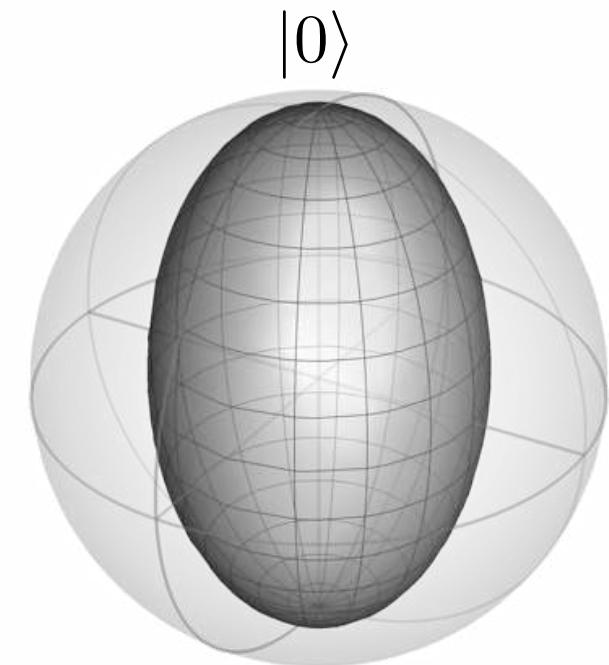
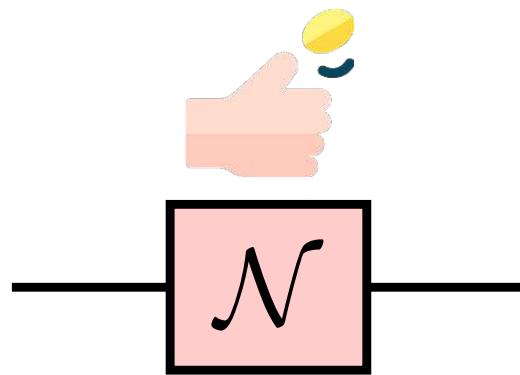
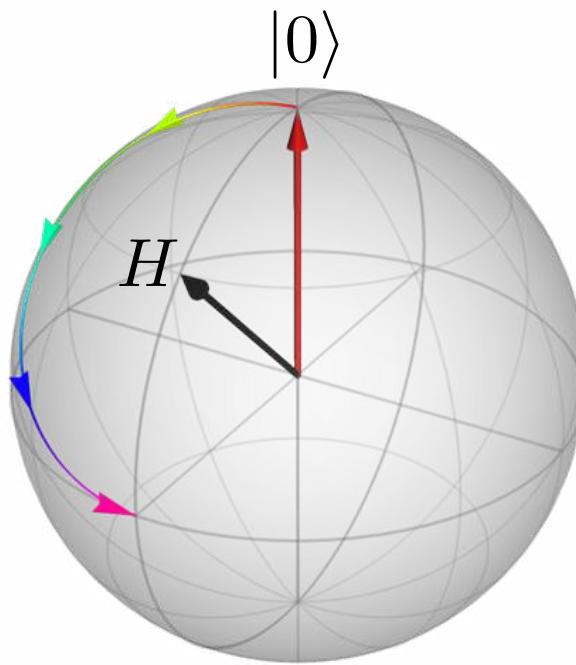


# Introduction to Quantum Noise



*Qiskit Global Summer School:  
Quantum Machine Learning*



## Zlatko K. Minev

IBM Quantum  
IBM T.J. Watson Research Center, Yorktown Heights, NY



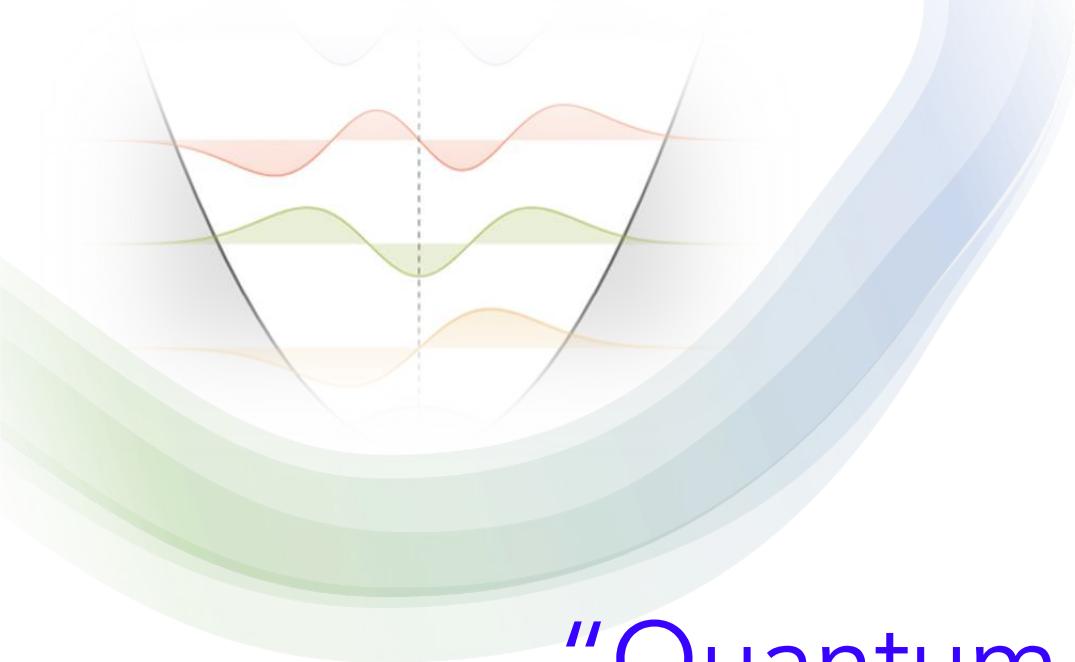
@zlatko\_minev



[zlatko-minev.com](http://zlatko-minev.com)

Image copyright:  
ZKM unless otherwise noted

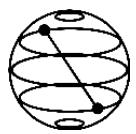
coin toss: flaticon  
spam: make it move



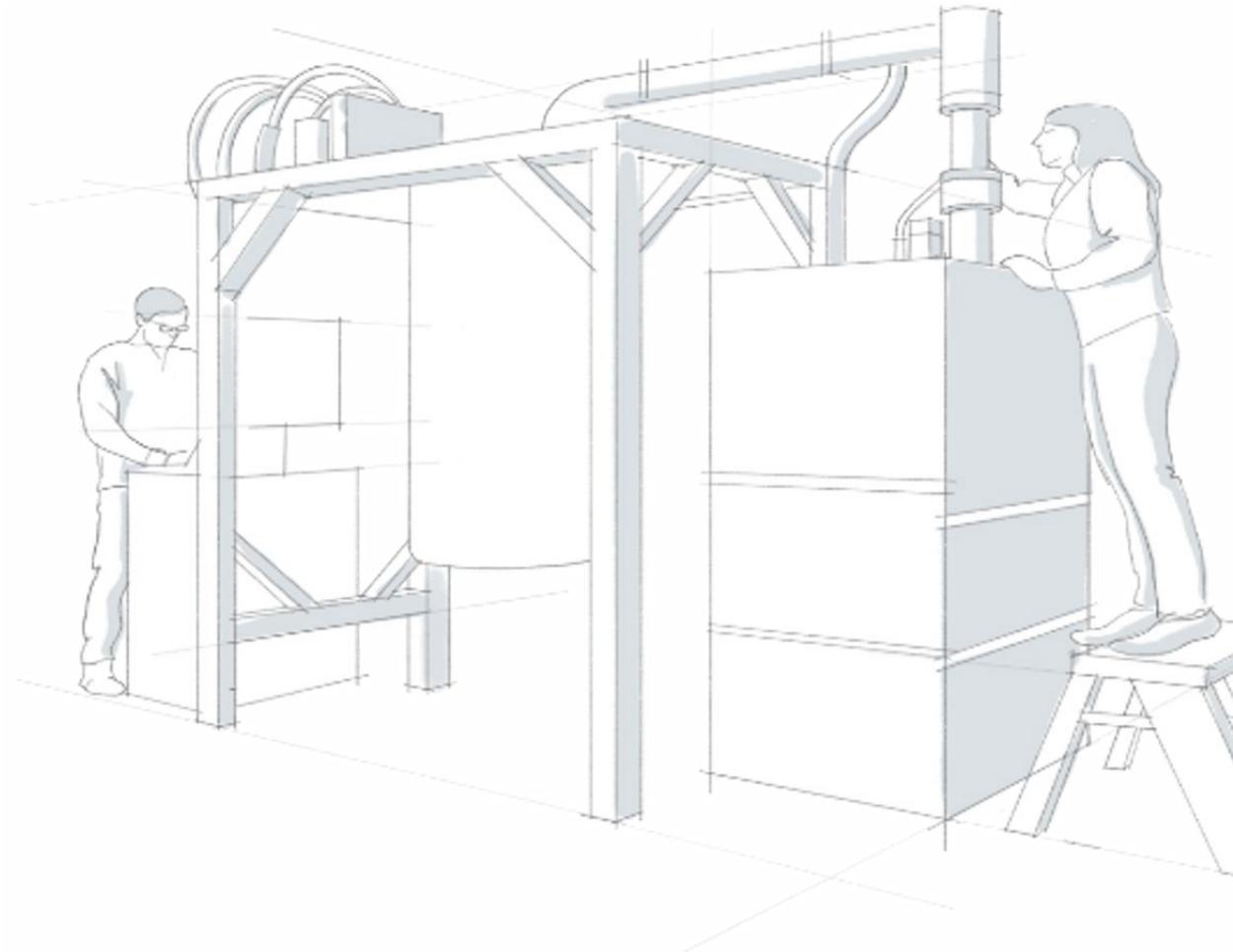
“Quantum phenomena  
do *not* occur in a Hilbert space,  
they occur in a laboratory.”

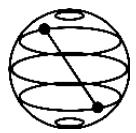
Asher Peres





# Hello World! to a real quantum experiment





# Hello World! to a real quantum experiment

Qubit computational  
basis states

$|0\rangle, |1\rangle$

Qubit gate

$X$

Qubit observable

$Z$

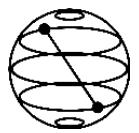
refresher:

$$X |0\rangle = |1\rangle$$

$$X |1\rangle = |0\rangle$$

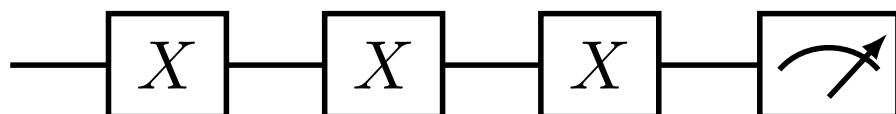
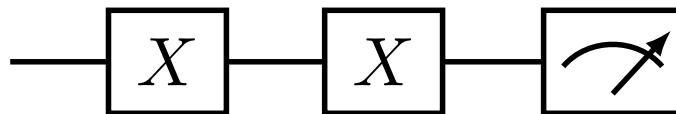
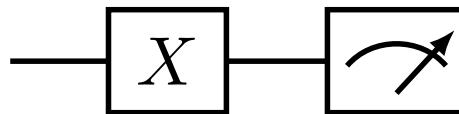
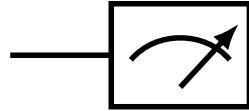
$$Z |0\rangle = +1 |0\rangle$$

$$Z |1\rangle = -1 |1\rangle$$



# Hello World! to a real quantum experiment

depth



⋮

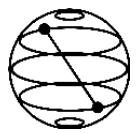
refresher:

$$X |0\rangle = |1\rangle$$

$$X |1\rangle = |0\rangle$$

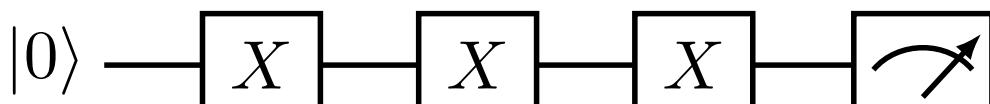
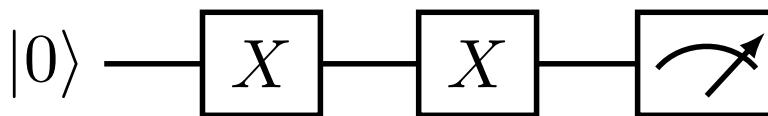
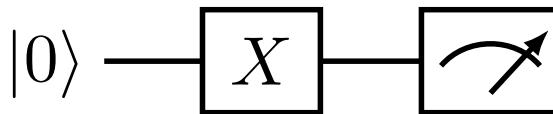
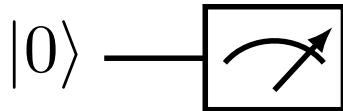
$$Z |0\rangle = +1 |0\rangle$$

$$Z |1\rangle = -1 |1\rangle$$



# Hello World! to a real quantum experiment

depth



⋮

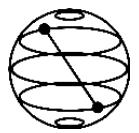
refresher:

$$X |0\rangle = |1\rangle$$

$$X |1\rangle = |0\rangle$$

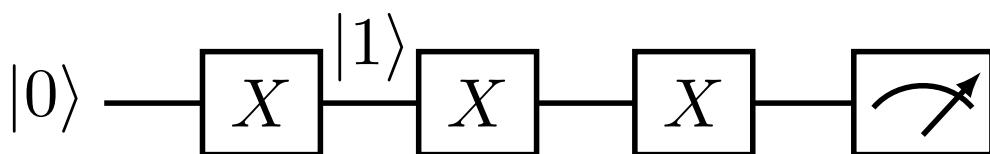
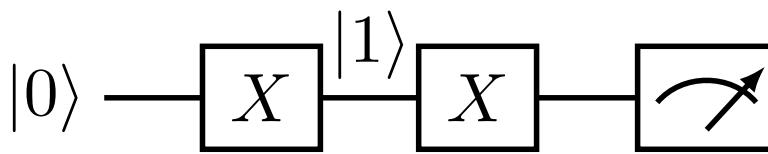
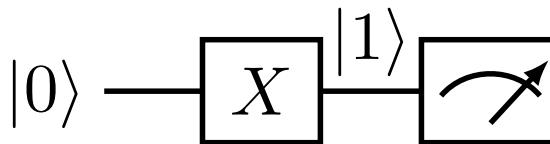
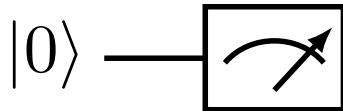
$$Z |0\rangle = +1 |0\rangle$$

$$Z |1\rangle = -1 |1\rangle$$



# Hello World! to a real quantum experiment

depth



⋮

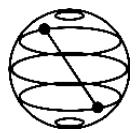
refresher:

$$X |0\rangle = |1\rangle$$

$$X |1\rangle = |0\rangle$$

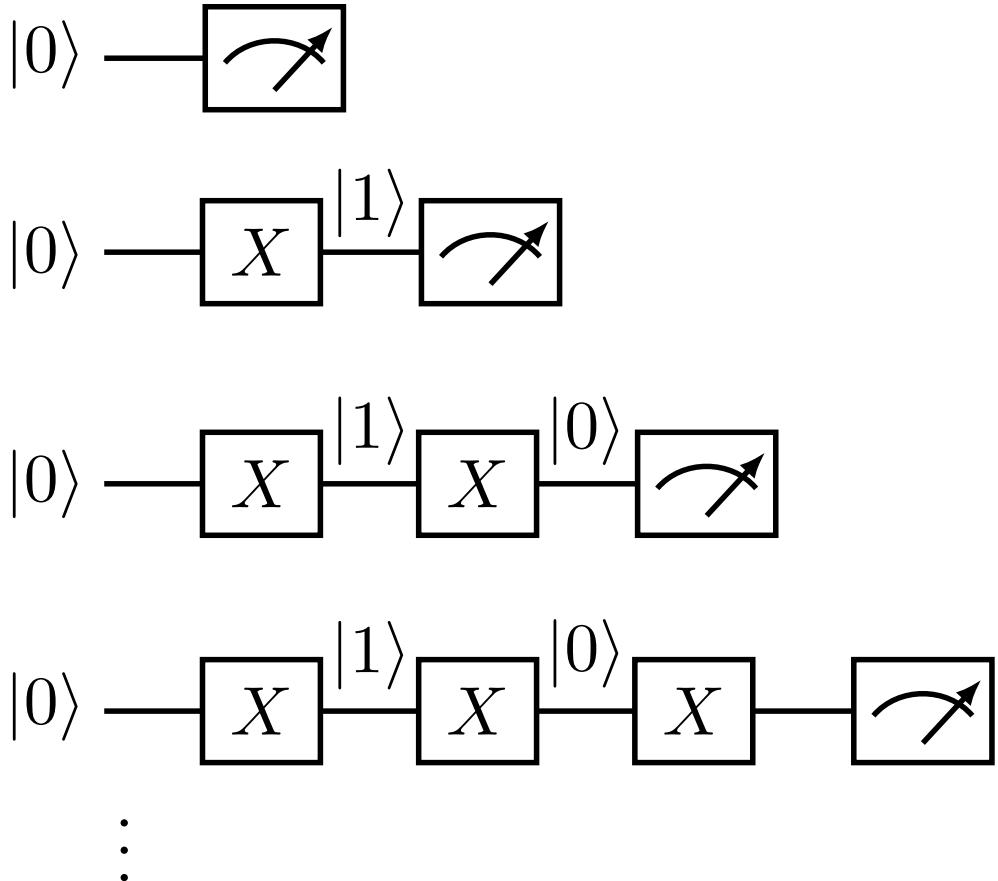
$$Z |0\rangle = +1 |0\rangle$$

$$Z |1\rangle = -1 |1\rangle$$



# Hello World! to a real quantum experiment

depth



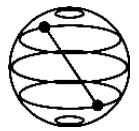
refresher:

$$X |0\rangle = |1\rangle$$

$$X |1\rangle = |0\rangle$$

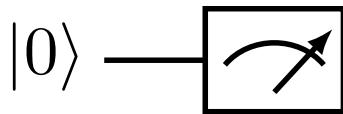
$$Z |0\rangle = +1 |0\rangle$$

$$Z |1\rangle = -1 |1\rangle$$



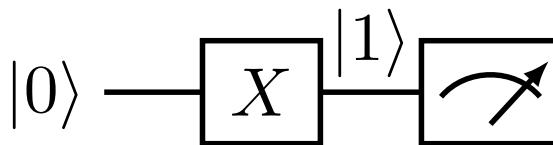
# Hello World! to a real quantum experiment

depth

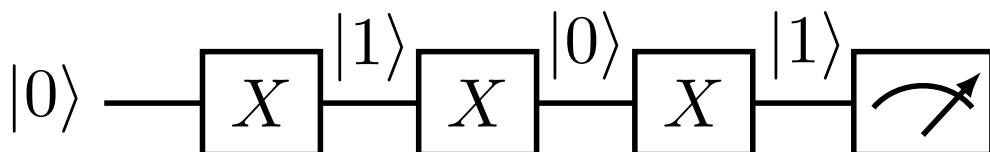
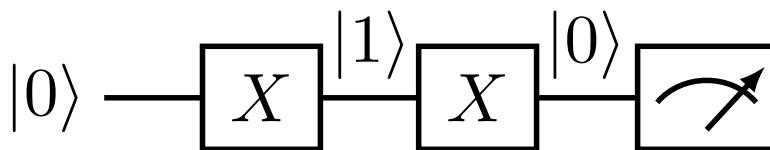


$$\langle 0|Z|0\rangle = +1$$

$\langle Z \rangle = (-1)^d$ ,  
where  $d$  is the circuit depth



$$\langle 1|Z|1\rangle = -1$$



⋮

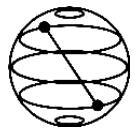
refresher:

$$X|0\rangle = |1\rangle$$

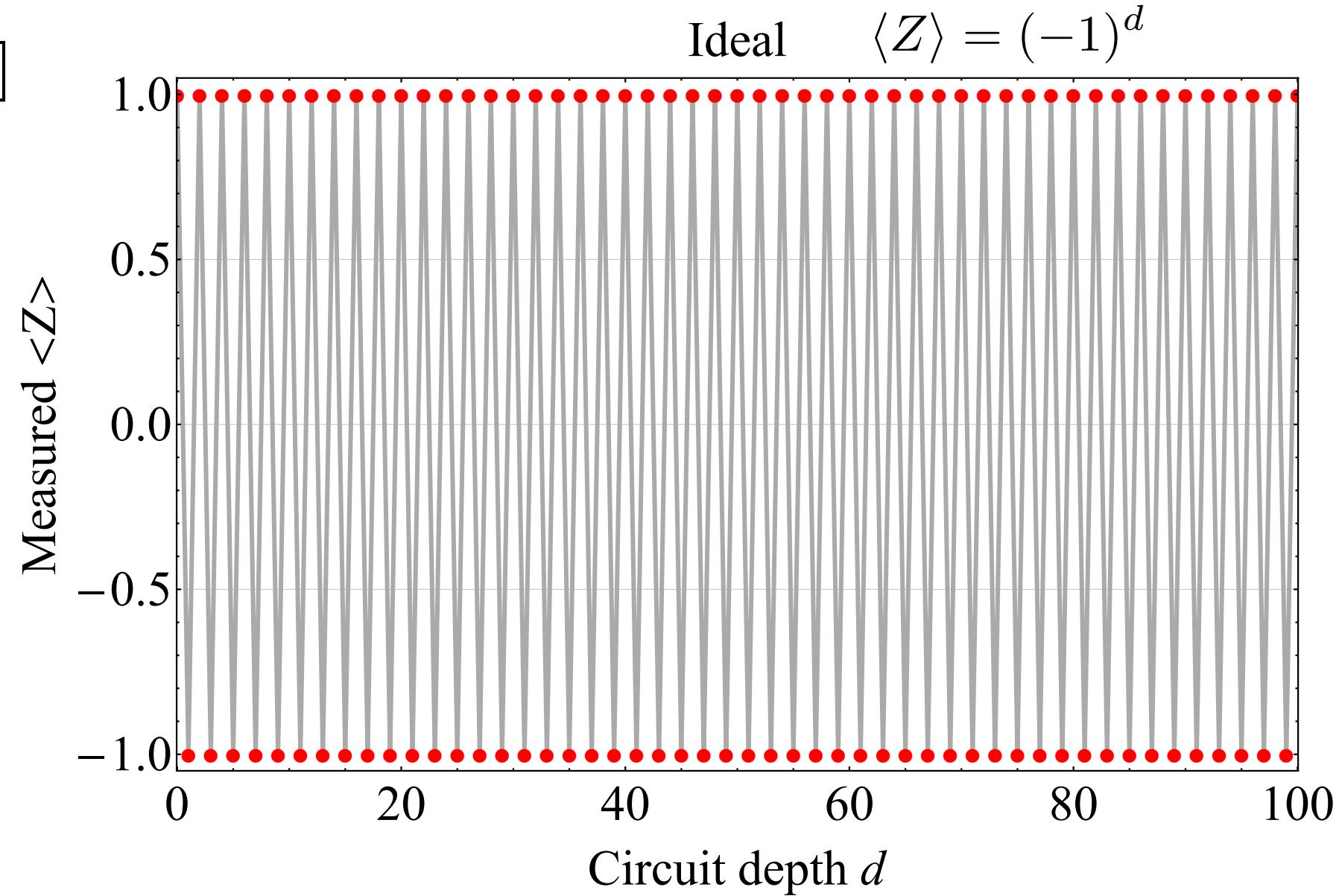
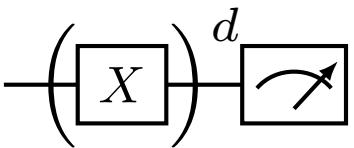
$$X|1\rangle = |0\rangle$$

