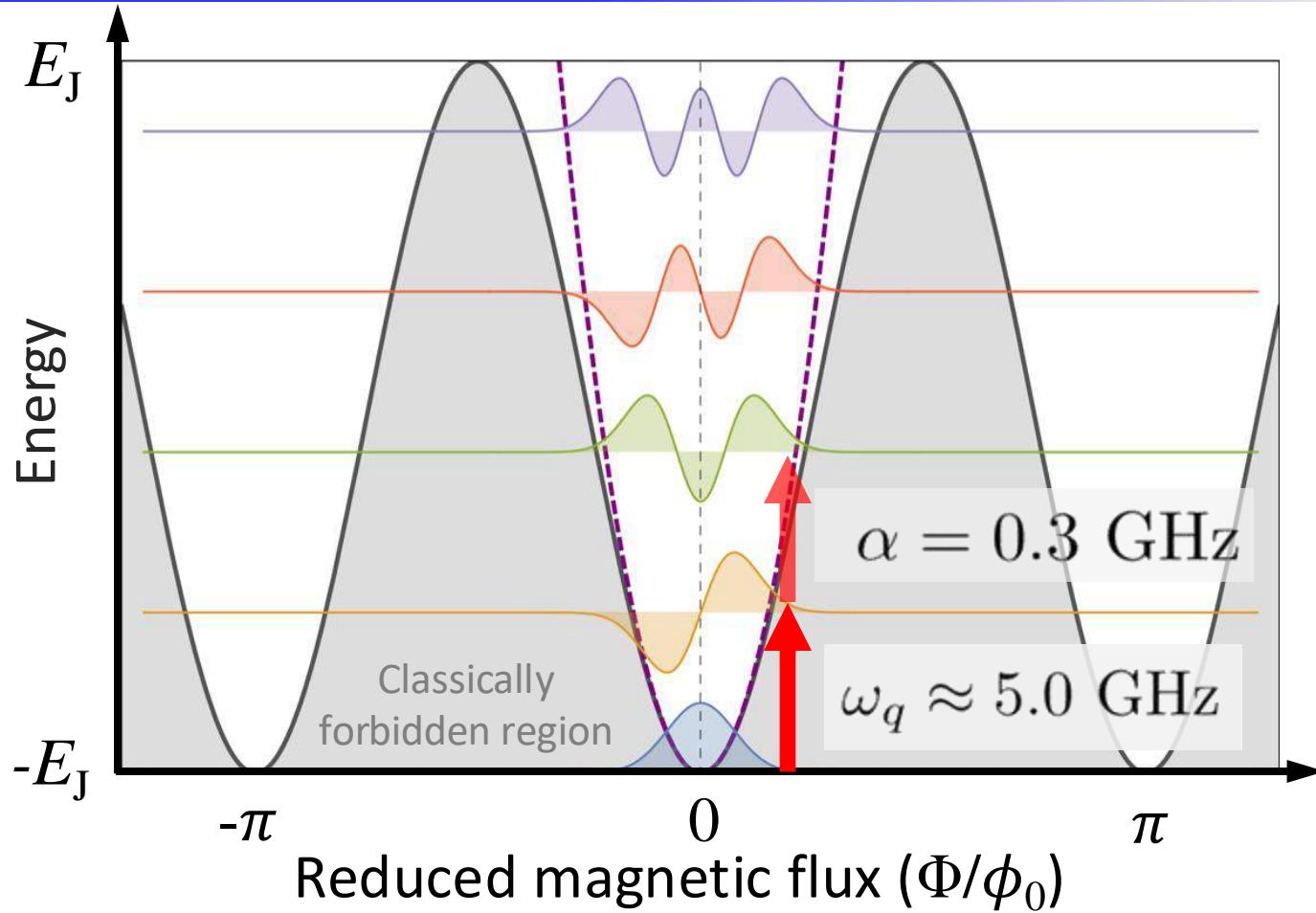
$$E_J = \frac{\phi_0^2}{L_J} = 12 \text{ GHz}$$

$$C_J = 65 \text{ fF}$$

$$E_C = \frac{e^2}{2C} = 0.3 \text{ GHz}$$

$$\omega_0 = \sqrt{\frac{1}{LC}} = 5.3 \text{ GHz}$$

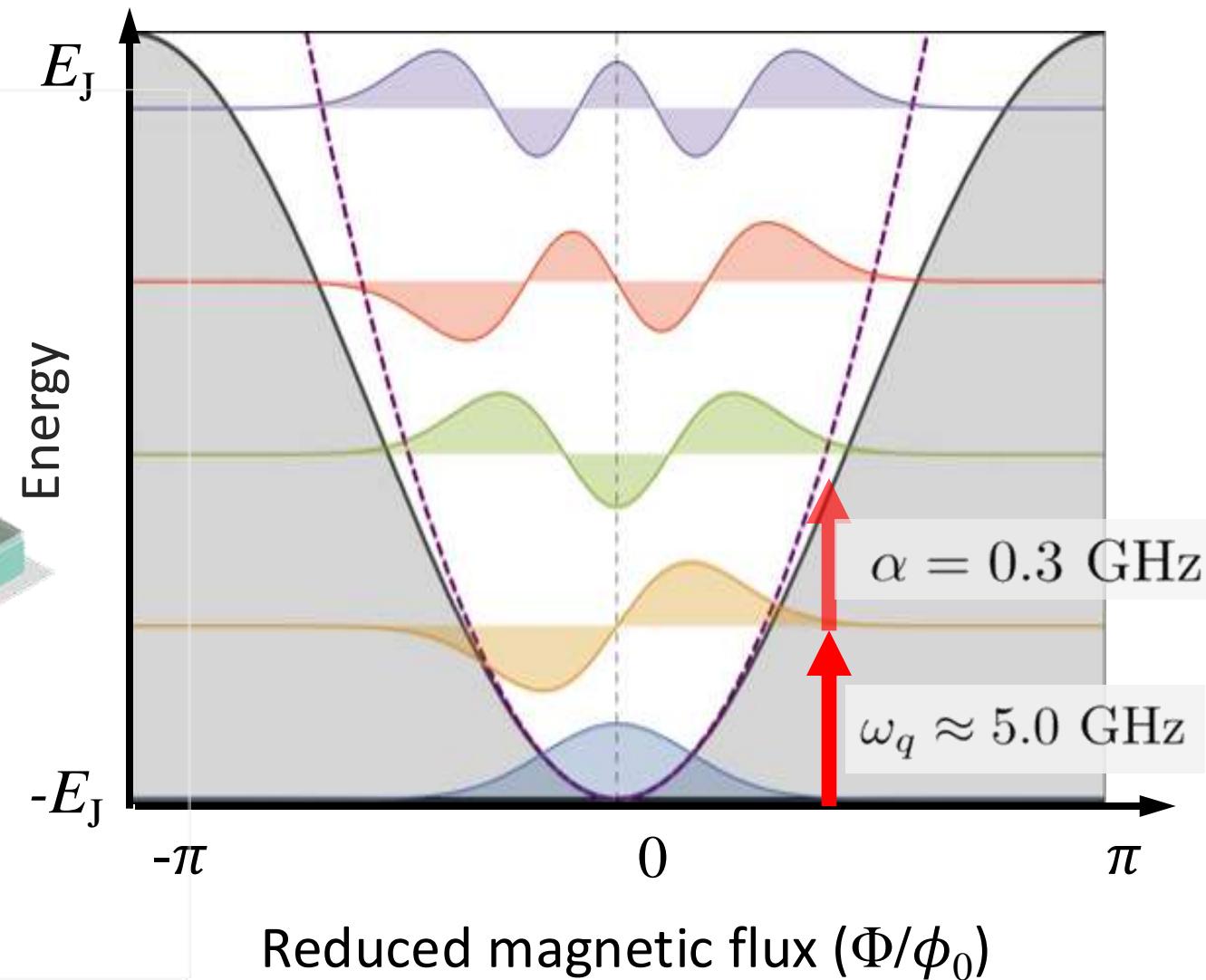
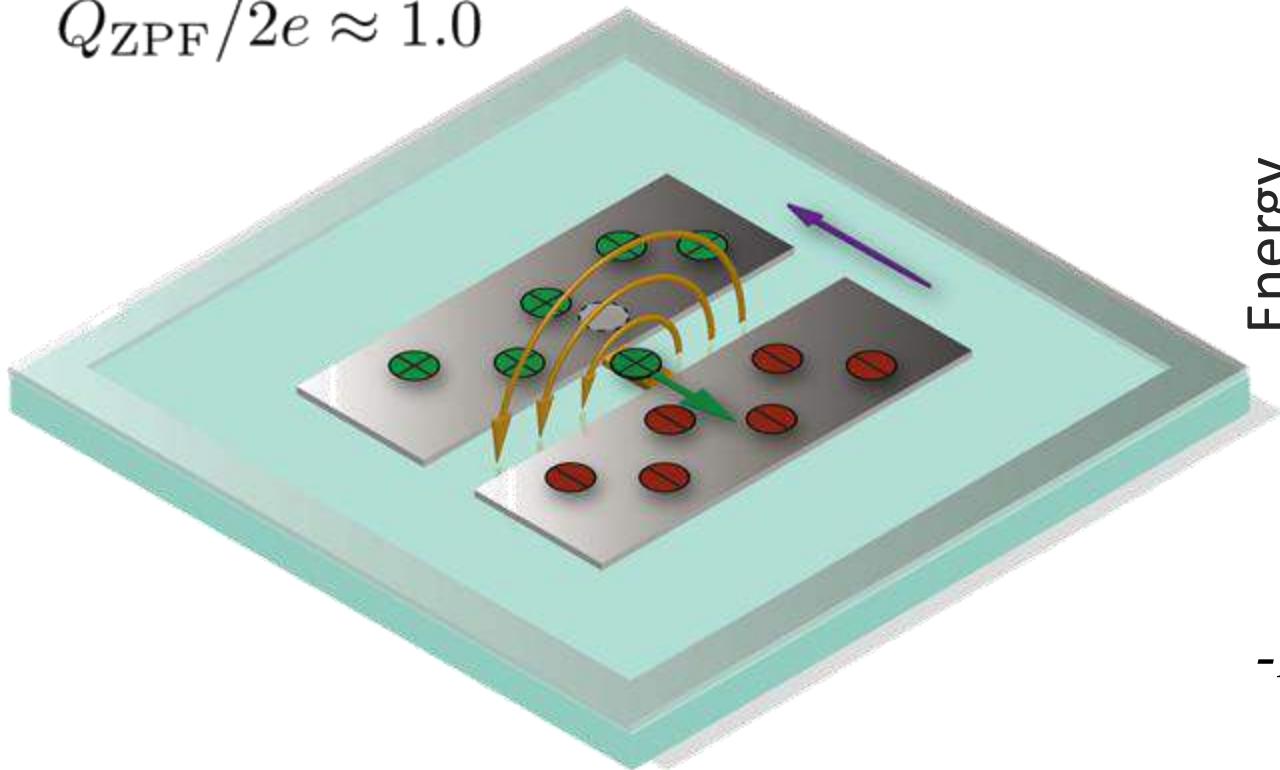
$$Z = \sqrt{\frac{L}{C}} \approx 450 \Omega$$



# Quantum fluctuations of the transmon qubit

$$\Phi_{\text{ZPF}}/\phi_0 \approx 0.5$$

$$Q_{\text{ZPF}}/2e \approx 1.0$$

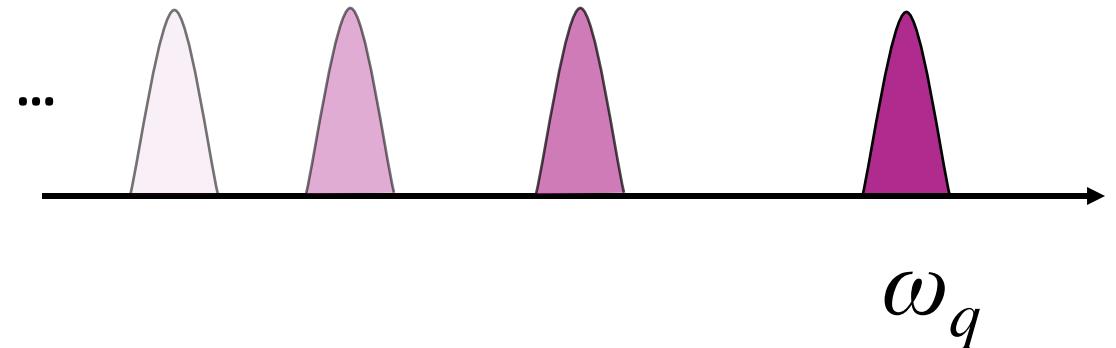


# Energy diagram and transition spectrum

Energy levels

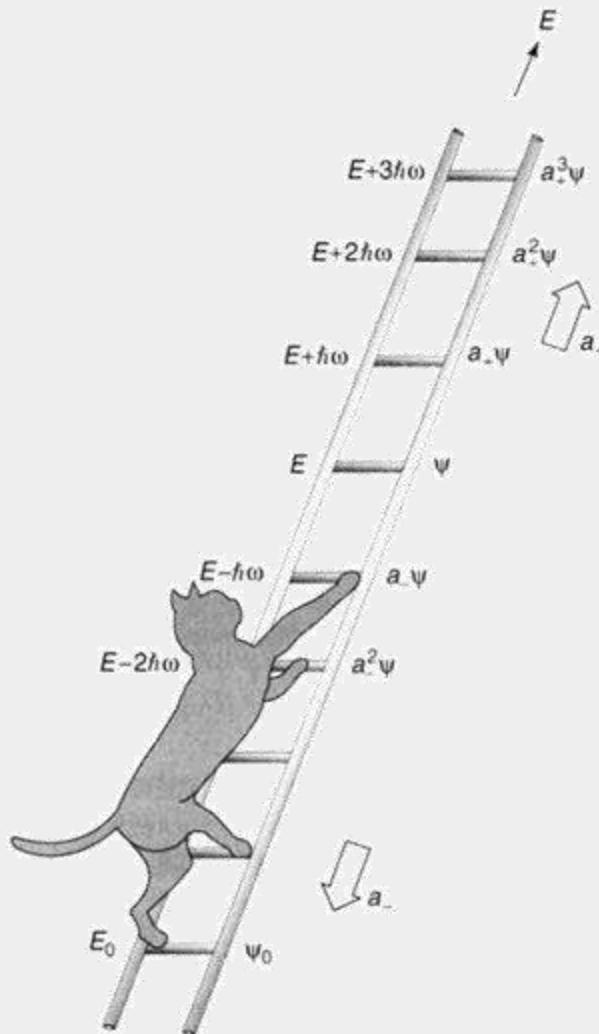
$$\hat{H} \approx \omega_0 \hat{a}^\dagger \hat{a} - \frac{\alpha}{2} \hat{a}^{\dagger 2} \hat{a}^2$$

Transition spectrum



$\omega_q$

# Ladder operators and matrix representation



annihilation

$$\hat{a} |0\rangle = 0$$

$$\hat{a} |1\rangle = \sqrt{1} |0\rangle$$

creation

$$\hat{a}^\dagger |0\rangle = \sqrt{1} |1\rangle$$

$$\hat{a}^\dagger |1\rangle = \sqrt{2} |2\rangle$$

general hopping

$$\hat{a} |n\rangle = \sqrt{n} |n-1\rangle$$

$$\hat{a}^\dagger |n\rangle = \sqrt{n+1} |n+1\rangle$$

# The Transmon qubit: restricting Hilbert space

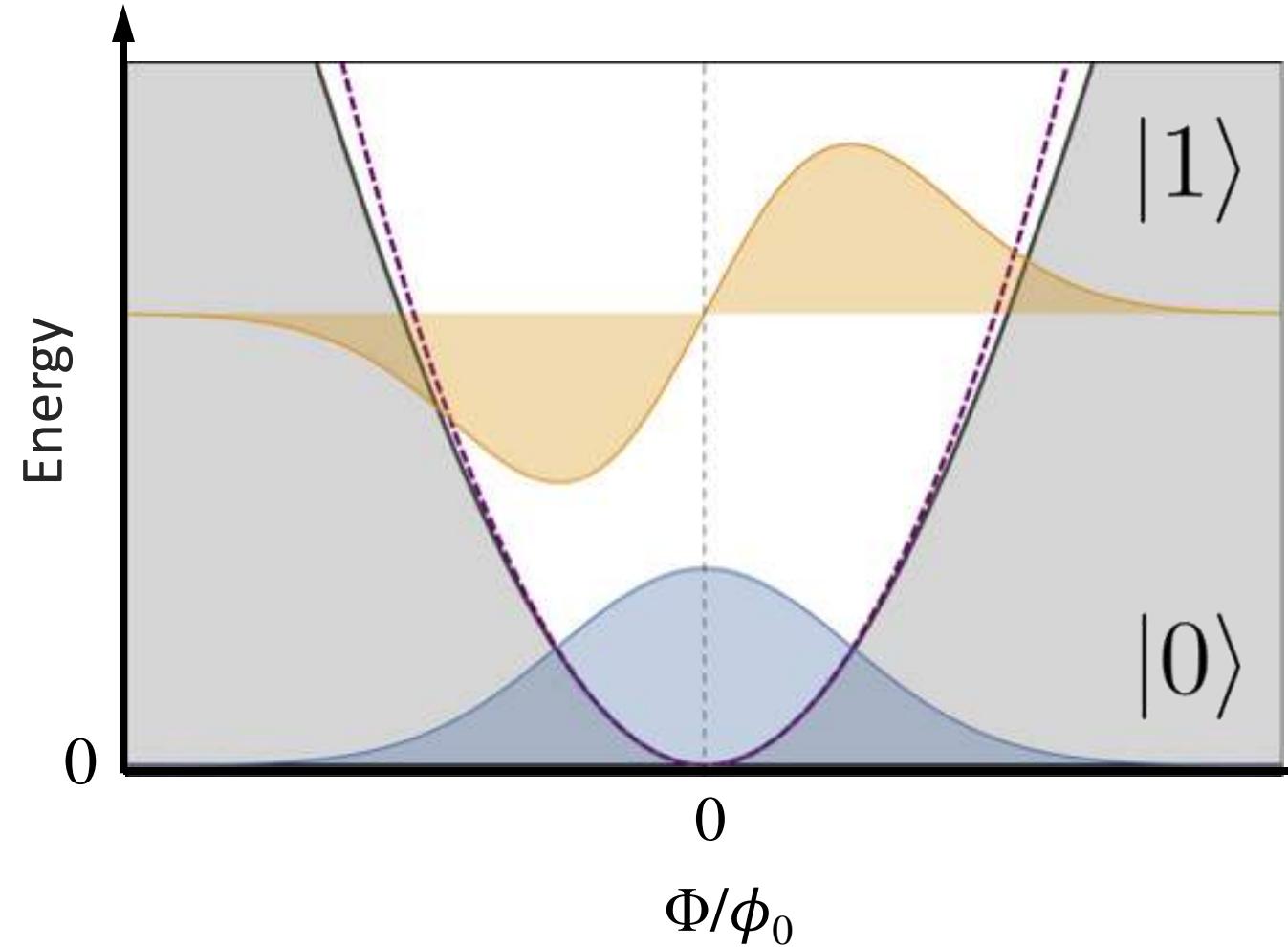
$$\hat{H}_4^{\text{RWA}} \approx \hbar\omega_q \hat{N} - \frac{\hbar\alpha}{2} \hat{N} (\hat{N} - 1)$$

$$\hat{N} \equiv \hat{a}^\dagger \hat{a}$$

Restrict to qubit subspace of  $|0\rangle$  and  $|1\rangle$

$$\hat{a}^\dagger \hat{a} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & \ddots \end{pmatrix}$$

$$\hat{a} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \sqrt{2} & 0 \\ 0 & 0 & 0 & \sqrt{3} \\ 0 & 0 & 0 & \ddots \end{pmatrix}$$



