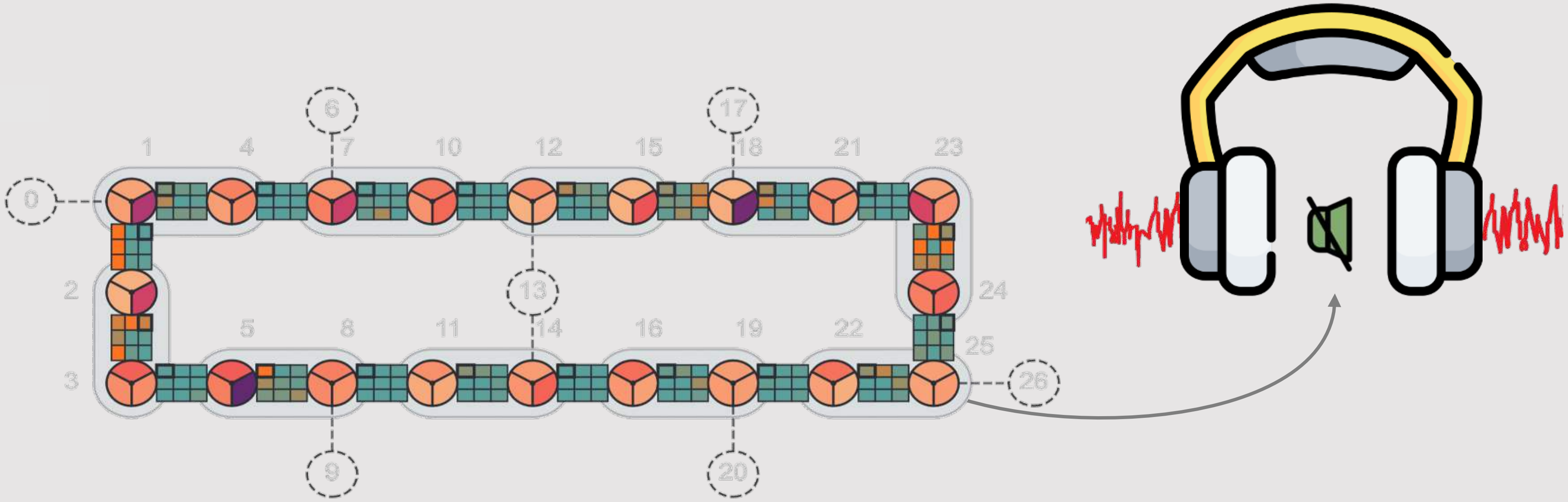
To learn and cancel quantum noise

IBM **Quantum**

Probabilistic error cancellation with sparse Pauli-Lindblad models on noisy quantum processors



Zlatko K. Minev

Ewout van den Berg, Zlatko K. Minev, Abhinav Kandala, **Kristan Temme**

arXiv:2201.09866 (2022)

Acknowledgements: broader IBM Quantum team

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Biggest challenge?

Reply in comment chat box

Biggest challenge?

Biggest challenge?

loss

hardware development

stability

decoherence

heat

algo development

Noise (Errors)

error correction overheads

high error rates

scalability

modularization

engineering

hype

need CS/EE talent

material quality

expectations

gravity

Biggest challenge

Noise

(Errors)

Cancel quantum noise

Icon: Flaticon

High-level message

Learn
accurate, efficient, scalable

Cancel
noise with noise, practical

Cost
more noise more cost

Icon: Flaticon

Outline

Idea

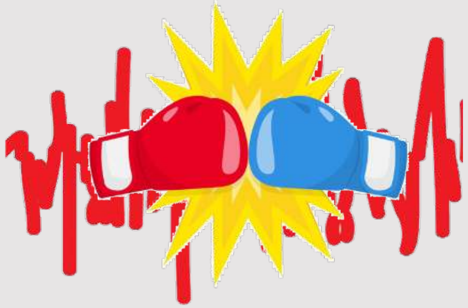
Probabilistic error cancelation (PEC) idea

Why not possible at scale until now?

Learn

Challenge

Sparse Pauli-Lindblad model in experiment



Cancel (realization)

Mitigation in practice with Ising

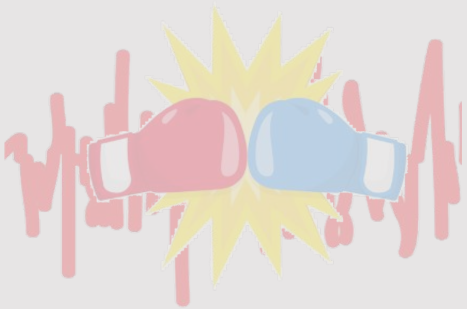
Cost

Icon: Flaticon

Outline

Idea

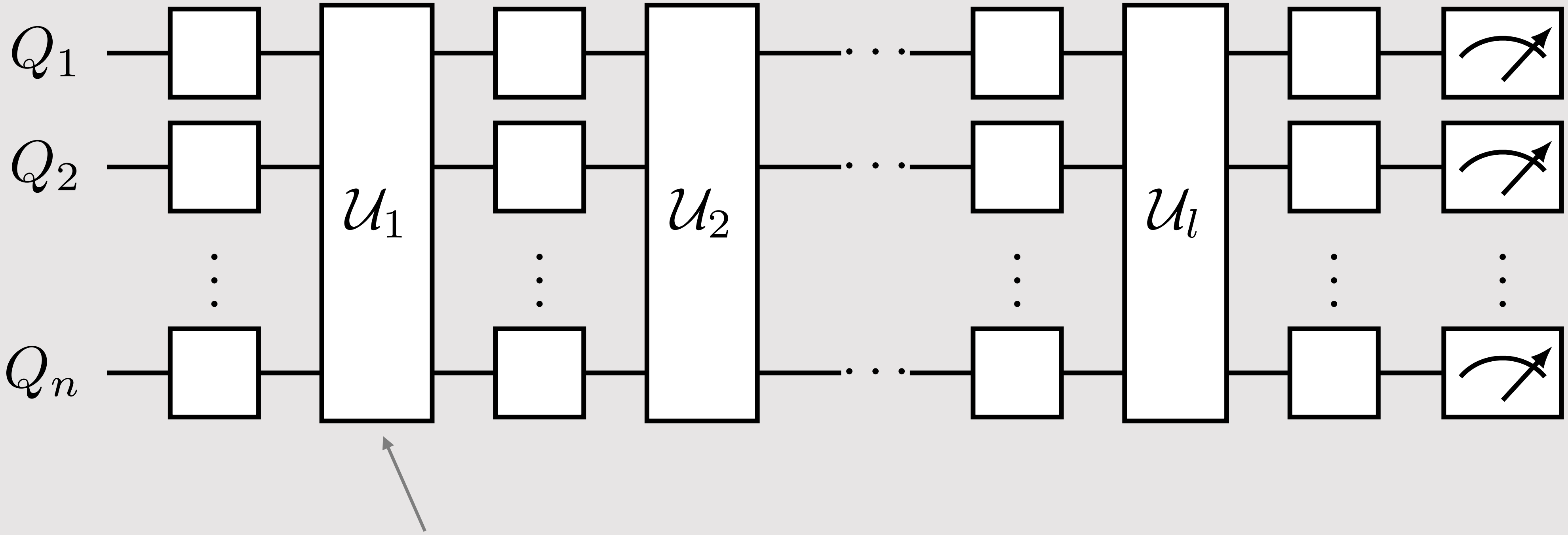
Learn



Cancel
(realization)

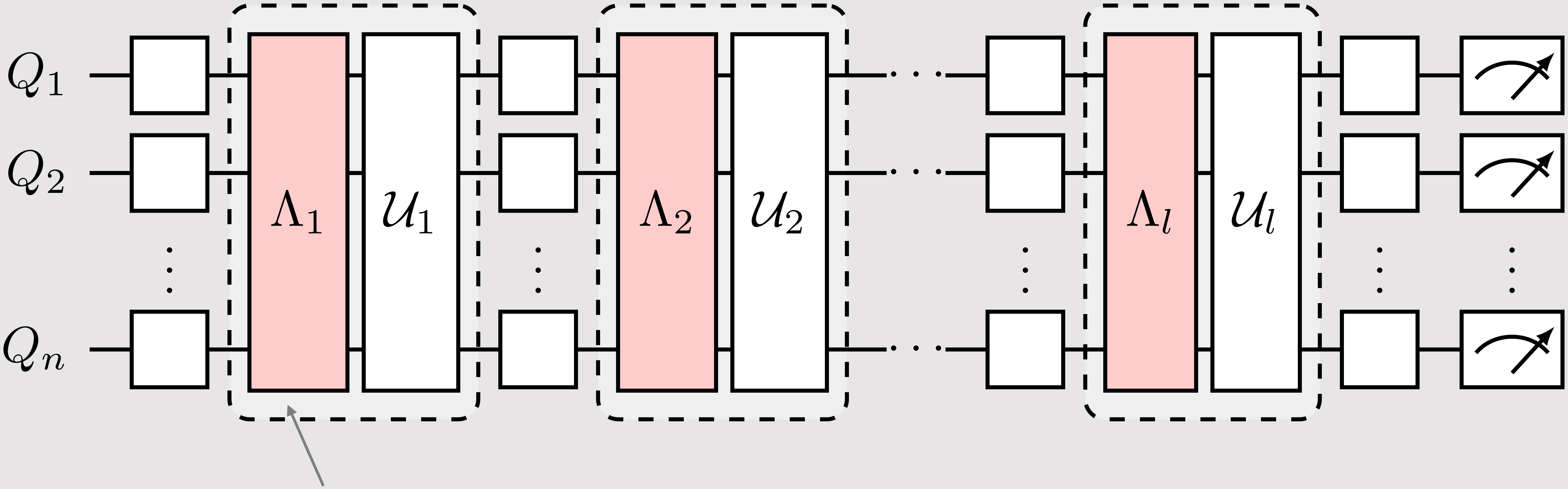
Icon: Flaticon

Ideal (noise-free) quantum circuit



A circuit can be decomposed into a layer construction
Example: Trotterization of Ising model simulation

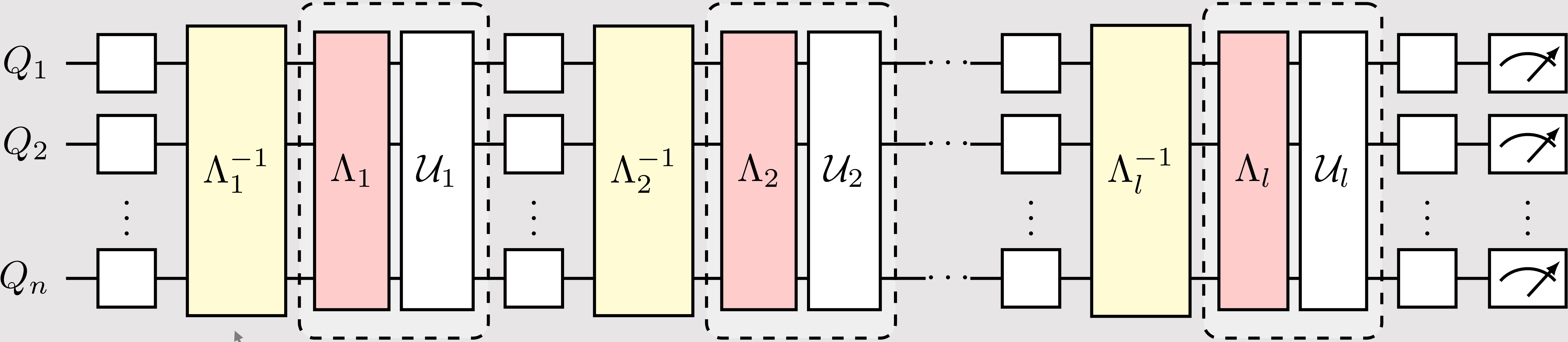
Real (noisy) quantum circuit



multi-qubit noise channel
inseparable from gate

completely positive and trace preserving (CPTP)
representable by a 4^n x 4^n matrix

Why not invert noise?

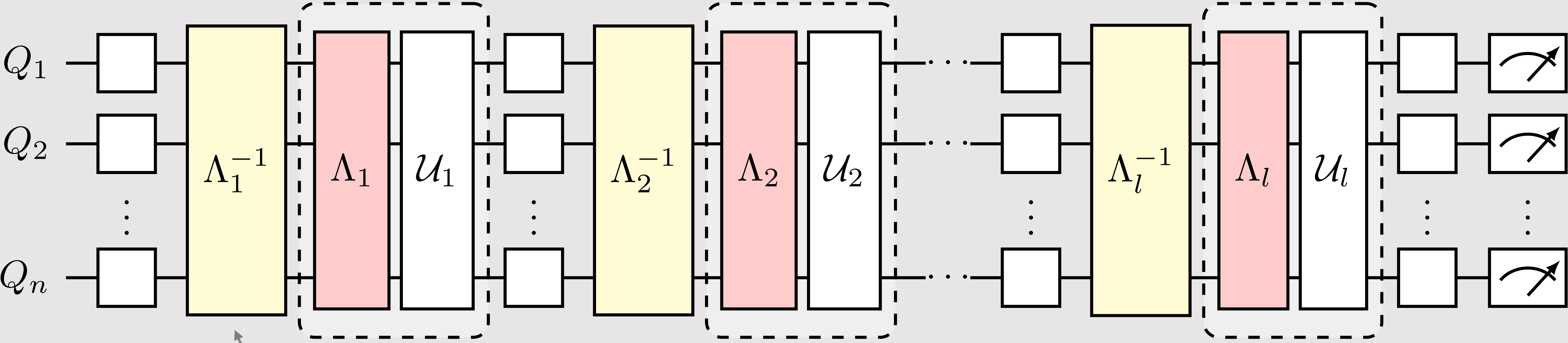


inverse operation of noise channel

Not possible?

unphysical
would need to know lost information due to noise
non CPTP map
has negative eigenvalues
…