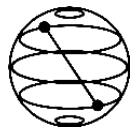
$$Z|0\rangle = +1|0\rangle$$

$$Z|1\rangle = -1|1\rangle$$

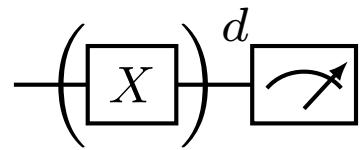


# Ideal experiment (warning: doesn't exist)



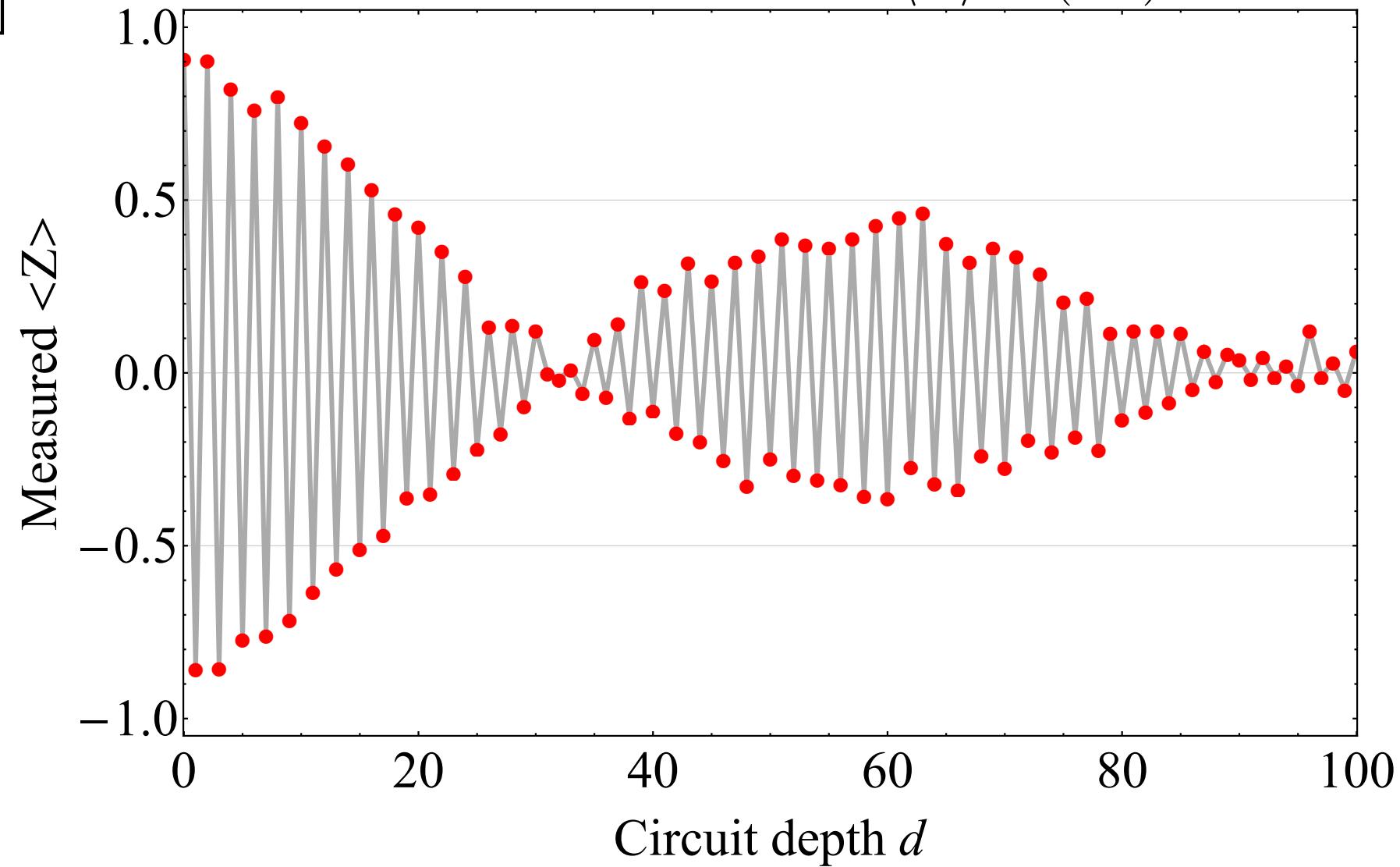


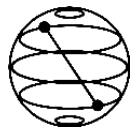
# Real experiment



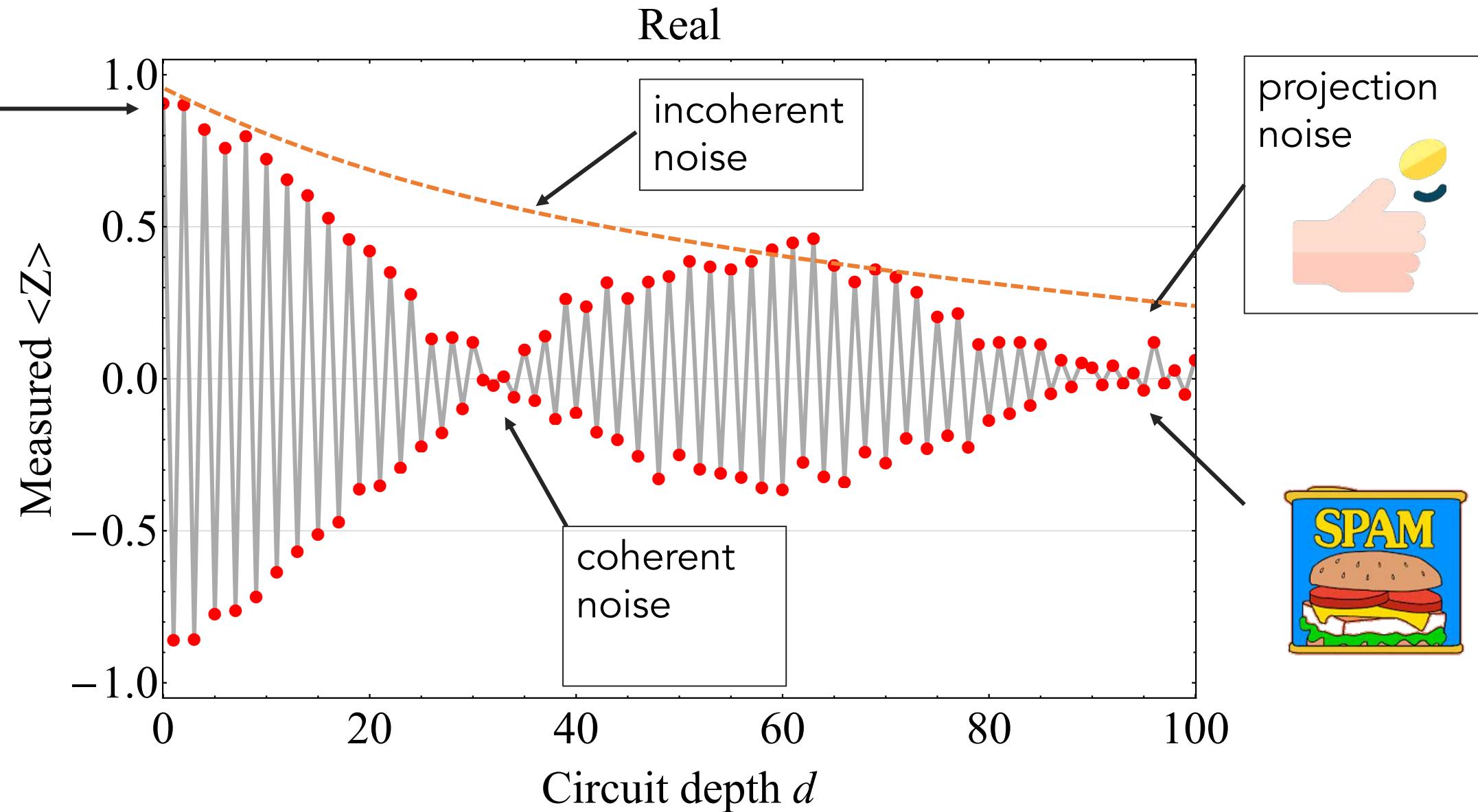
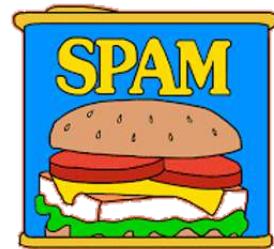
Real

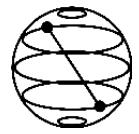
$$\langle Z \rangle = (-1)^d$$





# Elements of noise





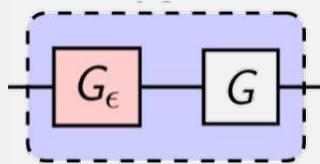
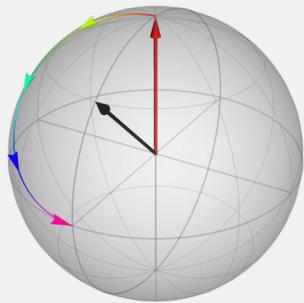
# Why study quantum noise?



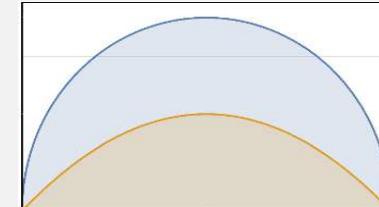
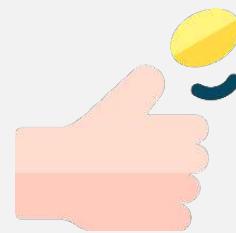
*“Well, your quantum computer is broken in  
every way possible simultaneously.”*

# Our road ahead

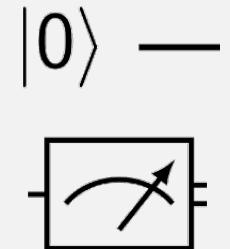
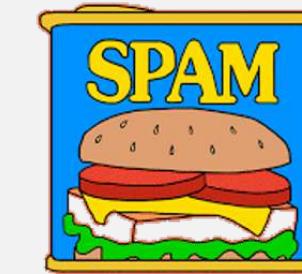
## Coherent



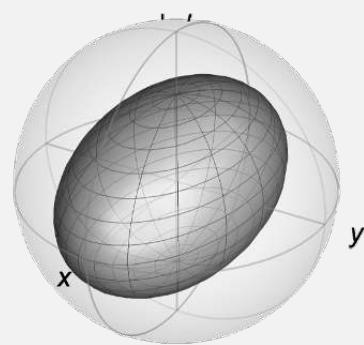
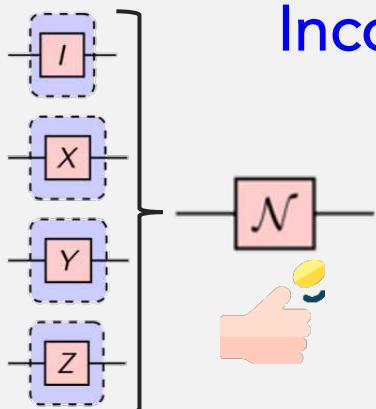
## Projection & measurement theory



## State preparation & measurement



## Incoherent

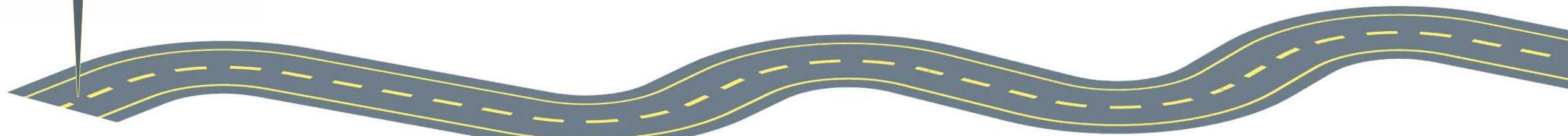
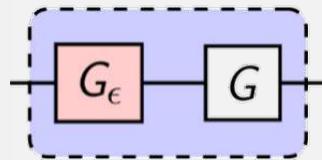
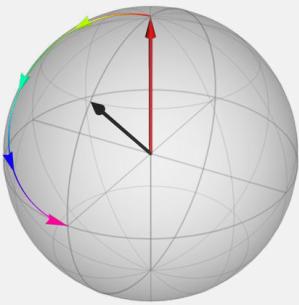


coin toss: flaticon; spam: make it move;  
road based on: freepik

Zlatko Minev, IBM Quantum (14)

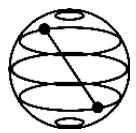
# Quantum gates meets coherent noise

Coherent

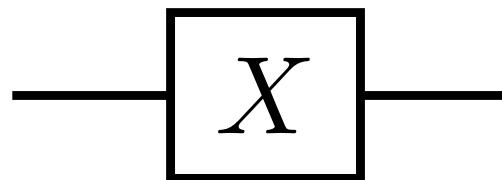


road based on: freepik

Zlatko Minev, IBM Quantum (15)



# Gate from time evolution



$$X = R_X(\pi)$$

(up to global phase)

refresher:

$$\hat{H} = \frac{\hbar\omega}{2} X$$

$$= \frac{\hbar\omega}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

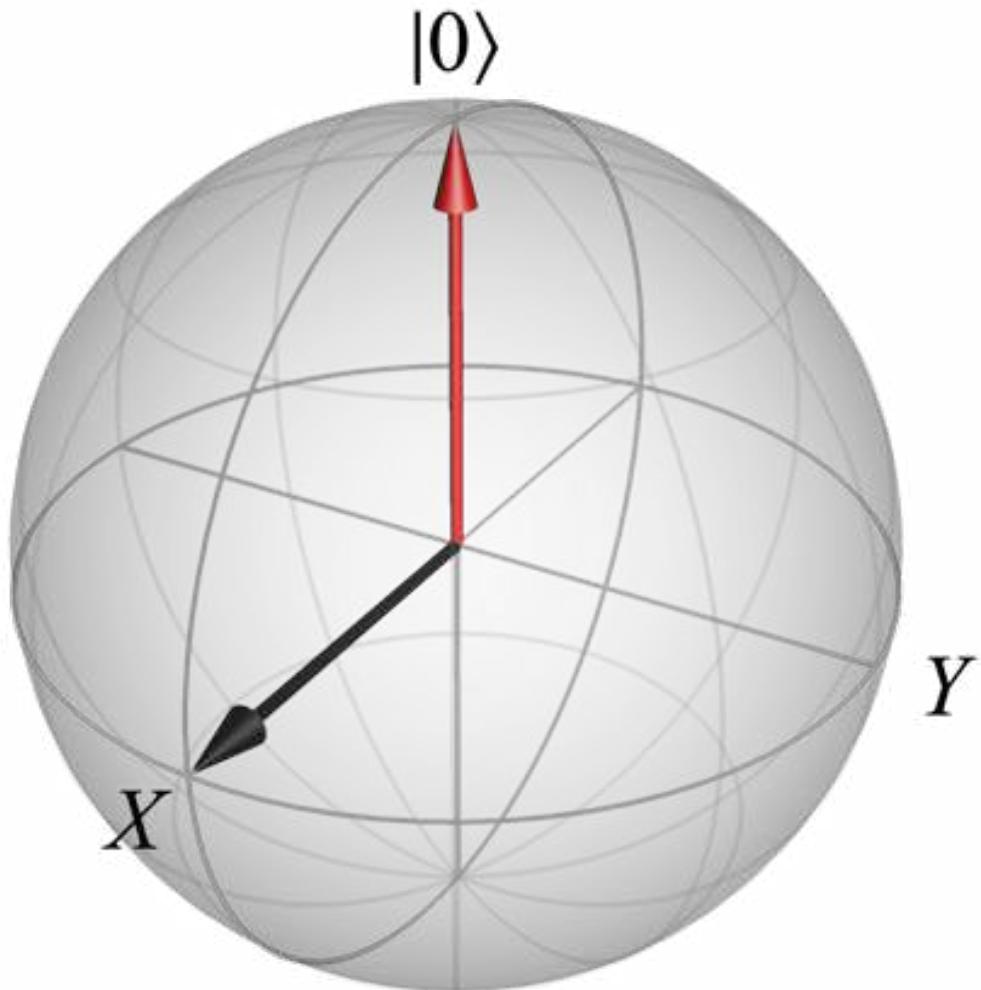
$$U(t) = \exp\left(-it\hat{H}/\hbar\right)$$

