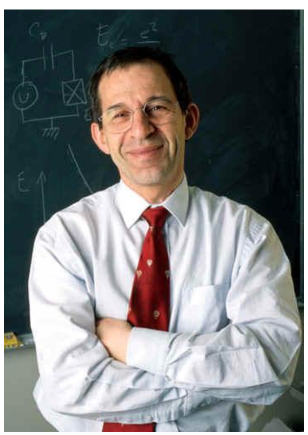
QUANTRONICS LABORATORY
Department of Applied Physics
Yale University

Parasitic Capacitance in Josephson Junction Arrays

Origins and Behavior

Zlatko Minev

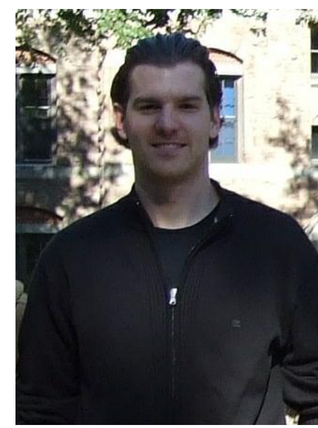


Michel H. Devoret

Nicholas Masluk

Ioan Pop

Archana Kamal



Flavius Schackert

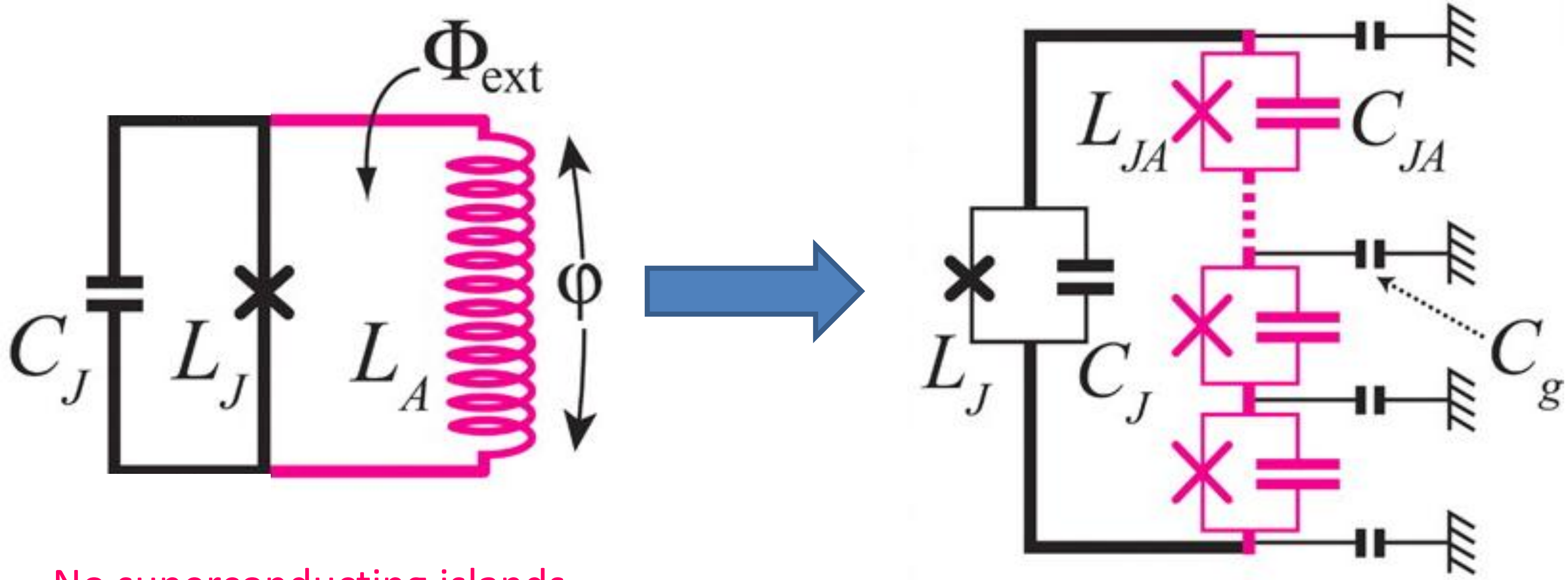
Outline

1. Fluxonium Junction Array
   - Motivation
   - Until now: Dolan Bridge Geometry & Constraints

2. Theory & Experiment v.s. Simulation
   - Design Study Conclusion & Simple Model
   - Resolution: Bridge Free (Ioan Pop)
   - Bridge Free Technique – Skinny & Long

3. In Progress:
   - Q Measurement Simulation & Chip Import
   - Ground Up Mathematica Assisted Exploration of Fluxonium
     - Aim Super-Fluxonium & Quantum Feedback on λ Energy Structure

Series Junction Array in Fluxonium



No superconducting islands,
No offset charges!

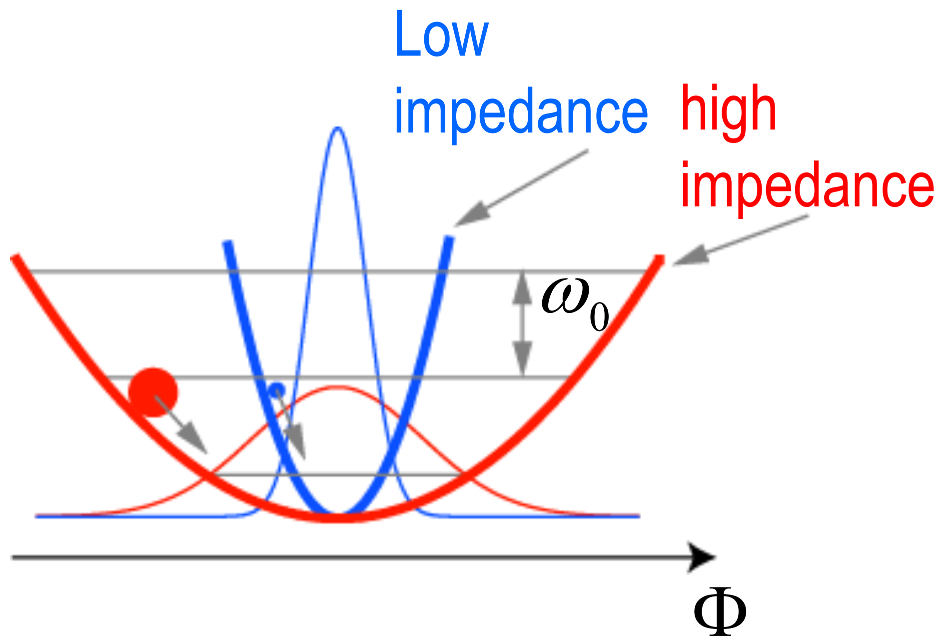
V. E. Manucharyan, J. Koch, M. H. Devoret, and L. I. Glazman, Science 326, 113 (2009)

