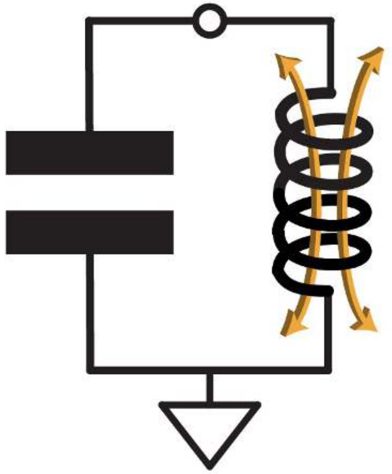
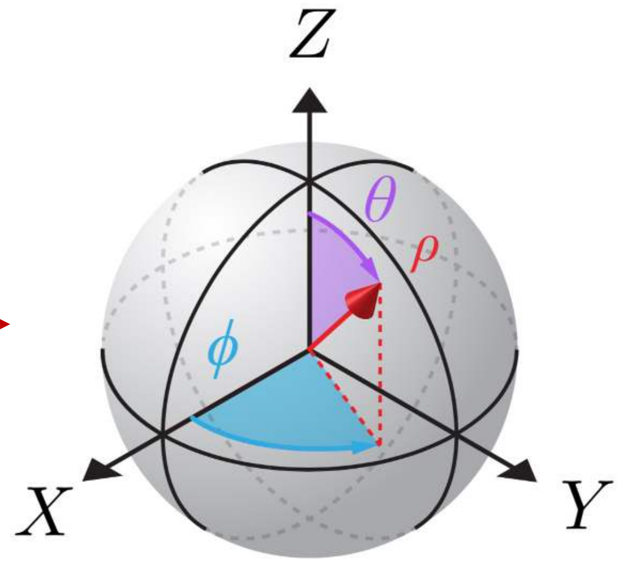
Superconducting Qubits 101

Making Your First Qubit From an Oscillator



Introduction to Circuit Quantum Electrodynamics (cQED)



Zlatko K. Minev

IBM Quantum
IBM T.J. Watson Research Center, Yorktown Heights, NY

@zlatko_minev zlatko-minev.com

What is a real qubit?

How can you design, control, and measure a real qubit?

Why?

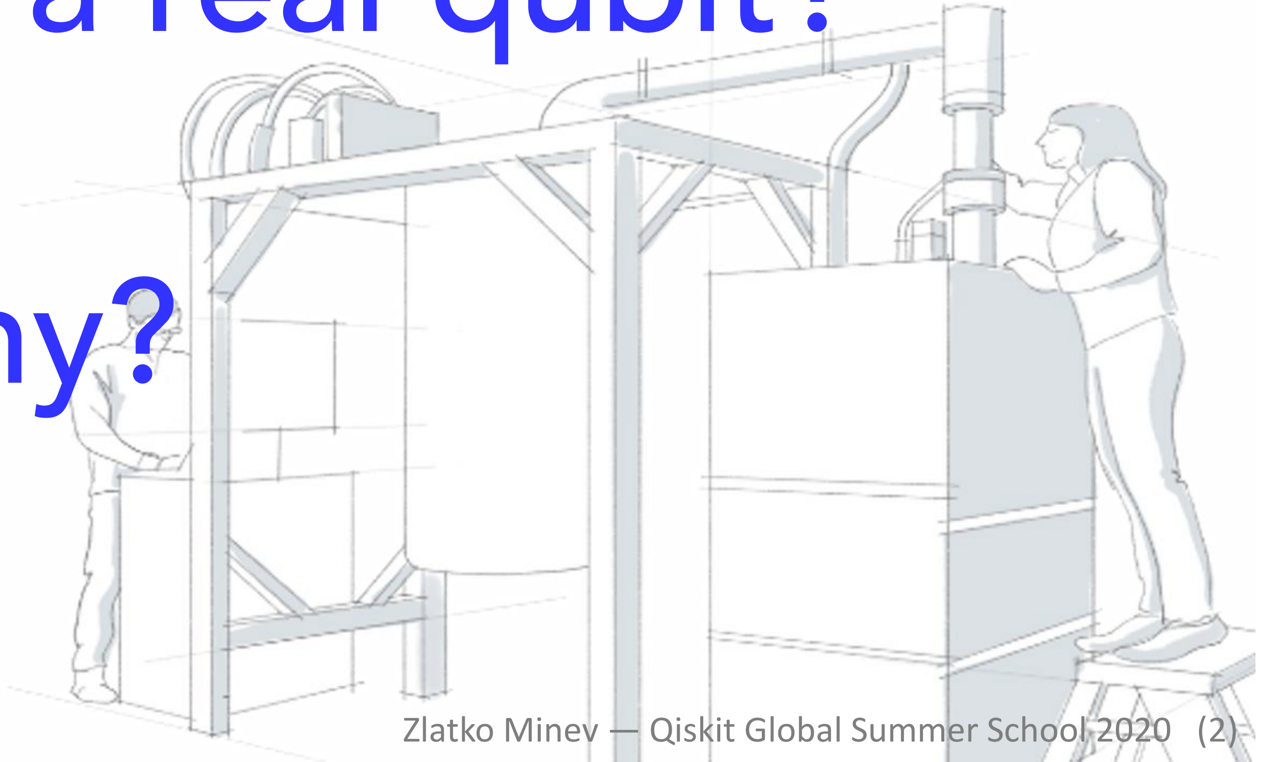
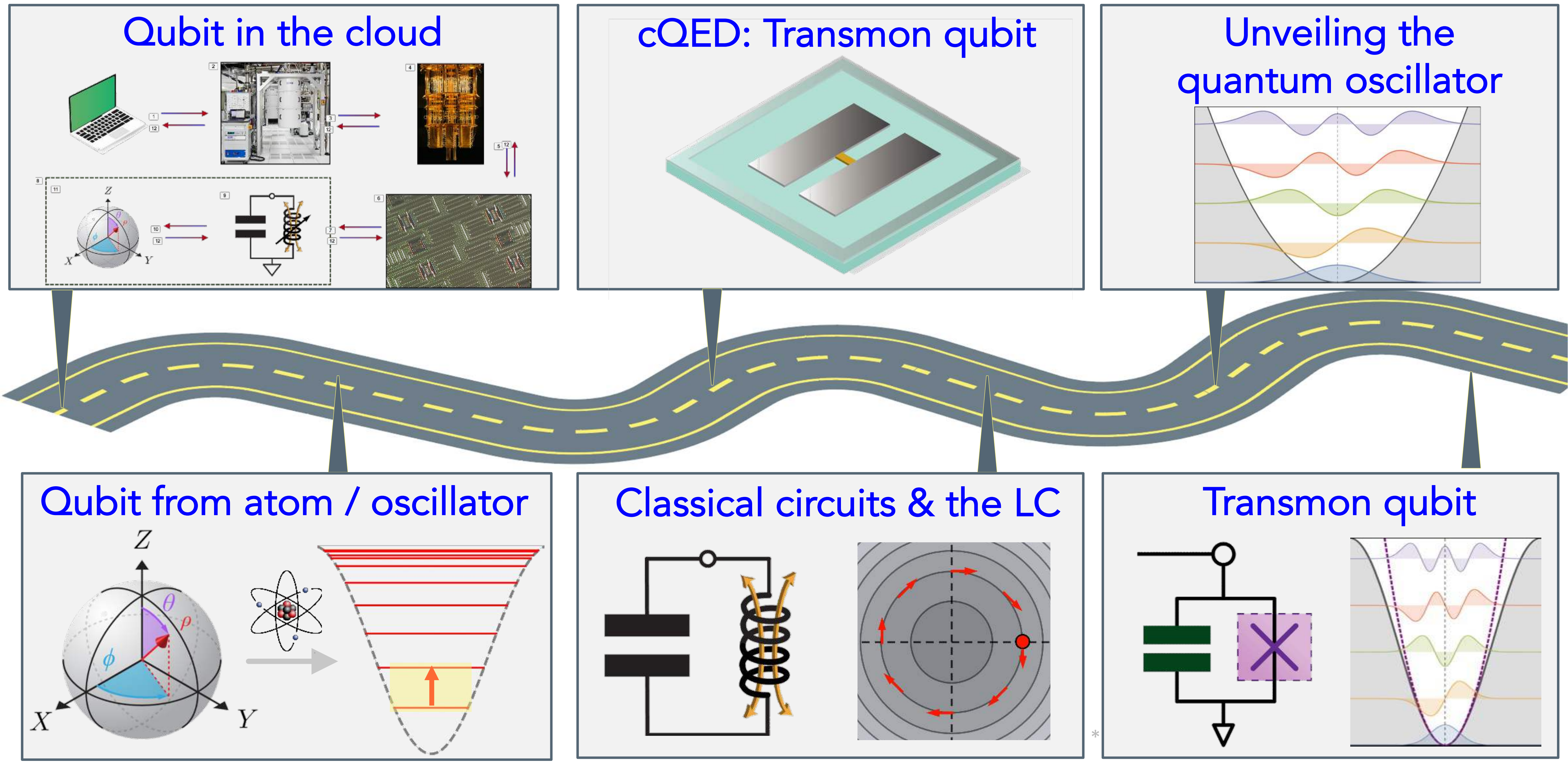


Illustration: IBM Qiskit Textbook

On the road ahead



This Lecture

Introductory and skill reaffirming

Don't need to know much going in, but we will go far

Advanced material

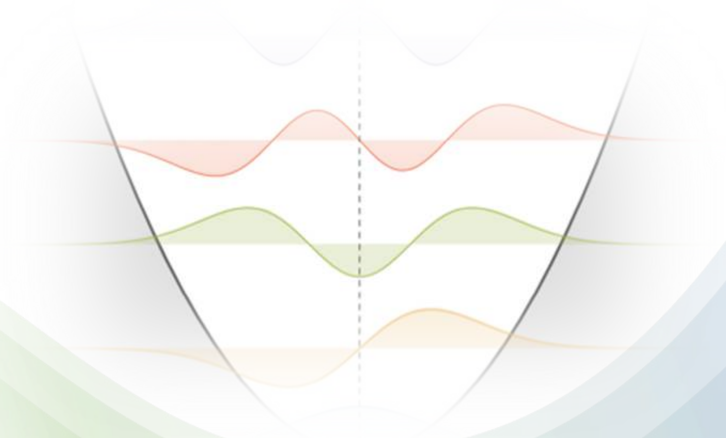
Examples: simplest, most practical examples

Step by step

Ask questions!

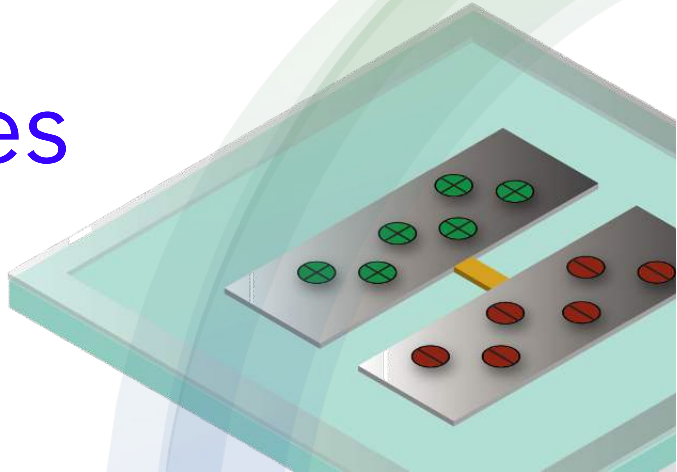
Tightly integrated work by Dr. Thomas McConkey!

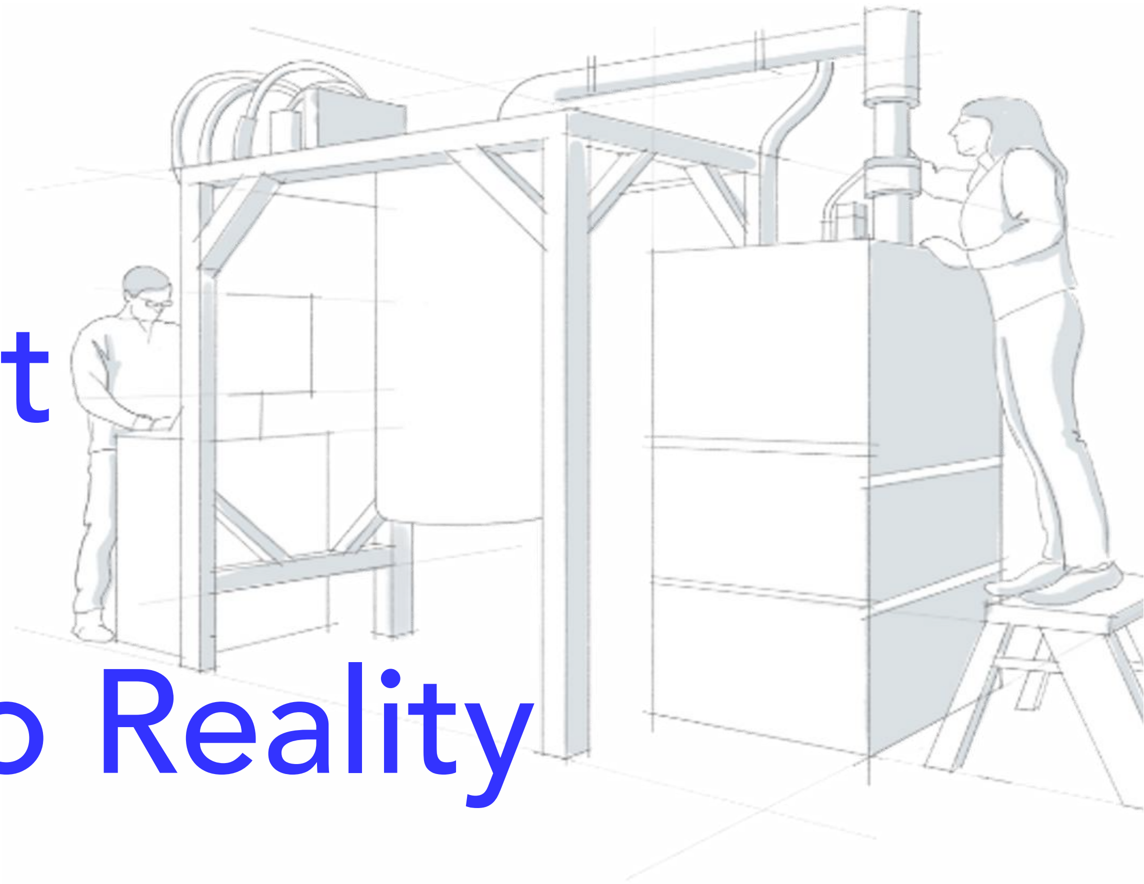
Icons here by FreePik. Dangerous bend symbol based on Donald Knuth.



“Quantum phenomena do *not* occur in a Hilbert space, they occur in a laboratory.”

Asher Peres





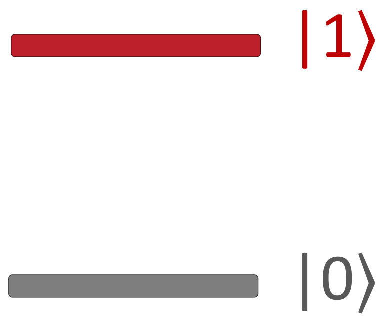
Qubit

From Idea to Reality

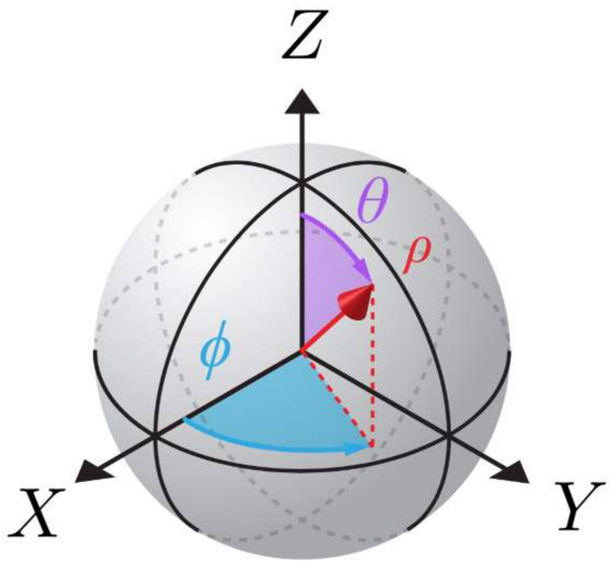
THE BIG PICTURE
before calculations

Qubit: idea

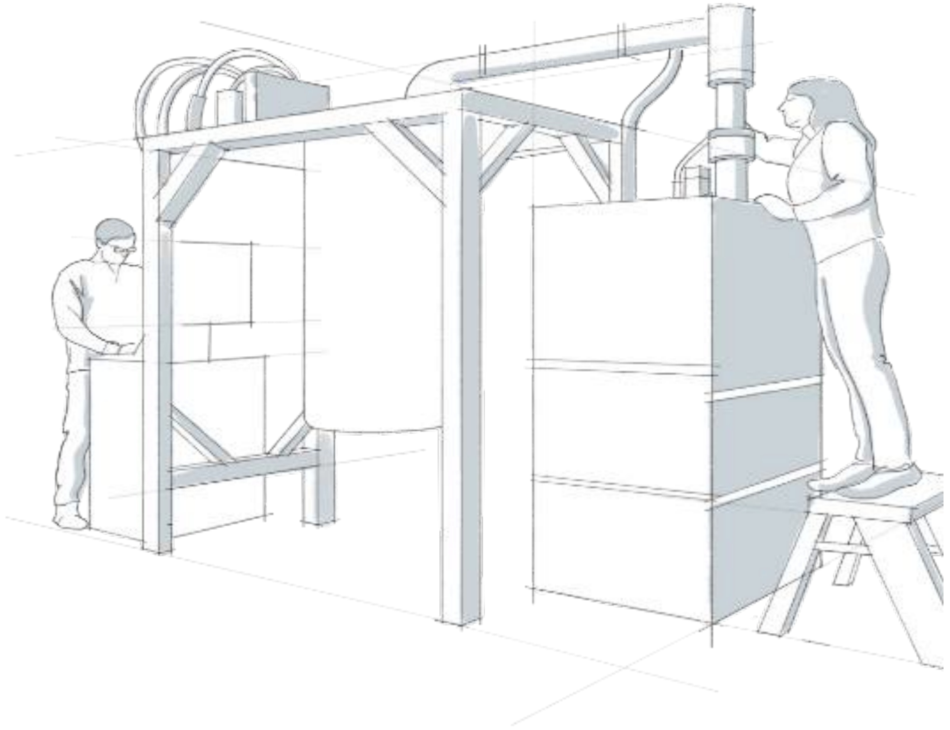
Energy levels



Hilbert space

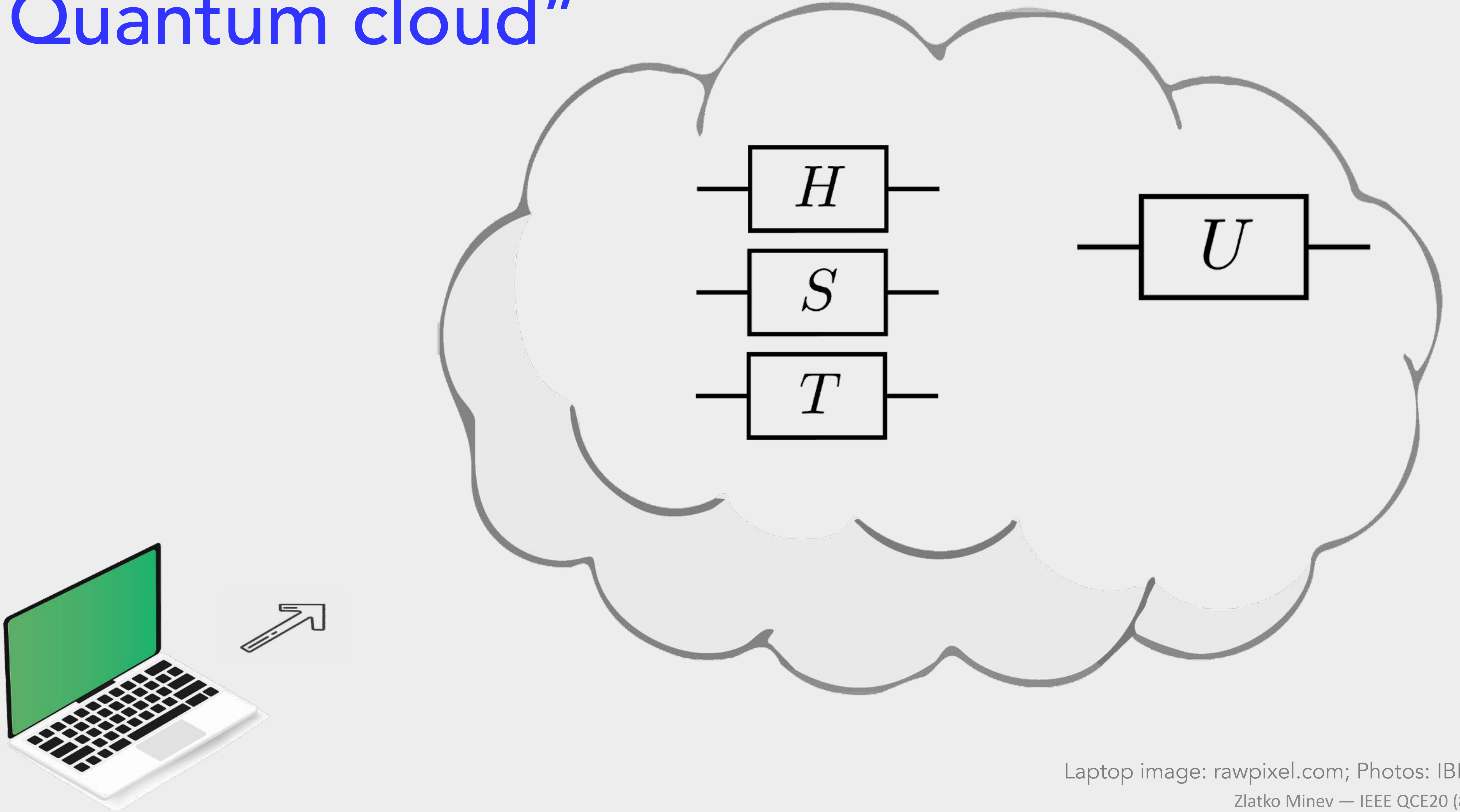


Quantum cloud?



* Images: Minev, arXiv:1902.10355; Illustration on right: IBM Qiskit Textbook;

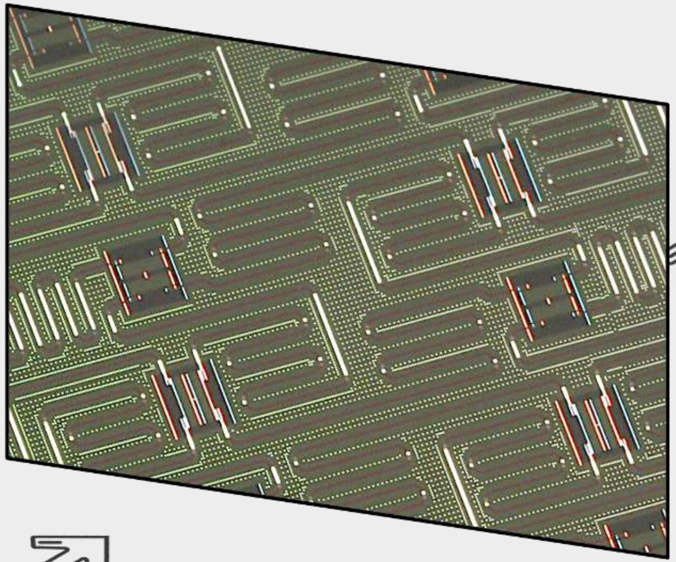
"Quantum cloud"



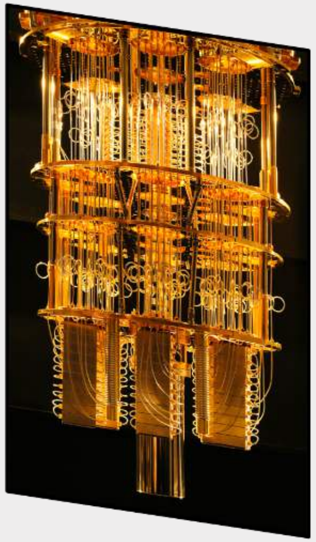
Laptop image: rawpixel.com; Photos: IBM

Quantum in the cloud or lab

Superconducting qubits

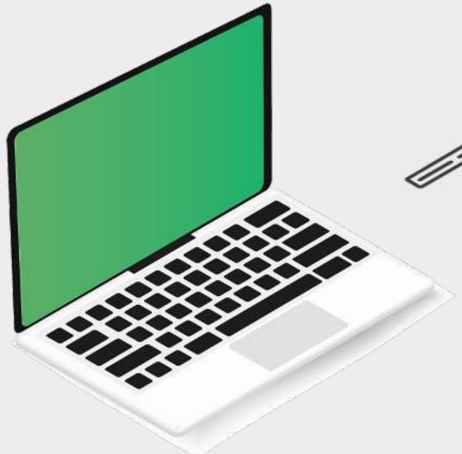


Quantum-device chip



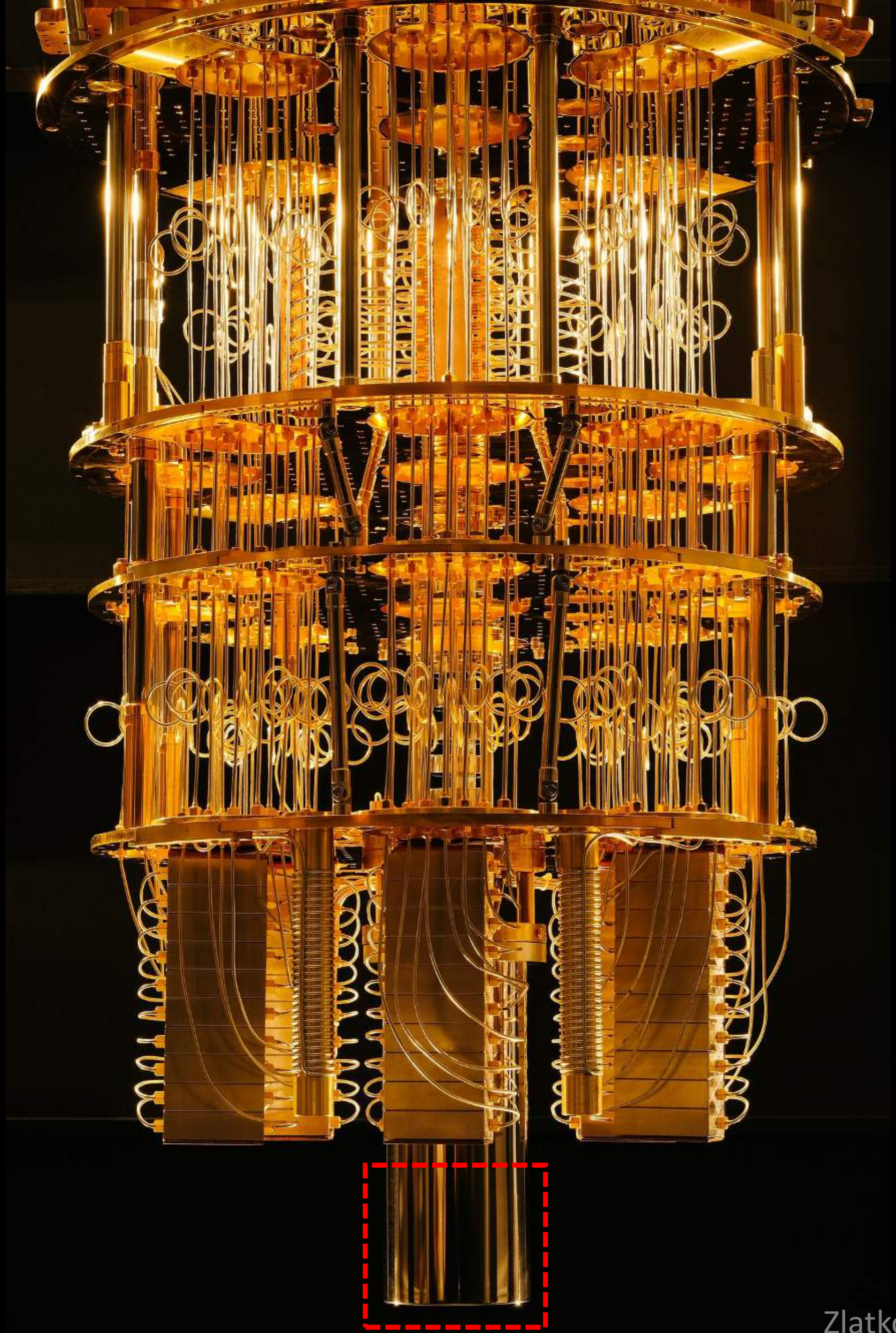
Cryogenic environment

Lab / cloud facility

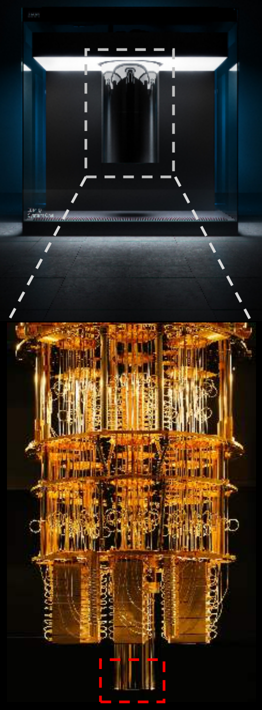


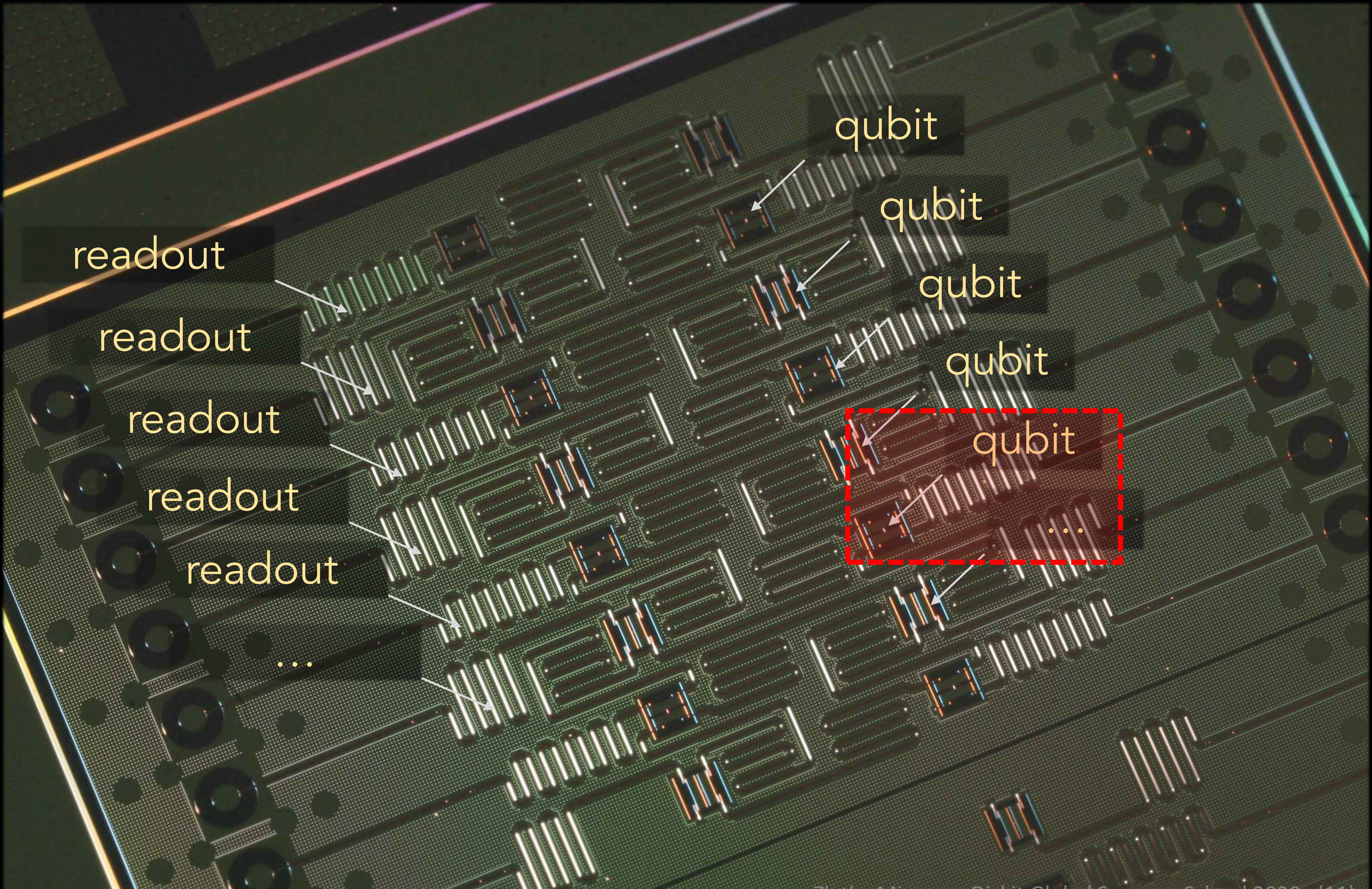
Quantum community

Laptop image: rawpixel.com; Photos: IBM



Operation at
15 mK (-273.13 °C)