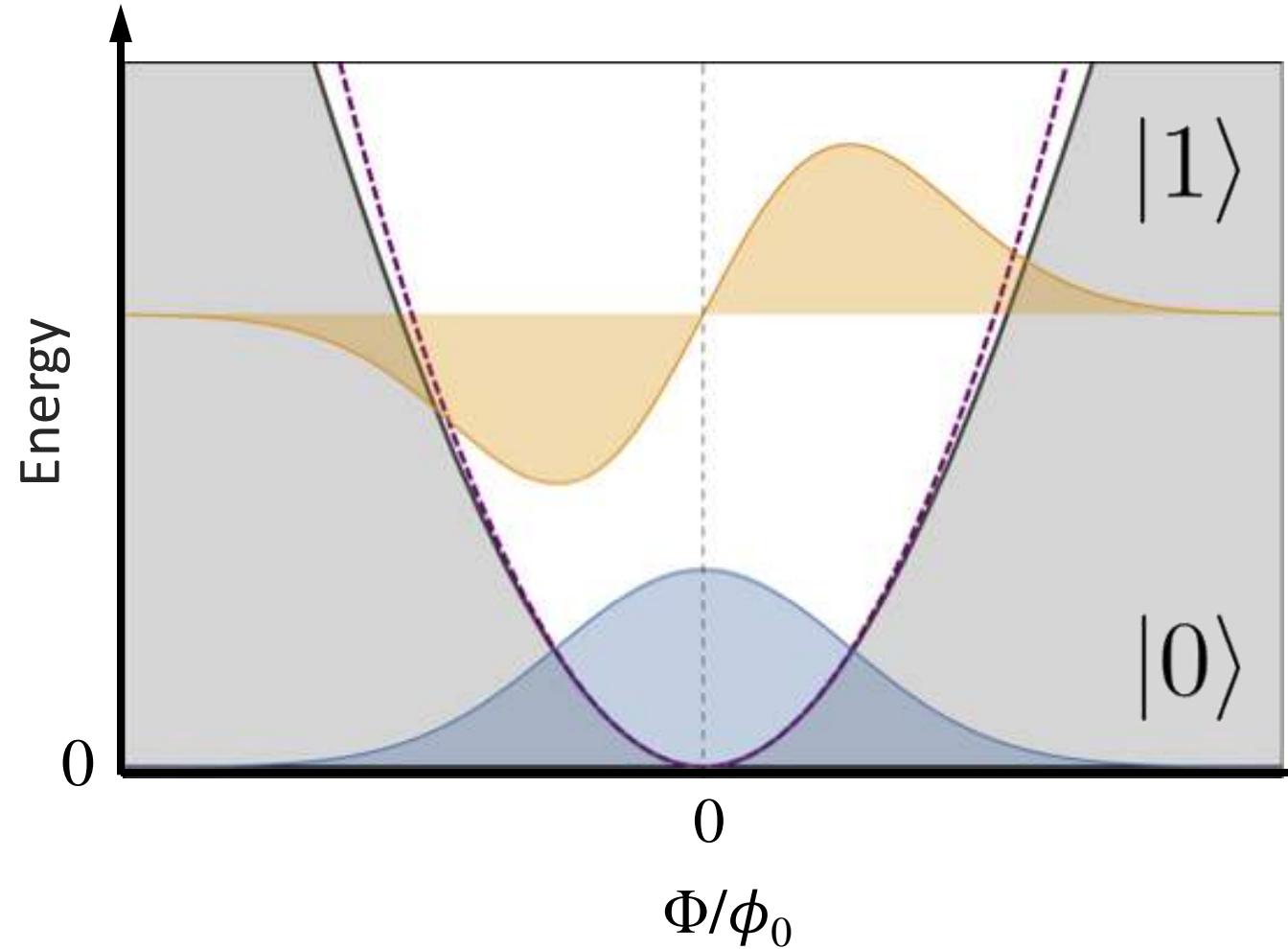
# The Transmon qubit: restricting Hilbert space

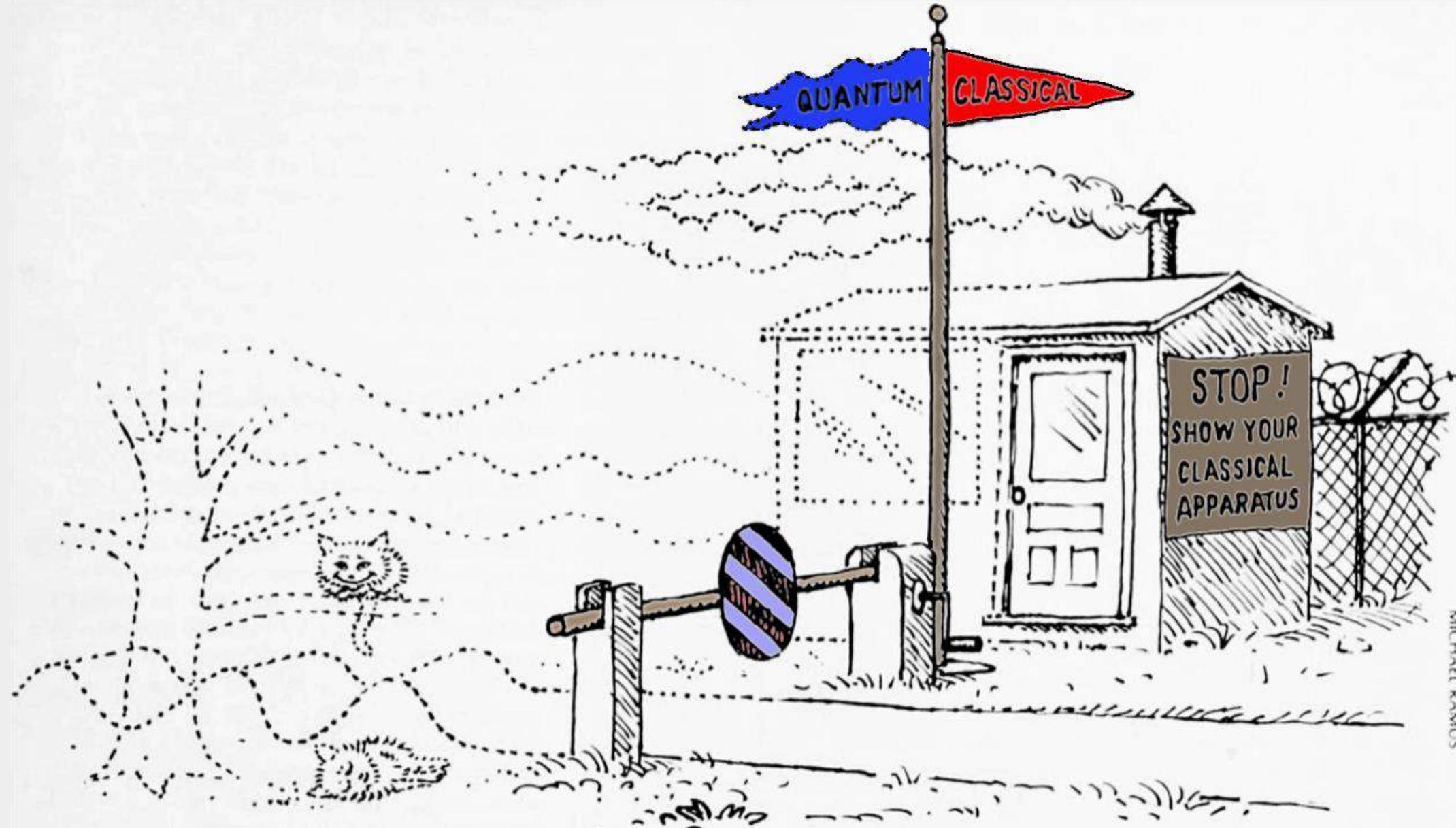
$$\hat{H}_4^{\text{RWA}} \approx \hbar\omega_q \hat{N} - \frac{\hbar\alpha}{2} \hat{N} (\hat{N} - 1)$$

$$\hat{N} \equiv \hat{a}^\dagger \hat{a}$$

Restrict to qubit subspace of  $|0\rangle$  and  $|1\rangle$

$$\begin{array}{c} \left( \hat{N} - \frac{1}{2} \hat{I} \right) \mapsto -\frac{1}{2} \hat{Z} \quad \hat{a} \mapsto \hat{\sigma}_- = \frac{1}{2} \left( \hat{X} - i\hat{Y} \right) \\ \uparrow \qquad \qquad \qquad \uparrow \qquad \qquad \qquad \uparrow \qquad \qquad \qquad \uparrow \\ \text{Fock number} \quad \text{Qubit Pauli Z} \quad \text{Qubit Pauli X and Y} \\ \text{operator} \qquad \text{operator} \qquad \text{operators} \\ \begin{pmatrix} 1 & 0 & \dots \\ 0 & 2 & \dots \\ \vdots & \vdots & \ddots \end{pmatrix} \quad \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \end{array}$$



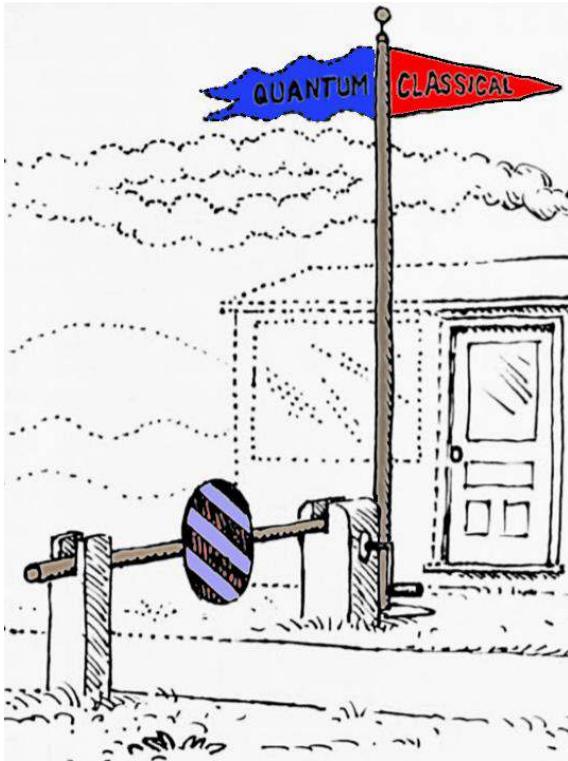


Drawing: Zurek, Physics Today (1991)

# Calculating the energy-participation ratio

$$p_m = \frac{\text{Energy stored in junction}}{\text{Inductive energy stored in mode } m}$$

$$0 \leq p_{mj} \leq 1.$$

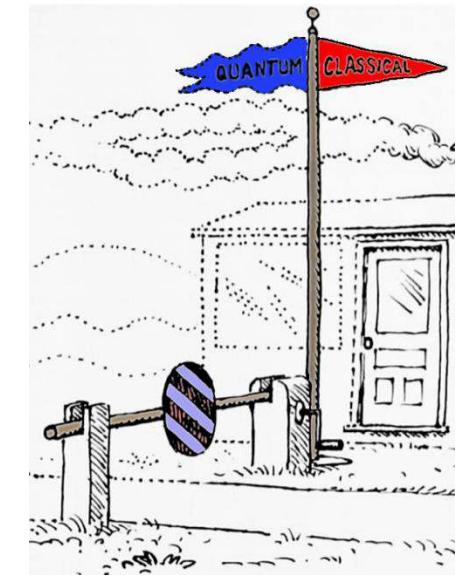


# Energy participation-ratio: a bridge

$\hat{H}_{\text{full}}$

What fraction of the energy of mode  $m$   
is stored in junction  $j$ ?

$$\frac{1}{\hbar} \phi_{mj}^2 = p_{mj} \frac{\omega_m}{2E_j}$$



for  $j > 1$ , root requires sign bit  $s_{mj} = \pm 1$

# All this explicated in

---

Minev dissertation Sec. 4.1 (arXiv: 1902.10355)

In detail, to appear soon in the EPR paper:

## Energy-participation quantization of Josephson circuits

Zlatko K. Minev,<sup>1</sup> Zaki Leghtas,<sup>1,2</sup> Shantanu O. Mundhada,<sup>1</sup>  
Lysander Christakis,<sup>1,3</sup> Ioan M. Pop,<sup>1,4</sup> and Michel H. Devoret<sup>1</sup>

<sup>1</sup>*Department of Applied Physics, Yale University, New Haven, Connecticut 06511, USA*<sup>\*</sup>

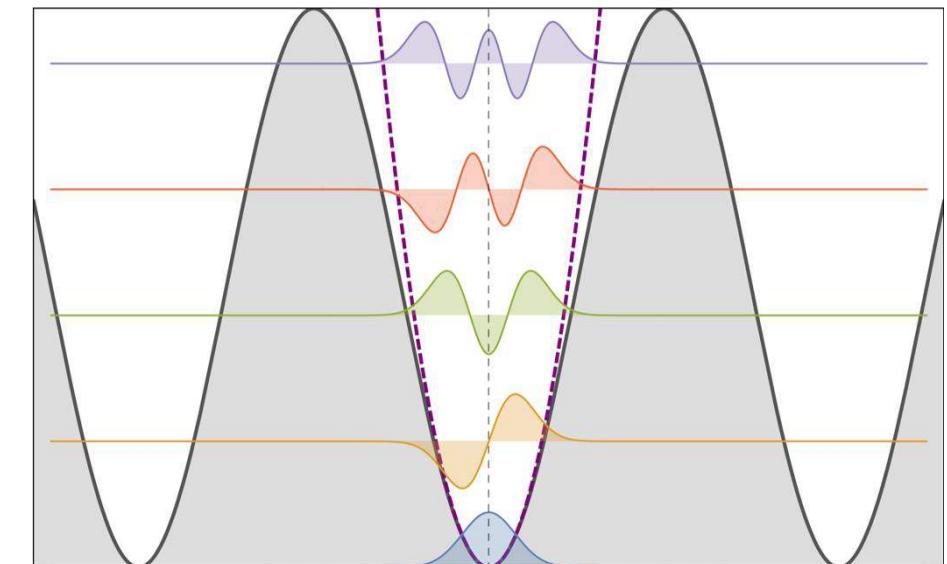
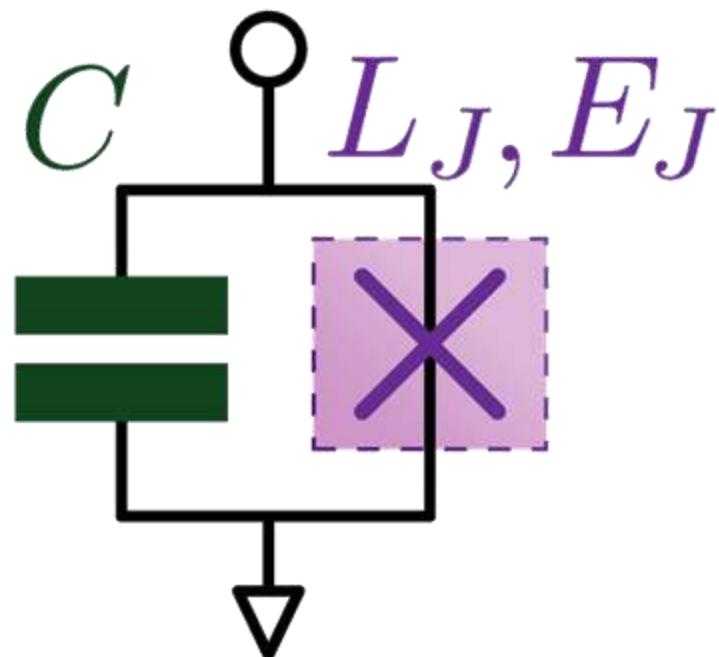
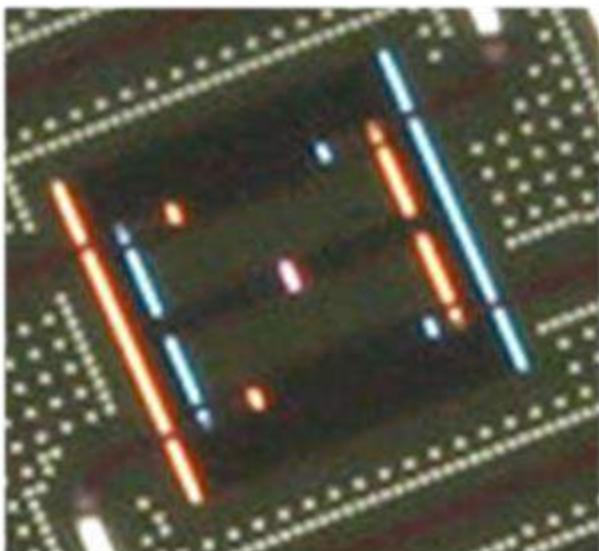
<sup>2</sup>*Automatique et Systèmes, Mines-ParisTech, PSL Research University, 60 Bd. Saint Michel, 75006 Paris, France*

<sup>3</sup>*Department of Physics, Princeton University, Princeton, NJ 08540, USA*

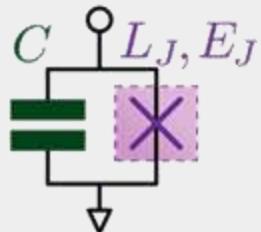
<sup>4</sup>*Physikalisches Institut, Karlsruhe Institute of Technology, 76131 Karlsruhe, Germany*

Superconducting microwave circuits incorporating nonlinear devices, such as Josephson junctions, are an appealing platform for emerging quantum technologies. Further increase of circuit complexity requires efficient numerical methods for the calculation and optimization of the spectrum, nonlinear interactions, and dissipation in multi-mode distributed quantum circuits. Here, we present a method based on a powerful concept—the energy-participation ratio (EPR) of a dissipative or nonlinear element in an electromagnetic mode. The EPR, a number between zero and one, quantifies how much of the energy of a mode is stored in each element. It is calculated from a unique, efficient electromagnetic eigenmode simulation of the linearized system, including lossy elements, and is the key to the modeling of the quantum Hamiltonian of the system. The method provides an intuitive and simple-to-use tool to quantize multi-junction circuits, and is especially well-suited for weakly anharmonic systems, such as transmon qubits coupled to resonators, or Josephson transmission lines. We experimentally tested this method on a variety of Josephson circuits in three-dimensional and flip-chip architectures, and demonstrated agreement within several percents for nonlinear couplings and modal Hamiltonian parameters, spanning five-orders of magnitude in energy, across a dozen samples.

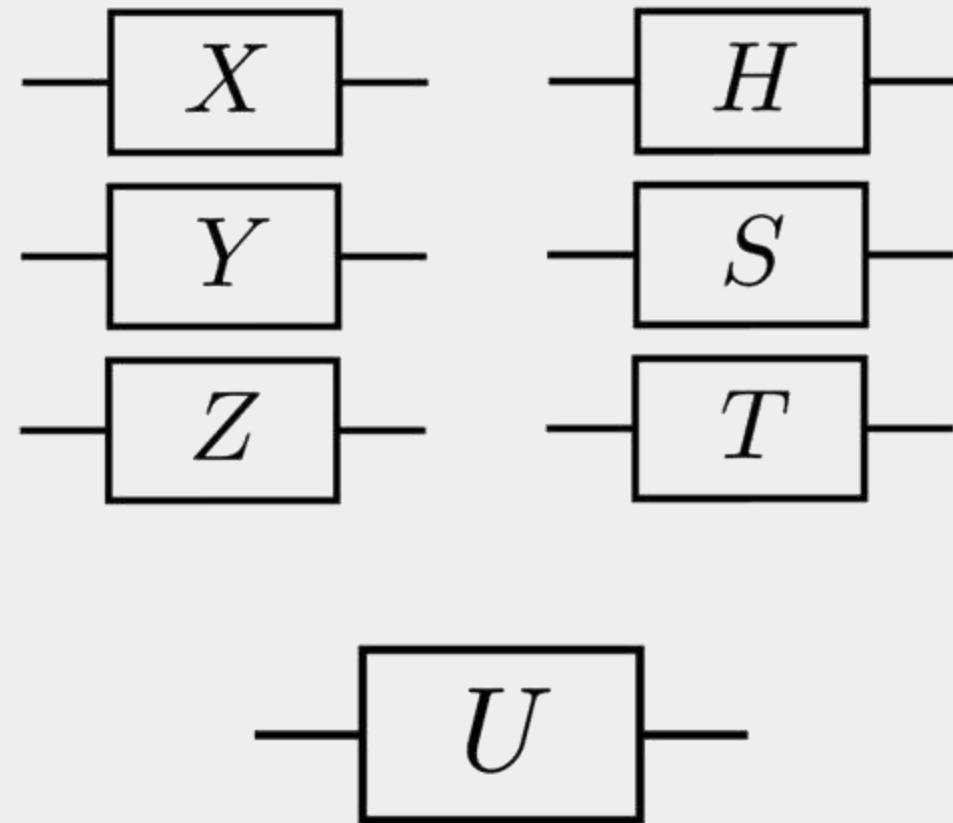
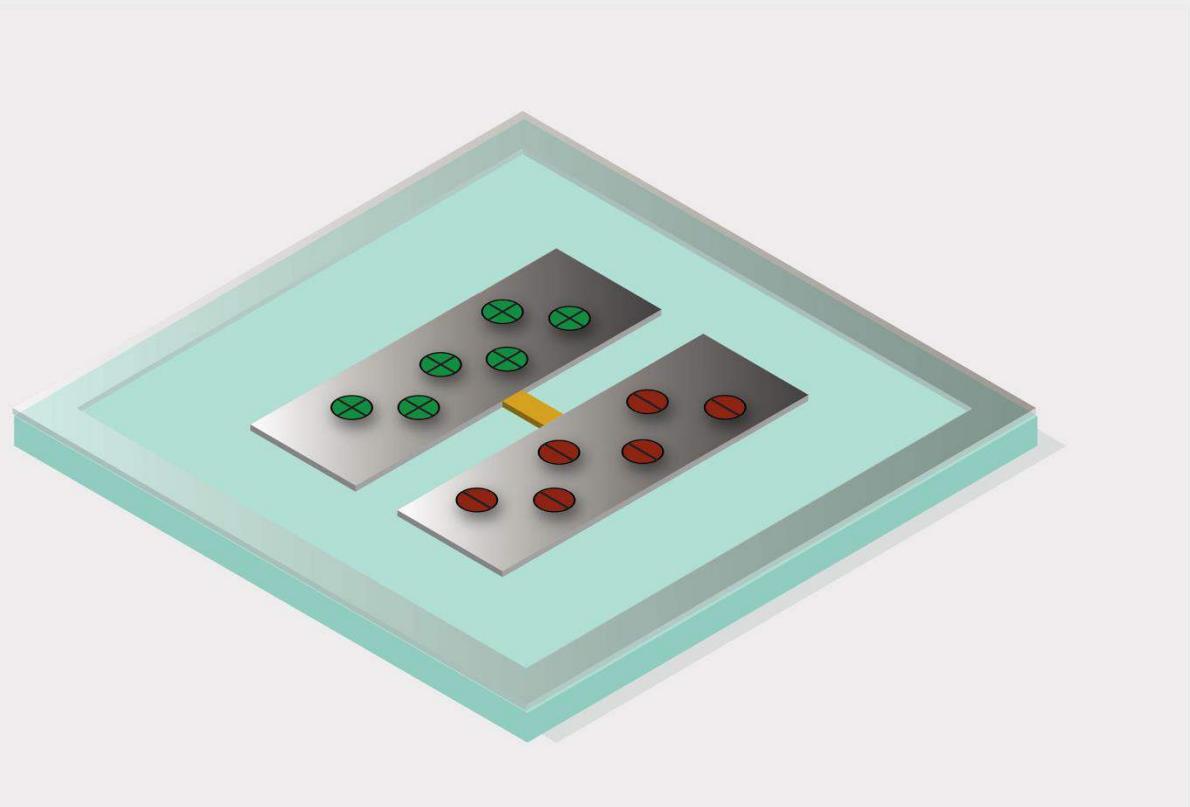
# Transmon Qubit