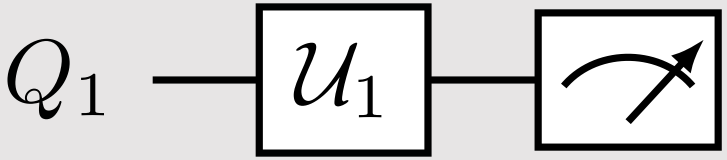
Probabilistic error cancelation

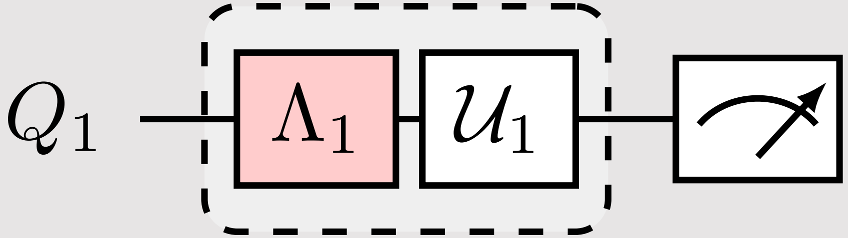


inverse operation of noise channel
implement on average

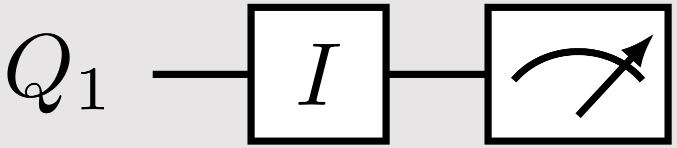
K. Temme, S. Bravyi, J. M. Gambetta,
Physical Review Letters 119, 180509 (2017)

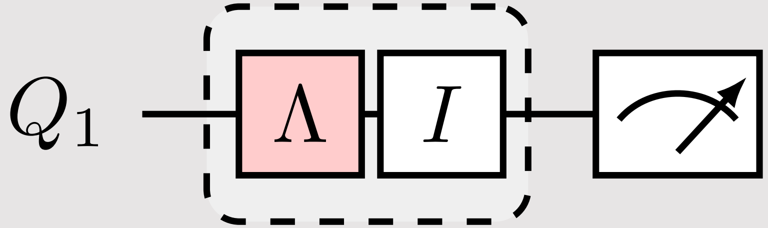
Toy model





Toy model: noise unraveling into quantum trajectories



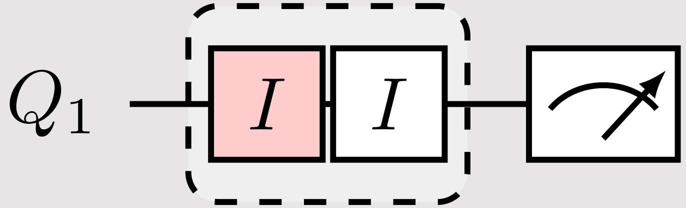


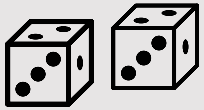
bit-flip noise

=

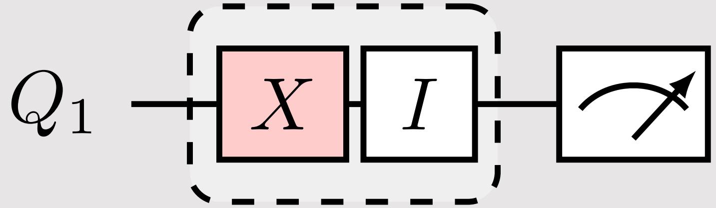
unraveling
(quantum trajectories)

probability 1-p



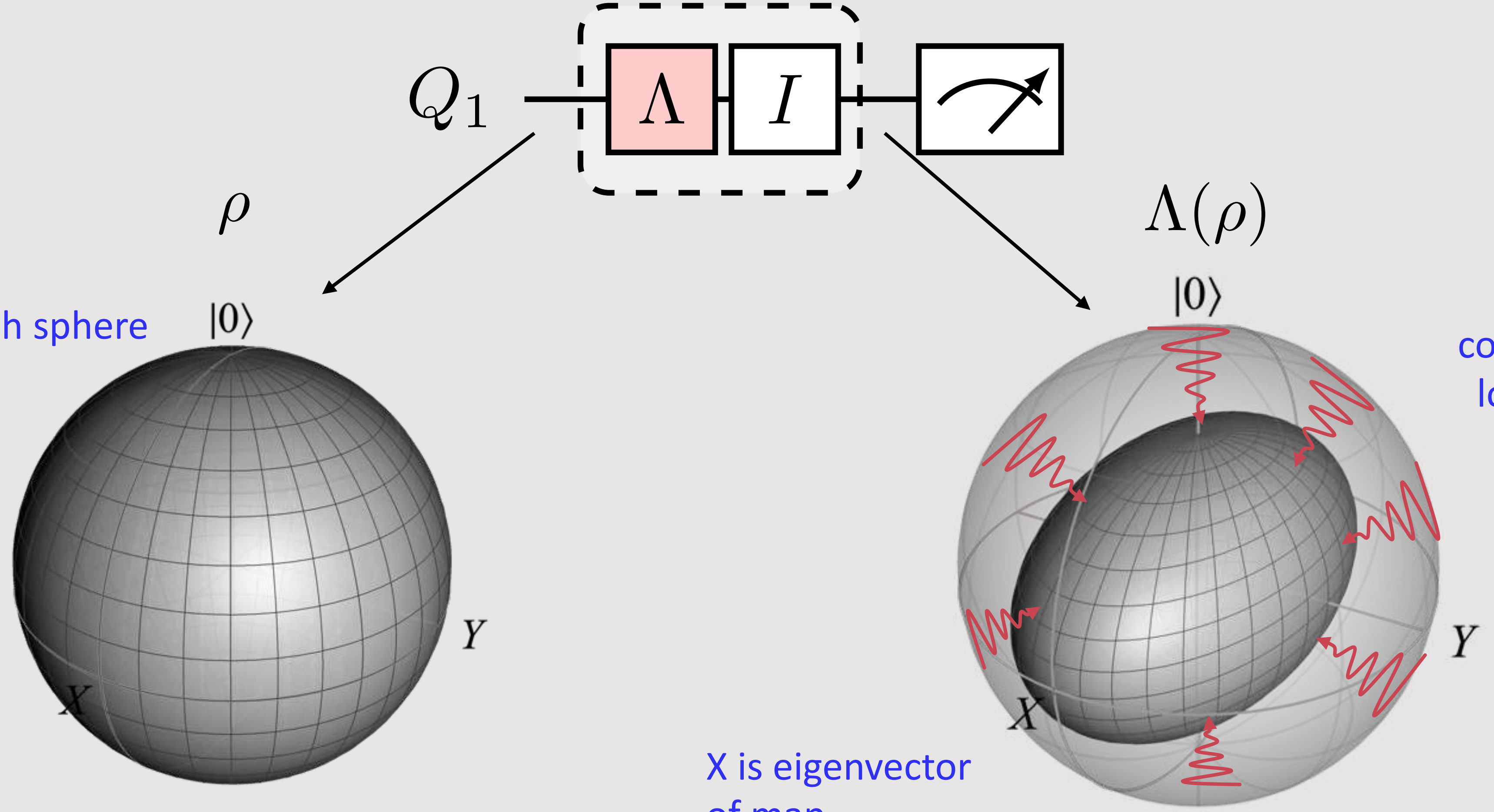


probability p

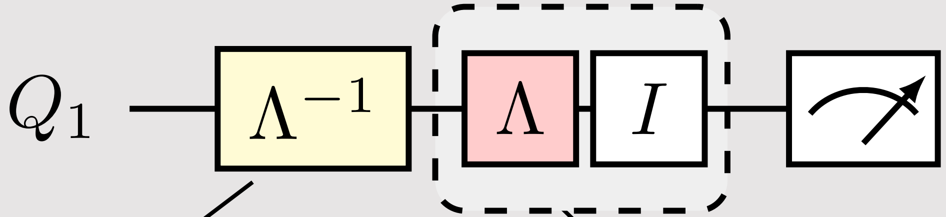


$$\Lambda\left(\rho\right) = (1-p)I\rho I + pX\rho X$$

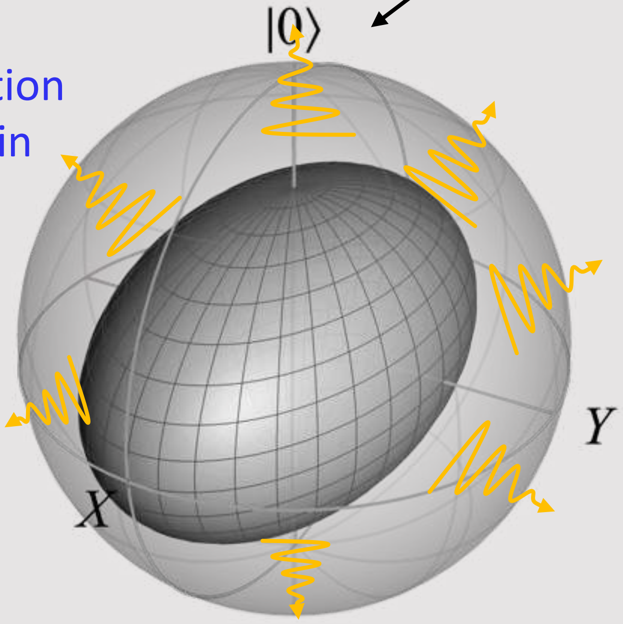
Toy model: noise unraveling into quantum trajectories



Inverse of noise map is not physical

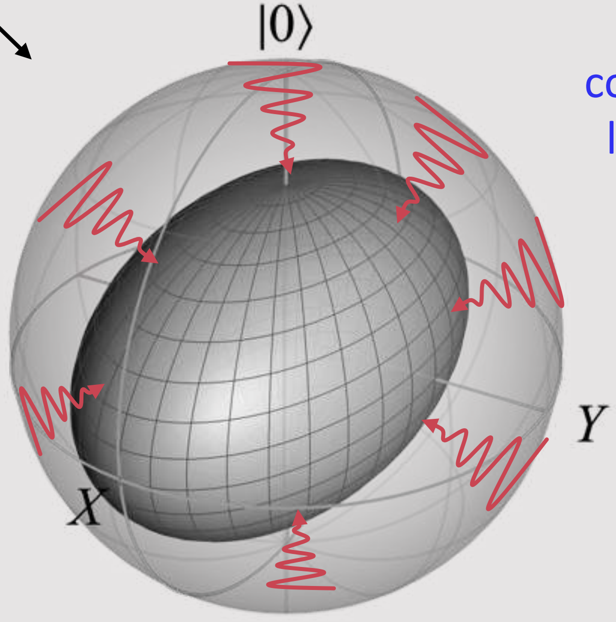


dilation
information gain
increase purity
eigenvalues < 0



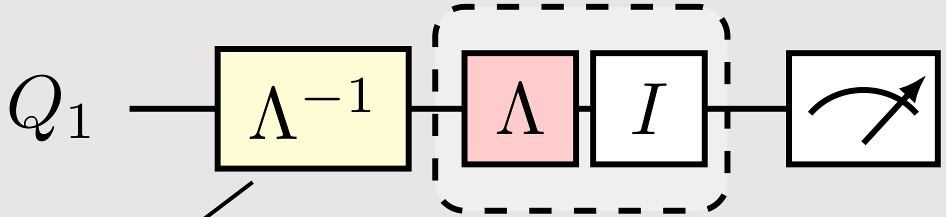
$$\Lambda^{-1}(\rho) = ?$$

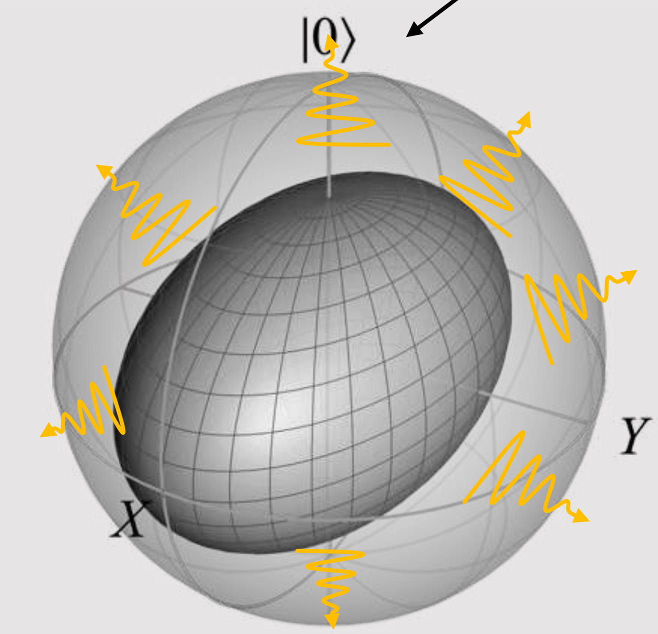
contraction
loss of information
reduced purity
eigenvalues > 0