qubit
qubit
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...
readout
readout
readout
readout
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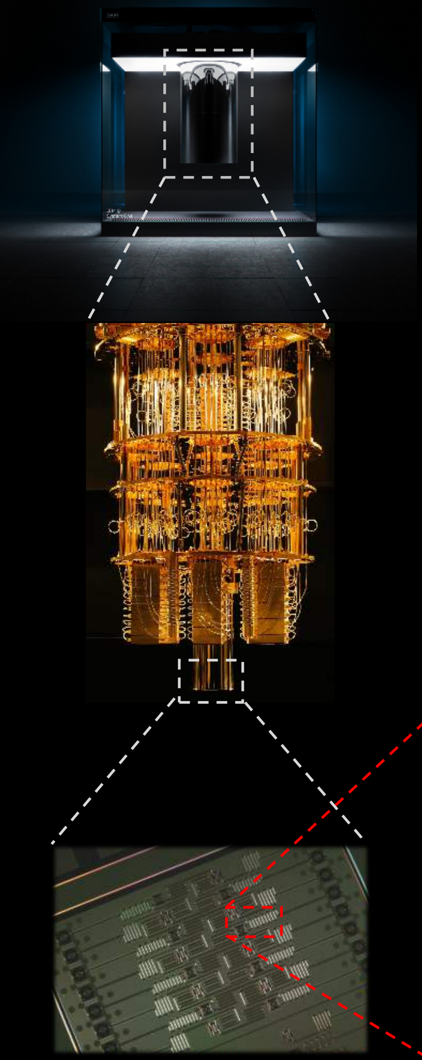
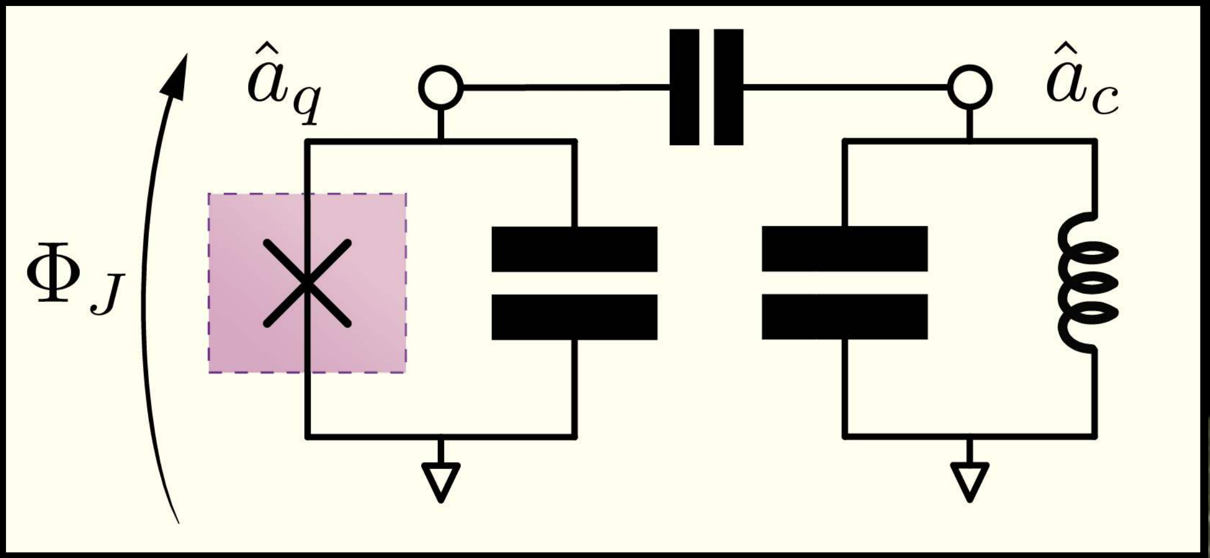
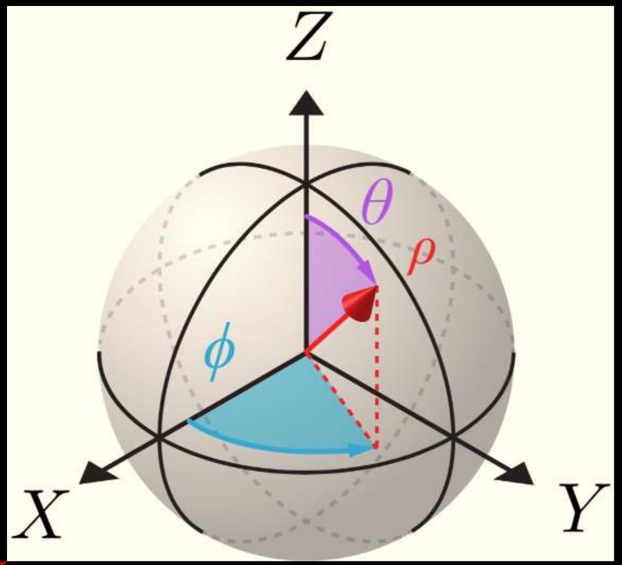
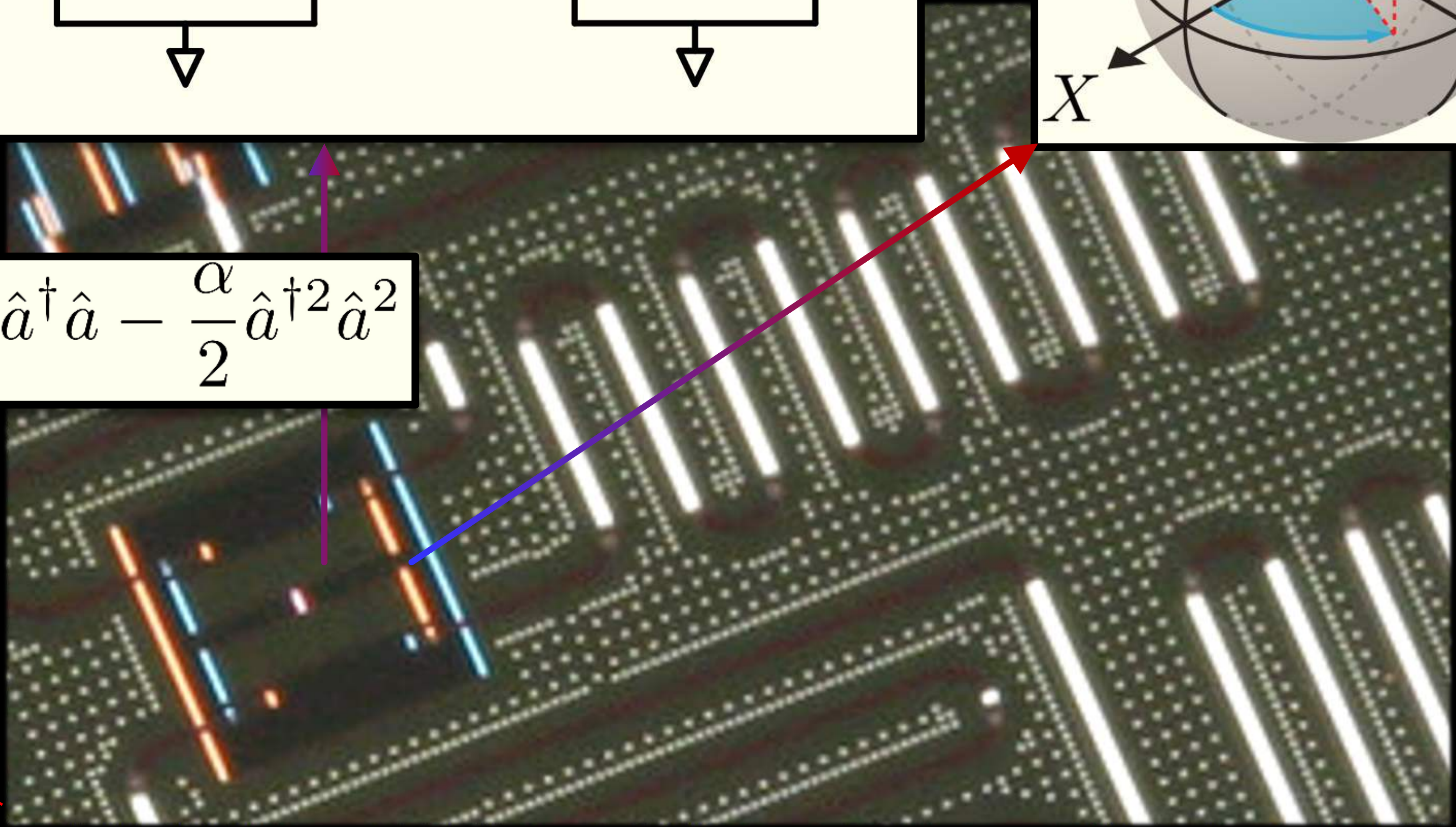
...





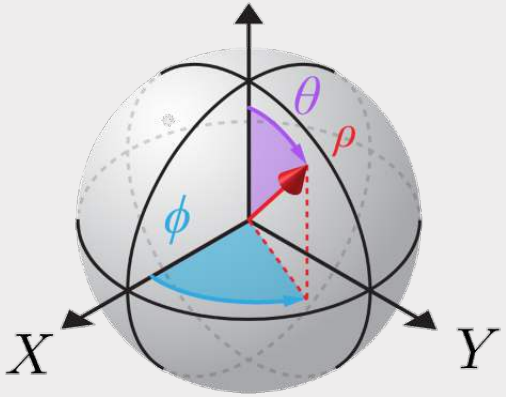




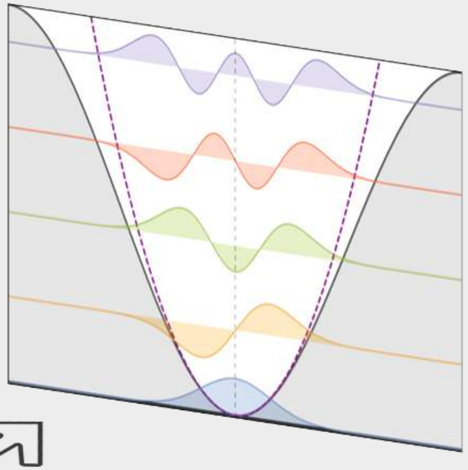
$$\hat{H} \approx \omega_0 \hat{a}^\dagger \hat{a} - \frac{\alpha}{2} \hat{a}^{\dagger 2} \hat{a}^2$$

Quantum Device Design

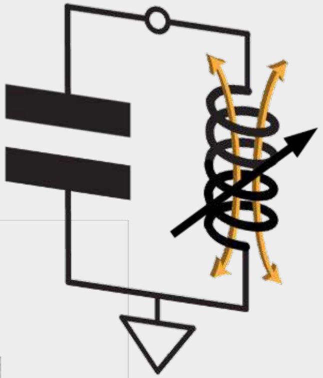
Superconducting circuits



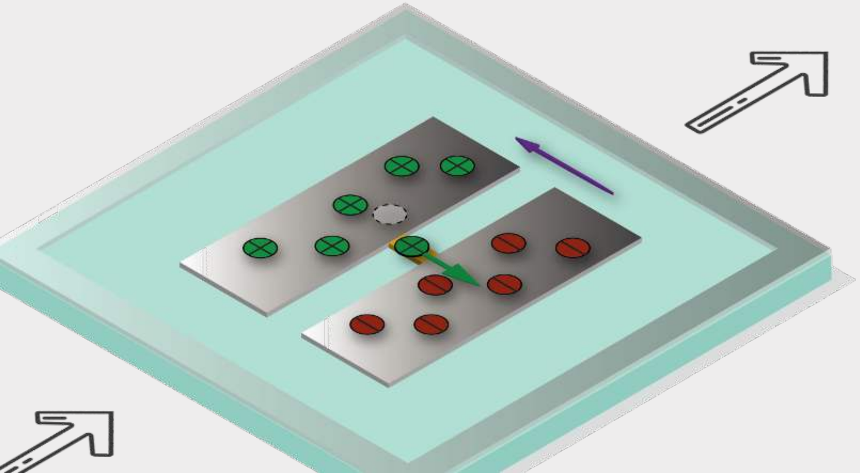
Qubit Hilbert space



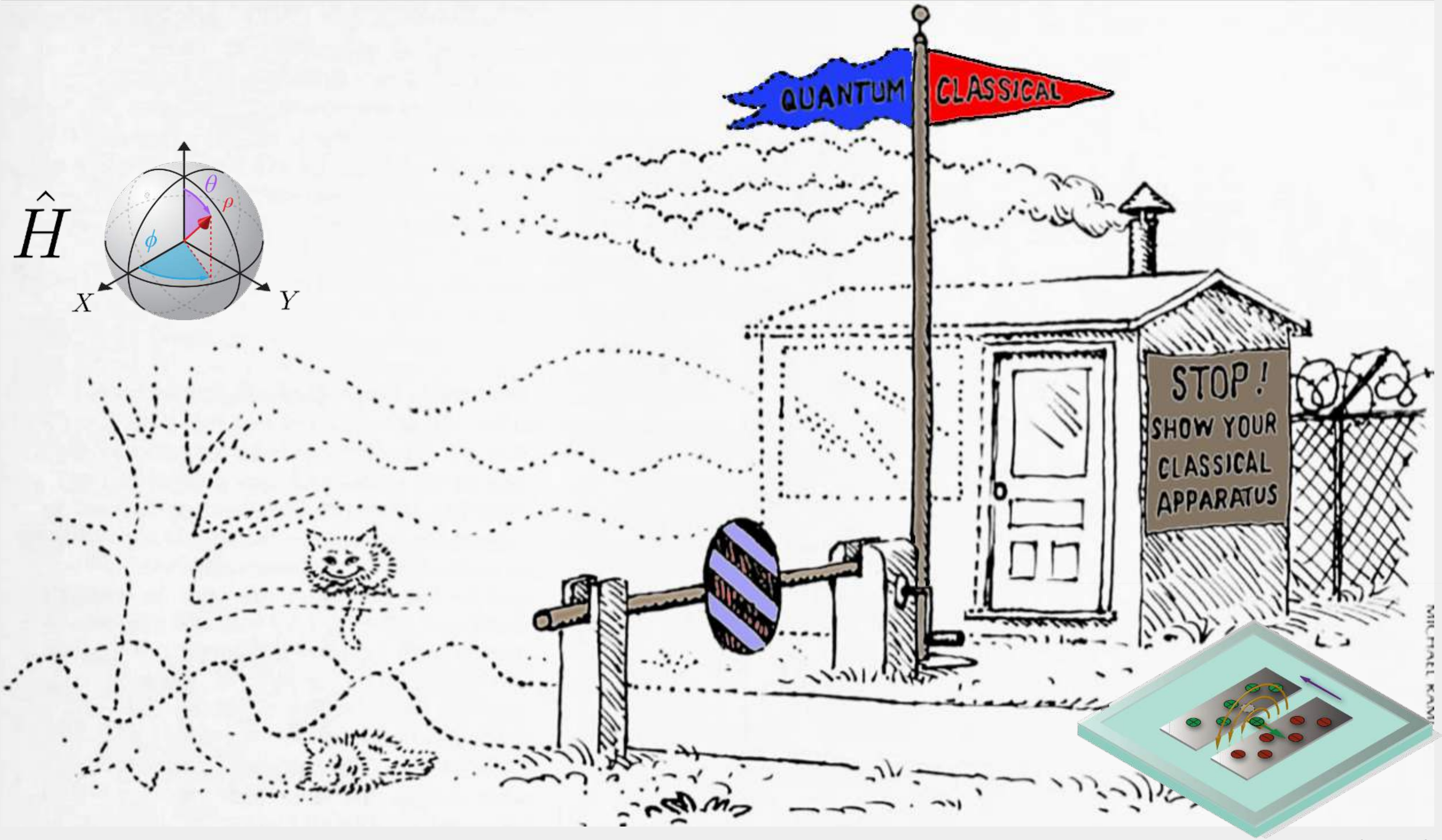
Quantum Hamiltonian analysis



Electromagnetic analysis

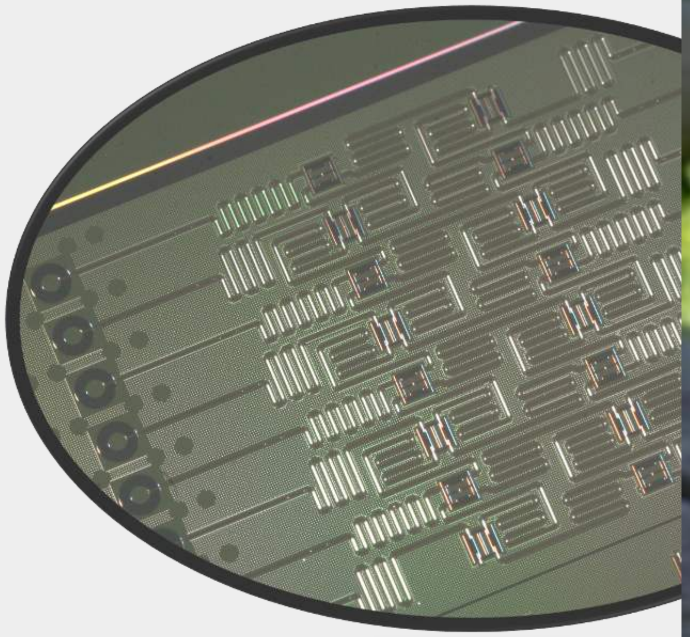


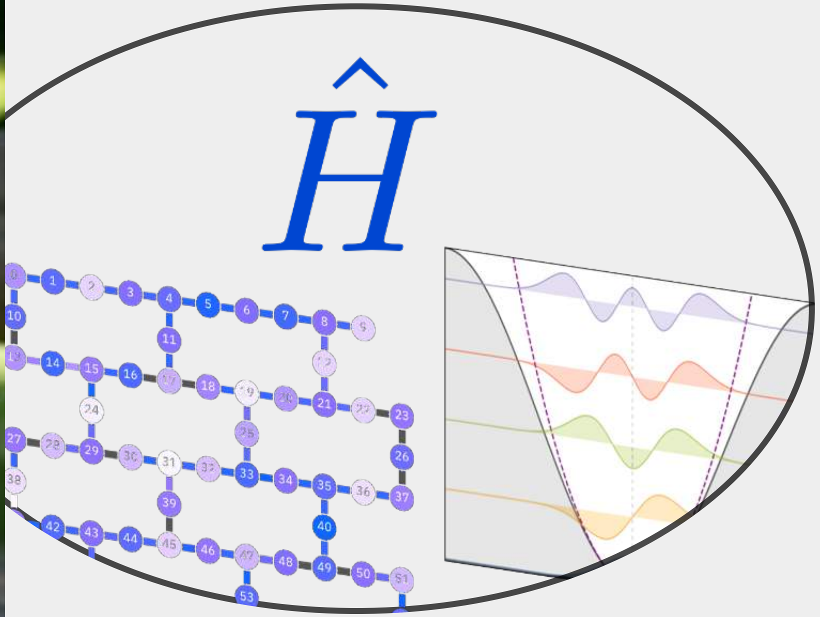
Physical layout



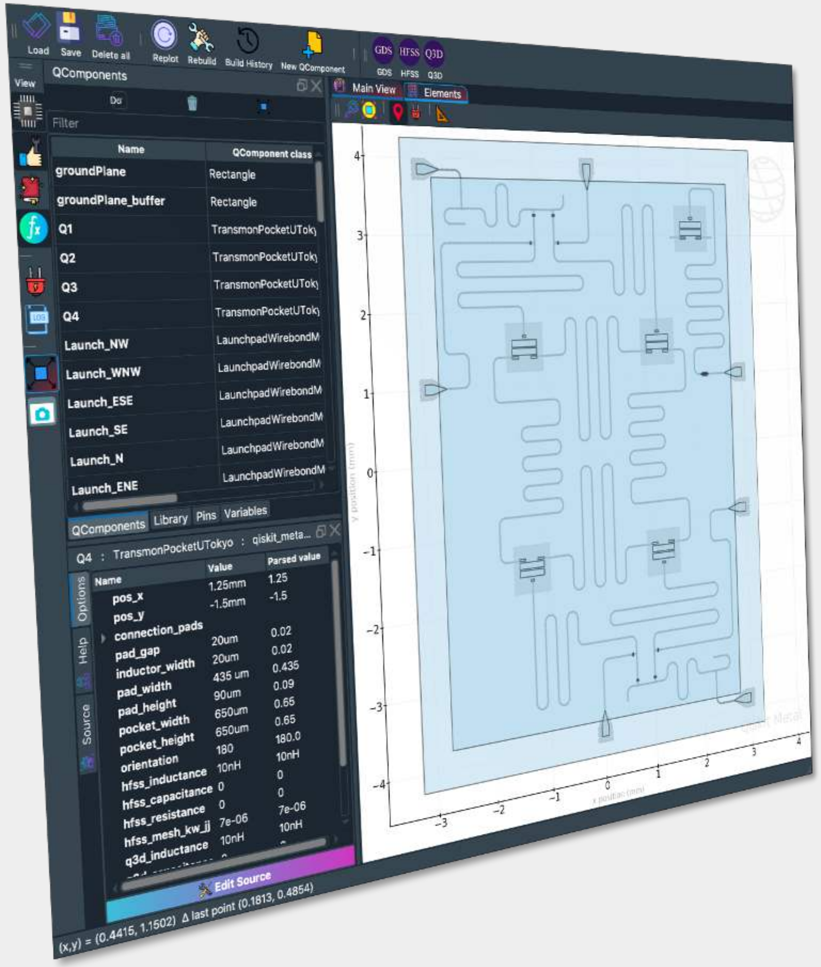
Drawing: Zurek, Physics Today (1991)

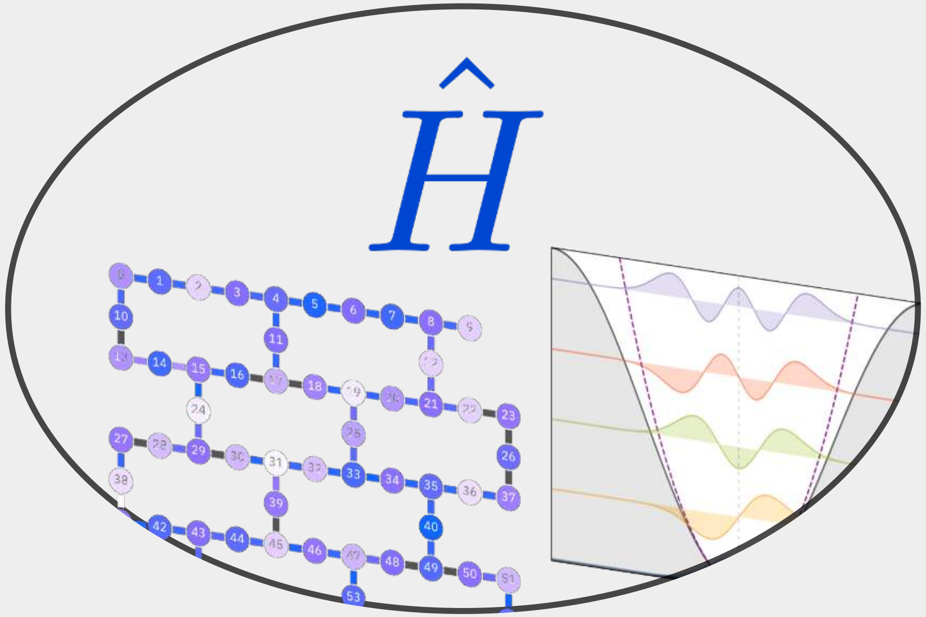
Physical Devices ↔ Quantum Hamiltonian





Make easy?

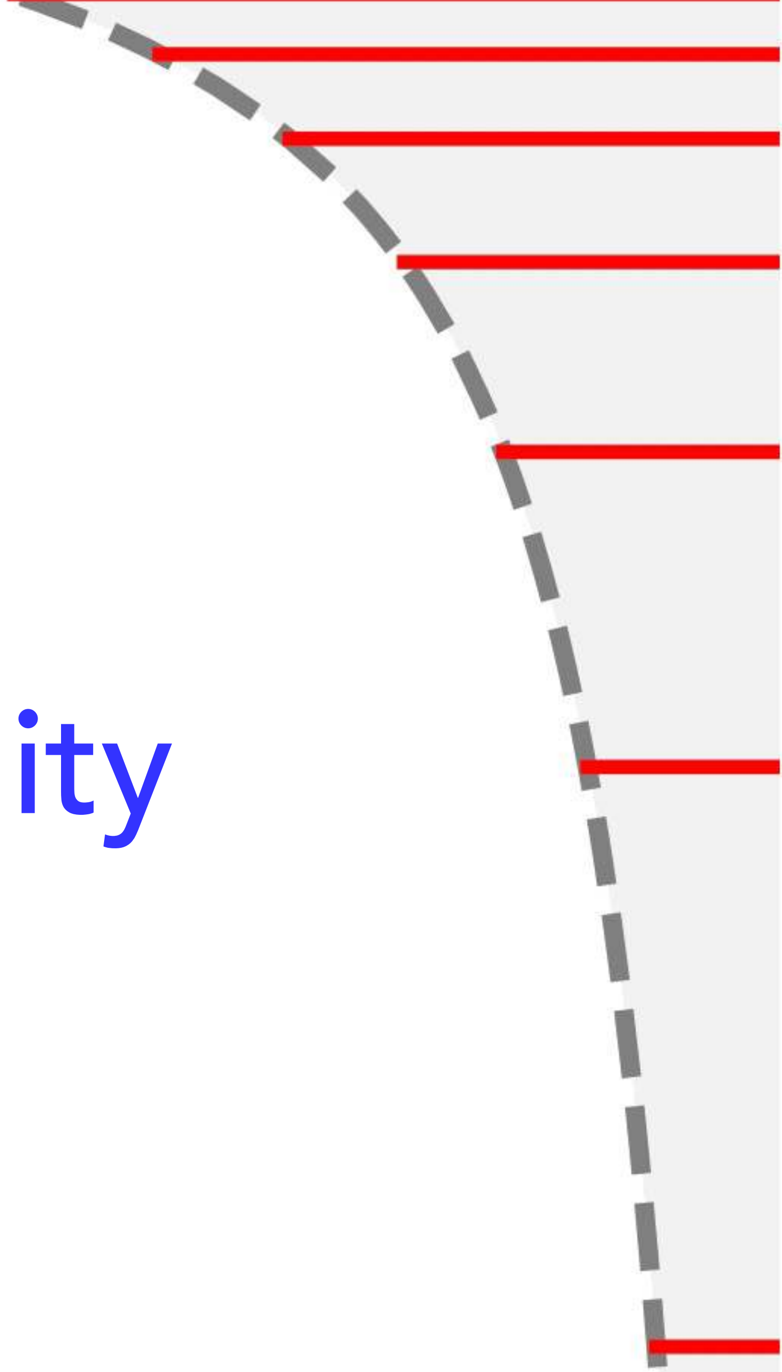

Load
Save
Delete all
Replot
Rebuild
Build History
New QComponent
GDS
HFSS
Q3D
QComponents
Main View
Elements
Filter
Name
QComponent class
groundPlane
Rectangle
groundPlane_buffer
Rectangle
Q1
Q2
Q3
Q4
Launch_NW
Launch_WNW
Launch_ESE
Launch_SE
Launch_N
Launch_ENE
QComponents
Library
Pins
Variables
Q4 : TransmonPocketUTokyo
Name
Value
Parsed value
pos_x
1.25mm
1.25
pos_y
-1.5mm
-1.5
connection_pads
pad_gap
inductor_width
pad_width
pad_height
pocket_width
pocket_height
orientation
hfss_inductance
hfss_capacitance
hfss_resistance
hfss_mesh_kw_jj
q3d_inductance
Edit Source
(x,y) = (0.4415, 1.1502) Δ last point (0.1813, 0.4854)


$\hat{H}$

Qubit

From Idea to Reality

Concepts

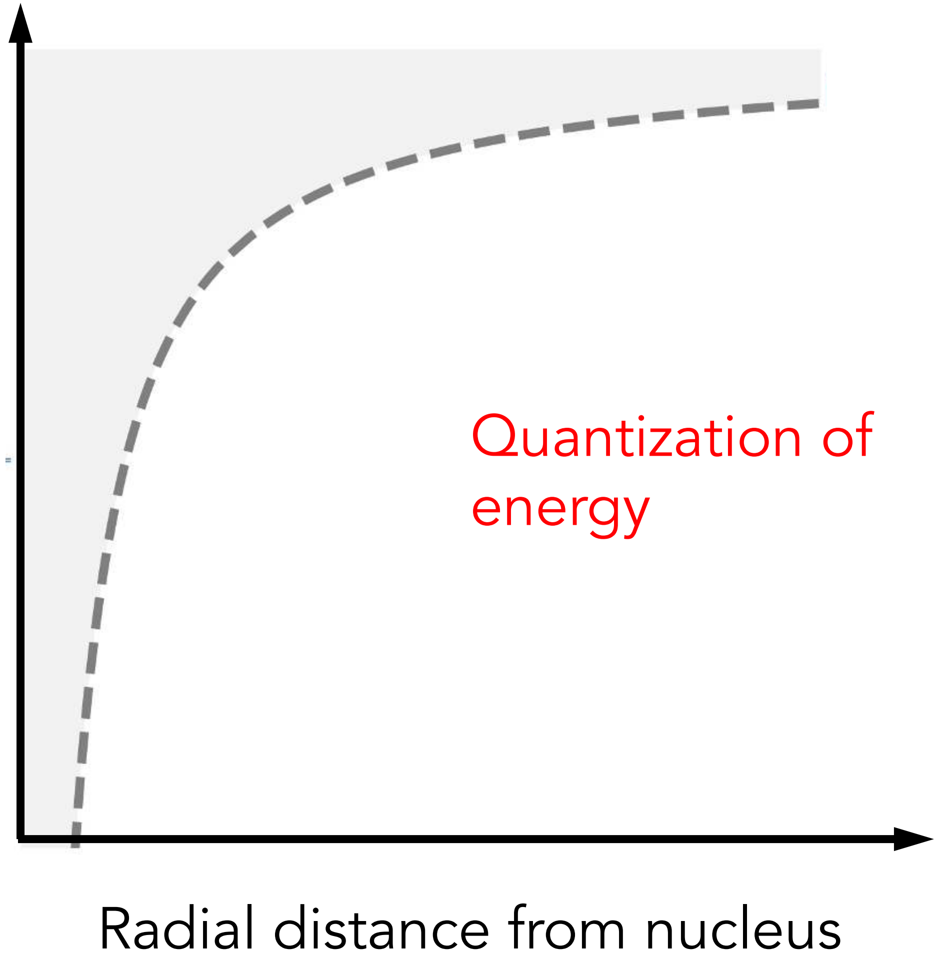


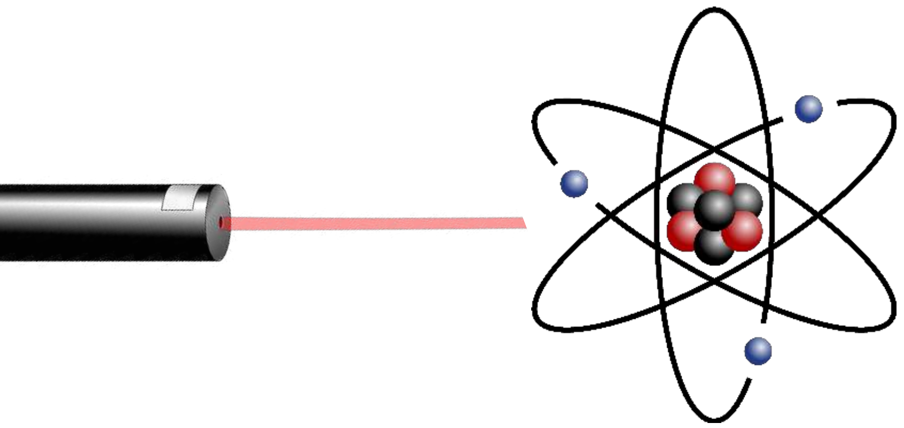
The light of atoms

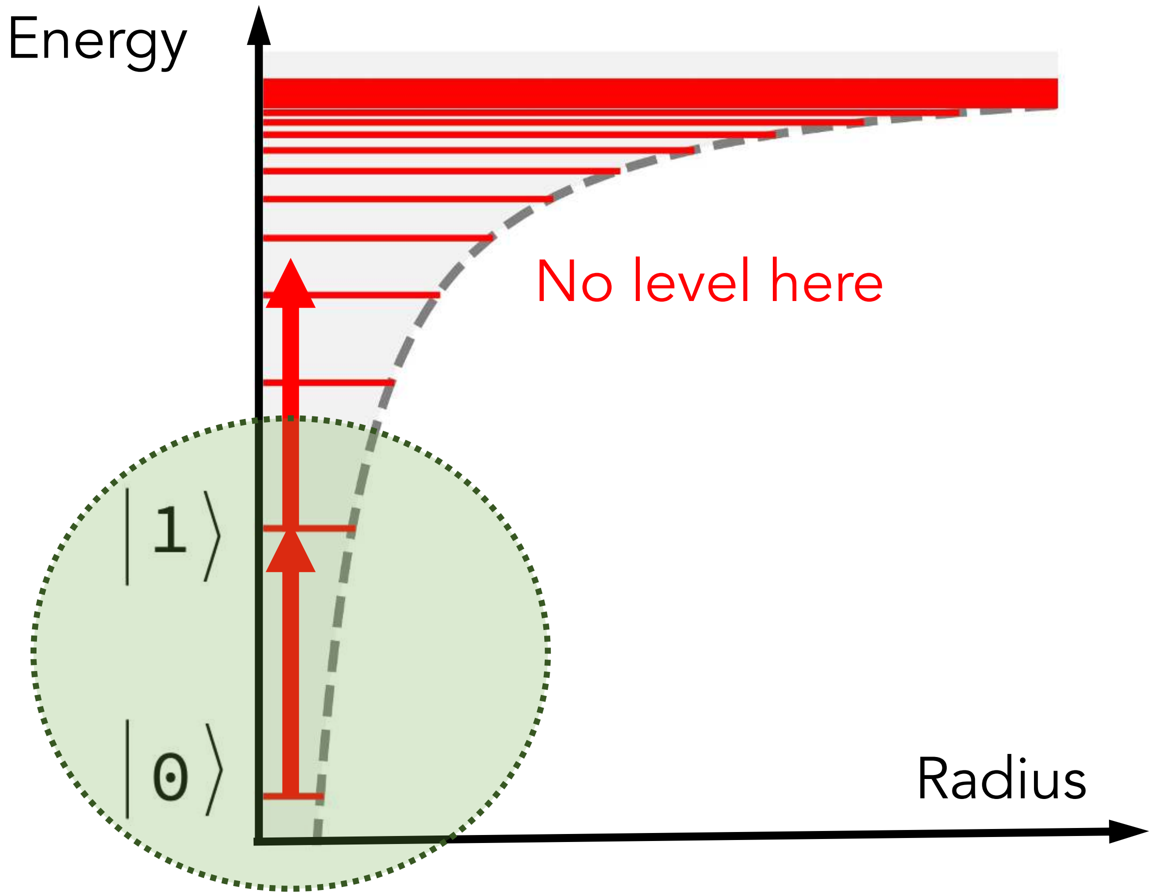
Image: Averill and Eldredge, Principles of General Chemistry

Atomic energy levels and transitions

Electron potential-energy landscape







Qubit from Atom

Anharmonic degree of freedom and spin

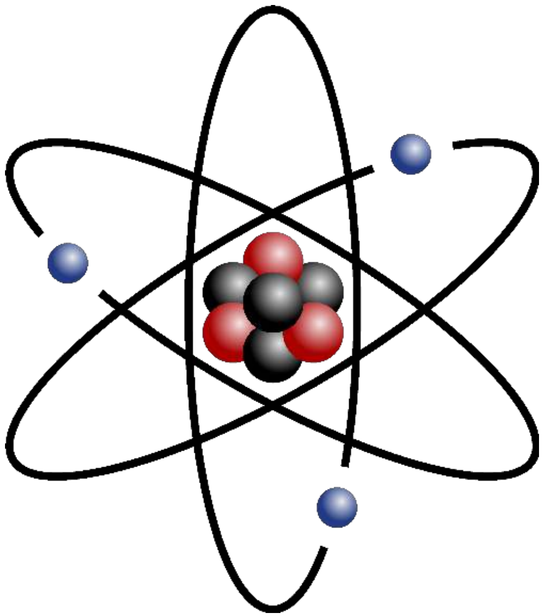
Isolated from environment and thermal bath