Control and beyond

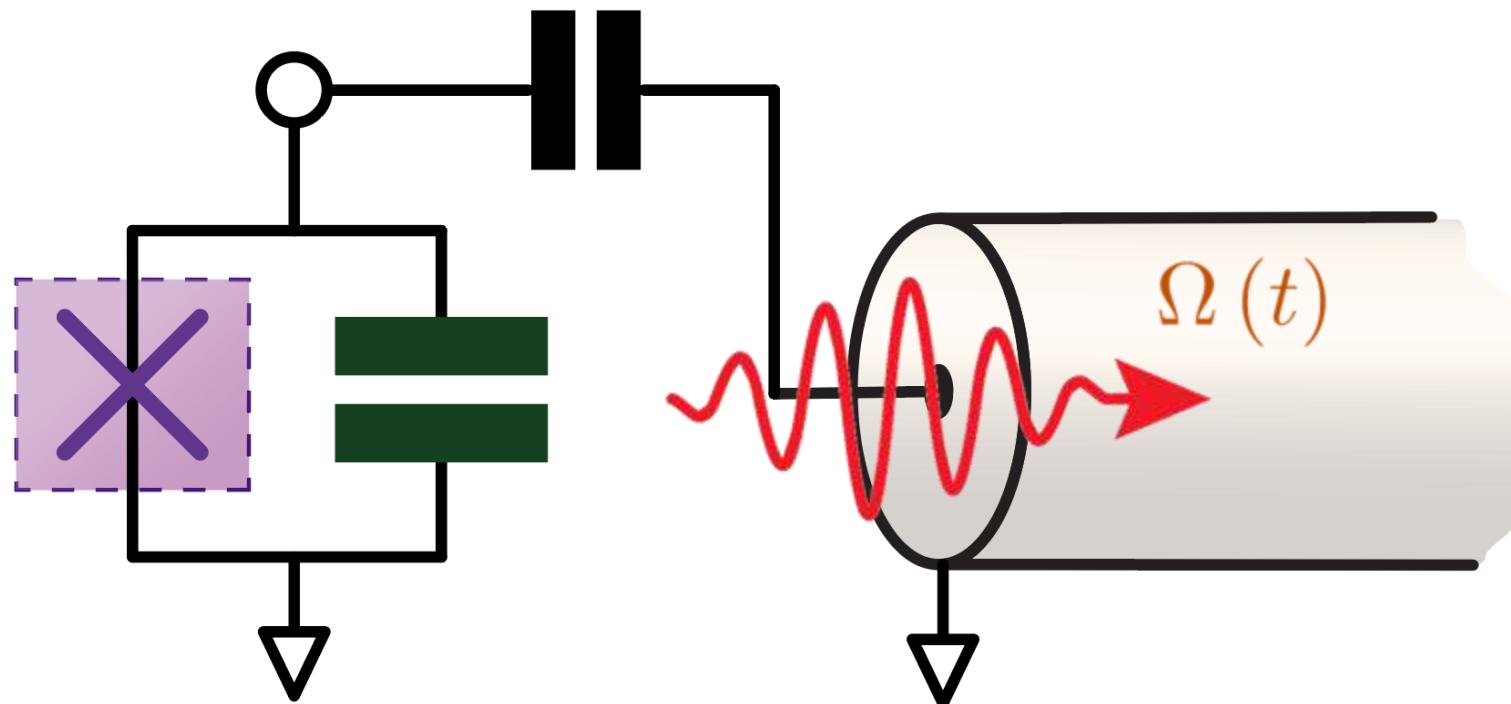


## Single-qubit quantum gates



# Qubit control

$$\hat{H}_{\text{drive}} = -i \frac{1}{2} \Omega(t) (\hat{a}^\dagger - \hat{a}) \quad \mapsto \quad \frac{1}{2} \Omega(t) \hat{Y}$$



input-output line  
(control, shaped signals, but also environment)

# Qubit control: overview

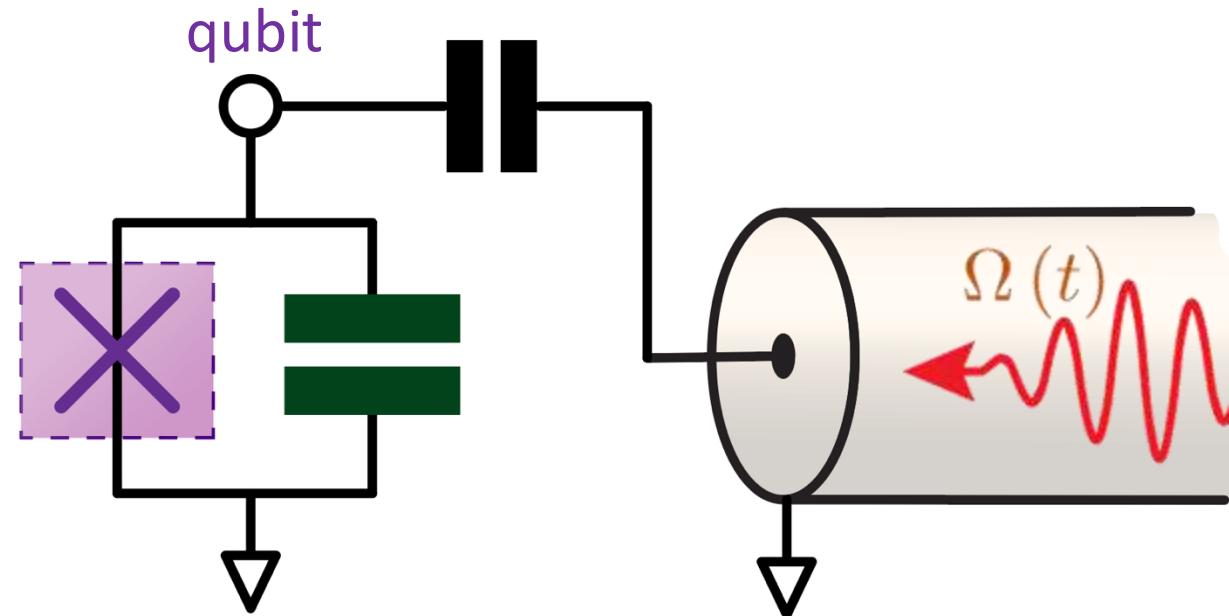
$$\hat{H}_{\text{int}} = -\frac{i}{2}\Omega(t)(\hat{a}^\dagger(t) - \hat{a}^\dagger(t))$$

$$\mapsto -\frac{i}{2}\Omega(t)(\hat{\sigma}^\dagger - \hat{\sigma}^\dagger)$$

$$\Omega(t) = \Omega_0 \sin(\omega_d t + \theta_d)$$

$$\sin(\omega_d t + \theta_d) = \frac{i}{2} (e^{-i(\omega_d t + \theta)} - e^{+i(\omega_d t + \theta)})$$

$$\hat{\sigma} \mapsto \hat{\sigma} e^{-i\omega_d t}$$



# Qubit control: overview

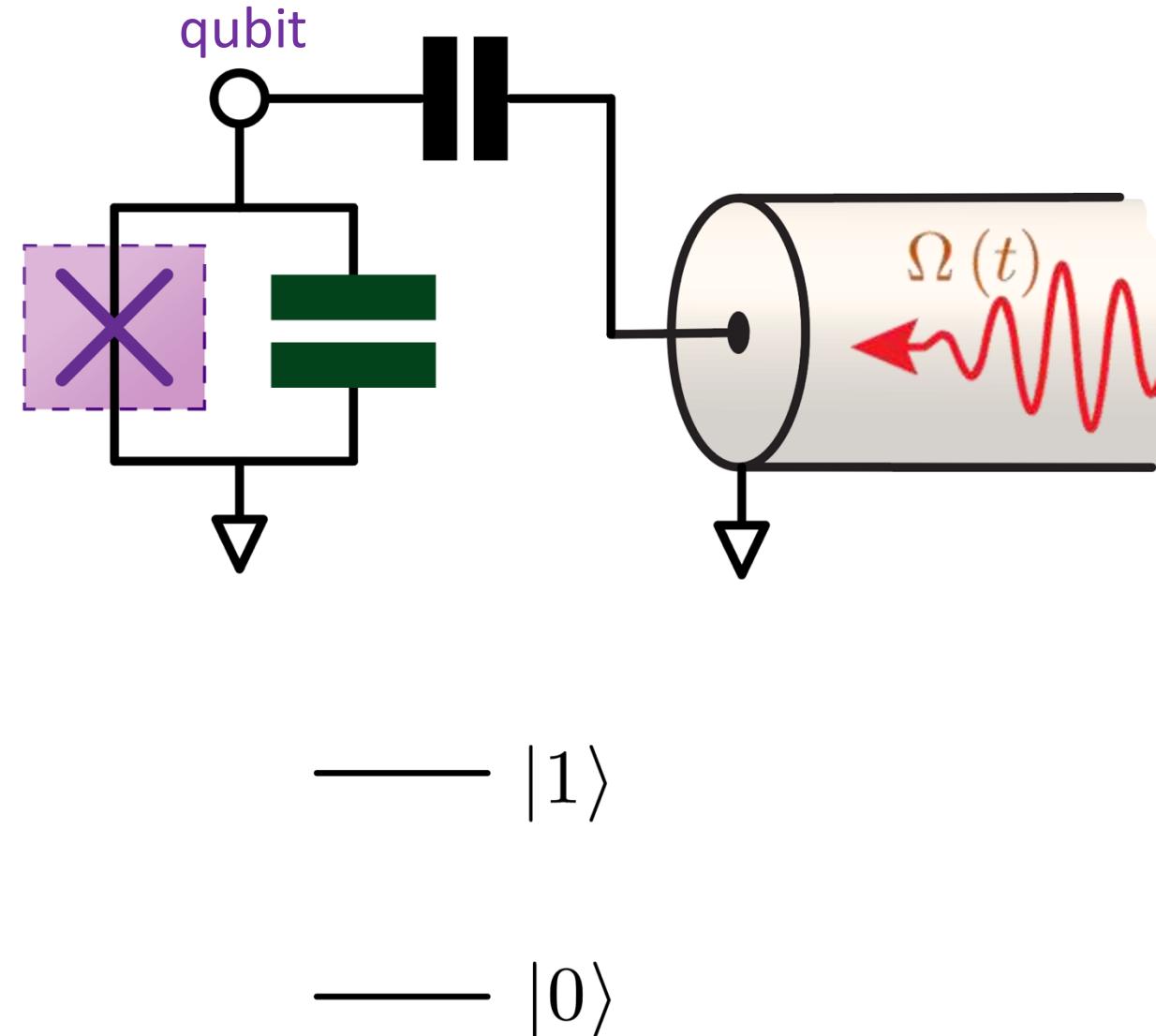
$$\hat{H}_q = -\frac{\hbar\omega_0}{2}\hat{Z}$$

$$\hat{U}(t) = \exp [\hat{u}(t)/i\hbar]$$

$$\hat{u}(t) = +\frac{\hbar\omega_d}{2}\hat{Z}$$

$$\hat{H}_R = U\hat{H}U^\dagger + \frac{d}{dt}\hat{u}(t)$$

$$= \frac{\hbar}{2}\Delta\hat{Z} \quad \Delta := \omega_d - \omega_0$$



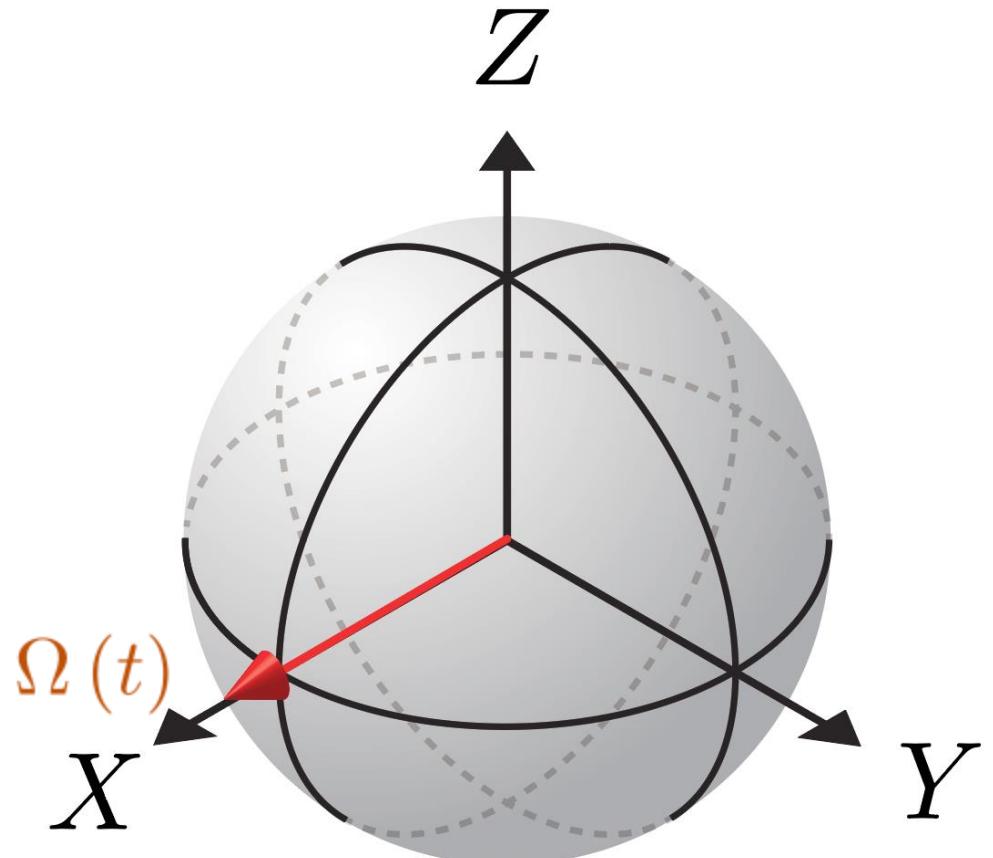
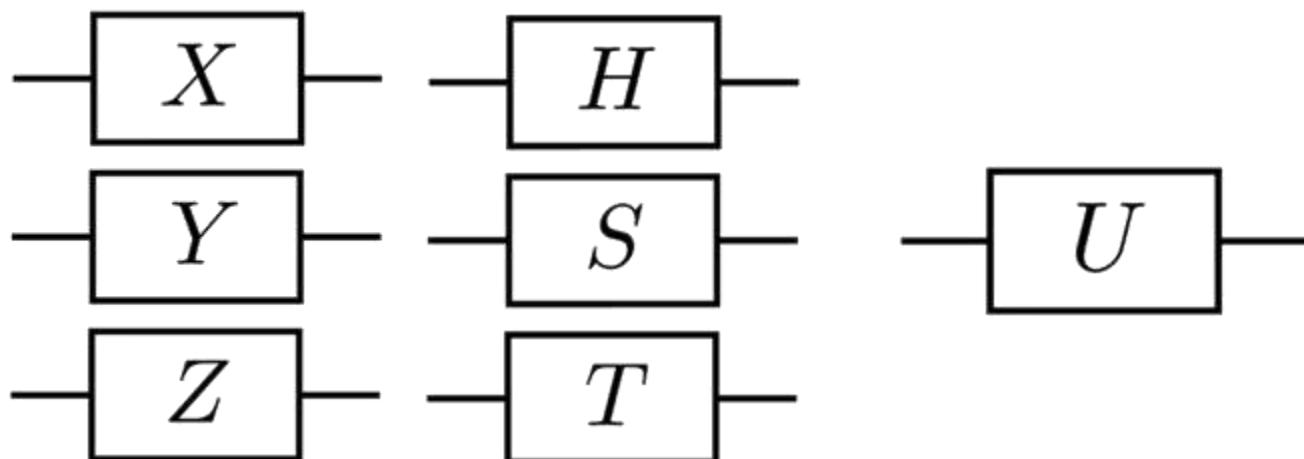


# Qubit control: overview

$$\hat{H}_R = \frac{\hbar}{2} \Delta \hat{Z} + \frac{\hbar}{2} \Omega (e^{-i\theta} \hat{\sigma} + e^{+i\theta} \hat{\sigma}^\dagger)$$

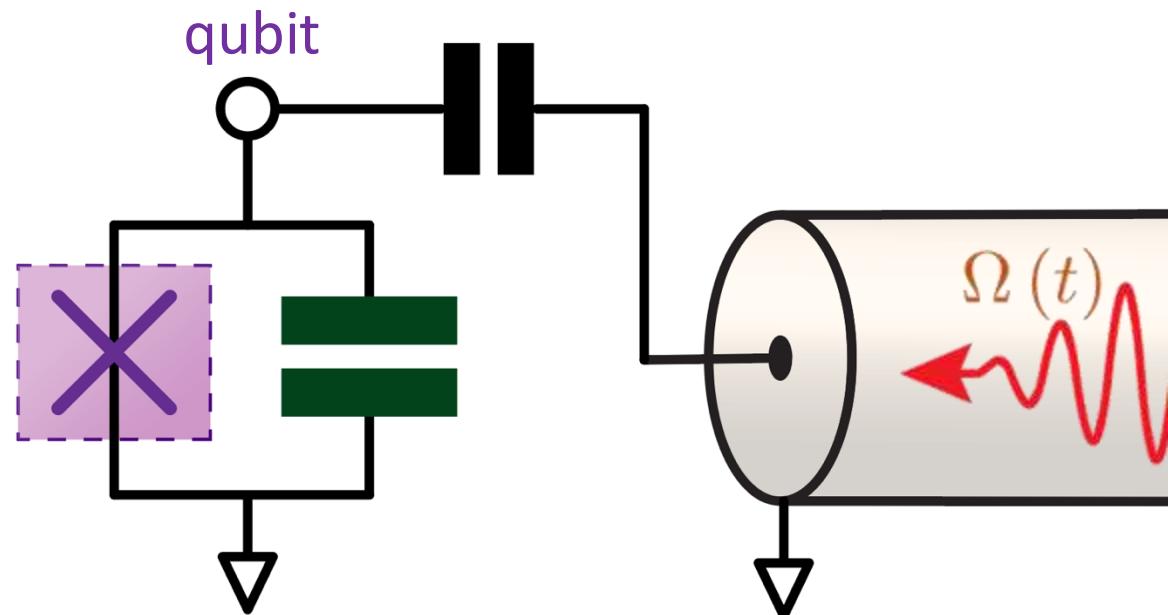
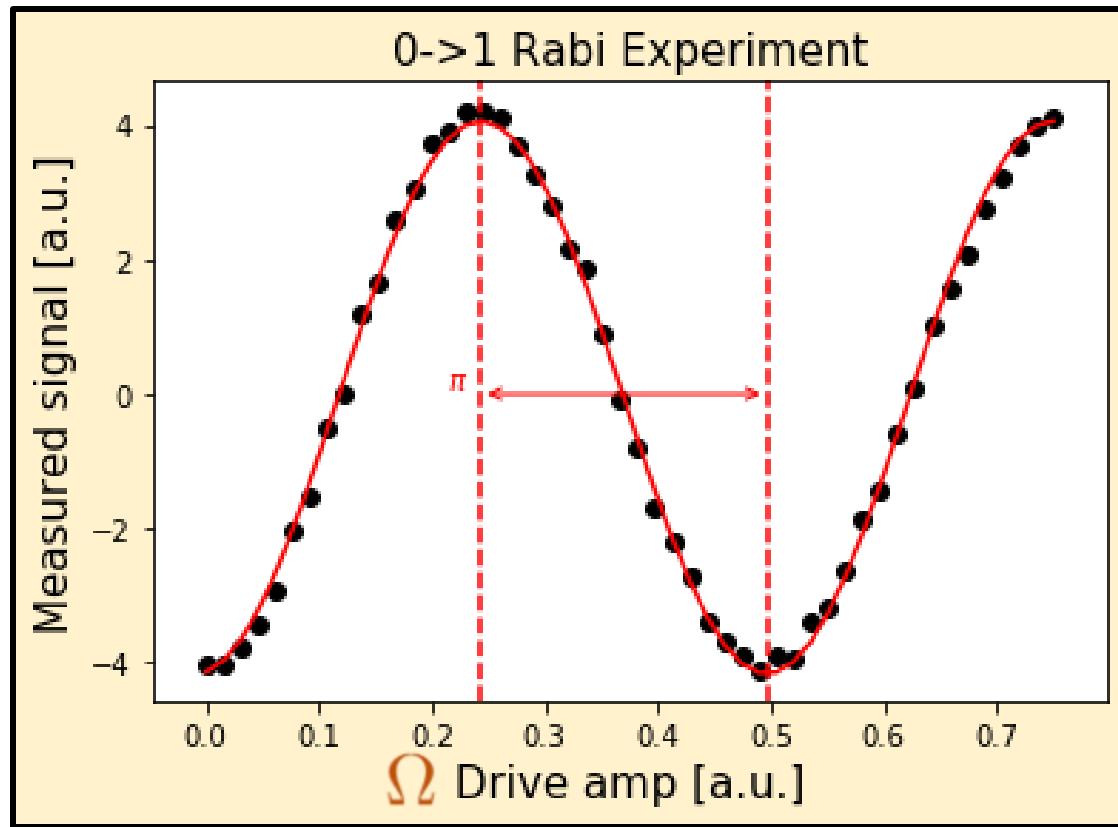
$$\hat{X} = \frac{1}{2} (\hat{\sigma}^\dagger - \hat{\sigma})$$

$$\hat{Y} = i \frac{1}{2} (\hat{\sigma}^\dagger + \hat{\sigma})$$



# Qubit control: Covered in Lab 1 by Nick Bronn & Co.

$$\hat{H}_{\text{drive}} = -i \frac{1}{2} \Omega(t) (\hat{a}^\dagger - \hat{a}) \quad \mapsto \quad \frac{1}{2} \Omega(t) \hat{Y}$$



input-output line  
(control, shaped signals, but also environment)

# Control, noise, and dissipation go hand-in-hand



Can lead to uncontrolled, random bit and phase flip.