$$\theta := \omega t$$

$$R_X(\theta) = \exp\left(-\frac{i\theta}{2} X\right)$$

$$= \cos(\theta/2)I - i \sin(\theta/2)X$$

# Visualize: Bloch sphere

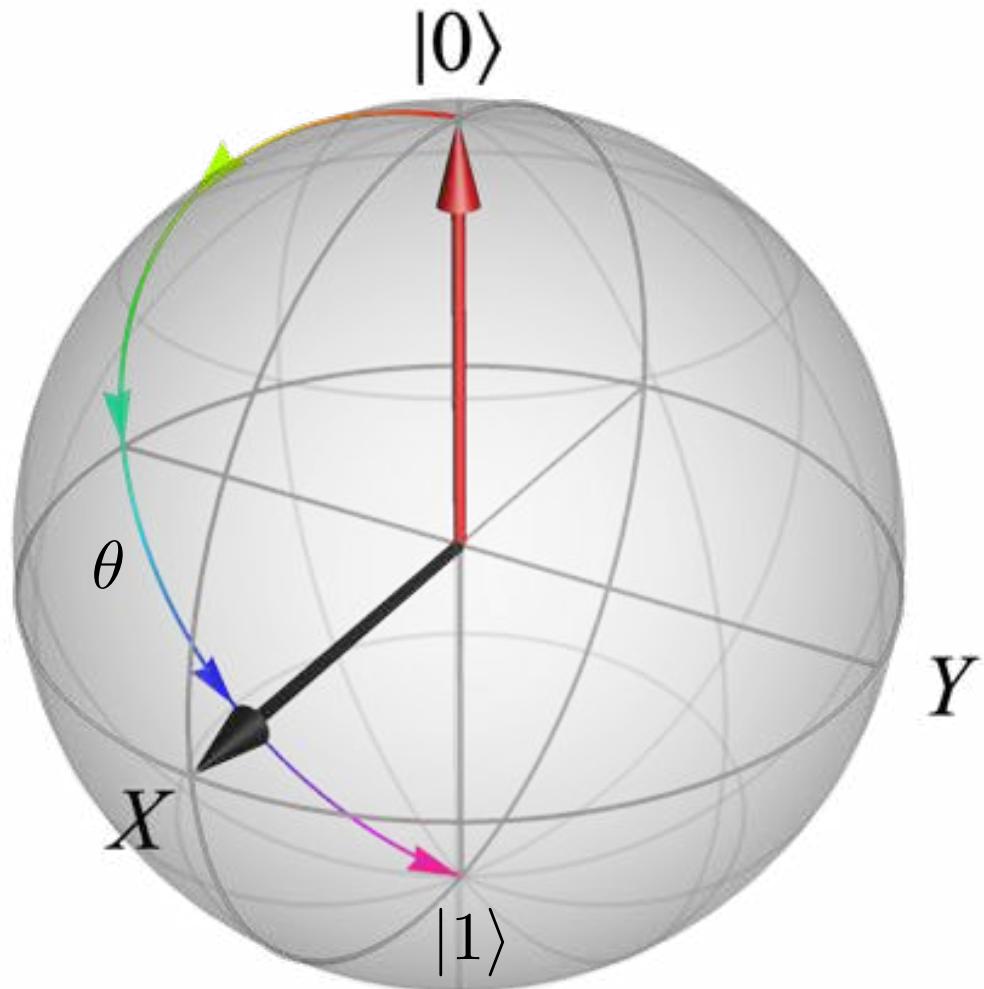


# Visualize: Evolution on the Bloch sphere

$$R_X(\theta) = \exp\left(-\frac{i\theta}{2}X\right)$$

$$X|0\rangle = |1\rangle$$

$$X|1\rangle = |0\rangle$$

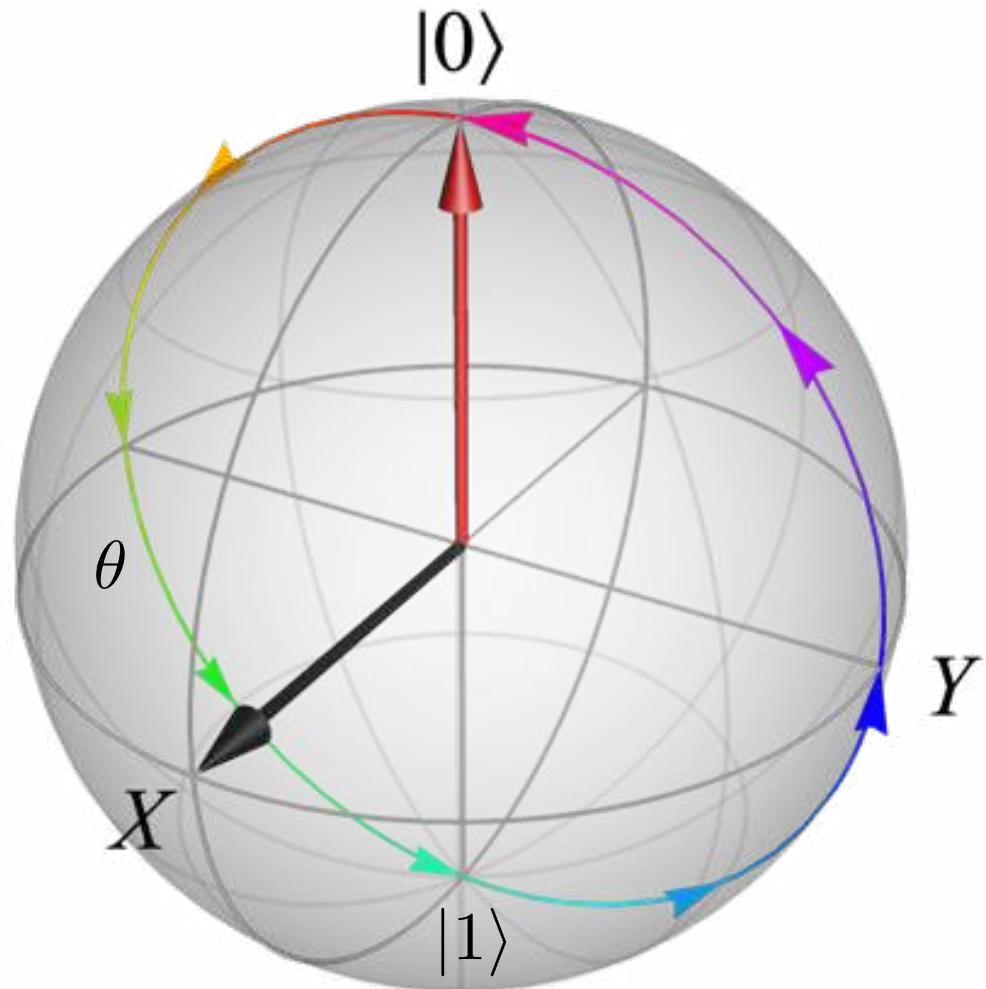


# Visualize: Evolution on the Bloch sphere

$$R_X(\theta) = \exp\left(-\frac{i\theta}{2}X\right)$$

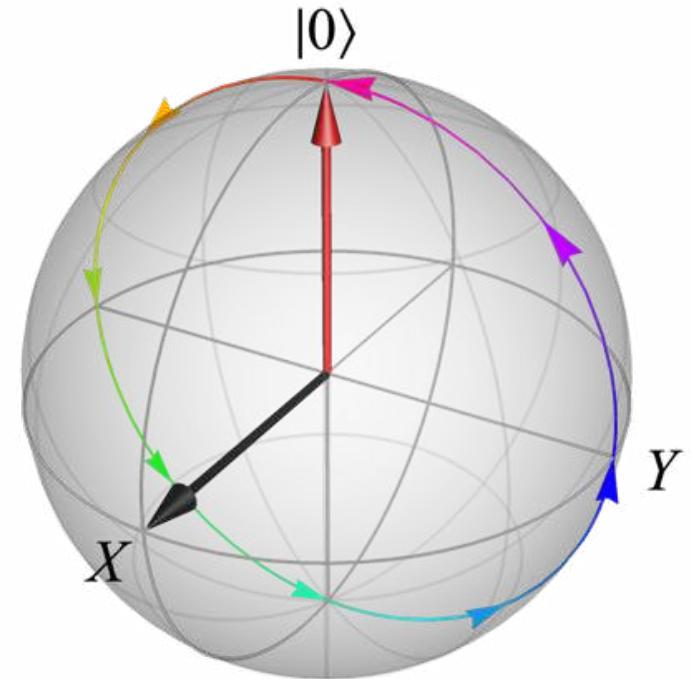
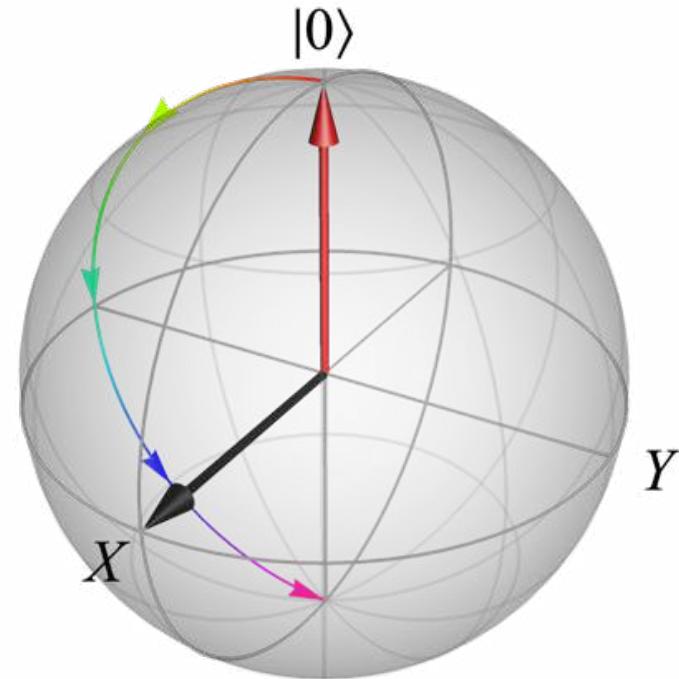
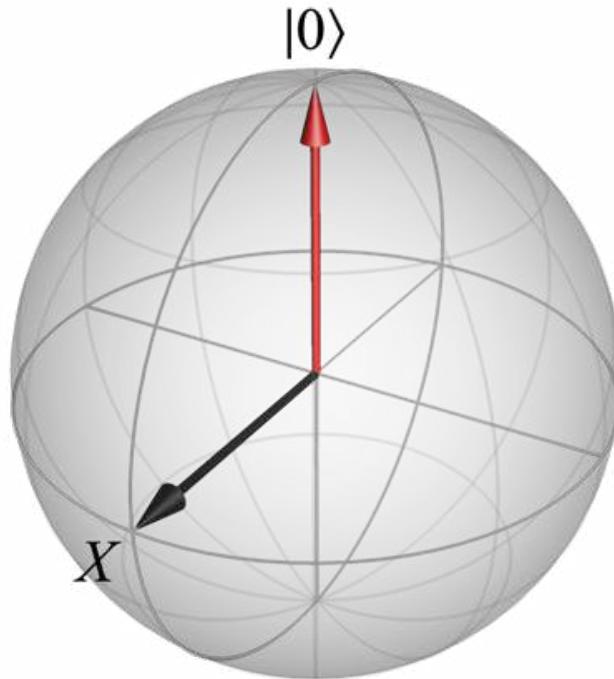
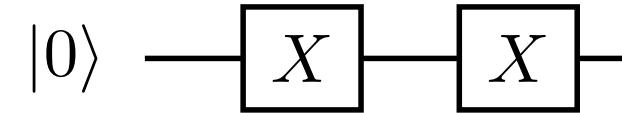
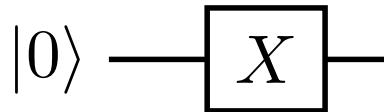
$$X|0\rangle = |1\rangle$$

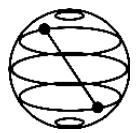
$$X|1\rangle = |0\rangle$$



# Evolution on the Bloch sphere

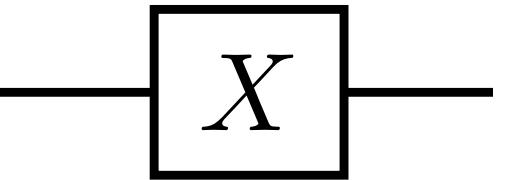
$|0\rangle$



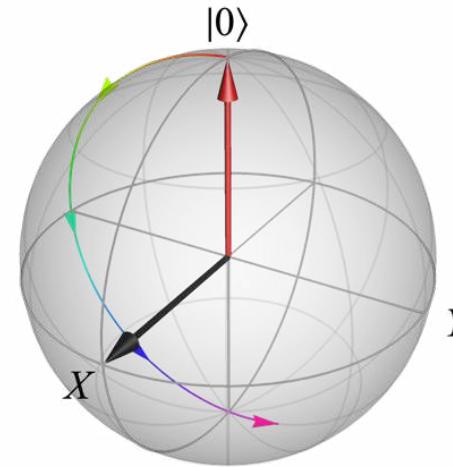
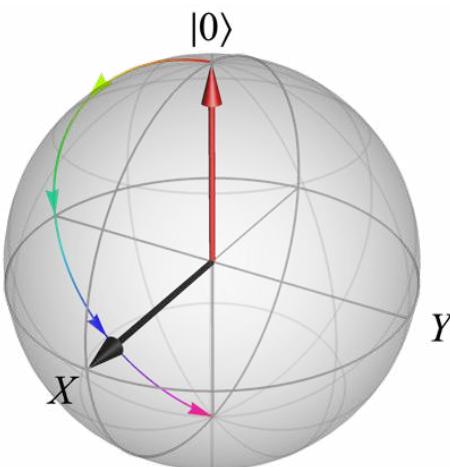
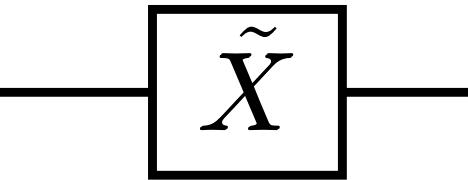


# Miscalibrated gate

Ideal gate



Noisy gate

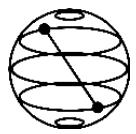


$$X = R_X(\pi)$$

$$\tilde{X} := R_X(\pi + \epsilon)$$

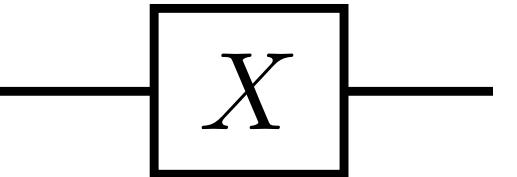
Show

$$R_X(\theta + \phi) = R_X(\theta) R_X(\phi)$$

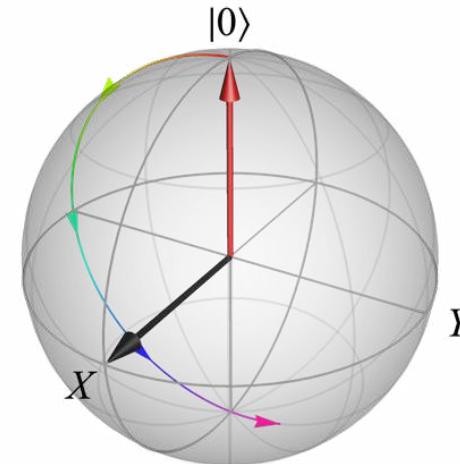
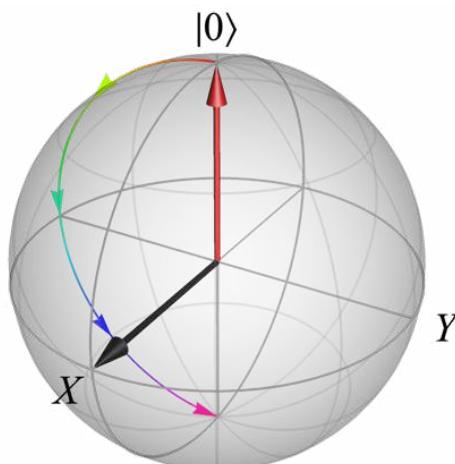
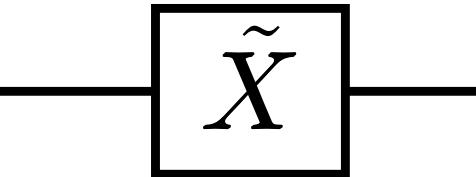


# Miscalibrated gate

Ideal gate



Noisy gate



Show

$$R_X(\theta + \phi) = R_X(\theta) R_X(\phi)$$

$$X = R_X(\pi)$$

$$\tilde{X} := R_X(\pi + \epsilon)$$

$$\begin{aligned} R_X(\theta) &= \exp\left(-\frac{i\theta}{2}X\right) \\ &= \cos(\theta/2)I - i \sin(\theta/2)X \end{aligned}$$

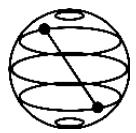
$$= \exp\left(-i\frac{\pi + \epsilon}{2}X\right)$$

$$= \exp\left(-i\frac{\pi}{2}X - i\frac{\epsilon}{2}X\right)$$

$$= \exp\left(-i\frac{\pi}{2}X\right) \exp\left(-i\frac{\pi}{2}X\right)$$

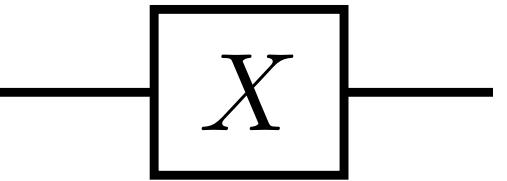
$$= R_X(\epsilon) R_X(\pi)$$

$$= X_\epsilon X$$

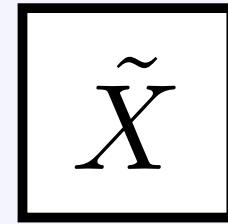
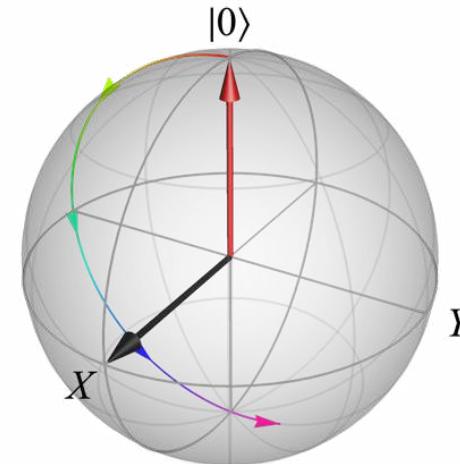
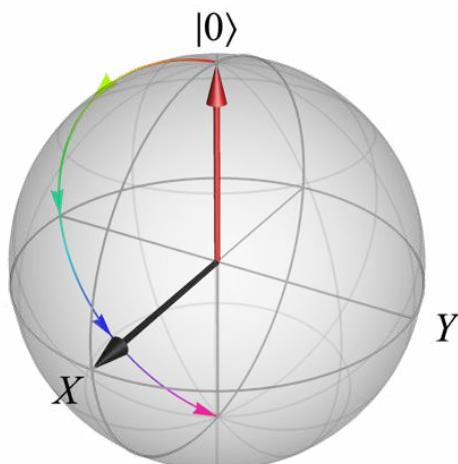
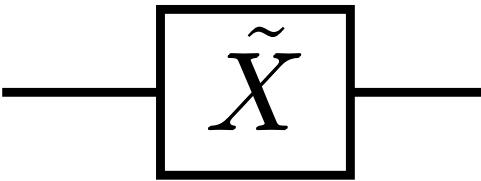


# Noisy gate decomposition

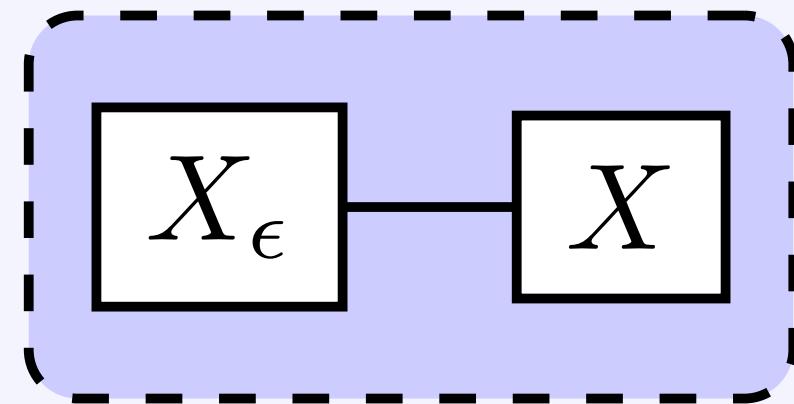
Ideal gate

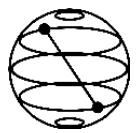


Noisy gate



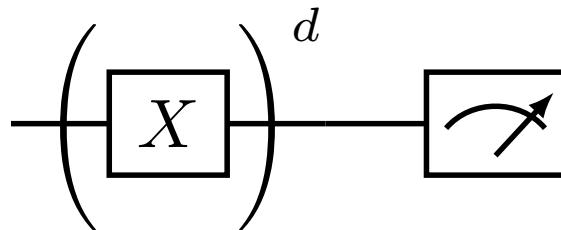
||





# Using a noisy gate in a quantum circuit

Ideal



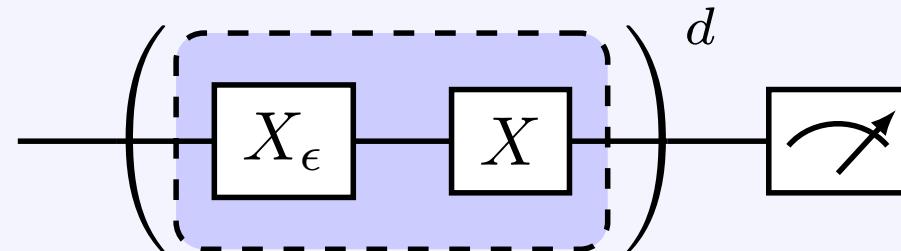
$$X = R_X(\pi)$$

$$U_{\text{total}} = X^d$$

$$= [R_X(\pi)]^d$$

$$= R_X(d\pi)$$

Noisy



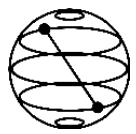
$$\tilde{X} := R_X(\pi + \epsilon) = X_\epsilon X$$

$$\tilde{U}_{\text{total}} = \tilde{X}^d$$

$$= [R_X(\epsilon) R_X(\pi)]^d$$

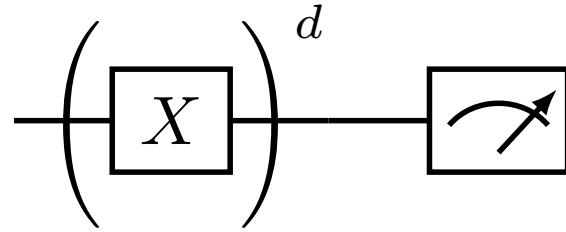
$$= R_X(d\epsilon) R_X(d\pi)$$

$$\tilde{U}_{\text{total}} = R_X(d\epsilon) U_{\text{total}}$$



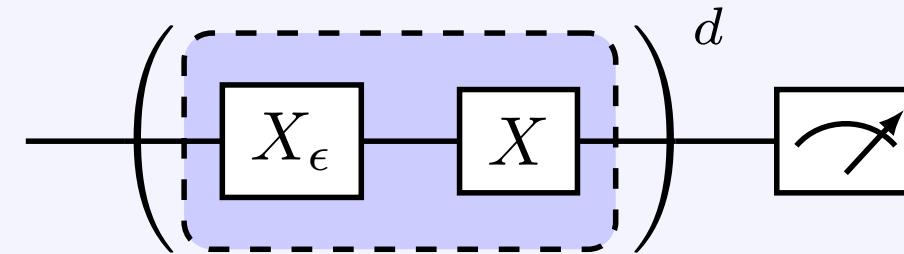
# Using a noisy gate in a quantum circuit: final state