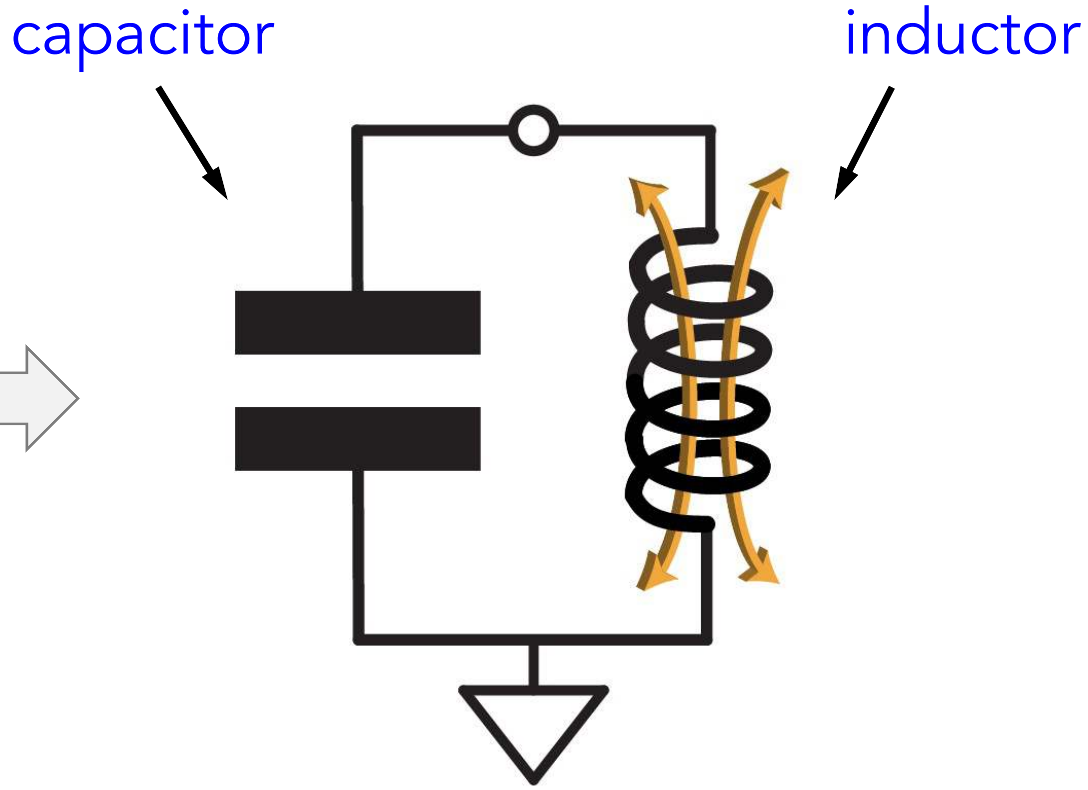
Low-loss

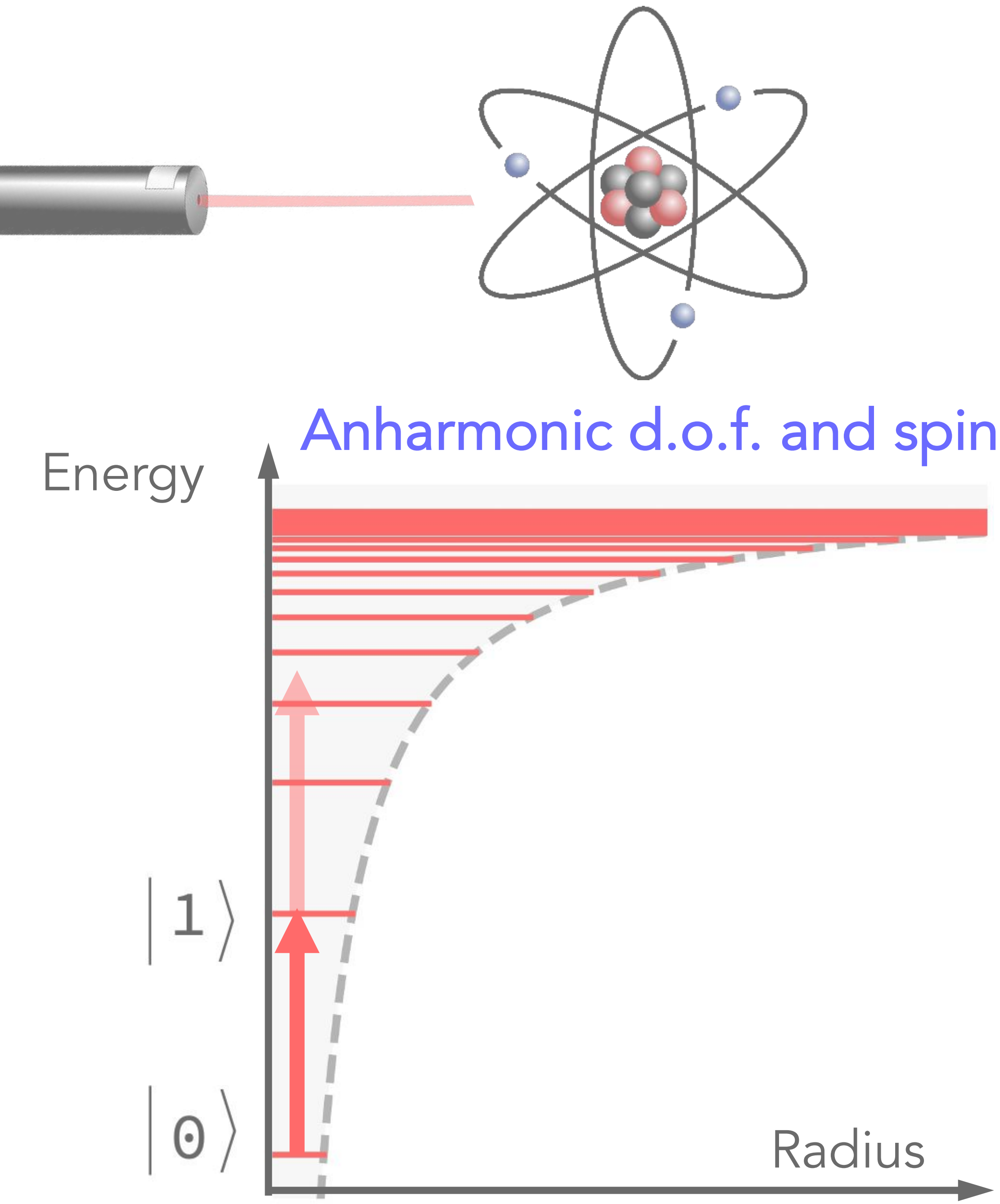
Level diagram allows for qubit-specific control and readout

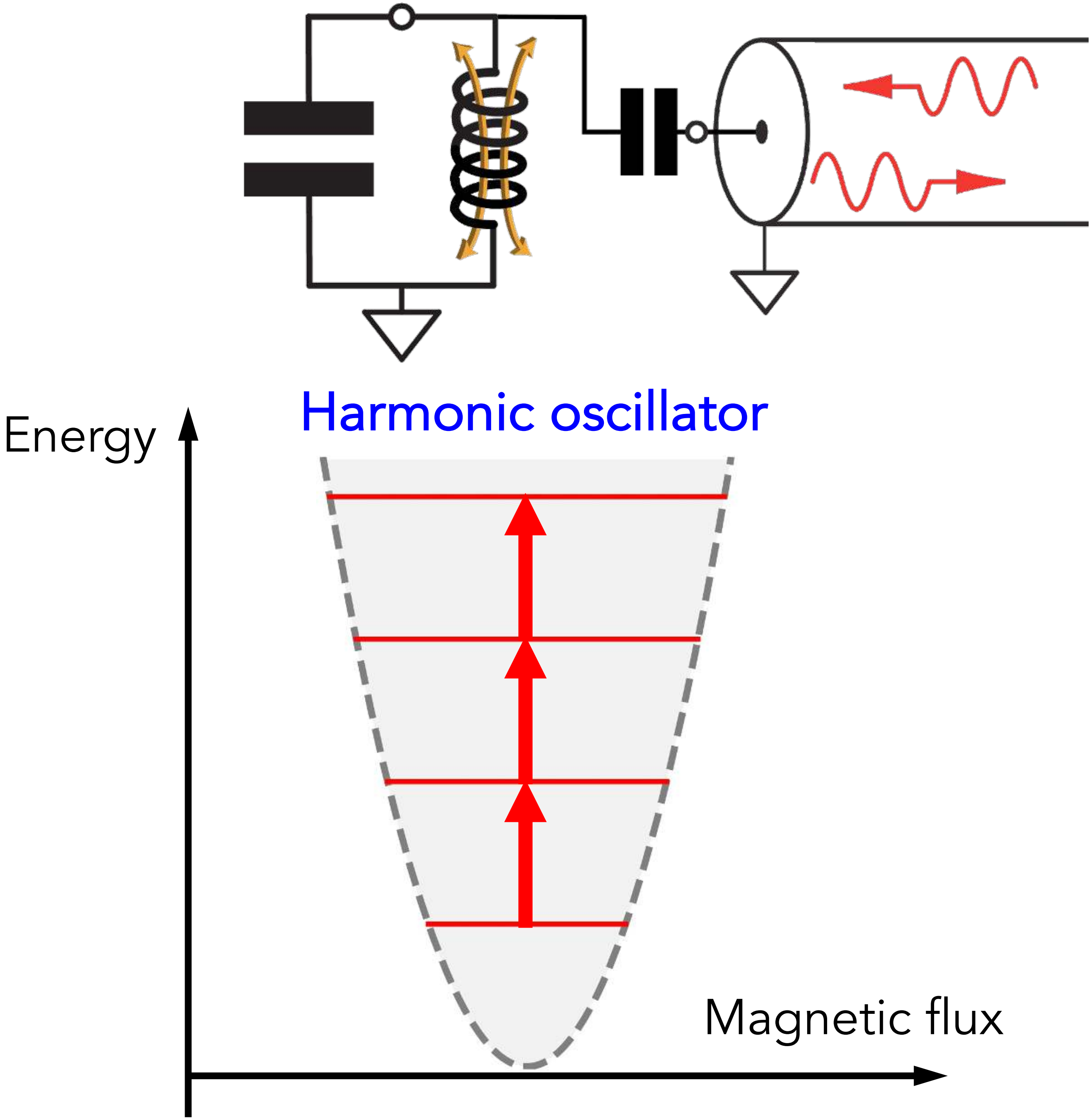
There are always more than two levels

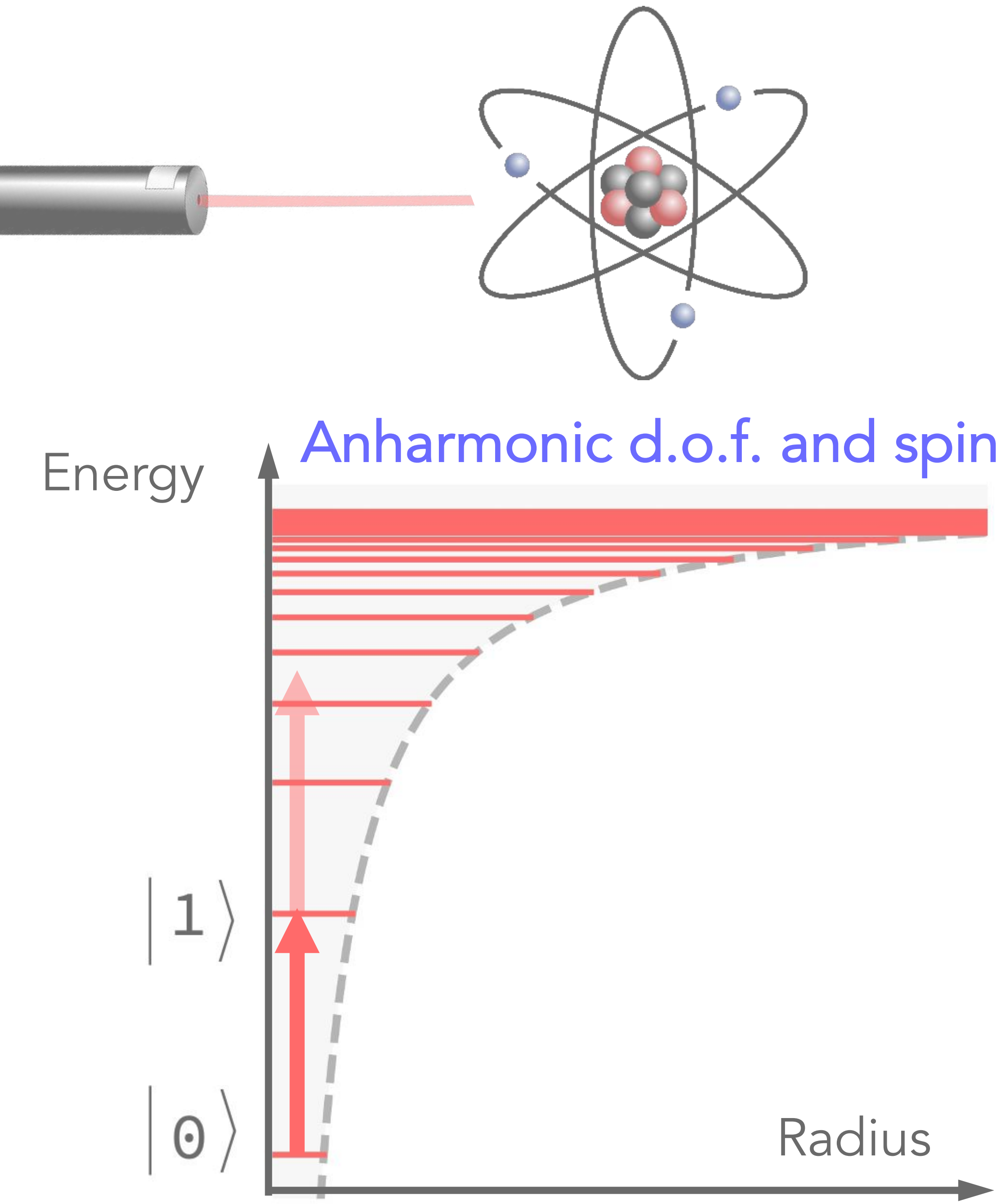
Artificial atoms

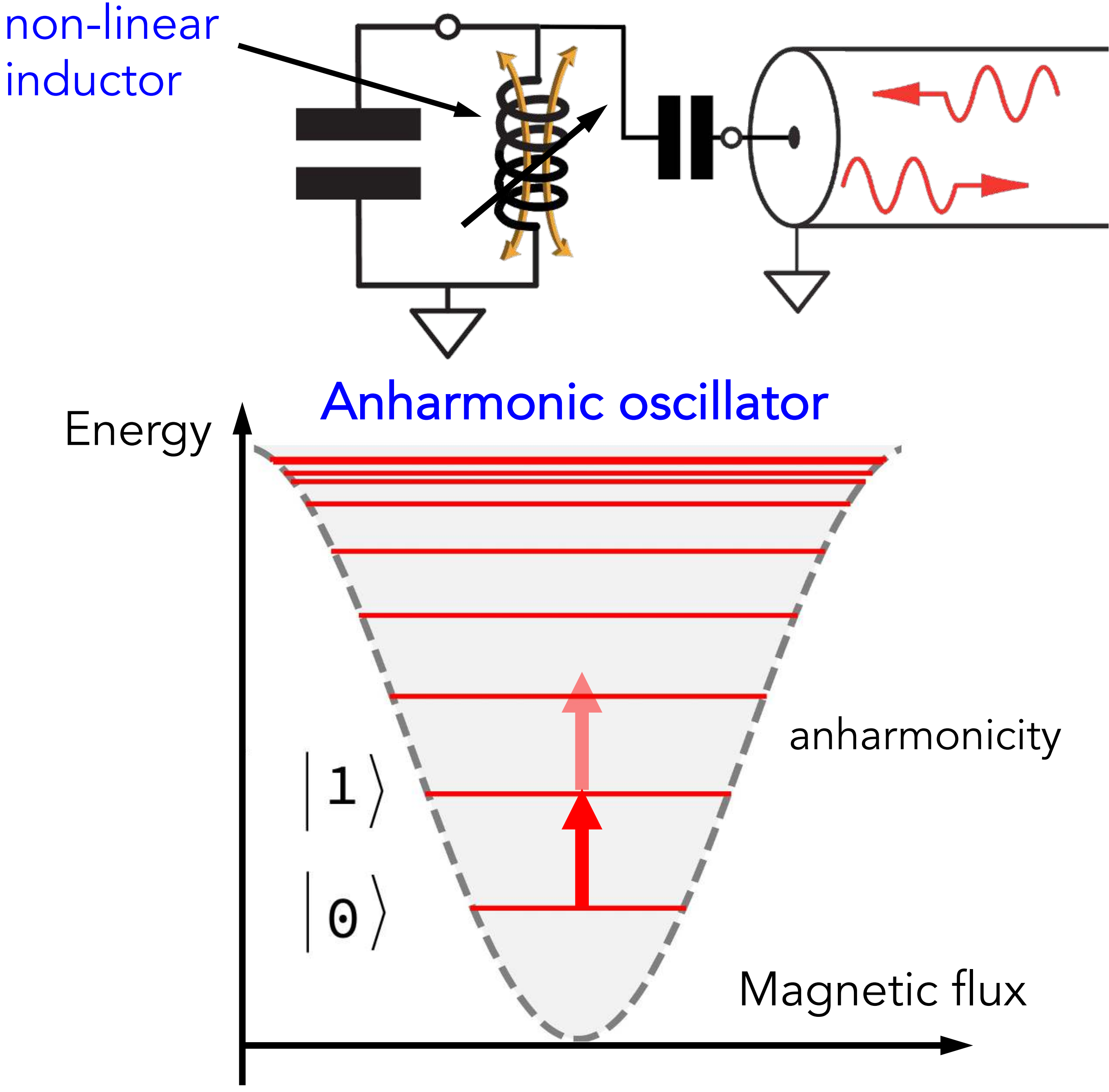





Anharmonic d.o.f. and spin
Energy
|1⟩
|0⟩
Radius

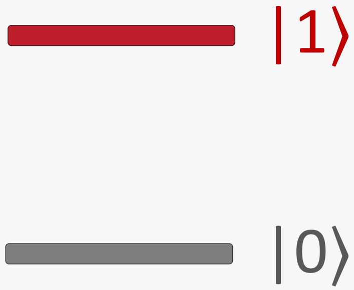

Harmonic oscillator
Energy
Magnetic flux


Anharmonic d.o.f. and spin
Energy
|1⟩
|0⟩
Radius

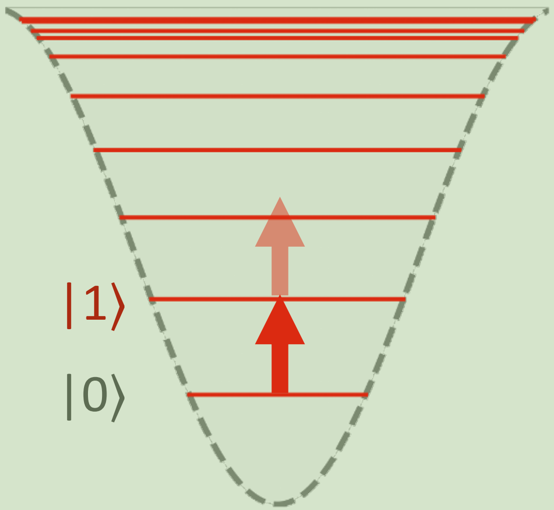

non-linear
inductor
Anharmonic oscillator
Energy
anharmonicity
|1⟩
|0⟩
Magnetic flux

Big-picture connections

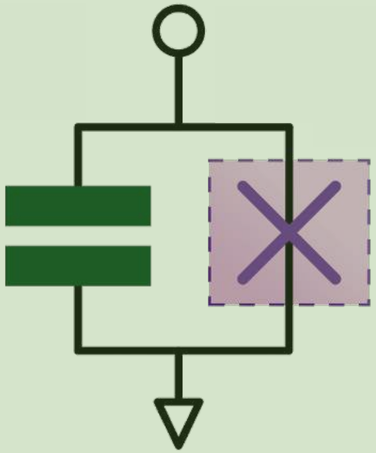
Idealization of *qubit*



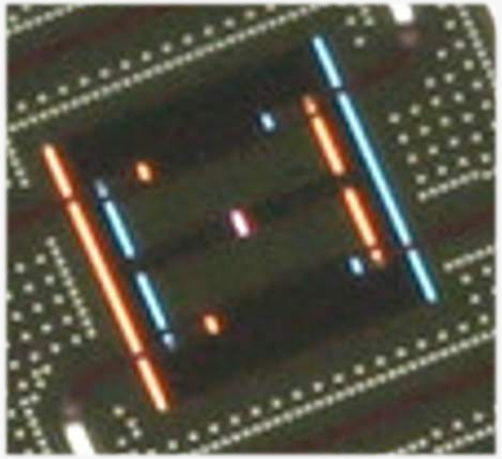
Anharmonic *oscillator*



Physical circuit model



Physical layout



Idealization → Physical reality

Circuit Quantum Electrodynamics (cQED)

Macro
overview

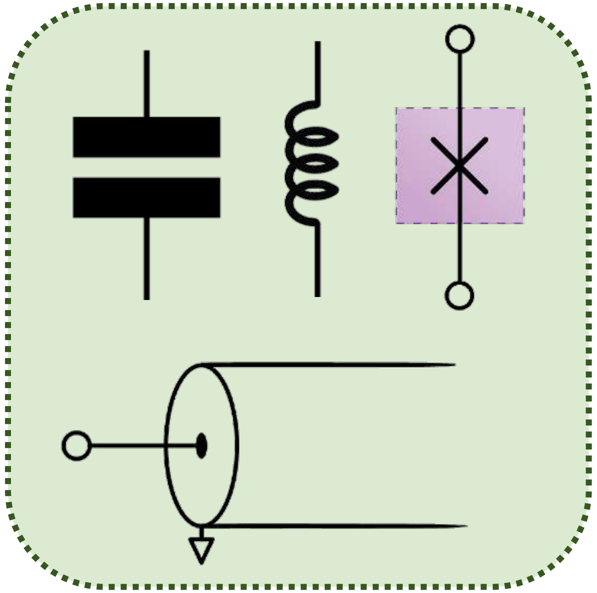
There are two kinds of physicists:

Those who believe all of physics is *spins*.

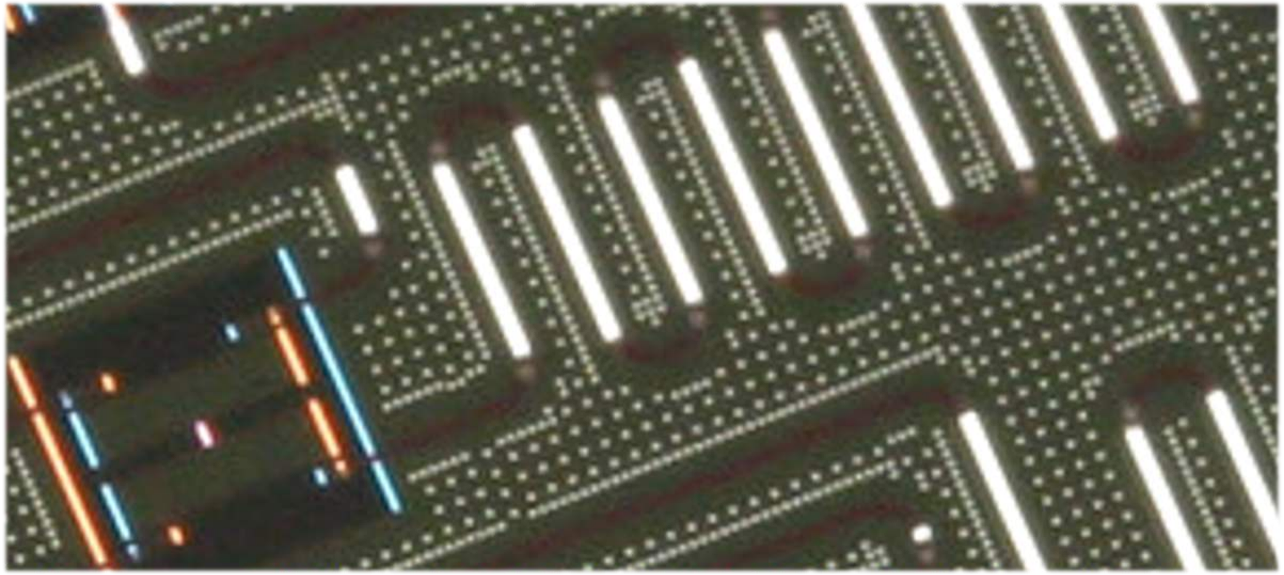
Those who believe all of physics is *oscillators*.

cQED Ingredients

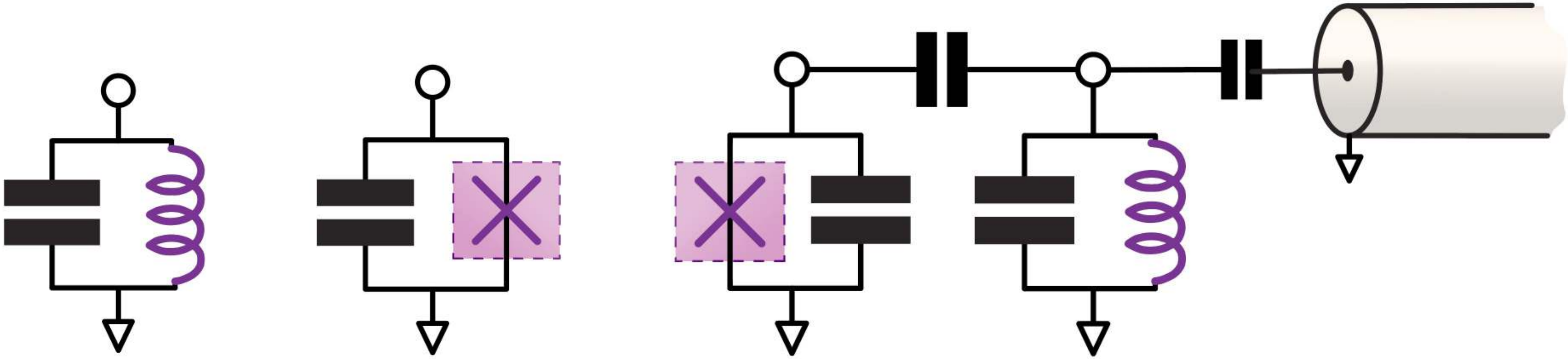
Circuit elements



Microwave oscillators

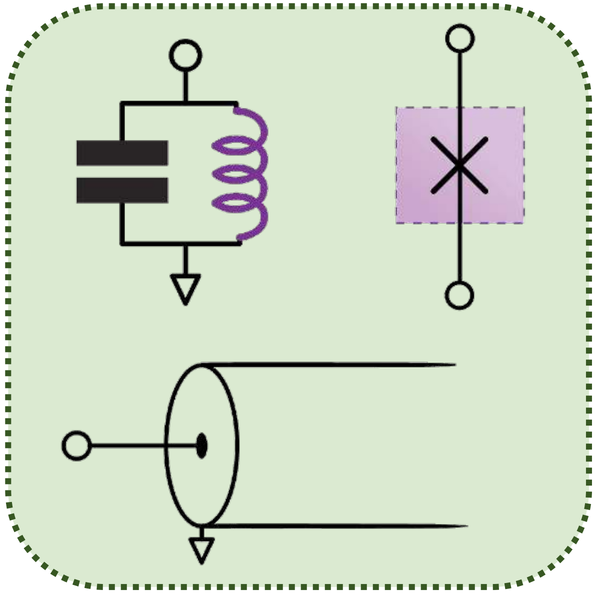


Combine



…

cQED Ingredients



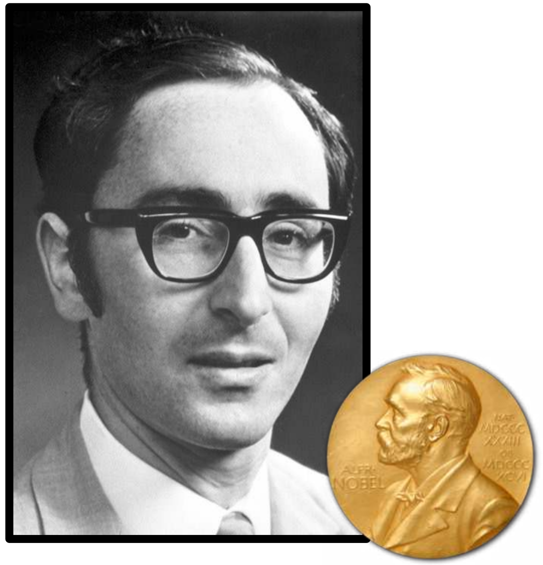
Small dissipation

Isolation from environment

Low temperature

Nonlinearity

Large vacuum fluctuations



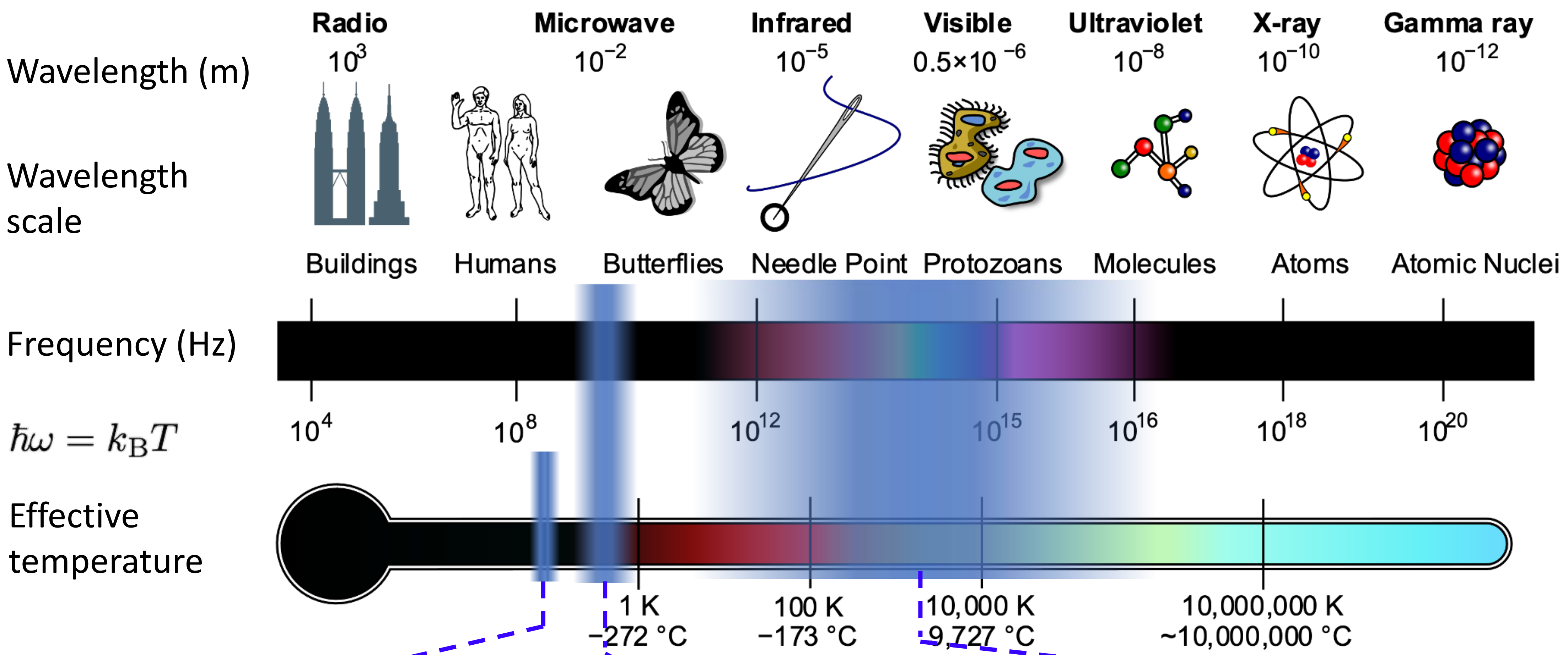
Superconductivity

Nominally zero intrinsic dissipation and heat

Nominal temperature far below energy level splitting

Non-linear, robust Josephson tunnel junction effect

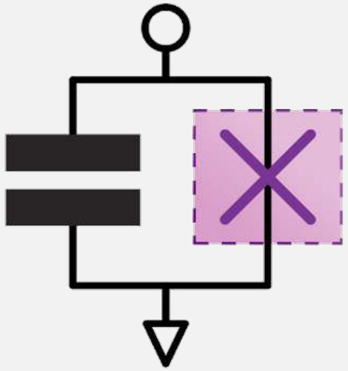
* Low energy dynamics. Microwave image: vectorpocket; Nobel photograph: Jonathunder; Josephson photograph: Nobel Foundation archive.

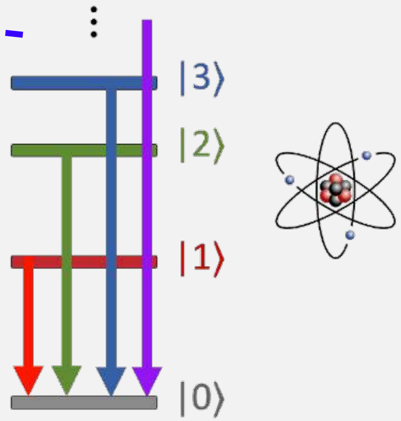


Order of magnitude

10^{-2} K

0.5 K, 10 GHz, 3 cm





Spectrum image: Inductiveload, NASA

A few introductory reviews

And many more… check online or ask us for specific topic

Qiskit Textbook (2020; more chapters coming)

Blais, A., Grimsmo, A. L., Girvin, S. M., & Wallraff, A. (2020)
Circuit Quantum Electrodynamics (arXiv:2005.12667)

Kjaergaard, M., Schwartz, … Oliver, W. D. *(2020)*
Superconducting Qubits: Current State of Play
Annual Reviews of Condensed Matter Physics 11, 369-395

Krantz, P., Kjaergaard, M., Yan, F., … & Oliver, W. D. (2019)
A quantum engineer's guide to superconducting qubits
Applied Physics Reviews, *6*(2), 021318

Corcoles, A. D., Kandala, A., … Gambetta, J. M. (2019)
Challenges and Opportunities of Near-Term Quantum Computing Systems. *Proceedings of the IEEE*, 1–15.

Wendin, G. (2017)
Quantum information processing with superconducting circuits. *Reports on Progress in Physics*, *80*(10), 106001

Gambetta, J. M., Chow, J. M., & Steffen, M. (2017)
Building logical qubits in a superconducting quantum computing system. *Npj Quantum Information*, *3*(1), 2

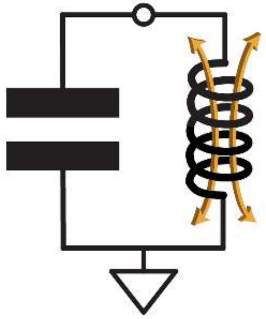
Girvin, S. M. (2011) Circuit QED: superconducting qubits coupled to microwave photons. *Quantum machines: measurement and control of engineered quantum systems*, *113*, 2.