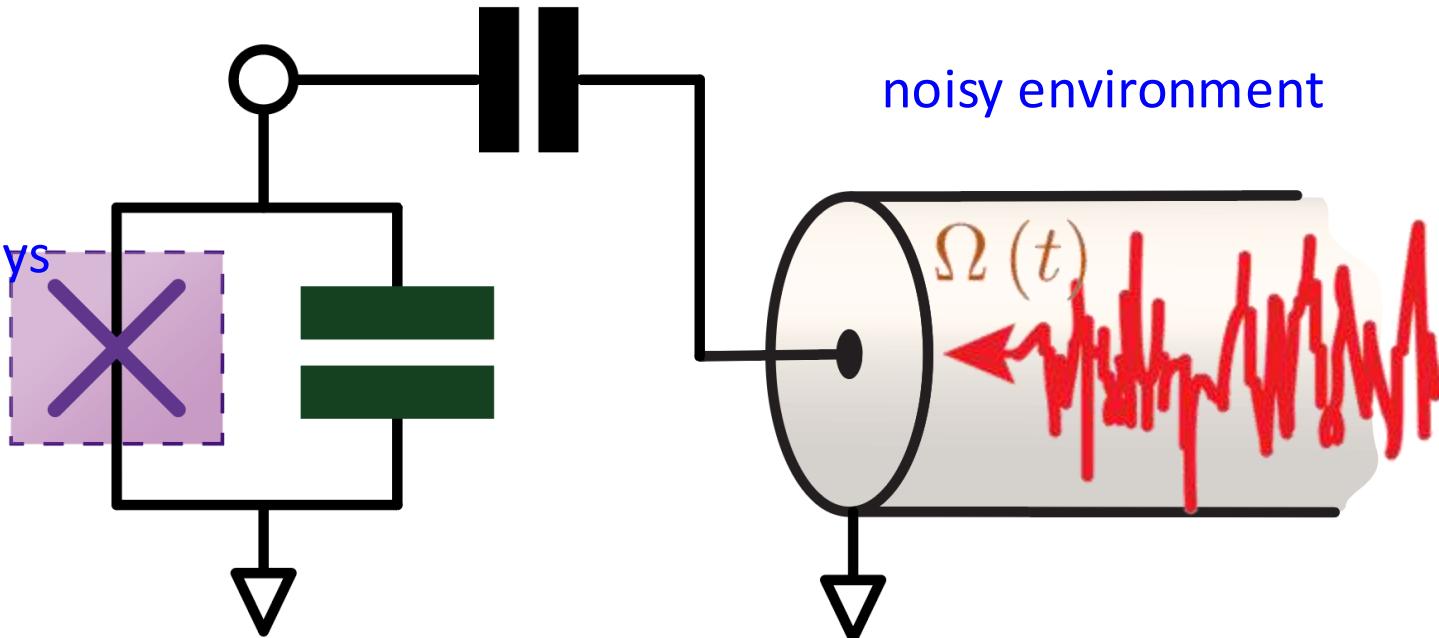
Fluctuation-dissipation theorem\*

susceptibility, noise, and dissipation always go hand-in-hand

Intrinsic and I-O channels, lead to relaxation times:

$T_1$ : energy

$T_2$ : (coherence) transverse



Noisy environment, and  
always zero-point quantum fluctuations

\* This is a major topic in condensed matter physics; we will only touch on it.

# Coherence in superconducting circuits

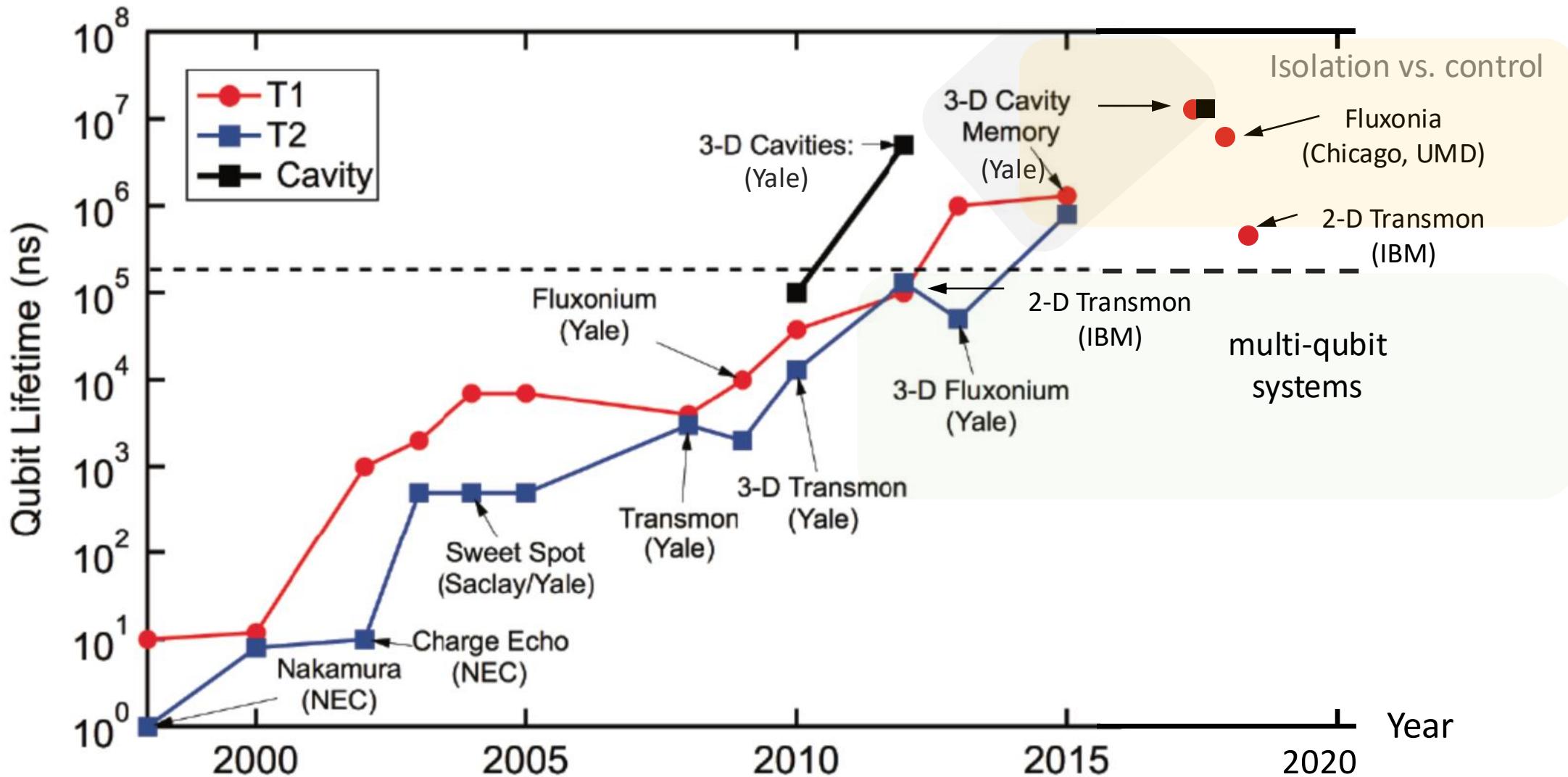
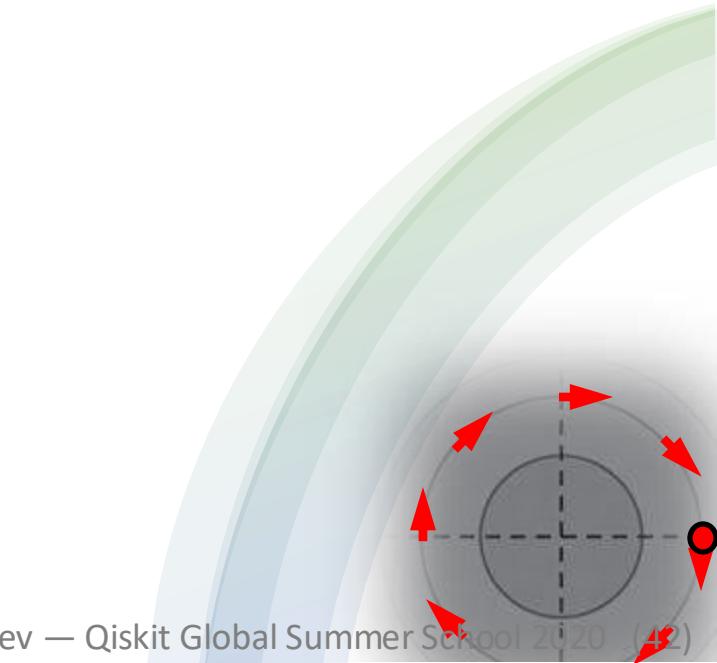
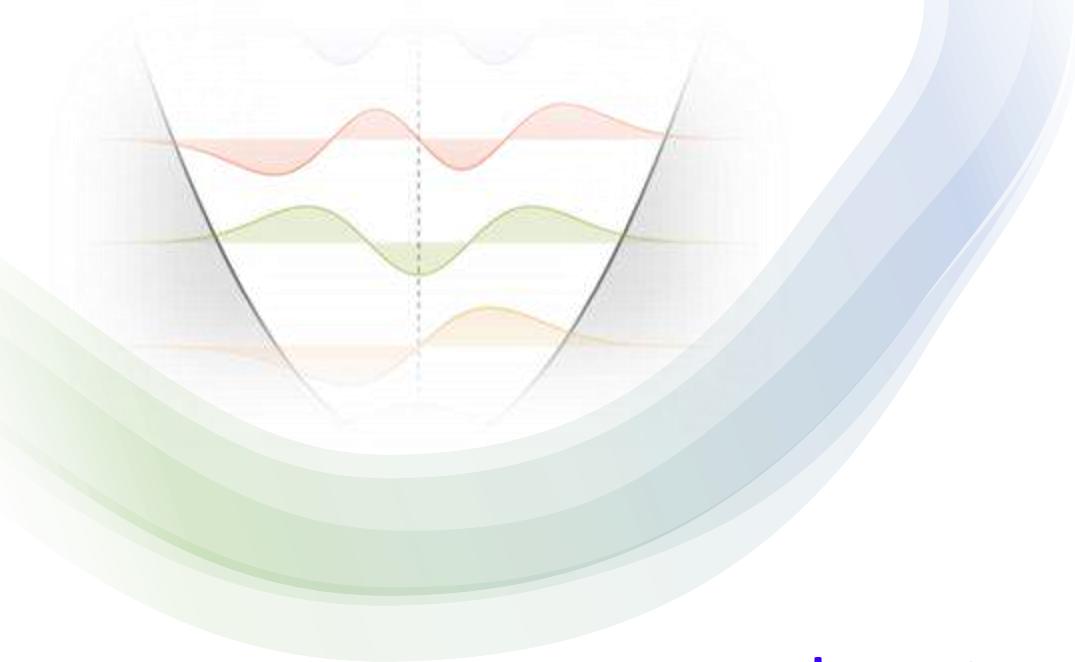


Image reproduced Reagor (2015), an update of Devoret and Schoelkopf (2013), and updated

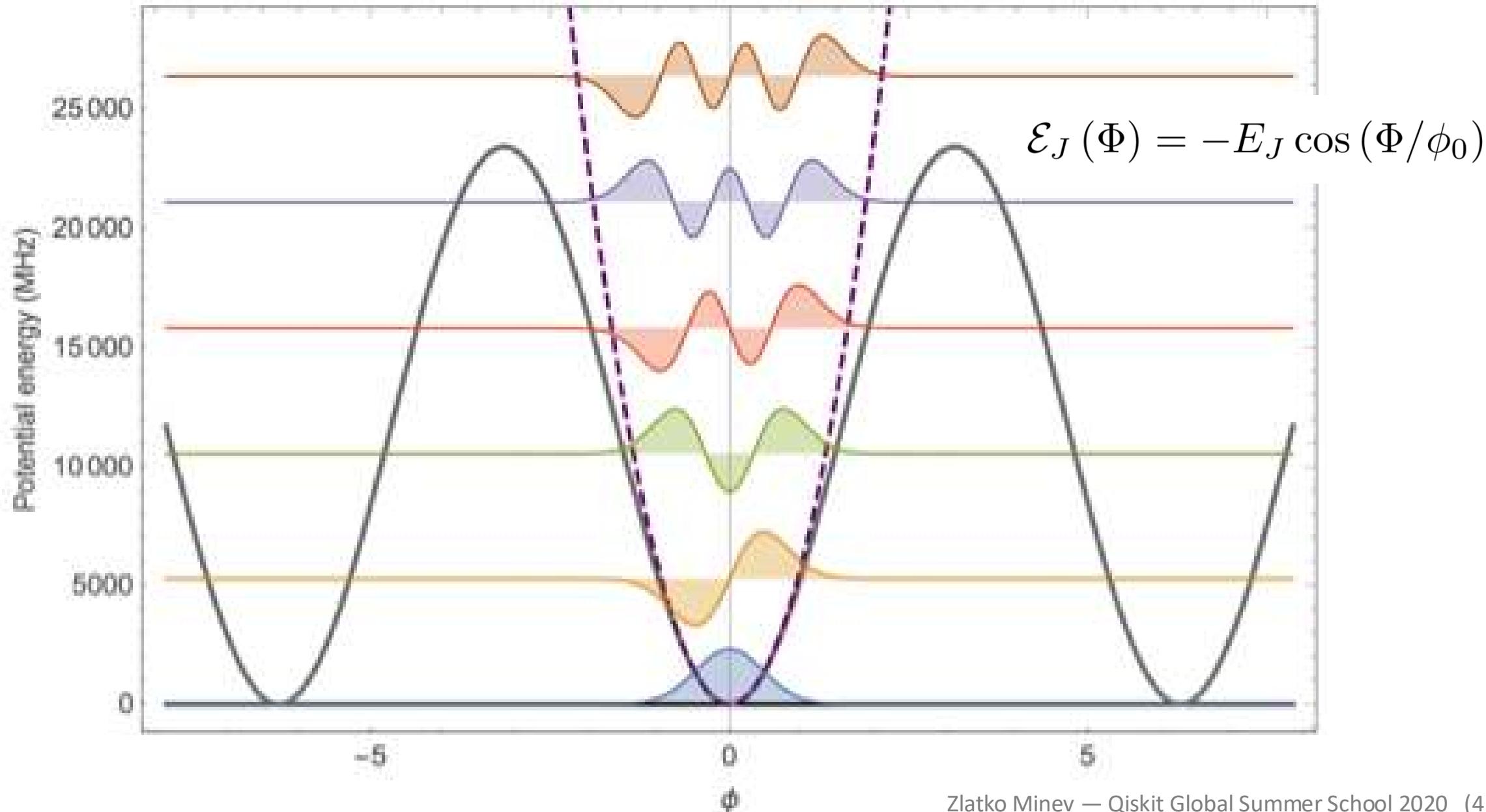


Is the junction phase/flux  
compact or not?

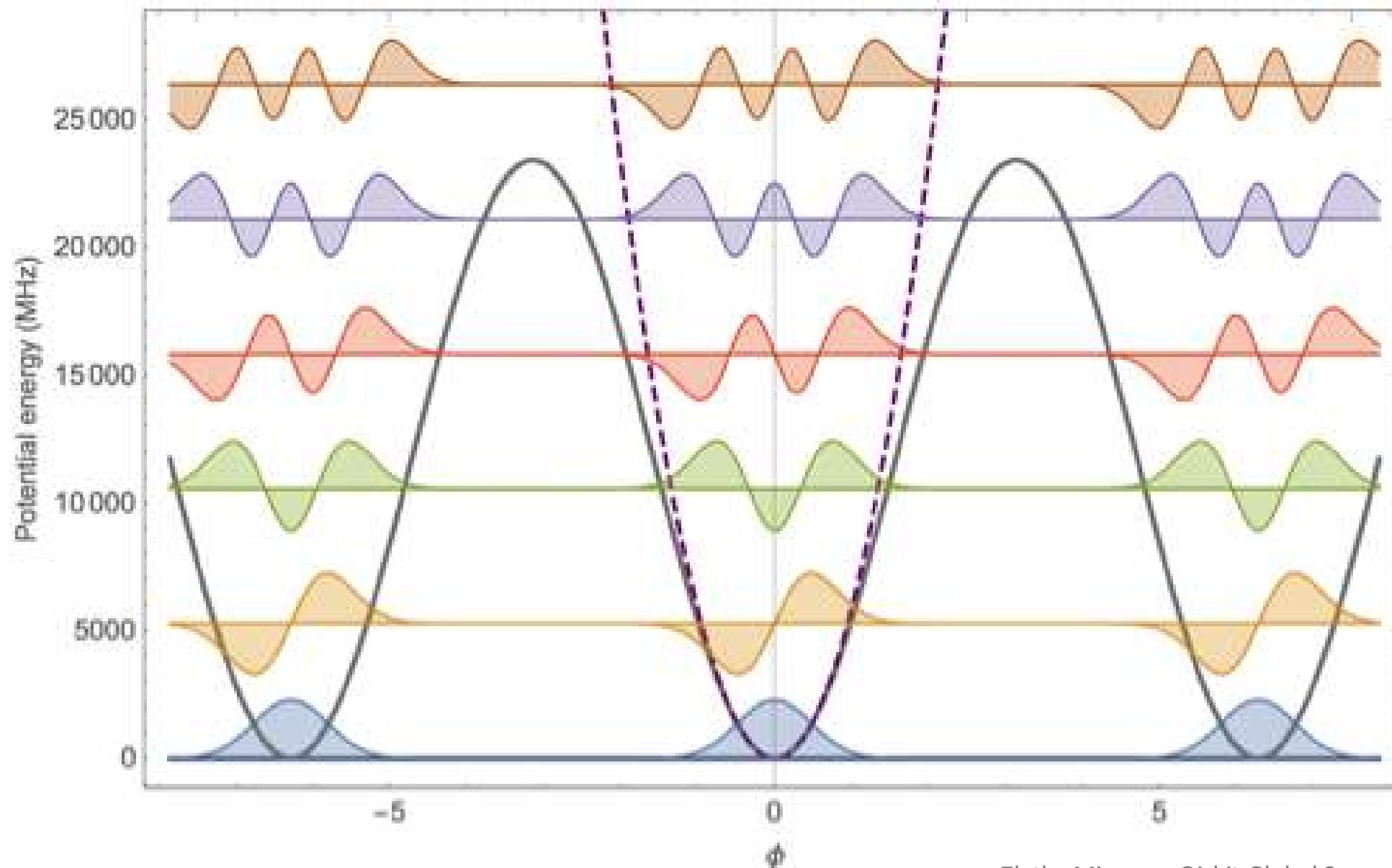




# Harmonic Approximation



# Tight-binding model



# Quantum measurement: *a very brief primer*

# Principal element of sensing: the measurement

THE PROJECT EXISTS  
IN A SIMULTANEOUS  
STATE OF BEING BOTH  
TOTALLY SUCCESSFUL  
AND NOT EVEN  
STARTED.