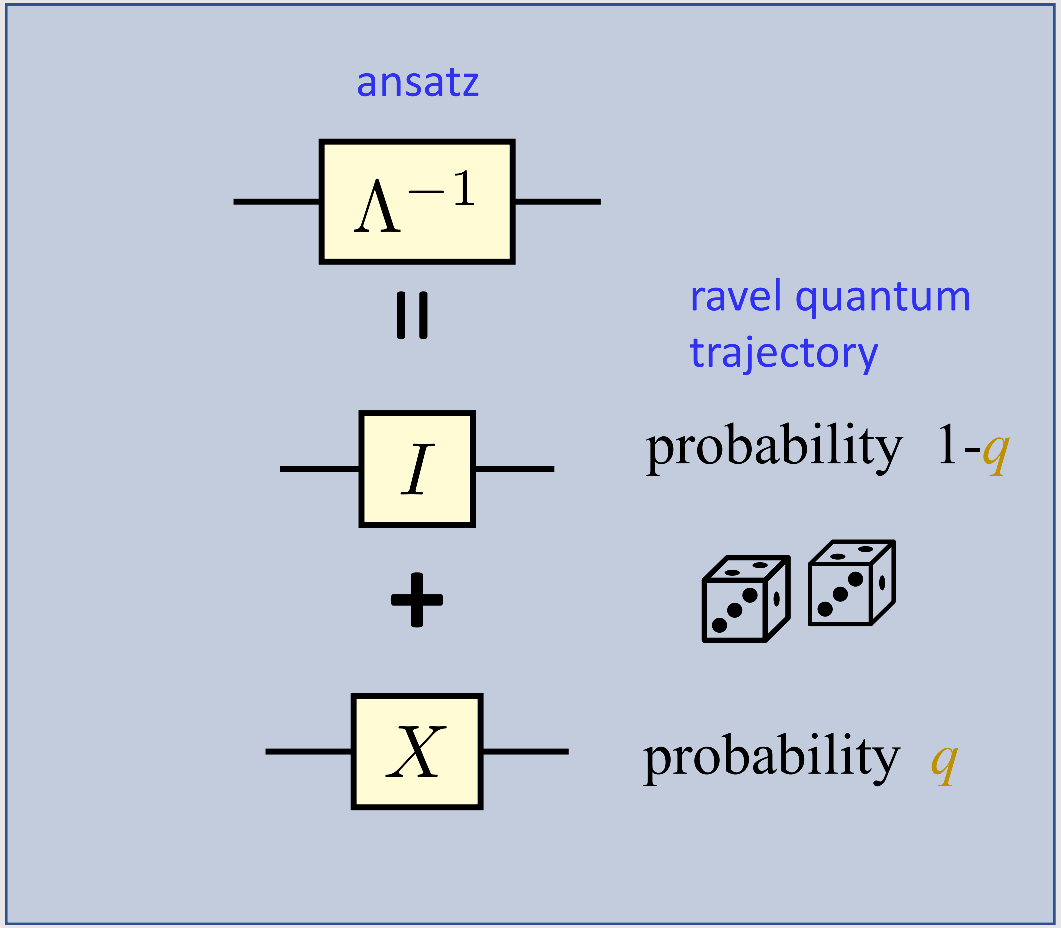
$$\Lambda(\rho)$$

Inverse of noise map is not physical





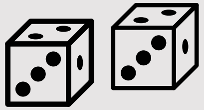
$$\Lambda^{-1}(\rho) = ?$$

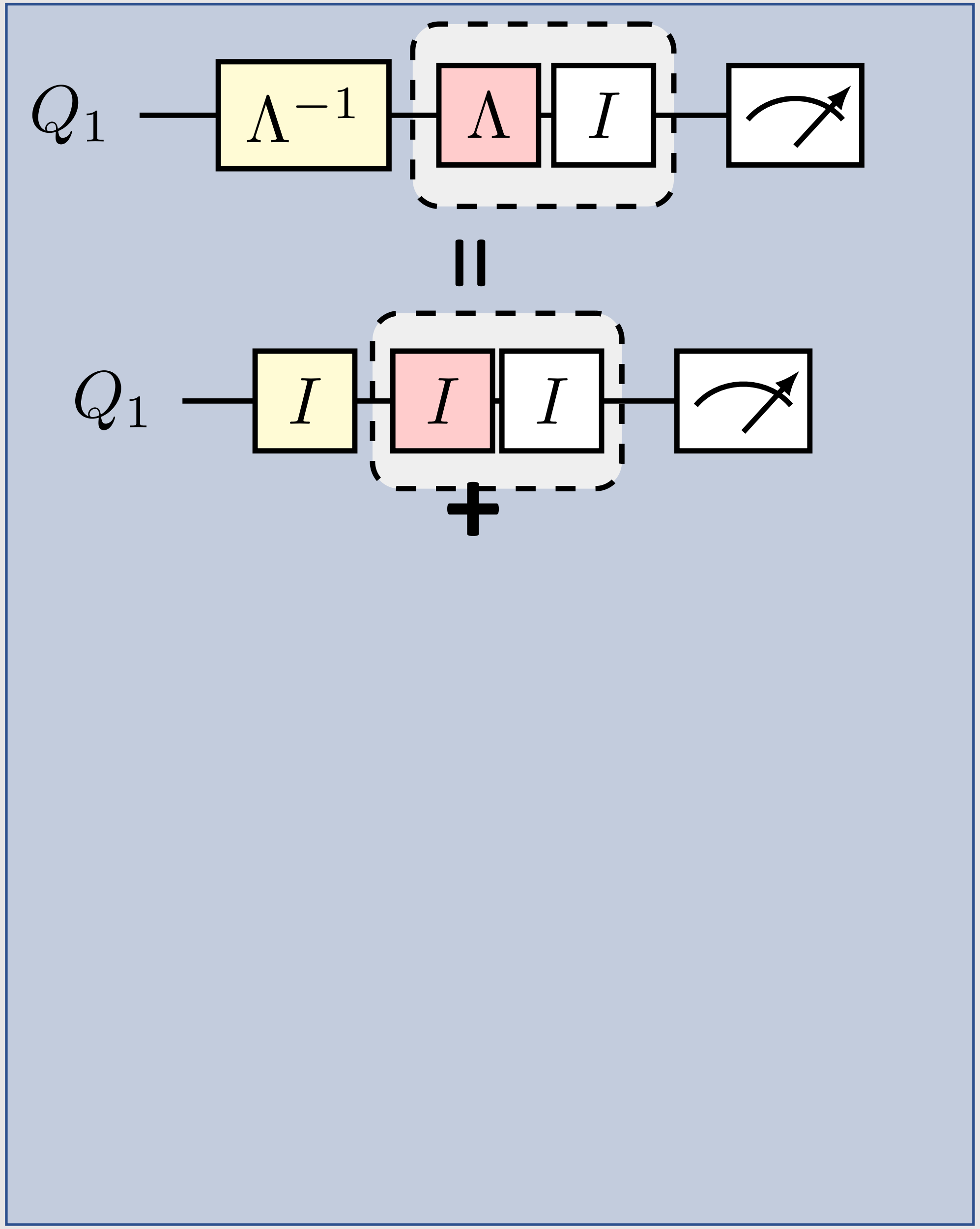


Raveling quantum trajectories to undo noise

probability

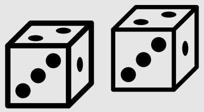
No error! (1-*q*) (1-*p*)

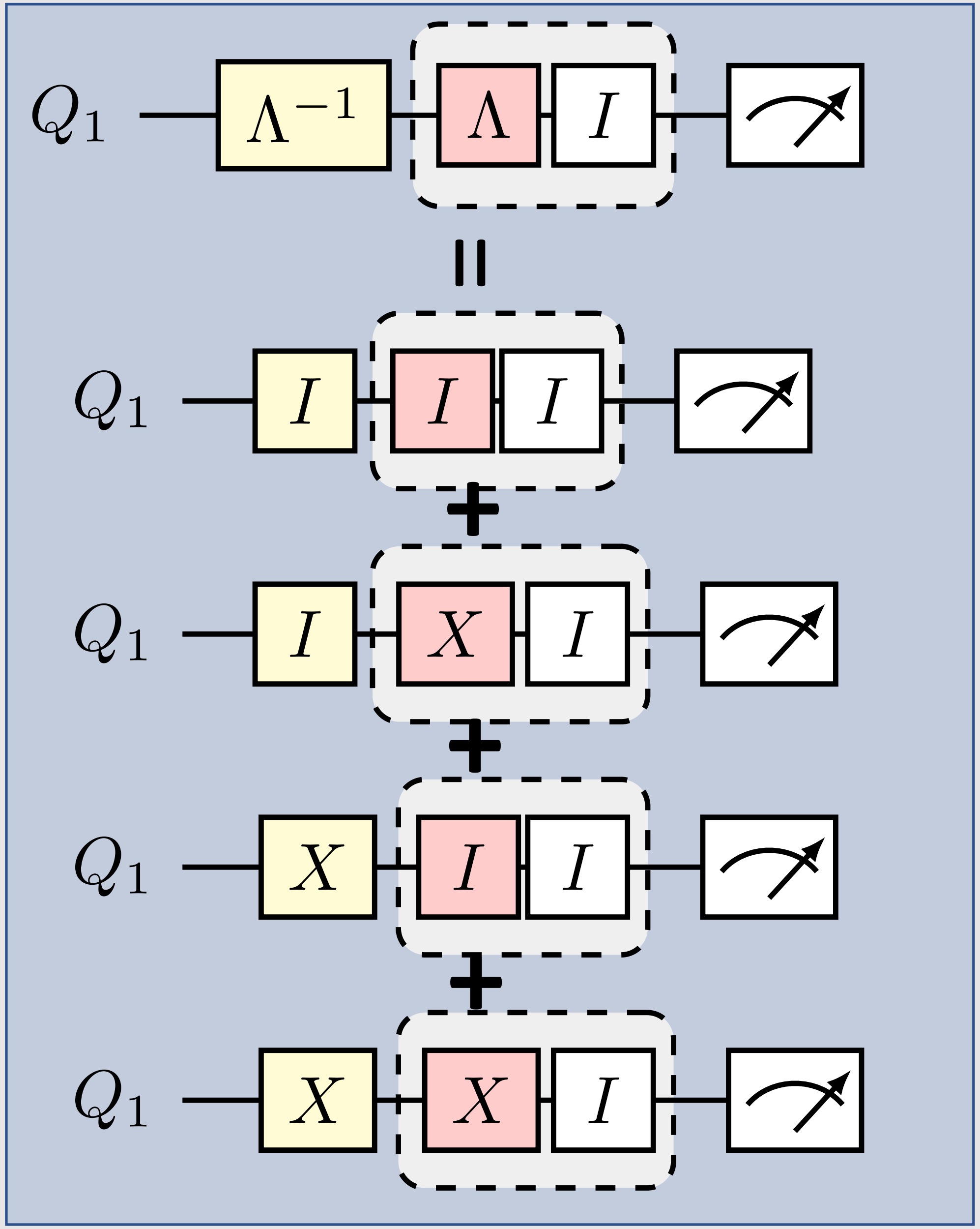




Raveling quantum trajectories to undo noise

	probability
No error!	$(1\text{-}q)\,(1\text{-}p)$
ERROR!	$(1\text{-}q)\,p$
ERROR!	$q\,(1\text{-}p)$
Error canceled!	$q\,p$

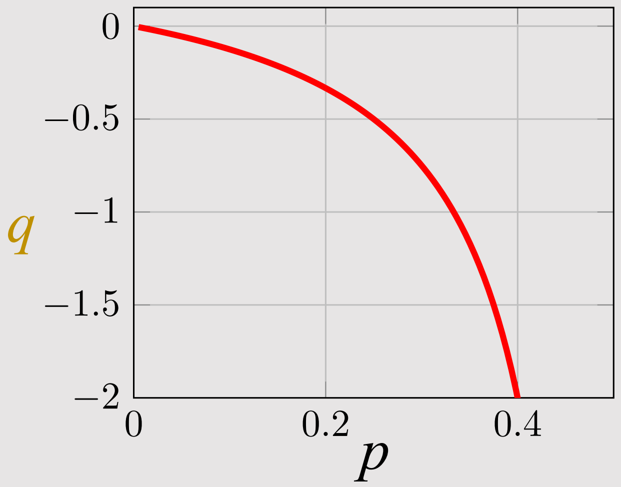




Solution to noise free!

$$q = \frac{-p}{1-2p}$$

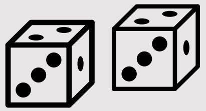
Sign & scale:
quasi-probability



How to implement?

probability

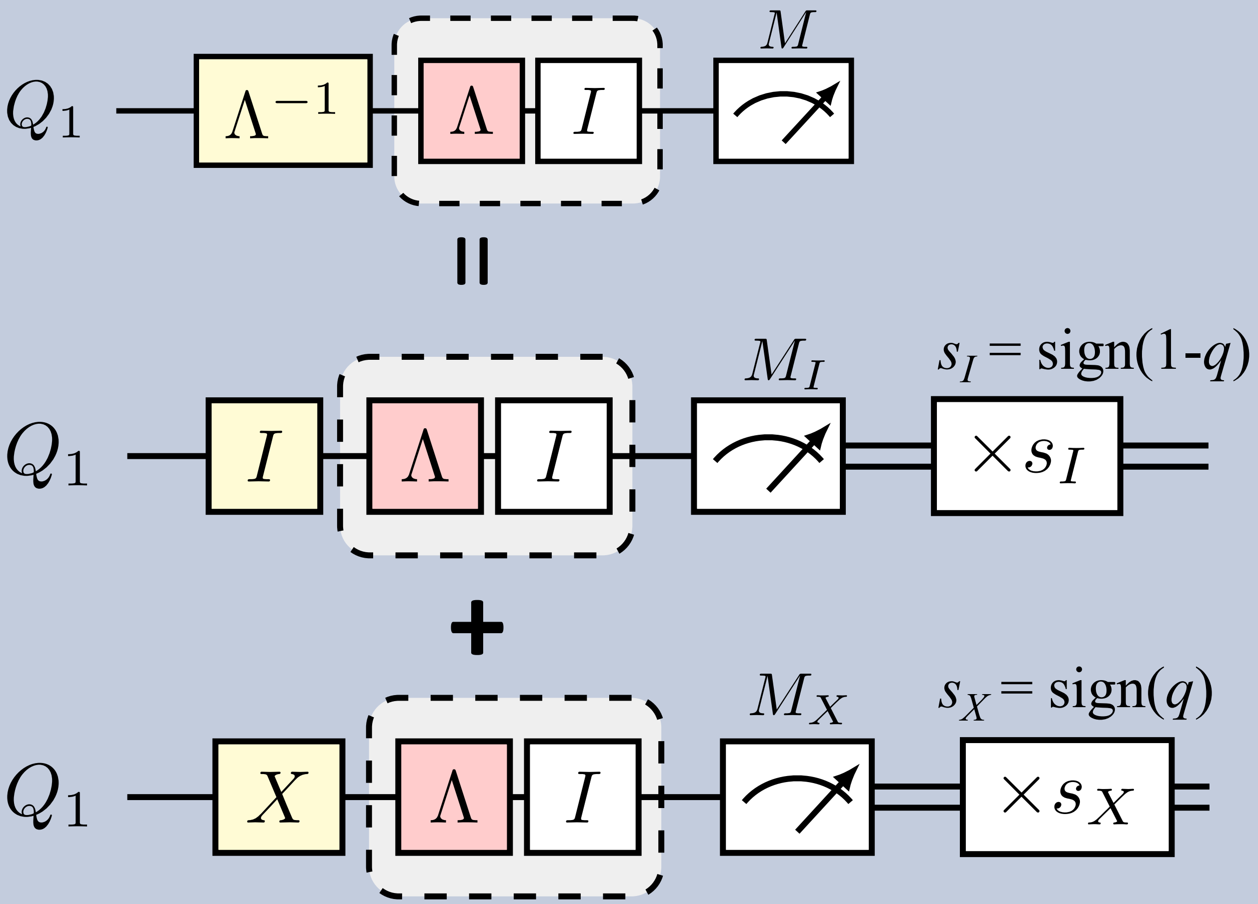
$P_I = |1\text{-}q|/\gamma$



$P_X = |q|/\gamma$

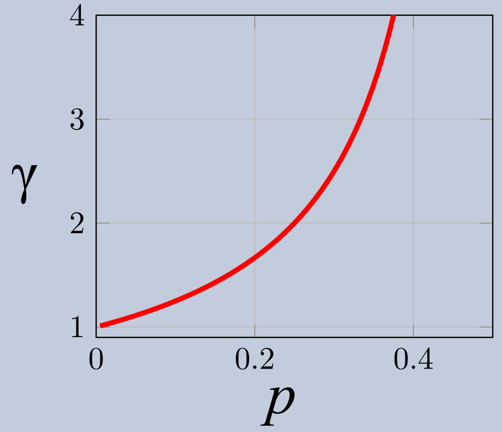
$P_I + P_X = 1$

random circuits
(quantum trajectories)