Clerk, A. A., Girvin, S. M., Marquardt, F., & Schoelkopf, R. J. (2010)
Introduction to quantum noise, measurement, and amplification
Reviews of Modern Physics, *82*(2), 1155–1208

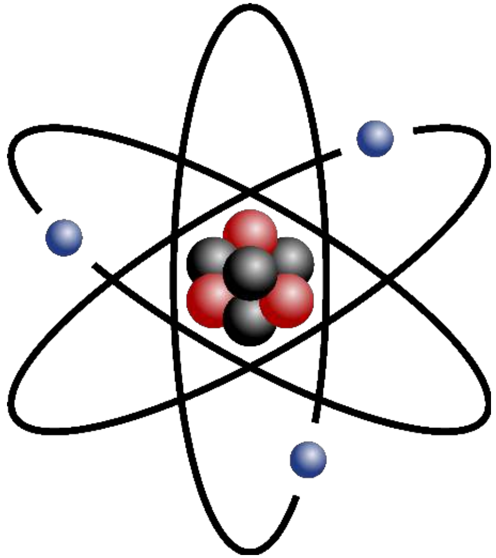
Clarke, J., & Wilhelm, F. K. (2008)
Superconducting quantum bits. *Nature*, *453*(7198), 1031–1042

Devoret, M. H. (1997)
Quantum Fluctuations in Electrical Circuits.
In *Fluctuations Quantiques/Quantum Fluctuations* (p. 351)

…



Circuit Quantum Electrodynamics

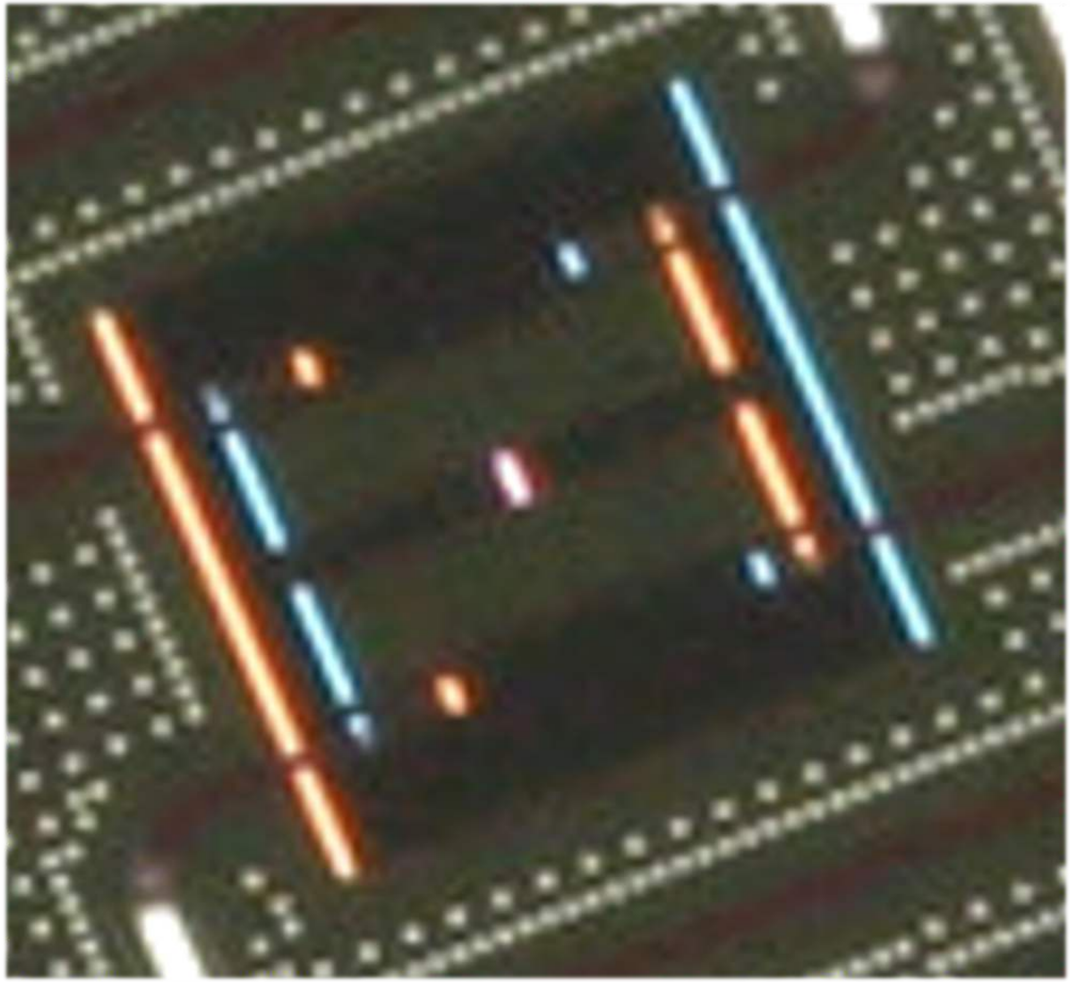


To do **quantum**,

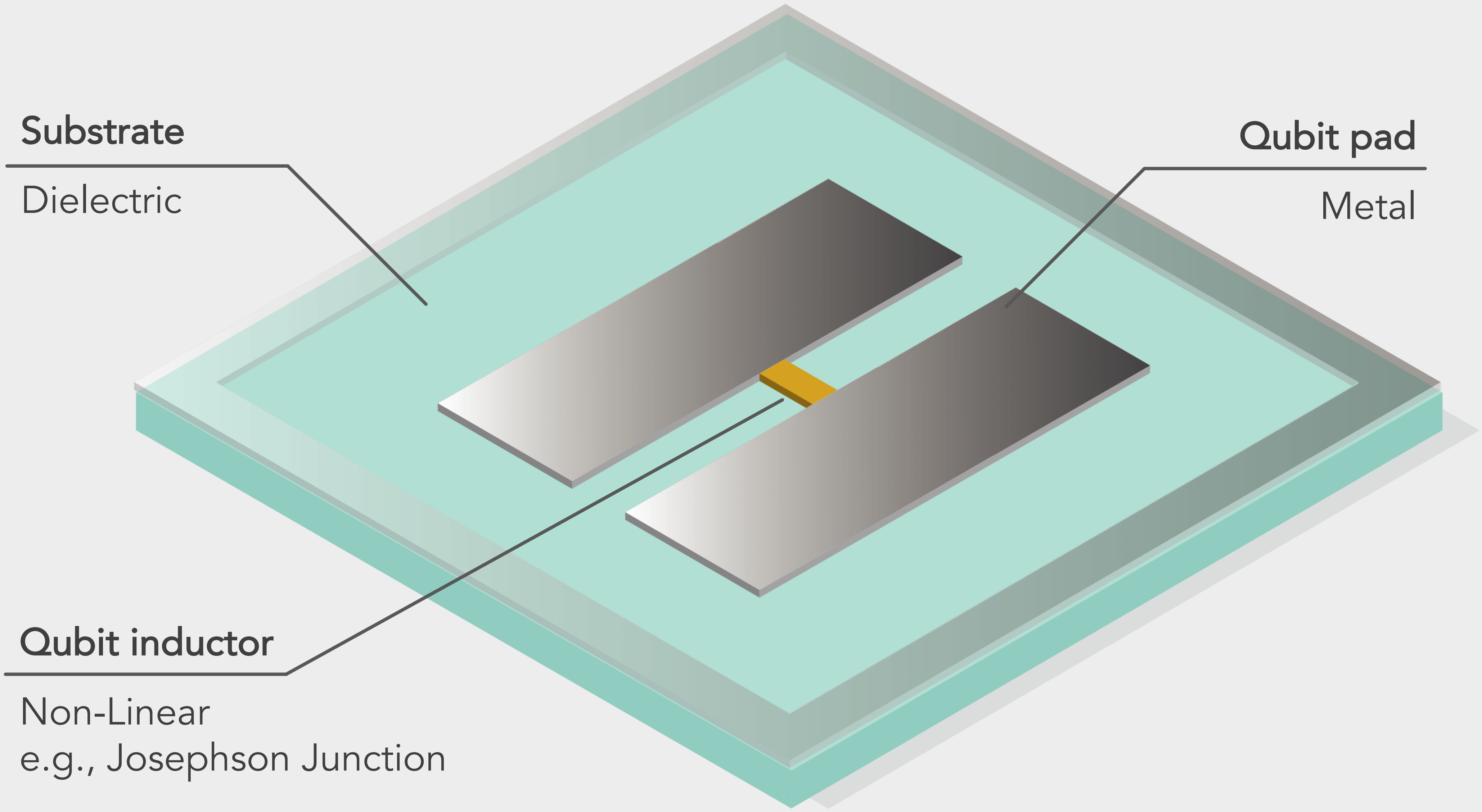
start with **classical**

Classical oscillator

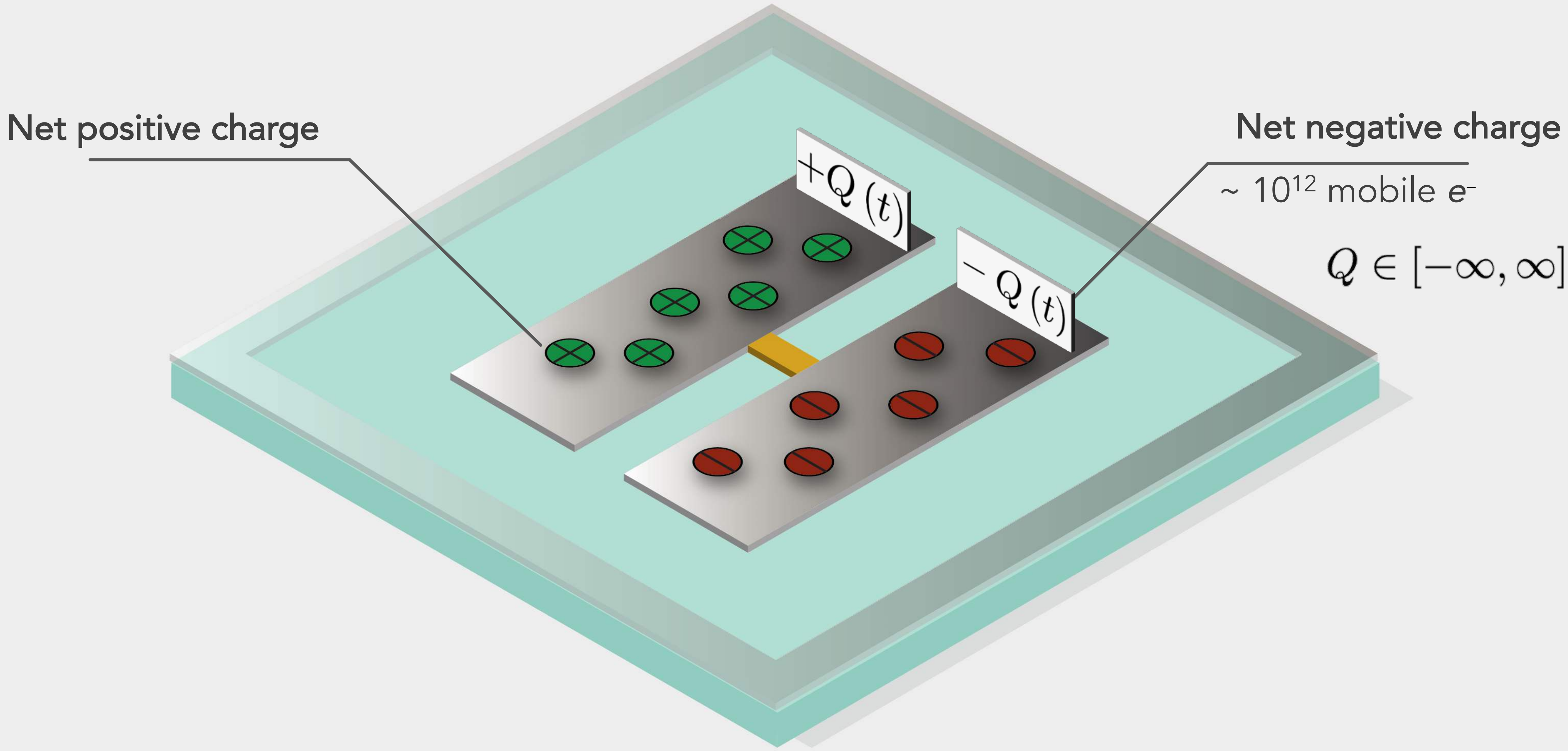
Transmon qubit



Transmon qubit

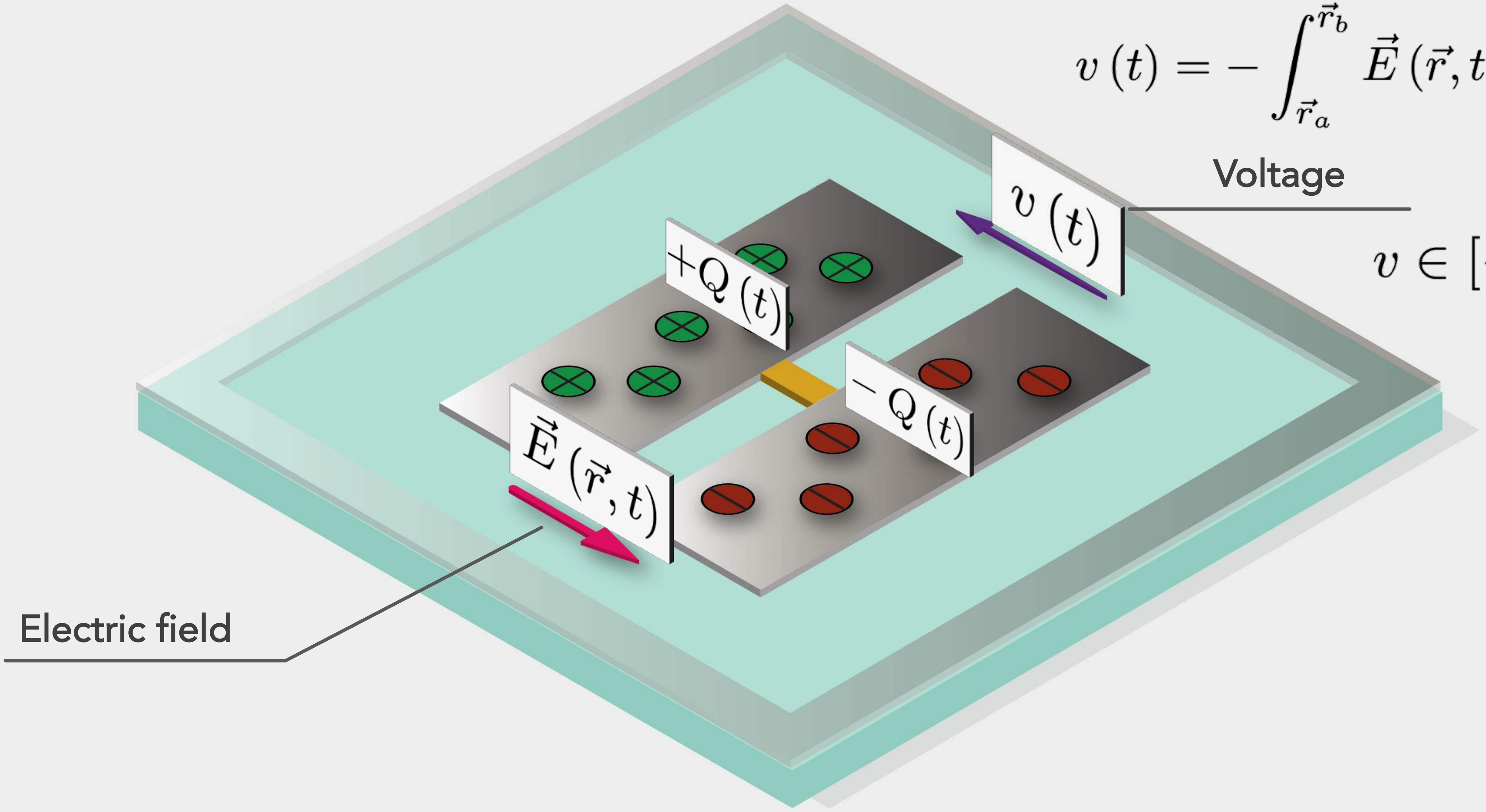


Transmon qubit: charge

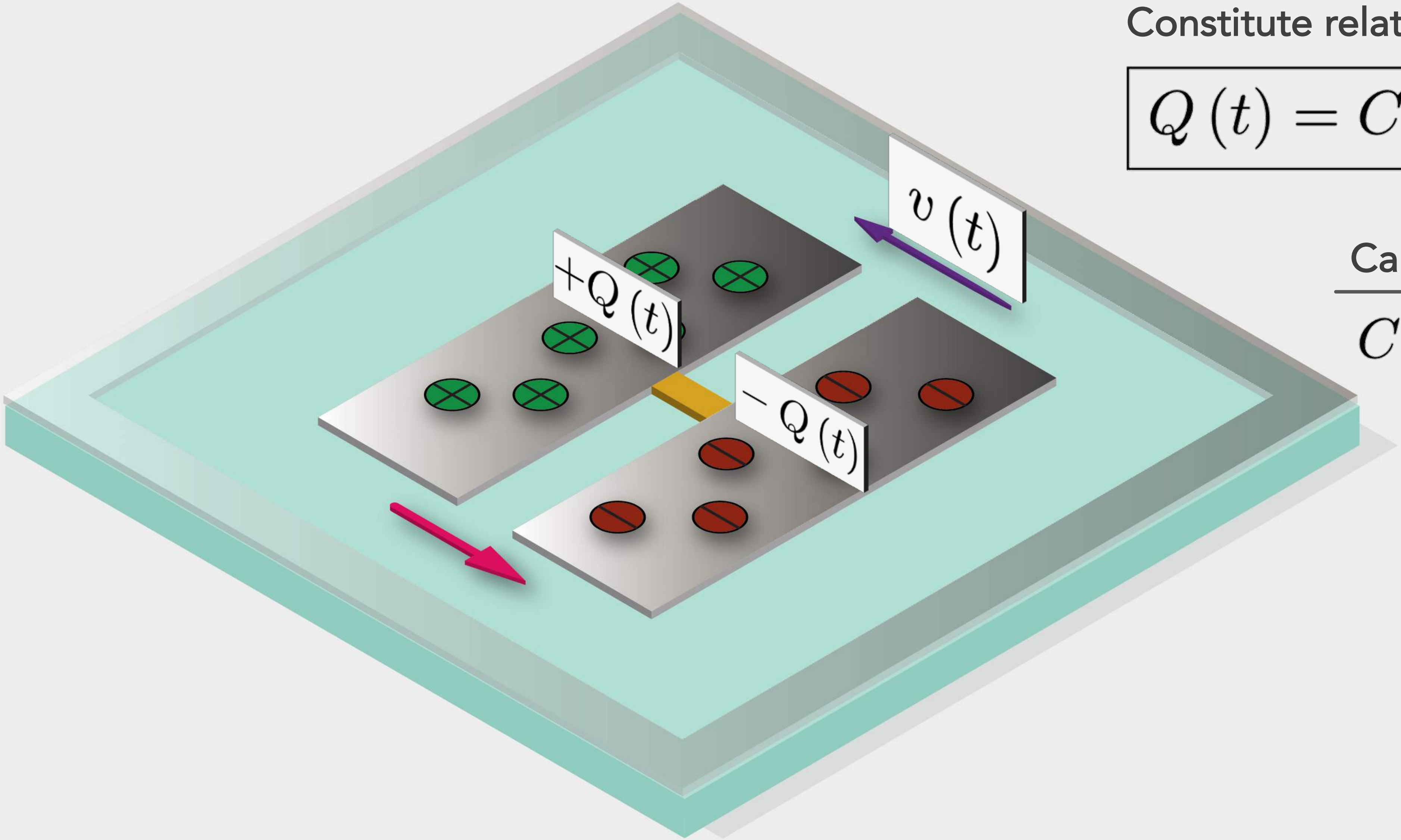


Electric field and voltage

$$v\left(t\right) = -\int_{\vec{r}_a}^{\vec{r}_b} \vec{E}\left(\vec{r}, t\right) \cdot \mathrm{d}\vec{l}\left(\vec{r}\right)$$



Charge and capacitance

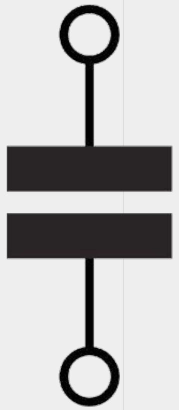


Constitute relationship

$$\boxed{Q(t) = Cv(t)}$$

Capacitance

$$C \in (0, \infty)$$



For a good discussion, see *"The Feynman Lectures on Physics Vol. II Ch. 22: AC Circuits."* Caltech.

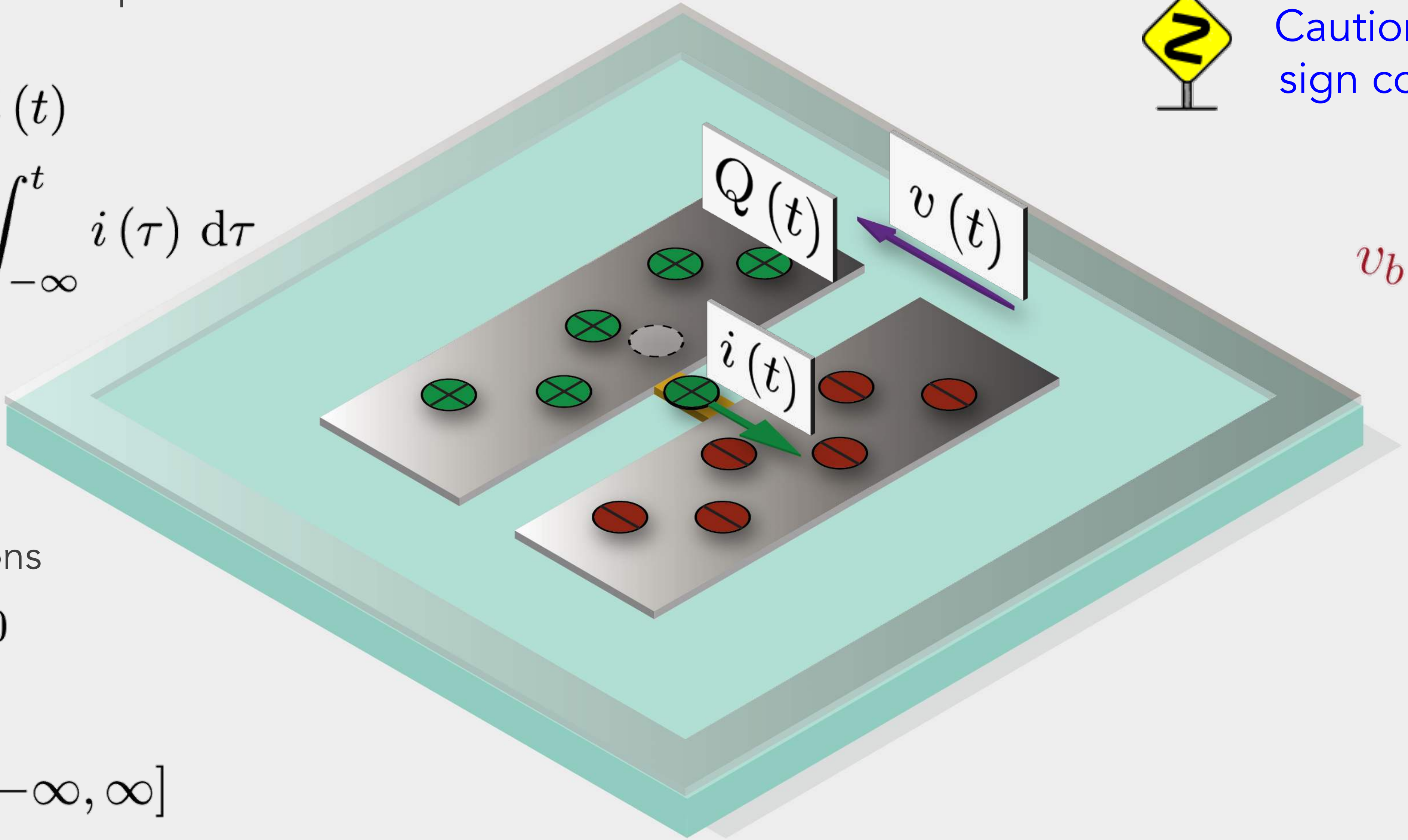
Charge and current

Conservation of charge

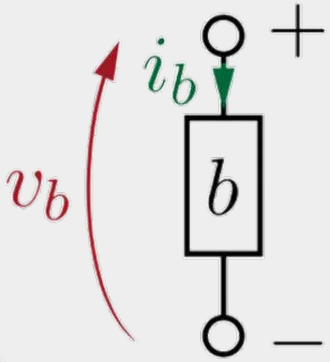
Universal relationship

$$\frac{\mathrm{d}}{\mathrm{d}t}Q(t) = i(t)$$

$$Q(t) = \int_{-\infty}^{t} i(\tau)\,\mathrm{d}\tau$$



Caution: Passive sign convention



Initial conditions

$$Q(-\infty) = 0$$

$$i \in [-\infty, \infty]$$

Magnetic flux and inductance

Faraday's law of induction

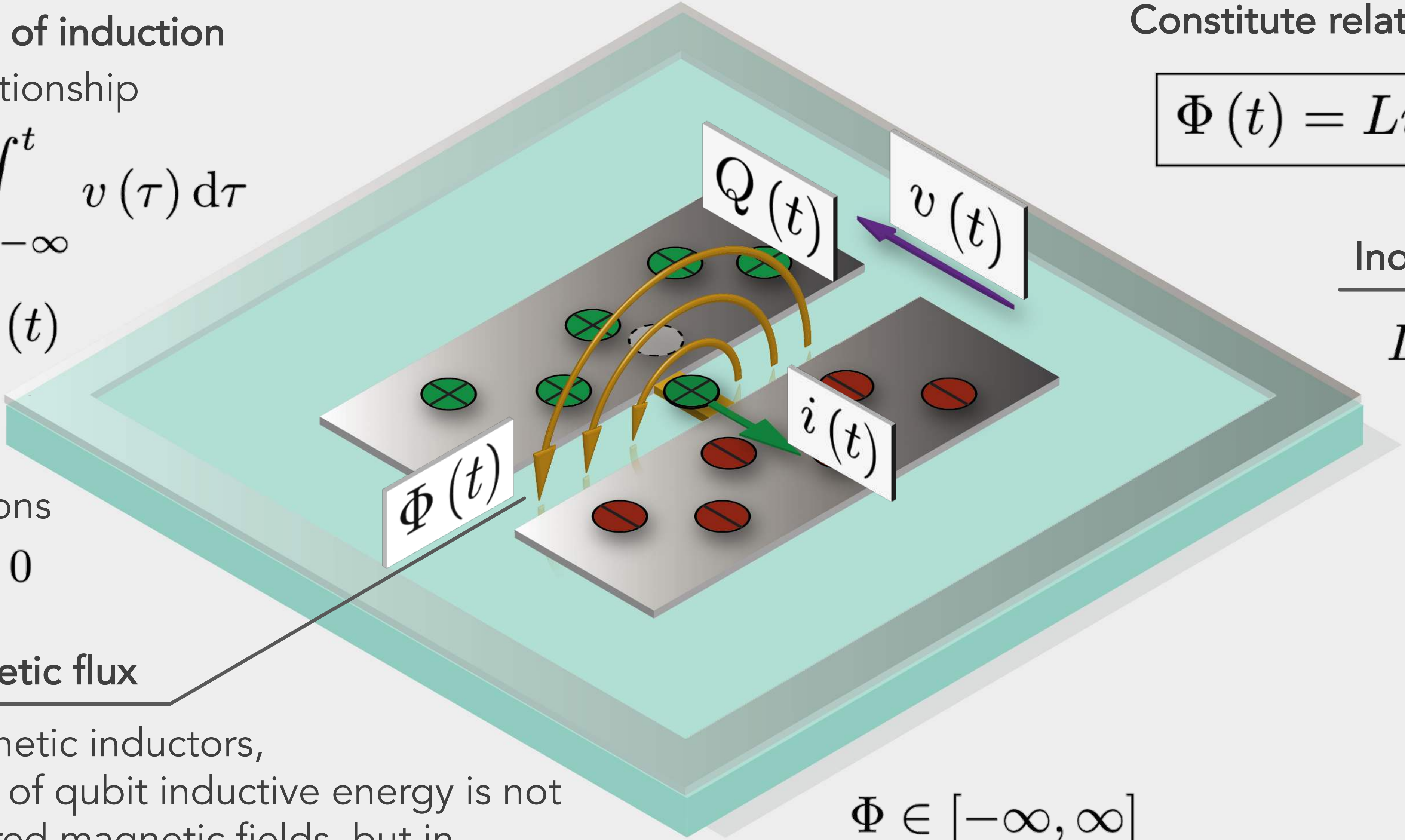
Universal relationship

$$\Phi\left(t\right)=\int_{-\infty}^{t}v\left(\tau\right)\mathrm{d}\tau$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\Phi\left(t\right)=v\left(t\right)$$

Initial conditions

$$\Phi\left(-\infty\right)=0$$



Magnetic flux

For kinetic inductors, ~98% of qubit inductive energy is not in stored magnetic fields, but in kinetic inductance

$$\Phi\in\left[-\infty,\infty\right]$$

Constitute relationship

$$\boxed{\Phi\left(t\right)=Li\left(t\right)}$$

Inductance

$$L\in\left(0,\infty\right)$$

Power and energy

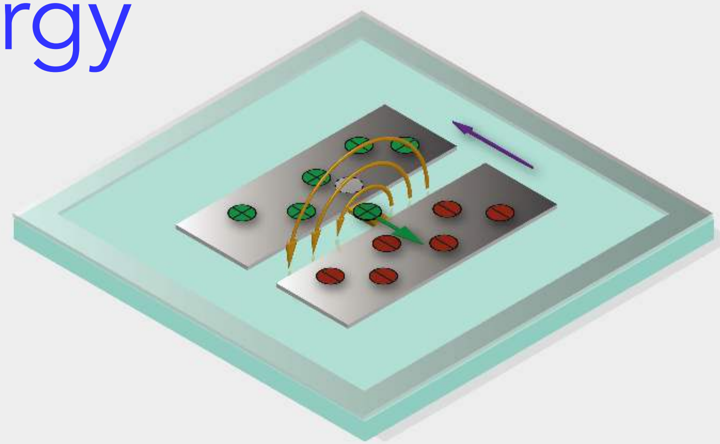
Universal

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{E}\left(t\right)=p\left(t\right)\equiv v\left(t\right)i\left(t\right)$$

Energy stored *in* (delivered *to*) component

Instantaneous power flowing *to* component

$$\mathcal{E}\left(t\right)=\int_{-\infty}^{t}p\left(\tau\right)\,\mathrm{d}\tau$$



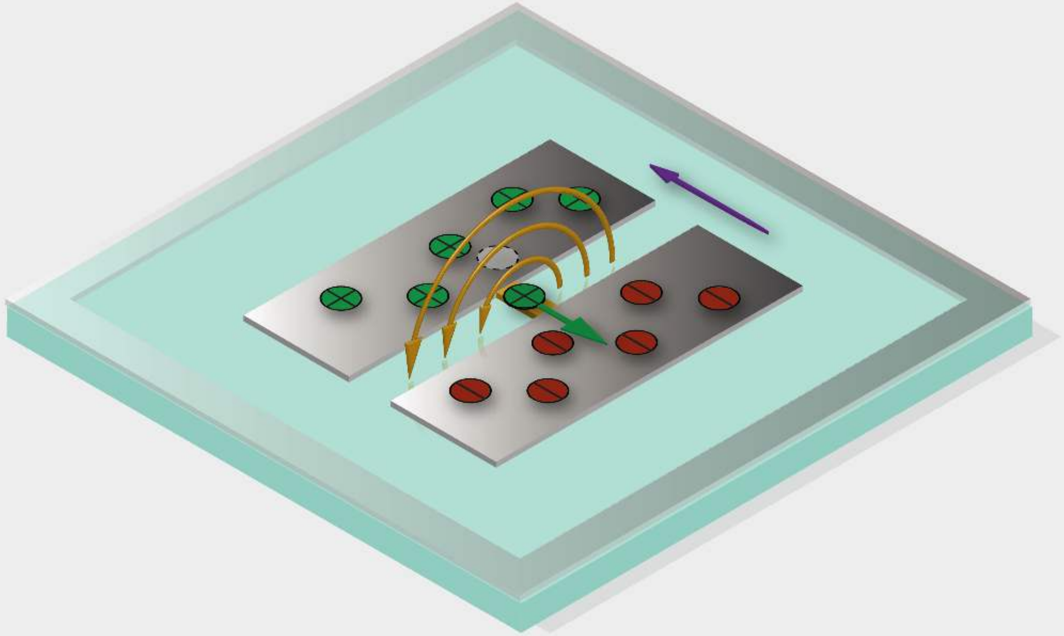
$$\mathcal{E}_{\mathrm{cap}}\left(\dot{\Phi}\right)=\frac{1}{2}C\dot{\Phi}^{2}$$

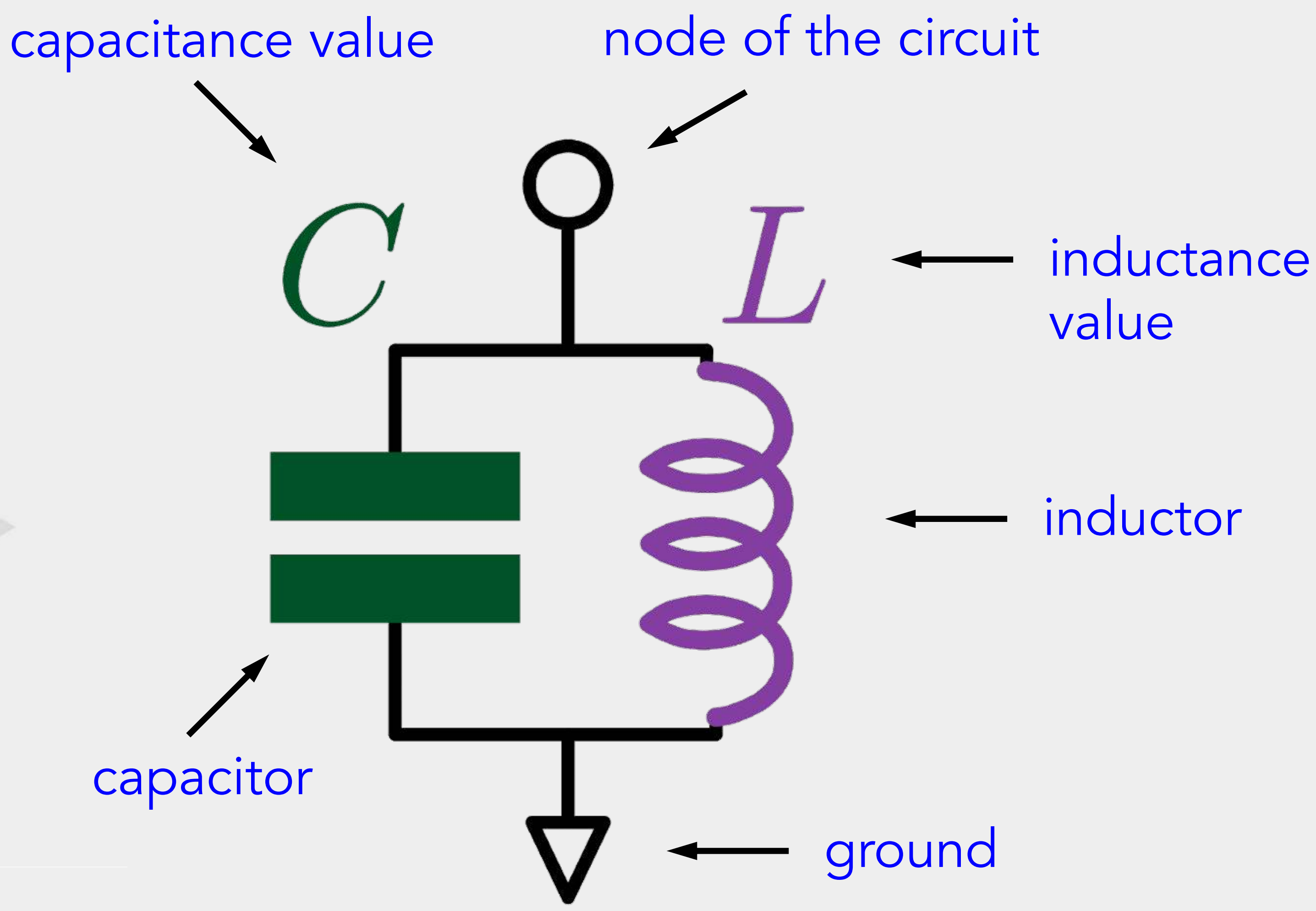
$$\mathcal{E}_{\mathrm{ind}}\left(\Phi\right)=\frac{\Phi^{2}}{2L}$$

$$\mathcal{E}_{\mathrm{cap}}\left(Q\right)=\frac{Q^{2}}{2C}$$

$$\mathcal{E}_{\mathrm{ind}}\left(\dot{Q}\right)=\frac{1}{2}L\dot{Q}^{2}$$

Electromagnetic oscillator

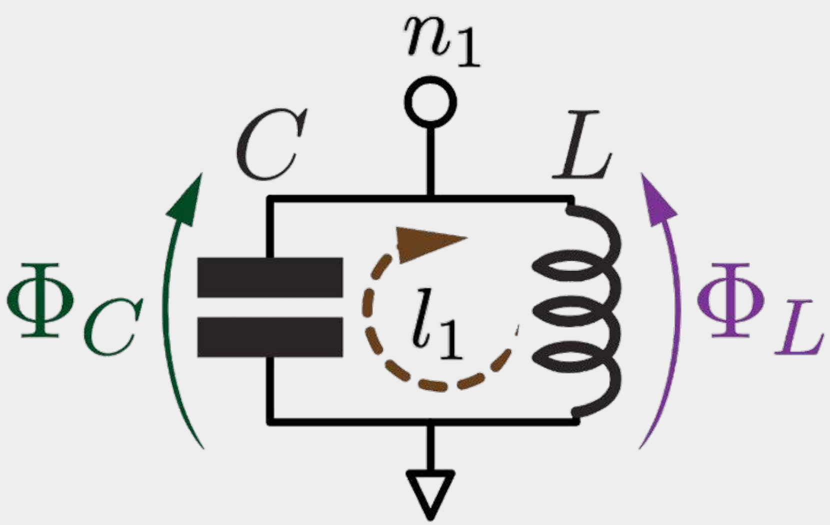




Kirchhoff's network laws*

Conservation of charge
Kirchhoff's current law

$$\sum_{b\in\text{node}} \pm\dot{Q}_b\left(t\right)=0$$



Faraday's law of induction
Kirchhoff's voltage law

$$\sum_{b\in\text{loop}} \pm\dot{\Phi}_b\left(t\right)=0$$

$$n_1: \quad \dot{Q}_C+\dot{Q}_L=0$$

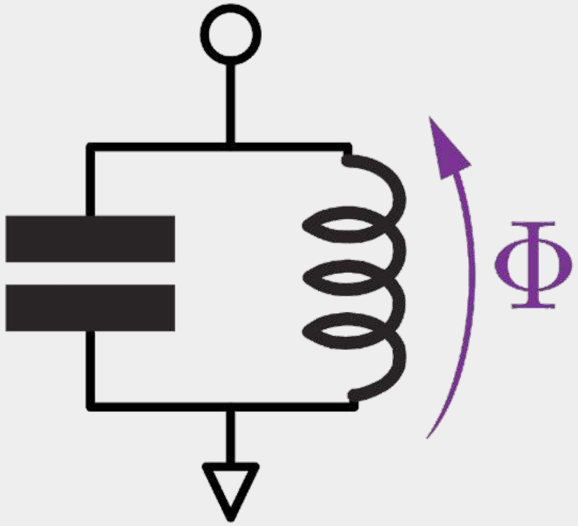
$$l_1: \quad \dot{\Phi}_C-\dot{\Phi}_L=0$$

$$C\ddot{\Phi}+L^{-1}\Phi=0$$

$$\Phi_C=\Phi_L$$

As we will see later, for the Lagrangian description in flux basis, KVL acts as a set of holonomic constraints and KCL as the equations of motion