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# A classical measurement example

Are there fumes in the oil barrel?



Example based on Wiseman and Milburn (2010)

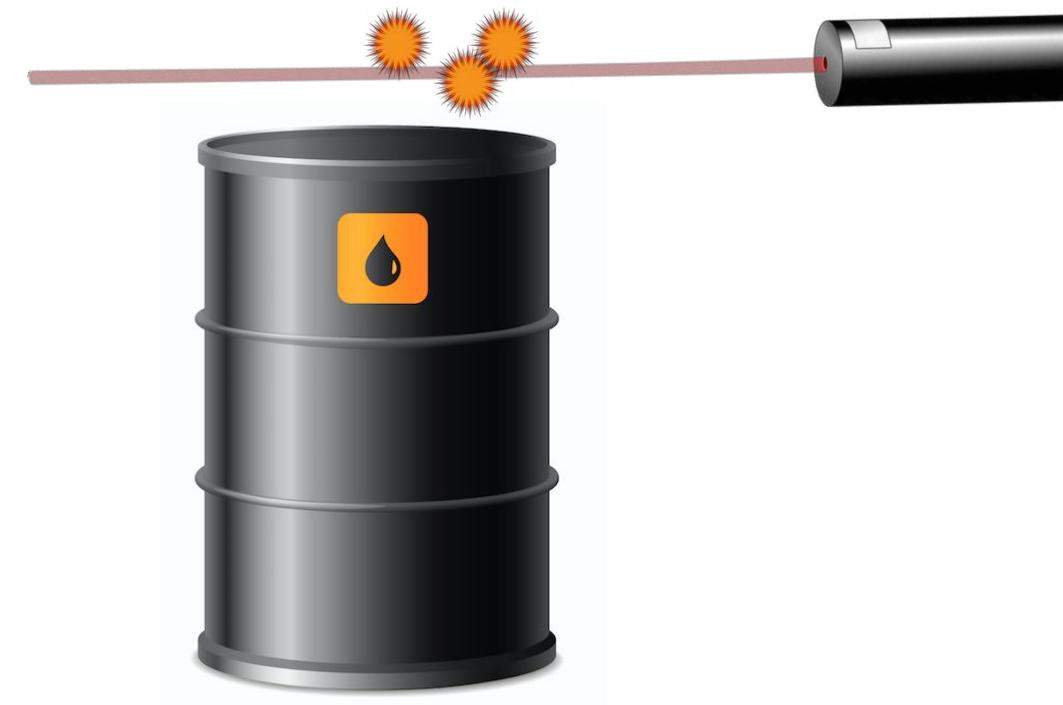


# Two basic classes of measurements

**Demolition**



**Non-demolition**

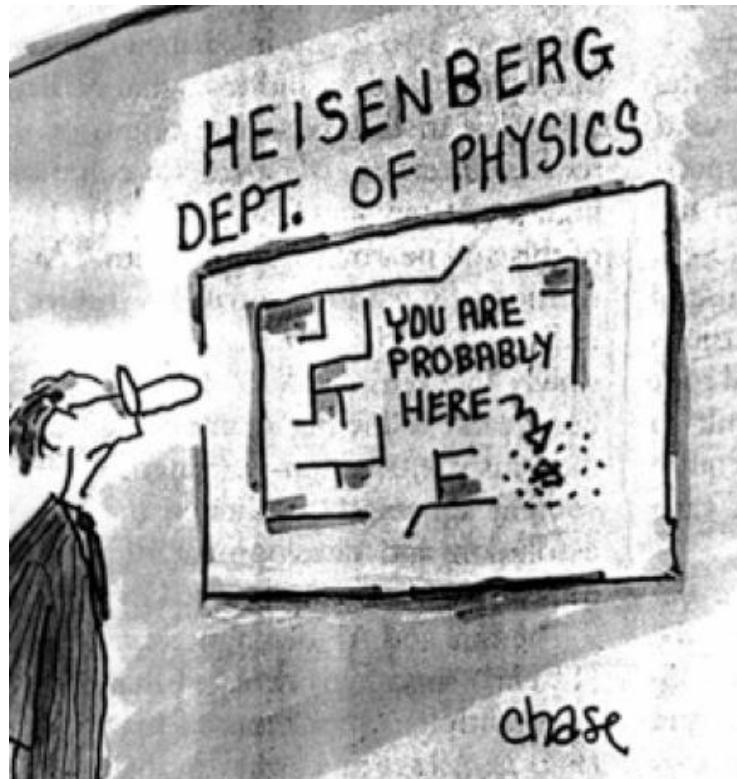


e.g., photon absorption

e.g., dispersive cavity

Both accessible in circuits, more on this later

# Takeaways: Basic character of quantum measurements



Necessarily *disturb* system (back-action)

non-commuting

Heisenberg uncertainty

fundamental limits to precision, e.g., SQL, ...

no joint probability distribution (over  $x, p$ )

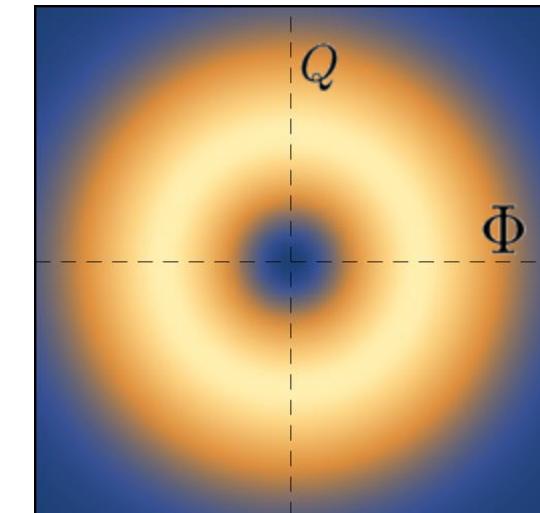
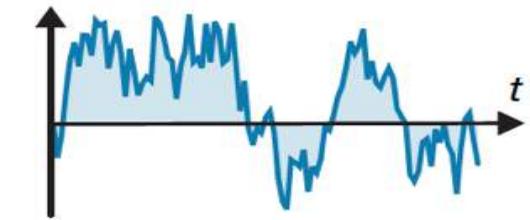
quasi-probabilities (Wigner,  $Q$ )

no classical Fisher information

entropy-increasing

contextuality

....



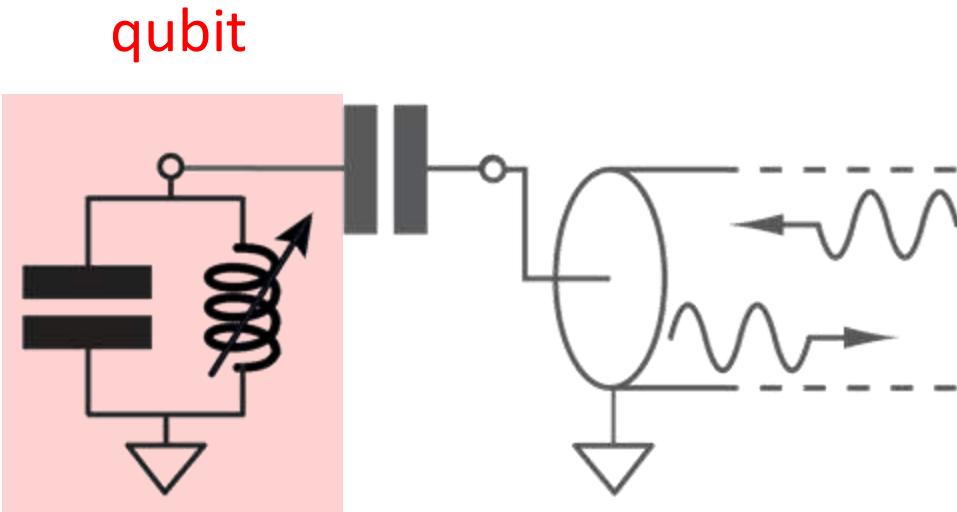
In quantum physics, you don't see  
what you get, you get what you see.

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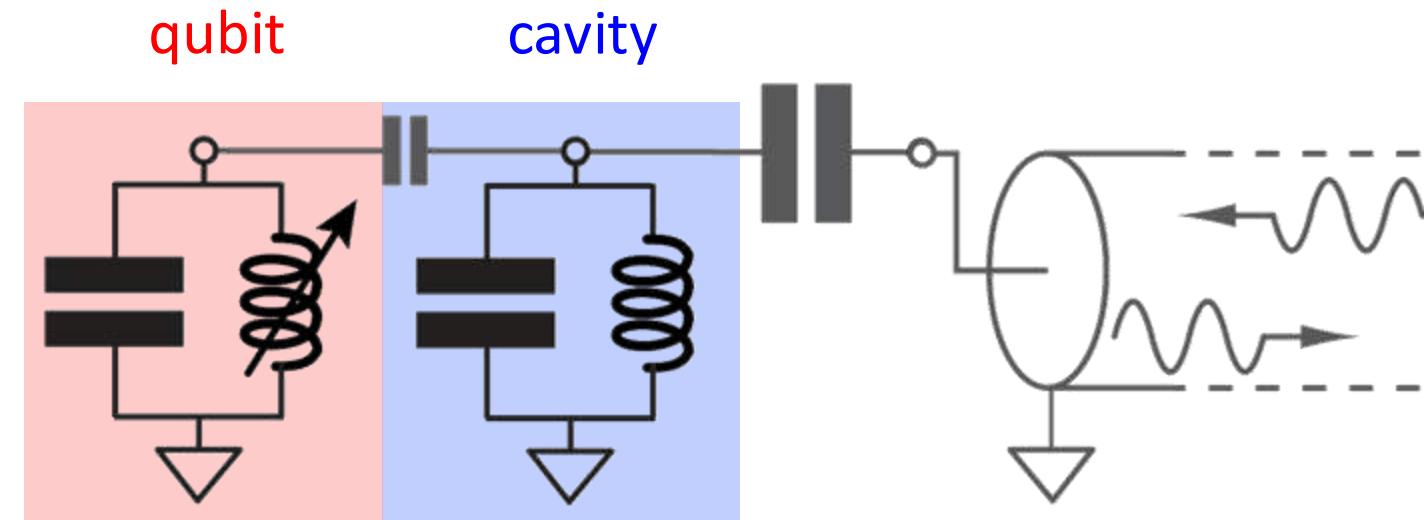
A.N. Korotkov  
Private communication

# Qubit measurement with circuits

## Direct monitoring



## cQED dispersive



Demolition



Spontaneous emission

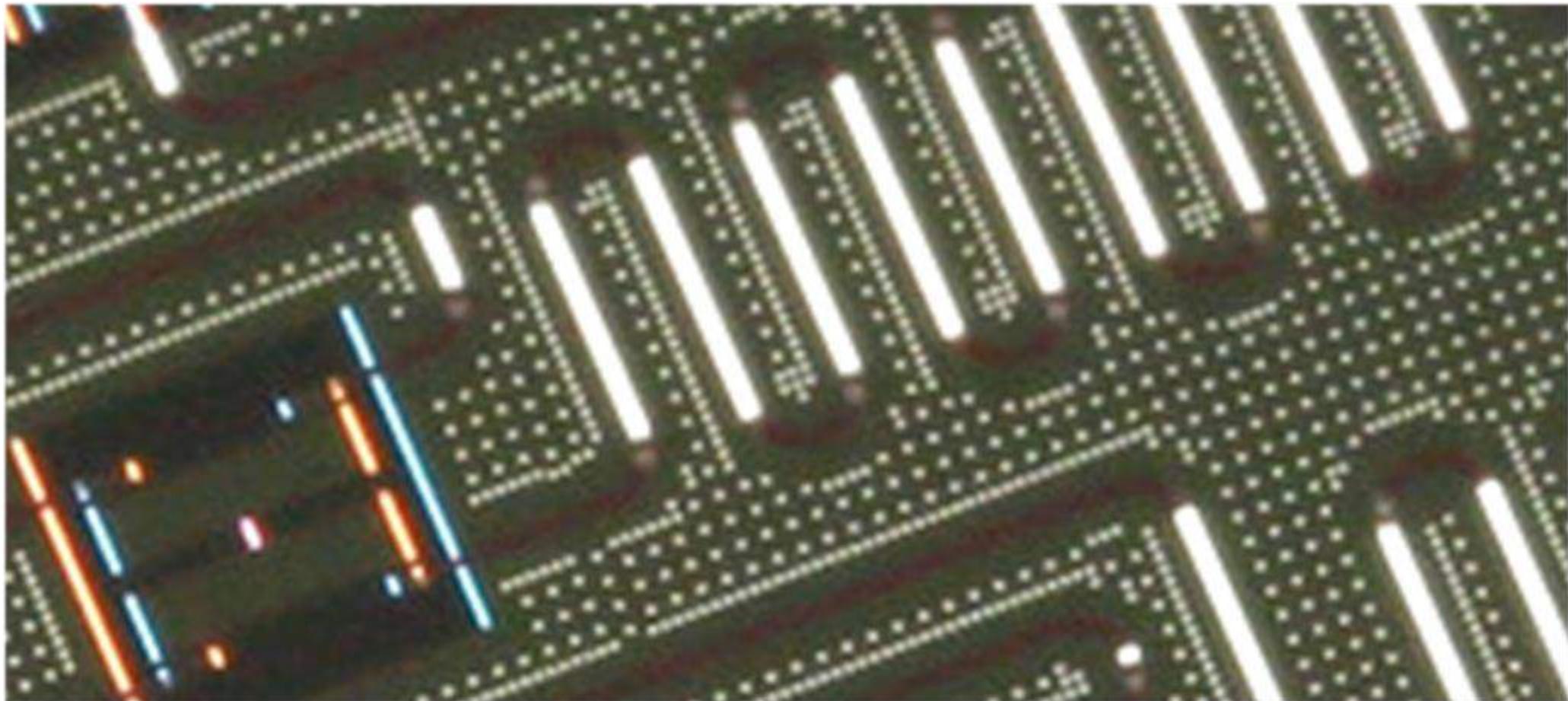


Quantum non-demolition\* (QND)