Ideal

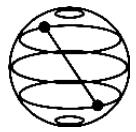


$$U_{\text{total}} = X^d = R_X(d\pi)$$

Noisy

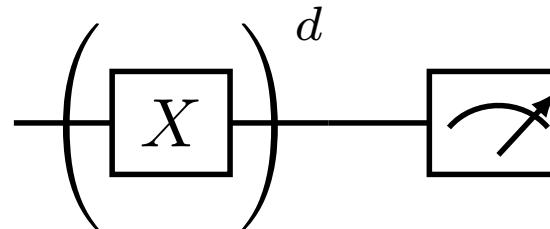


$$\tilde{U}_{\text{total}} = R_X(d\epsilon) U_{\text{total}}$$

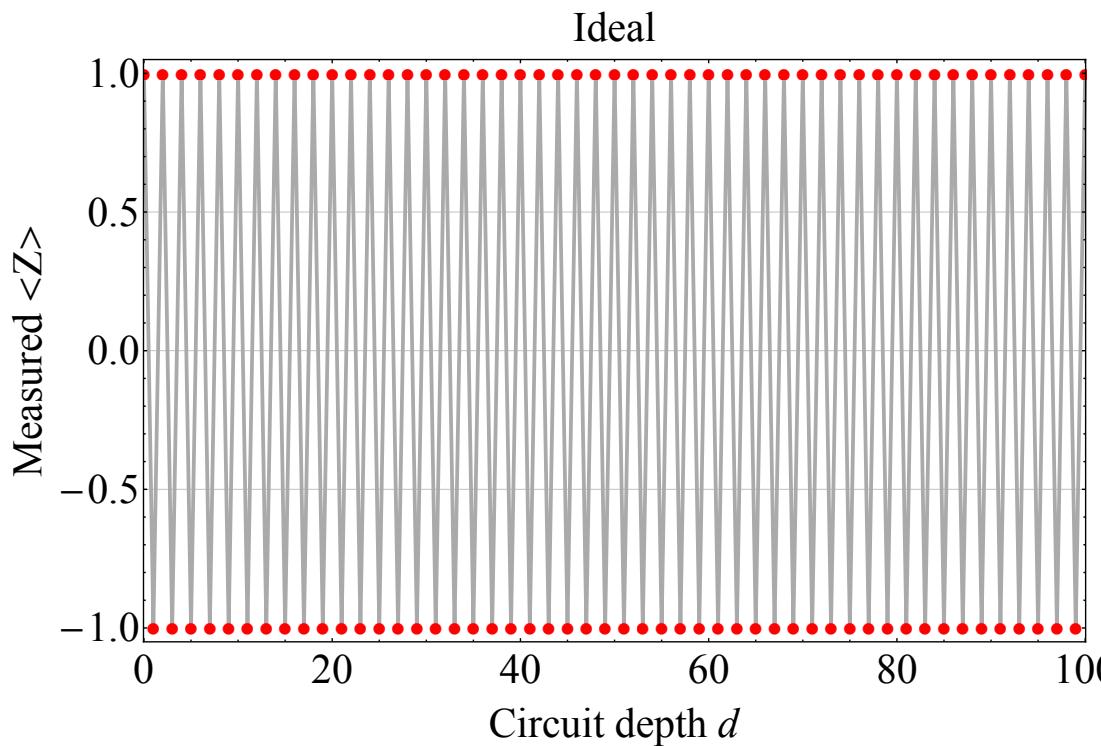


# Ideal vs. noisy observable

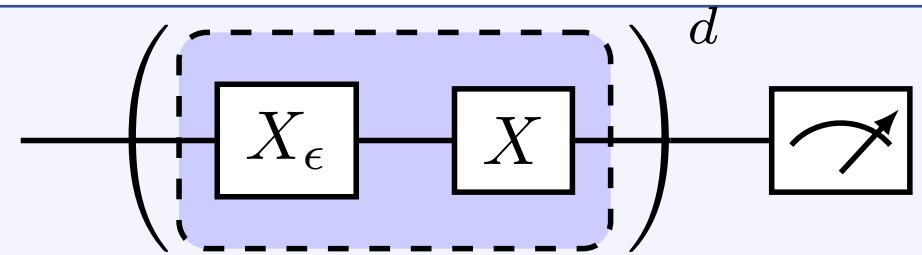
Ideal



$$\langle \psi_f | Z | \psi_f \rangle = \cos(d\pi) = (-1)^d$$

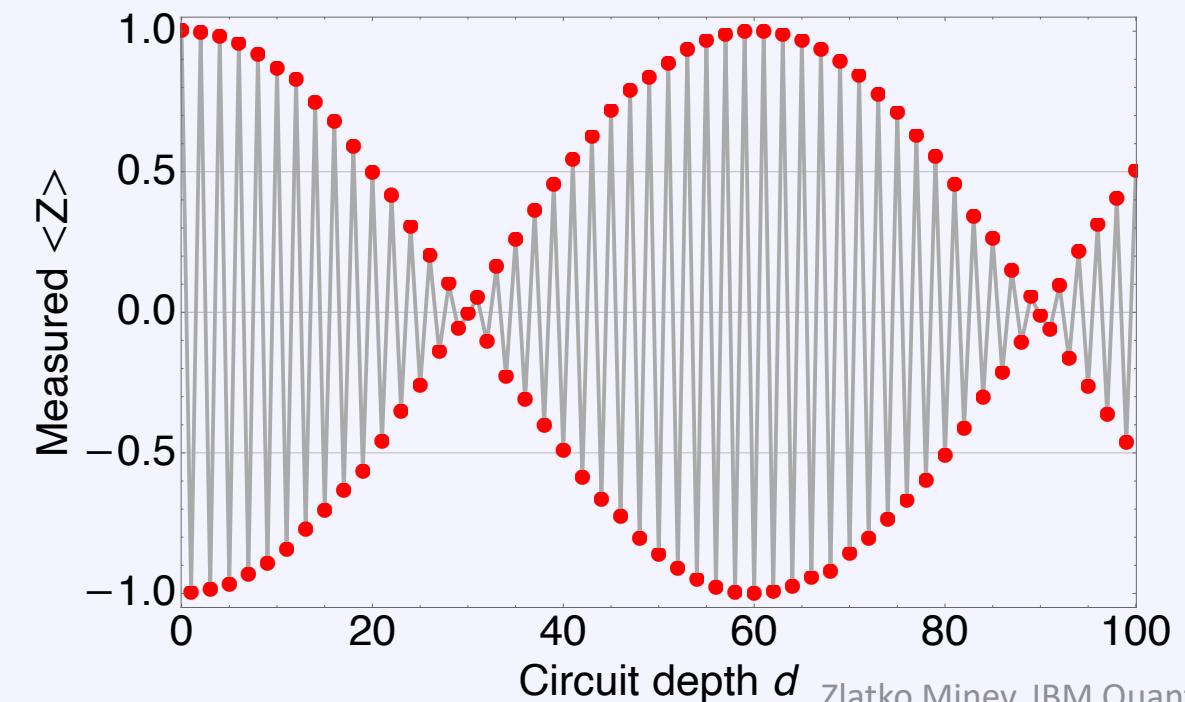


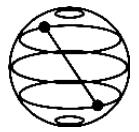
Noisy



$$\langle \tilde{\psi}_f | Z | \tilde{\psi}_f \rangle = \cos(d\pi + d\epsilon)$$

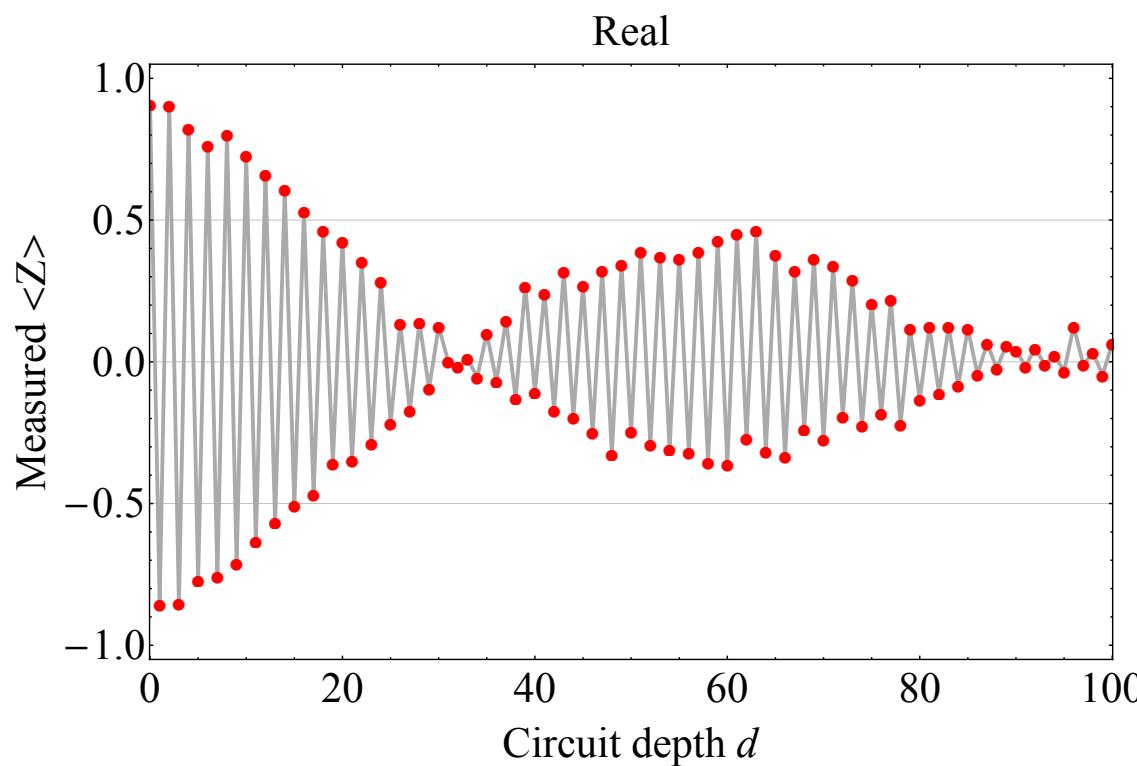
Gate error  $\epsilon = 3^\circ$



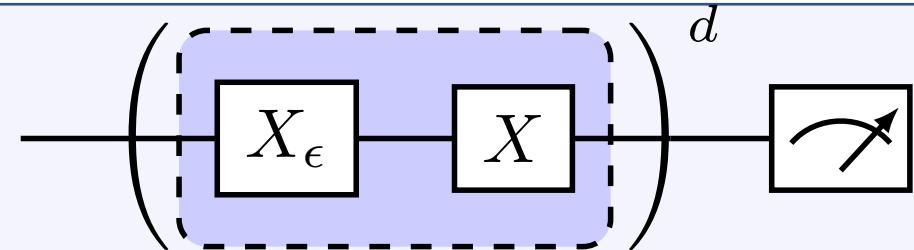


# Compare to full experiment

Full experiment

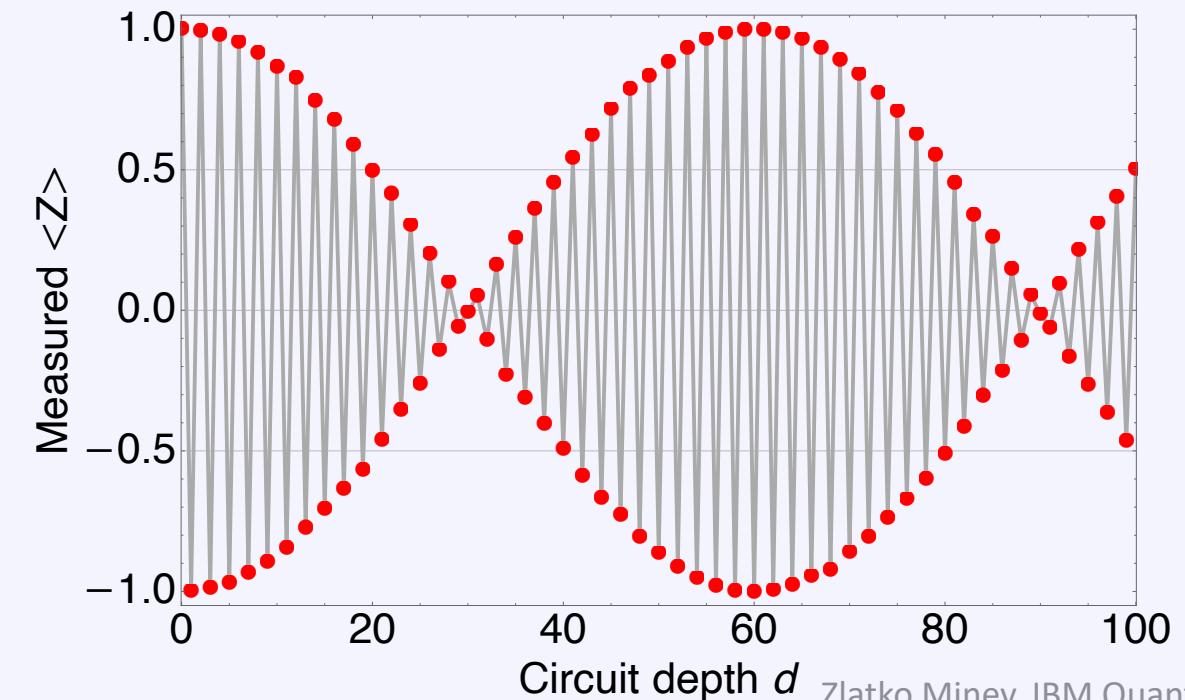


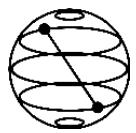
Noisy



$$\langle \tilde{\psi}_f | Z | \tilde{\psi}_f \rangle = \cos(d\pi + d\epsilon)$$

Gate error  $\epsilon=3^\circ$



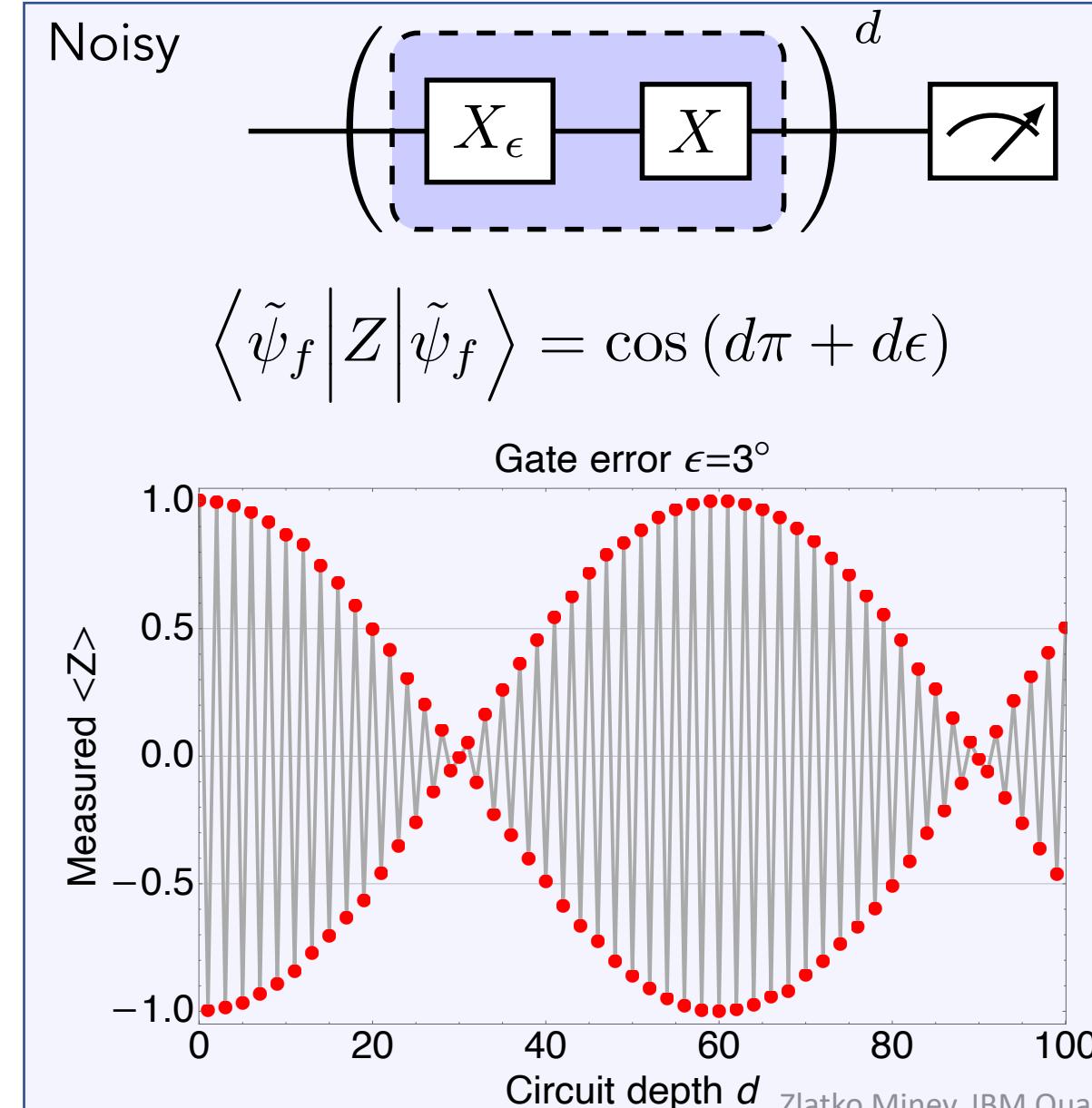


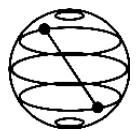
# Coherent error is bad, quadratically so

Coherent errors have a *quadratic* impact on algorithmic accuracy (worst-case error)

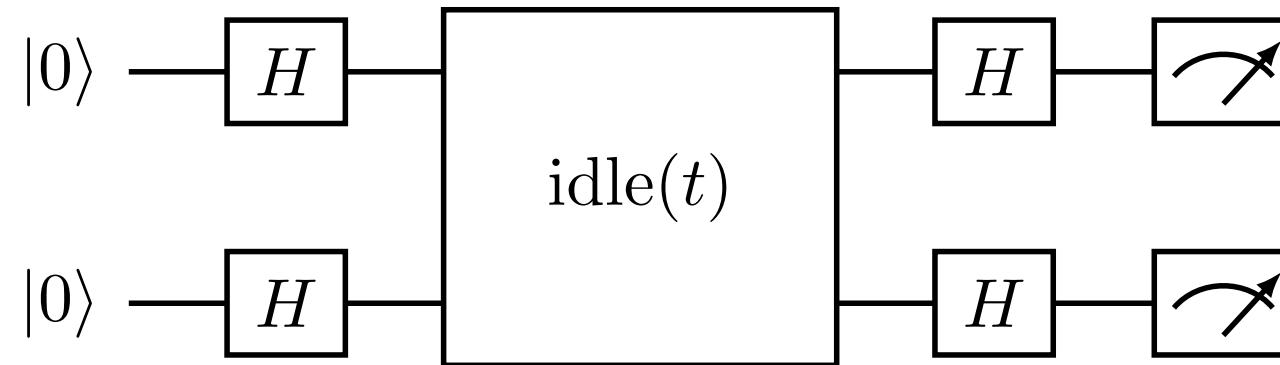
$$\langle \tilde{\psi}_f | Z | \tilde{\psi}_f \rangle - \langle \psi_f | Z | \psi_f \rangle$$

$$\cos(x) = 1 - \frac{1}{2}x^2 + \mathcal{O}(x^3)$$





# Example: two-qubit ZZ error



[Breakout to notebook](#)

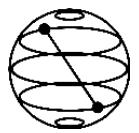
Hadamard gate



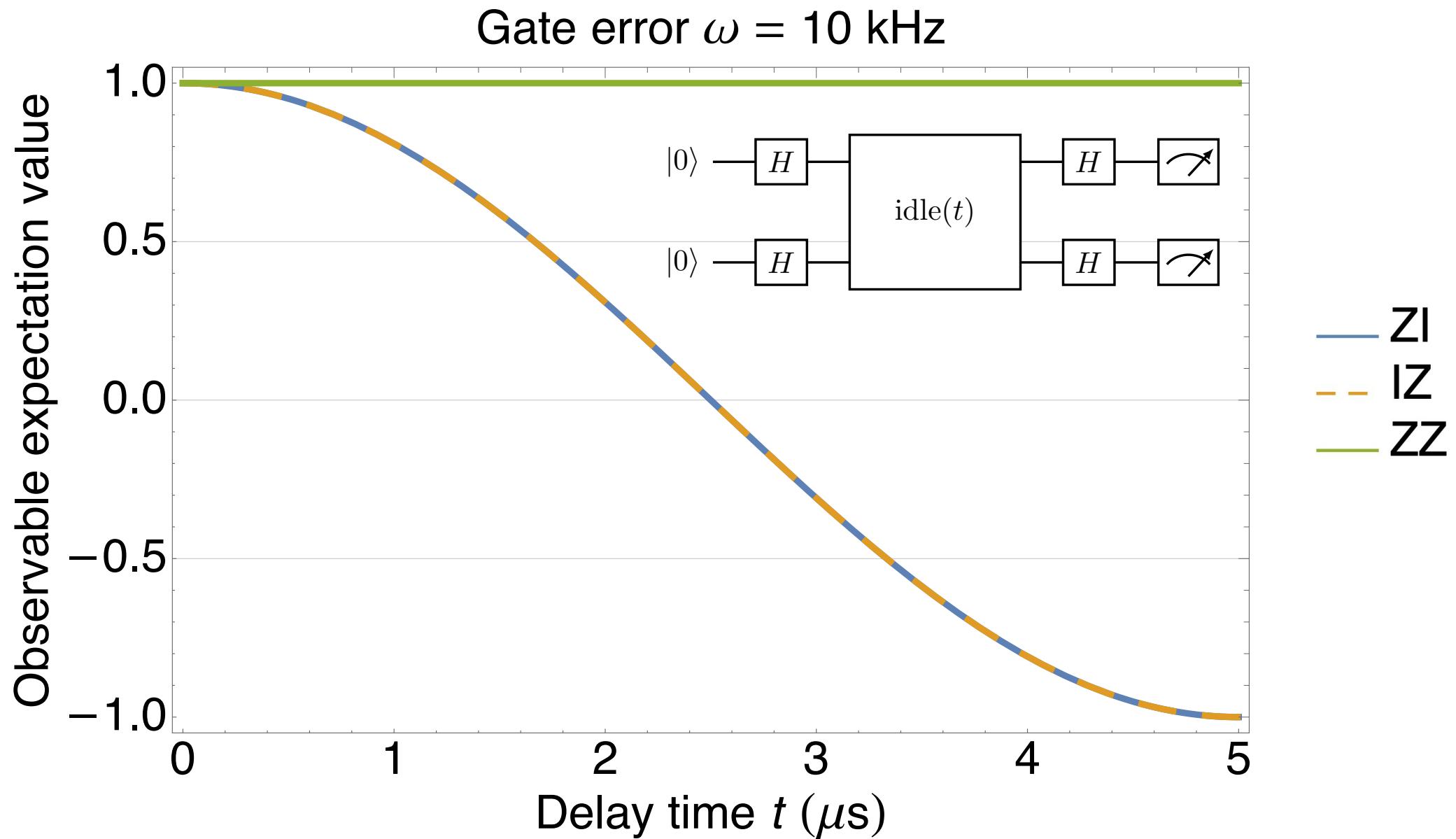
$$H = \begin{matrix} |0\rangle & \langle 0| \\ |1\rangle & \langle 1| \end{matrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{pmatrix}$$

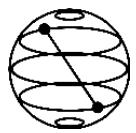
$$H |0\rangle = |+x\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$H |1\rangle = |-x\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$



## Example: two-qubit ZZ error





# Dive deeper? Try the following

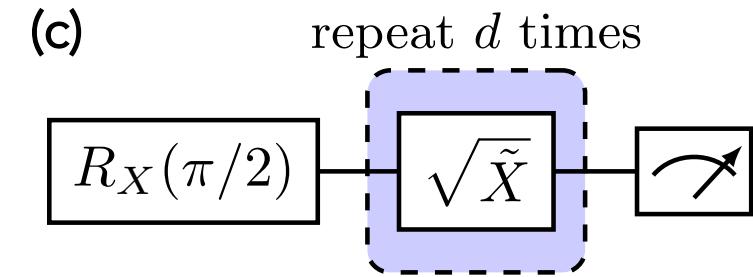
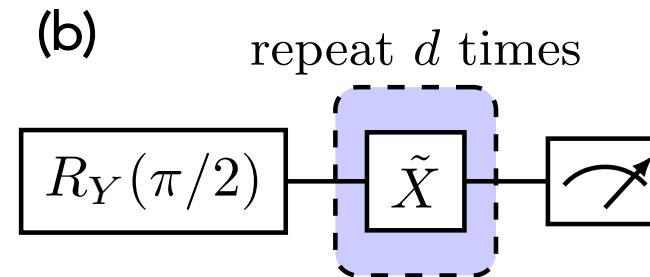
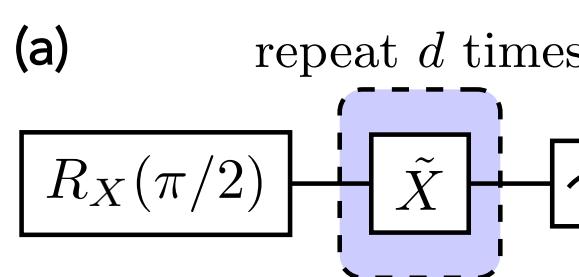


## 1. Amplitude-error amplification sequences.

(A) Calculate the expectation value  $\langle Z(d) \rangle$  as a function of depth  $d$  of the sequence for the following sequences. Assume the noise model  $\tilde{X} := R_X(\pi + \epsilon) = X_\epsilon X$

(B) How could you use this result to fine-tune your gates?

(C) Can you come up with an alternative or more clever error-amplification sequence?

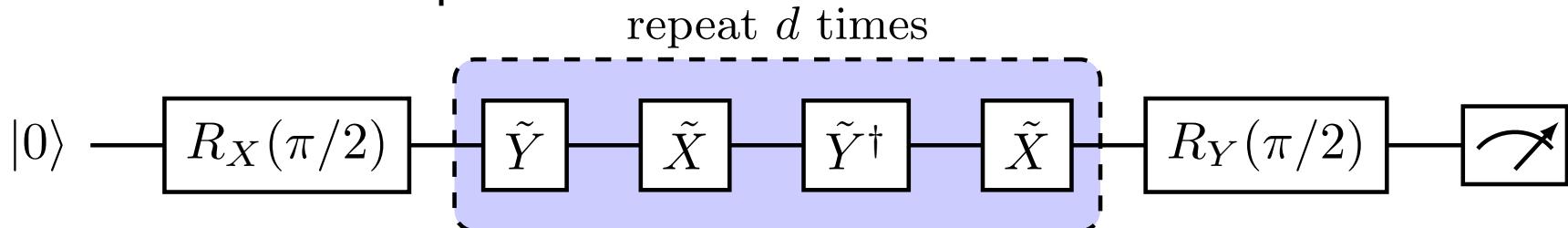


## 2. Phase-error amplification sequences. Use the noise model:

Repeat (A), (B), and (C) for Exercise (1) for the following sequences and assuming a phase error between the  $X$  and  $Y$  gates, rather than an amplitude error.