Oscillator analogy



$$C\ddot{\Phi} + L^{-1}\Phi = 0$$

$$\ddot{\Phi} = -\omega_0^2\Phi \,, \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

Resonance frequency

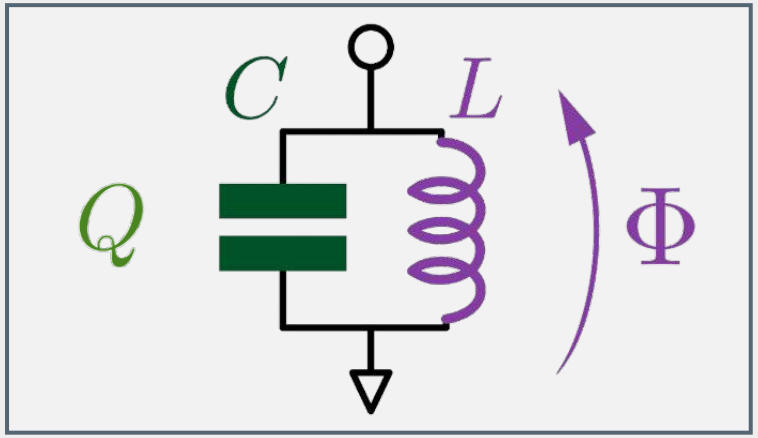
$\dot{\Phi}(t)$ Velocity

$\Phi(t)$ Deviation from equilibrium

Equilibrium position
$x = 0 \,, \quad \Phi = 0$

Spring: Svjo

The LC classical harmonic oscillator

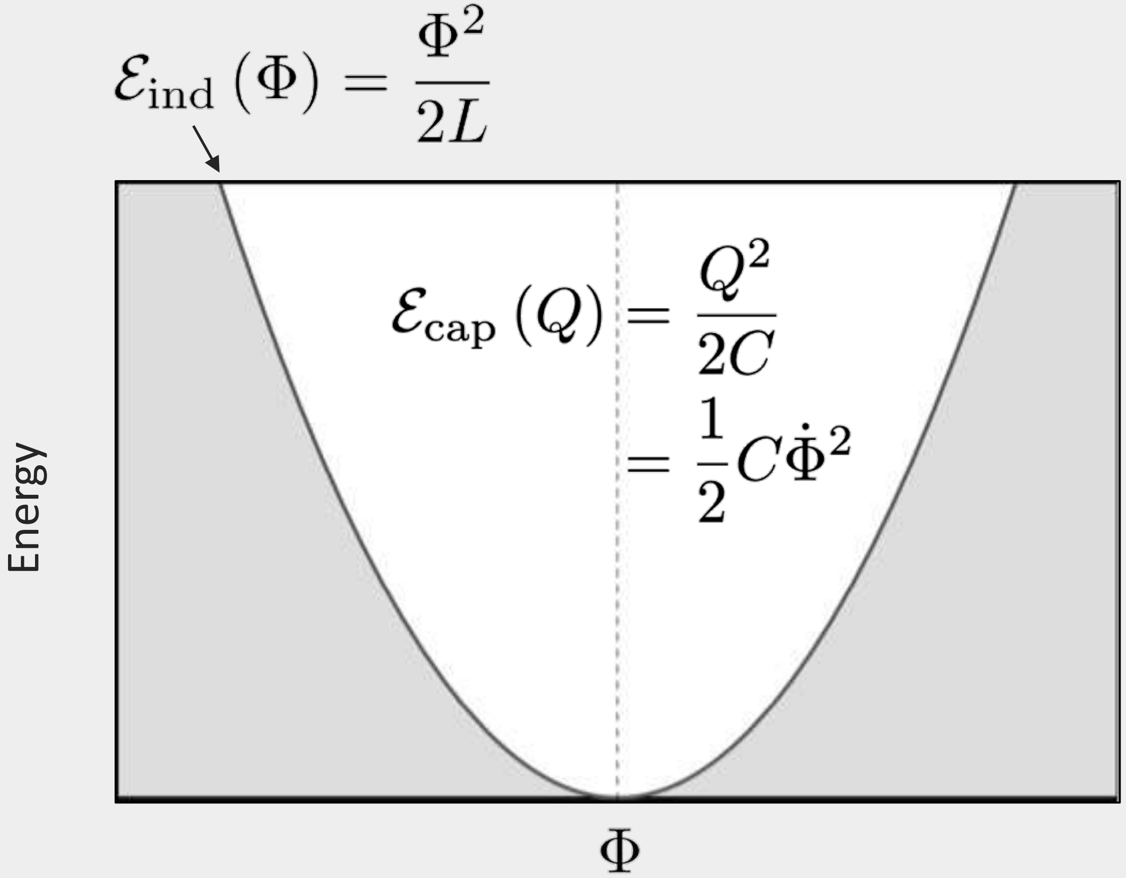


Position: $\Phi \mapsto x$ Momentum: $Q \mapsto p$

Mass: $C \mapsto m$ Spring constant: $L^{-1} \mapsto k$

$$\mathcal{H}(\Phi, Q) = \frac{Q^2}{2C} + \frac{\Phi^2}{2L}$$

$$\omega_0^2 = \frac{1}{LC} \qquad Z_0 = \frac{L}{C}$$



Graph: Minev; Spring-mass: Svjo

"It is by *logic* that we prove,
but by *intuition* that we discover."

Henri Poincaré

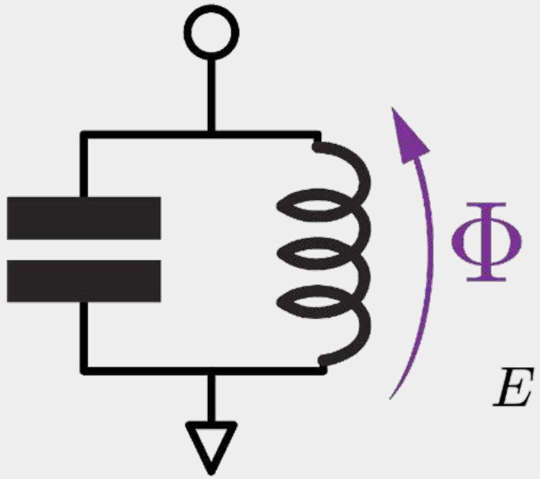
Photo by Eugène Pirou

Hamiltonian dynamics and phase space

$$\mathcal{H}(\Phi, Q) = \frac{Q^2}{2C} + \frac{\Phi^2}{2L}$$

with $L = C = 1$

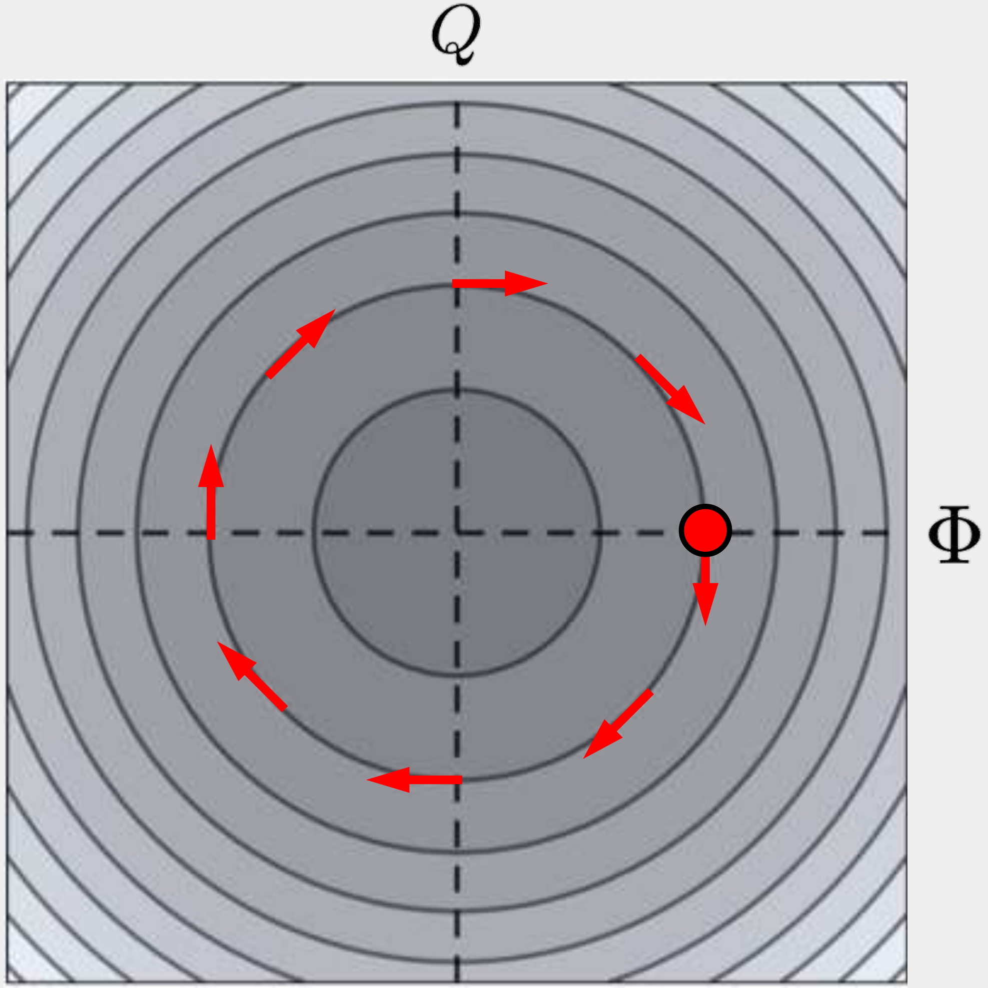
$$E = \hbar\omega_0 \left(n + \frac{1}{2}\right) = \frac{1}{2}\left(Q^2 + \Phi^2\right)$$



$\vdots$

$$E = \frac{3\hbar\omega_0}{2}$$

$$E = \frac{\hbar\omega_0}{2}$$



$$\dot{\Phi} = +\frac{\partial \mathcal{H}}{\partial Q} = +Q$$

$$\dot{Q} = -\frac{\partial \mathcal{H}}{\partial \Phi} = -\Phi$$

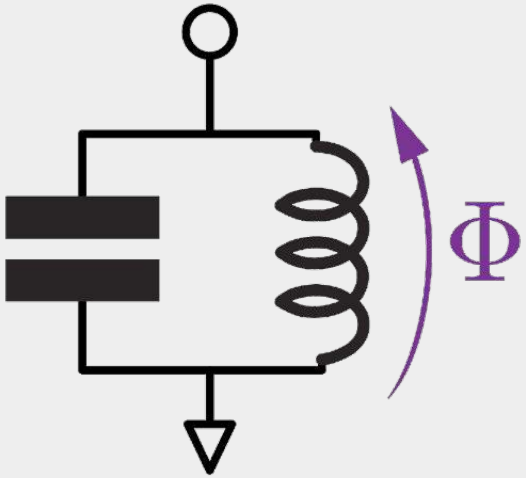
Complex action-angle variable

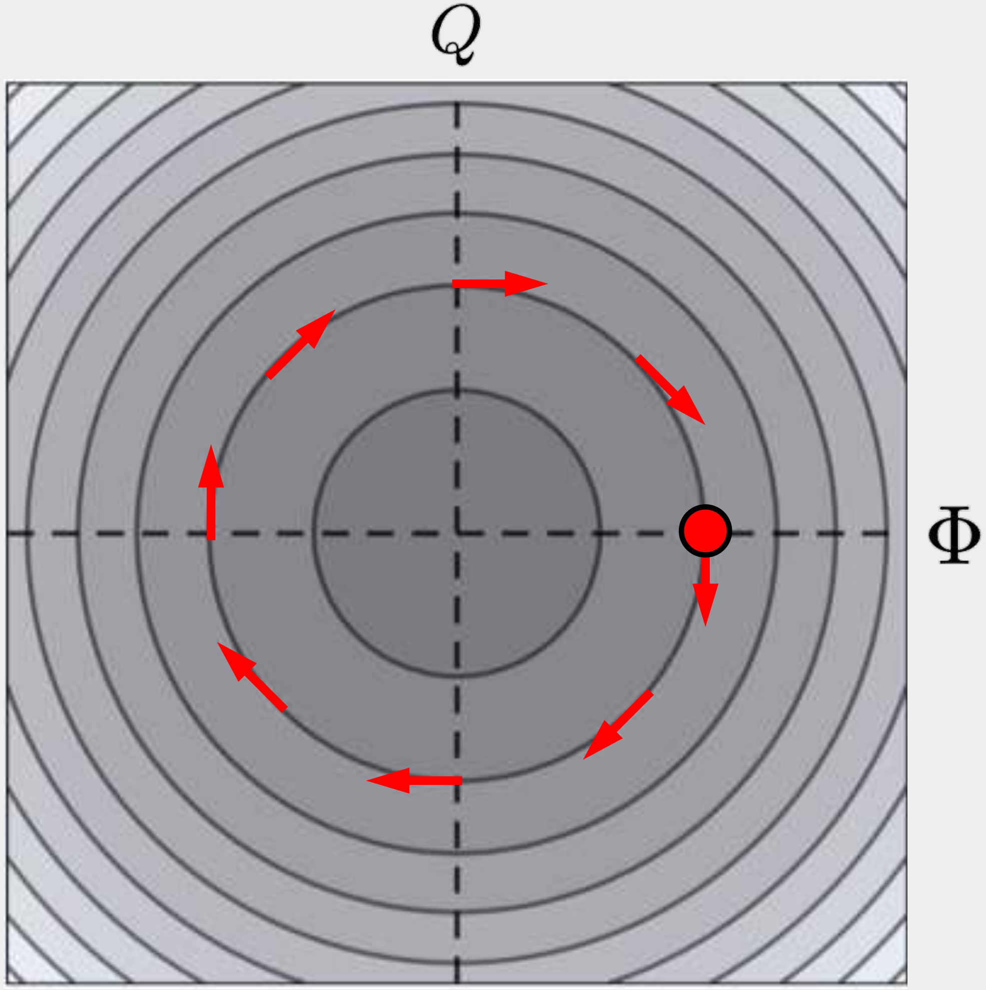
$$\mathcal{H}(\Phi, Q) = \frac{Q^2}{2C} + \frac{\Phi^2}{2L}$$

$$\mathcal{H}(\alpha, \alpha^*) = \frac{\hbar\omega_0}{2}(\alpha^*\alpha + \alpha\alpha^*)$$

$$E = \hbar\omega_0\left(n + \frac{1}{2}\right)$$

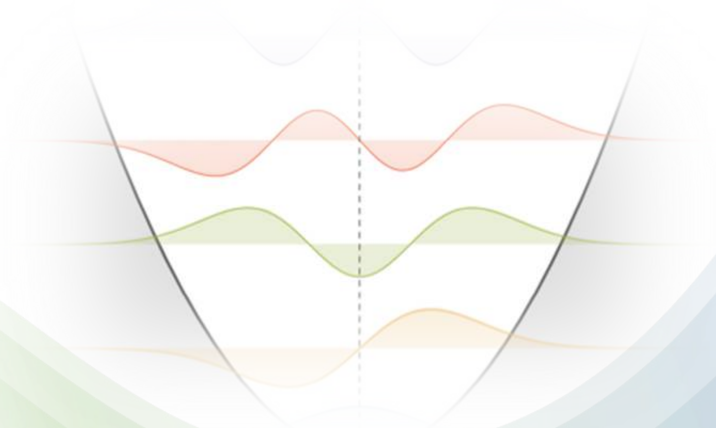
$$\alpha(t) = \sqrt{\frac{1}{2\hbar Z}}\left[\Phi(t) + iZQ(t)\right]$$





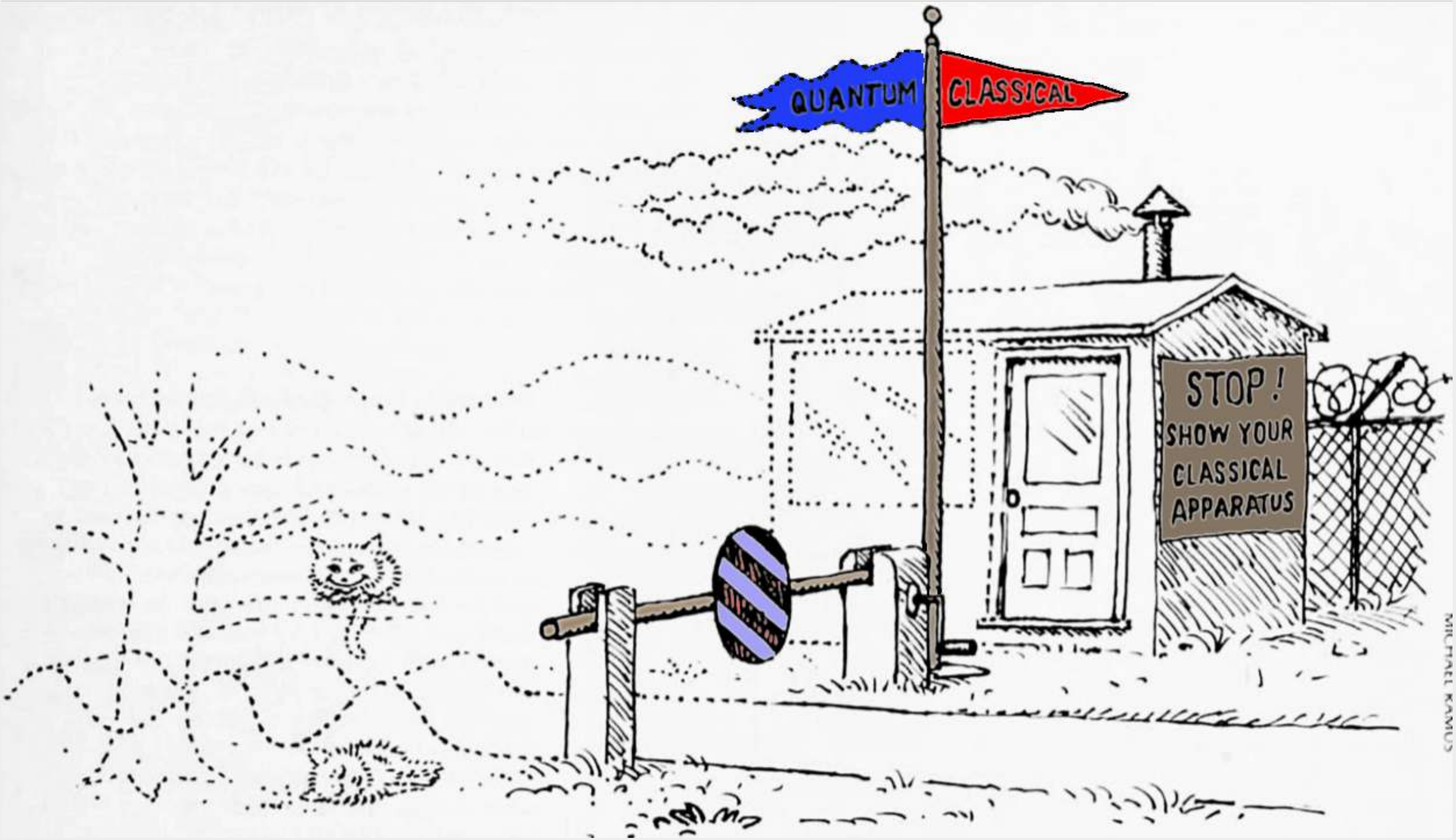
Classical analog of the bosonic ladder operator

$$\alpha(t) = \alpha(0)\, e^{-i\omega_0 t}$$

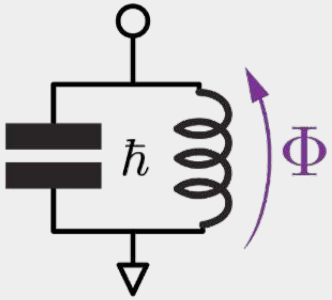


Unveiling the quantum

Quantum harmonic oscillator



Drawing: Zurek, Physics Today (1991)



The classical and quantum oscillator

Classical | Quantum

$$\Phi(t) \mapsto \hat{\Phi}$$

$$Q(t) \mapsto \hat{Q}$$

$$\mathcal{H}(\Phi, Q) \mapsto \hat{H}\left(\hat{\Phi}, \hat{Q}\right)$$

$$\{\Phi, Q\} = 1 \mapsto \left[\hat{\Phi}, \hat{Q}\right] = i\hbar\hat{1}$$

$$\{\alpha, \alpha^*\} = 1/(i\hbar) \mapsto \left[\hat{a}, \hat{a}^\dagger\right] = \hat{1}$$

$$\{A, B\} = \frac{\partial A}{\partial \Phi}\frac{\partial B}{\partial Q} - \frac{\partial B}{\partial \Phi}\frac{\partial A}{\partial Q} \qquad \left[\hat{A}, \hat{B}\right] = \hat{A}\hat{B} - \hat{B}\hat{A}$$

If I knew what I was doing, it wouldn't be called research.

Albert Einstein
See Hawken *et al.* (2010)

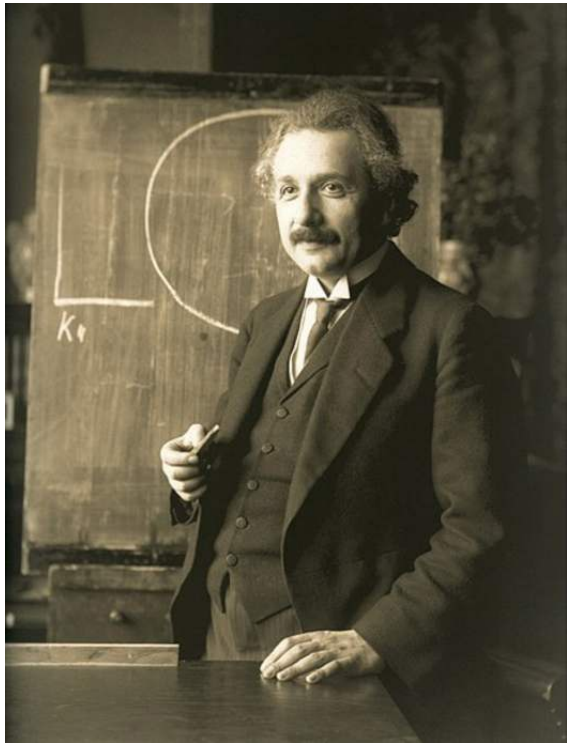
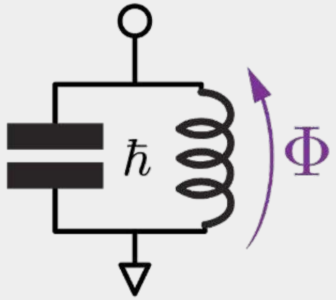
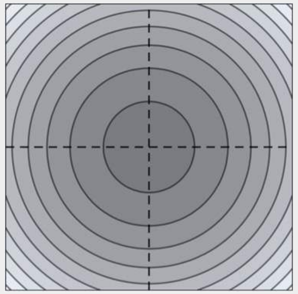


Photo: F. Schmutzer

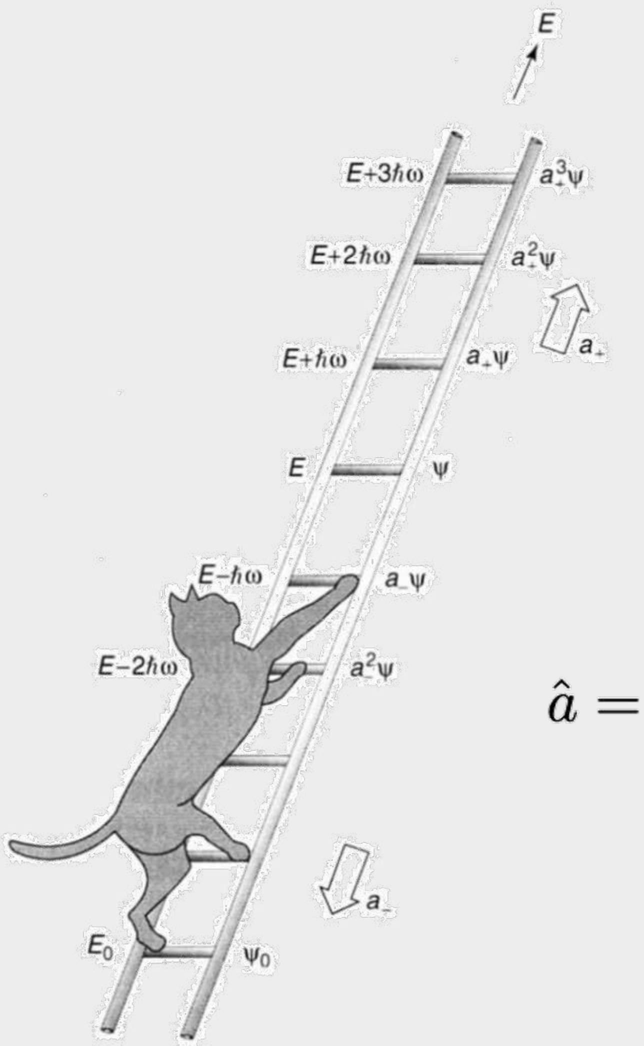


The classical and quantum oscillator



	Classical	Quantum
Hamiltonian	$\mathcal{H} = \frac{Q^2}{2C} + \frac{\Phi^2}{2L} = \frac{1}{2}\hbar\omega_0\left(\alpha^*\alpha + \alpha\alpha^*\right)$	$\hat{H} = \frac{\hat{\Phi}^2}{2L} + \frac{\hat{Q}^2}{2C} = \hbar\omega_0\left(\hat{a}^\dagger\hat{a} + \hat{a}\hat{a}^\dagger\right)$
Phase space	$\alpha(t) = \sqrt{\frac{1}{2\hbar Z}}\left[\Phi(t) + iZQ(t)\right]$ $\alpha(t) = \alpha(0)\, e^{-i\omega_0 t}$ $\Phi(t) = \sqrt{\frac{\hbar Z}{2}}\left(\alpha^*(t) + \alpha(t)\right)$ $Q(t) = i\sqrt{\frac{\hbar}{2Z}}\left(\alpha^*(t) - \alpha(t)\right)$	$\hat{a} = \sqrt{\frac{1}{2\hbar Z}}\left(\hat{\Phi} + iZ\hat{Q}\right)$ $\hat{a}(t) = \hat{a}(0)\, e^{-i\omega_0 t}$ (Heisenberg picture) $\hat{\Phi} = \Phi_{\mathrm{ZPF}}\left(\hat{a}^\dagger + \hat{a}\right)$ $\hat{Q} = iQ_{\mathrm{ZPF}}\left(\hat{a}^\dagger - \hat{a}\right)$

Ladder operators and matrix representation

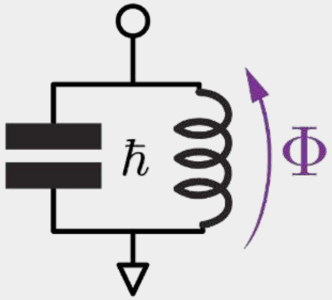


annihilation	creation	general hopping
$\hat{a}\,\|0\rangle = 0$	$\hat{a}^\dagger\,\|0\rangle = \sqrt{1}\,\|1\rangle$	$\hat{a}\,\|n\rangle = \sqrt{n}\,\|n-1\rangle$
$\hat{a}\,\|1\rangle = \sqrt{1}\,\|0\rangle$	$\hat{a}^\dagger\,\|1\rangle = \sqrt{2}\,\|2\rangle$	$\hat{a}^\dagger\,\|n\rangle = \sqrt{n+1}\,\|n+1\rangle$

$$\hat{a} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \sqrt{2} & 0 \\ 0 & 0 & 0 & \sqrt{3} \\ 0 & 0 & 0 & \ddots \end{pmatrix} \quad \hat{a}^\dagger = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & \sqrt{2} & 0 & 0 \\ 0 & 0 & \sqrt{3} & \ddots \end{pmatrix} \quad \hat{a}^\dagger\hat{a} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & \ddots \end{pmatrix}$$

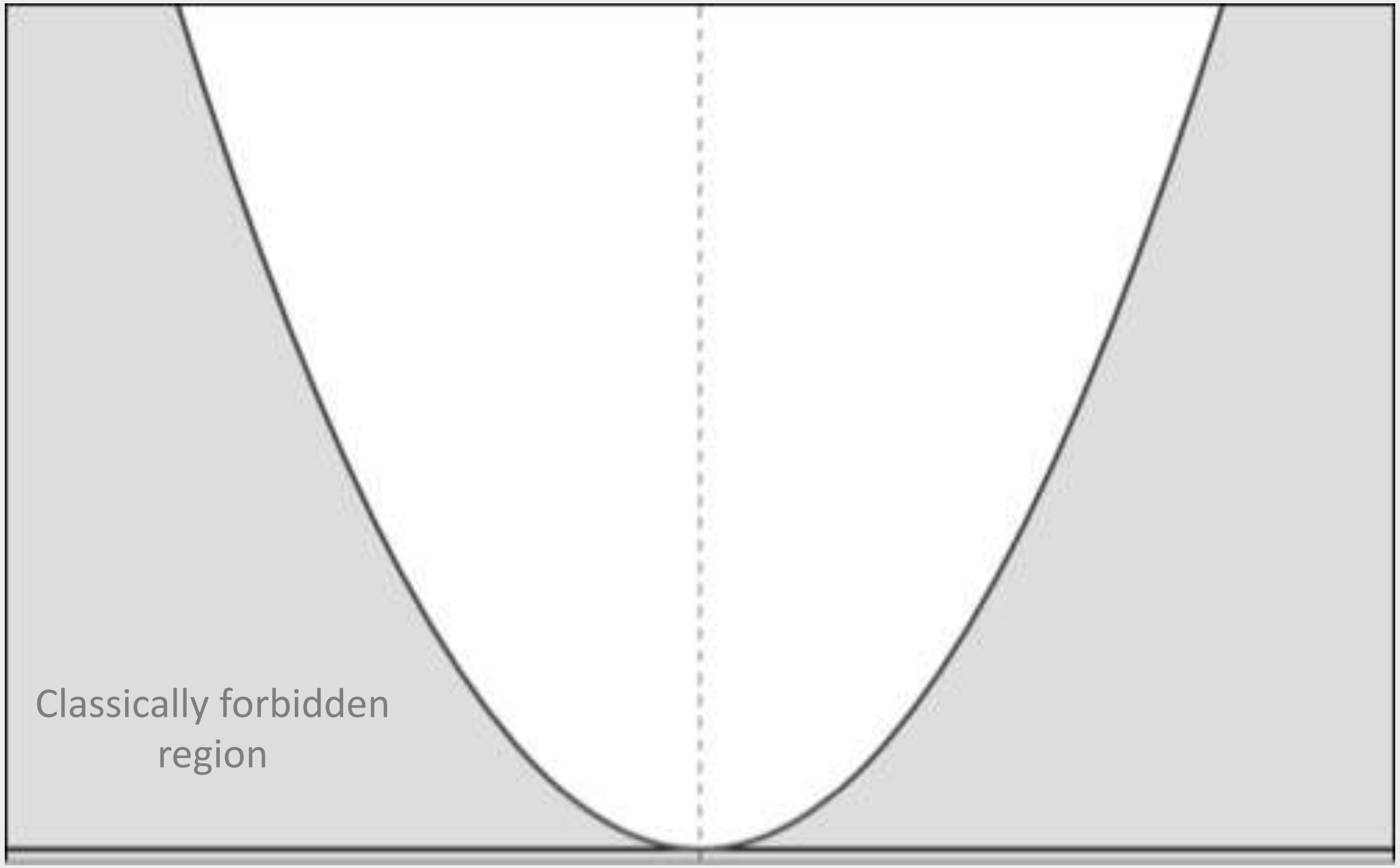
Image: Griffiths, D.J.