sampling overhead

$\gamma = |1\text{-}q| + |q|$



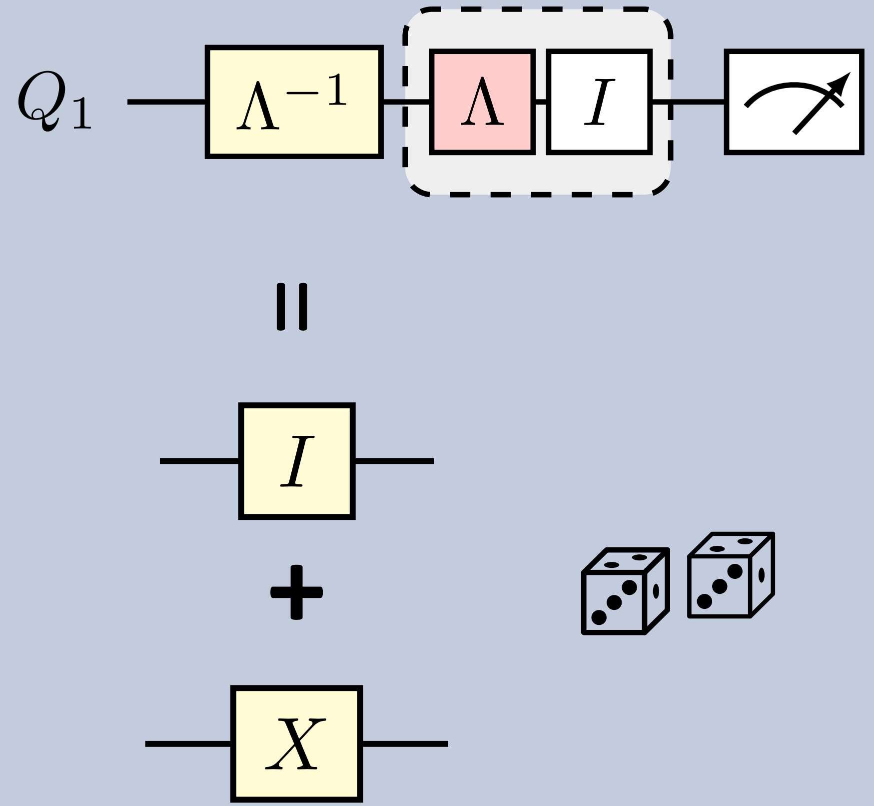
mitigated expectation

$$\langle M \rangle = \gamma\left(s_I P_I M_I + s_X P_X M_X\right)$$

Gain: bias-free estimate!

Cost: variance

Canceling noise with noise



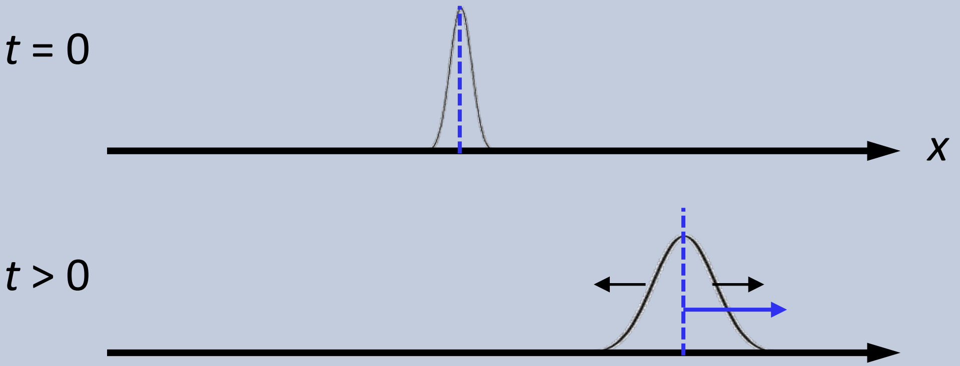
Canceling noise with noise: Drunkard's classical random walk analogy

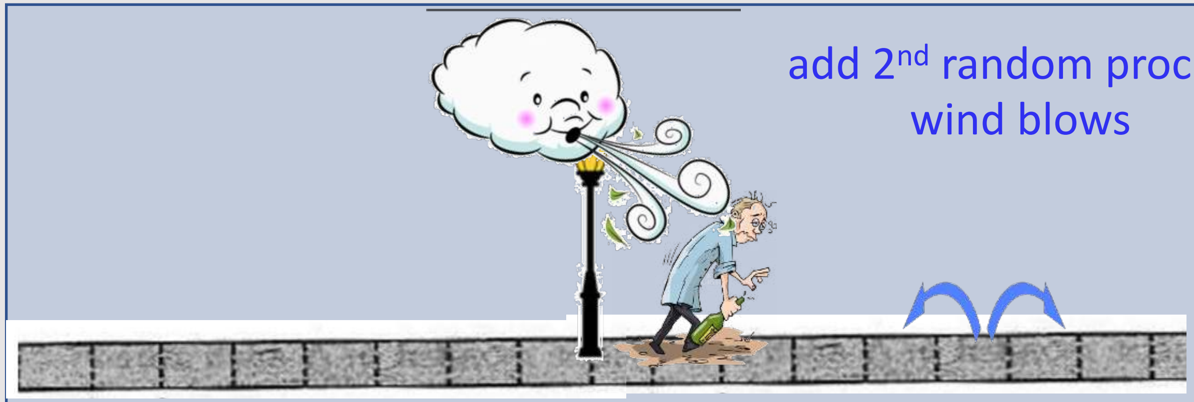
Random step

P(1 step left) = $\frac{1}{2}-p$

P(1 step right) = $\frac{1}{2}+p$

Distribution of random walk



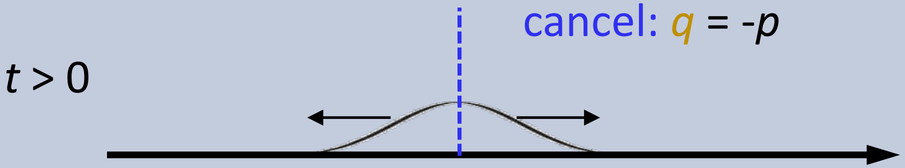


add 2nd random process
wind blows

P(1 step left) = $\frac{1}{2}+q$

P(1 step right) = $\frac{1}{2}-q$

Distribution of random walk with wind

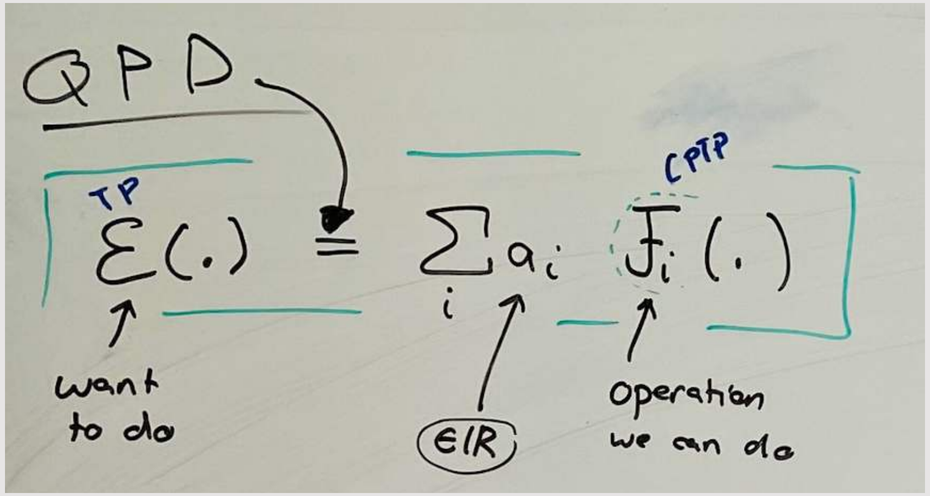


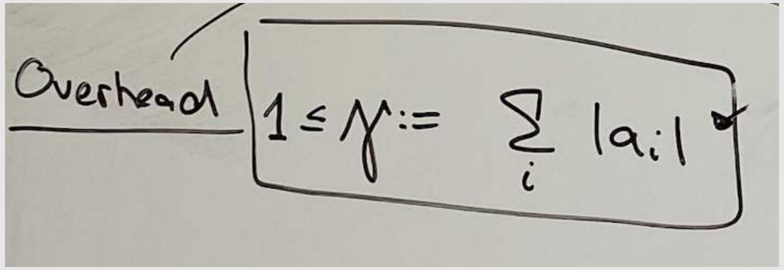
Gain: bias-free estimate!

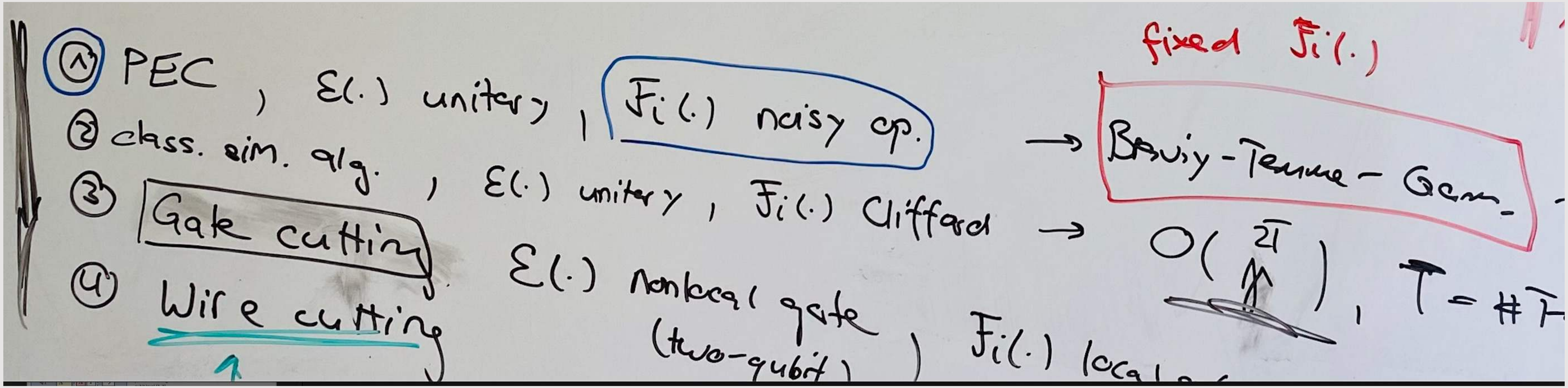
Cost: variance

Image: A. Brand

Putting them all on the same footing: David Sutter chat



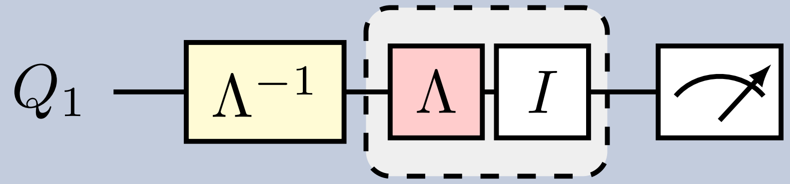




Nice, but why hasn't worked so far?

Challenges

Small scale

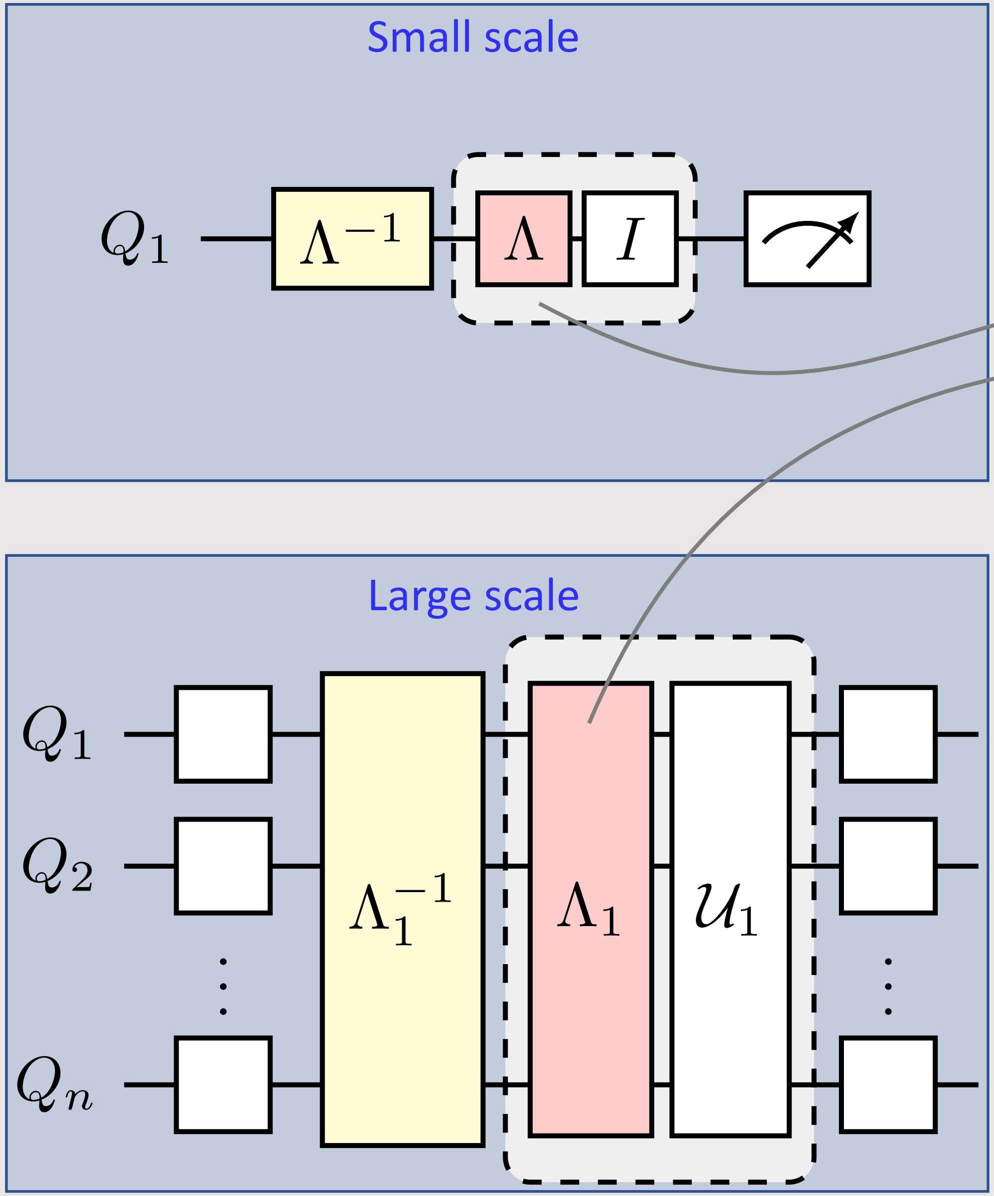


Critically hinges on knowing the full noise near perfectly

Despite the method's theoretical appeal (1-10),
practical challenges have limited its demonstration to the one and two-qubit level (2, 3)

1. S. Endo, S. C. Benjamin, Y. Li, Physical Review X 8, 031027 (2018).
2. C. Song, et al., Science Advances 5, arXiv:2109.04457(2019).
3. S. Zhang, et al., Nature Communications 11, 587 (2020).
4. C. Piveteau, D. Sutter, S. Woerner, arXiv:2101.09290 (2021).
5. S. Endo, et al., J. Physi Soc. of Japan 90, 032001 (2021).
6. C. Piveteau, et al., arXiv:2103.04915 (2021).
7. R. Takagi, Phys. Rev. Research 3, 033178 (2021).
8. R. Takagi, S. Endo, S. Minagawa, M. Gu, arXiv:2109.04457 (2021).
9. Y. Guo, S. Yang, arXiv preprint arXiv:2201.00752 (2022).
10. . . .