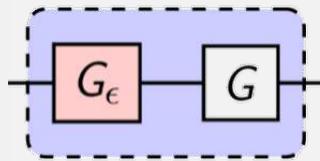
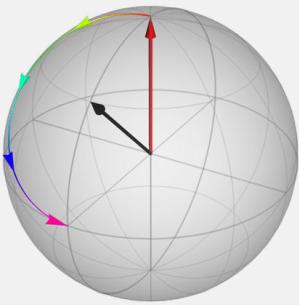
$$\tilde{X} = X ,$$

$$\tilde{Y} = \exp \left[ -i \frac{\pi}{2} (\cos \epsilon Y + \sin \epsilon X) \right] .$$

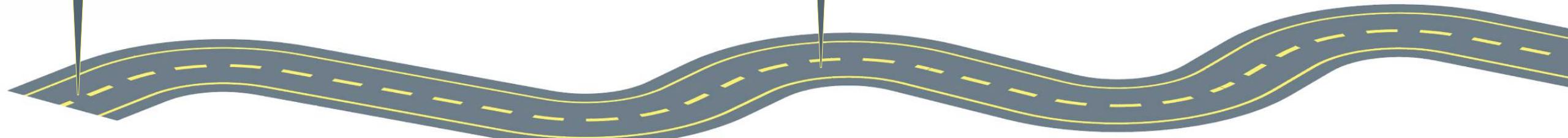
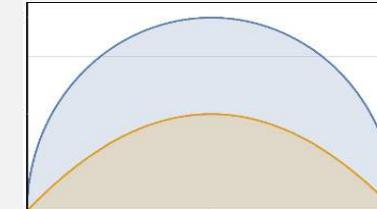
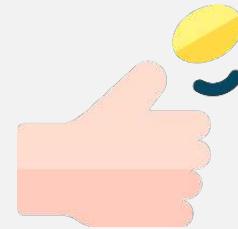


# Our road ahead

Coherent

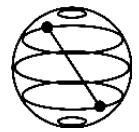


Projection &  
measurement theory

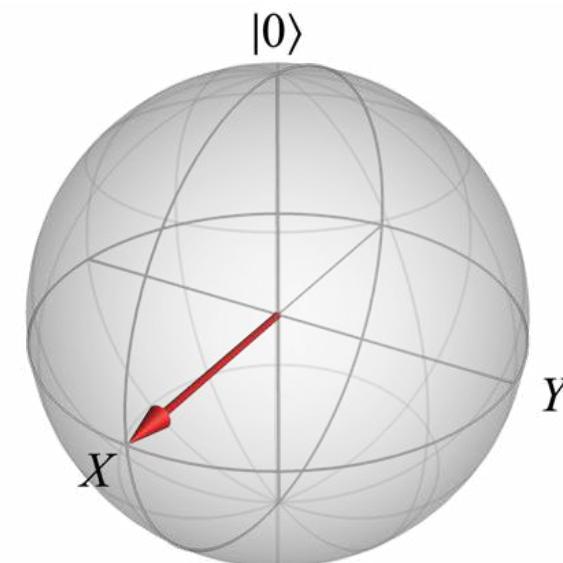
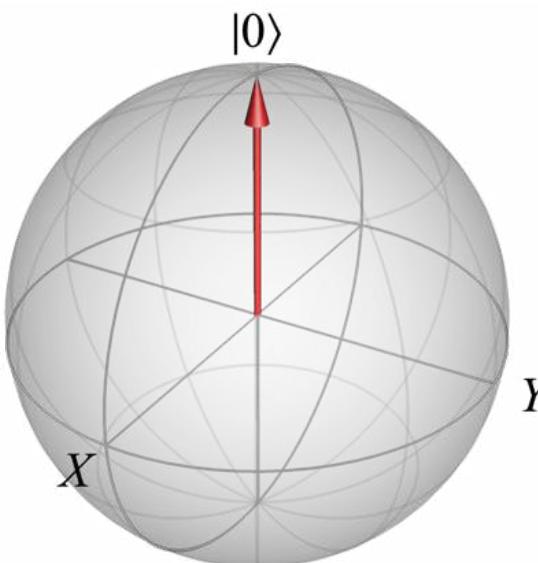
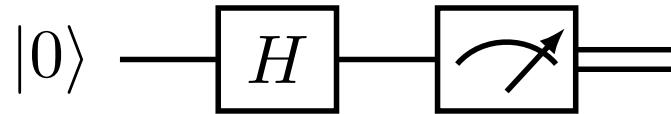


coin toss: flaticon; spam: make it move;  
road based on: freepik

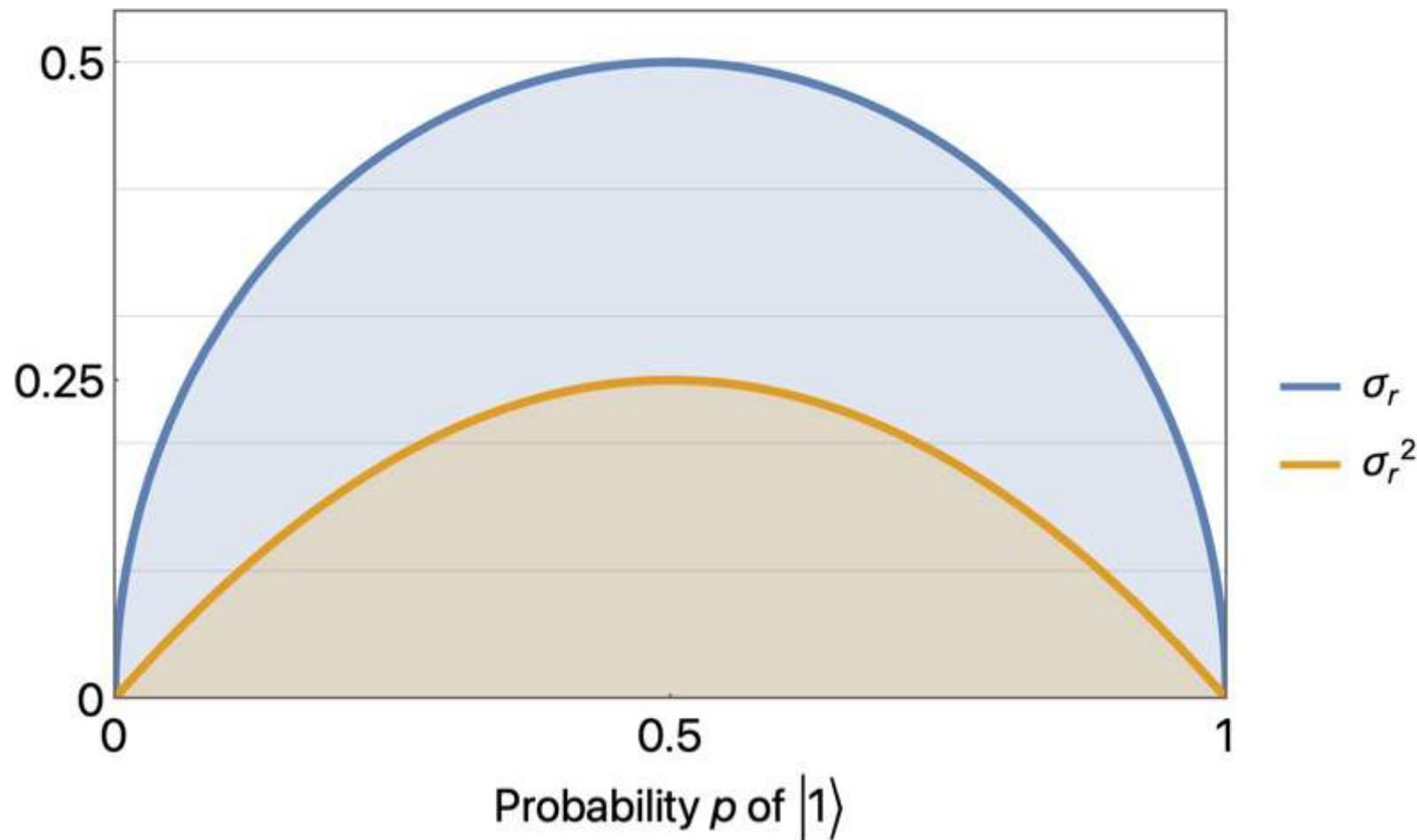
Zlatko Minev, IBM Quantum (32)

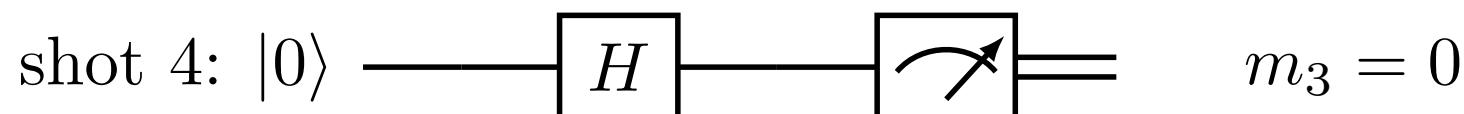
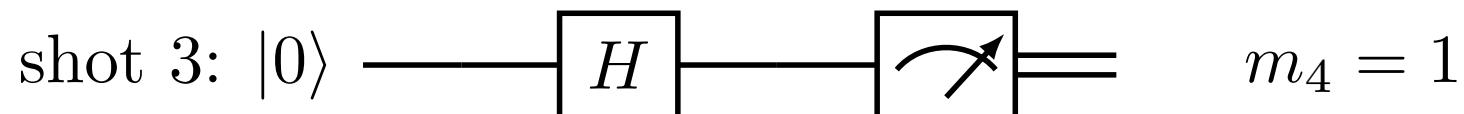
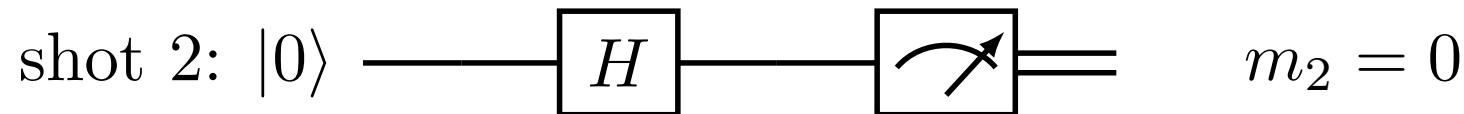


# Ideal circuit

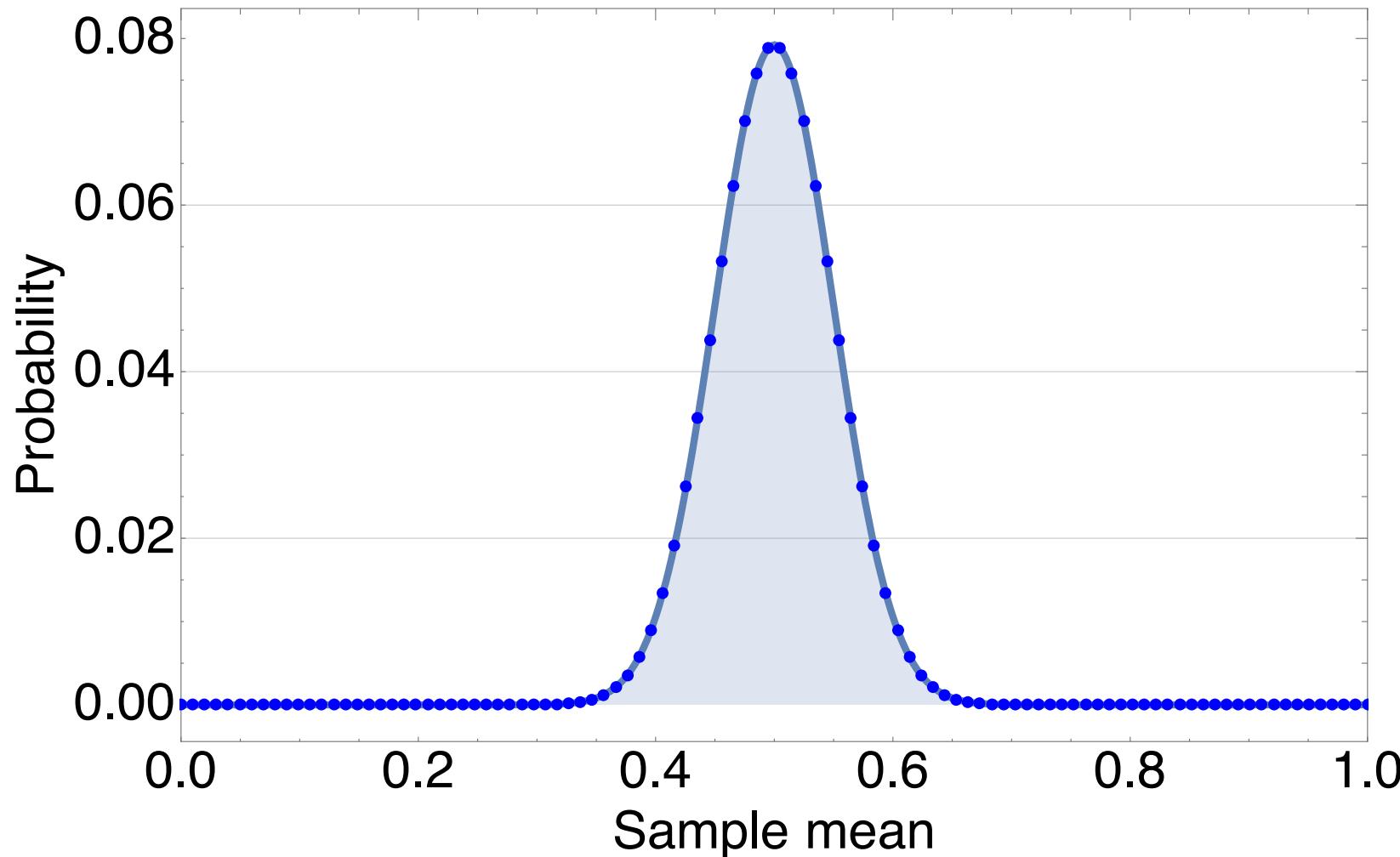


# Variance

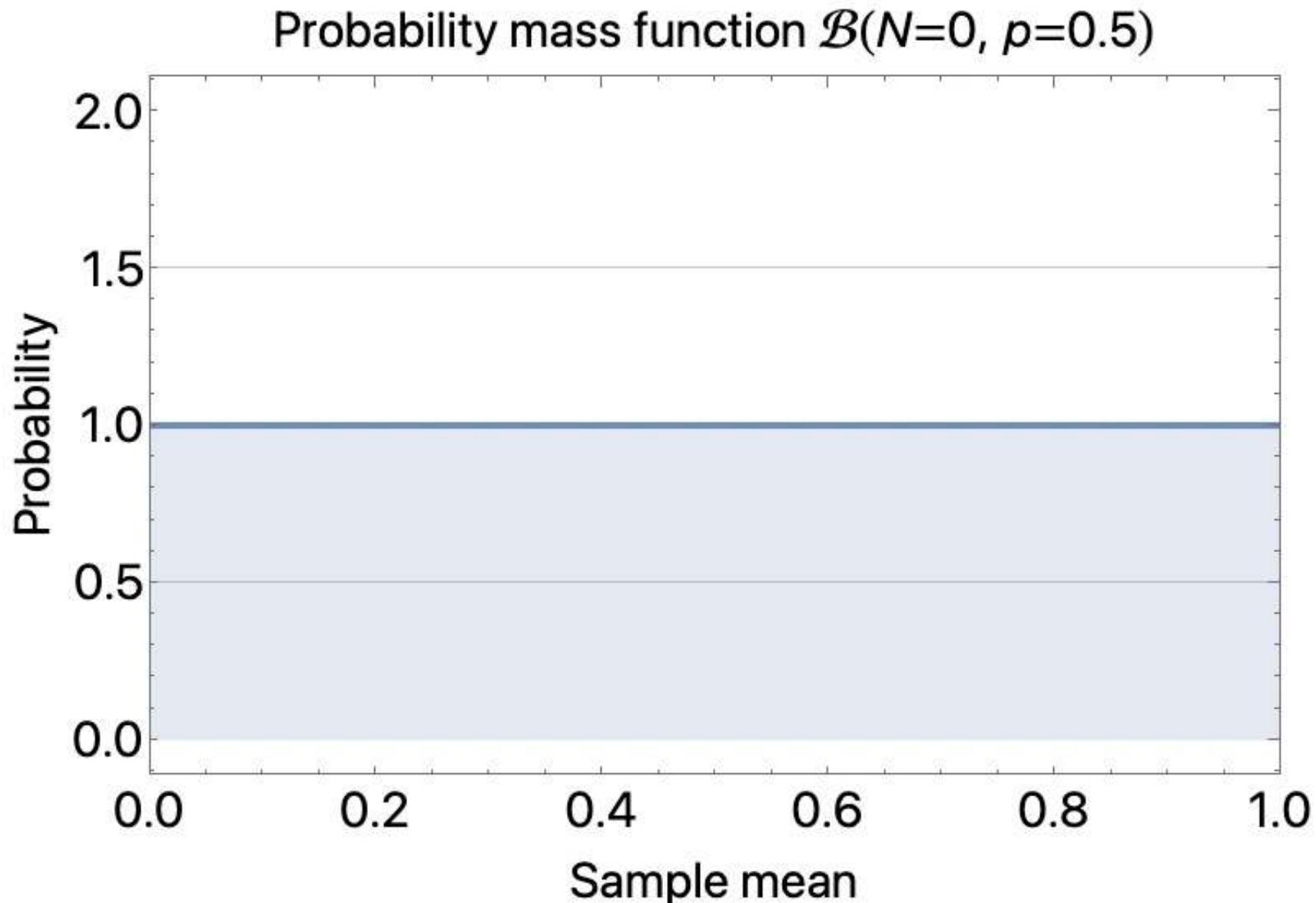




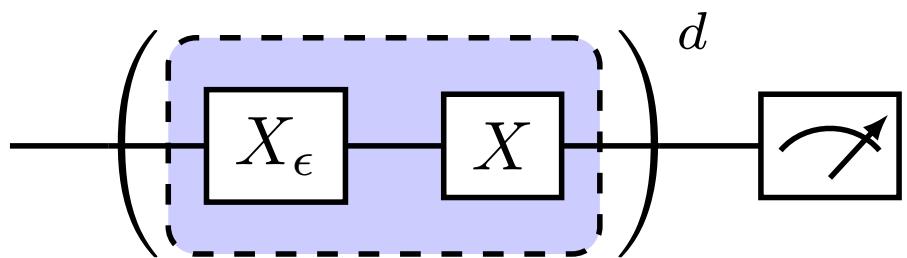
## Probability mass function $\mathcal{B}(N=101, p=0.5)$



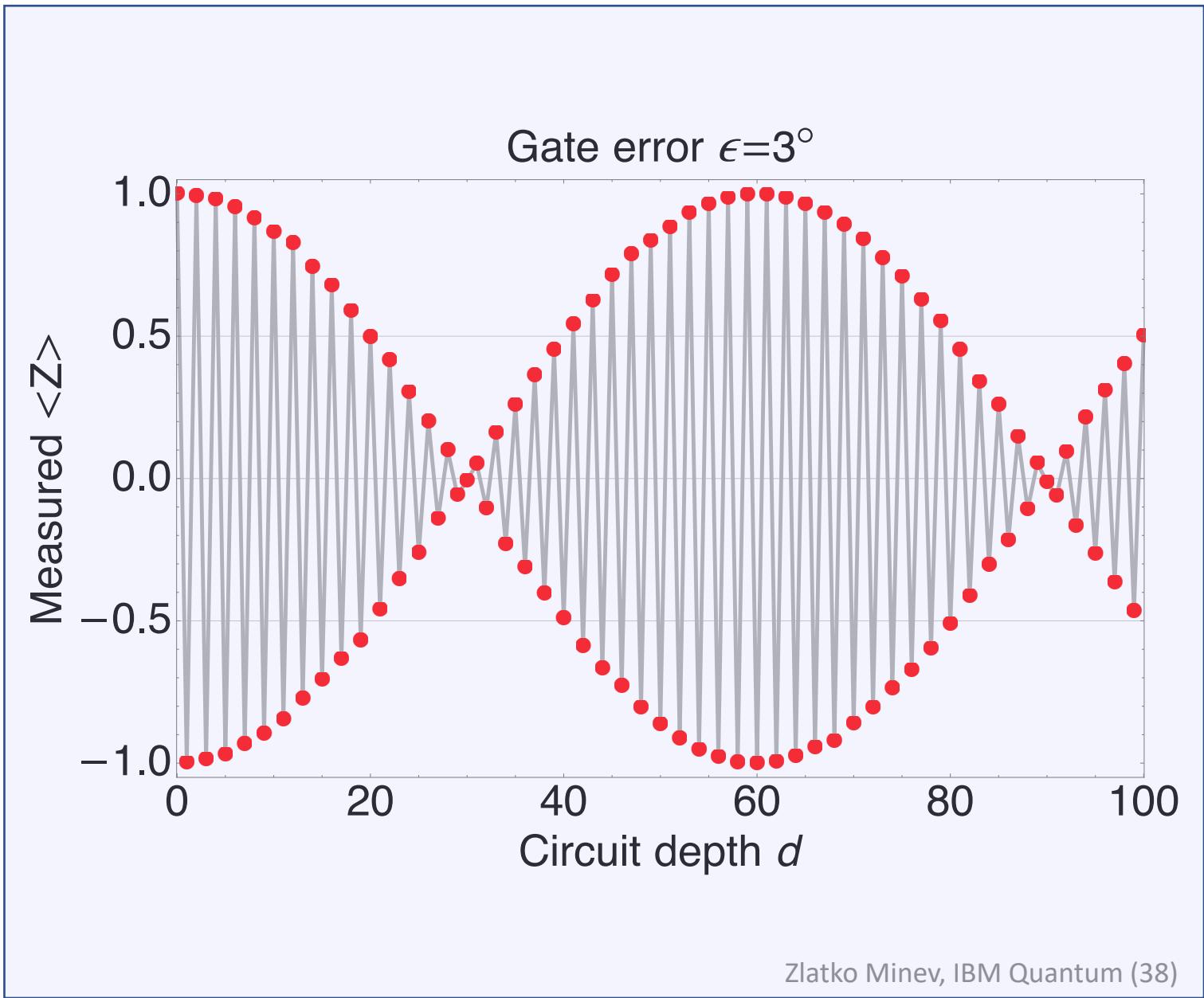
# Animation of convergence of shots expectation value and mean