Wavefunctions of the quantum oscillator

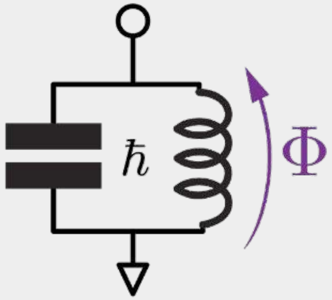


$$\mathcal{E}_{\text{ind}}(\Phi) = \frac{\Phi^2}{2L}$$

Energy



Φ

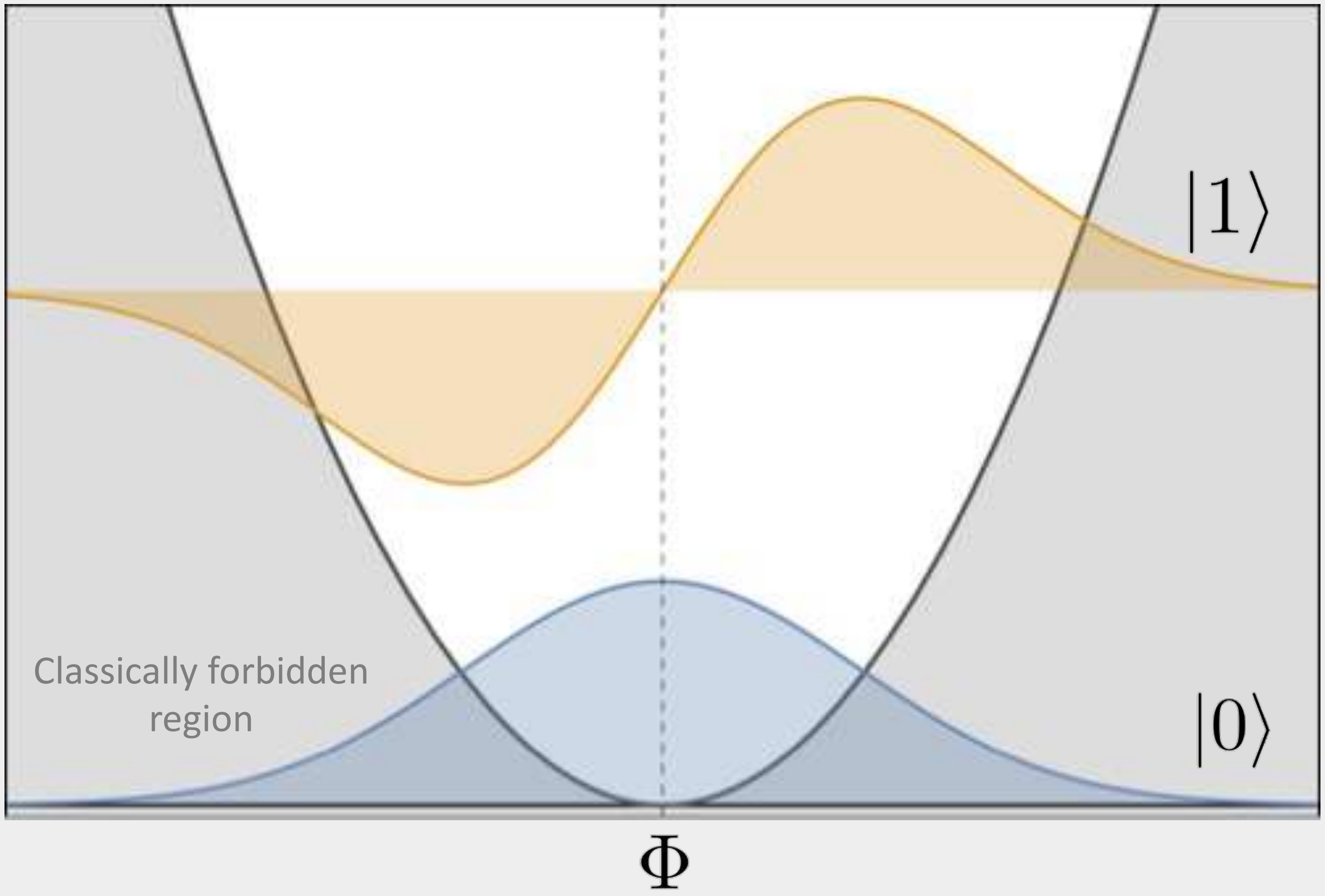


Wavefunctions of the quantum oscillator

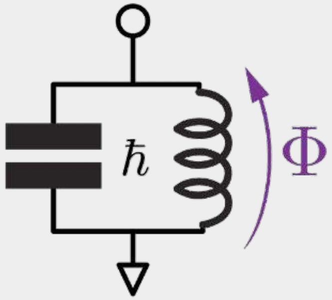
$$\mathcal{E}_{\text{ind}}\left(\Phi\right) = \frac{\Phi^2}{2L} \qquad \psi_n\left(\Phi\right) \equiv \langle \Phi | n \rangle$$

Energy /

Scaled wavefunction amplitude



Wavefunctions of the quantum oscillator

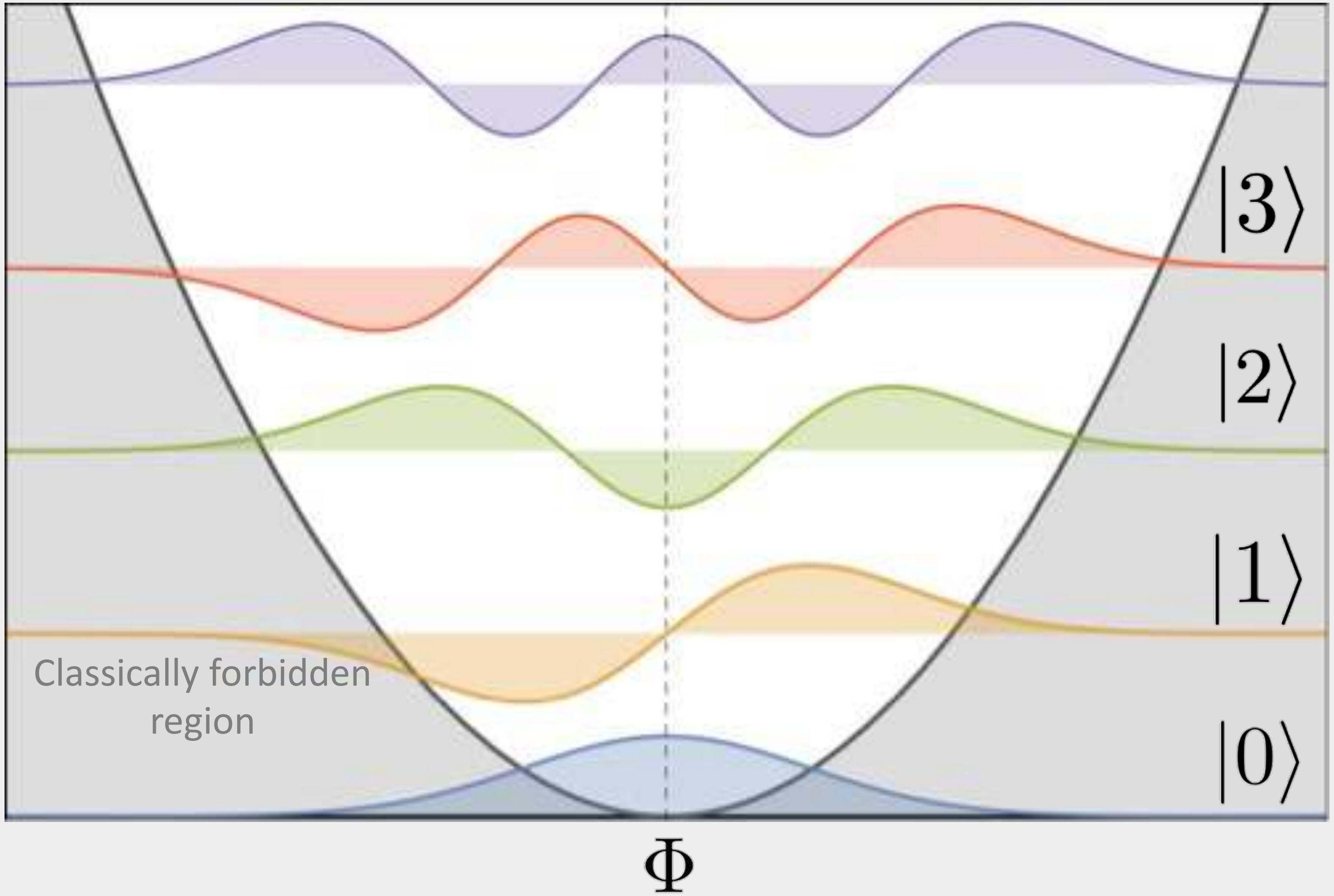


$$\mathcal{E}_{\mathrm{ind}}(\Phi) = \frac{\Phi^2}{2L}$$

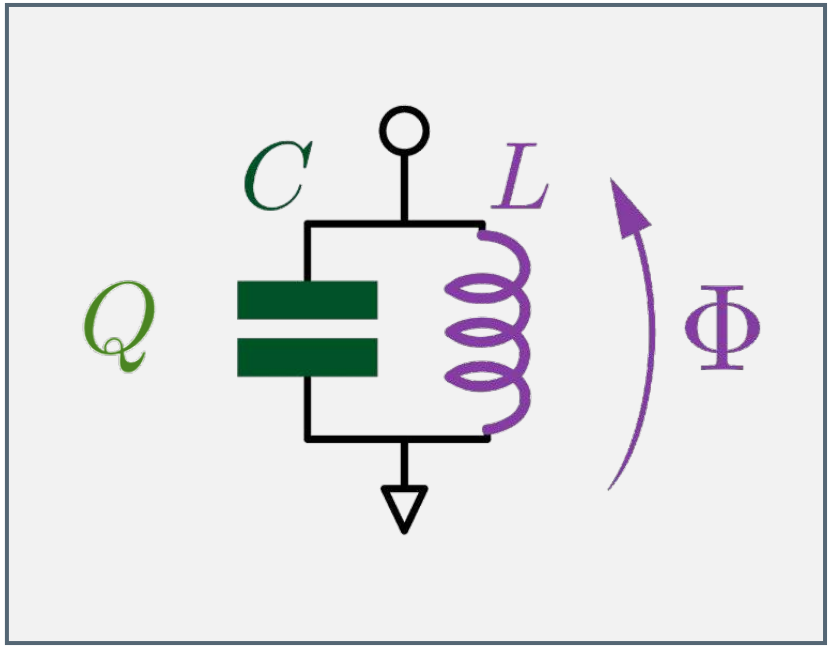
$$\psi_n(\Phi) \equiv \langle \Phi | n \rangle$$

Energy /

Scaled wavefunction amplitude



Pop-up question



The flux and charge operators are Hermitian observables.

How can some expectations, such as

$$\left\langle 0 \middle| \hat{\Phi}\hat{Q} \middle| 0 \right\rangle = \frac{1}{4} i \,,$$

be imaginary?

Or, others be negative…?

Advanced questions

1. What is $\hat{\Phi}(t)$ in terms of $\hat{a}$ in the Heisenberg picture?

2. **Autocorrelation.** Find the autocorrelation operator $\hat{C}_{\Phi\Phi}(t) = \hat{\Phi}(t)\,\hat{\Phi}(0)$. What frequency components does it have?

   (a) What is the flux autocorrelation expectation value $C_{\Phi\Phi}(t) = \left\langle n \middle| \hat{\Phi}(t)\,\hat{\Phi}(0) \middle| n \right\rangle$ for the Fock state $|n\rangle$? Is it real? Why not? What is the frequency spectrum?

   (b) Repeat for a coherent state $|\alpha\rangle$.

3. **Thermal state.** The thermal state is $\hat{\rho}_{\mathrm{th}} = \exp\left[-\beta\hat{a}^{\dagger}\hat{a}\right] / \operatorname{Tr}\left[\exp\left(-\beta\hat{a}^{\dagger}\hat{a}\right)\right]$, where $\beta = \hbar\omega_0/k_{\mathrm{B}}T$, and T is the temperature of the oscillator.

   (a) What is the mean and variance of the flux $\hat{\Phi}$ and charge $\hat{Q}$ operators?

   (b) How does the frequency spectrum of the autocorrelation $\left\langle \hat{C}_{\Phi\Phi}(t) \right\rangle = \operatorname{Tr}\left[\hat{\rho}_{\mathrm{th}}\hat{C}_{\Phi\Phi}(t)\right]$ change when with that for the state $|n\rangle$ and $|\alpha\rangle$?

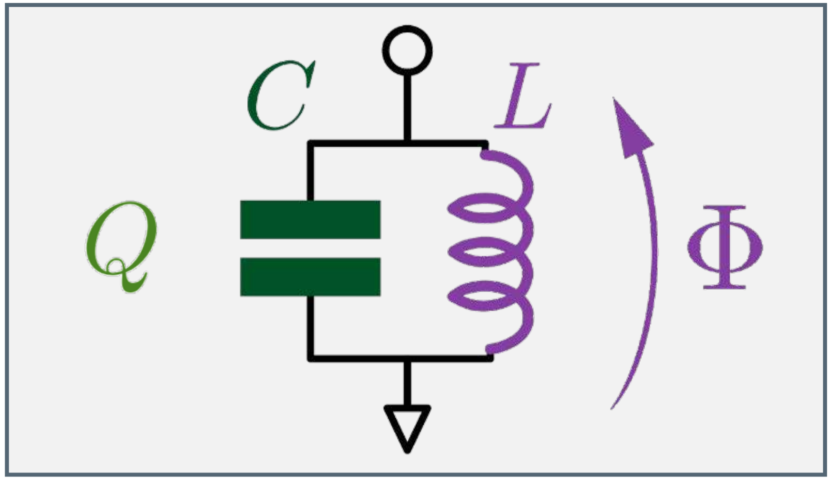
   (c) The spectrum is not symmetric in frequency. How can you interpret that positive and negative frequencies have different weights? How is this related to absorption and emission of the oscillator? (Consider Fermi's golden rule).

   (d) What happens to the spectrum in the limit of high temperature, $k_{\mathrm{B}}T \gg \hbar\omega_0$? How about low temperature, $k_{\mathrm{B}}T \gg \hbar\omega_0$?

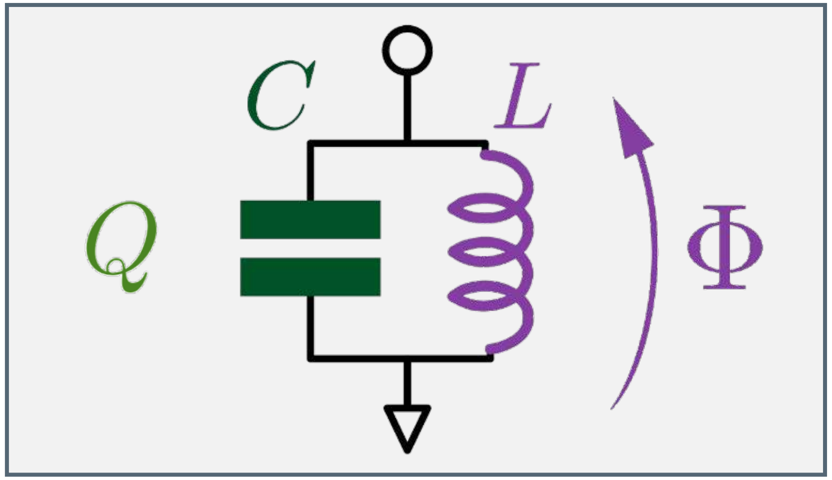
   (e) How do the above conclusions change for charge; i.e., for $\left\langle \hat{C}_{QQ}(t) \right\rangle = \operatorname{Tr}\left[\hat{\rho}_{\mathrm{th}}\hat{C}_{QQ}(t)\right]$?

Will discuss answers on my blog sometime soon

Linear harmonic oscillator summary



The LC quantum harmonic oscillator



Position: $\Phi \mapsto x$ Momentum: $Q \mapsto p$

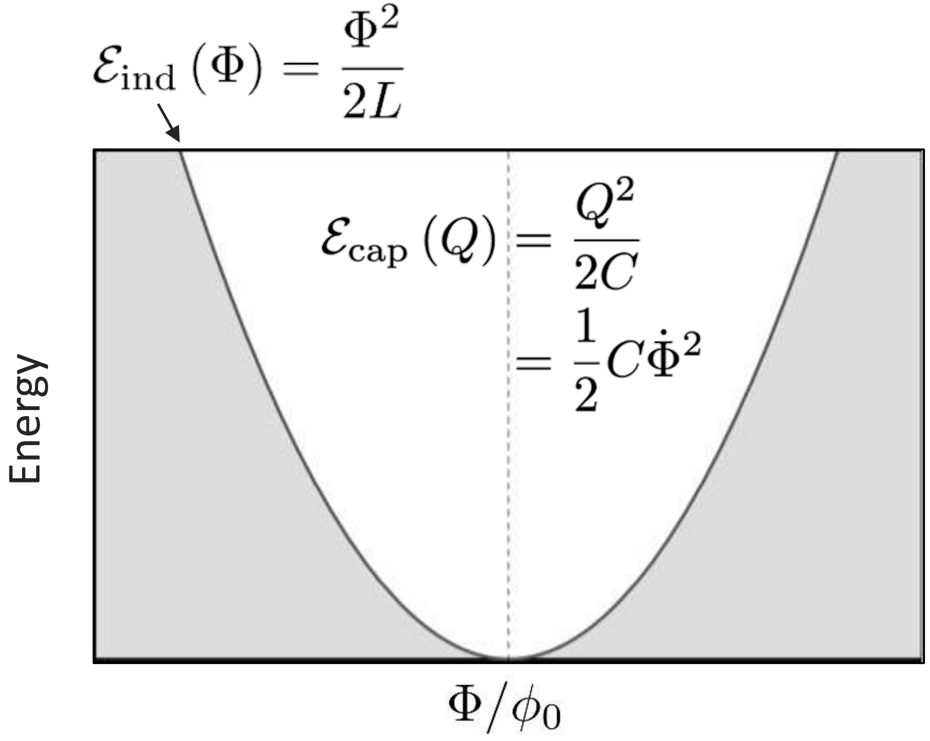
Mass: $C \mapsto m$ Spring constant: $L^{-1} \mapsto k$

$$\hat{H}\left(\hat{\Phi},\hat{Q}\right) = \frac{\hat{\Phi}^2}{2L} + \frac{\hat{Q}^2}{2C} = \hbar\omega_0\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right) \qquad \omega_0^2 = \frac{1}{LC}$$

$$\hat{H}\left|n\right\rangle = \hbar\omega_0\left(n + \frac{1}{2}\right)\left|n\right\rangle \qquad Z_0 = \frac{L}{C}$$

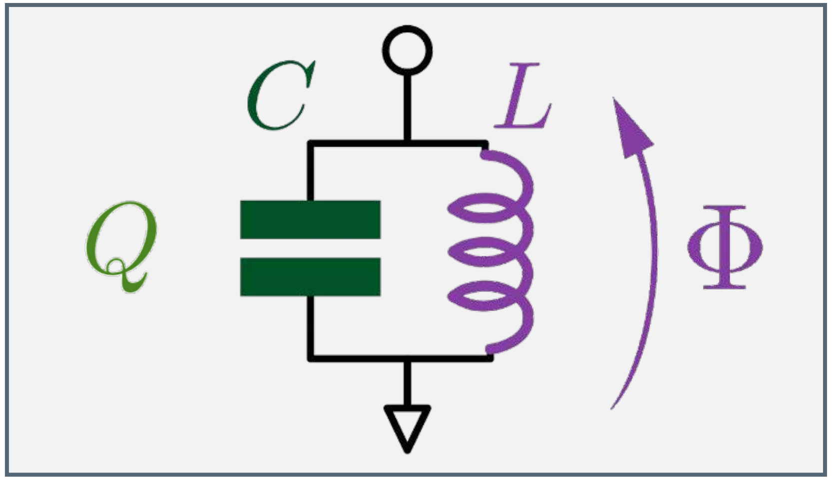
$$\hat{\Phi} = \Phi_{\mathrm{ZPF}}\left(\hat{a}^\dagger + \hat{a}\right) \qquad \Phi_{\mathrm{ZPF}} = \sqrt{\frac{\hbar}{2}Z_0} = \Phi_0\sqrt{\frac{z_0}{2\pi}},$$

$$\hat{Q} = iQ_{\mathrm{ZPF}}\left(\hat{a}^\dagger - \hat{a}\right) \qquad Q_{\mathrm{ZPF}} = \sqrt{\frac{\hbar}{2}Z_0^{-1}} = (2e)\sqrt{\frac{1}{2\pi z_0}},$$

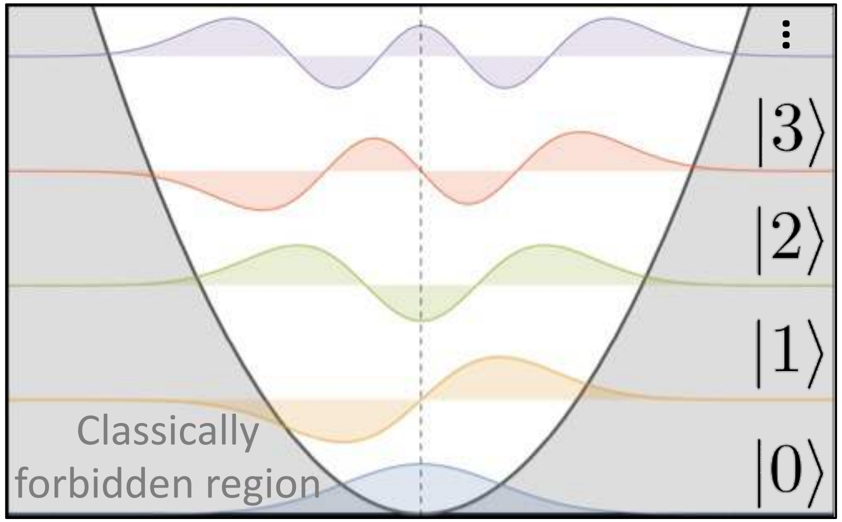


Graph: Minev; Spring-mass: Svjo

The LC quantum harmonic oscillator



$$\mathcal{E}_{\text{ind}}\left(\Phi\right) = \frac{\Phi^2}{2L} \qquad \psi_n\left(\Phi\right) \equiv \langle \Phi | n \rangle$$



Energy / Scaled wavefunction amplitude

Φ/ϕ_0

Position: $\Phi \mapsto x$ Momentum: $Q \mapsto p$

Mass: $C \mapsto m$ Spring constant: $L^{-1} \mapsto k$

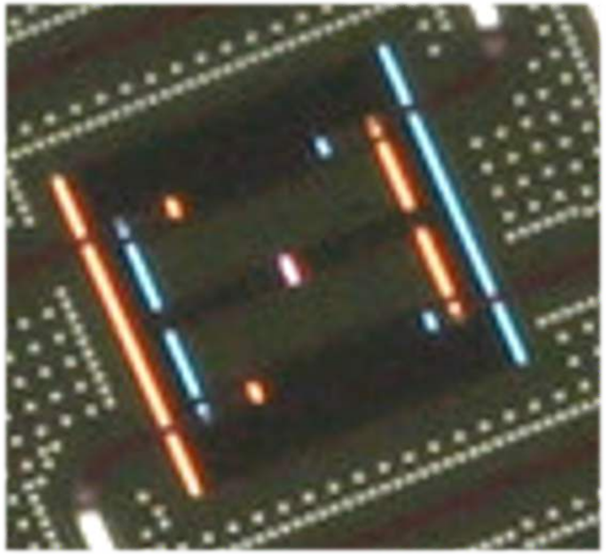
$$\hat{H}\left(\hat{\Phi}, \hat{Q}\right) = \frac{\hat{\Phi}^2}{2L} + \frac{\hat{Q}^2}{2C} = \hbar\omega_0\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right) \qquad \omega_0^2 = \frac{1}{LC}$$

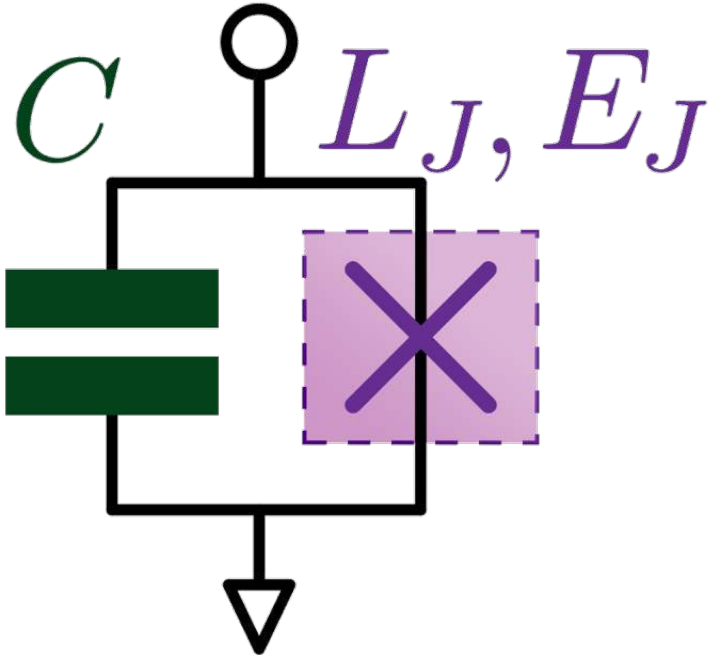
$$\hat{H}\left|n\right\rangle = \hbar\omega_0\left(n + \frac{1}{2}\right)\left|n\right\rangle \qquad \Phi_{\text{ZPF}}Q_{\text{ZPF}} = \frac{\hbar}{2} \qquad Z_0 = \frac{L}{C}$$

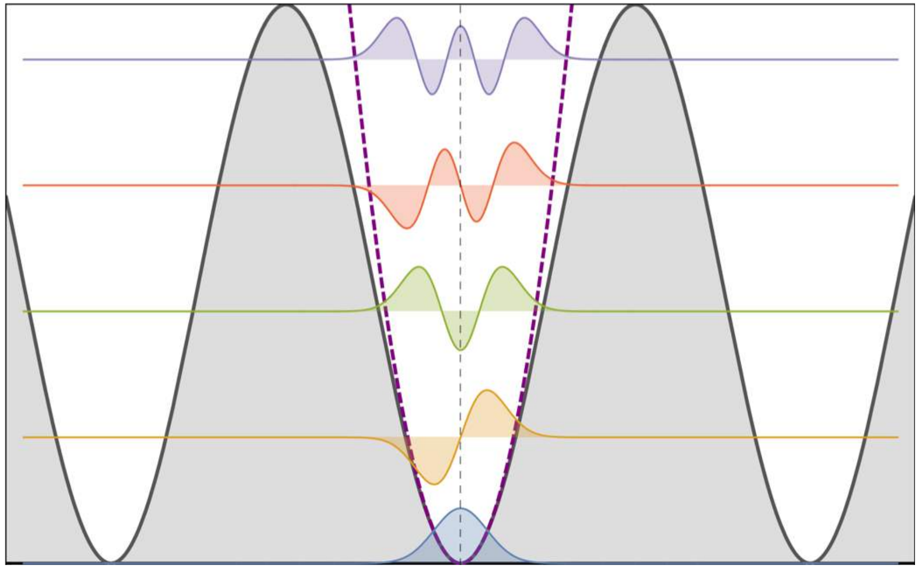
$$\hat{\Phi} = \Phi_{\text{ZPF}}\left(\hat{a}^\dagger + \hat{a}\right) \qquad \Phi_{\text{ZPF}} = \sqrt{\frac{\hbar}{2}Z_0} \qquad \left\langle 0\middle|\hat{\Phi}^2\middle|0\right\rangle = \Phi_{\text{ZPF}}^2$$

$$\hat{Q} = iQ_{\text{ZPF}}\left(\hat{a}^\dagger - \hat{a}\right) \qquad Q_{\text{ZPF}} = \sqrt{\frac{\hbar}{2}Z_0^{-1}} \qquad \left\langle 0\middle|\hat{Q}^2\middle|0\right\rangle = Q_{\text{ZPF}}^2$$

Transmon Qubit

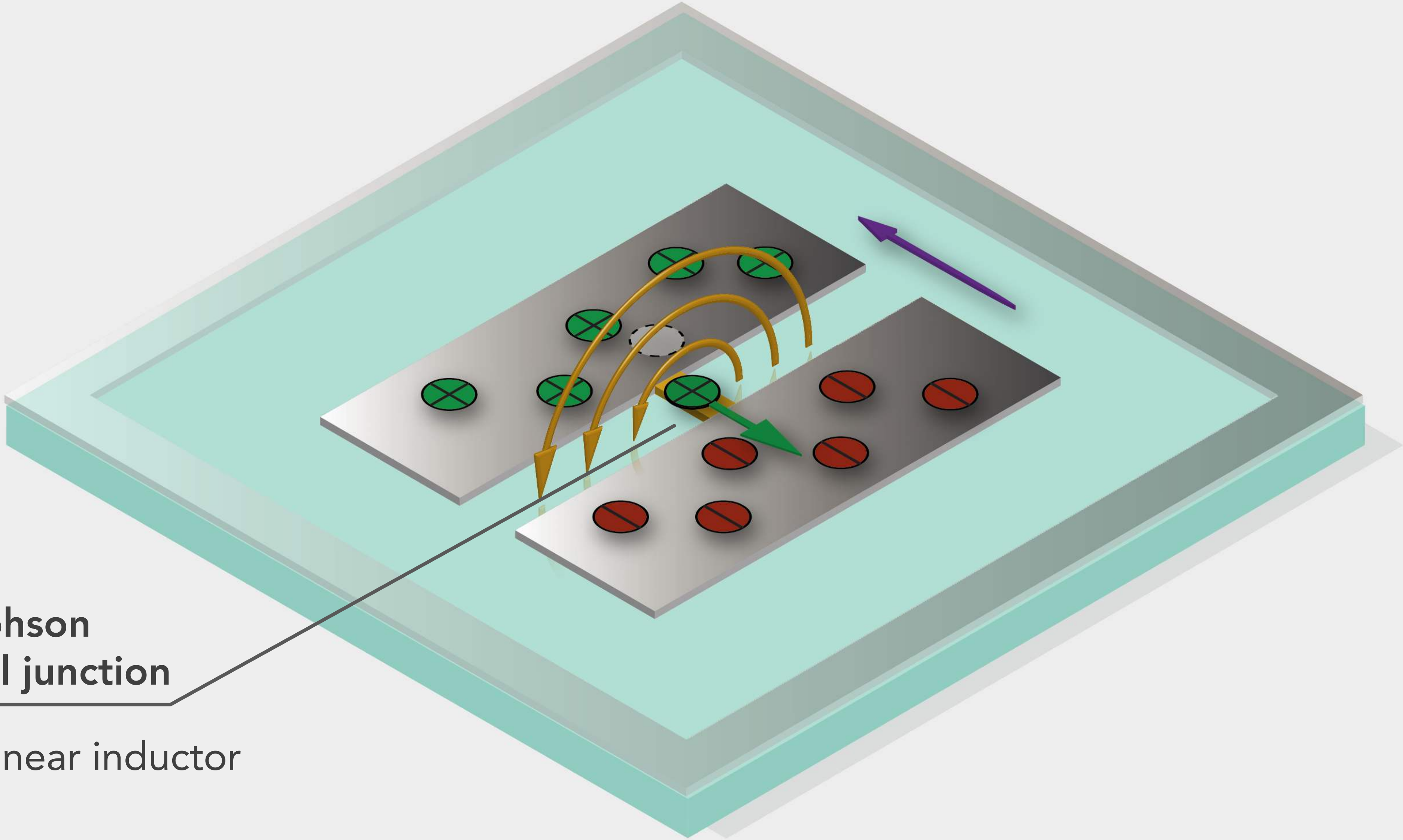






Introduction

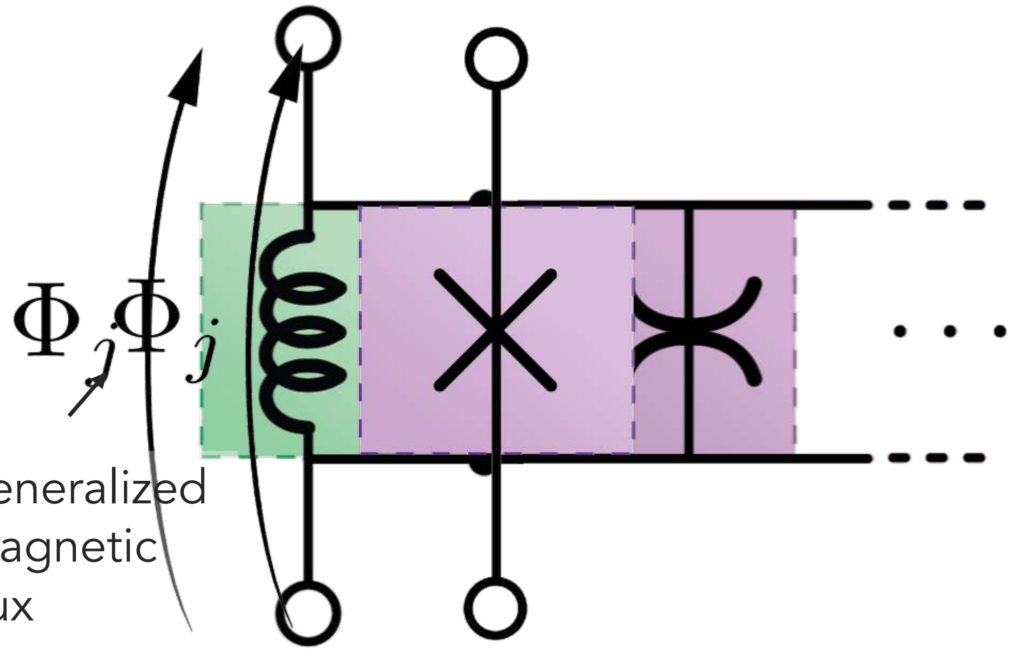
The transmon as a non-linear oscillator

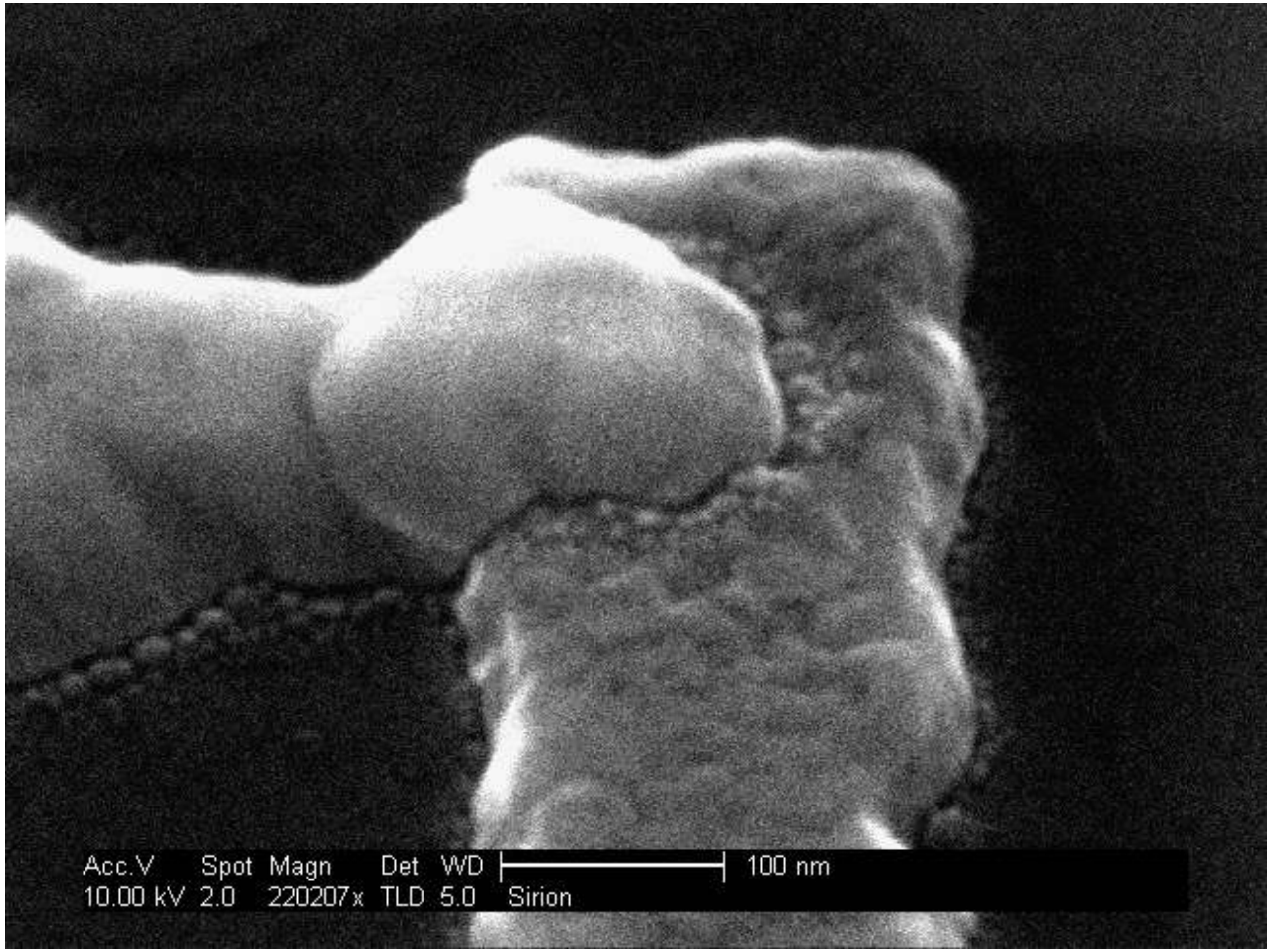


Josephson tunnel junction

Non-linear inductor

Josephson tunnel junction





SEM image: L. Frunzio

$$\mathcal{E}_j\left(\Phi_j\right) = E_j\left(1 - \cos\left(\Phi_j/\phi_0\right)\right)$$

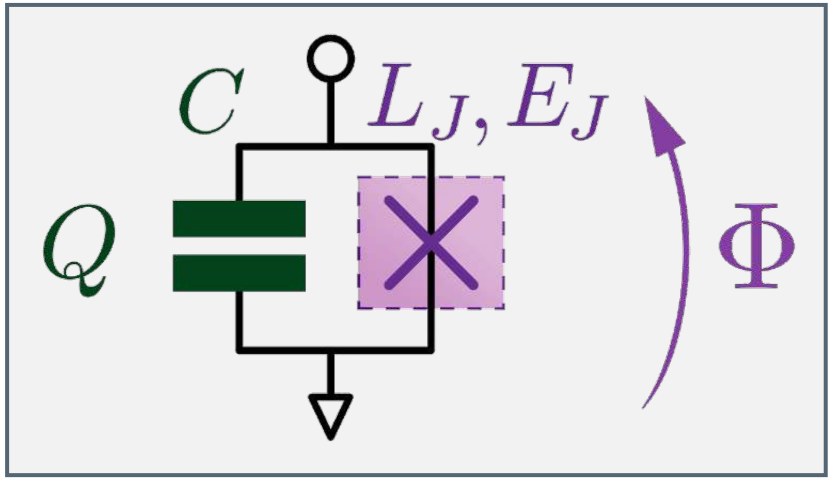
$$= \mathcal{E}_j^{\mathrm{lin}}\left(\Phi_j\right) + \mathcal{E}_j^{\mathrm{nl}}\left(\Phi_j\right)$$

$$\phi_0 \equiv \hbar/2e$$

$$= \frac{E_j}{2}\left(\frac{\Phi_j}{\phi_0}\right)^2 - \frac{E_j}{4!}\left(\frac{\Phi_j}{\phi_0}\right)^4 + \mathcal{O}\left(\Phi_j^6\right)$$