0.5 2.5

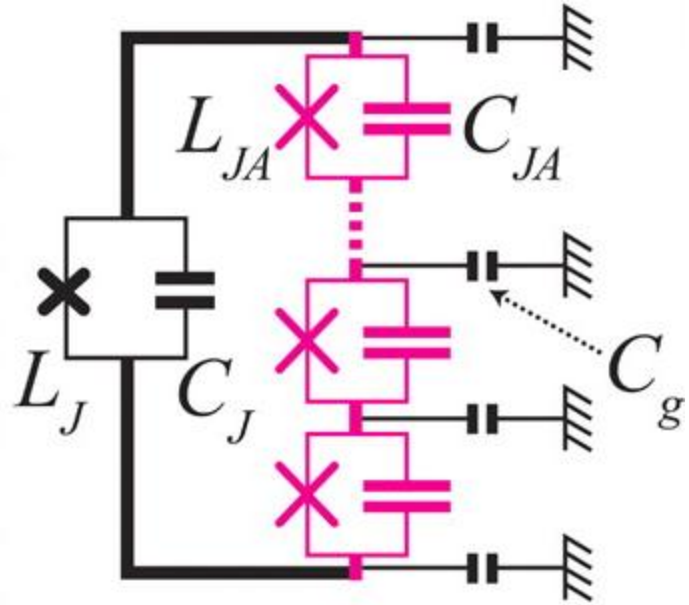
Fx

9.0

Impedance Control ZPF



Capacitance to Ground



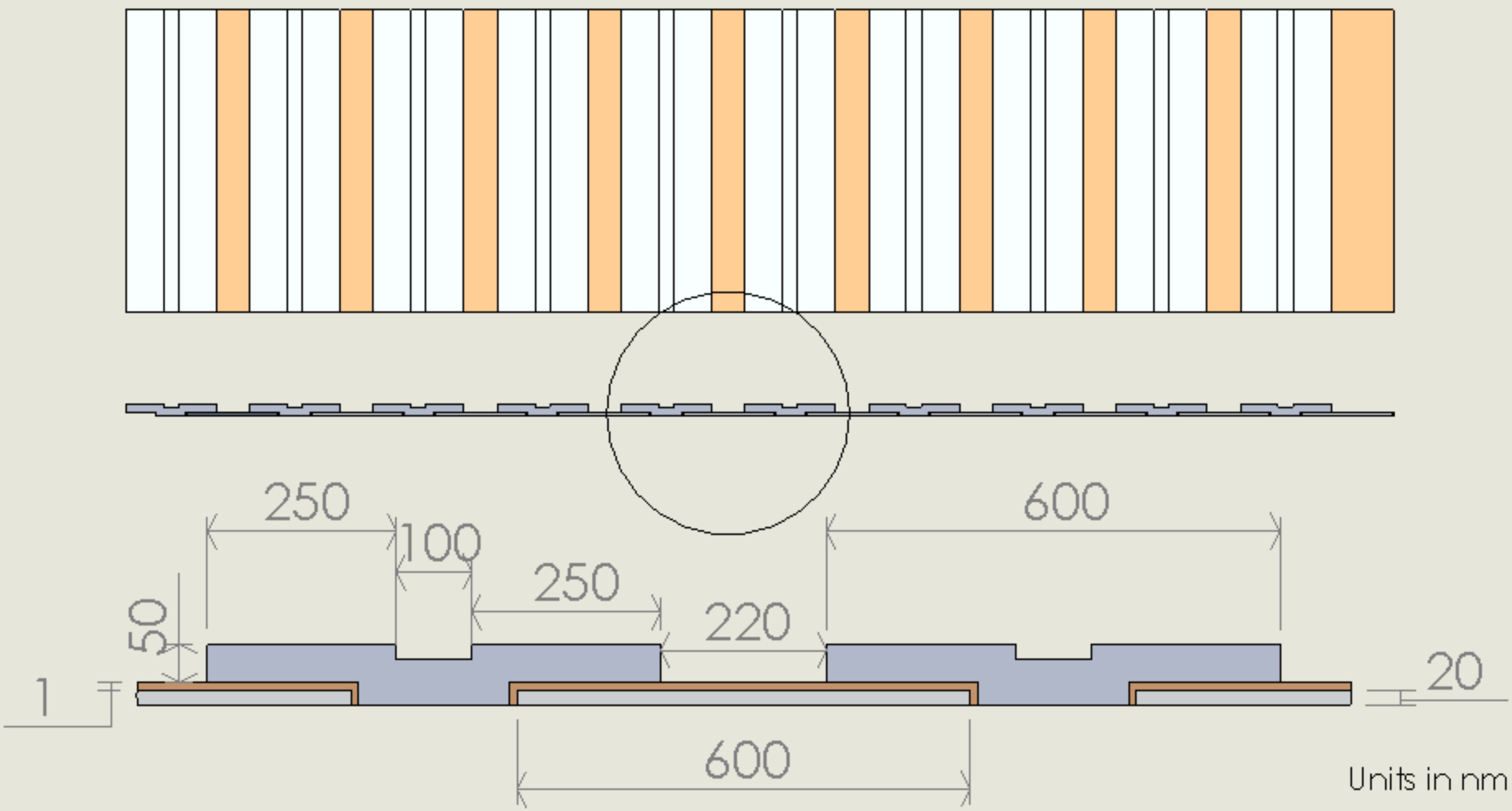
$$\frac{C_{JA}}{N} > N\,C_g$$

$$N_{\max} \approx \sqrt{C_{JA} / C_g} \approx \sqrt{10^3}$$

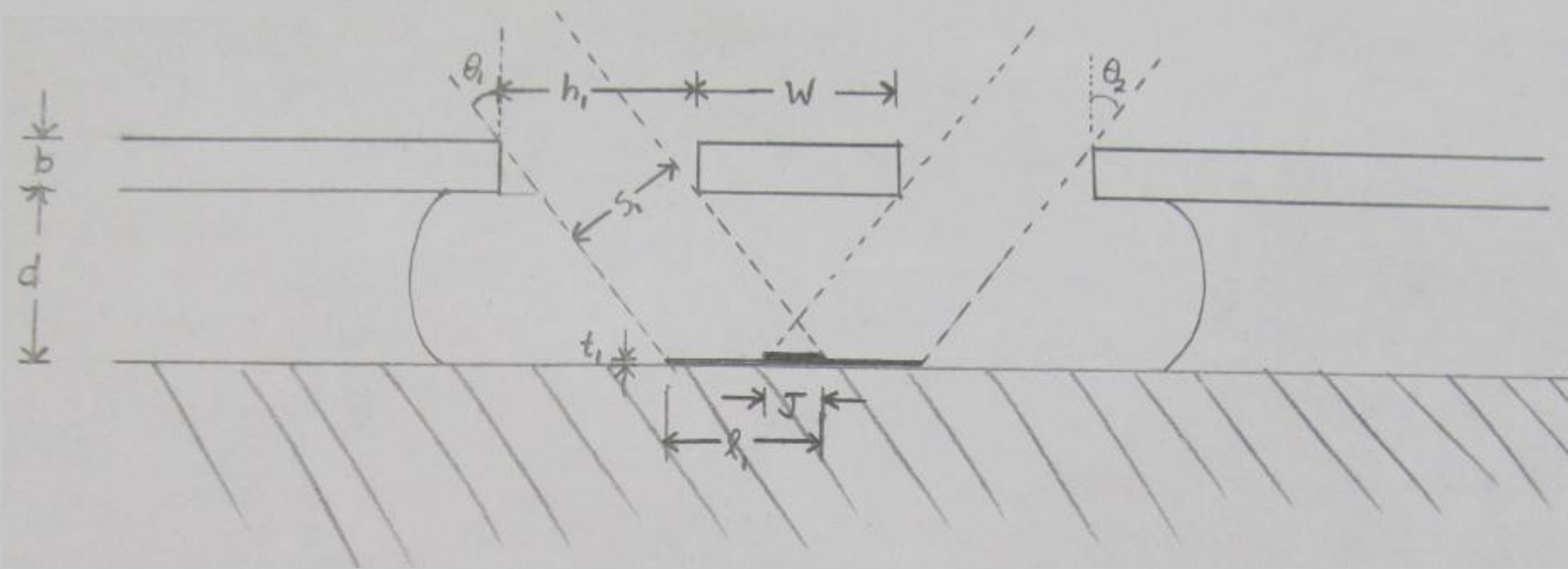
V. E. Manucharyan, J. Koch, M. H. Devoret, and L. I. Glazman, Science 326, 113 (2009)