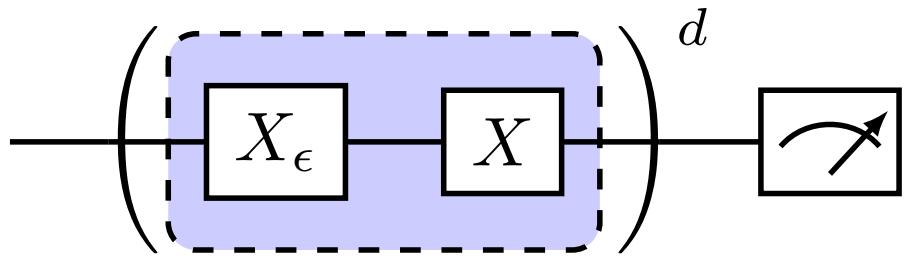
# Recall gate error result



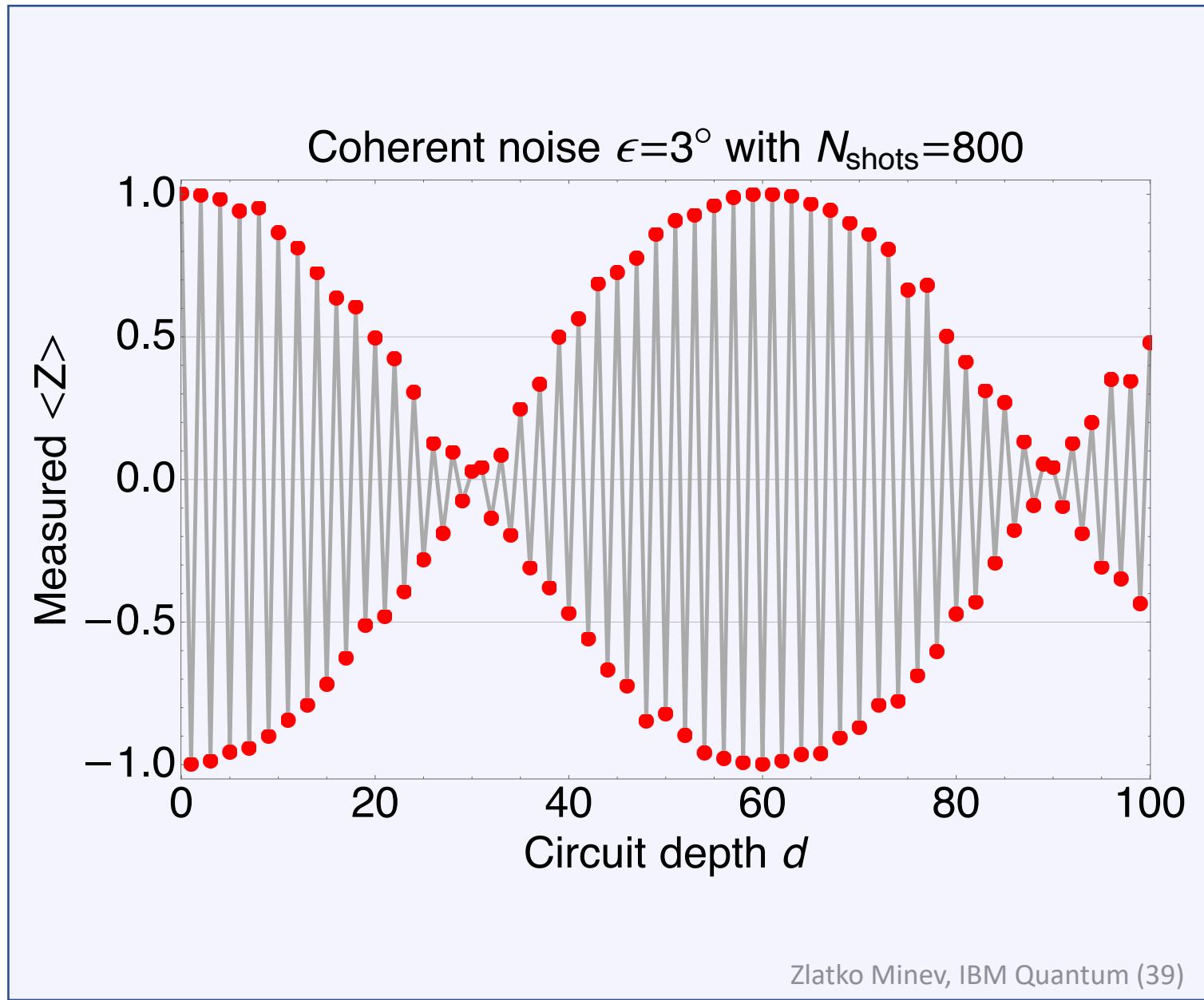
$$\langle \tilde{\psi}_f | Z | \tilde{\psi}_f \rangle = \cos(d\pi + d\epsilon)$$

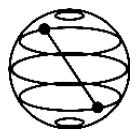


# Projection & sampling noise



$$\langle \tilde{\psi}_f | Z | \tilde{\psi}_f \rangle = \cos(d\pi + d\epsilon)$$





# Dive deeper? Try the following



1. Calculate the following for a qubit

1. The expectation value of the sample variance for N shots of the observable  $|1\rangle\langle 1|$ .

where the sample mean is defined as

$$S = \frac{1}{N} \sum_{n=1}^N M_n$$

and the sample variance is defined as

$$V = \frac{1}{N} \sum_{n=1}^N (M_n - S)^2$$

2. Is the estimate biased?

3. The variance of V.

4. Can you find an expression for an unbiased estimate of the sample variance?

2. What about two qubits?

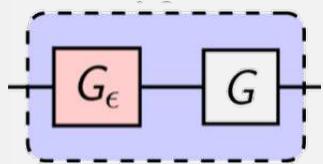
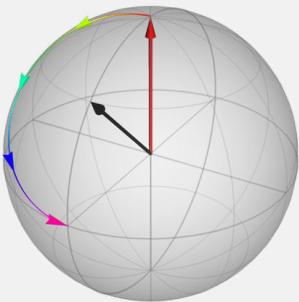
1. Can you find the projection operators for the observable ZZ?

2. Find the probability distribution for the observables ZI, IZ, and ZZ for a general state.

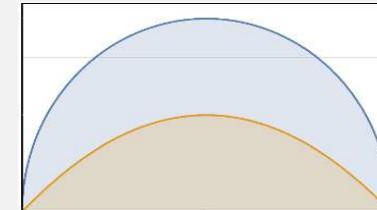
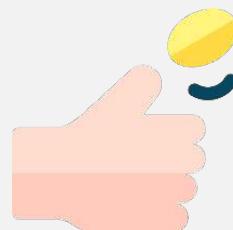
3. If you take 10 shots and find all 10 outcomes to be 1, what is the probability the qubit is in the  $|0\rangle$  state? (hint: it's not zero!)

# Our road ahead

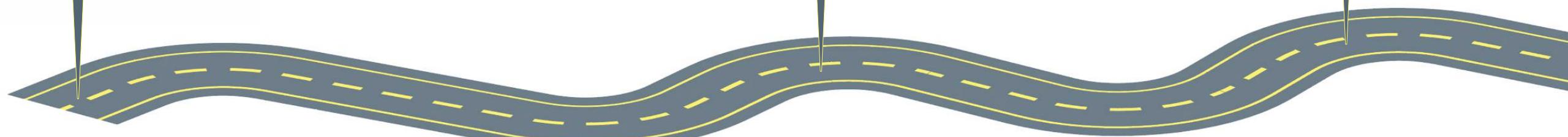
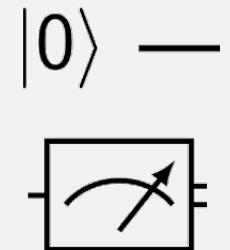
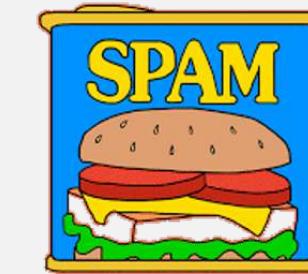
Coherent



Projection &  
measurement theory



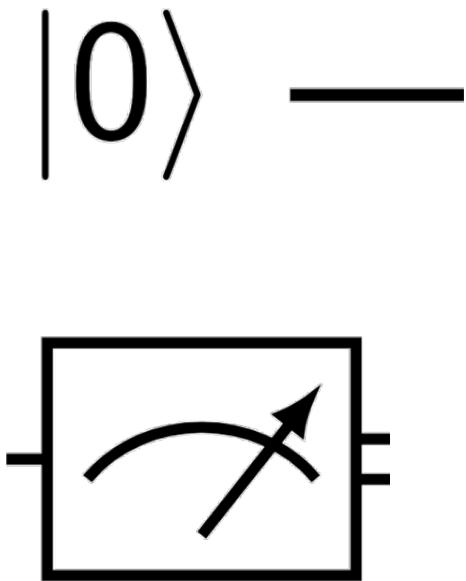
State preparation  
& measurement



coin toss: flaticon; spam: make it move;  
road based on: freepik

Zlatko Minev, IBM Quantum (41)

# State preparation & measurement errors

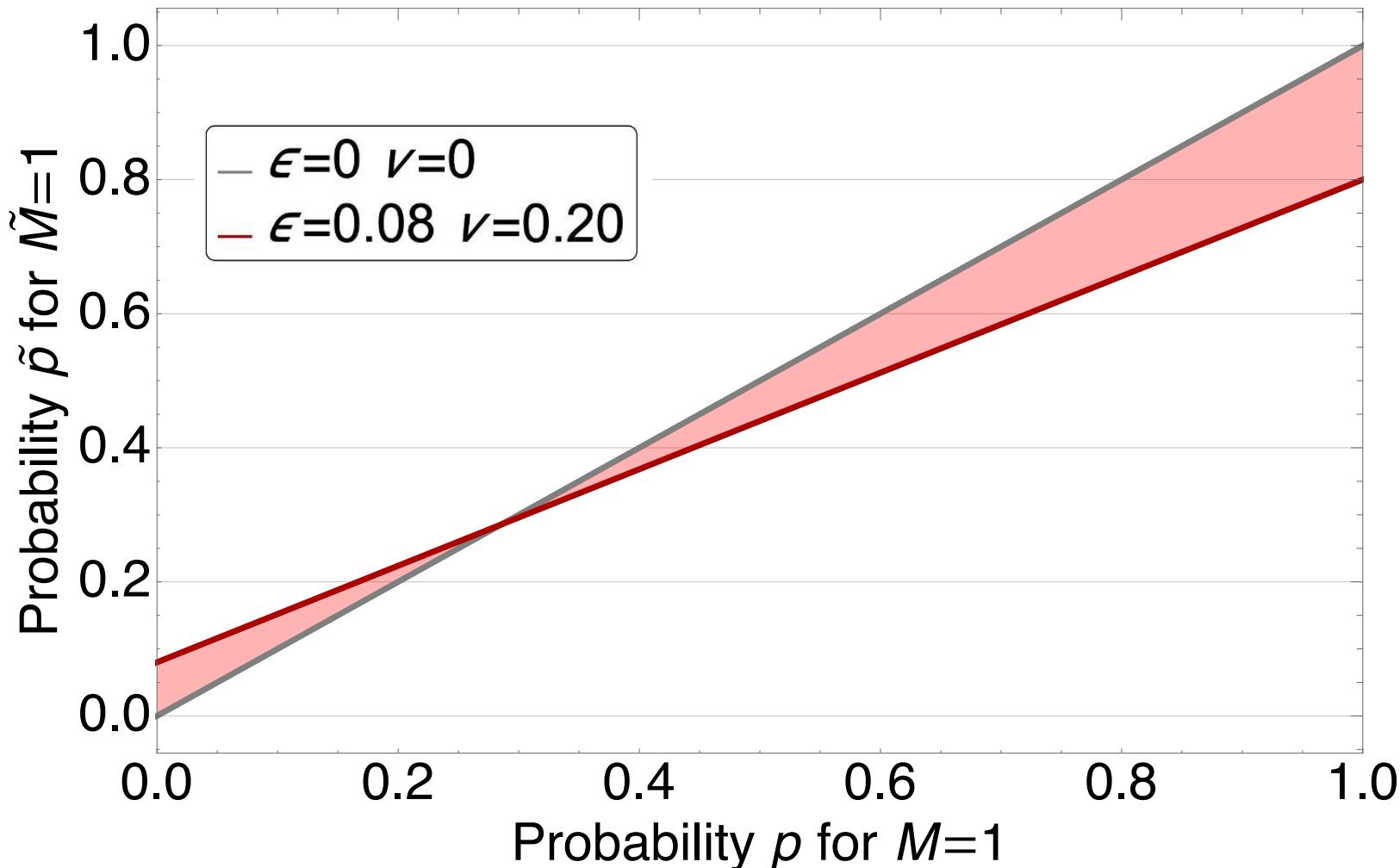


# Qubit example

$$|0\rangle \xrightarrow{\boxed{U}} \xrightarrow{0/1} \left\{ \begin{array}{lll} \text{outcome} & \text{probability} & \text{quantum} \\ \mathbb{P}(M = 0) = & 1 - p & = |\langle 0|\psi \rangle|^2 \\ \mathbb{P}(M = 1) = & p & = |\langle 1|\psi \rangle|^2 \end{array} \right.$$

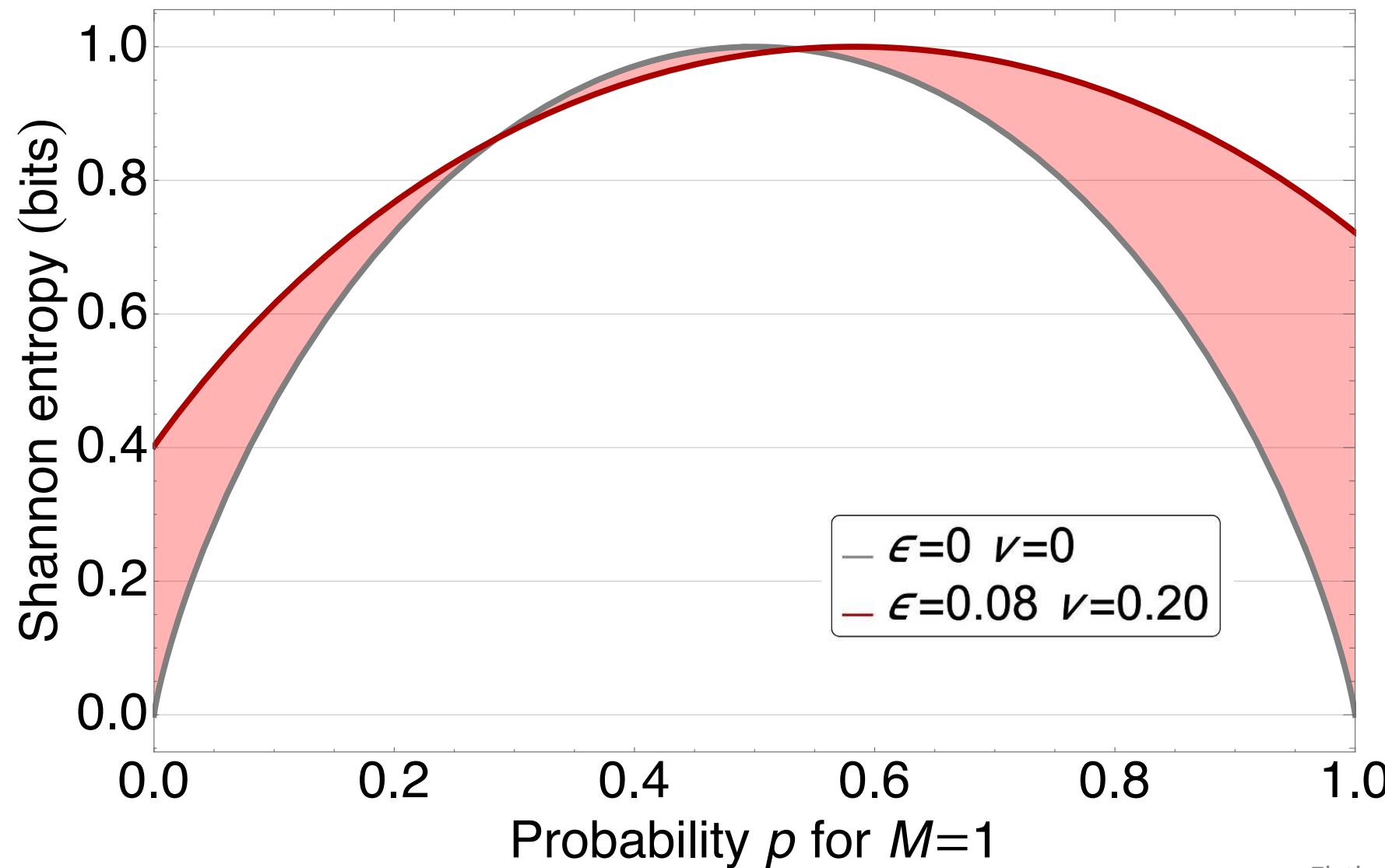
# Readout error

Readout error expectation bias

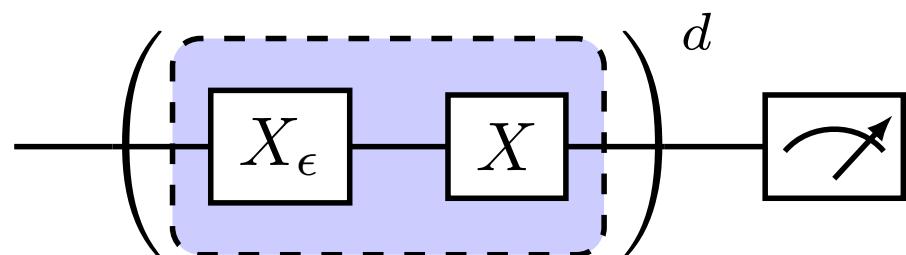


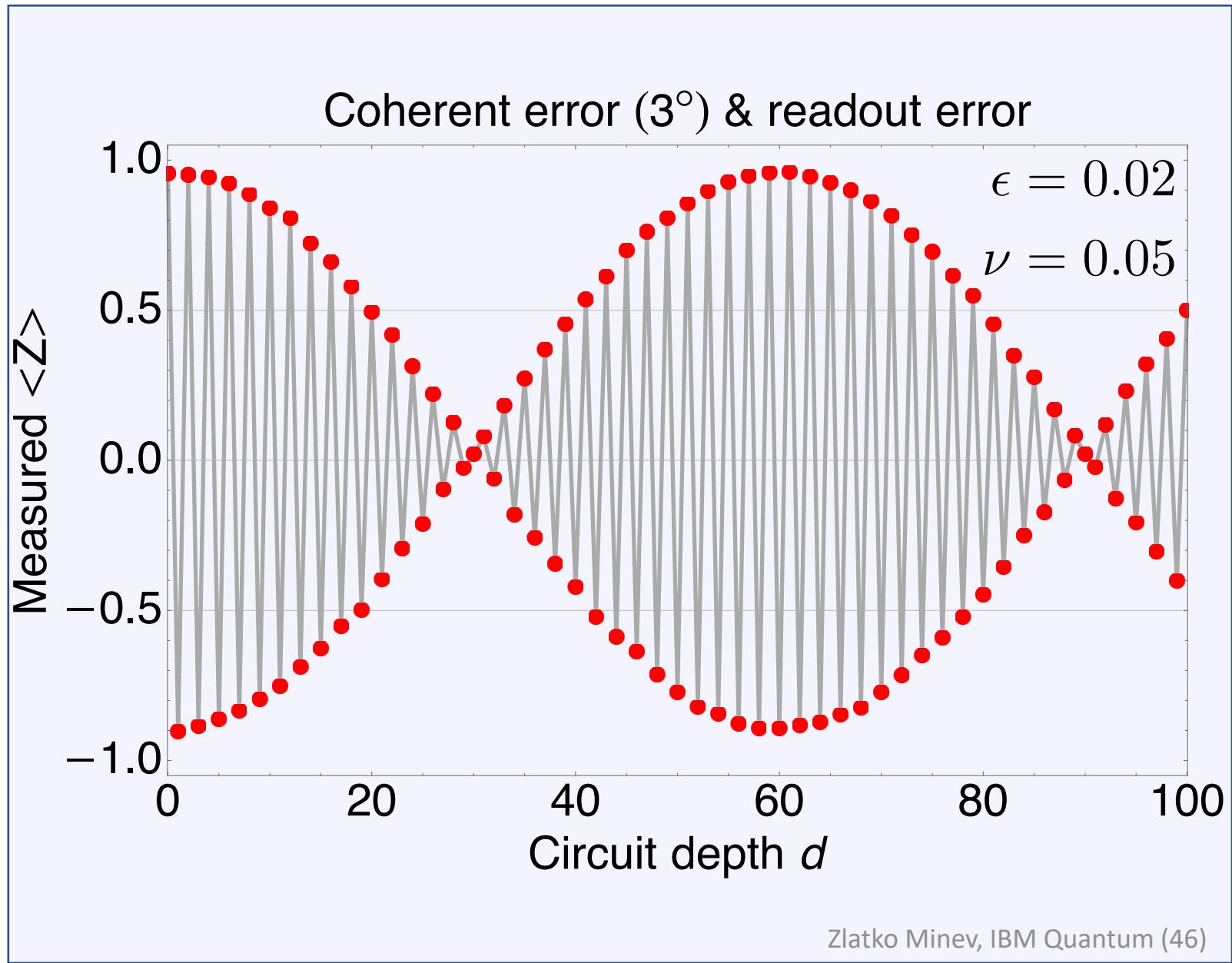
# Entropy

## Shannon entropy and readout error

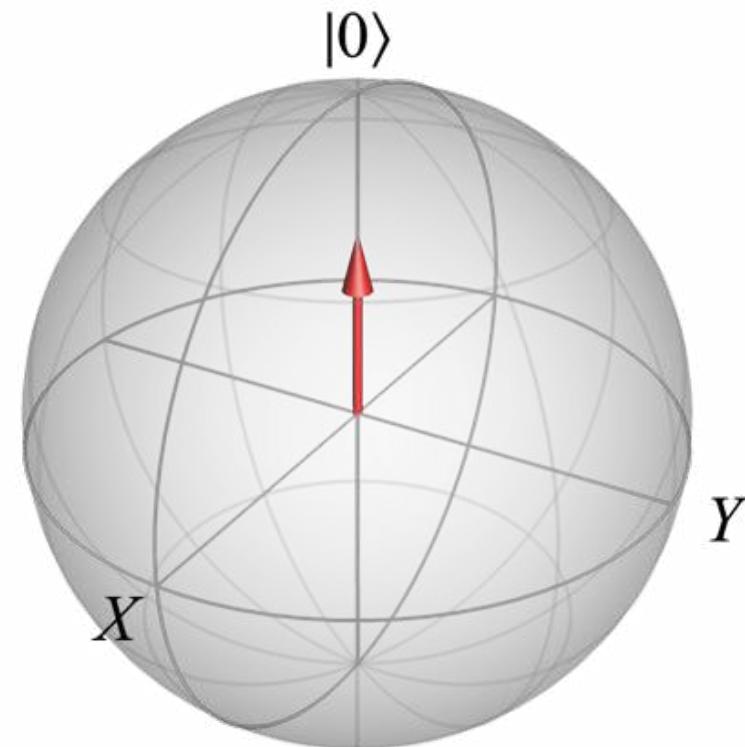
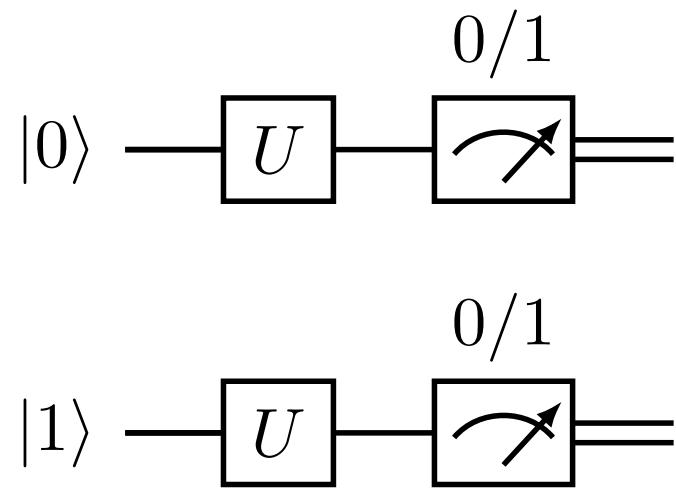