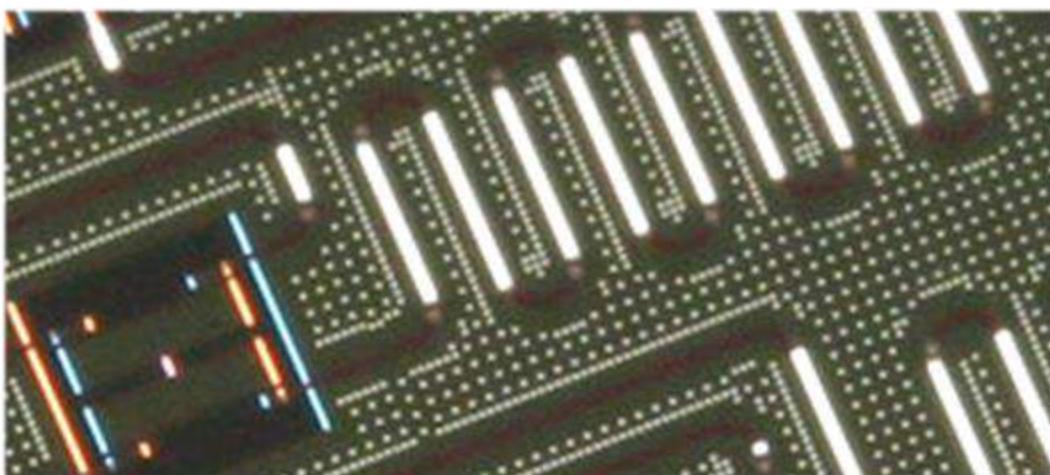


Inhibited spontaneous emission

# Qubit coupled to resonator

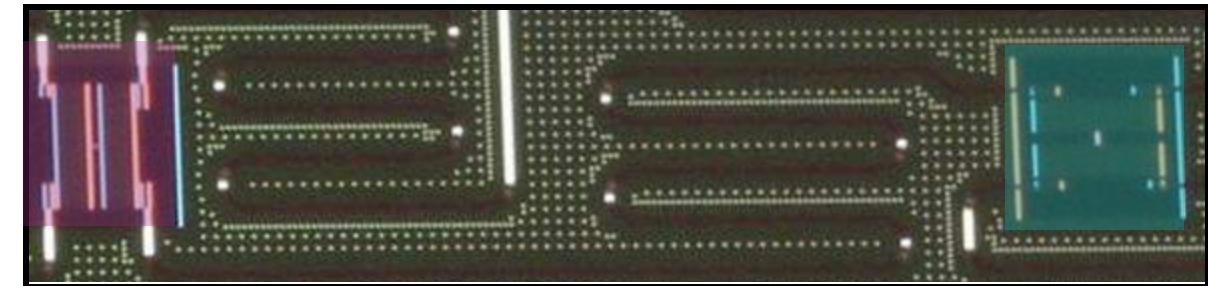


# Qubit coupled to resonator



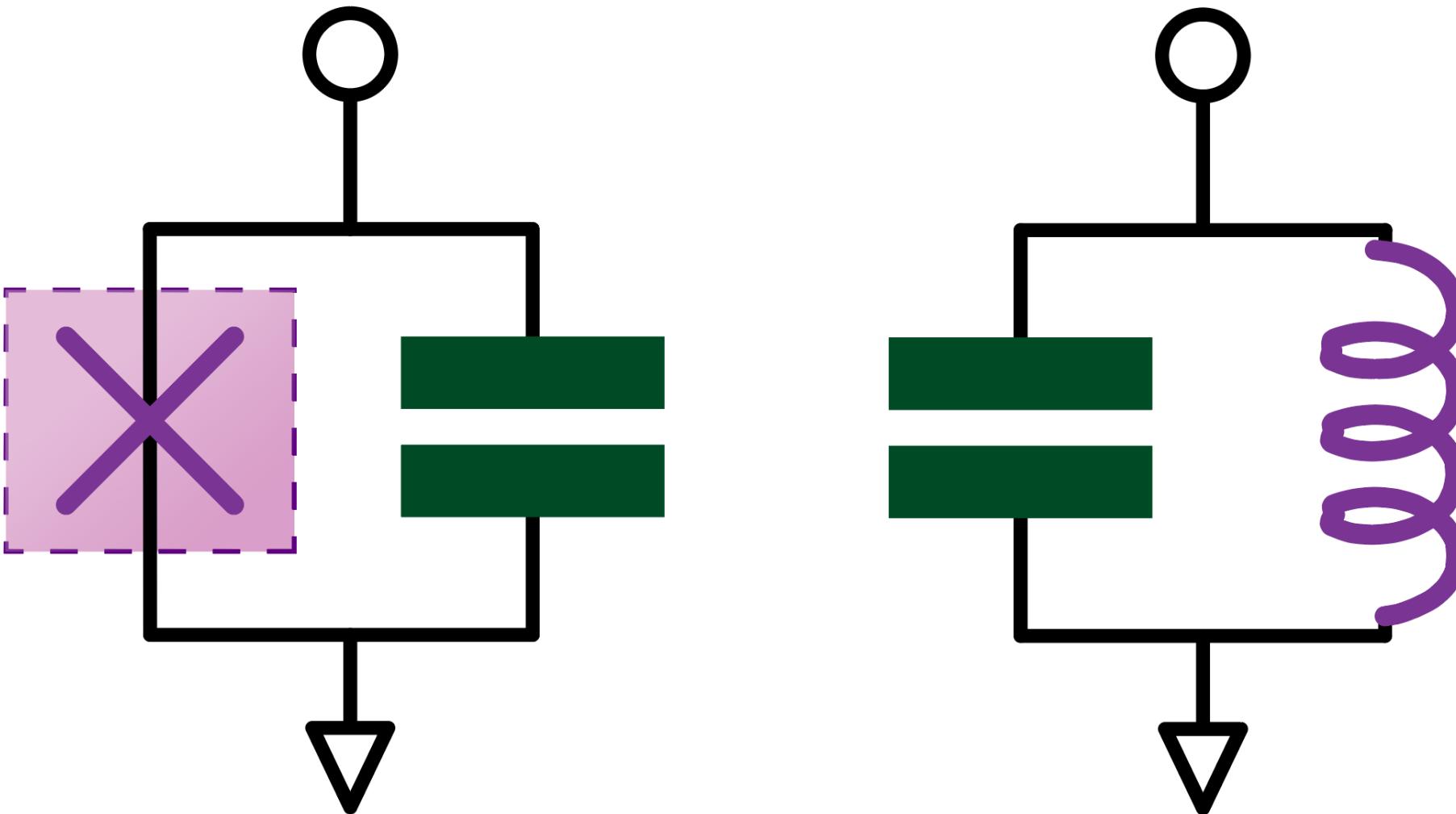
Readout

# Qubit coupled to qubit

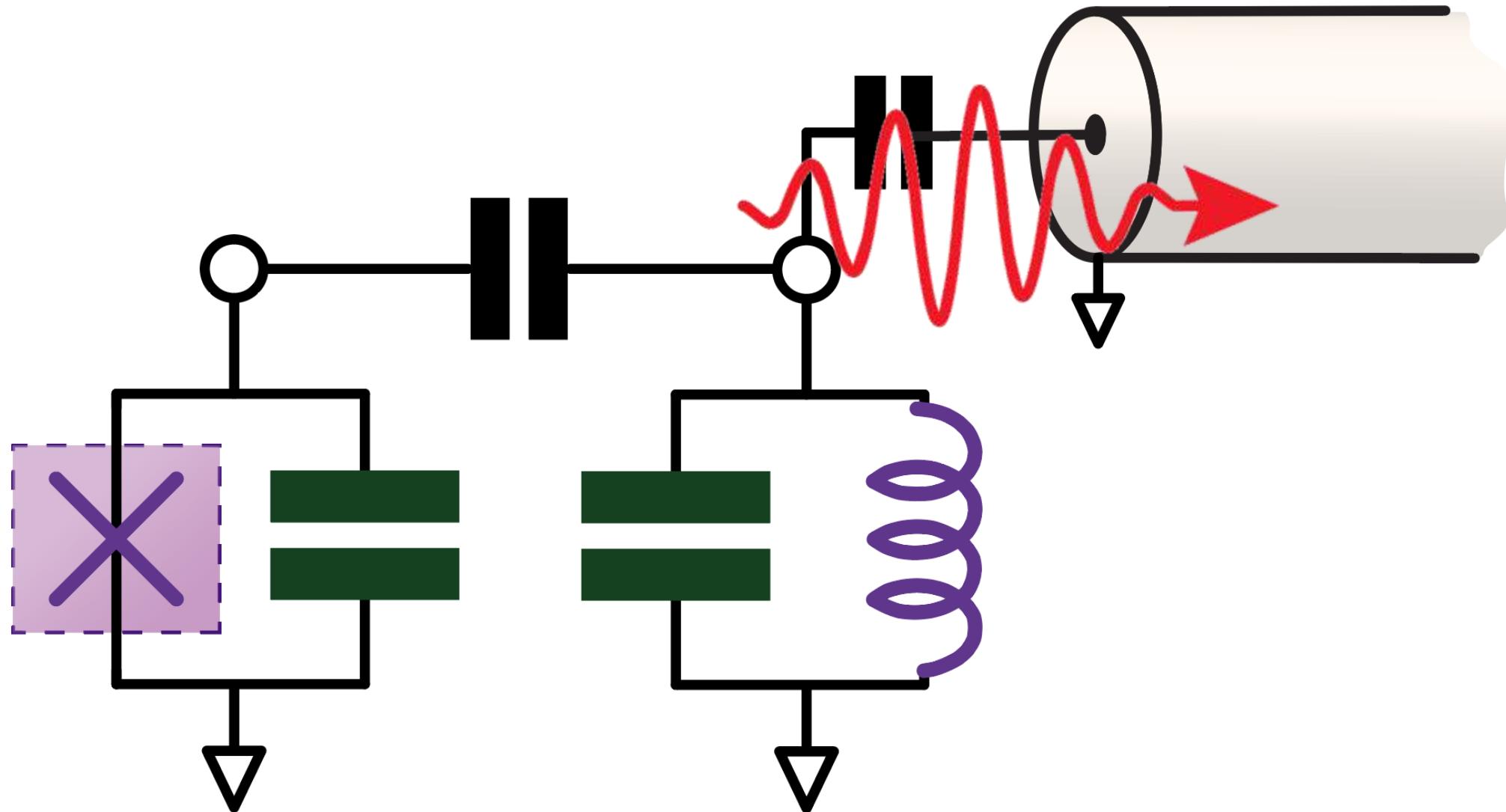


Two-qubit gates

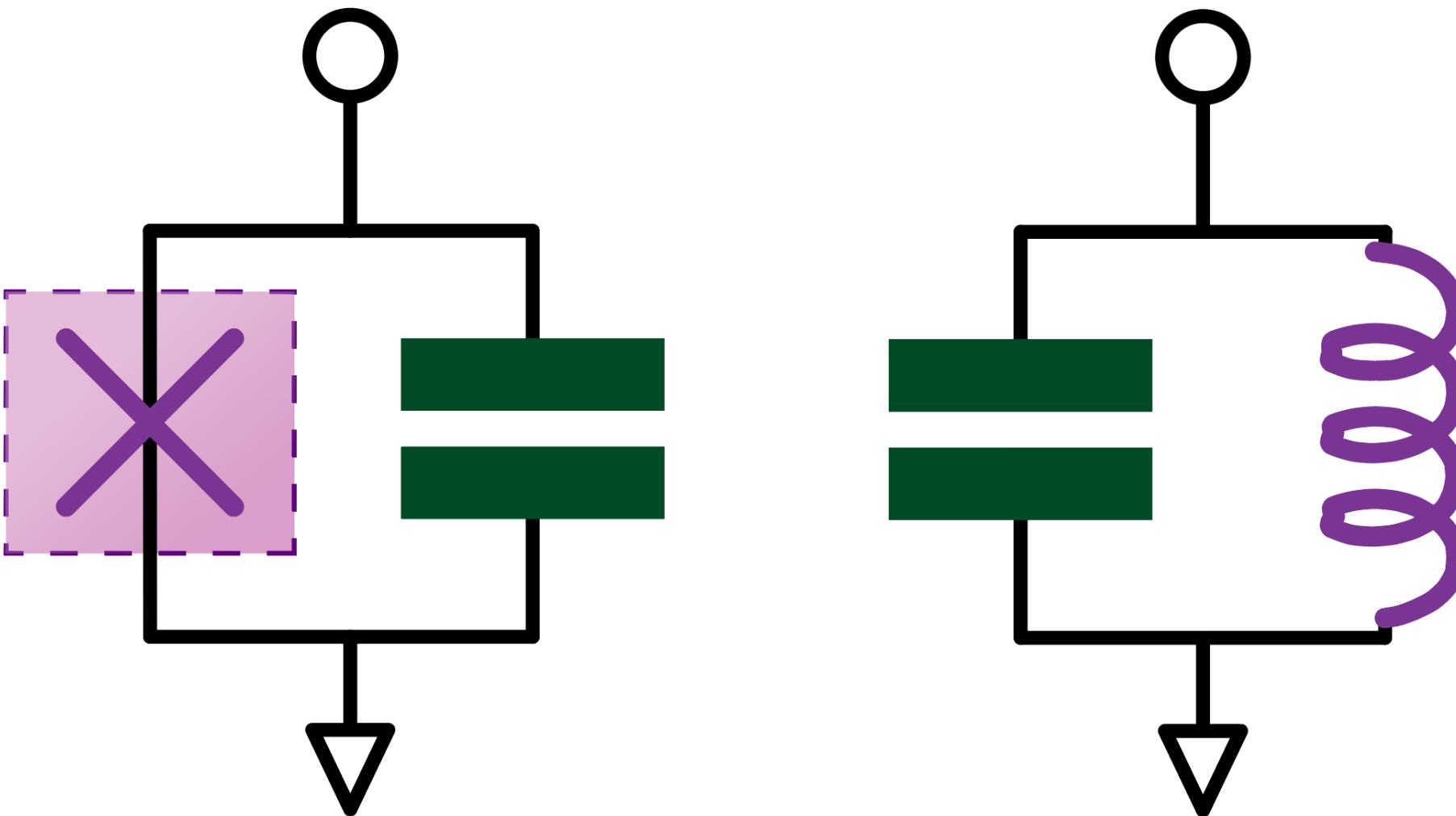
# Qubit and oscillator



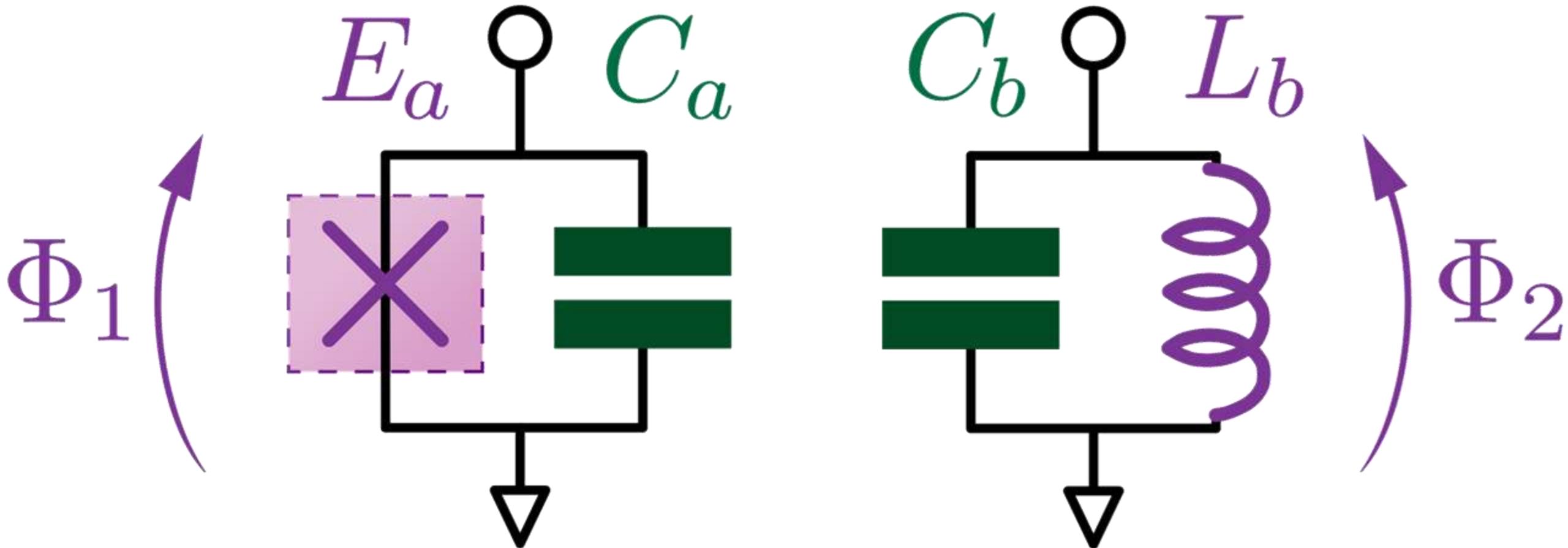
# Qubit measurement with circuits



# Qubit and oscillator



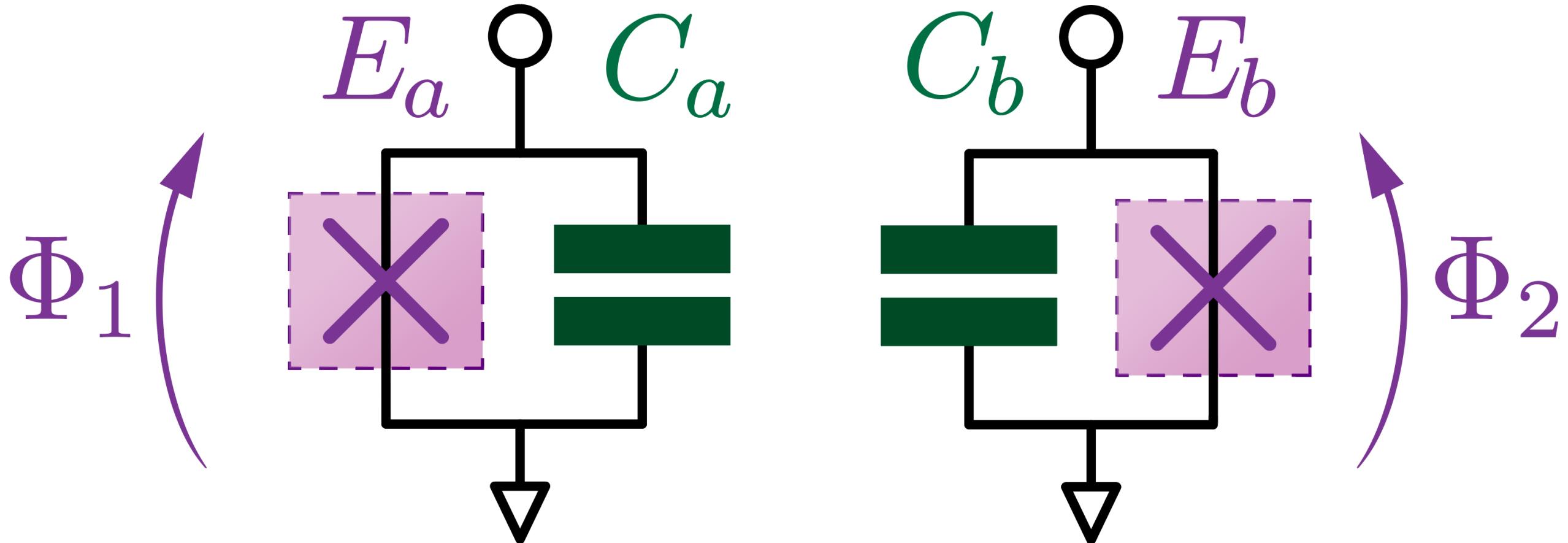
# Qubit and oscillator



$$\hat{H}_1 = \frac{\hat{Q}_1^2}{2C_a} - E_a \cos\left(\hat{\Phi}_1/\phi_0\right)$$

$$\hat{H}_2 = \frac{\hat{Q}_2^2}{2C_b} + \frac{\hat{\Phi}_1^2}{2L_b}$$

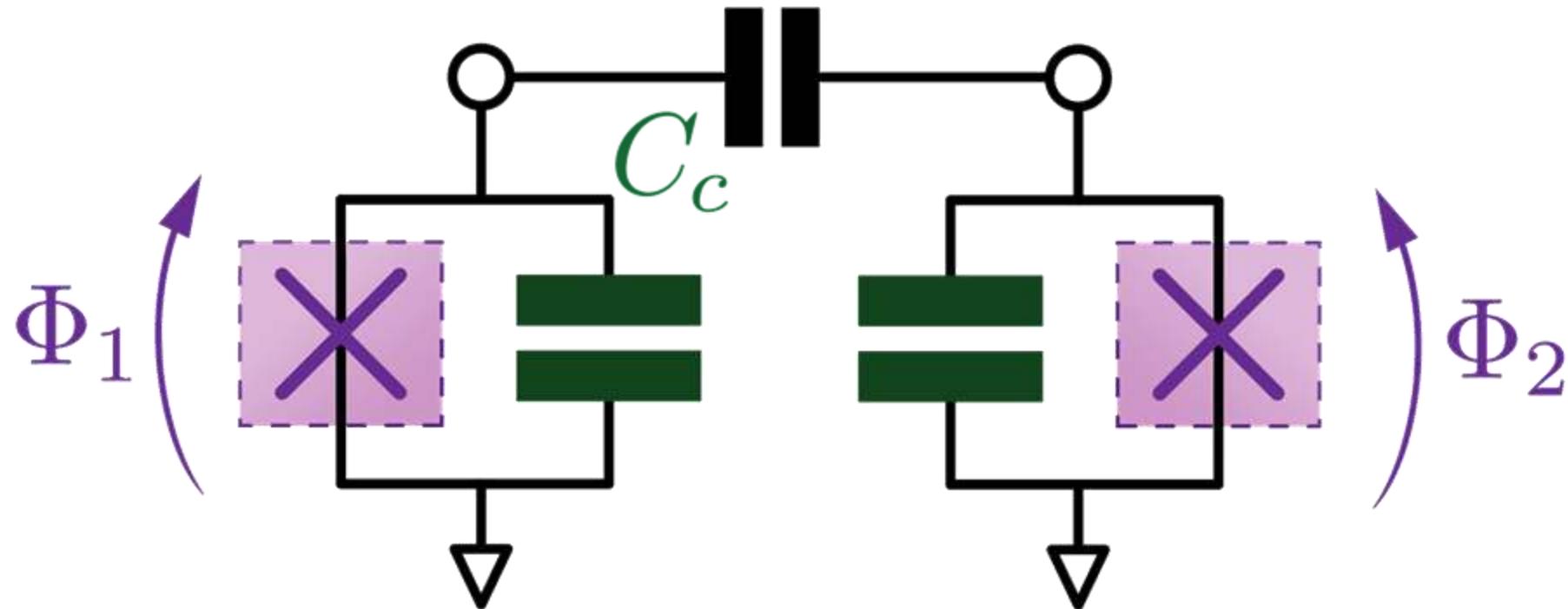
# Qubit and oscillator



$$\hat{H}_1 = \frac{\hat{Q}_1^2}{2C_a} - E_a \cos\left(\frac{\Phi_1}{\phi_0}\right)$$

$$\hat{H}_2 = \frac{\hat{Q}_2^2}{2C_b} - E_b \cos\left(\frac{\Phi_2}{\phi_0}\right)$$

# Two coupled transmon qubits



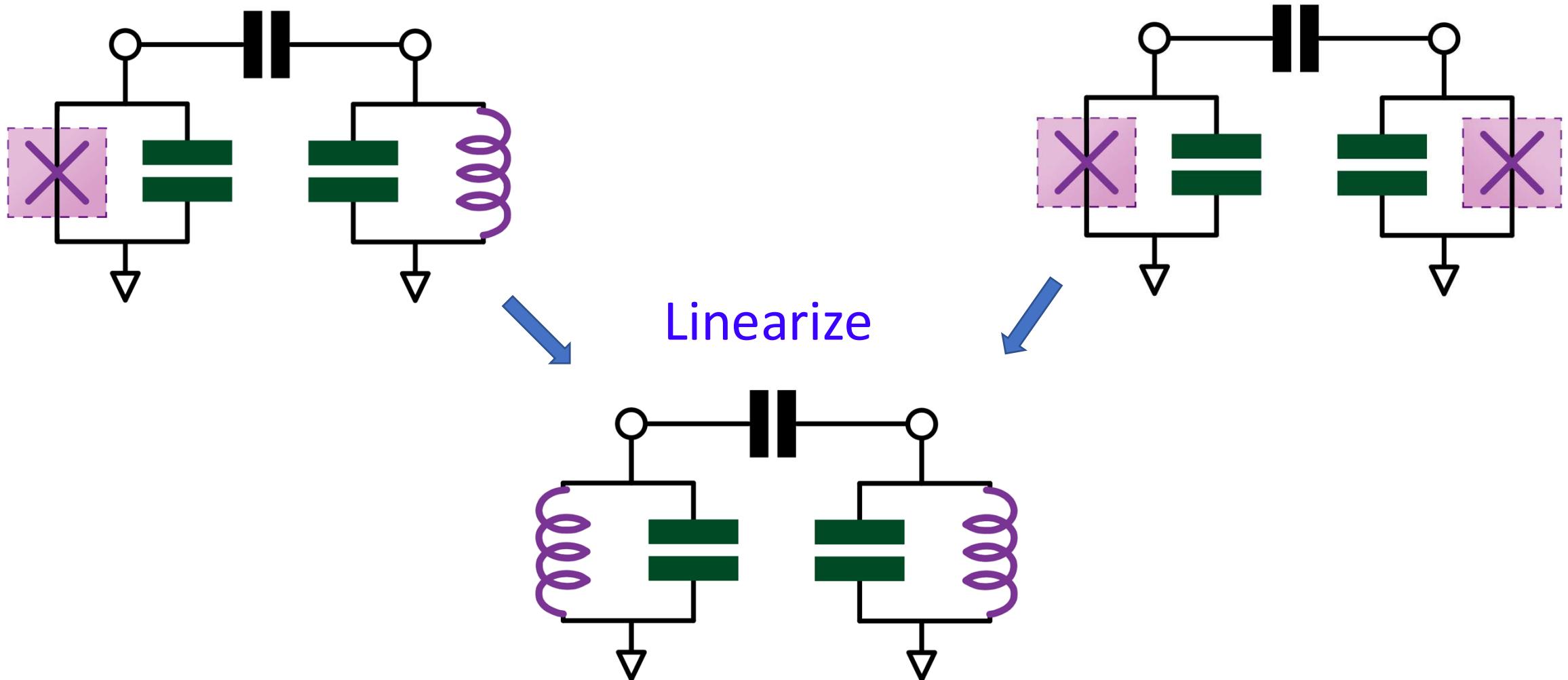
Canonical charge is modified by coupling capacitor

Can use EPR to avoid issues

$$\hat{H}_1 = \frac{\hat{Q}_1^2}{2C_a} - E_a \cos\left(\hat{\Phi}_1/\phi_0\right)$$

$$\hat{H}_2 = \frac{\hat{Q}_2^2}{2C_b} - E_b \cos\left(\hat{\Phi}_2/\phi_0\right)$$