Nice, but why hasn't worked so far? Challenges



2 qubits	255 parameters
10 qubits	10^{12} parameters
50 qubits	10^{60} parameters

noise param values 10^{-2} - 10^{-5}
additive error sampling cost ($>10^2 - 10^{10}$)

Challenges

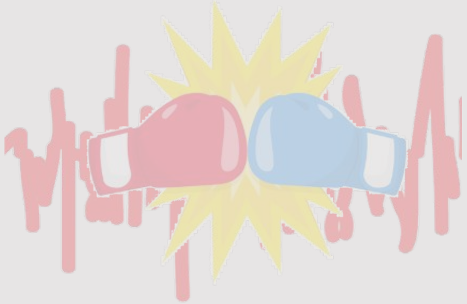
learning complexity
- efficient
- scalable
- accurate
- compact, tractable representation

noise in full device
- cross-talk
- correlated errors
- parallel gates

Outline

Idea

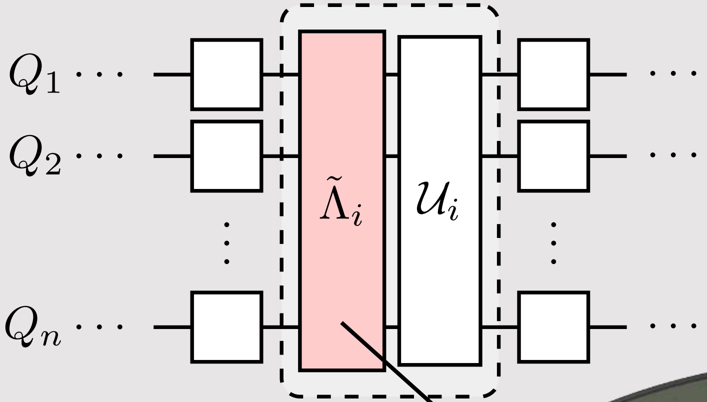
Learn



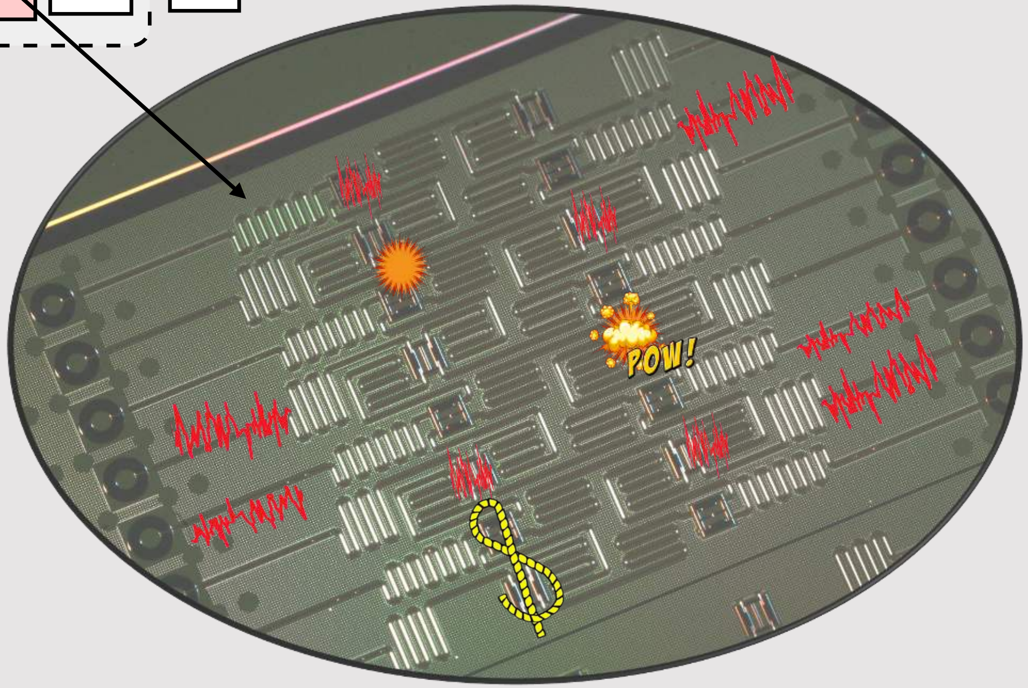
Cancel
(realization)

Icon: Flaticon

Is it possible to learn the noise with accuracy, efficiency, and scalability?

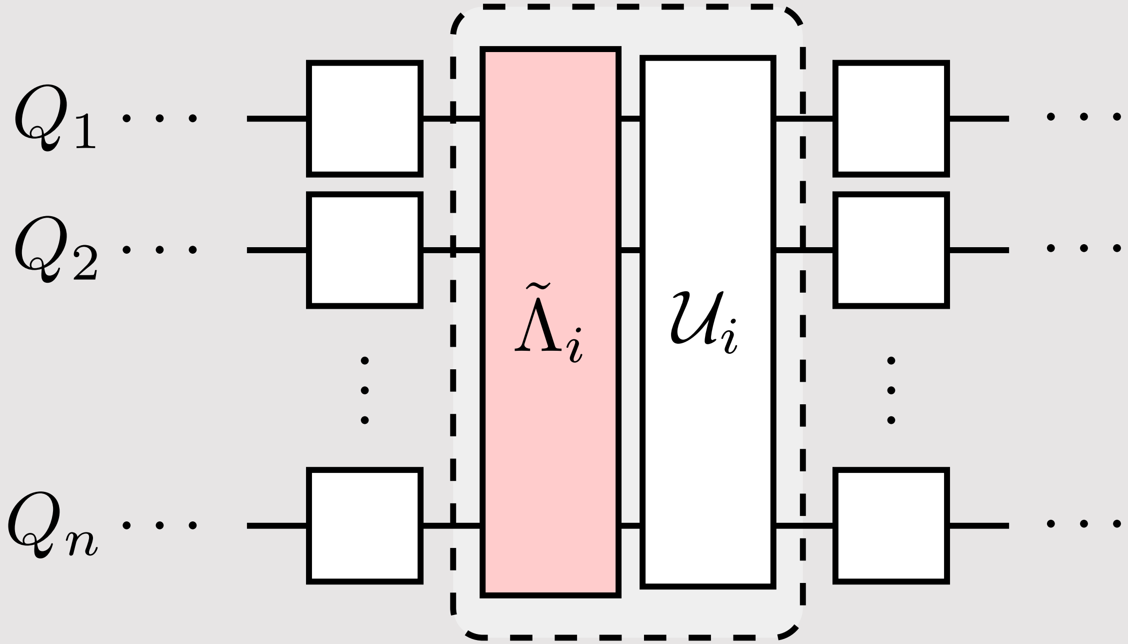


Energy relaxation T_1
Dephasing T_2
Coherent errors ZZ
Classical crosstalk
Quantum crosstalk
State preparation error
Measurement correlated errors
…



Control errors
Photon shot noise
1/f charge noise
1/f flux noise
Nonequilibrium quasiparticles
Leakage
Cosmic rays
…

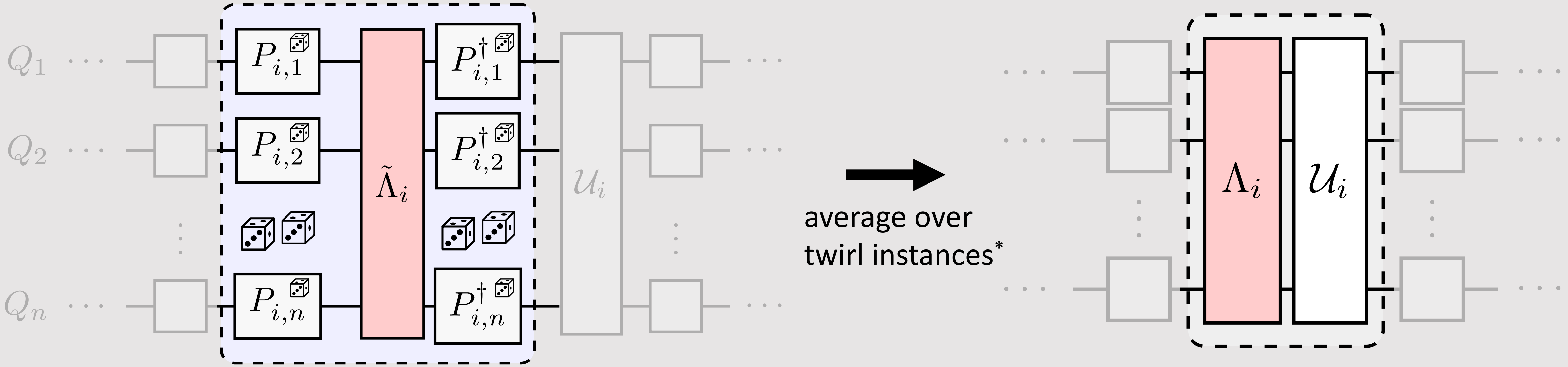
Step 1: Simplify the noise



noise that includes cross-talk errors, etc.
characterized by some 4^n x 4^n matrix

Step 1: Simplify the noise: twirl

twirl reduces to noise 4^n x 4^n matrix to diagonal one with 4^n entries in Pauli basis



Stochastic Pauli channel

$$\Lambda_i(\rho) = \sum_{a=0}^{4^n-1} c_{ia} P_a \rho P_a^\dagger$$

$$\Lambda(P_a) = f_a P_a$$

eigenvecs are Paulis

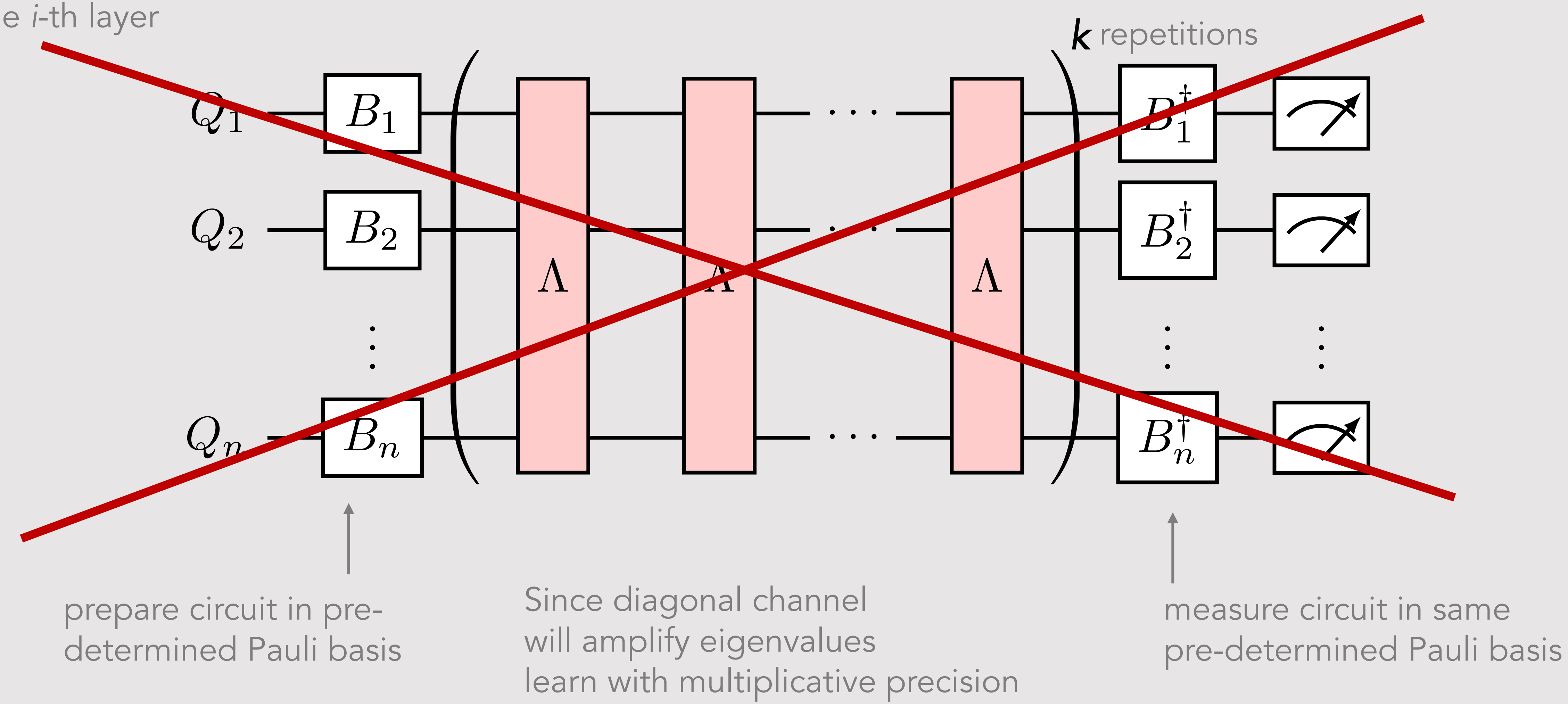
* some sub-Clifford twirl group (use Paulis)

Twirling references

1. C. H. Bennett, et al., Phys. Rev. Lett. 76, 722 (1996).
2. E. Knill, arXiv:0404104 (2004).
3. O. Kern, G. Alber, D. L. Shepelyansky, EPJ D 32, 153 (2005).
4. M. R. Geller, Z. Zhou, Physical Review A 88, 012314 (2013).
5. J. J. Wallman, J. Emerson, Physical Review A 94, 052325 (2016)
6. Hashim *et al.*, Phys. Rev. X 11, 041039 (2021)
7. Z. Minev et al., Qiskit Global Summer School Lecture Notes (in prep; see zlatko-minev.com/blog for update) (2022)
8. ...