2. Fluxonium Junction Array

Junction Array Geometry

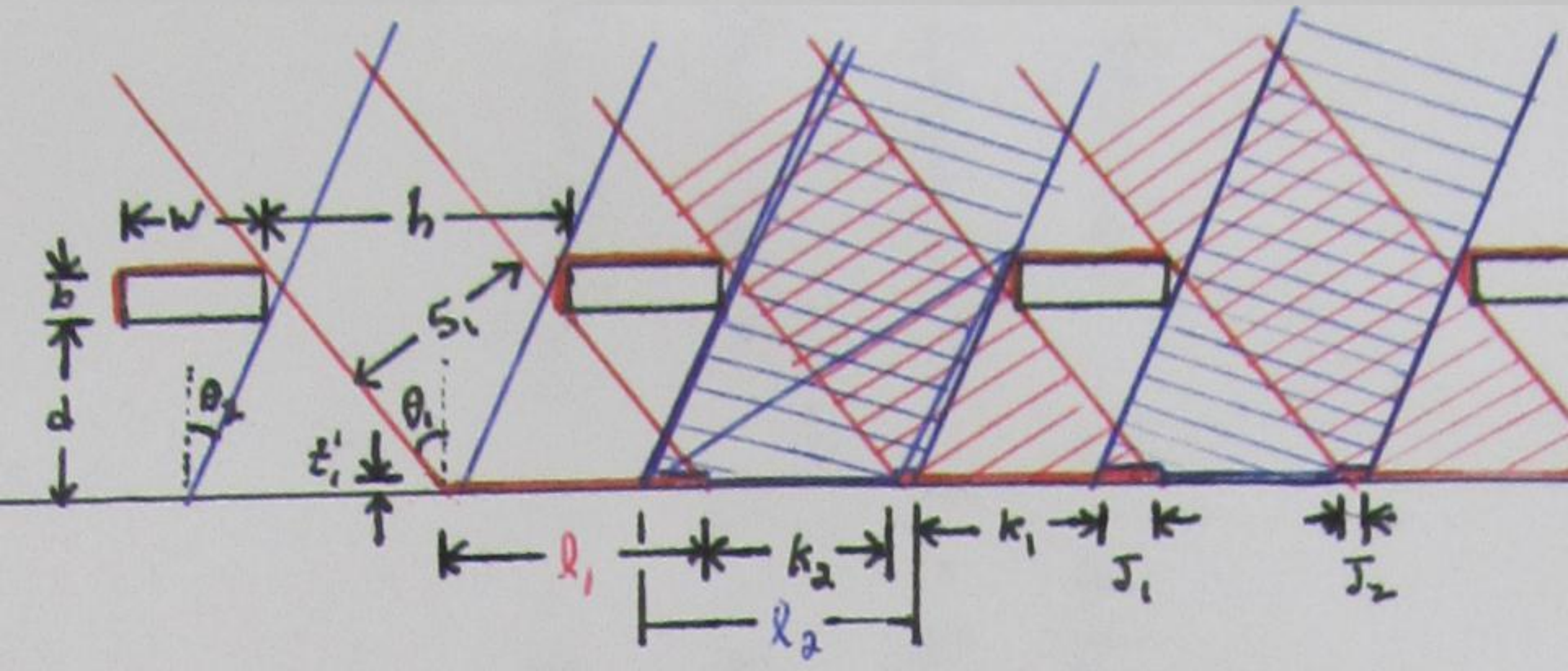


Dolan Bridge Technique



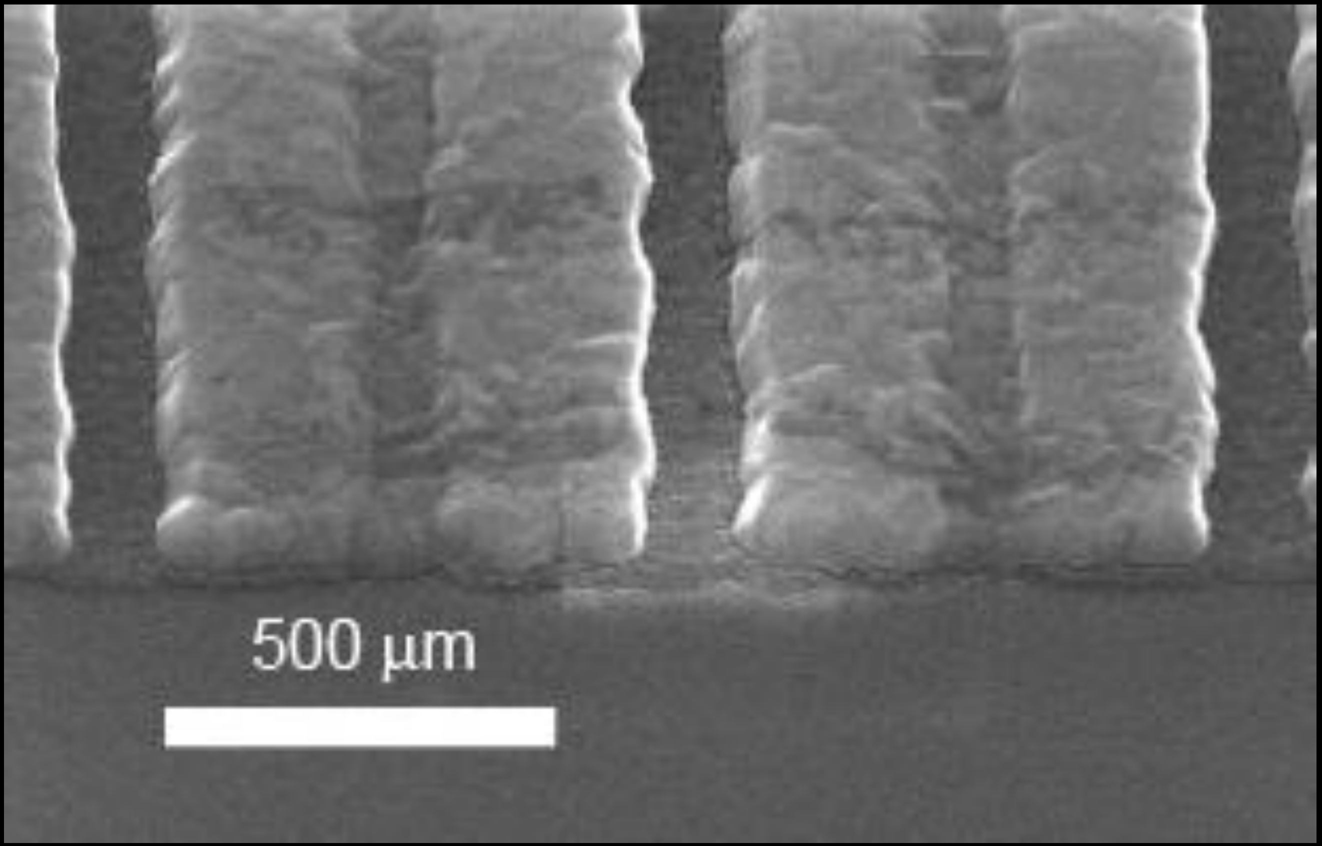
Evaporation 1
Evaporation 2

Dolan Bridge



Nicholas Masluk

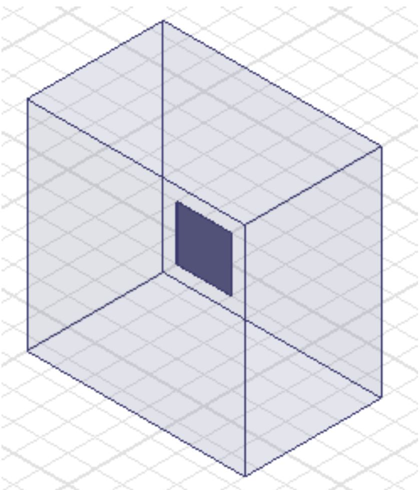
Reality of Junction Array



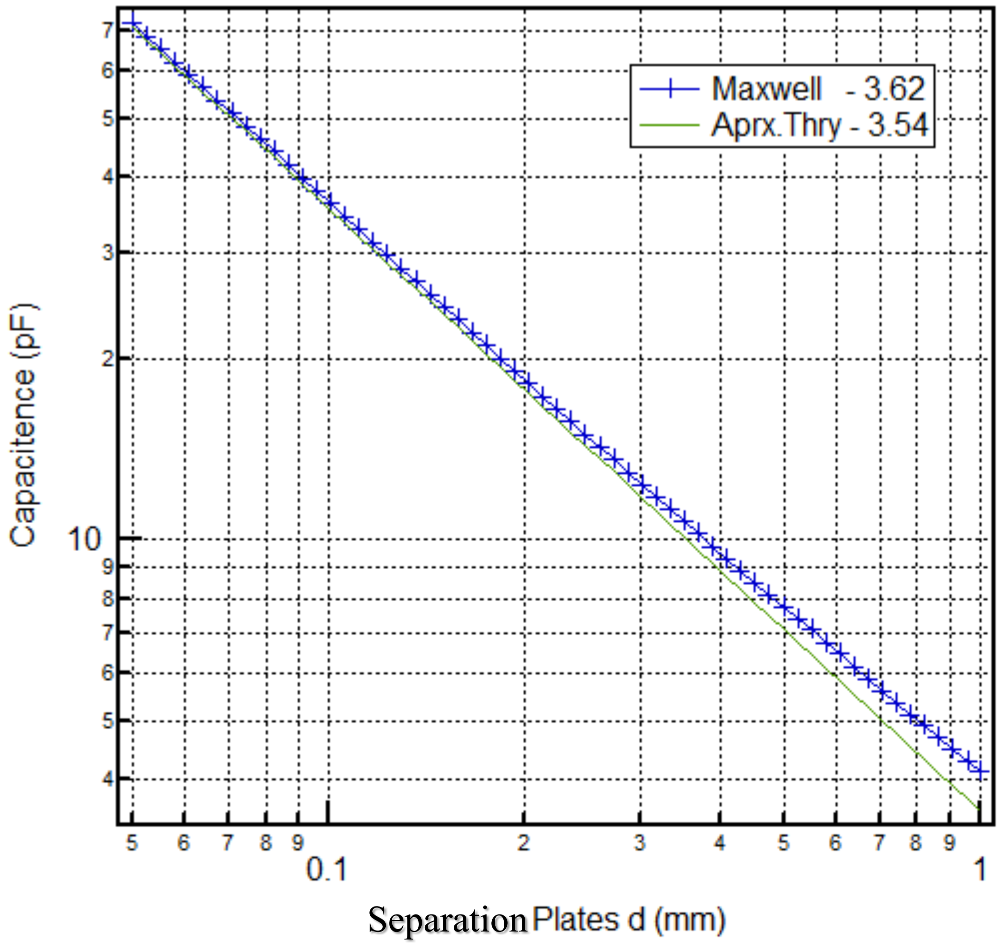
Simulation

Simulation v. Theory

Parallel Plate Capacitor

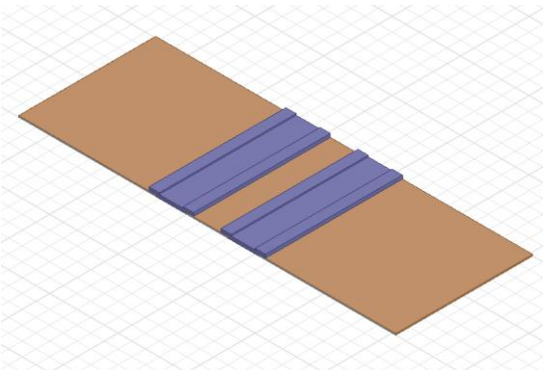


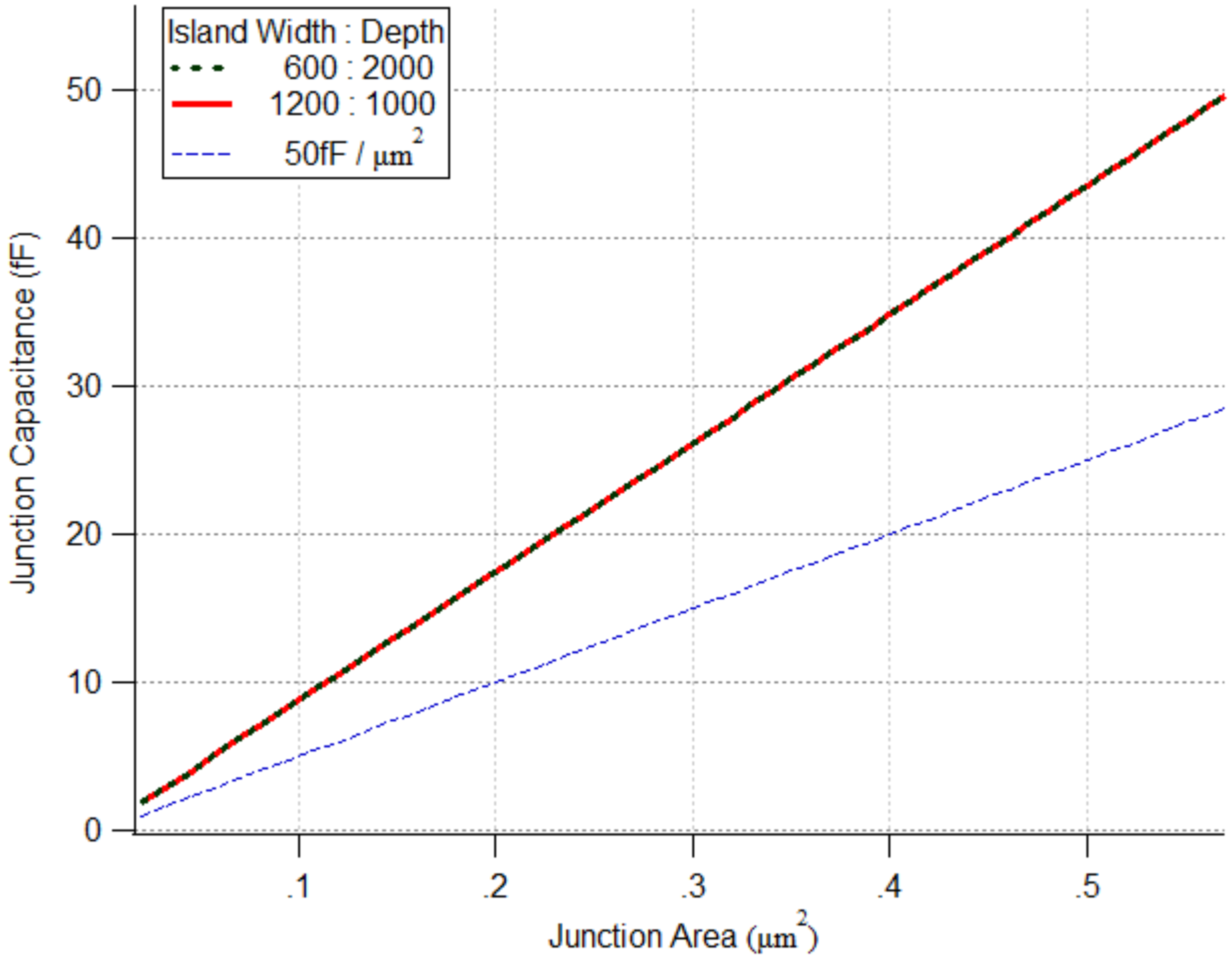
Area = 20 x 20 mm



Simulation v. Empirical Data

Junction Capacitance v. Junction Area



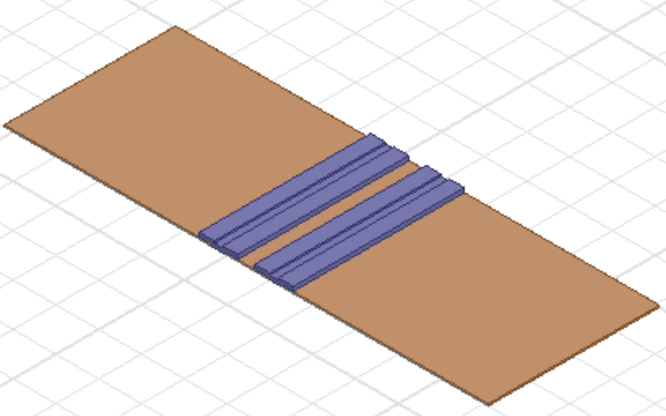


Simulated junction capacitance per junction area:

~ 86 fF / (μm)^2

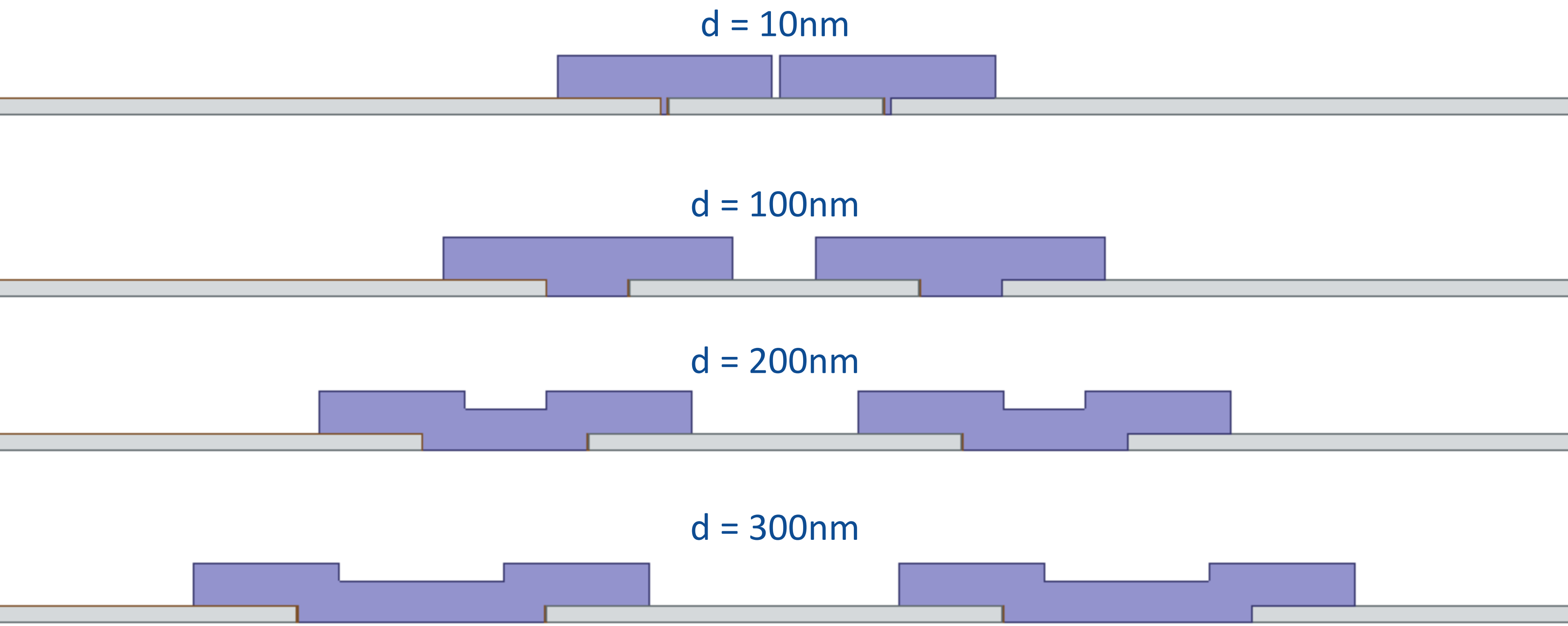
Empirically:

~ 50 fF / (μm)^2



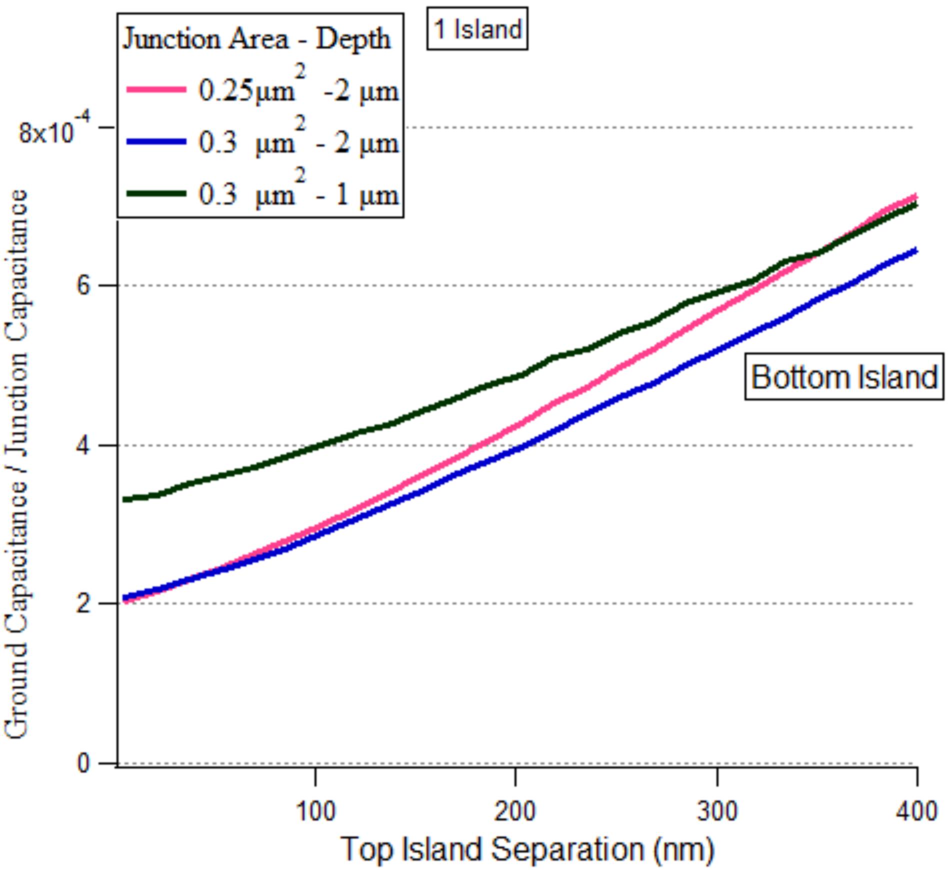
Dolan Bridge Geometry

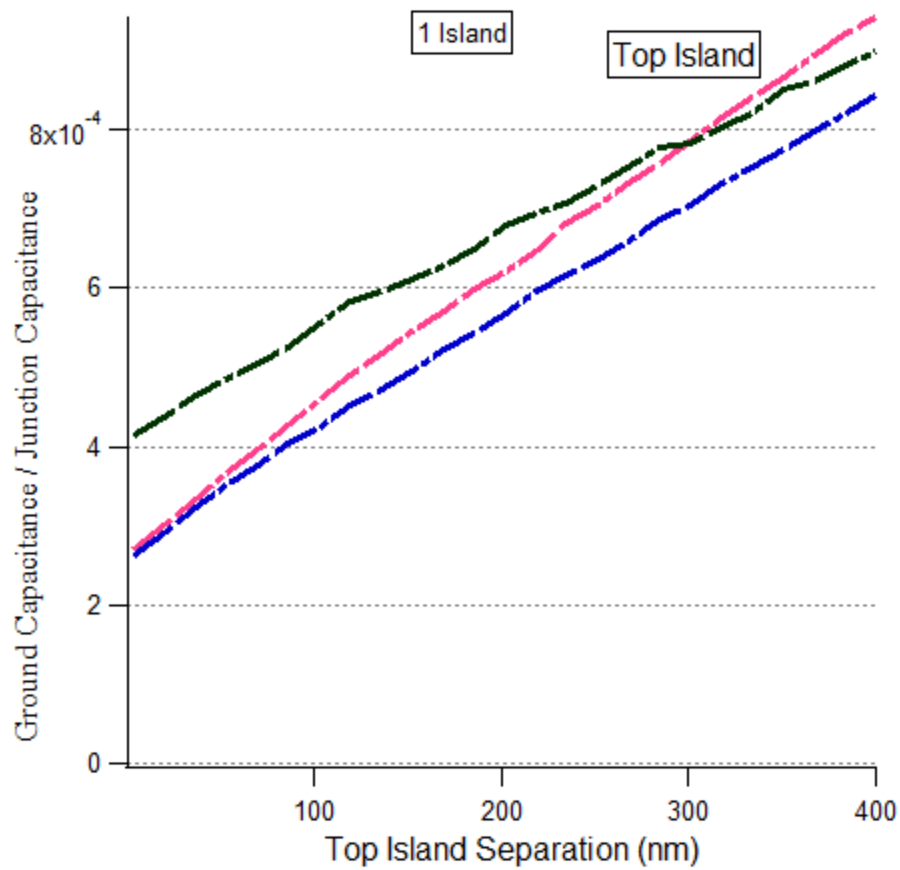
Vary Island Separation by Varying Island Width



Dolan Bridge Geometry

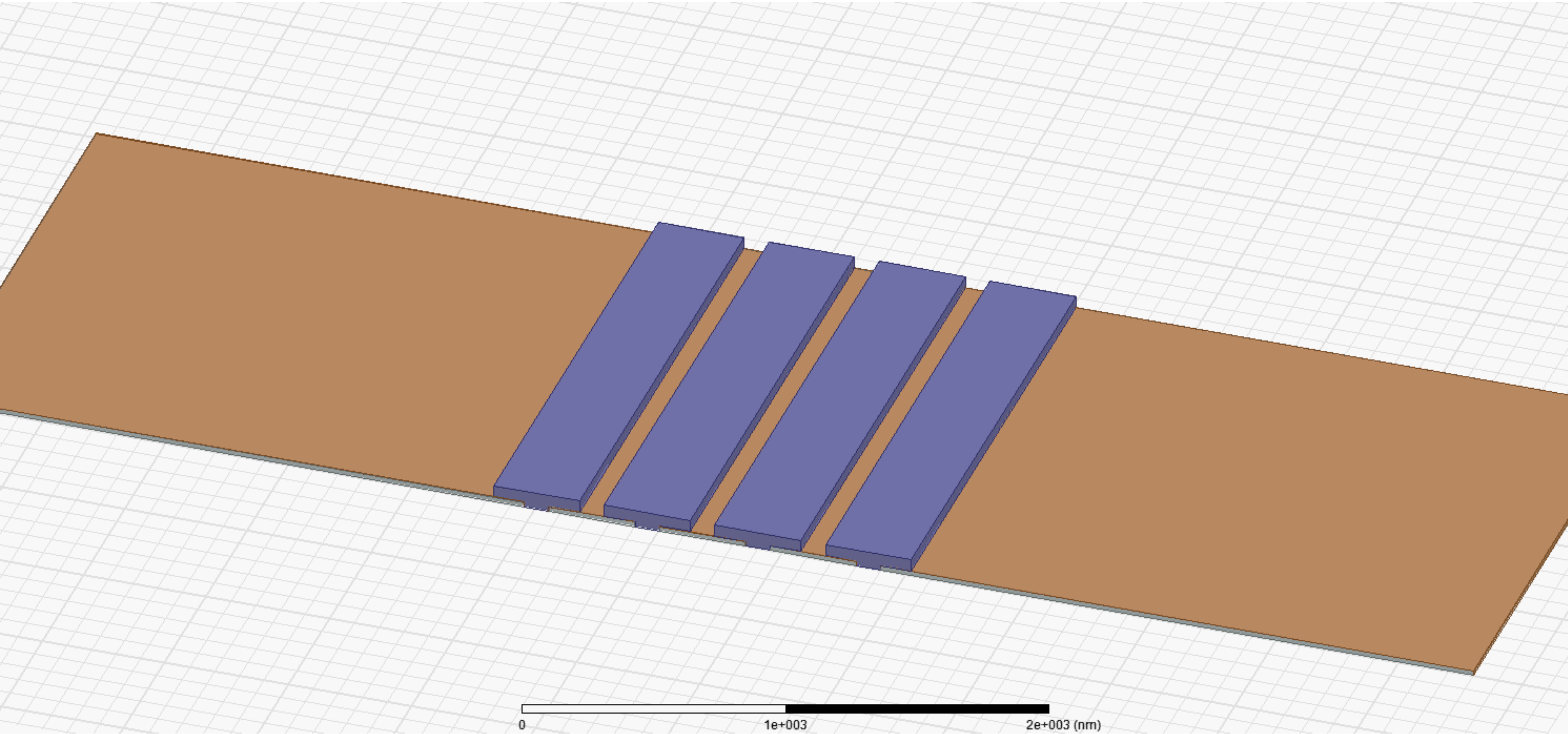
Vary Island Separation by Varying Island Width