Circuit image: Minev et al., EPR to appear (2020)

The Transmon qubit



$$\boxed{E_J = \frac{\phi_0^2}{L_J}}$$

$$\phi_0 = \frac{\hbar}{2e}$$

$$\approx 3.3 \times 10^{-16}\ \mathrm{Wb}$$

$$\mathcal{E}_J\left(\Phi\right) = -E_J \cos\left(\Phi/\phi_0\right)$$

$$\mathcal{E}_J^{(2)}\left(\Phi\right) \approx \frac{\Phi^2}{2L_J} \qquad \mathcal{E}_{\mathrm{cap}}\left(\dot{\Phi}\right) \approx \frac{1}{2}C\dot{\Phi}^2$$

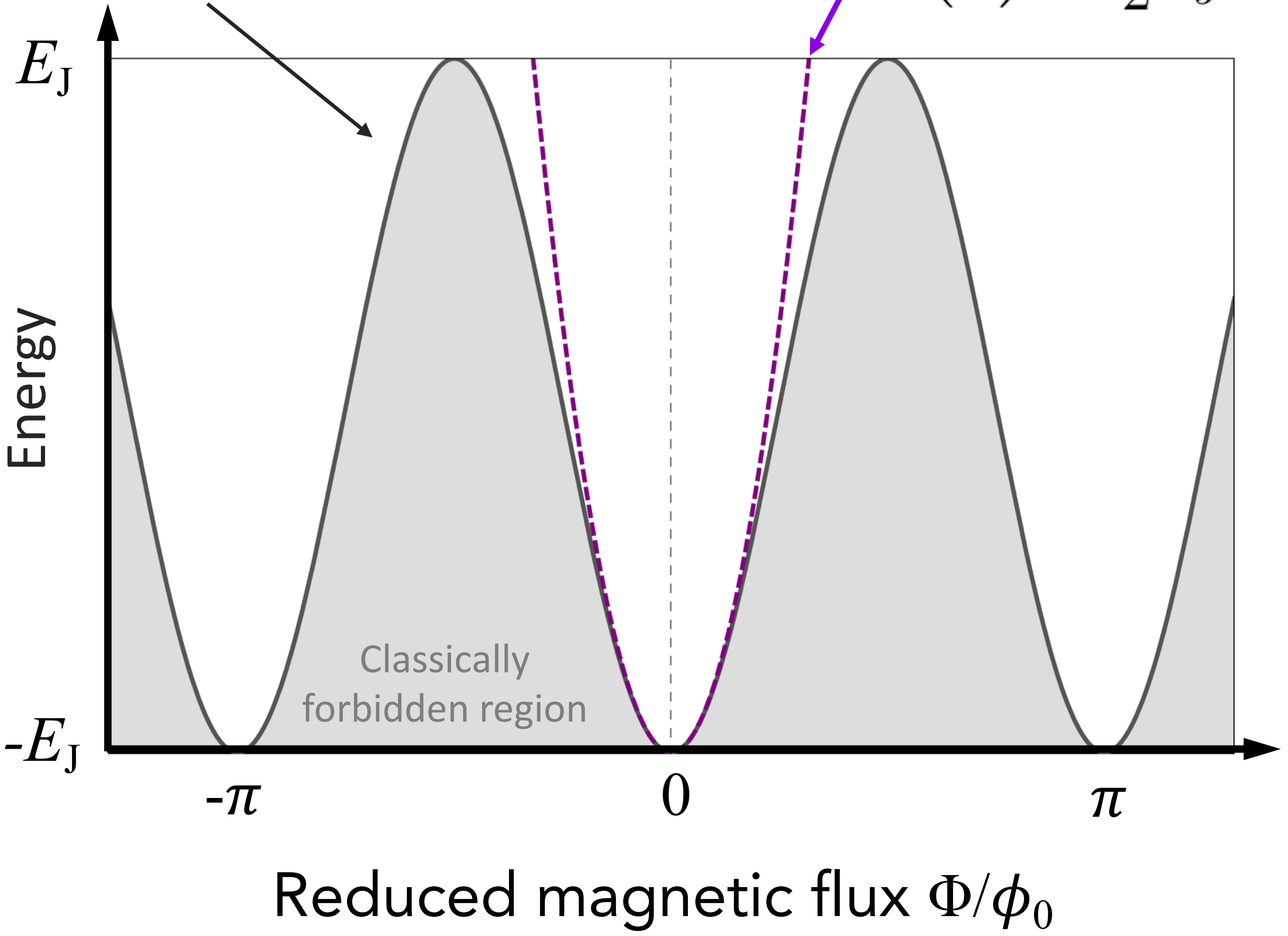
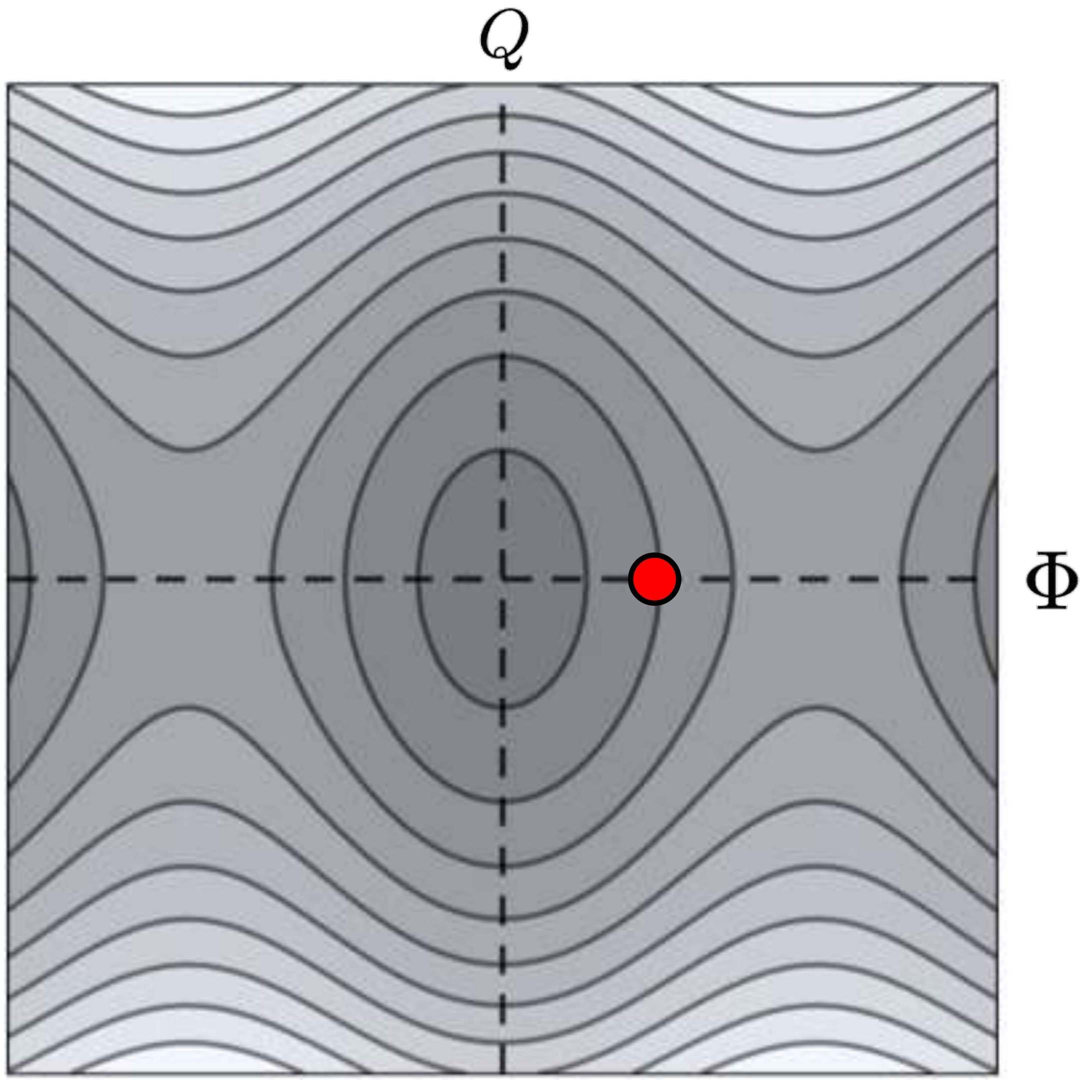


Image: Minev et al., EPR to appear (2020)

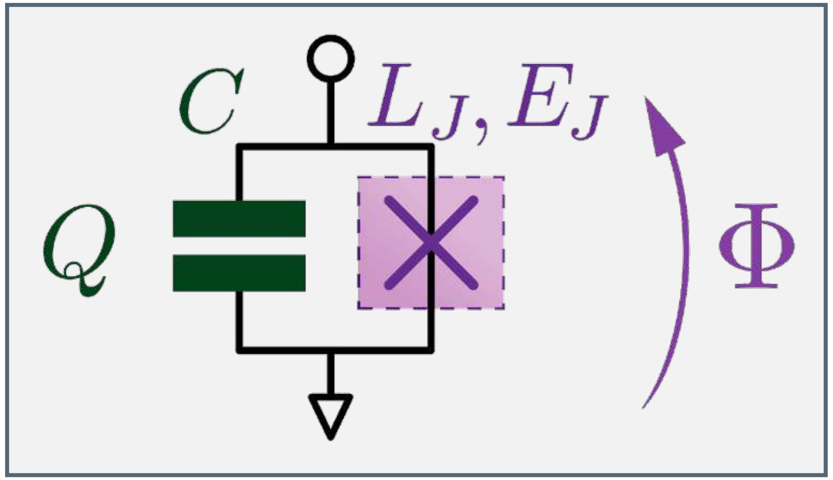
Semi-classical intuition: phase space picture

$$\mathcal{H}(\Phi, Q) = \frac{Q^2}{2C} - E_J \cos(\Phi/\phi_0)$$

$$E = \hbar\omega_0 \left(n + \frac{1}{2}\right)$$
$$= \frac{Q^2}{2C} - E_J \cos(\Phi/\phi_0)$$



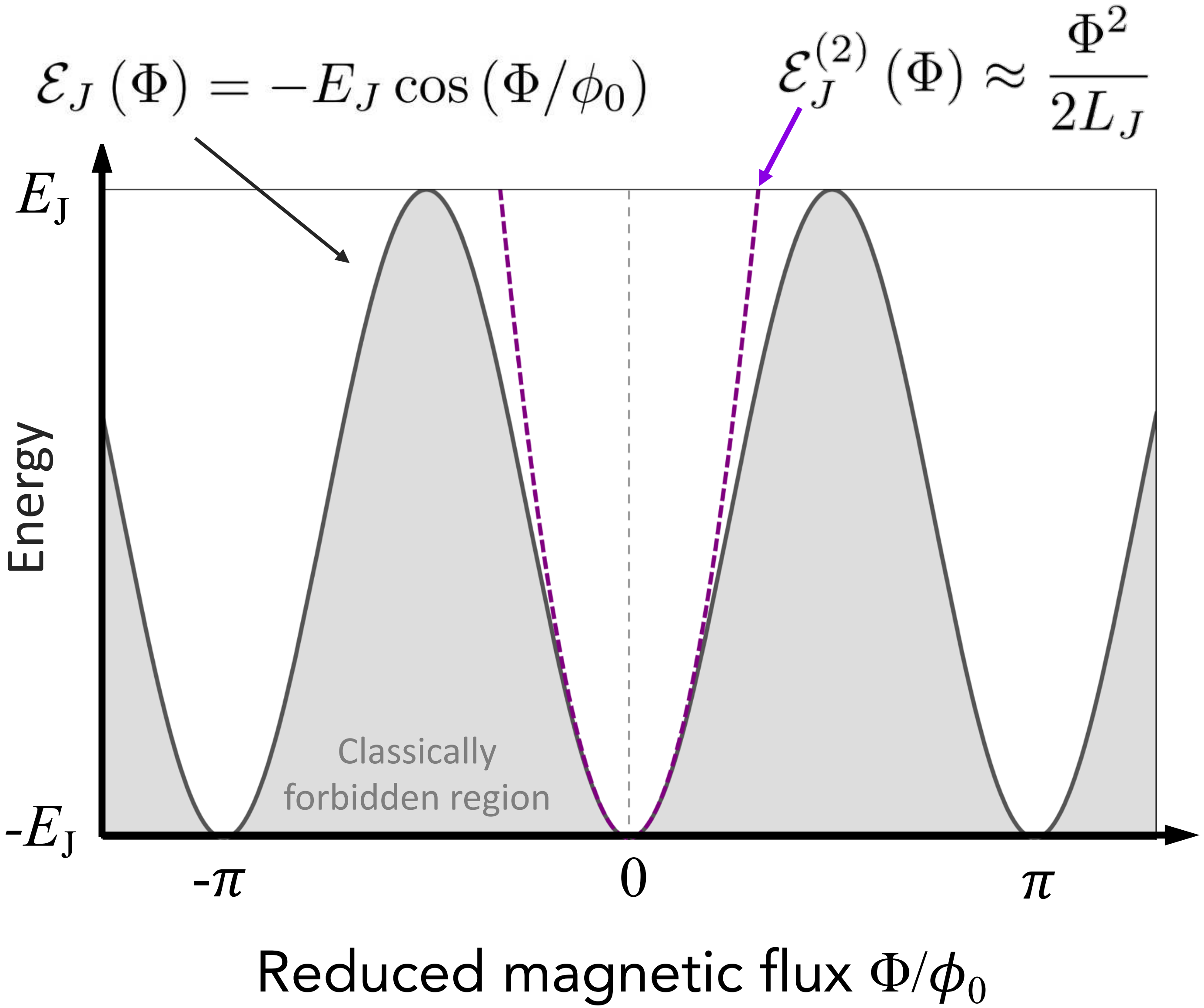
The Transmon qubit

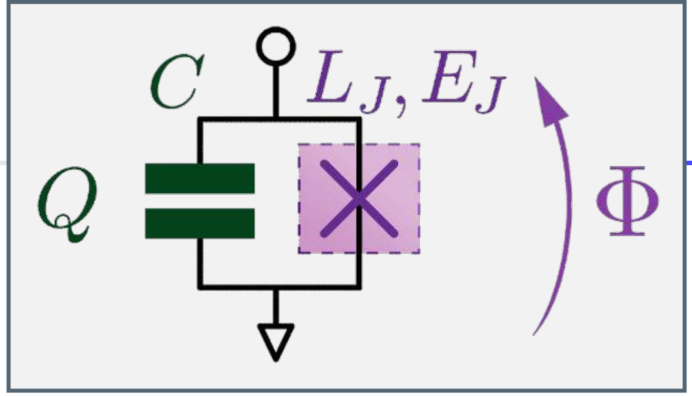


$$\hat{H} = \frac{\hat{Q}^2}{2C} - E_J \cos\left(\hat{\Phi}/\phi_0\right)$$

$$\approx \frac{\hat{Q}^2}{2C} + \frac{\hat{\Phi}^2}{2L_J} - \frac{E_J}{4!}\left(\frac{\hat{\Phi}}{\phi_0}\right)^4$$

$$= \hbar\omega_0 \hat{a}^\dagger \hat{a} - \frac{E_J \phi_{\mathrm{ZPF}}^4}{4!}\left(\hat{a} + \hat{a}^\dagger\right)^4$$





The Transmon qubit

$$= \hbar\omega_0 \hat{a}^\dagger \hat{a} - \frac{E_J \phi_{\mathrm{ZPF}}^4}{4!}\left(\cancel{3} + 6\cancel{\hat{a}^2} + 12\hat{a}^\dagger\hat{a} + 6\cancel{\hat{a}^{\dagger 2}}\right.$$
$$\left. + \cancel{\hat{a}^4} + 4\cancel{\hat{a}^\dagger\hat{a}^3} + 6\hat{a}^{\dagger 2}\hat{a}^2 + 4\cancel{\hat{a}^{\dagger 3}\hat{a}} + \cancel{\hat{a}^{\dagger 4}}\right)$$

RWA or 1st order PT

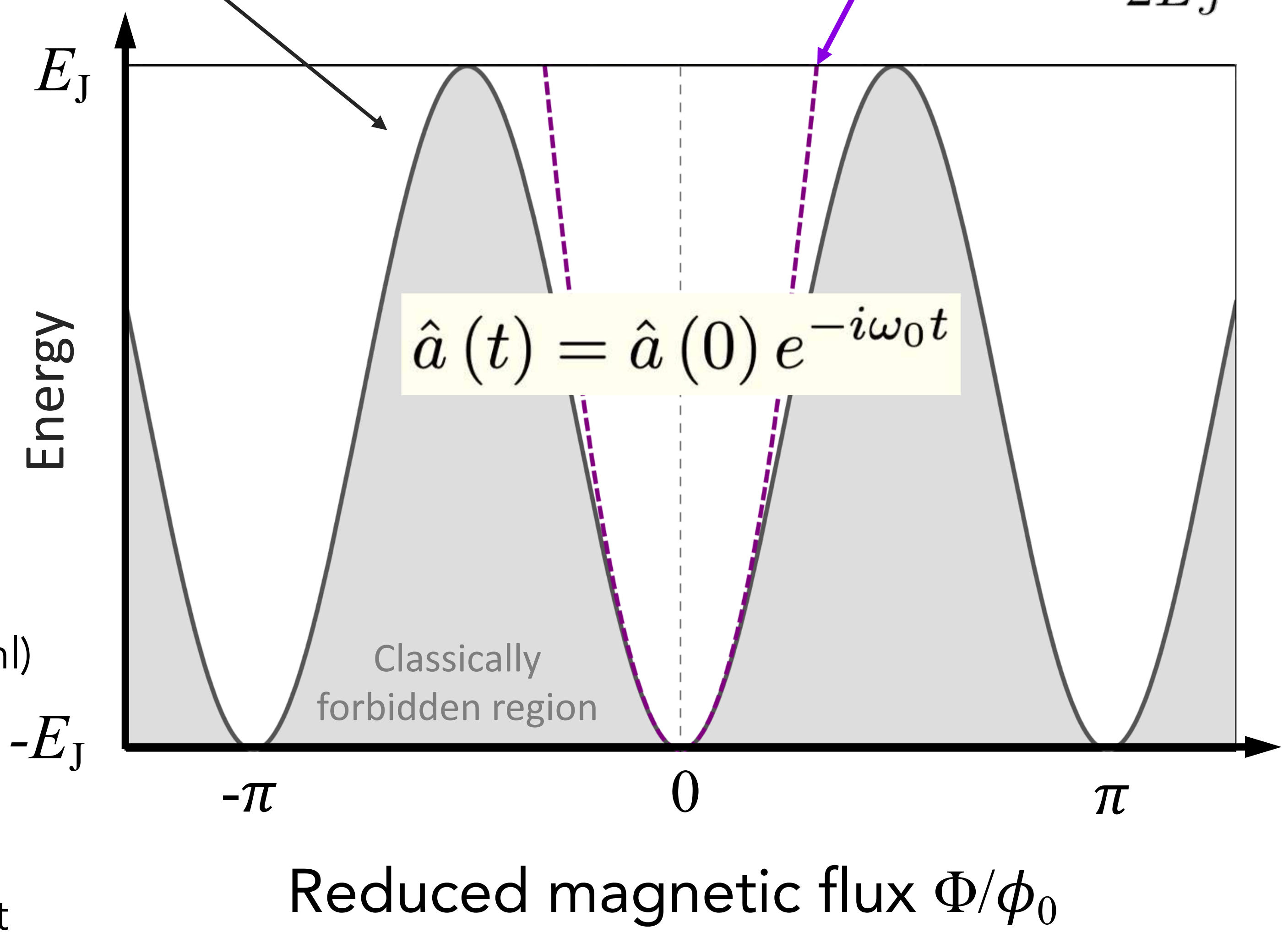
$$\hat{H}_4^{\mathrm{RWA}} = \hbar\omega_0 \hat{a}^\dagger \hat{a} - \frac{E_J}{4!}\phi_a^4\left(12\hat{a}^\dagger\hat{a} + 6\hat{a}^{\dagger 2}\hat{a}^2\right)$$
$$= \boxed{\hbar\left(\omega_0 - \Delta_q\right)\hat{a}^\dagger\hat{a} - \frac{\hbar\alpha}{2}\hat{a}^{\dagger 2}\hat{a}^2}$$

Due to nl & commutator   Due to non-linearity (nl)

where $\hbar\Delta_q = \hbar\alpha \equiv \frac{1}{2}E_J\phi_{\mathrm{ZPF}}^4$

"Lamb shift" due to ZPF   Anharmonicity   Zero-point fluctuations

$$\mathcal{E}_J(\Phi) = -E_J\cos(\Phi/\phi_0) \qquad \mathcal{E}_J^{(2)}(\Phi) \approx \frac{\Phi^2}{2L_J}$$



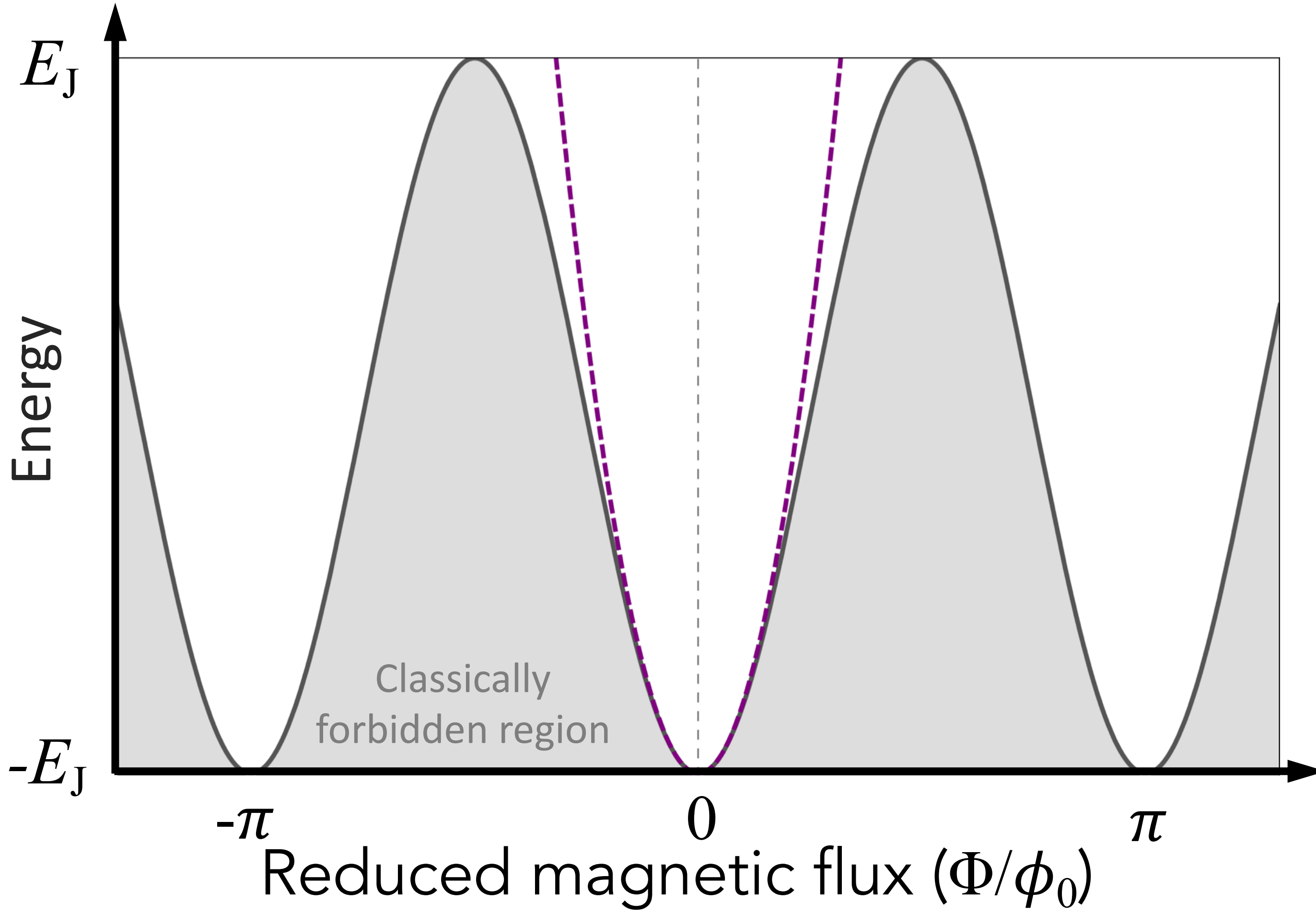
The Kerr Hamiltonian of the transmon

$$\hat{N} \equiv \hat{a}^\dagger \hat{a}$$

$$\boxed{\hat{H}_4^{\mathrm{RWA}} \approx \underbrace{\hbar\omega_q \hat{N}}_{\hat{H}_{\mathrm{lin}}} \underbrace{- \frac{\hbar\alpha}{2}\hat{N}\left(\hat{N}-1\right)}_{\hat{H}_{\mathrm{nl}}}}$$

Solution known (SHO) New from nonlinearity

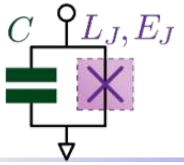
$$\hat{H}_4^{\mathrm{RWA}}\left|n\right\rangle = \hbar\omega_q n\left(1 - \frac{\hbar\alpha_a}{2}(n-1)\right)\left|n\right\rangle$$



1st order correction to energy: $E_n^{(1)} = \left\langle n^{(0)}\right|\hat{H}_{\mathrm{nl}}\left|n^{(0)}\right\rangle$ and to eigenstates: $\left|n^{(1)}\right\rangle = \sum_{k\neq n} \frac{\left\langle k^{(0)}\right|\hat{H}_{\mathrm{nl}}\left|n^{(0)}\right\rangle}{E_n^{(0)} - E_k^{(0)}}\left|k^{(0)}\right\rangle.$

To first order perturbation theory the eigenstates do not change! Only the energy changes. Dispersive.

Exploring a real transmon qubit

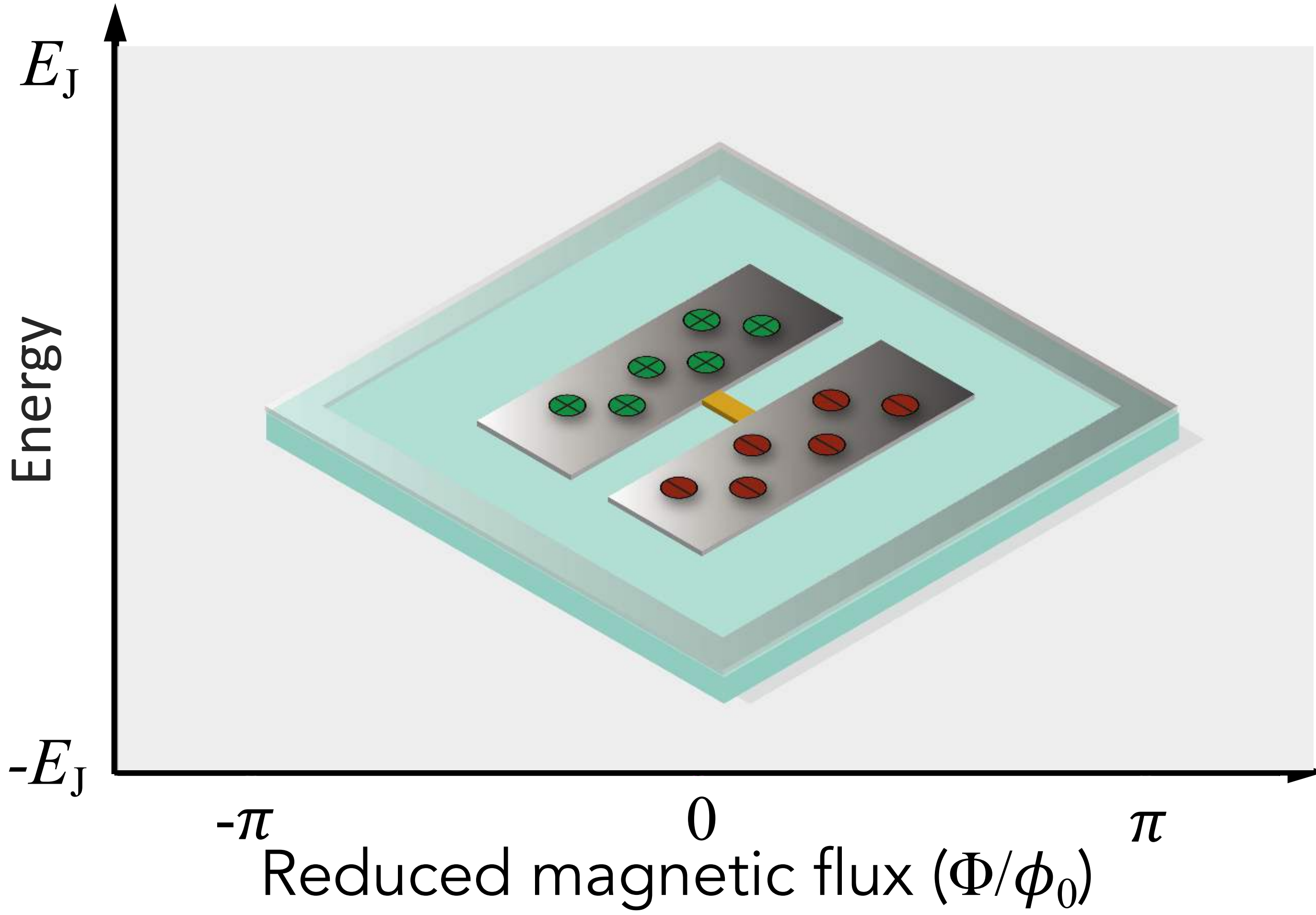


$$\hat{N} \equiv \hat{a}^\dagger \hat{a}$$

$$\boxed{\hat{H}_4^{\mathrm{RWA}} \approx \underbrace{\hbar\omega_q \hat{N}}_{\hat{H}_{\mathrm{lin}}} - \underbrace{\frac{\hbar\alpha}{2}\hat{N}\left(\hat{N}-1\right)}_{\hat{H}_{\mathrm{nl}}}}$$

Solution known (SHO) → $\hat{H}_{\mathrm{lin}}$

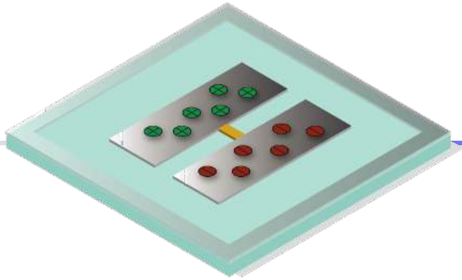
$\hat{H}_{\mathrm{nl}}$ ← New from nonlinearity



Experimental parameters

$$L_J = 14\ \mathrm{nH} \qquad E_J = \frac{\phi_0^2}{L_J} = 12\ \mathrm{GHz}$$

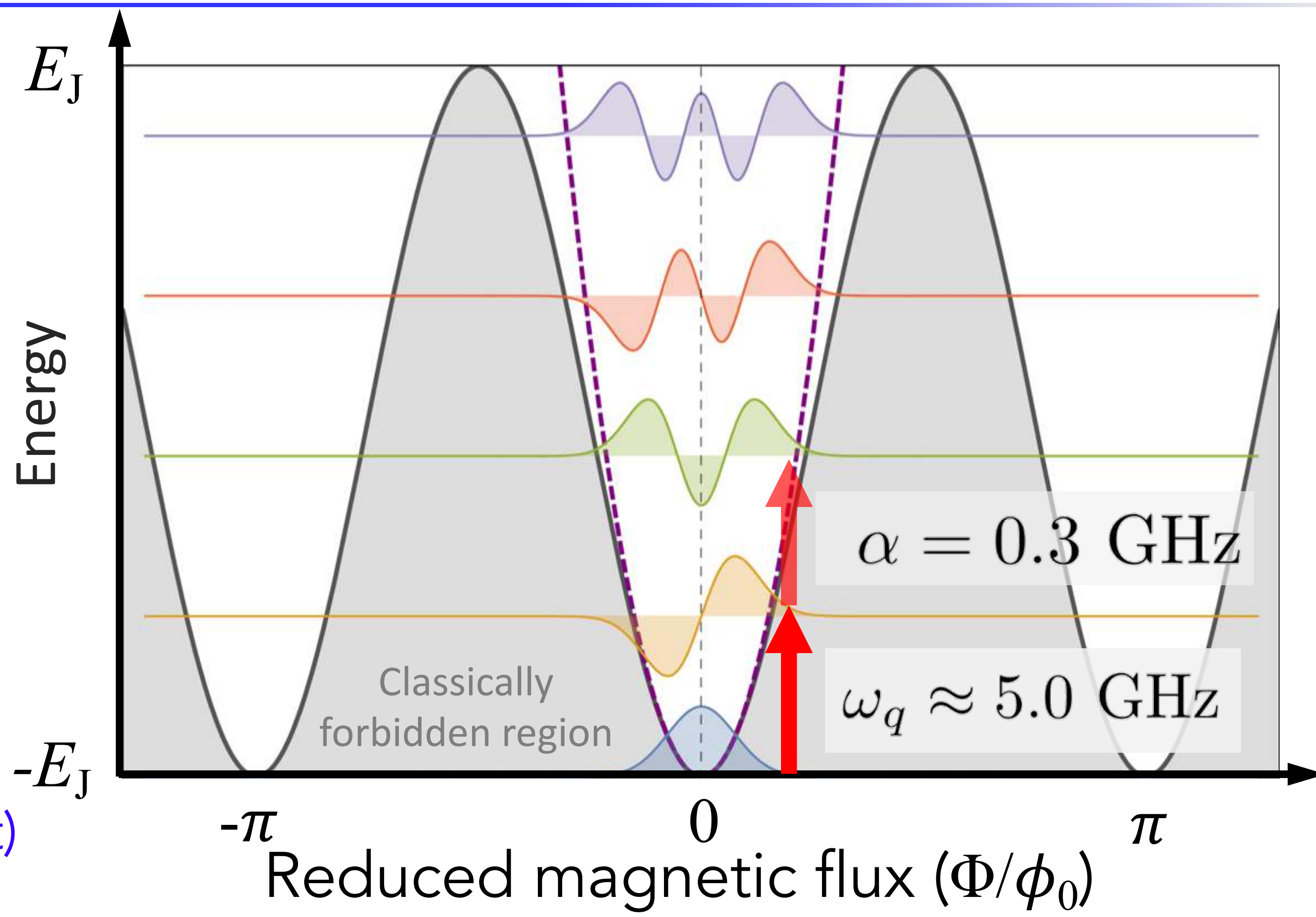
$$C_J = 65\ \mathrm{fF} \qquad E_C = \frac{e^2}{2C} = 0.3\ \mathrm{GHz}$$



Exploring a real transmon qubit

$$\hat{H}_4^{\mathrm{RWA}} \approx \hbar\omega_q \hat{N} - \frac{\hbar\alpha}{2}\hat{N}\left(\hat{N}-1\right)$$

Dispersive, states didn't change to 1st order
"Lamb shift" due to ZPF



Parameters used in figure (of a measured qubit)