# Projection & sampling noise

$$A = \begin{matrix} M=0 & M=1 \\ \tilde{M}=0 & \begin{pmatrix} 1-\epsilon & \nu \\ \epsilon & 1-\nu \end{pmatrix} \\ \tilde{M}=1 \end{matrix}$$




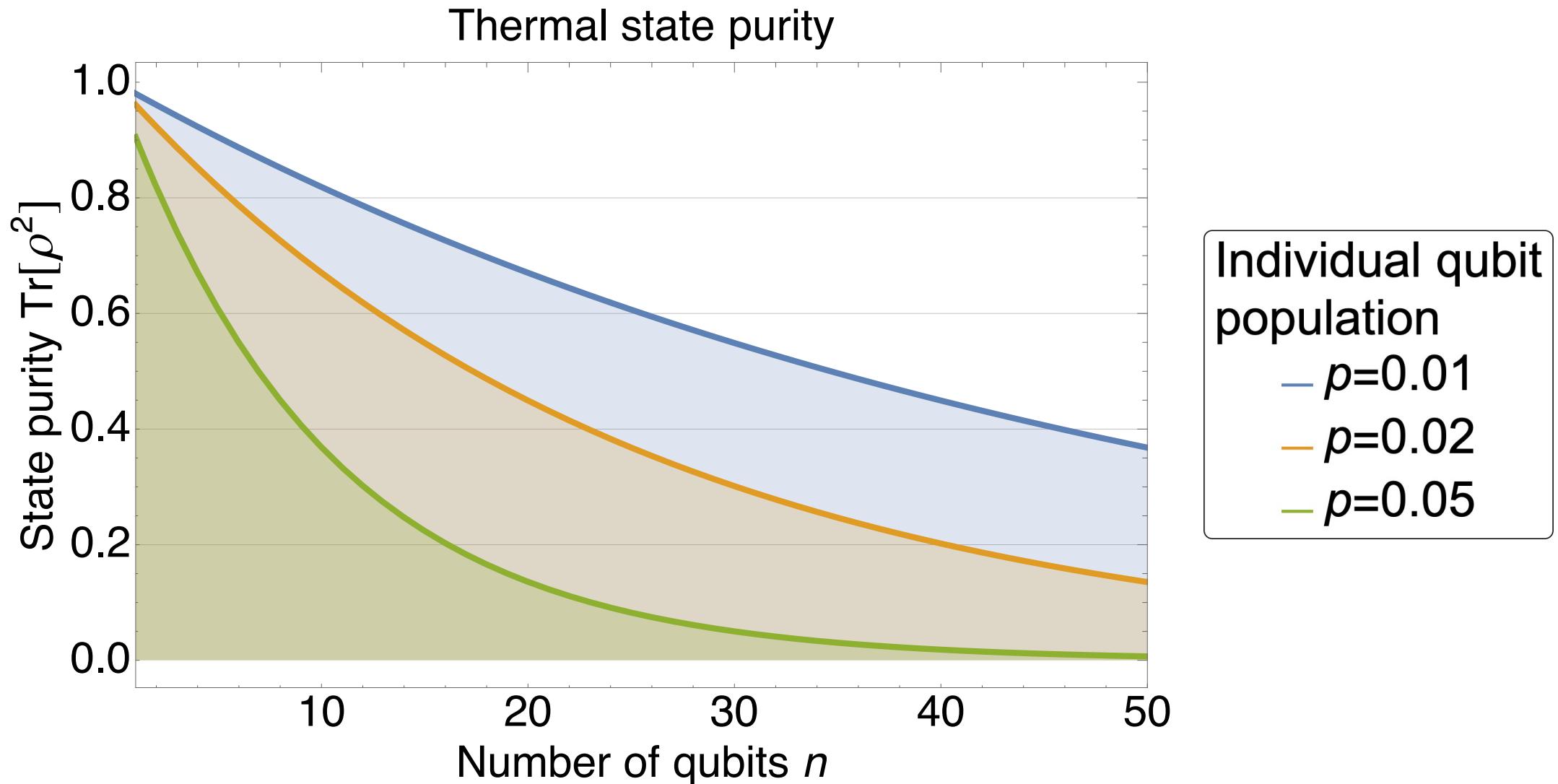
# State prep



# Multiple qubits

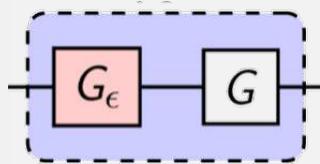
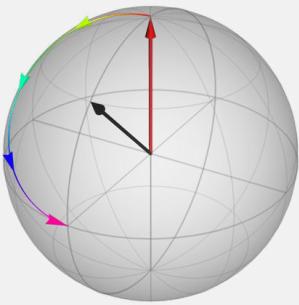
$$[(1 - p) |0\rangle\langle 0| + p |1\rangle\langle 1|]^{\otimes n} \equiv \boxed{U} \equiv \boxed{\curvearrowright}$$

# Thermal state

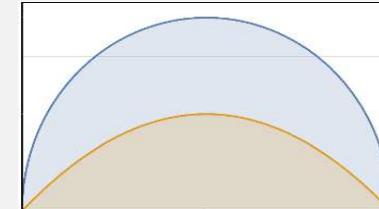
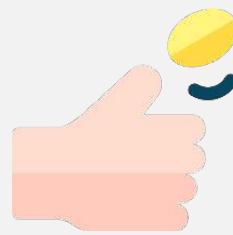


# Our road ahead

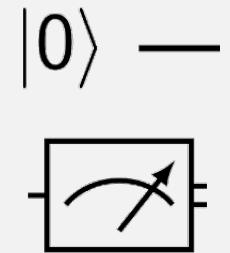
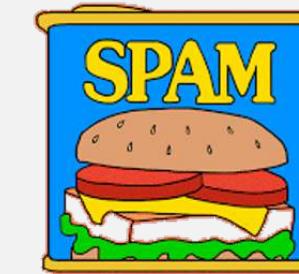
Coherent



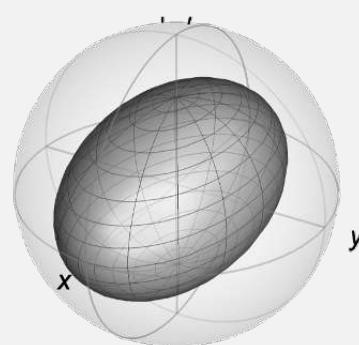
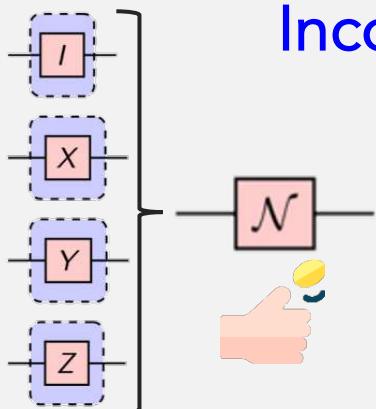
Projection &  
measurement theory



State preparation  
& measurement



Incoherent

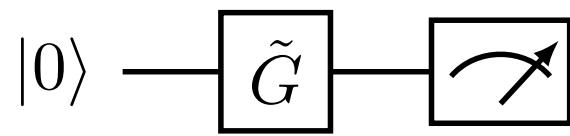


$y$

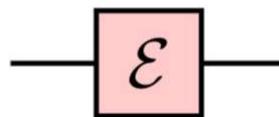
$x$

coin toss: flaticon; spam: make it move;  
road based on: freepik

Zlatko Minev, IBM Quantum (50)

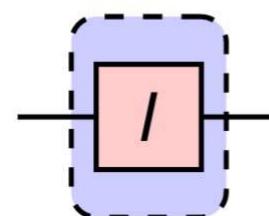


X bit-flip noise

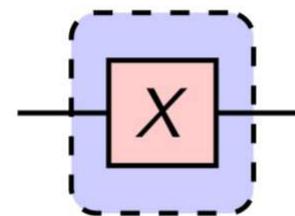


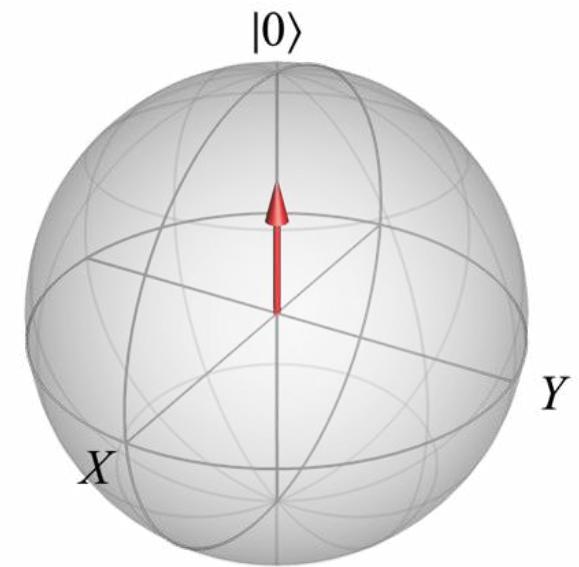
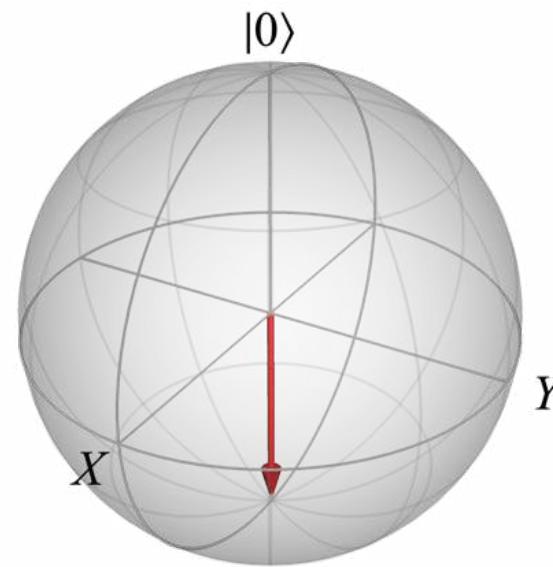
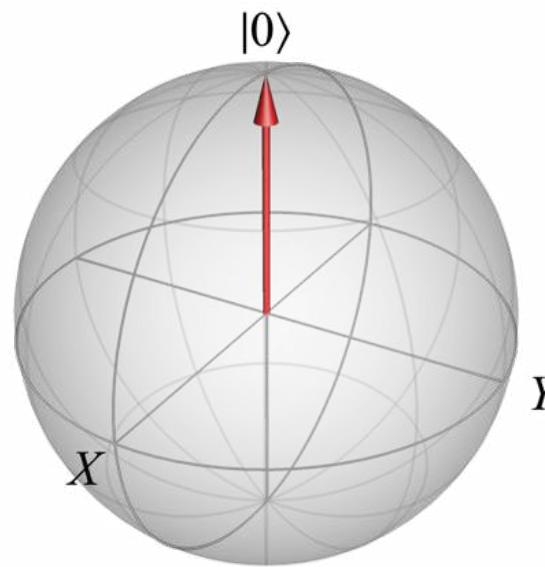
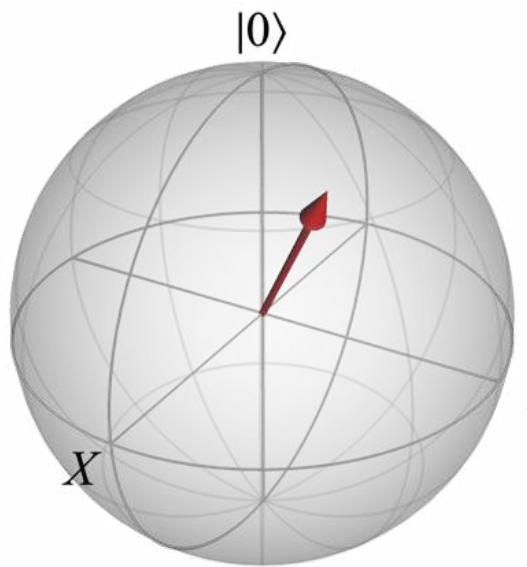
probability      circuit instance