Step 2 wish: amplify noise

for the *i*-th layer



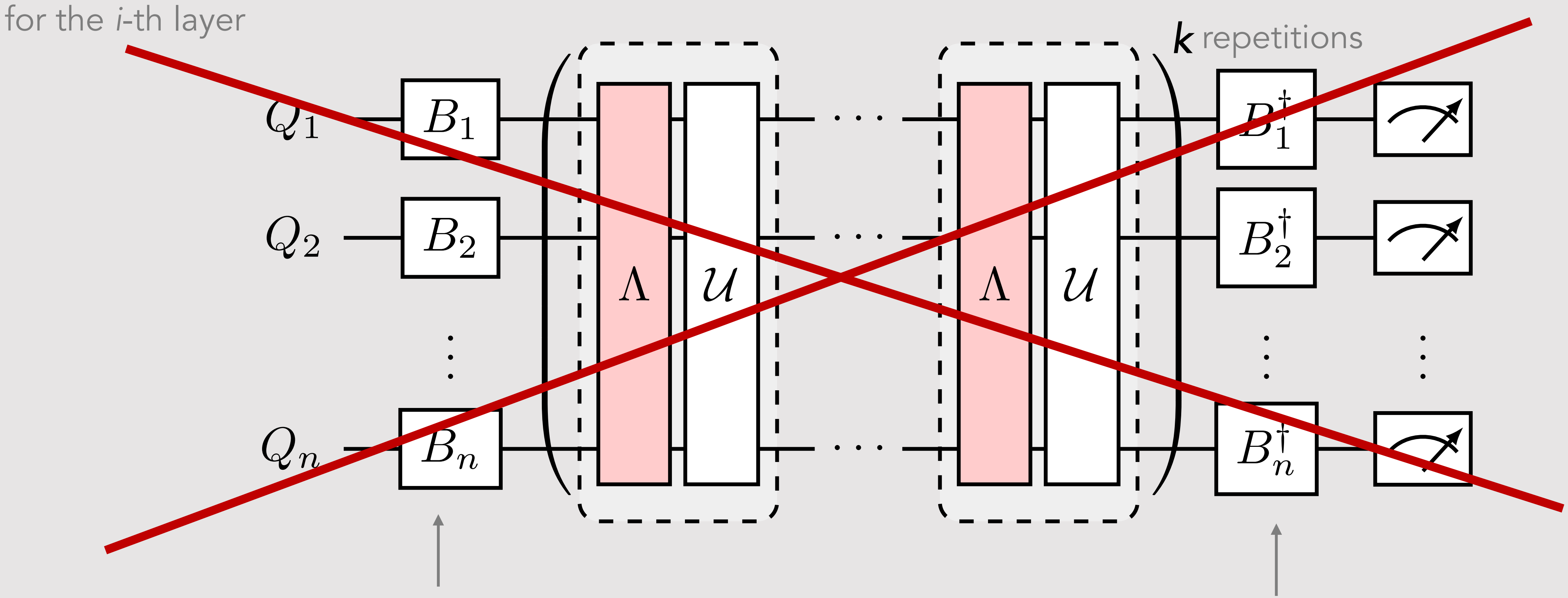
Akin to:

RB, Cycle RB, K-body noise reconstruction, ...

S.T. Flammia and J.J. Wallman ACM Trans QC 1, 3 (2020), ...

For something of a review of protocols, see Helsen, *et al.*, *A general framework for randomized benchmarking* (arXiv:2010.07974)

Step 2 wish: amplify noise with gates?



prepare circuit in pre-determined Pauli basis

Since diagonal channel will amplify eigenvalues learn with multiplicative precision

measure circuit in same pre-determined Pauli basis

Akin to:

RB, Cycle RB, K-body noise reconstruction, …

S.T. Flammia and J.J. Wallman ACM Trans QC 1, 3 (2020), …

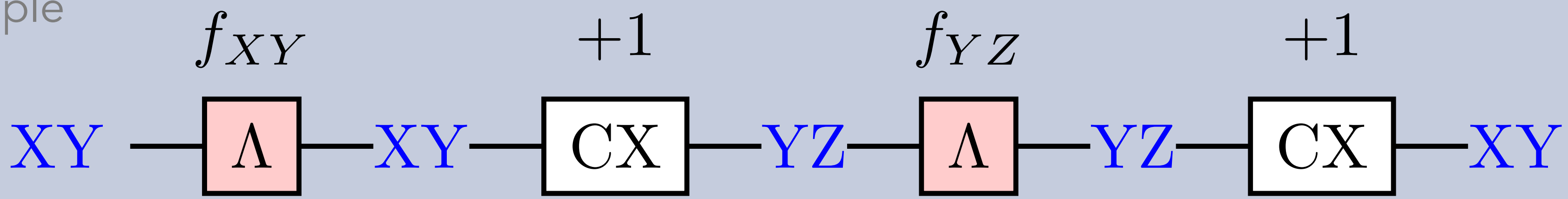
Erhard *et al.*, arXiv:1902.08543; Ferracin *et al.*, arXiv:2201.10672, …

For something of a review of protocols, see Helsen, *et al.*, arXiv:2010.07974

Let's see how the amplification works with gates: no-go theorem

$$\Lambda(P_a) = f_a P_a$$

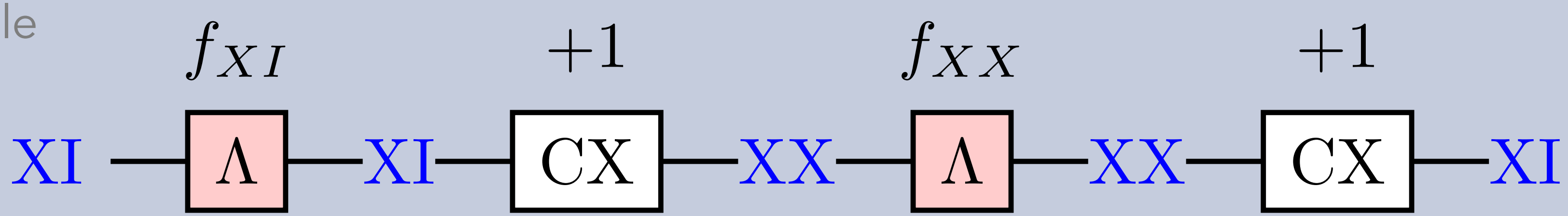
2Q example



action of CX here can be undo by SQ gates

$f_{XY} f_{YZ} XY$

2Q example

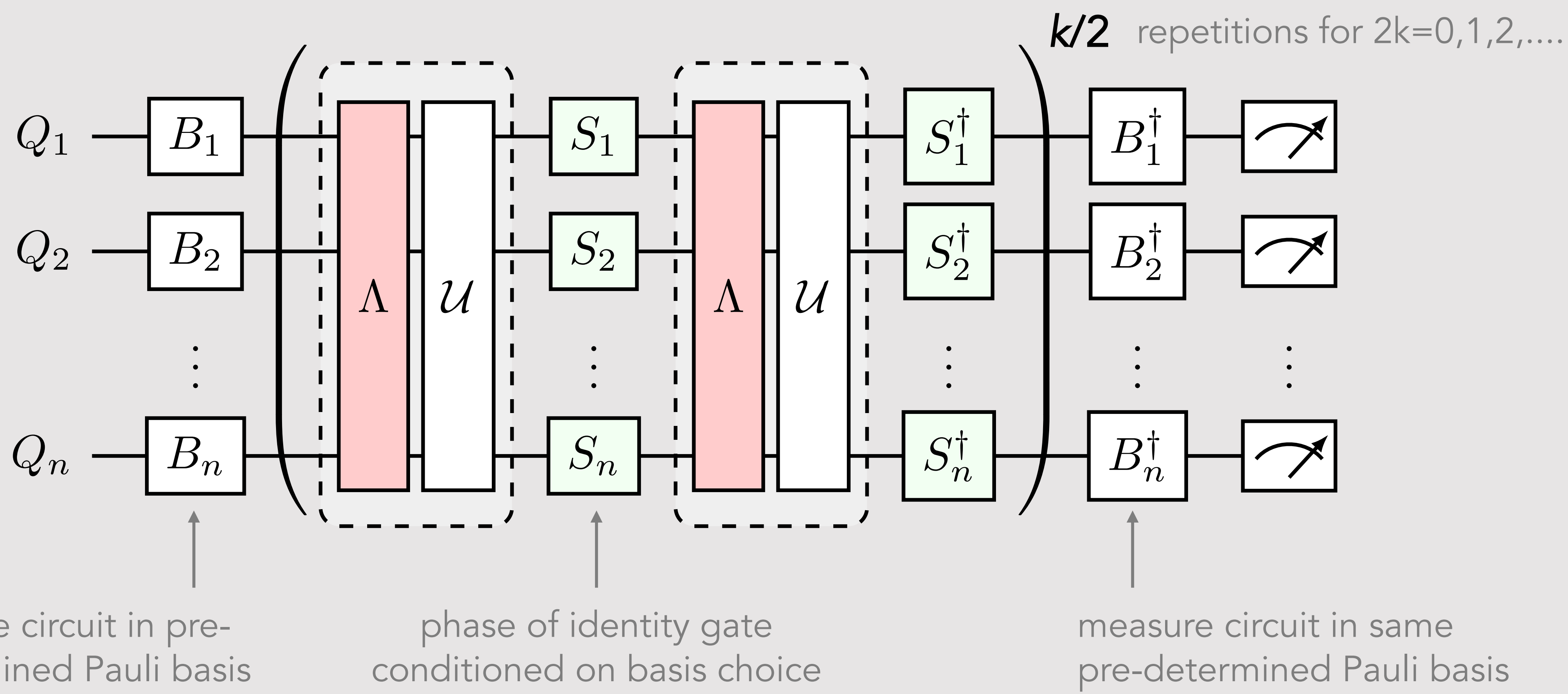


action of CX here can **not** be undone by SQ gates

$f_{XI} f_{XX} XI$

fundamental degeneracy – can not undo some non-local – need entangling operation

Step 2: Amplify & learn noise per layer



Akin to:

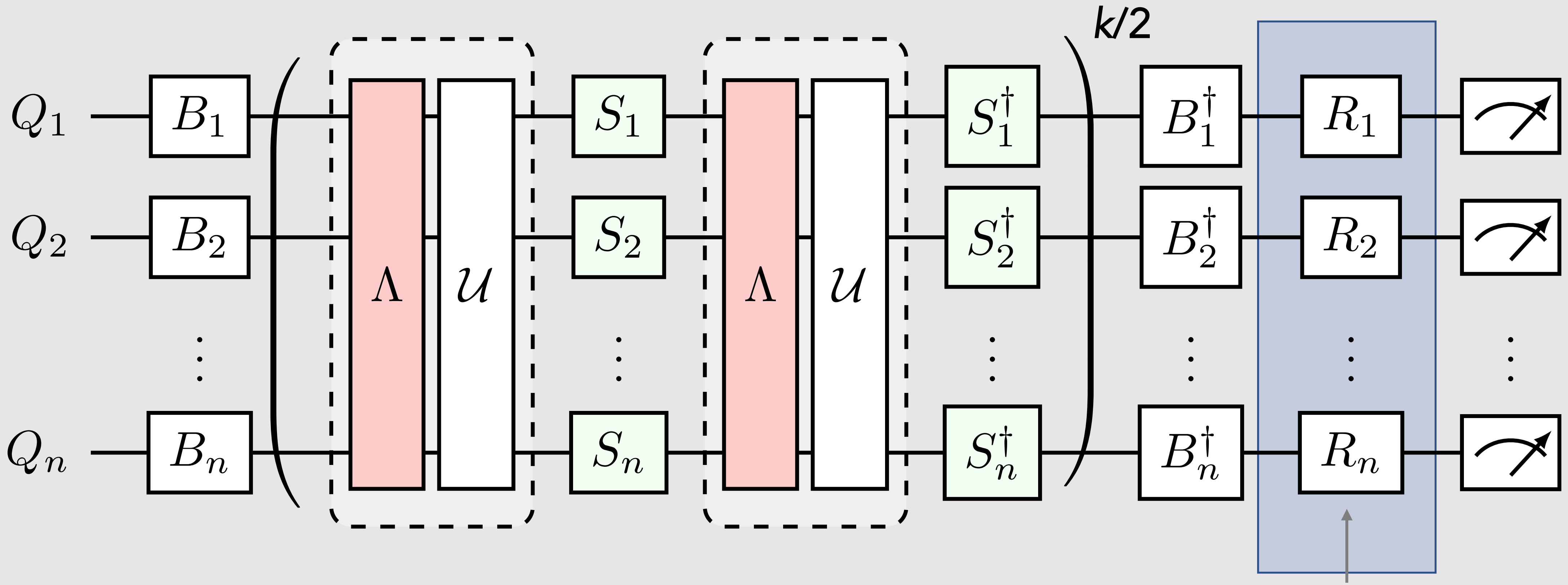
RB, Cycle RB, K-body noise reconstruction, ...

S.T. Flammia and J.J. Wallman ACM Trans QC 1, 3 (2020), ...

Erhard *et al.*, arXiv:1902.08543; Ferracin *et al.*, arXiv:2201.10672, ...

...