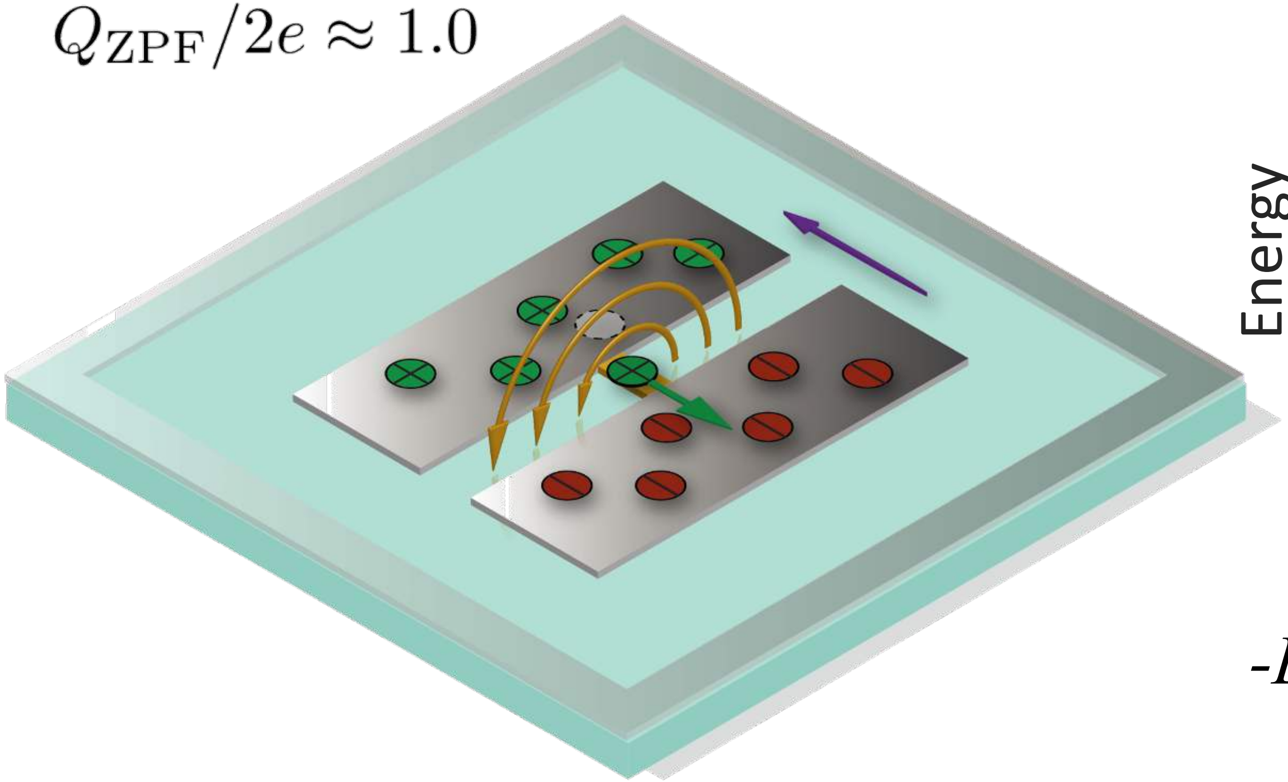
$$L_J = 14\ \mathrm{nH} \qquad E_J = \frac{\phi_0^2}{L_J} = 12\ \mathrm{GHz} \qquad \omega_0 = \sqrt{\frac{1}{LC}} = 5.3\ \mathrm{GHz}$$

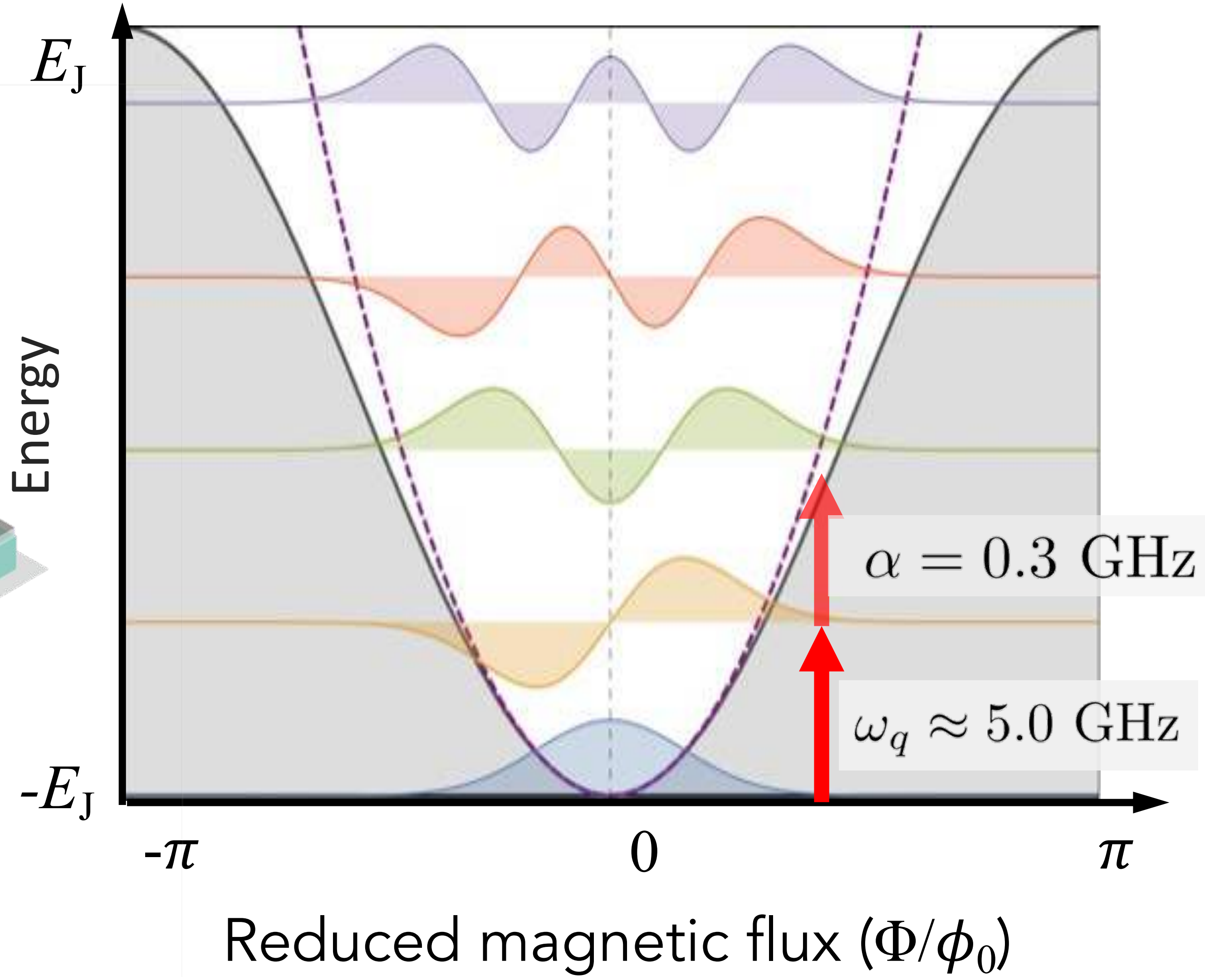
$$C_J = 65\ \mathrm{fF} \qquad E_C = \frac{e^2}{2C} = 0.3\ \mathrm{GHz} \qquad Z = \sqrt{\frac{L}{C}} \approx 450\ \Omega$$

Quantum fluctuations of the transmon qubit

$$\Phi_{\mathrm{ZPF}}/\phi_0 \approx 0.5$$

$$Q_{\mathrm{ZPF}}/2e \approx 1.0$$



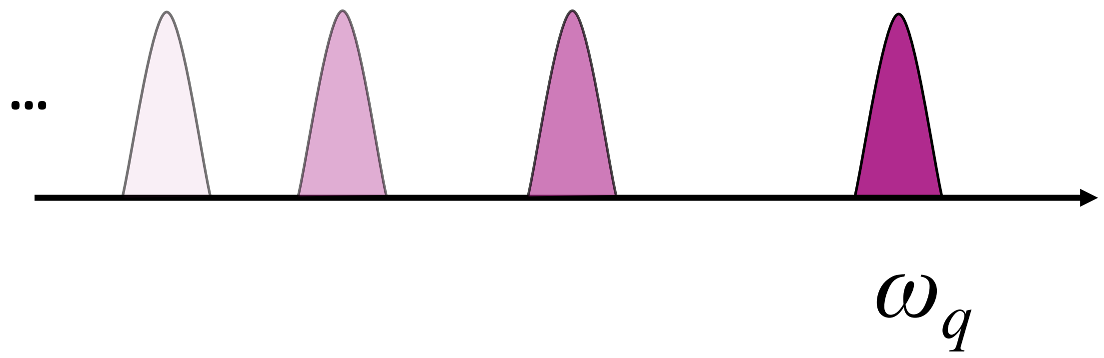


Energy diagram and transition spectrum

Energy levels

$$\hat{H} \approx \omega_0 \hat{a}^\dagger \hat{a} - \frac{\alpha}{2} \hat{a}^{\dagger 2} \hat{a}^2$$

Transition spectrum



The Transmon qubit: restricting Hilbert space

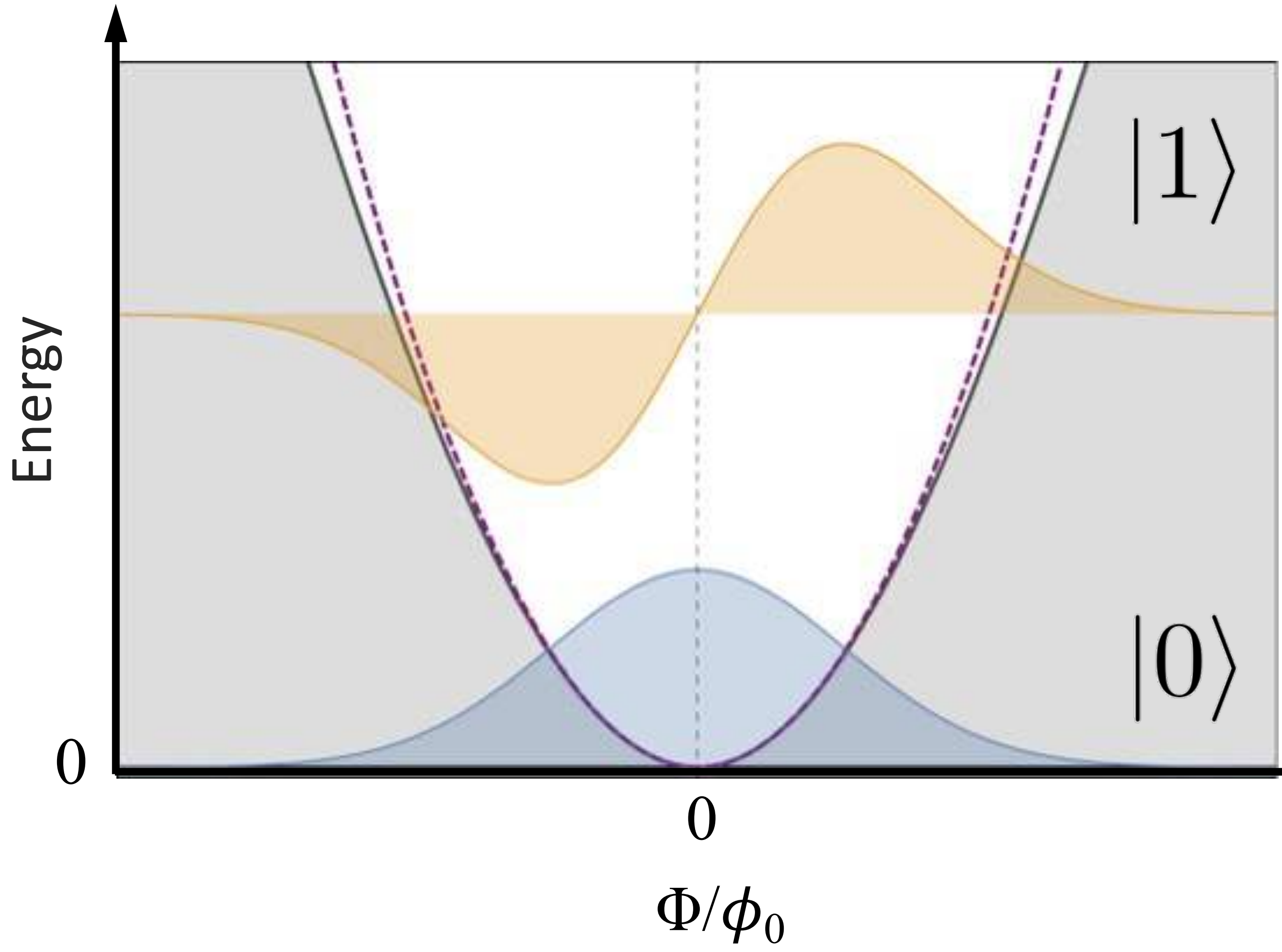
$$\boxed{\hat{H}_4^{\mathrm{RWA}} \approx \hbar\omega_q \hat{N} - \frac{\hbar\alpha}{2}\hat{N}\left(\hat{N}-1\right)}$$

$$\hat{N} \equiv \hat{a}^\dagger \hat{a}$$

Restrict to qubit subspace of |0> and |1>

$$\hat{a}^\dagger \hat{a} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & \ddots \end{pmatrix}$$

$$\hat{a} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \sqrt{2} & 0 \\ 0 & 0 & 0 & \sqrt{3} \\ 0 & 0 & 0 & \ddots \end{pmatrix}$$



The Transmon qubit: restricting Hilbert space

$$\hat{H}_4^{\mathrm{RWA}} \approx \hbar\omega_q \hat{N} - \frac{\hbar\alpha}{2}\hat{N}\left(\hat{N}-1\right)$$

$$\hat{N} \equiv \hat{a}^\dagger \hat{a}$$

Restrict to qubit subspace of |0> and |1>

$$\left(\hat{N} - \frac{1}{2}\hat{I}\right) \mapsto -\frac{1}{2}\hat{Z} \qquad \hat{a} \mapsto \hat{\sigma}_- = \frac{1}{2}\left(\hat{X} - i\hat{Y}\right)$$

Fock number operator

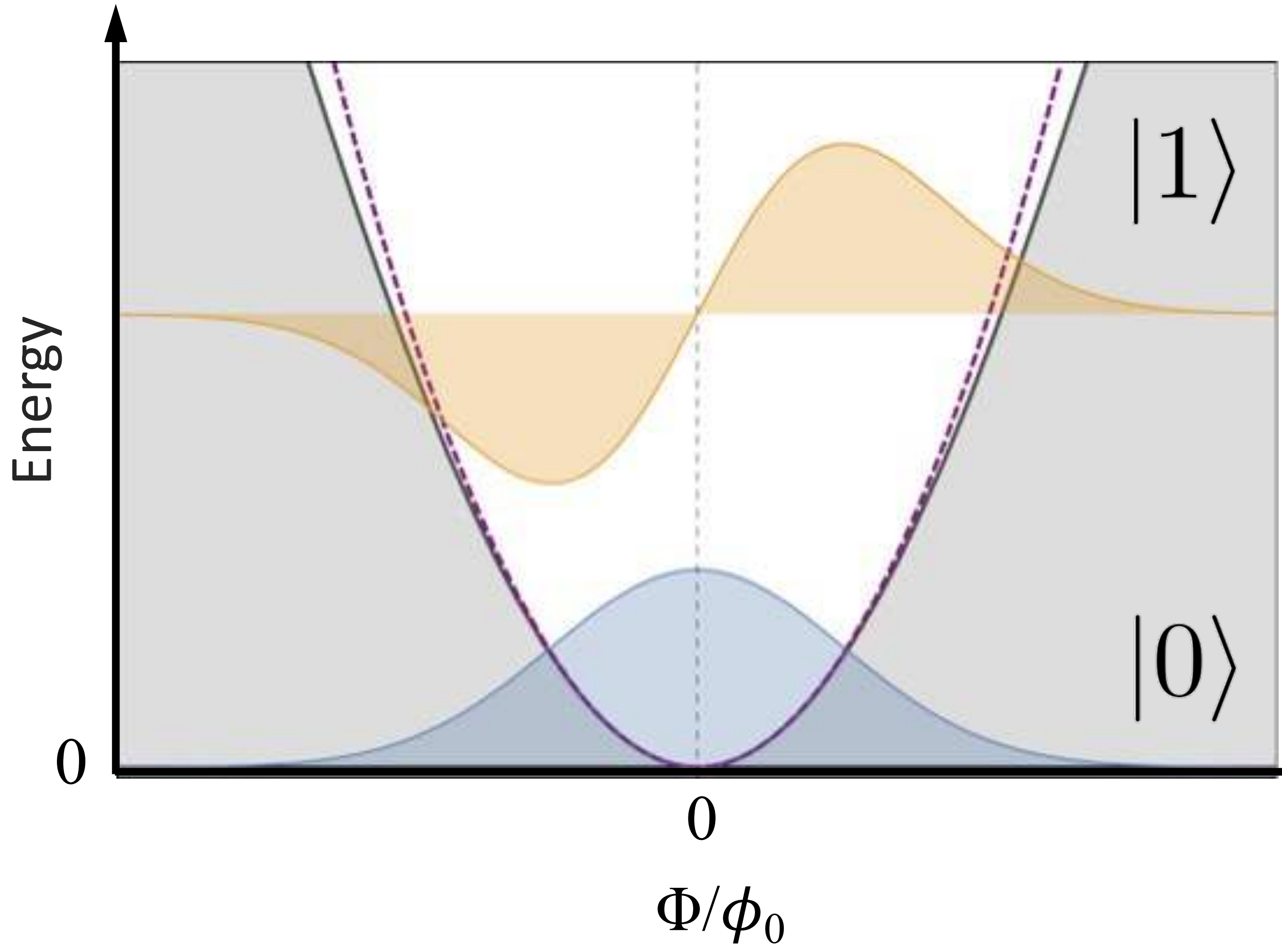
$$\begin{pmatrix} 1 & 0 & \cdots \\ 0 & 2 & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}$$

Qubit Pauli Z operator

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Qubit Pauli X and Y operators

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$



arXiv.org > quant-ph > arXiv:2010.00620

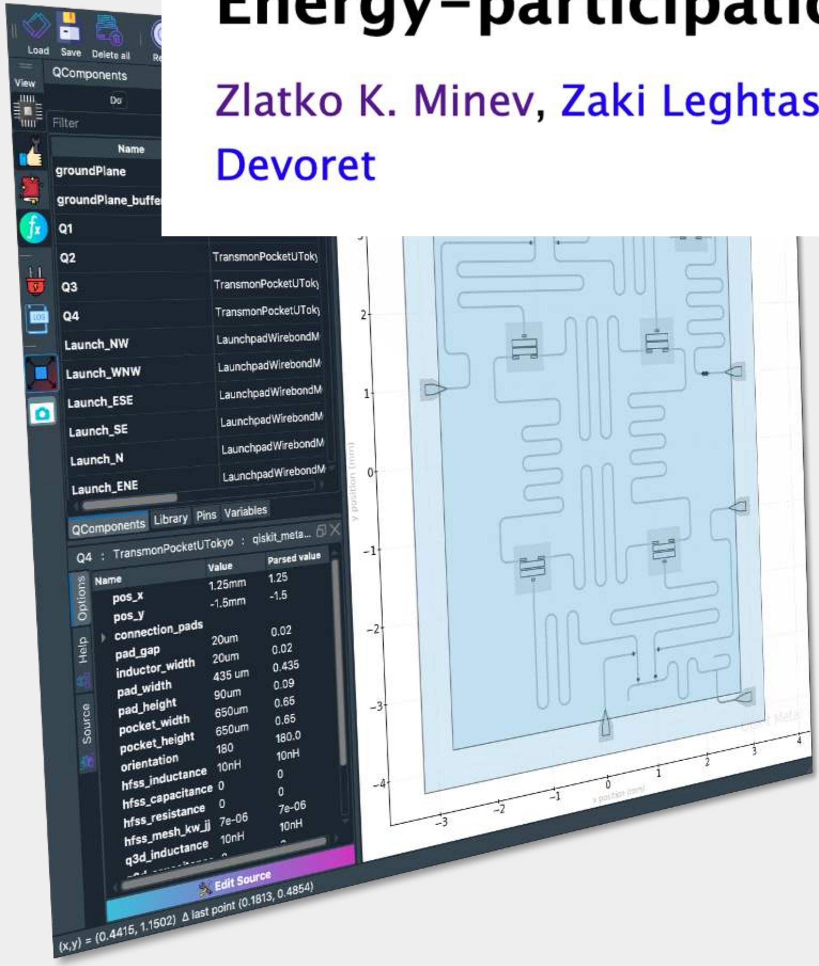
Quantum Physics

[Submitted on 1 Oct 2020]

Energy-participation quantization of Josephson circuits

Zlatko K. Minev, Zaki Leghtas, Shantanu O. Mundhada, Lysander Christakis, Ioan M. Pop, Michel H. Devoret

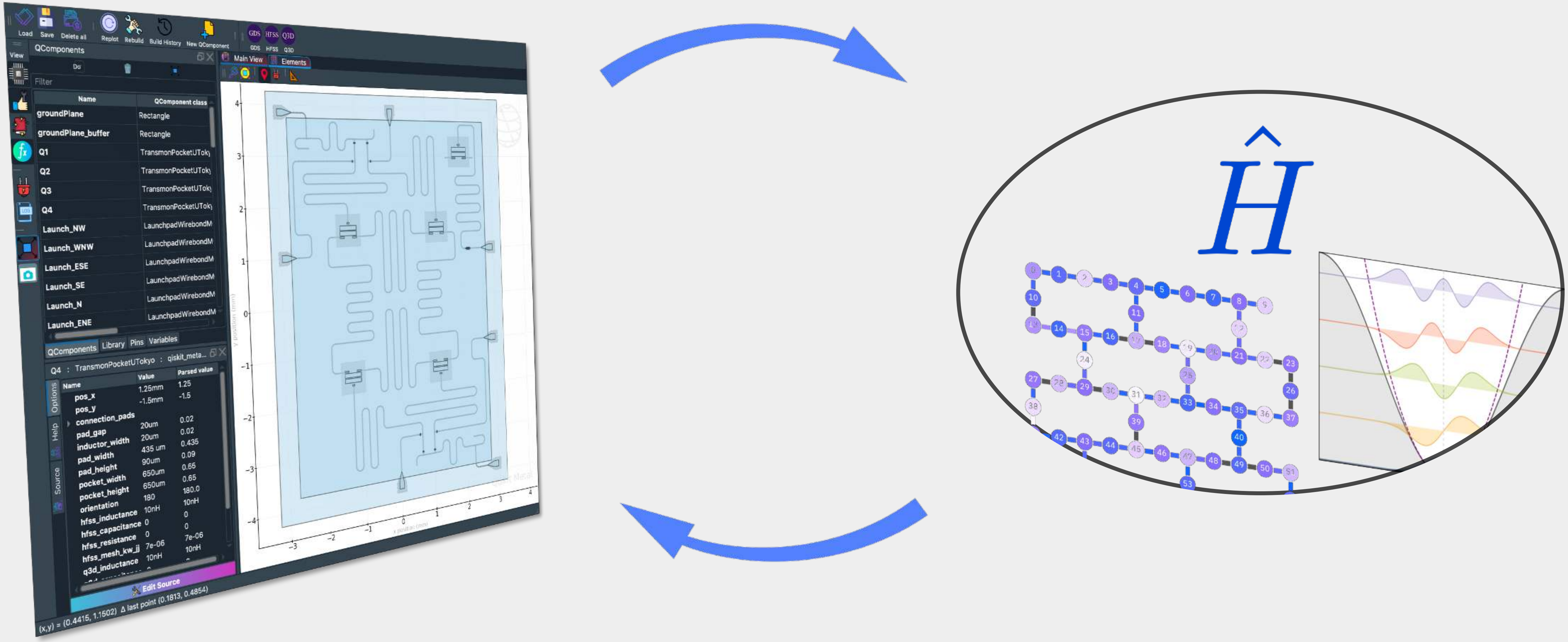
Accepted: *Nature npj qi*



$$\hat{H}_{\mathrm{tot}} = \hat{H}_{\mathrm{lin}} + \hat{H}_{\mathrm{nl}}$$

See also Minev
Yale dissertation Sec. 4.1
(arXiv: 1902.10355)

Make easy?


Load
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Build History
New QComponent
GDS
HFSS
Q3D
QComponents
Main View
Elements
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QComponent class
groundPlane
Rectangle
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Q4
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Library
Pins
Variables
Q4 : TransmonPocketUTokyo
Value
Parsed value
pos_x
1.25mm
1.25
pos_y
-1.5mm
-1.5
connection_pads
pad_gap
inductor_width
pad_width
pad_height
pocket_width
pocket_height
orientation
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hfss_capacitance
hfss_resistance
hfss_mesh_kw_jj
q3d_inductance
Edit Source
(x,y) = (0.4415, 1.1502) Δ last point (0.1813, 0.4854)
$\hat{H}$

https://github.com/Qiskit/qiskit-metal

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Qiskit Metal

Open Source ❤ | awesome | DOI 10.5281/zenodo.4618153

> Quantum hardware design and analysis

Qiskit | quantum device design

Tutorials so far

1 

Qiskit Metal E01 - Overview
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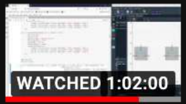
2 

Qiskit Metal E02 - End to end example of Quantum Chip Design - Part 1 of 2
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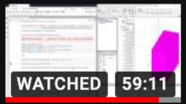
3 

Qiskit Metal E03 End to end example of Quantum Chip Design - Part 2 of 2
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Qiskit Metal E04 - QComponents for parametric design
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New analysis
(pre-recorded)

5 

Qiskit Metal E05.1 - Analysis - Capacitance and Frequency Control
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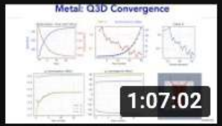
6 

Qiskit Metal E05.2 - Analysis - Eigenmode and Energy Participation
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Qiskit Metal E05.3 - Analysis - EPR Theory
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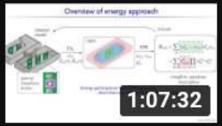
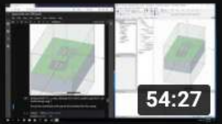
8 

Qiskit Metal E05.4 - Analysis - Summary EPR Quantization with Code Example
Qiskit

9 

Qiskit Metal E05.5 - Analysis - Finish Eigenmode Start Impedance Analysis
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Modeling transmon qubit Cooper-pair box Hamiltonian in the charge basis

(Zlatko Minev, Christopher Warren, Nick Lanzillo 2021)

This module models the transmon qubit in the cooper-pair charge basis, assuming wrapped junction phase variable. The Hamiltonian is given by:

$$\hat{H} = 4E_C\left(\hat{n} - n_g\right) - E_J\cos\left(\hat{\phi}\right),$$

where E_C is the charging energy, E_J is the Josephson energy, $\hat{n}$ is the number of Cooper pairs transferred between charge islands, $\hat{\phi}$ is the gauge-i phase difference between charge islands, and n_g is effective offset charge of the device. Expressions for the charging energy, Josephson energy and charge can be written as:

$$E_C = \frac{e^2}{2C_\Sigma}, \qquad n_g = -\frac{C_d\Phi_s(t)}{2e}, \qquad E_J = \frac{\phi_0^2}{L_J},$$

where $C_\Sigma = C_J + C_B + C_g$ (the sum of the Josephson capacitance, shunting capacitance and gate capacitance), L_J is the inductance of the Jose junction, and ϕ is the magnetic flux.

The variables are

$$\hat{\phi} \equiv \frac{\hat{\Phi}}{\phi_0}, \qquad \hat{n} \equiv \frac{\hat{Q}}{2e},$$

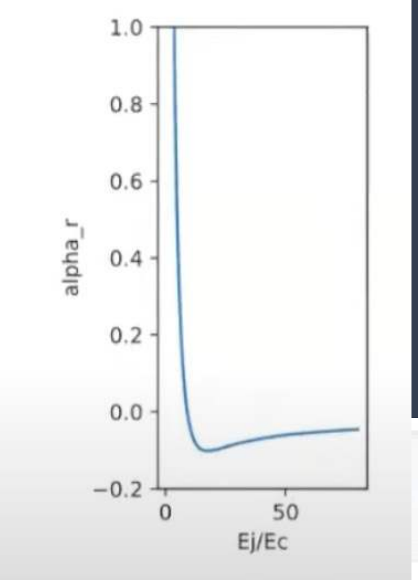
Observe that $\hat{\phi}$ and $\hat{n}$ are both dimensiuonless, and they obey the commutation relationship:

$$[\hat{\phi}, \hat{n}] = i$$

The Hamiltonian can be written in the charge ($\hat{n}$) basis as:

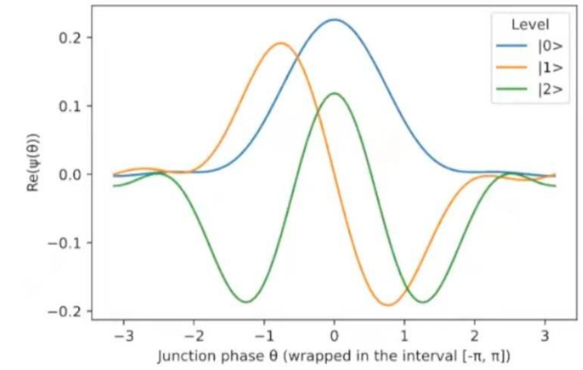
$$H = 4E_C(\hat{n} - n_g)^2 - \frac{1}{2}E_J\sum_n(|n\rangle\langle n+1| + \text{h.c.}),$$

Where $\hat{n} = \sum^{\text{inf}} |n\rangle\langle n|$




```python
import matplotlib.pyplot as plt
for k in range (3):
    ψ, θ = H.psi_k(k,101)
    plt.plot(θ, ψ.real+ψ.imag, label=f"|{k}
plt.xlabel("Junction phase θ (wrapped in the
plt.ylabel("Re(ψ(θ))")
plt.legend(title="Level")
```

<matplotlib.legend.Legend at 0x1fbfc29d7c8>



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Qiskit Metal E08 - Physics of the Cooper Pair Box Transmon Qubit

Unlisted

image: *pikpng*

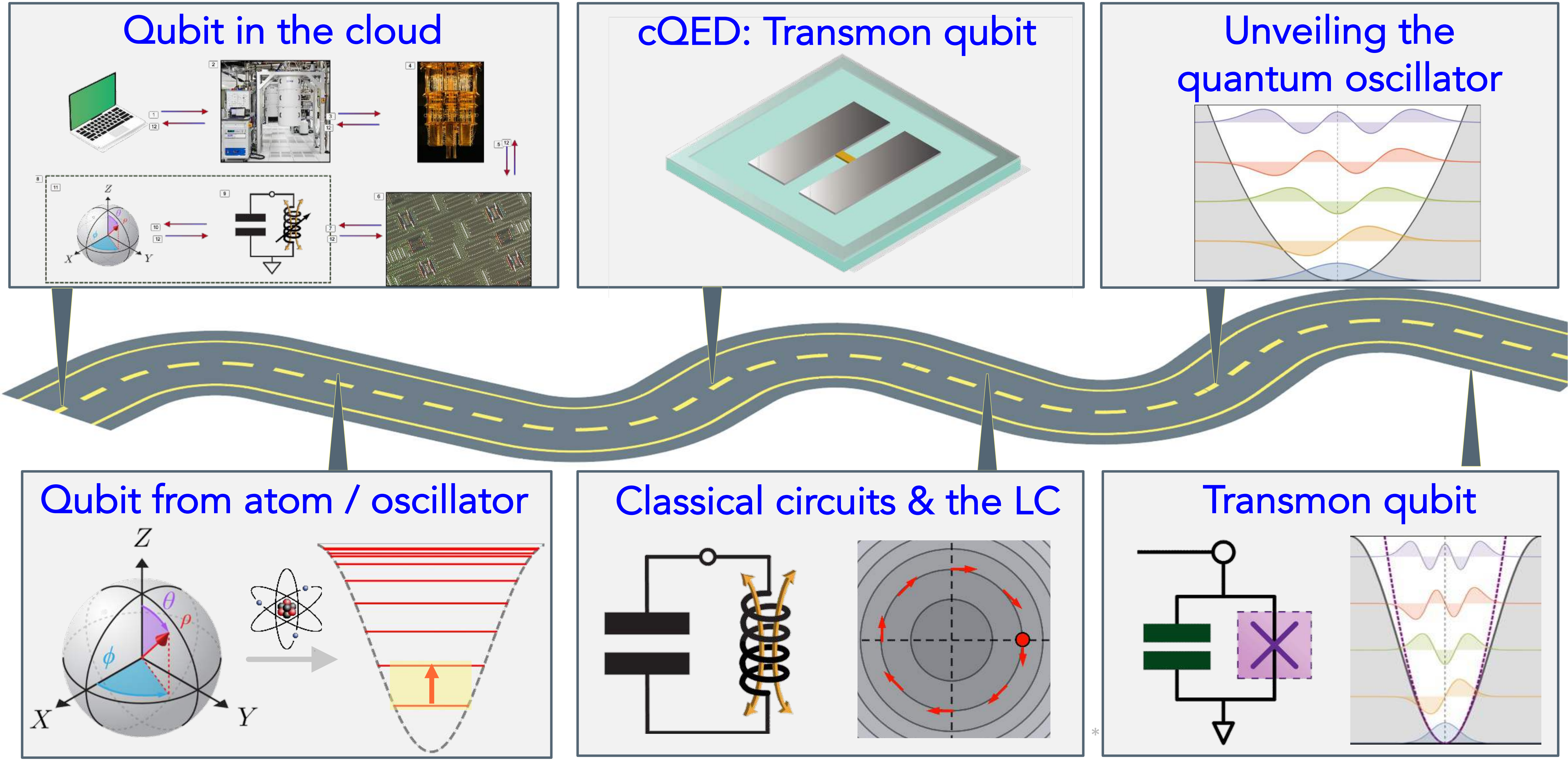
Docs & tutorials
qiskit.org/documentation/metal

Tutorial videos
YouTube – see docs

Slack
#metal (qiskit workspace)

Live weekly tutorials

The road behind and ahead



Next steps

Tightly integrated lab work with Dr. Thomas McConkey!

More depth on qubit control

Run experiments on real devices

Check out references, problems given in the lecture, dangerous bends

Break away from the rules of today

Thank you!

Zlatko K. Minev

The important thing is not to stop questioning. Curiosity has its own reason for existence.

One cannot help but be in awe when he contemplates the mysteries of eternity, of life, of the marvelous structure of reality.

It is enough if one tries merely to comprehend a little of this mystery each day.

Albert Einstein

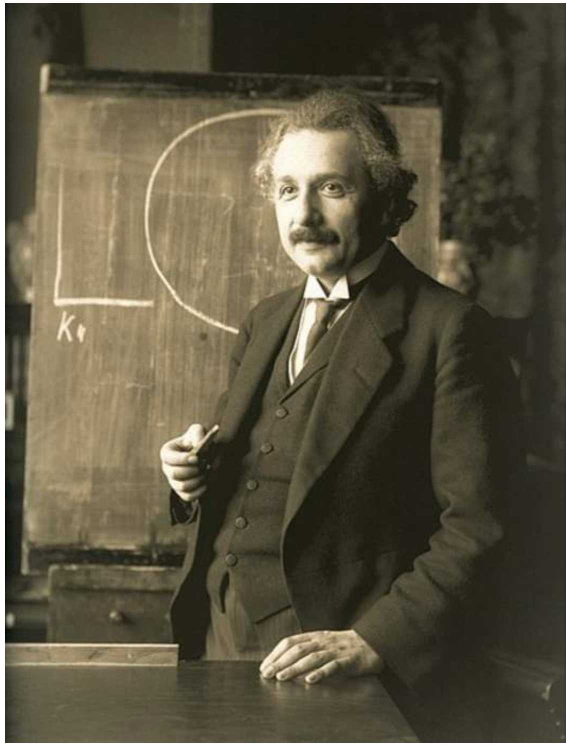


Photo: F. Schmutzer