# Energy-participation eigenmode approach



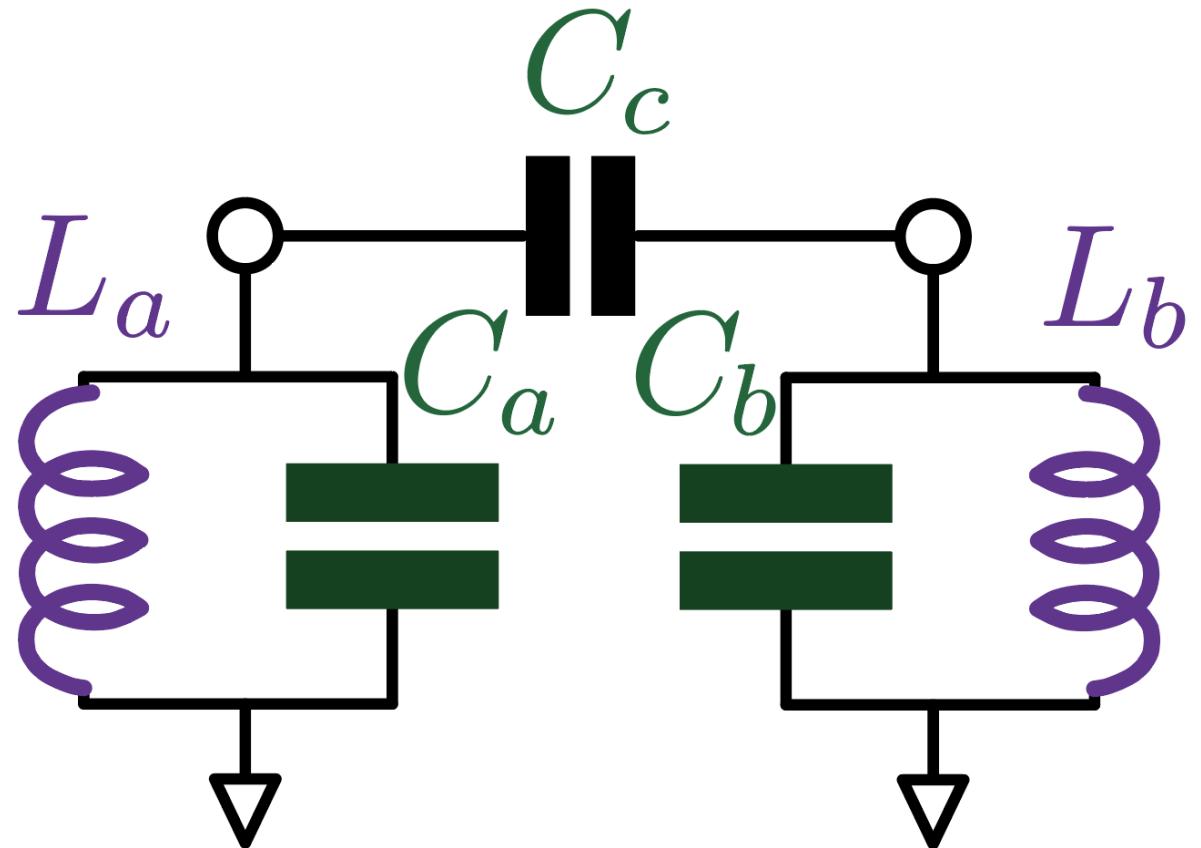
# Linearize

$$\hat{H}_{\text{full}} = \boxed{\hat{H}_{\text{lin}}} + \boxed{\hat{H}_{\text{nl}}}$$

All linear

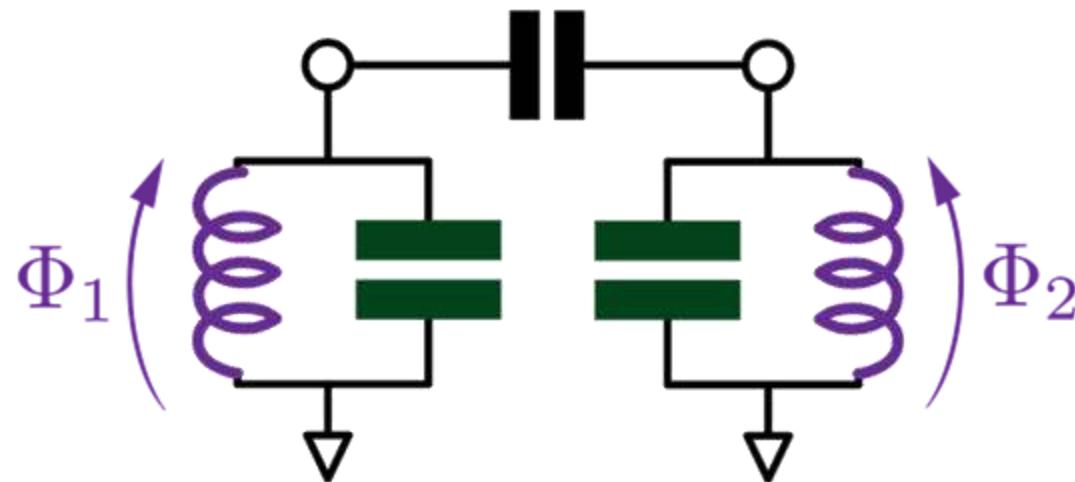
Easy to solve, classical problem

Eigenmode solutions



# Linearize

$$\hat{H}_{\text{full}} = \boxed{\hat{H}_{\text{lin}}} + \boxed{\hat{H}_{\text{nl}}}$$



Recall

$$\begin{aligned} \mathcal{E}_J(\Phi) &= -E_J \cos(\Phi/\phi_0) \\ &\approx \frac{E_J}{2!} \left(\hat{\Phi}/\phi_0\right)^2 - \frac{E_J}{4!} \left(\hat{\Phi}/\phi_0\right)^4 \\ &\quad + \mathcal{O}\left(\hat{\Phi}^6\right) \end{aligned}$$

Solutions are eigenmodes (normal mode), just more SHO. Their Eigenfrequencies are

$$\omega_a, \omega_b$$

The Hamiltonian of two SHO is

$$\hat{H}_{\text{lin}} = \hbar\omega_a \hat{a}^\dagger \hat{a} + \hbar\omega_b \hat{b}^\dagger \hat{b}$$

The non-linear part of the Hamiltonian is

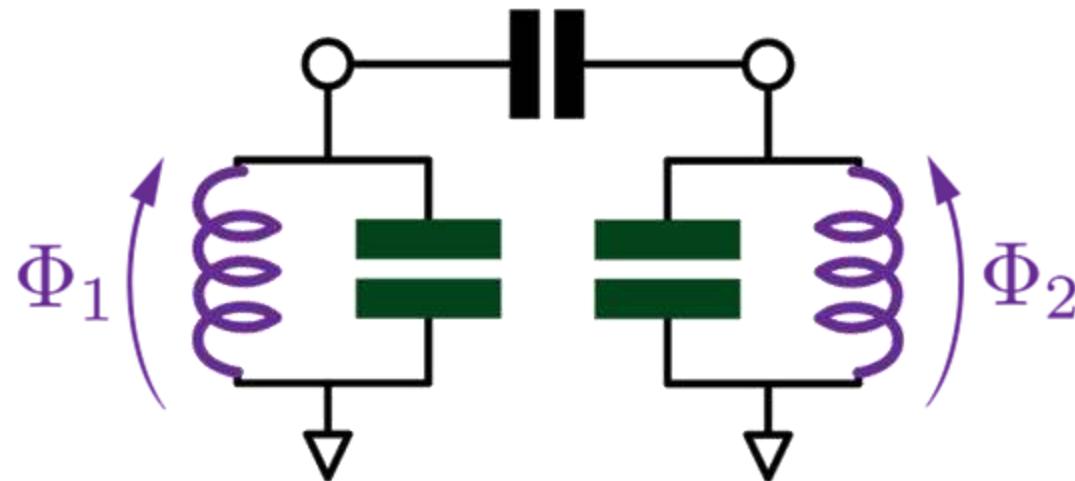
$$\hat{H}_{\text{nl}} \approx -\frac{E_a}{4!} \left(\hat{\Phi}_1/\phi_0\right)^4 - \frac{E_b}{4!} \left(\hat{\Phi}_2/\phi_0\right)^4$$

The coupling capacitor  $C_C$  effect is accounted for in the definition of the eigenmode  $a$  and  $b$  operators.

We need the junction flux in mode operators.

# Eigen-decomposition of junction flux

$$\hat{H}_{\text{full}} = \hat{H}_{\text{lin}} + \hat{H}_{\text{nl}}$$



“Bare” coordinates

Eigen (dressed) coordinates

$$\begin{aligned} \Phi_1 &= e_{1a}\Phi_a + e_{1b}\Phi_b \\ \Phi_2 &= e_{2a}\Phi_a + e_{2b}\Phi_b \end{aligned}$$

Canonical transformation eigenvectors (found classically)

Use in

$$\hat{H}_{\text{nl}} \approx -\frac{E_a}{4!} \left( \hat{\Phi}_1/\phi_0 \right)^4$$

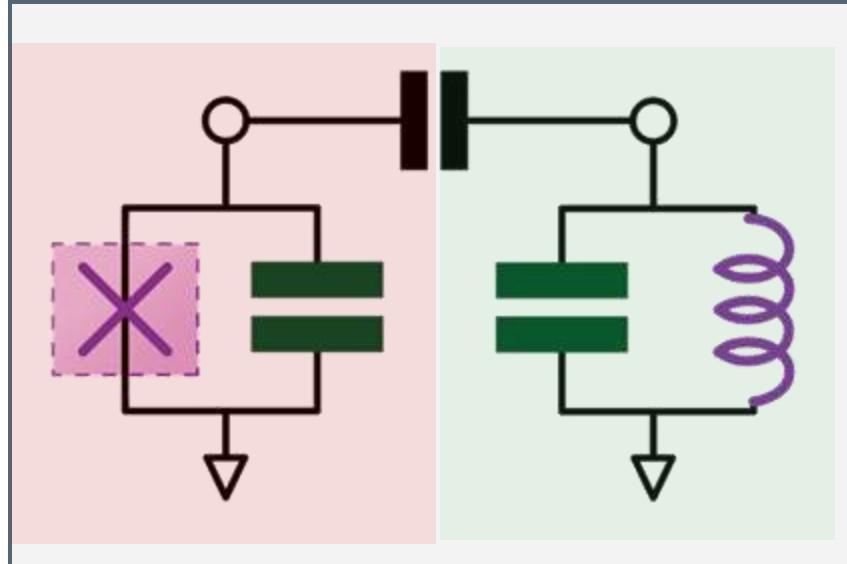
It follows

$$\Phi_1 = \Phi_{1a}^{\text{ZPF}} (\hat{a}^\dagger + \hat{a}) + \Phi_{1b}^{\text{ZPF}} (\hat{b}^\dagger + \hat{b})$$

and, similarly, for the second junction flux

(second quantization in eigen basis of linearized circuit)

# Partitioning the Hamiltonian



What fraction of the energy  
of a mode  $m$   
is stored in the junction?

$$\frac{1}{\hbar} \phi_{mj}^2 = p_{mj} \frac{\omega_m}{2E_j}$$

$$\hat{H}_{\text{full}} = \hat{H}_{\text{lin}} + \hat{H}_{\text{nl}}$$

$$\hbar\omega_c \hat{a}_c^\dagger \hat{a}_c + \hbar\omega_q \hat{a}_q^\dagger \hat{a}_q$$

$$-\frac{E_a}{4!} \left[ \phi_{1a}^{\text{ZPF}} (\hat{a}^\dagger + \hat{a}) + \phi_{1b}^{\text{ZPF}} (\hat{b}^\dagger + \hat{b}) \right]^4$$

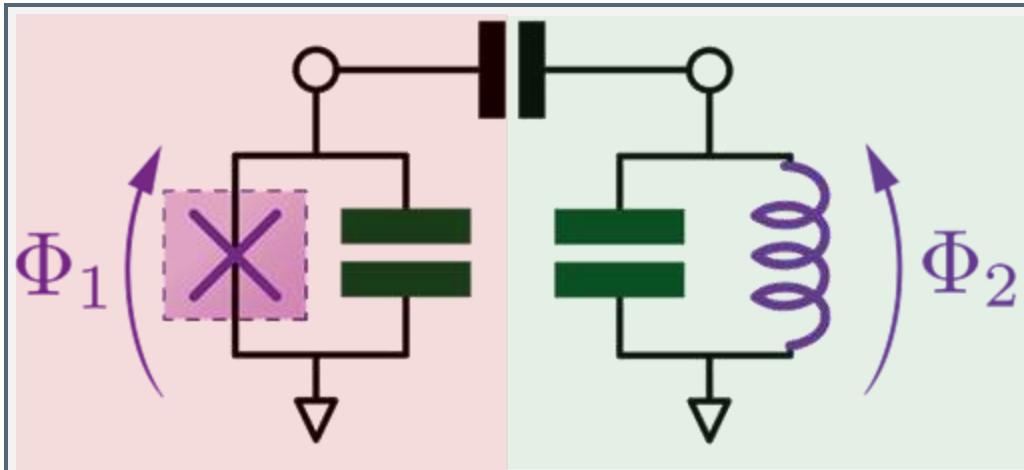
where  $\phi_{1a}^{\text{ZPF}} = \Phi_{1a}^{\text{ZPF}} / \phi_0$ , and so on

and for weak coupling

$$\phi_{1a}^{\text{ZPF}} \gg \phi_{1b}^{\text{ZPF}}$$



# Breakdown of the Hamiltonian



Terms like those of isolated transmon

Drop small terms (cross-participation)

$$\phi_b \ll \phi_a$$

Only new addition is this non-linear, photon-number dependent coupling, called the cross-Kerr.

RWA. Notation: dropping ZPF and index for 1

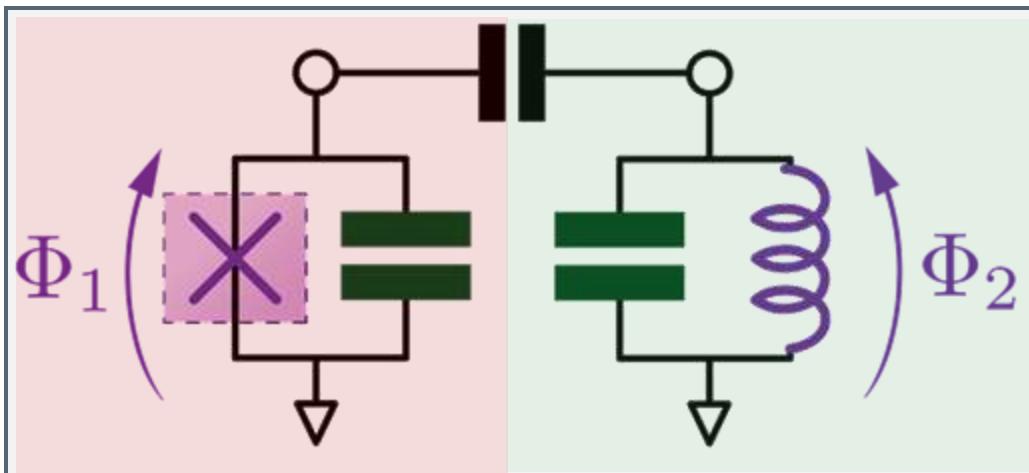
$$\left[ \phi_a^{\text{ZPF}} (\hat{a}^\dagger + \hat{a}) + \phi_b^{\text{ZPF}} (\hat{b}^\dagger + \hat{b}) \right]^4$$

$$= 12 (\phi_a^4 + \phi_a^2 \phi_b^2) \hat{a}^\dagger \hat{a} + 6 \phi_a^4 \hat{a}^{\dagger 2} \hat{a}^2 +$$

...

$$24 \phi_a^2 \phi_b^2 \hat{a}^\dagger \hat{a} \hat{b}^\dagger \hat{b}$$

# Linearize system and find eigenmodes



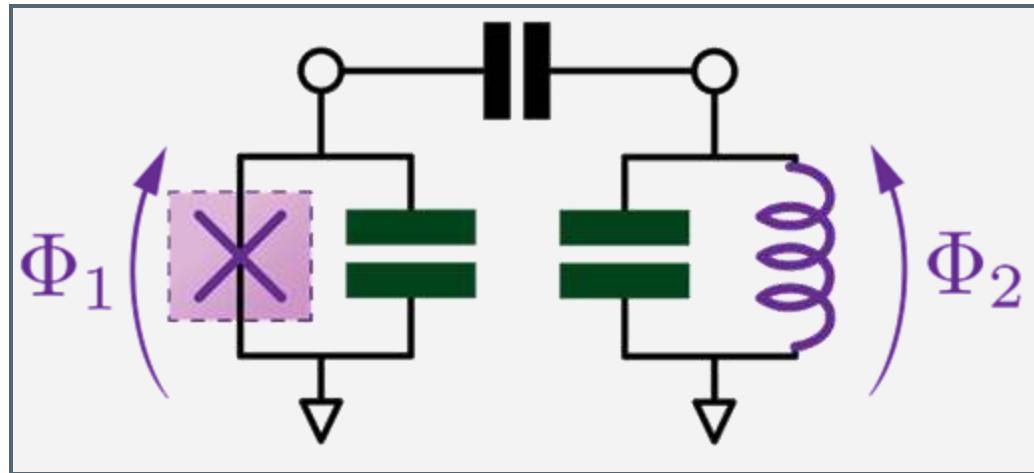
Review of steps

1. Linearize and find eigenmodes
2. Taylor series of nl. potential to 4<sup>th</sup> order
3. RWA and normal order
4. Dispersive approximation

$$\hat{H}_{\text{eff}} = (\omega_q - \Delta_q) \hat{n}_q + (\omega_q \hat{n}_q \Delta_c) \hat{n}_c \hat{n}_c - \chi_{qc} \hat{n}_q \hat{n}_c \\ - \frac{1}{2} \alpha_q \hat{n}_q (\hat{n}_q - 1)$$

Using labels to  $q$  for qubit eigenmode and  $c$  for cavity eigenmode, and  $\hat{n}_q = \hat{a}^\dagger \hat{a}$   $\hat{n}_c = \hat{b}^\dagger \hat{b}$

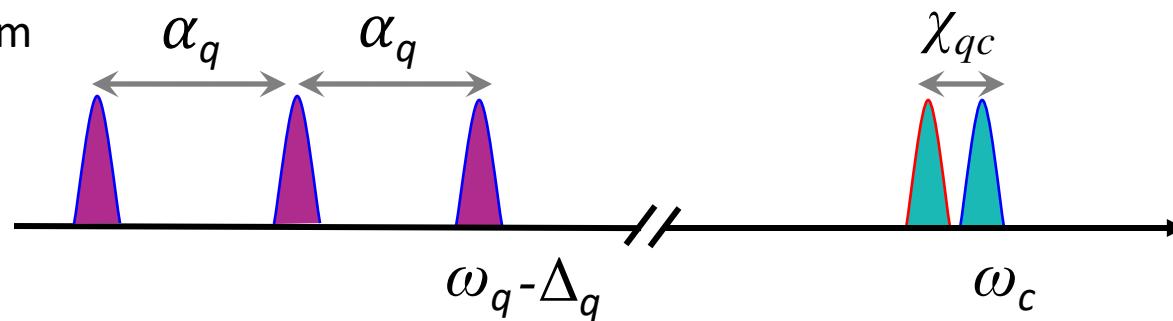
# Joint spectrum



$$\hat{H}_{\text{eff}} = (\omega_q - \Delta_q) \hat{n}_q + (\omega_c - \Delta_c) \hat{n}_c - \chi_{qc} \hat{n}_q \hat{n}_c - \frac{1}{2} \alpha_q \hat{n}_q (\hat{n}_q - 1) - \frac{1}{2} \alpha_c \hat{n}_c (\hat{n}_c - 1),$$

$$\hat{H}_{\text{cav}}^{\text{eff}} = \underbrace{\hbar (\omega_c - \chi \hat{a}^\dagger \hat{a})}_{\text{cavity frequency}} \hat{b}^\dagger \hat{b}$$