$1 - p$

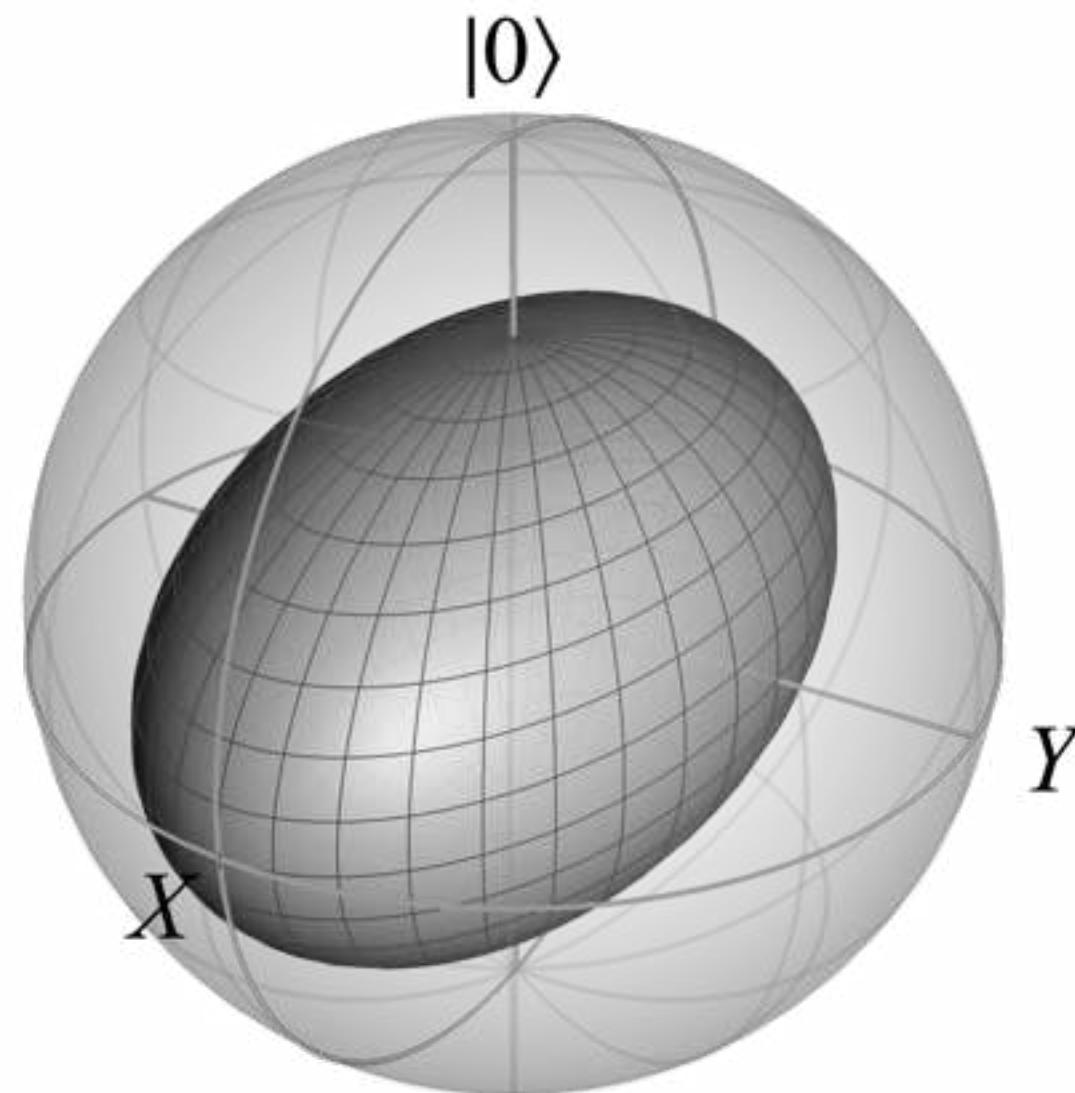


$p$

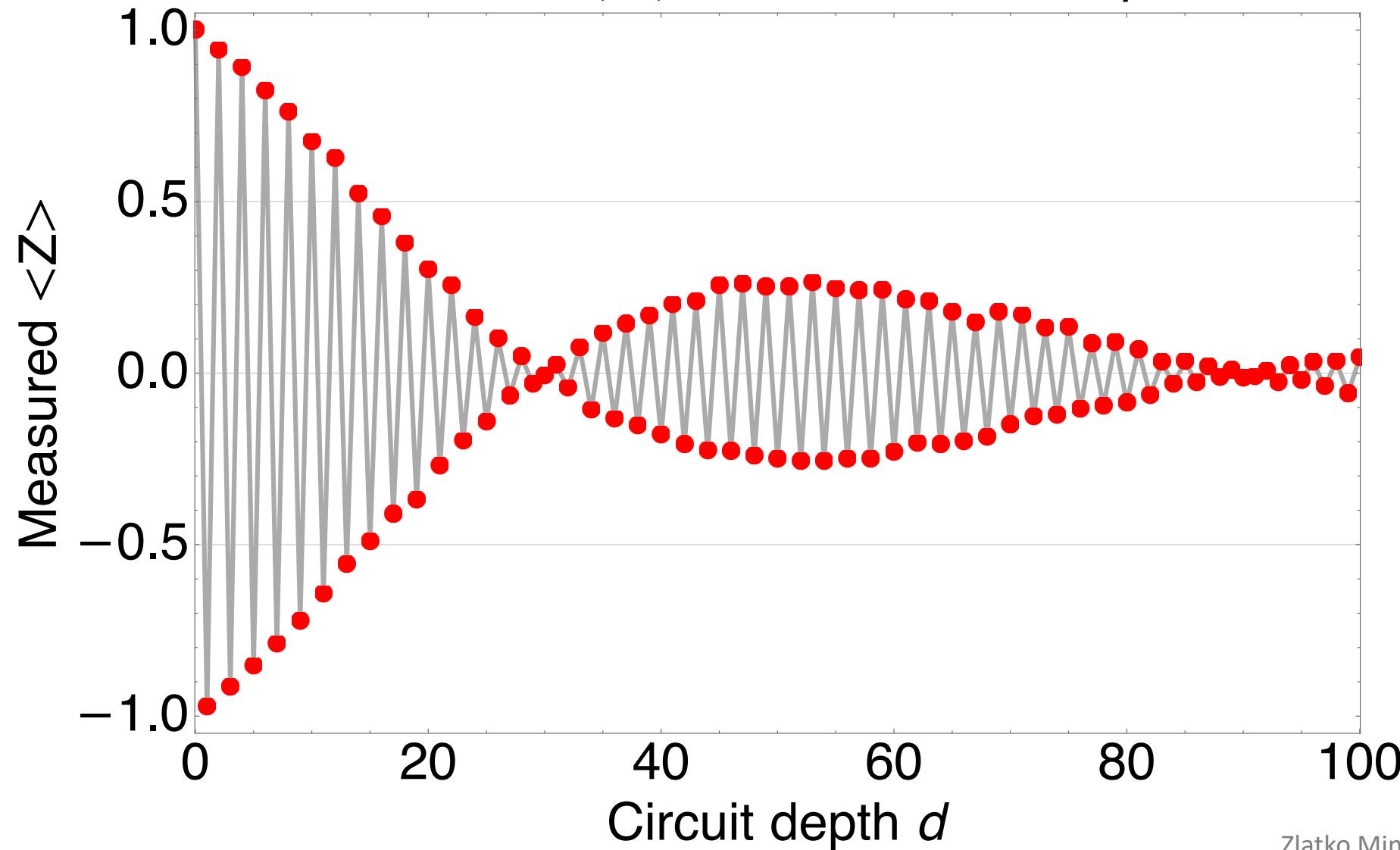




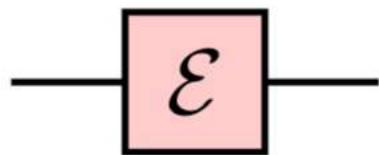
# Bit-flip noise



## Coherent error ( $3^\circ$ ) & incoherent error $p=0.012$

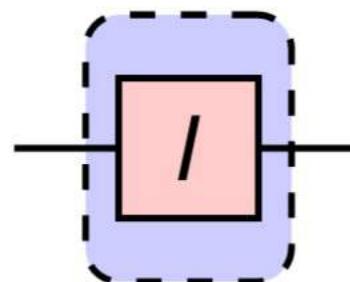


## Phase noise

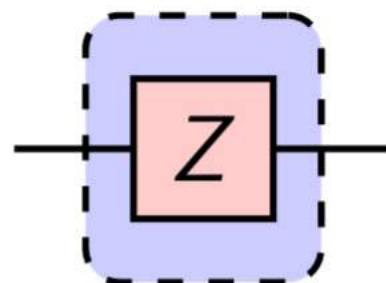


probability      circuit instance

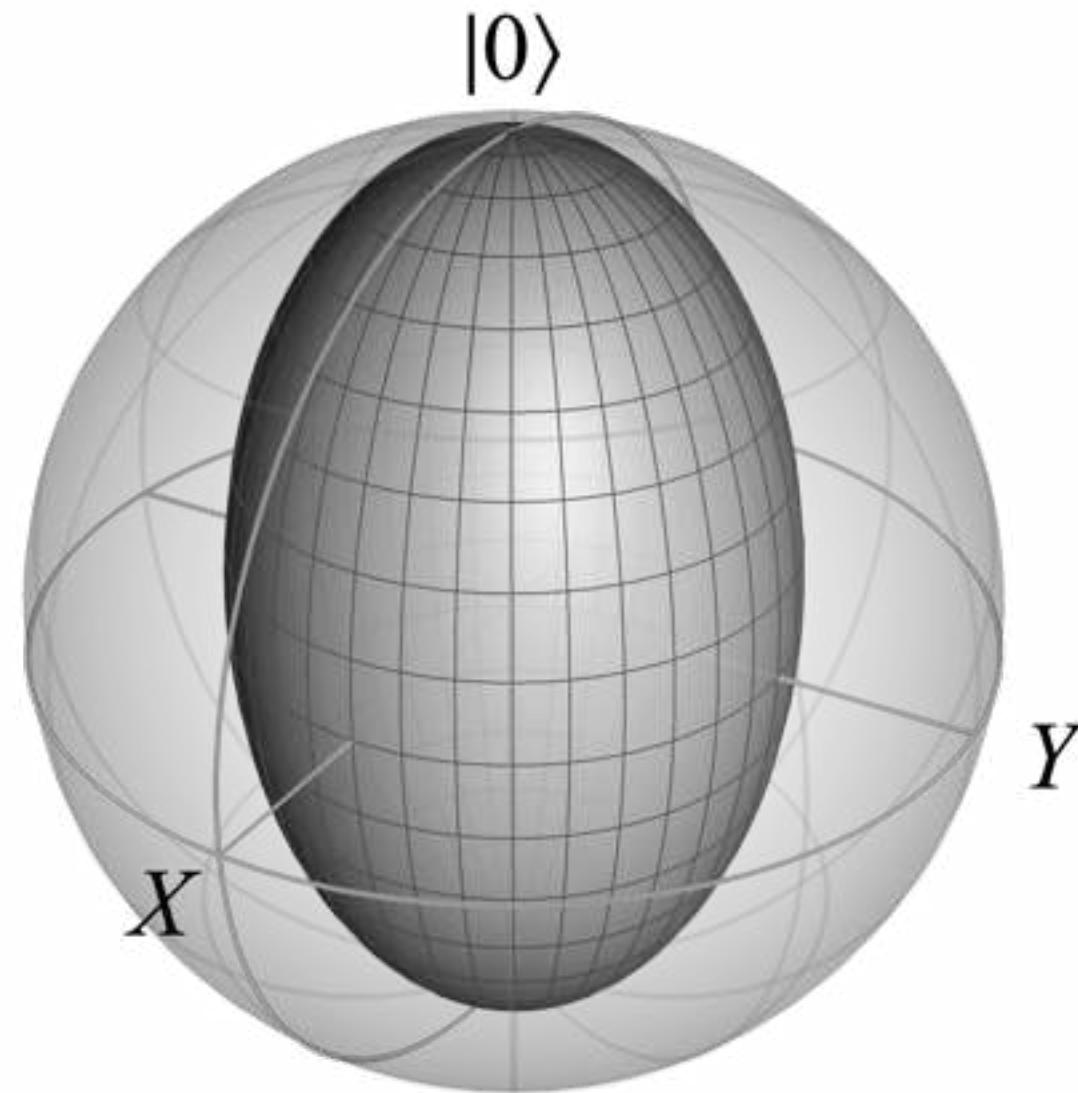
$1 - p$



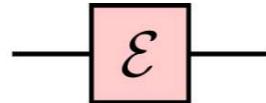
$p$



# Phase damping noise

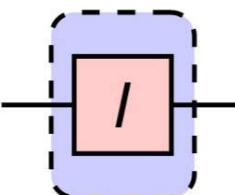


Depolarizing noise

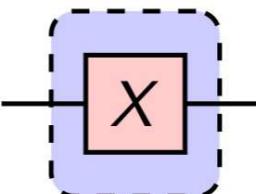


probability    circuit instance

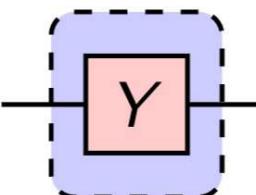
$1 - 3p/4$



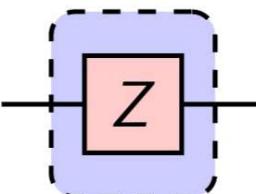
$p/4$



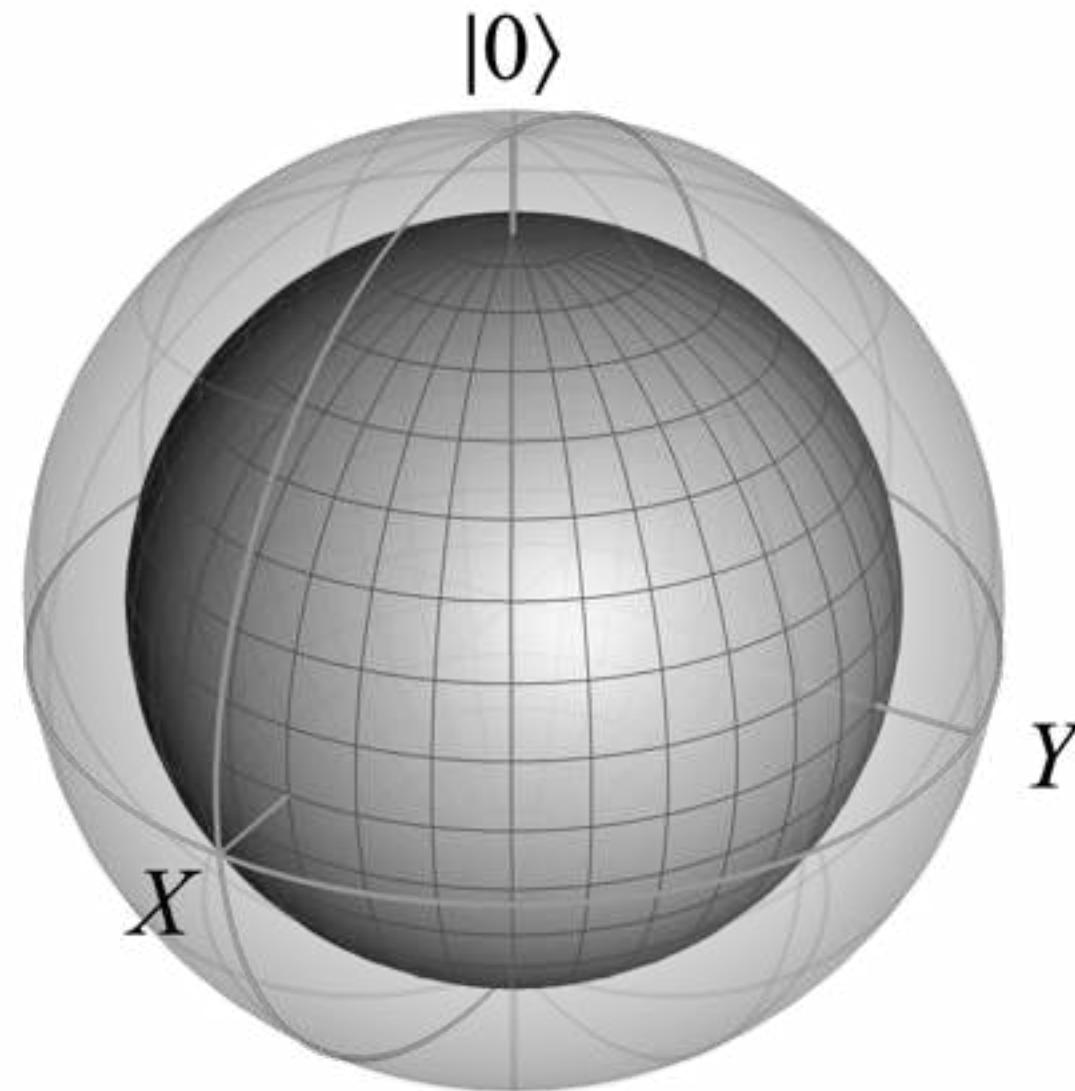
$p/4$



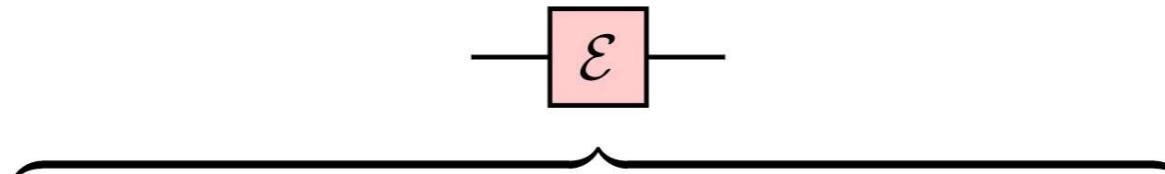
$p/4$



# Depolarizing noise



## Stochastic Pauli noise



probability

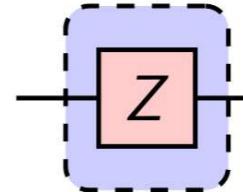
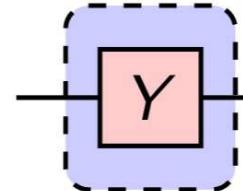
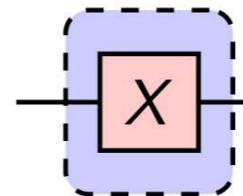
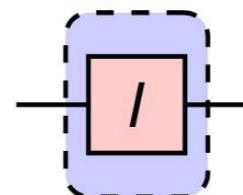
$$1 - p_X - p_Y - p_Z$$

$$p_X$$

$$p_Y$$

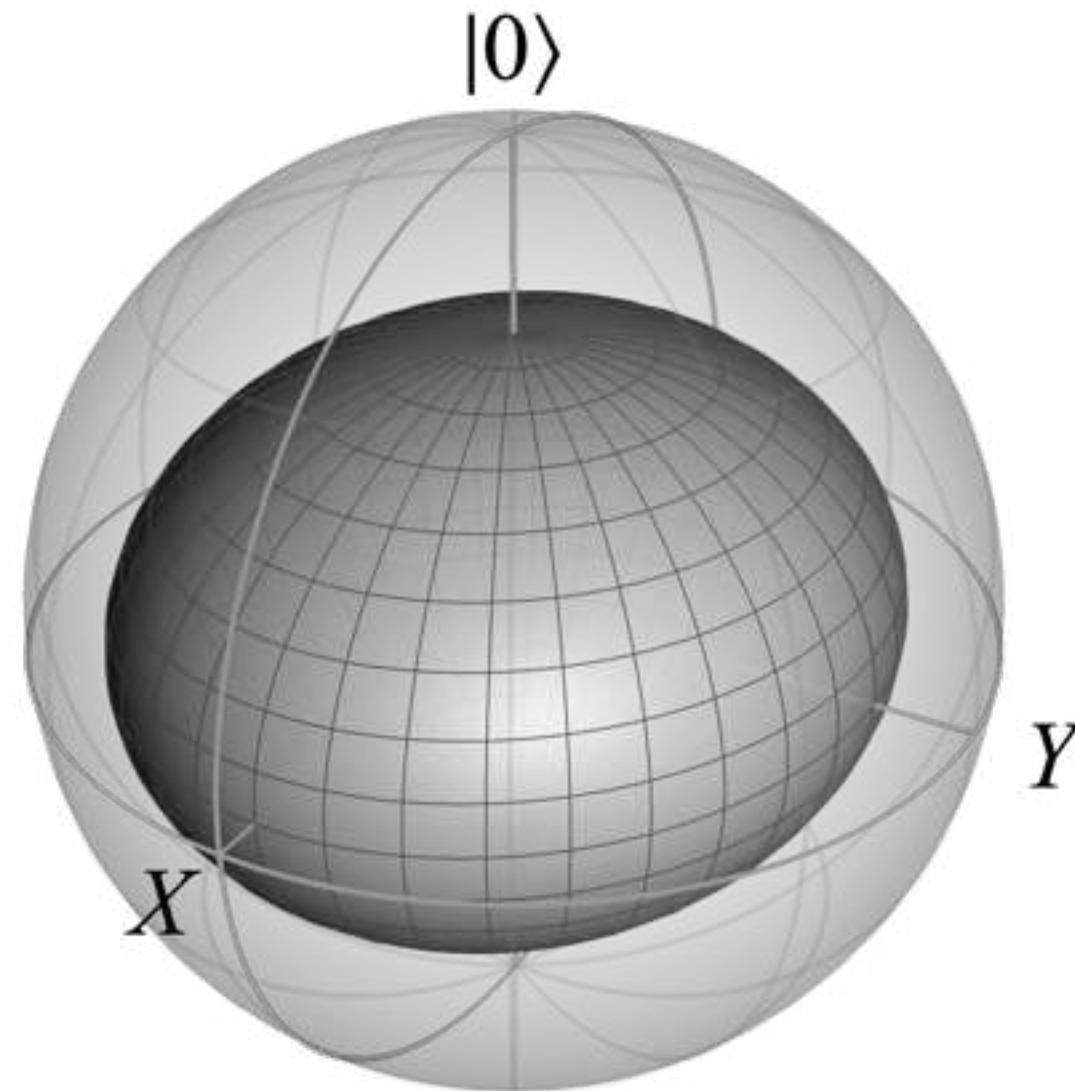
$$p_Z$$

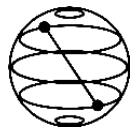
circuit instance



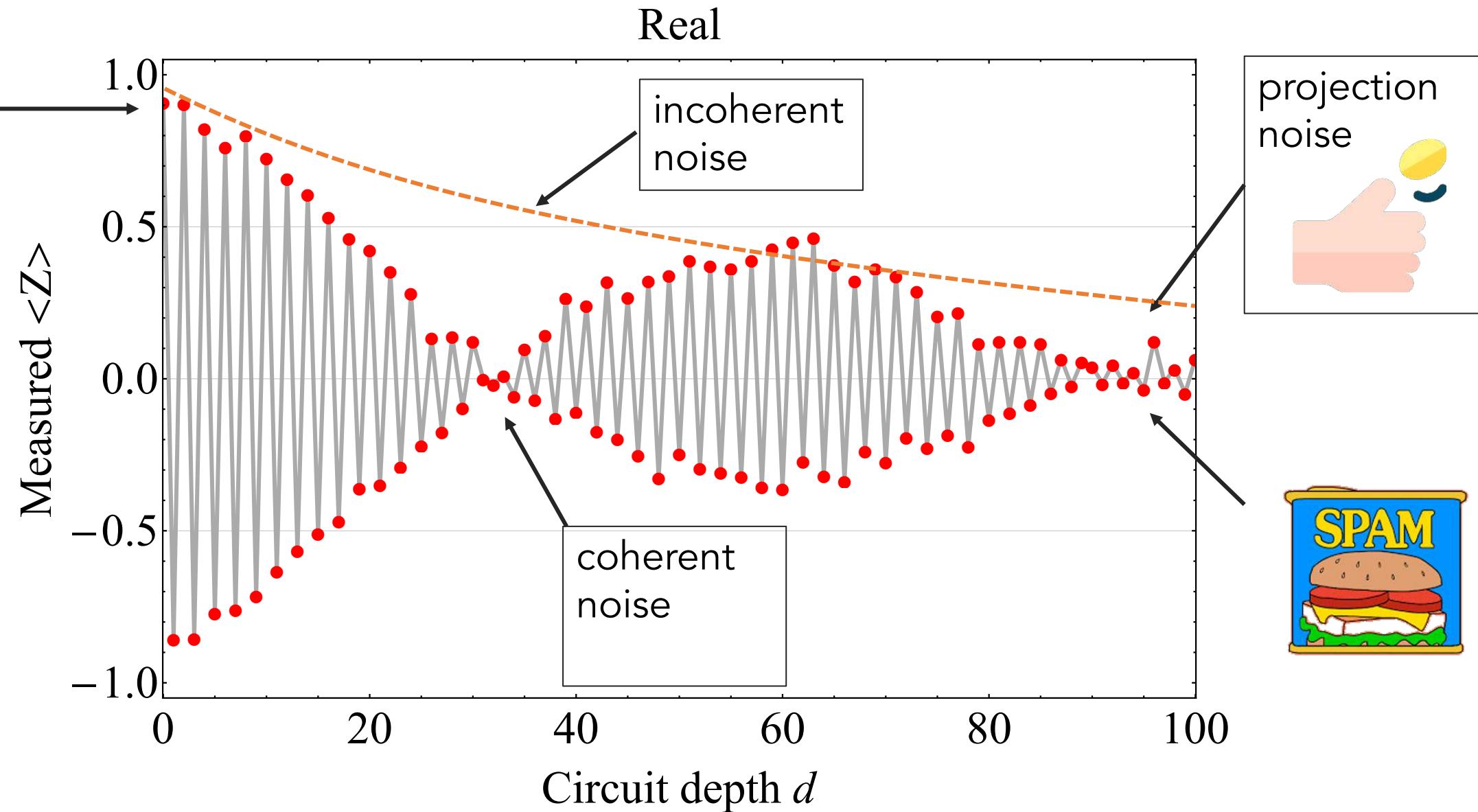
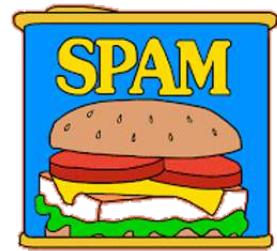
# Stochastic Pauli Noise

$p_x = 0.1$   
 $p_y = 0.2$   
 $p_z = 0.4$



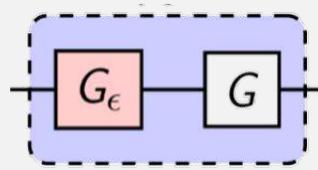
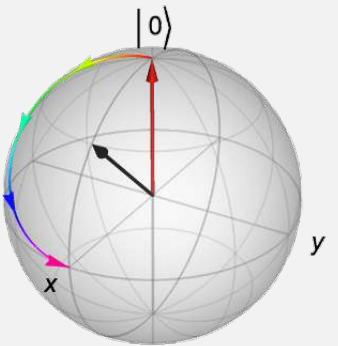


# Elements of noise

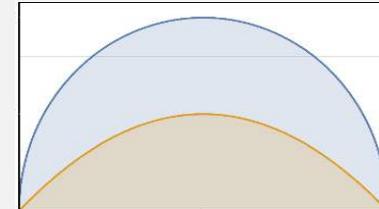
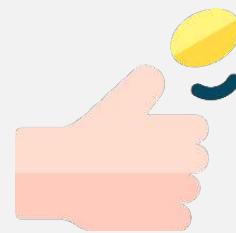


# The journey behind us

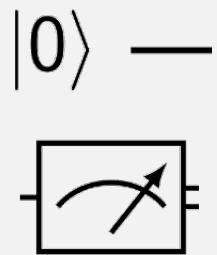
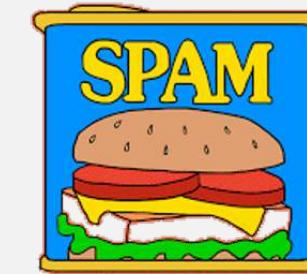
## Coherent noise



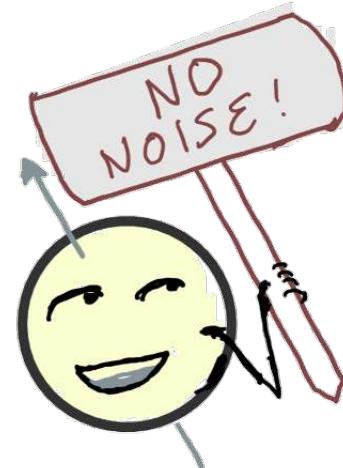
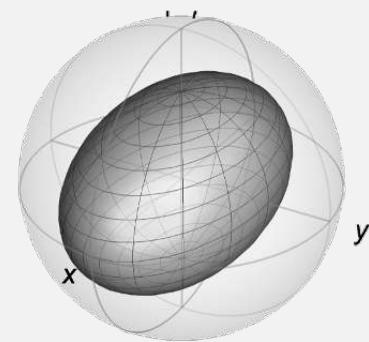
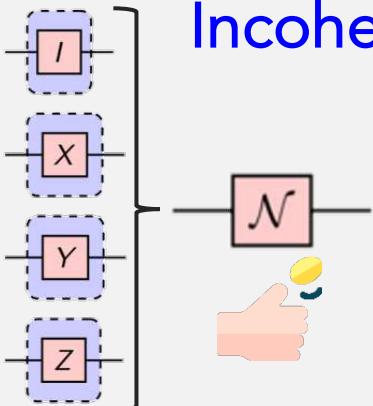
## Quantum measurement theory Projection & sampling noise



## State prep & measure



## Incoherent noise



coin toss: flaticon; spam: make it move;  
road based on: freepik

Zlatko Minev, IBM Quantum (63)

# Next steps

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Lab work with Qiskit

Run experiments on real devices

Check out references, problems given in the lecture,  
dangerous bends



Stay in touch

Thank you!

**Zlatko K. Minev**



@zlatko\_minev



zlatko-minev.com

IBM Quantum

The important thing is not to stop questioning.  
Curiosity has its own reason for existence.

One cannot help but be in awe when he  
contemplates the mysteries of eternity, of life, of the  
marvelous structure of reality.

It is enough if one tries merely to comprehend a  
little of this mystery each day.

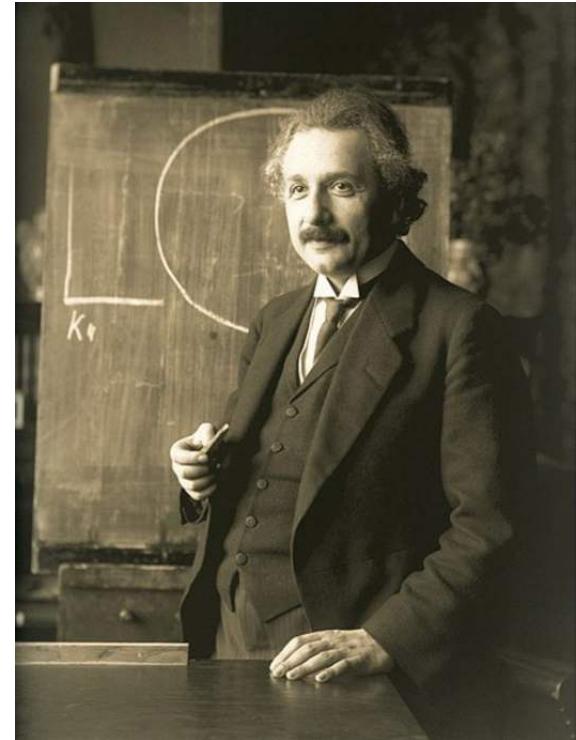


Photo: F. Schmutz

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Albert Einstein



@zlatko\_minev



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IBM Quantum