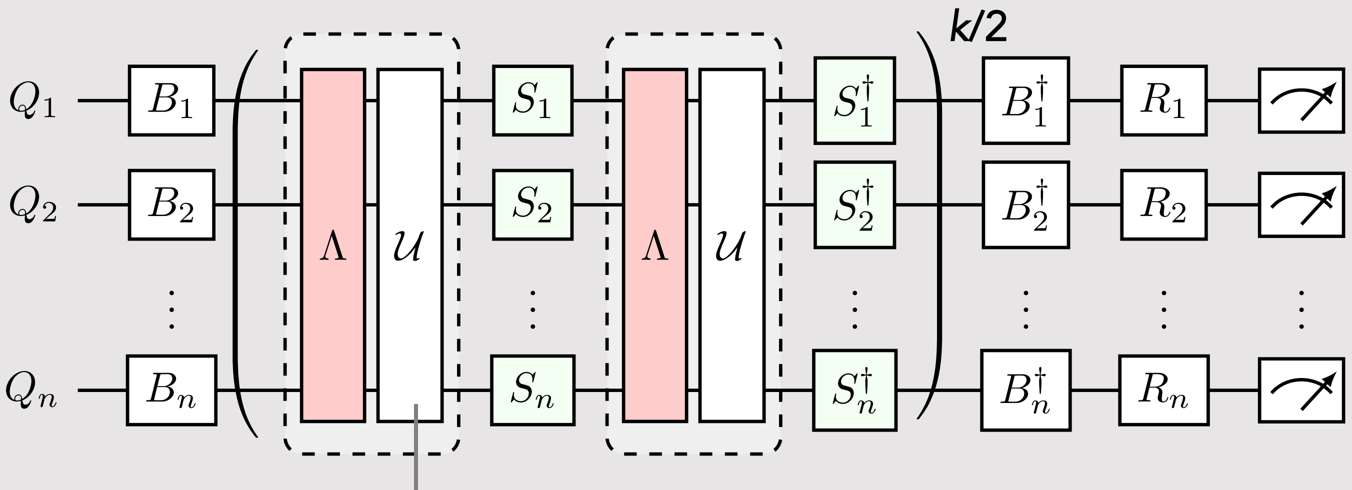
Step 3: Twirl readout-error mitigation



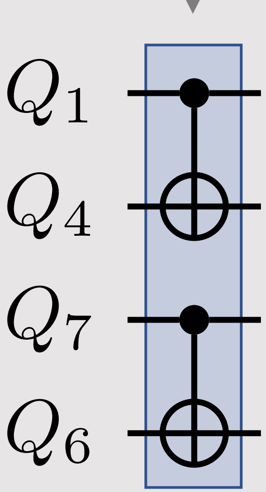
twirl readout to do readout error mitigation in-situ*

* Model-free readout-error mitigation for quantum expectation values
Ewout van den Berg, Zlatko K. Minev, and Kristan Temme
Phys. Rev. A 105, 032620 (2022)

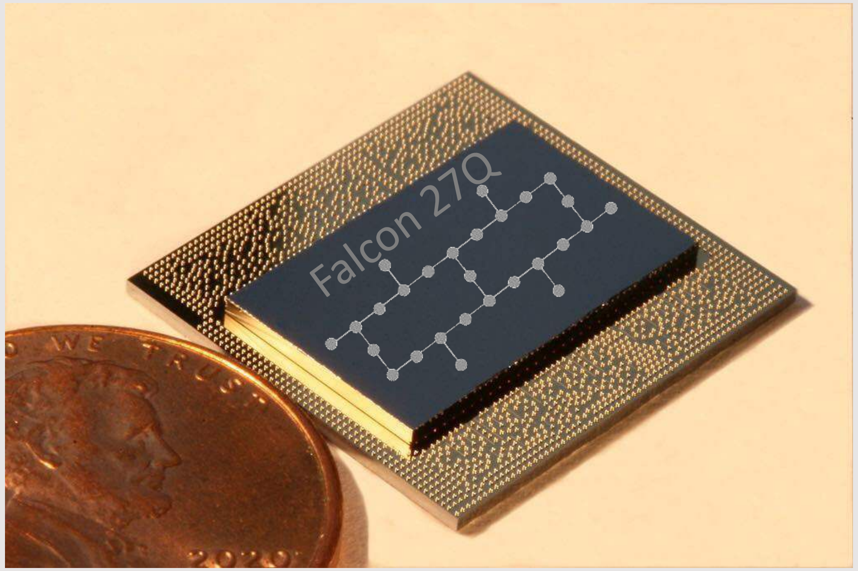
Experiment



Q_1
Q_2
Q_n
B_1
B_2
B_n
Λ
$\mathcal{U}$
S_1
S_2
S_n
Λ
$\mathcal{U}$
$S_1^\dagger$
$S_2^\dagger$
$S_n^\dagger$
k/2
$B_1^\dagger$
$B_2^\dagger$
$B_n^\dagger$
R_1
R_2
R_n