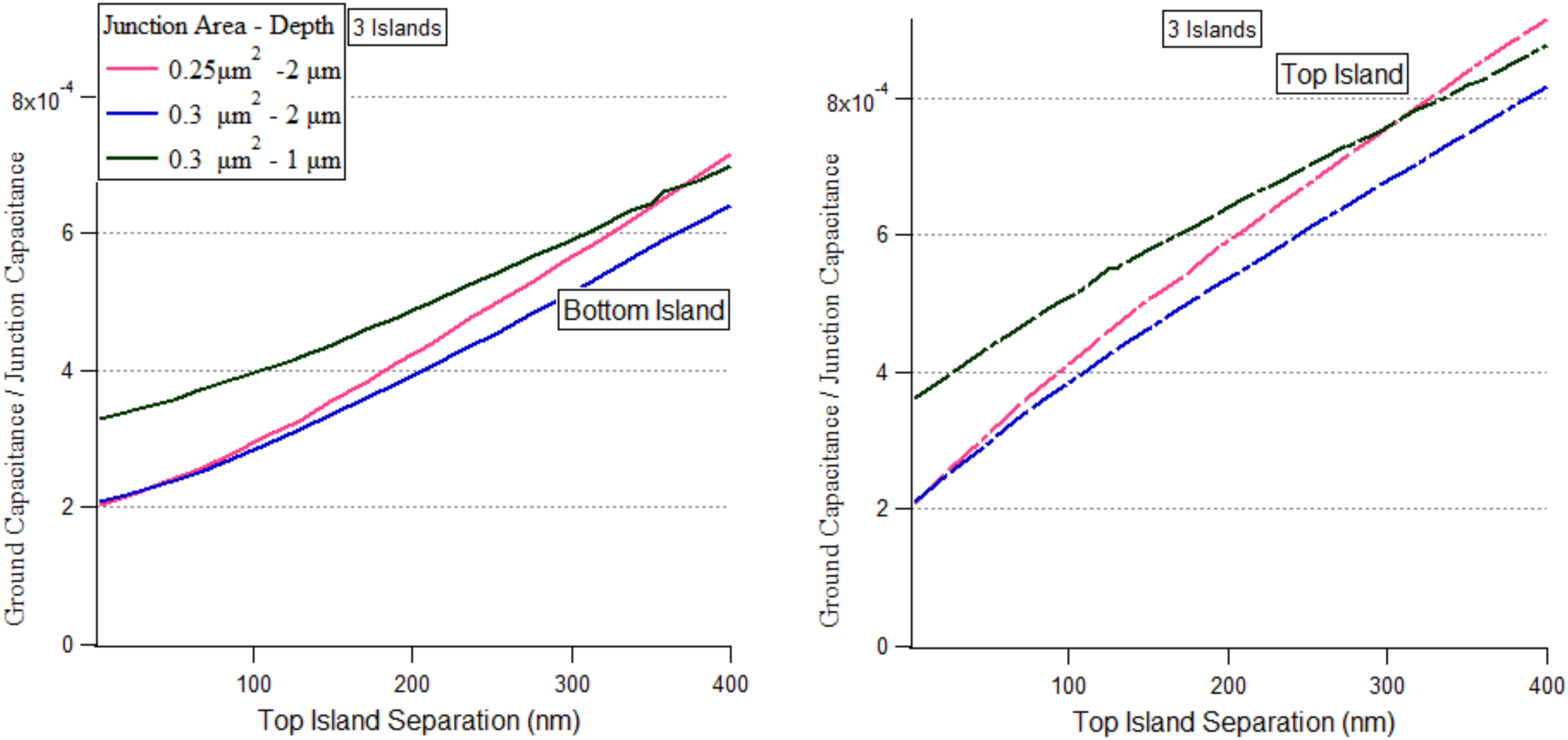


Robustness of Results

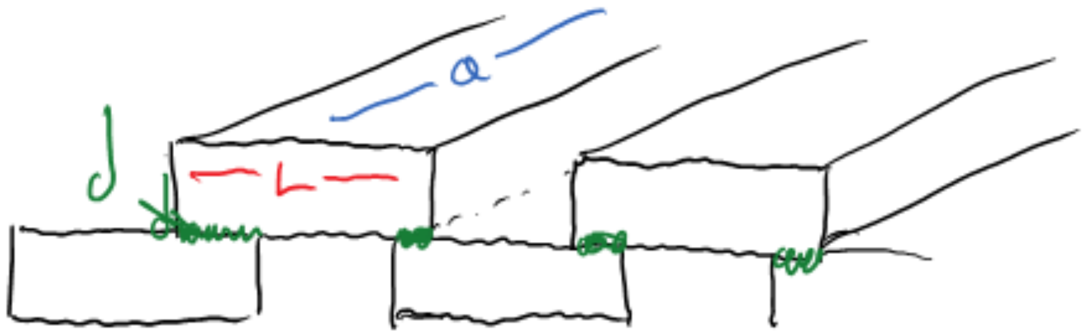
3 v. 1 Middle Islands

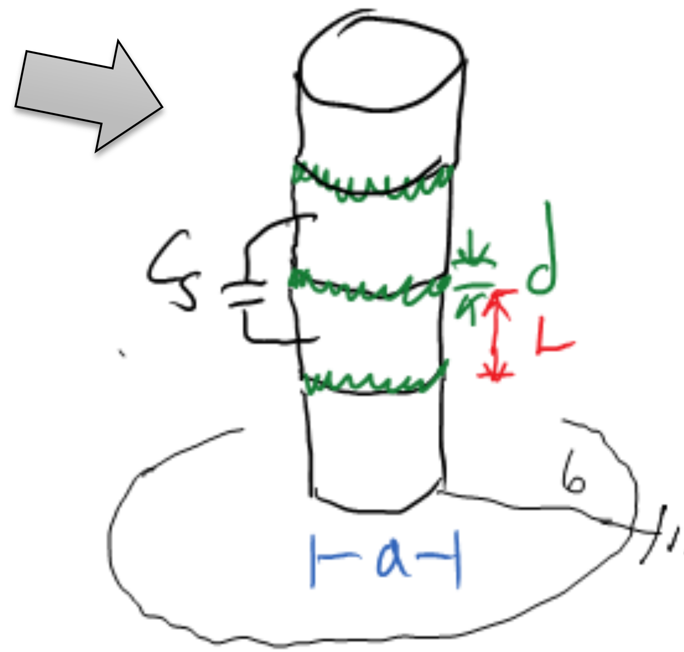


Dolan Bridge Geometry



Model

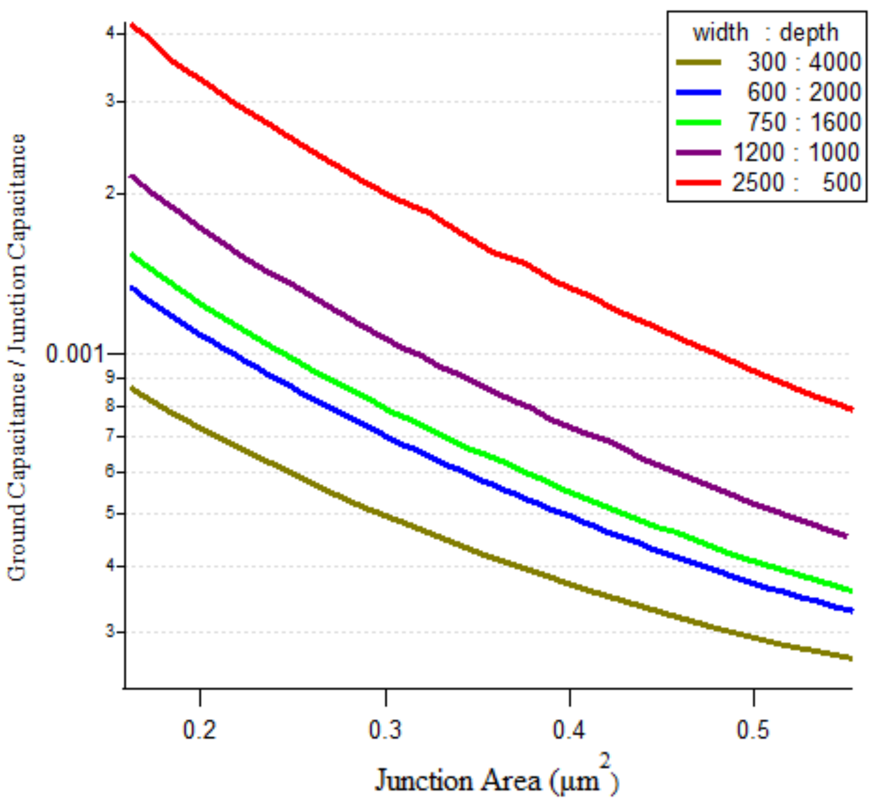




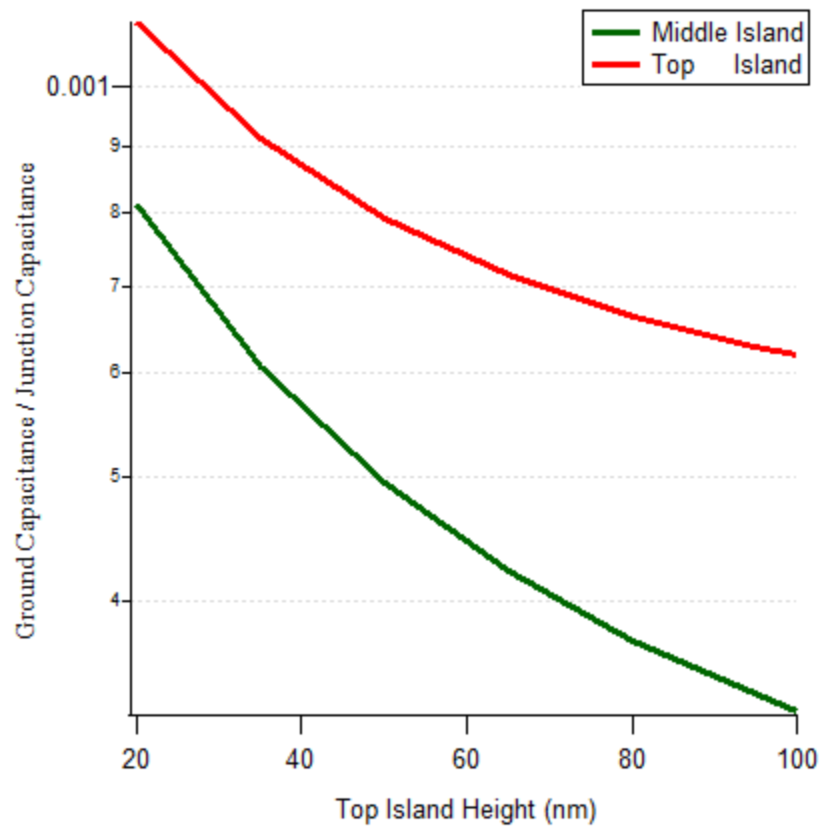
$$\frac{C_g}{C_{ss}} \sim \# L + \#$$

Other Geometrical Variations

Vary Junction Area, Fix Island Size



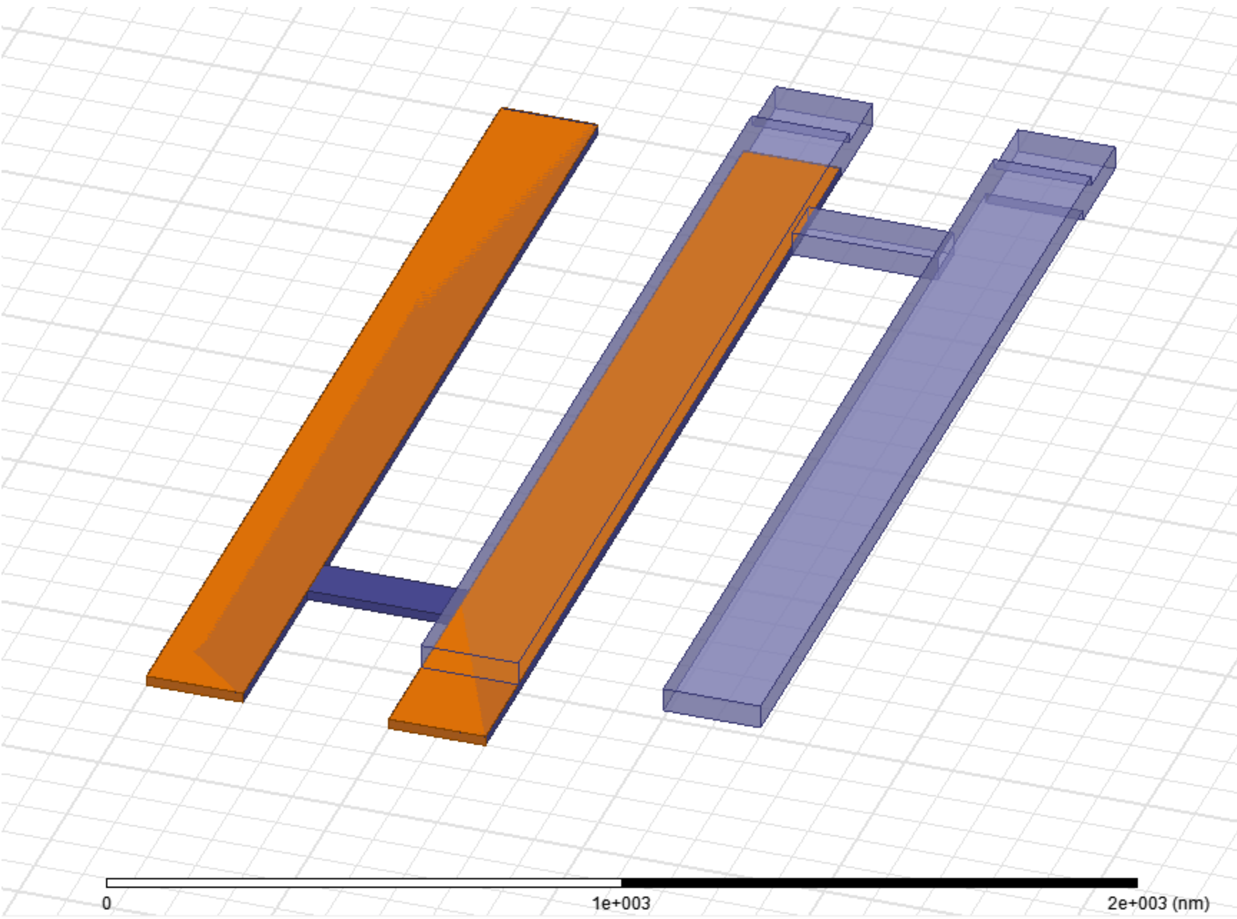
Vary Top Island Height