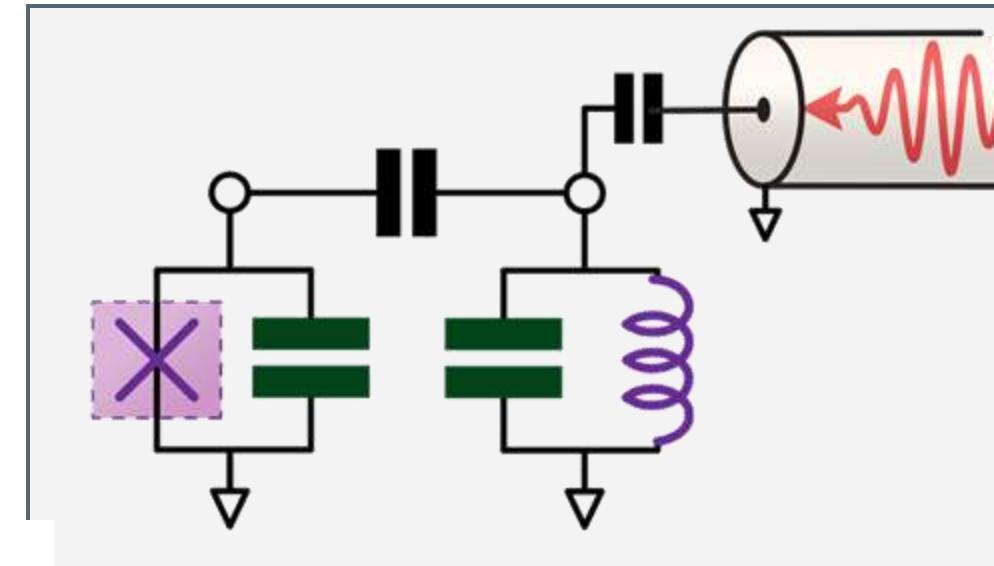
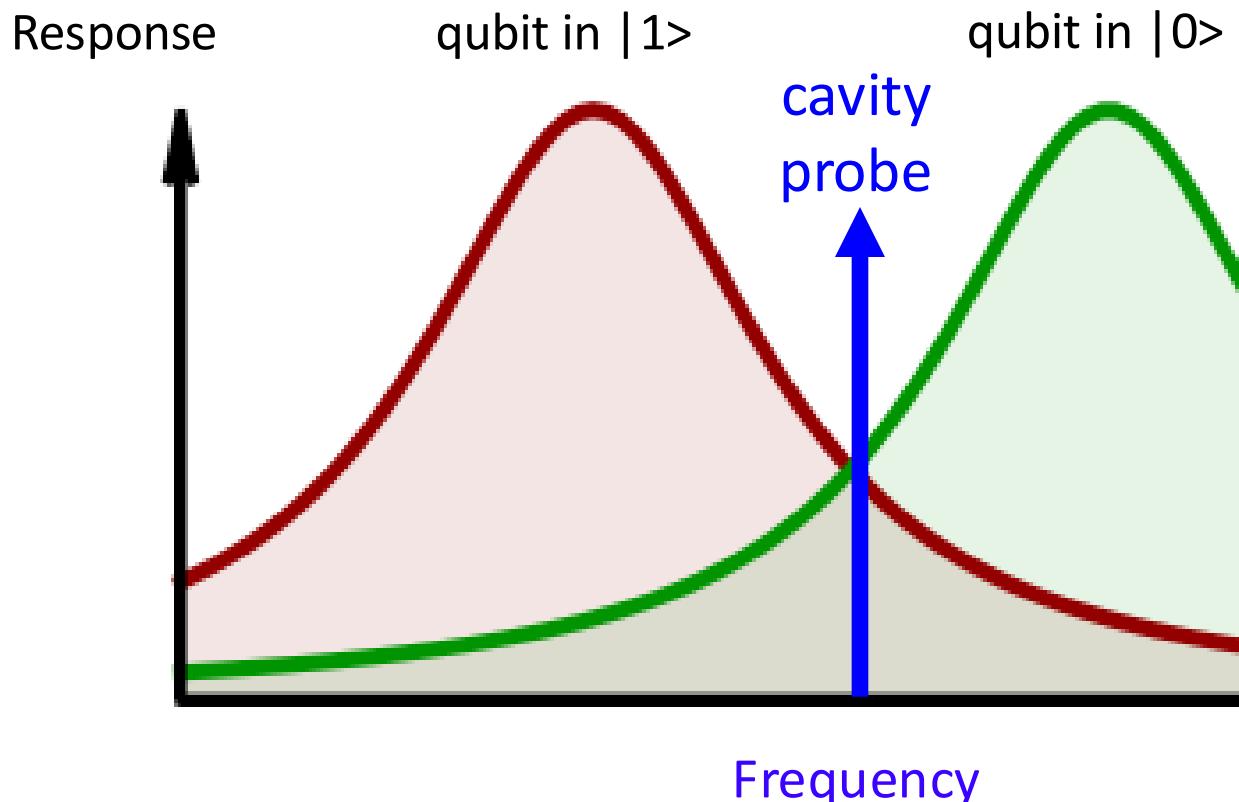
Transition  
spectrum



Recall that harmonic oscillator has evenly spaced levels

# Conditional cavity spectrum

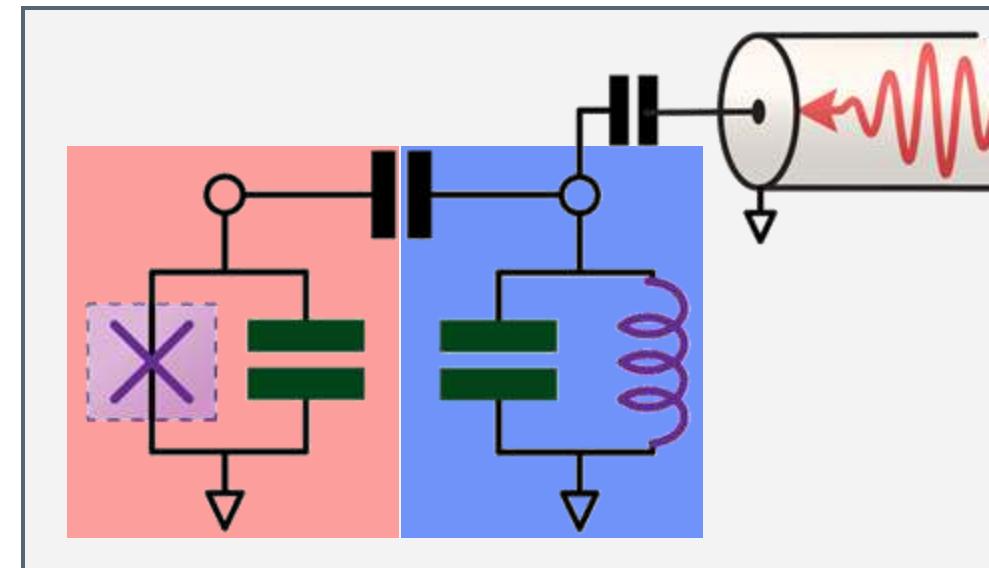
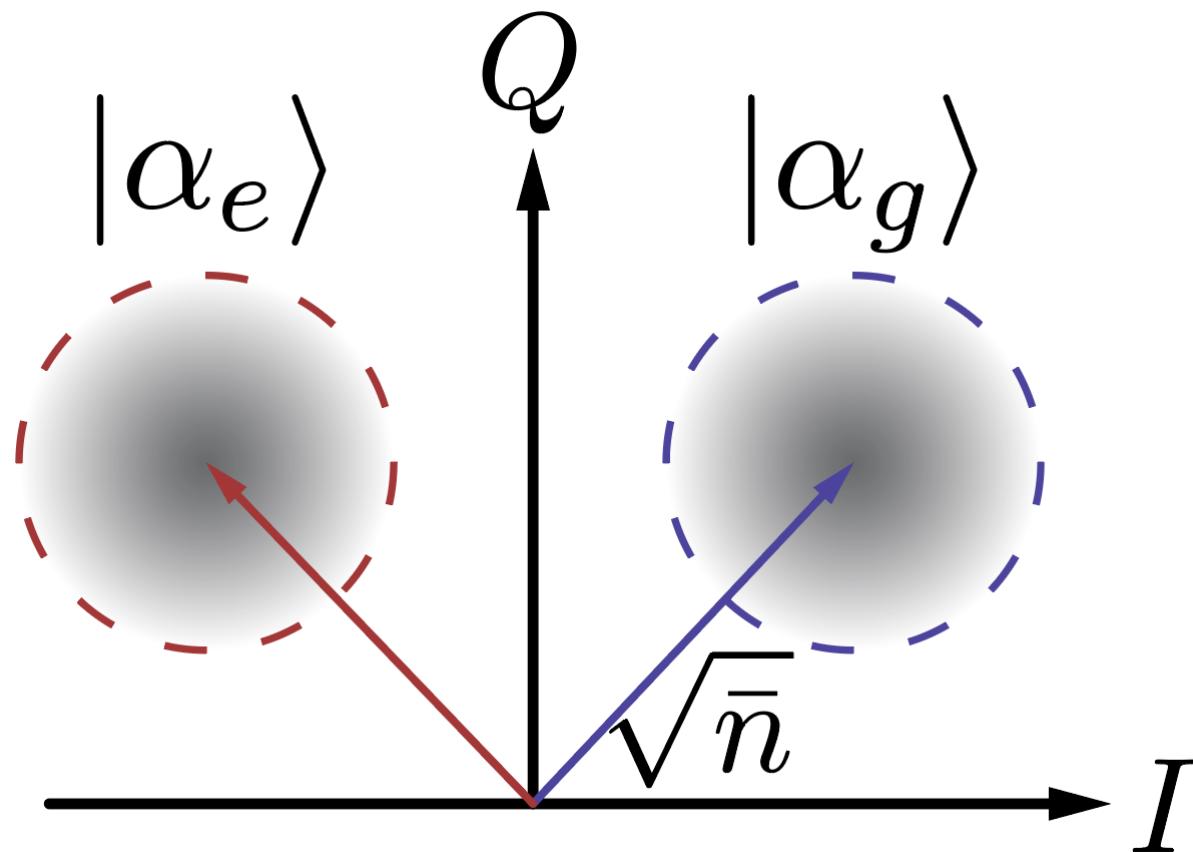
$$\hat{H}_{\text{cav}}^{\text{eff}} = \hbar \underbrace{(\omega_c - \chi \hat{a}^\dagger \hat{a})}_{\text{cavity frequency}} \hat{b}^\dagger \hat{b}$$



Lorentzian peak  
(response of cavity on irradiation)

# cQED dispersive readout: measuring Z

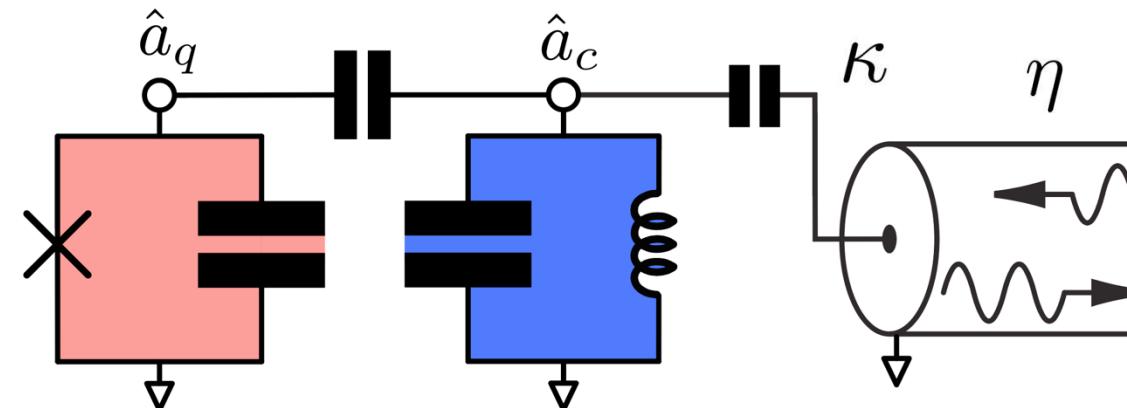
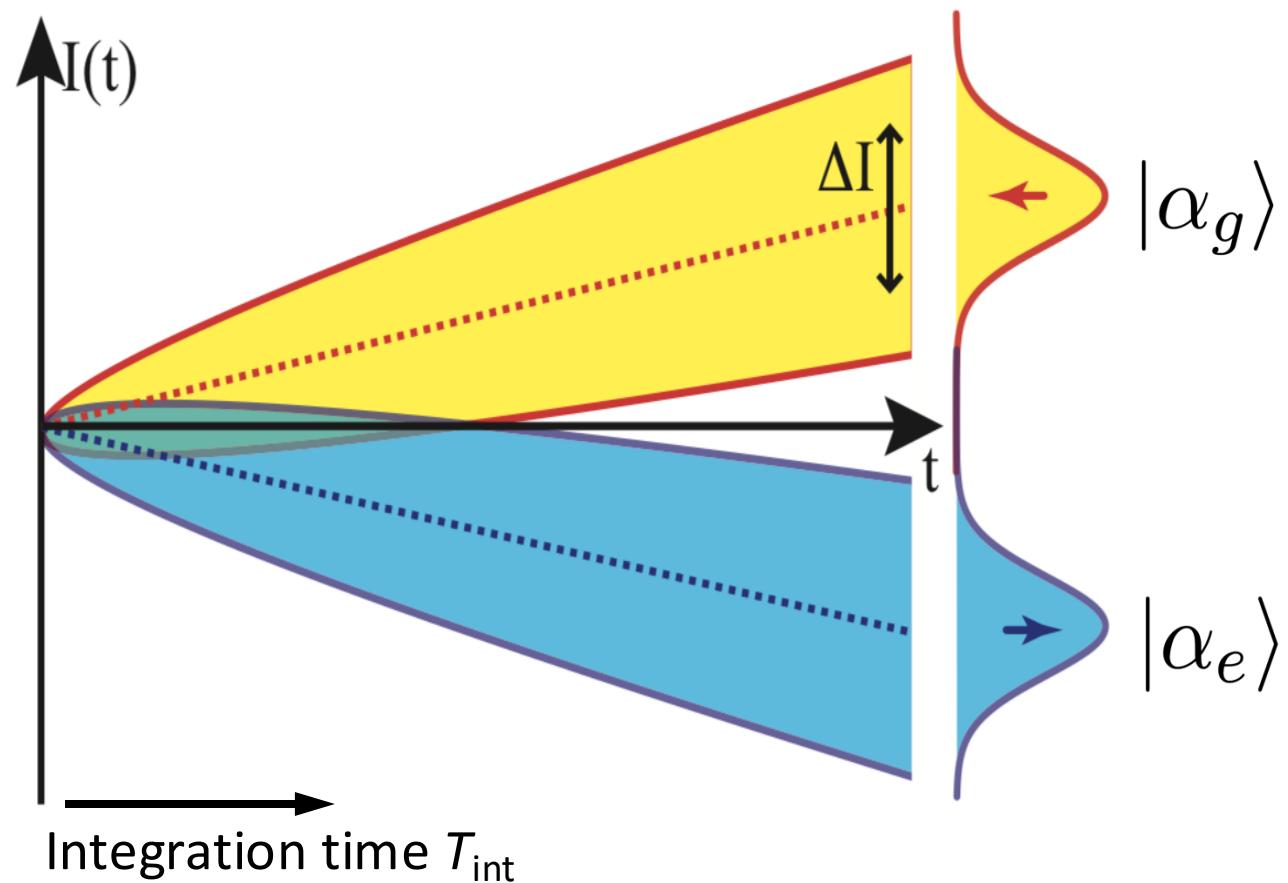
$$\hat{H}_{\text{int}} = \hbar\chi \hat{a}_q^\dagger \hat{a}_q \hat{a}_c^\dagger \hat{a}_c$$



*Pointer states* of measurement apparatus (cavity) corresponding to the qubit in the ground ( $g$ ) and excited ( $e$ ) states

$$\hat{I} \equiv \hat{c} + \hat{c}^\dagger \text{ and } \hat{Q} \equiv -i(\hat{c} - \hat{c}^\dagger)$$

# cQED dispersive readout: measuring Z



$$\text{SNR} = \frac{1}{2} \eta \kappa T_{\text{int}} |\alpha_e - \alpha_g|^2$$

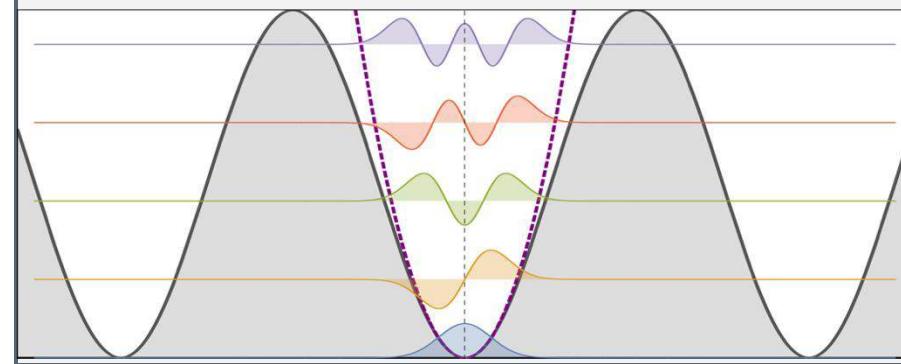
distinguishability

$$\eta_{\text{disc}} = \frac{1}{2} \operatorname{erfc} \left[ -\sqrt{\frac{\text{SNR}}{2}} \right]$$

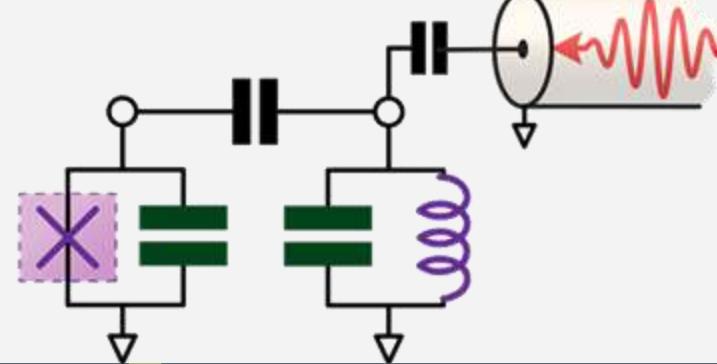
efficient, fast measurements

# The road behind

## Transmon qubit



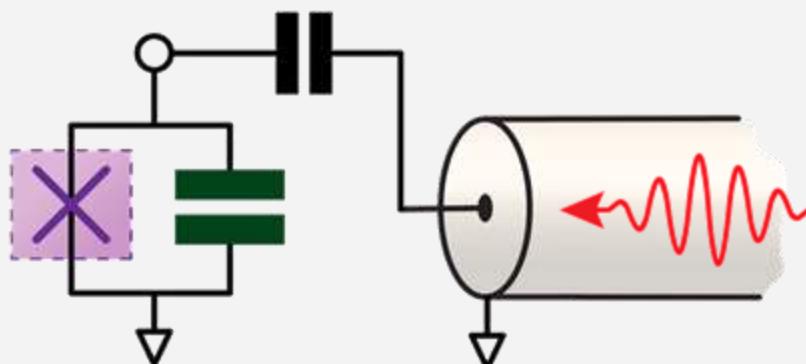
## Qubit and readout (/qubit)



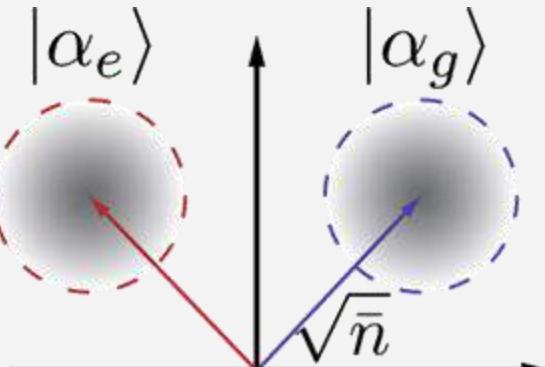
## Lab



## Qubit control



## On measurements



# Next steps

Tightly integrated lab work with Dr. Nick Bronn and Co.!

Run experiments on real devices



Check out references, problems given in the lecture,  
dangerous bends

Break away from the rules of today



Thank you!

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IBM Quantum

The important thing is not to stop questioning. Curiosity has its own reason for existence.

One cannot help but be in awe when he contemplates the mysteries of eternity, of life, of the marvelous structure of reality.

It is enough if one tries merely to comprehend a little of this mystery each day.

---

Albert Einstein

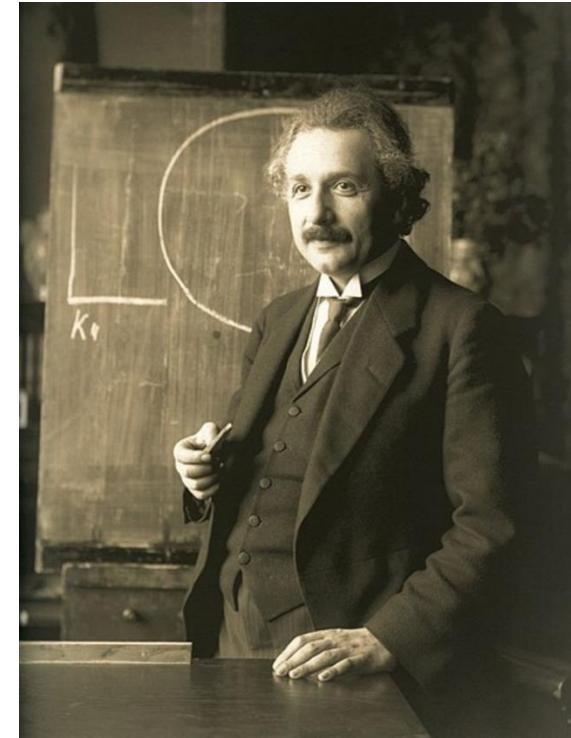


Photo: F. Schmutzler



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