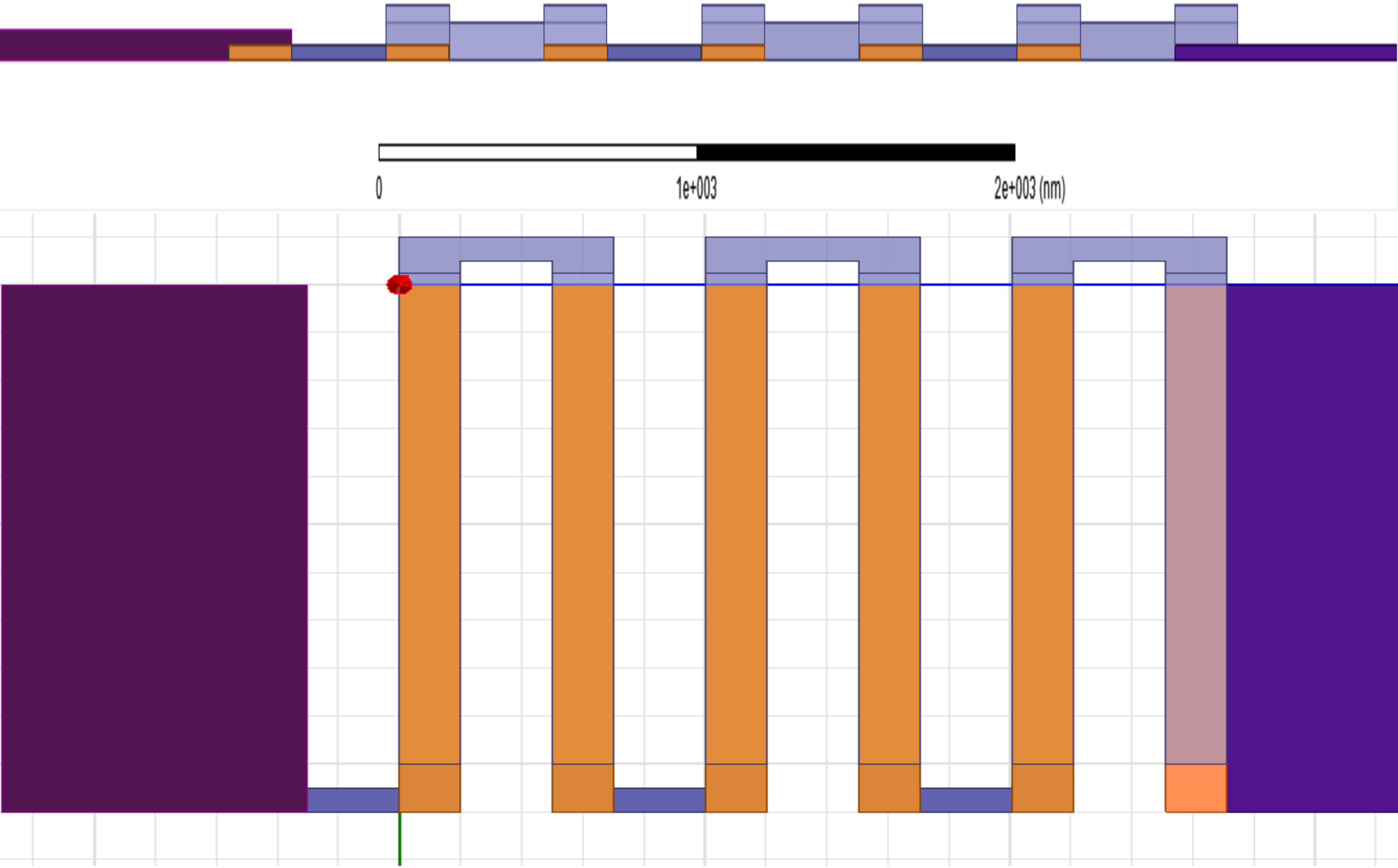


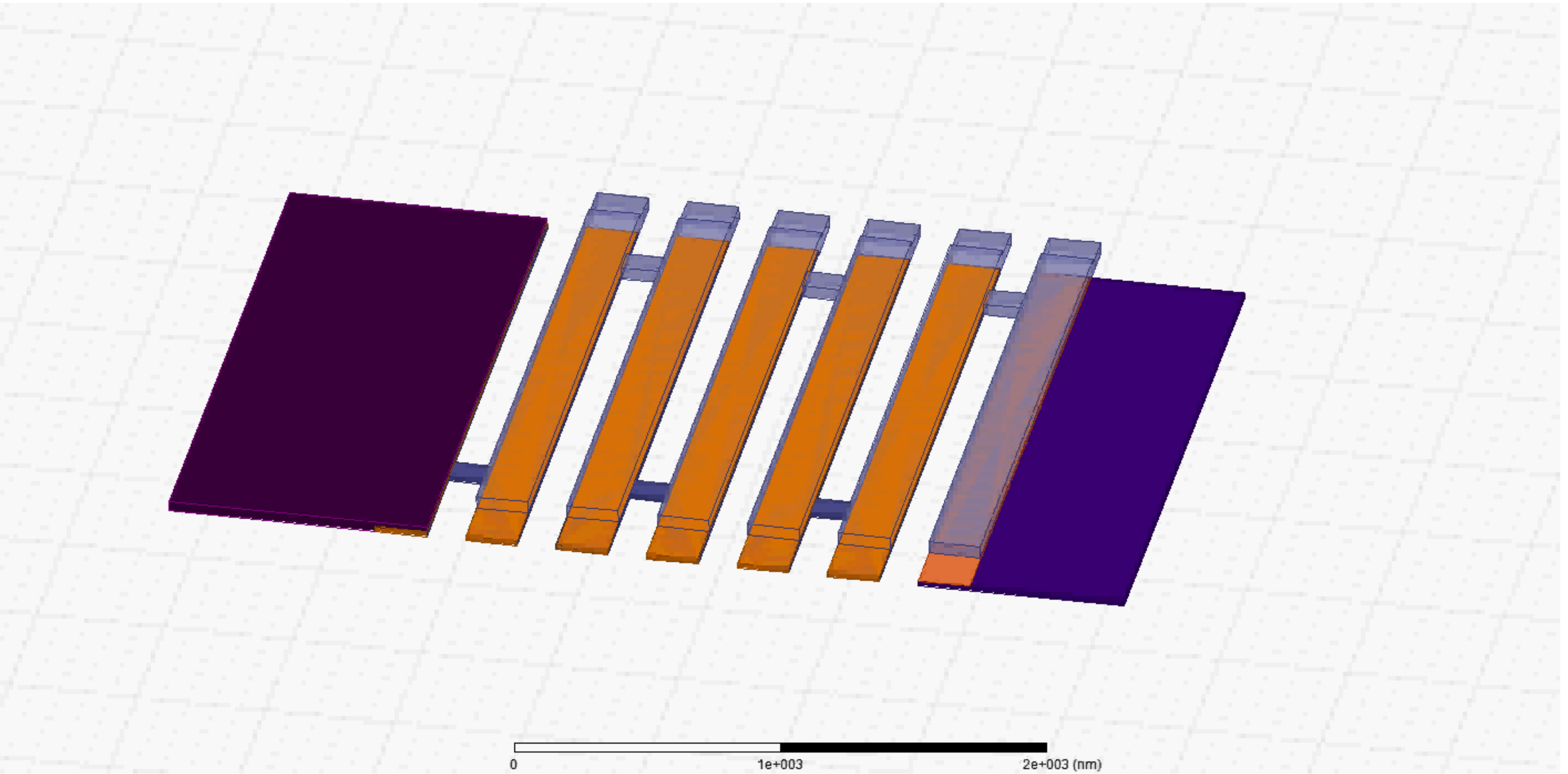
Dolan Bridge Conclusions

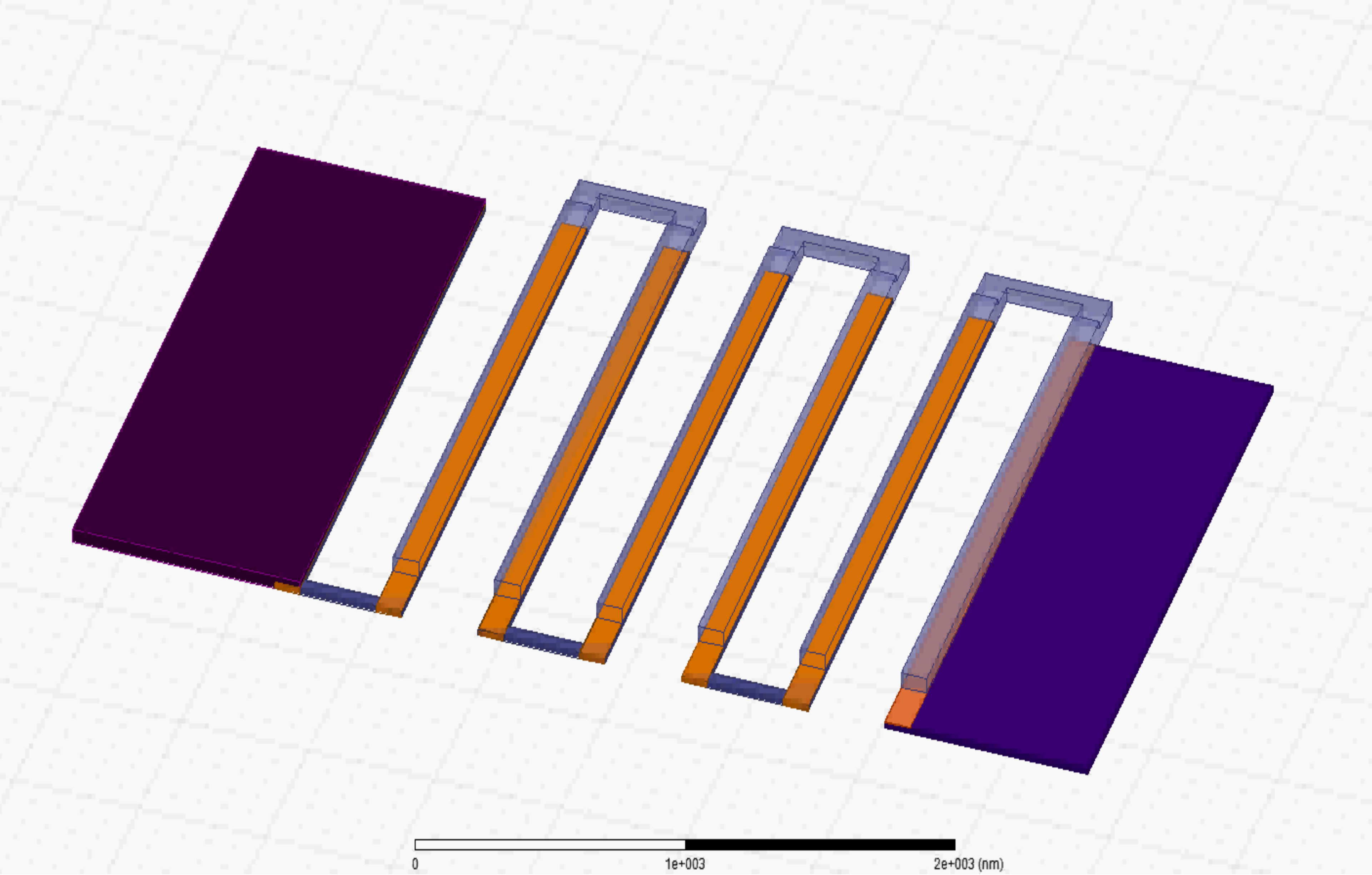
- To minimize parasitic capacitance to ground in Josephson junction arrays use:
  - Deeper junction (more rectangular, less square)
  - Larger junctions
  - Thicker second layer
  - There is a lower bound on the parasitic capacitance
  - Capacitance to ground varies linearly with island separation

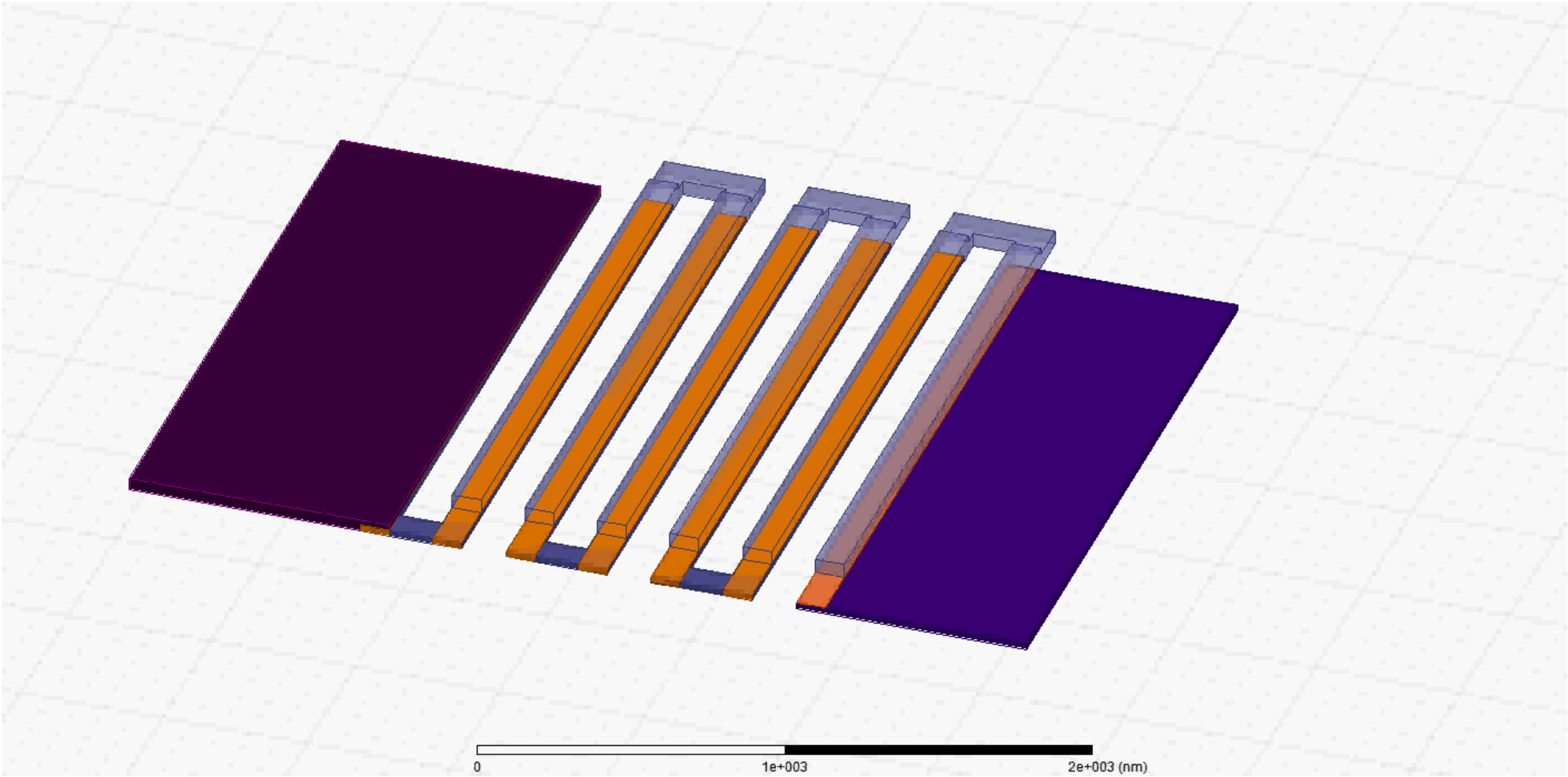
Bridge Free


0
1e+003
2e+003 (nm)

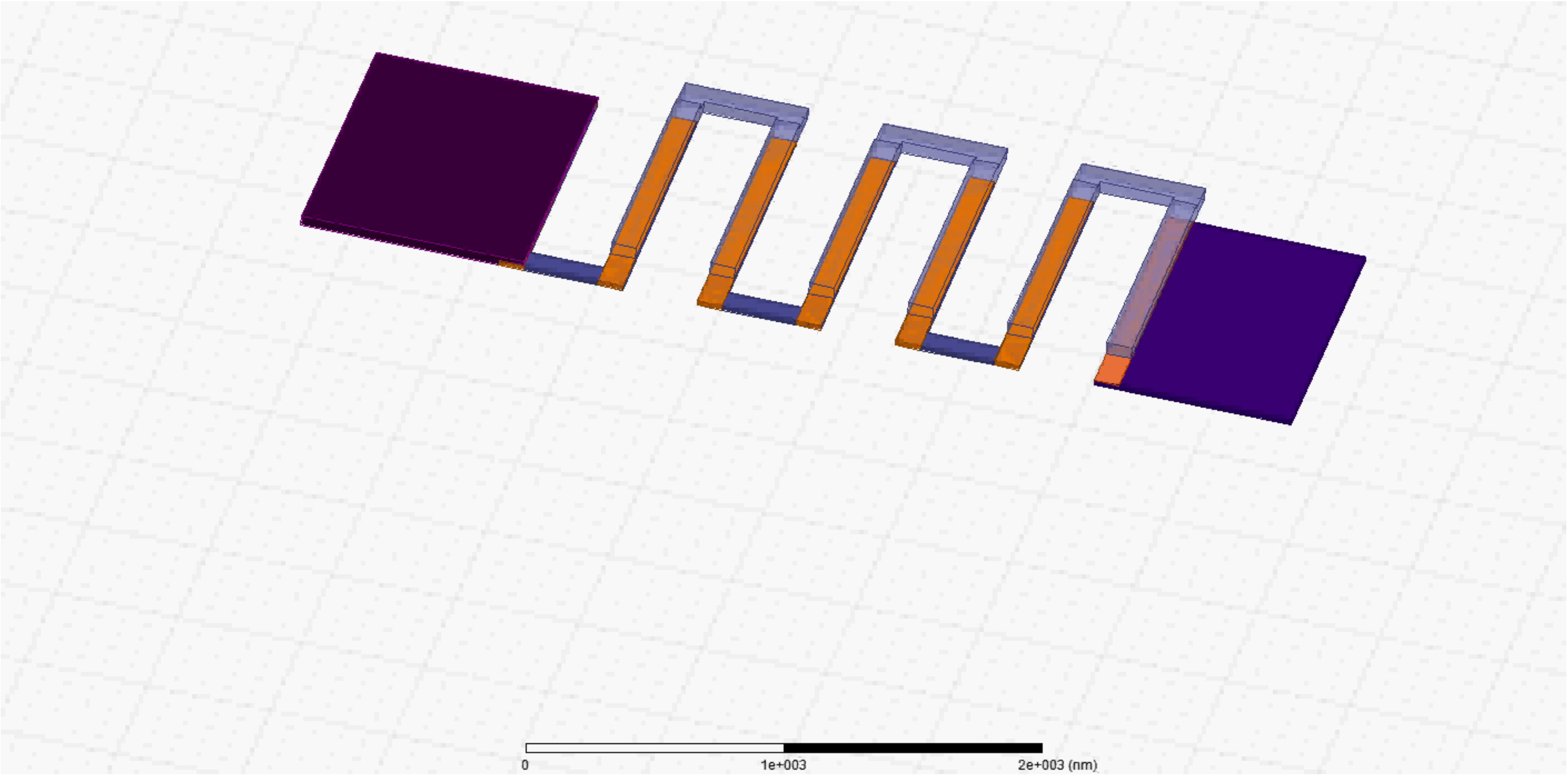

0
1e+003
2e+003 (nm)


0
1e+003
2e+003 (nm)


0
1e+003
2e+003 (nm)



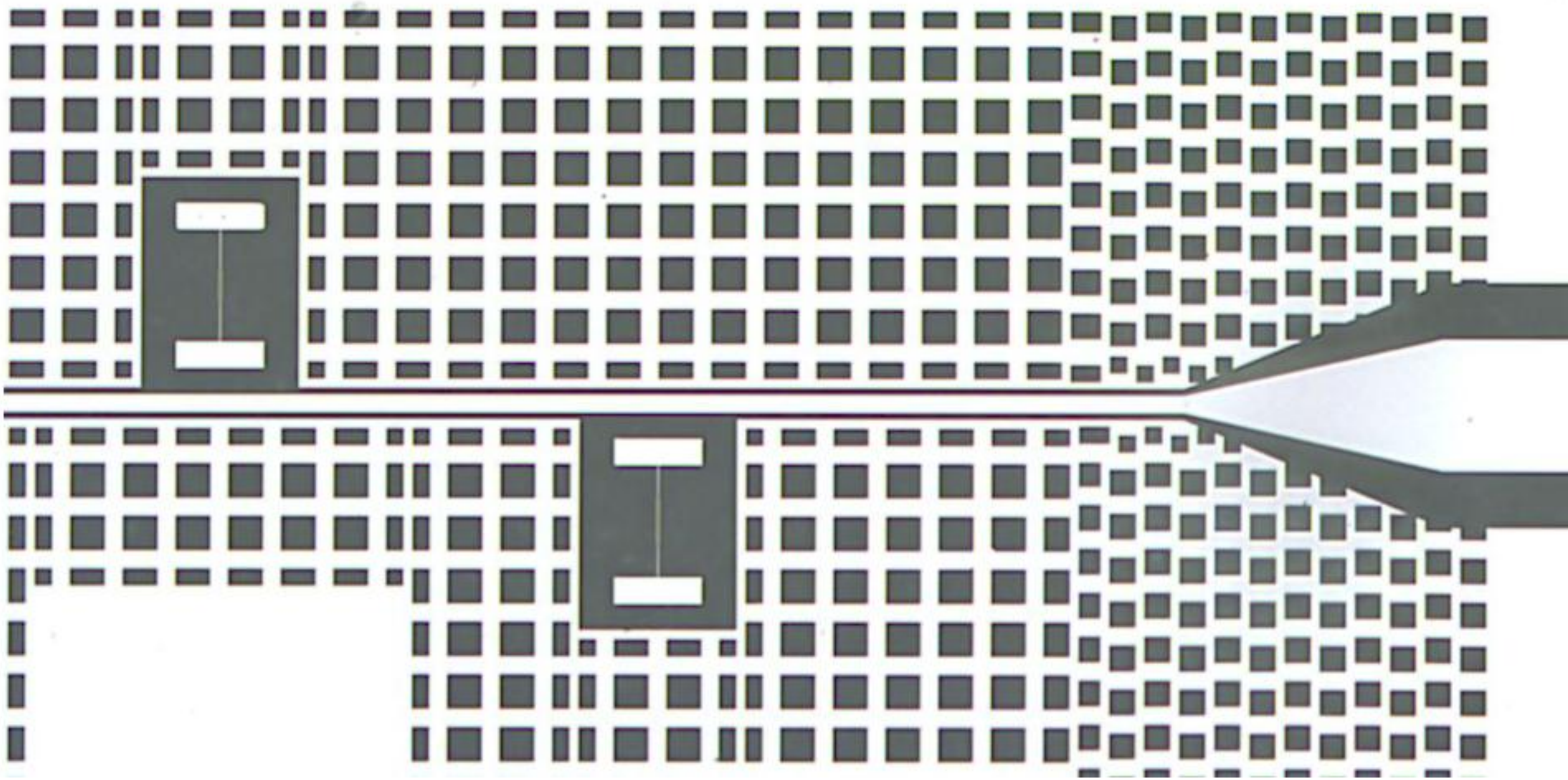
0
1e+003
2e+003 (nm)


0
1e+003
2e+003 (nm)

Maximum Number of Junctions for Junction Width of 100nm
jw100,vary jD
Curve Info
3*Sqrt(Matrix1.C(TI2,BI2)/Matrix1.C(Ground,BI2))
Standard : LastAdaptive
Maximum Number of Junctions
170.00
160.00
150.00
140.00
130.00
120.00
110.00
2.00
3.00
4.00
5.00
6.00
7.00
8.00
9.00
10.00
Junction Length [um]

Junction Array Q

Junction Array Loss



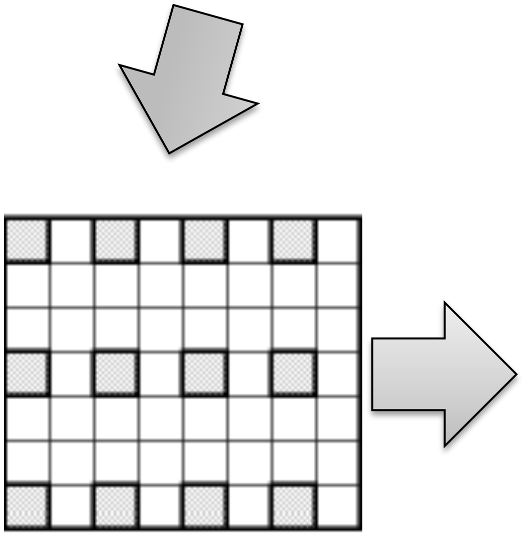
		W													
		Resonator C			*Top Ground*			*Bottom C-Pin*			*Bottom Grnd*				
S	L	10	20	30	10	20	30	10	20	30	10	20	30	L	S
60	10	0.05	0.11	0.21	2.21	3.21	3.98	0.75	0.86	0.95	3.01	3.67	4.32	10	60
	30	0.14	0.30	0.53	4.08	5.40	6.51	1.11	1.30	1.43	4.46	5.58	6.47	30	
	50	0.26	0.49	0.88	5.68	7.31	8.58	1.49	1.64	1.82	5.99	7.04	8.28	50	
	70	0.39	0.74	1.25	7.26	9.13	10.56	1.78	2.01	2.17	7.30	8.78	10.02	70	
	90	0.52	0.94	1.59	8.73	10.85	12.42	2.05	2.25	2.43	8.62	10.06	11.52	90	
	110	0.64	1.18	1.94	10.28	12.69	14.45	2.27	2.54	2.72	9.83	11.80	13.35	110	
	120	0.69	1.28	2.15	11.01	13.66	15.63	2.34	2.63	2.85	10.33	12.55	14.50	120	
50	10	0.04	0.07	0.14	2.25	3.23	4.08	0.83	0.92	1.03	2.82	3.31	4.03	10	50
	30	0.11	0.20	0.37	4.13	5.57	6.73	1.34	1.46	1.64	4.50	5.25	6.36	30	
	50	0.18	0.35	0.60	5.85	7.50	8.87	1.70	1.88	2.06	5.69	6.81	7.99	50	
	70	0.28	0.50	0.86	7.44	9.38	10.97	2.10	2.25	2.48	7.16	8.19	9.71	70	
	90	0.37	0.67	1.12	8.94	11.24	13.03	2.43	2.65	2.89	8.46	9.96	11.50	90	
	110	0.46	0.85	1.37	10.56	13.23	15.12	2.67	3.03	3.21	9.58	11.77	13.31	110	
	120	0.49	0.90	1.48	11.31	14.10	16.17	2.75	3.09	3.35	10.03	12.27	14.22	120	
40	10	0.02	0.06	0.10	2.26	3.28	4.18	0.77	1.10	1.19	2.04	3.37	3.93	10	40
	30	0.06	0.14	0.25	4.25	5.73	6.89	1.36	1.71	1.86	3.64	5.08	6.03	30	
	50	0.12	0.25	0.42	5.99	7.74	9.20	1.87	2.25	2.41	5.00	6.76	7.79	50	
	70	0.18	0.36	0.60	7.58	9.68	11.33	2.32	2.68	2.90	6.31	8.06	9.38	70	
	90	0.26	0.48	0.80	9.30	11.62	13.50	2.90	3.13	3.40	8.06	9.55	11.23	90	
	110	0.33	0.59	0.97	10.90	13.61	15.69	3.22	3.55	3.79	9.27	11.20	12.93	110	
	120	0.36	0.65	1.07	11.74	14.62	16.85	3.38	3.72	4.02	9.93	11.99	14.06	120	
30	10	0.01	0.04	0.07	2.29	3.40	4.25	0.83	1.26	1.38	1.70	3.04	3.68	10	30
	30	0.04	0.10	0.17	4.32	5.87	7.06	1.53	2.01	2.18	3.13	4.76	5.64	30	
	50	0.08	0.17	0.30	6.15	8.01	9.47	2.21	2.69	2.88	4.56	6.34	7.47	50	
	70	0.12	0.26	0.43	7.84	10.03	11.80	2.73	3.28	3.52	5.71	7.84	9.18	70	
	90	0.17	0.35	0.57	9.63	12.13	14.13	3.34	3.86	4.13	7.17	9.36	10.91	90	
	110	0.23	0.43	0.70	11.31	14.09	16.46	3.87	4.32	4.65	8.58	10.75	12.68	110	
	120	0.25	0.47	0.77	12.18	15.25	17.69	4.08	4.59	4.95	9.21	11.62	13.79	120	
20	10	0.01	0.02	0.05	2.38	3.44	4.42	1.01	1.37	1.74	1.51	2.39	3.65	10	20
	30	0.03	0.06	0.13	4.52	6.07	7.37	1.91	2.36	2.75	2.85	4.11	5.59	30	
	50	0.05	0.12	0.22	6.45	8.40	9.95	2.70	3.29	3.67	4.05	5.80	7.41	50	
	70	0.08	0.17	0.31	8.29	10.59	12.46	3.45	4.06	4.47	5.25	7.18	8.99	70	

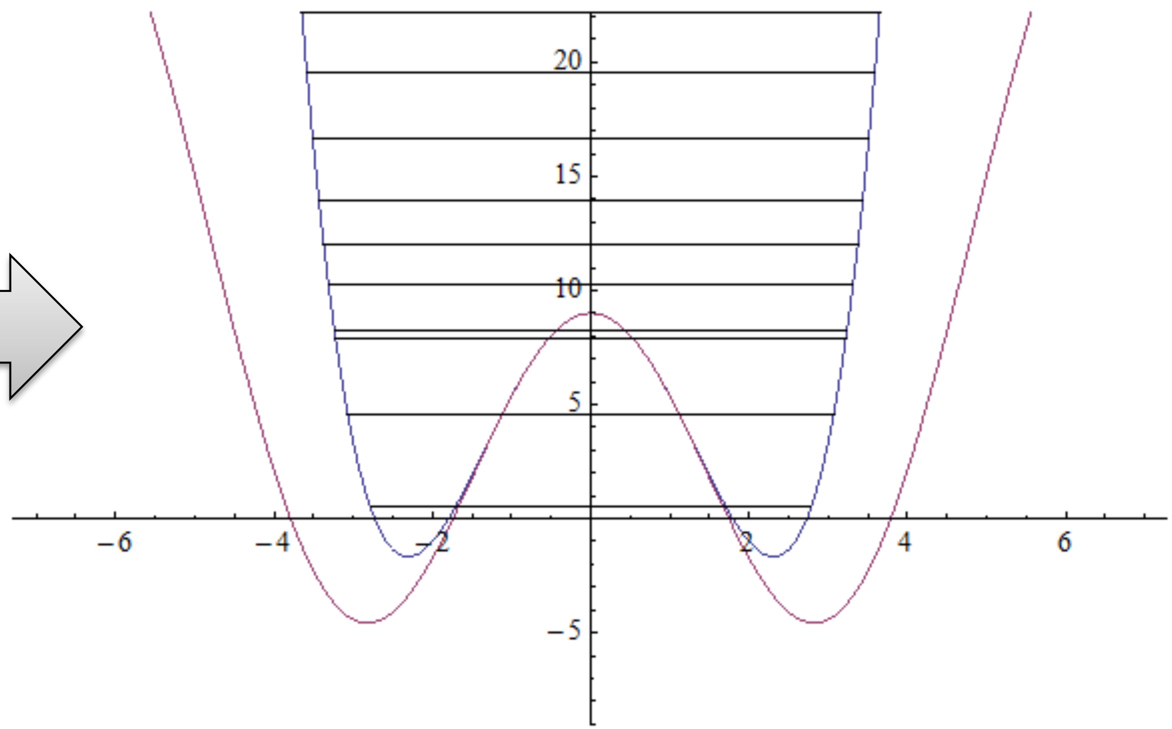
Fluxonium and Beyond

From Scratch = Faster Diagonalization

$$\tilde{H} = \tilde{E}_c\,\tilde{q}^2 + \tilde{E}_L\,\tilde{\phi}^2 - \tilde{E}_J\,\mathrm{Cos}[\tilde{\phi} - \tilde{\phi_e}]$$

Turn analytic problem into algebraic problem





Non-Commutative Algebra in Mathematica

```
(*break up cosine *)
cosList[N_] := ExpandXPower @ (List @@ (Cos[x] // Series[#, {x, 0, N}] & // Normal))
cos = cosList[5] /. x → (aL + aR) // ExpandNCM
  (*give me taylor expanded cos up to 5th order and then x replaced by aL +
   aR then fully power expanded by noncommutative algebra*)
```

$$\left\{1, \frac{1}{2}(-aL ** aL - aL ** aR - aR ** aL - aR ** aR), \frac{1}{24}(aL ** aL ** aL ** aL + aL ** aL ** aL ** aR + aL ** aL ** aR ** aL + aL ** aL ** aR ** aR + aL ** aR ** aL ** aL + aL ** aR ** aL ** aR + aL ** aR ** aR ** aL + aL ** aR ** aR ** aR + aR ** aL ** aL ** aL + aR ** aL ** aL ** aR + aR ** aL ** aR ** aL + aR ** aL ** aR ** aR + aR ** aR ** aL ** aL + aR ** aR ** aL ** aR + aR ** aR ** aR ** aL + aR ** aR ** aR ** aR)\right\}$$

```
CL = 5;
BL = 100;
ξ = Sqrt[1/2 (Ec/El)^(1/4)] /. {Ec → 2.5, El → 0.5};
ϕe = π;
T
cosList[N_] := ExpandXPower @ (List @@ (Cos[ξ x] // Series[#, {x, 0, N}] & // Normal))
cos0 = cosList[CL] /. x → (aL + aR);
cos1 = cos0 // ExpandNCM
(*COSelIJ[i_,j_] := Plus@@(bra[i]**#&/@    ((#**ket[j]&/@ cos1)//ExpandNCM)) *)
COSelIJ[i_, j_] := Evaluate[
   cos2 = (# ** ket[j] & /@ cos1) // ExpandNCM;
   cos2 = (bra[i] ** # & /@ cos2) // ExpandNCM;
   Plus @@ cos2
  ];
sinList[N_] := ExpandXPower @ (List @@ (Sin[ξ x] // Series[#, {x, 0, N}] & // Normal))
sin0 = sinList[CL] /. x → (aL + aR);
sin1 = sin0 // ExpandNCM;
SINelIJ[i_, j_] := Evaluate[
   sin2 = (# ** ket[j] & /@ sin1) // ExpandNCM;
   sin2 = (bra[i] ** # & /@ sin2) // ExpandNCM;
   Plus @@ sin2
  ];
(*can keep the numbers factored out might run faster *)

HoMtrx = Table[Cos[ϕe] COSelIJ[n, j] - Sin[ϕe] * SINelIJ[n, j], {n, 0, BL}, {j, 0, BL}];
HoMtrx // MatrixForm // TraditionalForm

evals = Eigenvalues[HoMtrx];
```

$$
\begin{matrix}
-\frac{1}{2\sqrt{2}} & 0 & \frac{1}{2\sqrt{6}} & 0 \\
0 & \frac{5}{2\sqrt{6}} - \sqrt{\frac{3}{2}} & 0 & \frac{\sqrt{\frac{5}{6}}}{2} \\
\frac{1}{8} & 0 & \frac{7}{2\sqrt{3}} - \sqrt{3} & 0 \\
0 & \frac{5}{8} & 0 & \frac{\sqrt{5}}{2} \\
\frac{7}{2\sqrt{3}} - \sqrt{3} & 0 & \frac{13}{8} & 0 \\
0 & \frac{\sqrt{5}}{2} & 0 & \frac{25}{8} \\
\frac{\sqrt{\frac{5}{2}}}{2} & 0 & \frac{11\sqrt{\frac{5}{6}}}{2} - \sqrt{\frac{15}{2}} & 0 \\
0 & \frac{\sqrt{\frac{35}{6}}}{2} & 0 & \frac{13\sqrt{\frac{7}{6}}}{2} - \sqrt{\frac{21}{2}} \\
0 & 0 & \frac{\sqrt{\frac{35}{3}}}{2} & 0 \\
0 & 0 & 0 & \frac{\sqrt{21}}{2} \\
0 & 0 & 0 & 0
\end{matrix}
$$

From Scratch = Faster Diagonalization

$$\tilde{H} = \tilde{E}_c\, \tilde{q}^2 + \tilde{E}_L\, \tilde{\phi}^2 - \tilde{E}_J\, \mathrm{Cos}[\tilde{\phi} - \tilde{\phi_e}]$$

- Turn analytic problem into algebraic problem