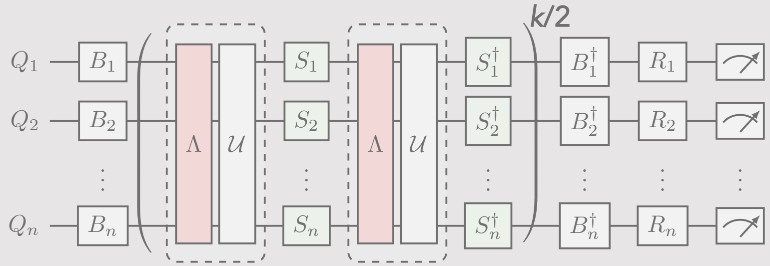


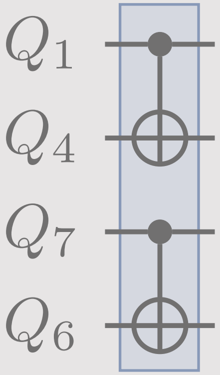
Q_1
Q_4
Q_7
Q_6



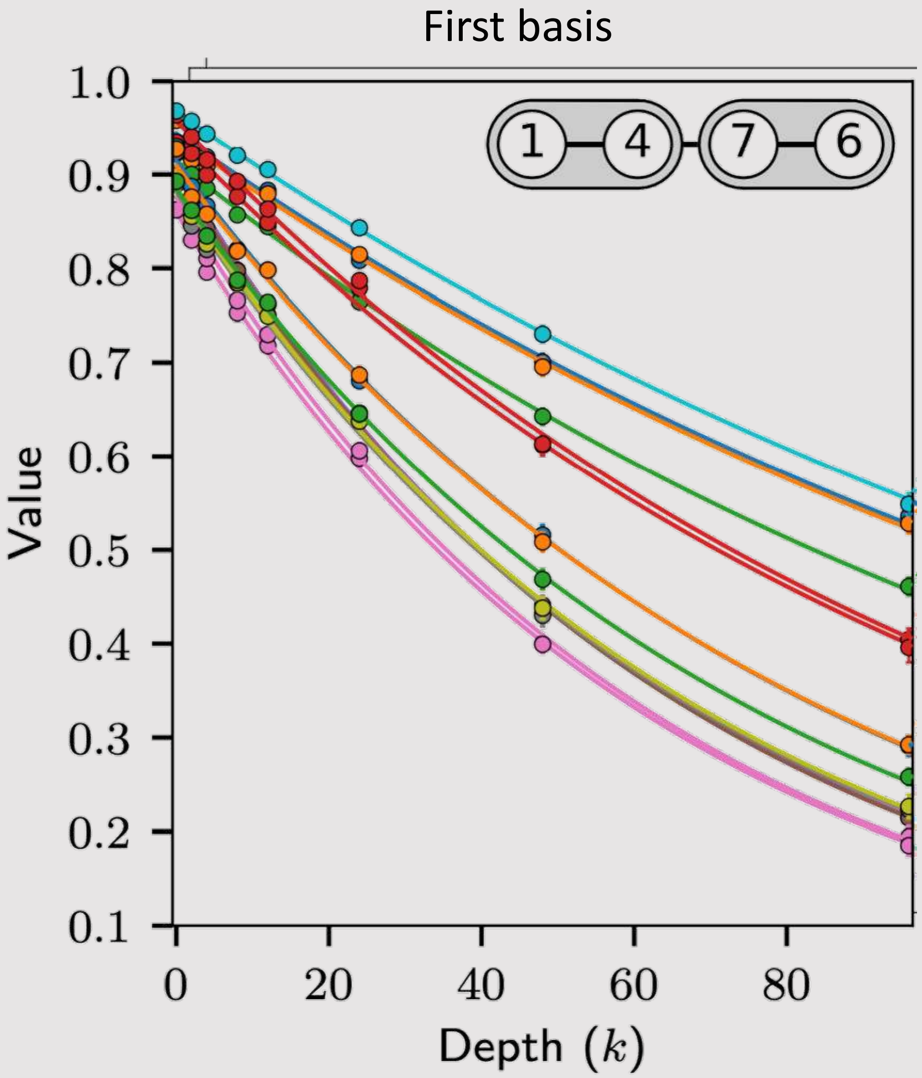
Falcon 27Q

Learning the noise: raw data

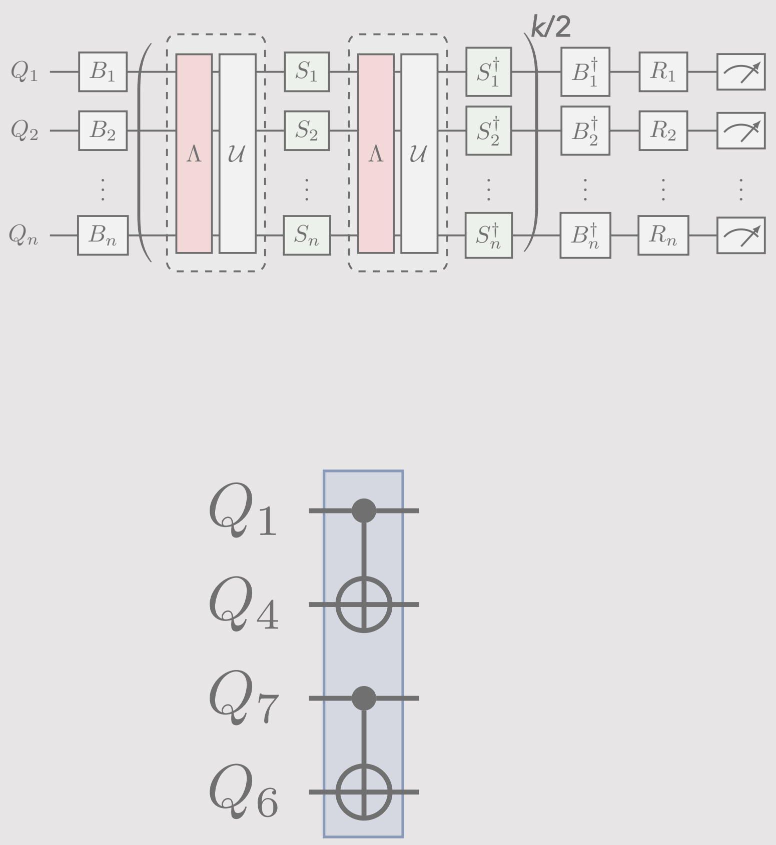




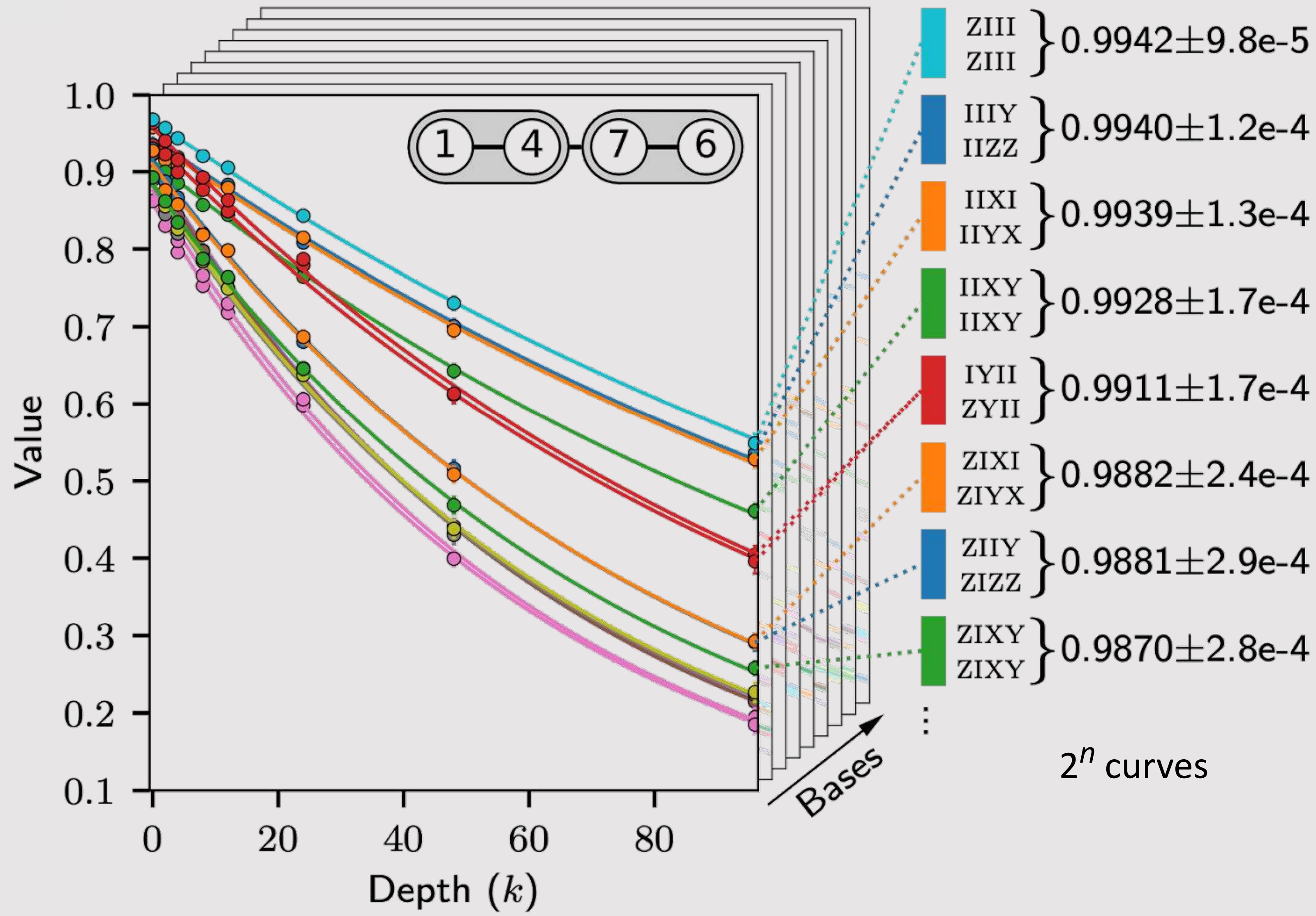
9 bases, 100 randomizations, 216 shots



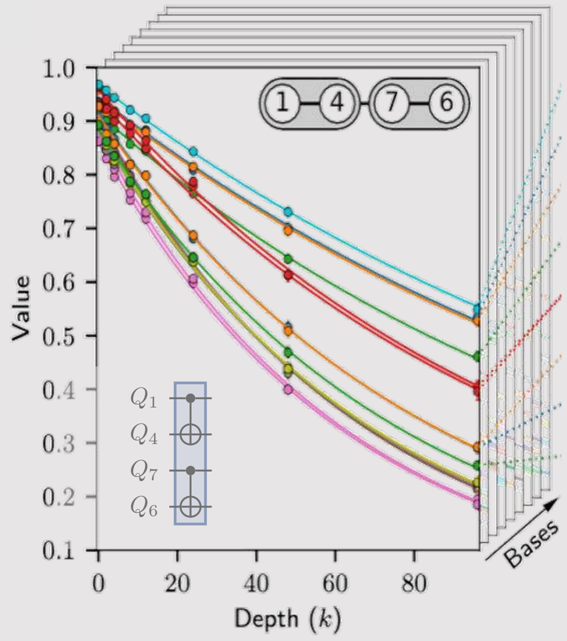
Learning the noise: raw data



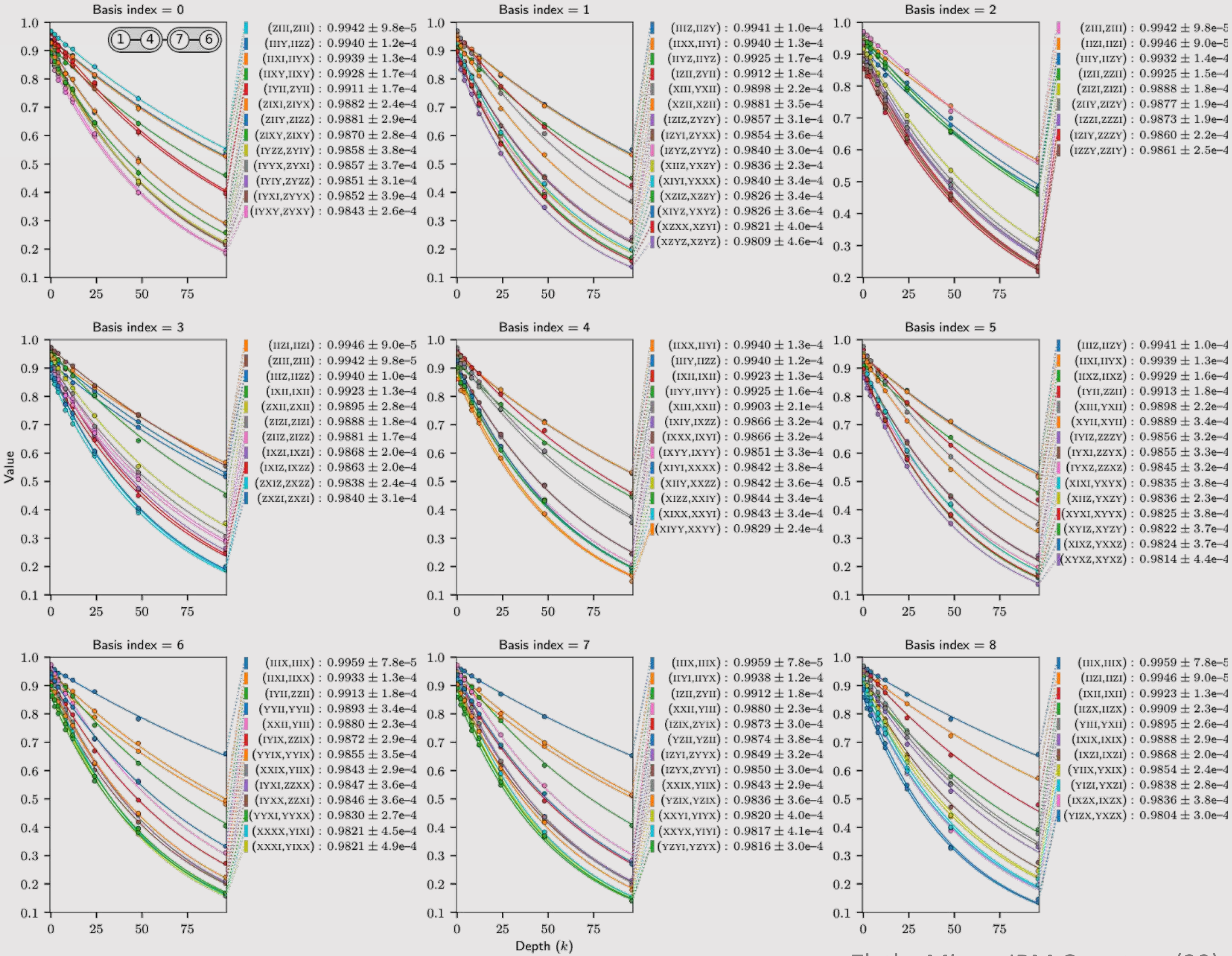
9 bases, 100 randomizations, 216 shots



Raw data

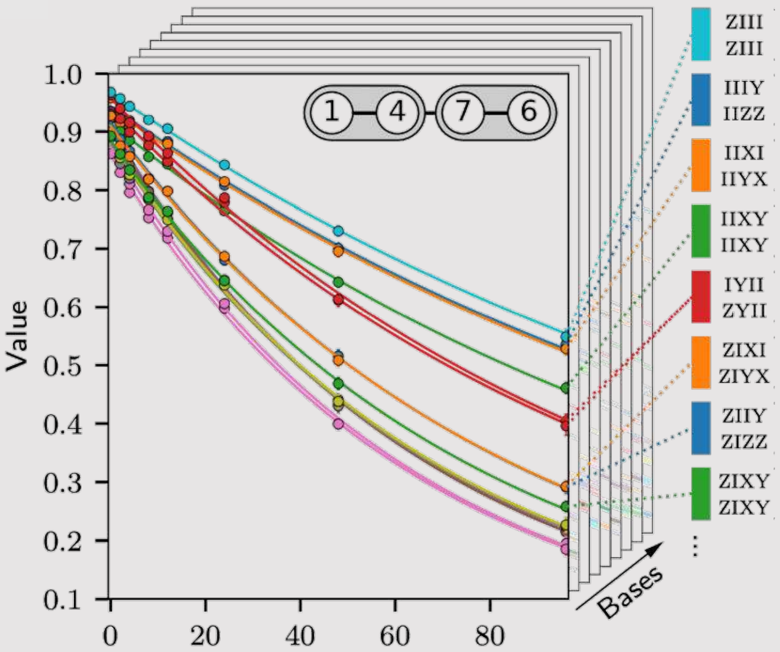


1
4
7
6
Value
Depth (k)
Bases
Q_1
Q_4
Q_7
Q_6

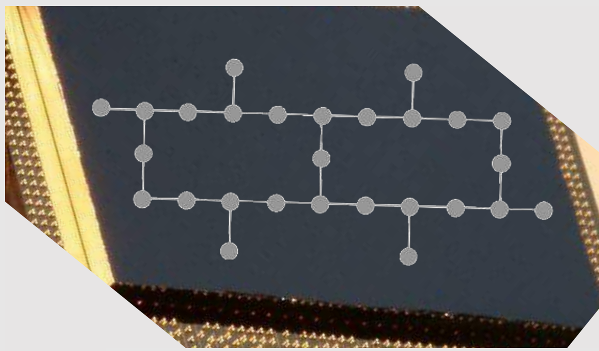


Basis index = 0
(ZIII,ZIII) : 0.9942 ± 9.8e-5
(IIIY,IIZZ) : 0.9940 ± 1.2e-4
(IIXI,IIYX) : 0.9939 ± 1.3e-4
(IIXY,IIXY) : 0.9928 ± 1.7e-4
(IYII,ZYII) : 0.9911 ± 1.7e-4
(ZIXI,ZIYX) : 0.9882 ± 2.4e-4
(ZIIY,ZIZZ) : 0.9881 ± 2.9e-4
(ZIXY,ZIXY) : 0.9870 ± 2.8e-4
(IYZZ,ZYIY) : 0.9858 ± 3.8e-4
(IYYX,ZYXI) : 0.9857 ± 3.7e-4
(IYIY,ZYZZ) : 0.9851 ± 3.1e-4
(IYXI,ZYYX) : 0.9852 ± 3.9e-4
(IYXY,ZYXY) : 0.9843 ± 2.6e-4
Basis index = 1
(IIIZ,IIZY) : 0.9941 ± 1.0e-4
(IIXX,IIYI) : 0.9940 ± 1.3e-4
(IIYZ,IIYZ) : 0.9925 ± 1.7e-4
(IZII,ZYII) : 0.9912 ± 1.8e-4
(XIII,YXII) : 0.9898 ± 2.2e-4
(XZII,XZII) : 0.9881 ± 3.5e-4
(IZIZ,ZYZY) : 0.9857 ± 3.1e-4
(IZYI,ZYXX) : 0.9854 ± 3.6e-4
(IZYZ,ZYYZ) : 0.9840 ± 3.0e-4
(XIIZ,YXZY) : 0.9836 ± 2.3e-4
(XIYI,YXXX) : 0.9840 ± 3.4e-4
(XZIZ,XZZY) : 0.9826 ± 3.4e-4
(XIYZ,YXYZ) : 0.9826 ± 3.6e-4
(XZXX,XZYI) : 0.9821 ± 4.0e-4
(XZYZ,XZYZ) : 0.9809 ± 4.6e-4
Basis index = 2
(ZIII,ZIII) : 0.9942 ± 9.8e-5
(IIZI,IIZI) : 0.9946 ± 9.0e-5
(IIIY,IIZY) : 0.9932 ± 1.4e-4
(IZII,ZZII) : 0.9925 ± 1.5e-4
(ZIZI,ZIZI) : 0.9888 ± 1.8e-4
(ZIIY,ZIZY) : 0.9877 ± 1.9e-4
(IZZI,ZZZI) : 0.9873 ± 1.9e-4
(IZIY,ZZZY) : 0.9860 ± 2.2e-4
(IZZY,ZZIY) : 0.9861 ± 2.5e-4
Basis index = 3
(IIZI,IIZI) : 0.9946 ± 9.0e-5
(ZIII,ZIII) : 0.9942 ± 9.8e-5
(IIIZ,IIZZ) : 0.9940 ± 1.0e-4
(IXII,IXII) : 0.9923 ± 1.3e-4
(ZXII,ZXII) : 0.9895 ± 2.8e-4
(ZIZI,ZIZI) : 0.9888 ± 1.8e-4
(ZIIZ,ZIZZ) : 0.9881 ± 1.7e-4
(IXZI,IXZI) : 0.9868 ± 2.0e-4
(IXIZ,IXZZ) : 0.9863 ± 2.0e-4
(ZXIZ,ZXZZ) : 0.9838 ± 2.4e-4
(ZXZI,ZXZI) : 0.9840 ± 3.1e-4
Basis index = 4
(IIXX,IIYI) : 0.9940 ± 1.3e-4
(IIIY,IIZZ) : 0.9940 ± 1.2e-4
(IXII,IXII) : 0.9923 ± 1.3e-4
(IIYY,IIYY) : 0.9925 ± 1.6e-4
(XIII,XXII) : 0.9903 ± 2.1e-4
(IXIY,IXZZ) : 0.9866 ± 3.2e-4
(IXXX,IXYI) : 0.9866 ± 3.2e-4
(IXYY,IXYY) : 0.9851 ± 3.3e-4
(XIYI,XXXX) : 0.9842 ± 3.8e-4
(XIIY,XXZZ) : 0.9842 ± 3.6e-4
(XIZZ,XXIY) : 0.9844 ± 3.4e-4
(XIXX,XXYI) : 0.9843 ± 3.4e-4
(XIYY,XXYY) : 0.9829 ± 2.4e-4
Basis index = 5
(IIIZ,IIZY) : 0.9941 ± 1.0e-4
(IIXI,IIYX) : 0.9939 ± 1.3e-4
(IIXZ,IIXZ) : 0.9929 ± 1.6e-4
(IYII,ZZII) : 0.9913 ± 1.8e-4
(XIII,YXII) : 0.9898 ± 2.2e-4
(XYII,XYII) : 0.9889 ± 3.4e-4
(IYIZ,ZZZY) : 0.9856 ± 3.2e-4
(IYXI,ZZYX) : 0.9855 ± 3.3e-4
(IYXZ,ZZXZ) : 0.9845 ± 3.2e-4
(XIXI,YXYX) : 0.9835 ± 3.8e-4
(XIIZ,YXZY) : 0.9836 ± 2.3e-4
(XYXI,XYYX) : 0.9825 ± 3.8e-4
(XYIZ,XYZY) : 0.9822 ± 3.7e-4
(XIXZ,YXXZ) : 0.9824 ± 3.7e-4
(XYXZ,XYXZ) : 0.9814 ± 4.4e-4
Basis index = 6
(IIIX,IIIX) : 0.9959 ± 7.8e-5
(IIXI,IIXX) : 0.9933 ± 1.3e-4
(IYII,ZZII) : 0.9913 ± 1.8e-4
(YYII,YYII) : 0.9893 ± 3.4e-4
(XXII,YIII) : 0.9880 ± 2.3e-4
(IYIX,ZZIX) : 0.9872 ± 2.9e-4
(YYIX,YYIX) : 0.9855 ± 3.5e-4
(XXIX,YIIX) : 0.9843 ± 2.9e-4
(IYXI,ZZXX) : 0.9847 ± 3.6e-4
(IYXX,ZZXI) : 0.9846 ± 3.6e-4
(YYXI,YYXX) : 0.9830 ± 2.7e-4
(XXXX,YIXI) : 0.9821 ± 4.5e-4
(XXXI,YIXX) : 0.9821 ± 4.9e-4
Basis index = 7
(IIIX,IIIX) : 0.9959 ± 7.8e-5
(IIYI,IIYX) : 0.9938 ± 1.2e-4
(IZII,ZYII) : 0.9912 ± 1.8e-4
(XXII,YIII) : 0.9880 ± 2.3e-4
(IZIX,ZYIX) : 0.9873 ± 3.0e-4
(YZII,YZII) : 0.9874 ± 3.8e-4
(IZYI,ZYYX) : 0.9849 ± 3.2e-4
(IZYX,ZYYI) : 0.9850 ± 3.0e-4
(XXIX,YIIX) : 0.9843 ± 2.9e-4
(YZIX,YZIX) : 0.9836 ± 3.6e-4
(XXYI,YIYX) : 0.9820 ± 4.0e-4
(XXYX,YIYI) : 0.9817 ± 4.1e-4
(YZYI,YZYX) : 0.9816 ± 3.0e-4
Depth (k)
Basis index = 8
(IIIX,IIIX) : 0.9959 ± 7.8e-5
(IIZI,IIZI) : 0.9946 ± 9.0e-5
(IXII,IXII) : 0.9923 ± 1.3e-4
(IIZX,IIZX) : 0.9909 ± 2.3e-4
(YIII,YXII) : 0.9895 ± 2.6e-4
(IXIX,IXIX) : 0.9888 ± 2.9e-4
(IXZI,IXZI) : 0.9868 ± 2.0e-4
(YIIX,YXIX) : 0.9854 ± 2.4e-4
(YIZI,YXZI) : 0.9838 ± 2.8e-4
(IXZX,IXZX) : 0.9836 ± 3.8e-4
(YIZX,YXZX) : 0.9804 ± 3.0e-4
Value

Sparse Pauli-Lindblad model



$$\Lambda(\rho) = \sum_{a=0}^{4^n-1} c_a P_a \rho P_a^\dagger$$



$$\Lambda(\rho) = \exp[\mathcal{L}](\rho)$$

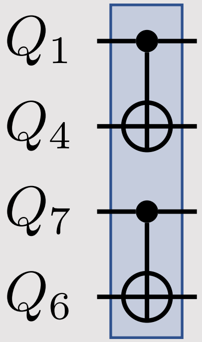
$$\mathcal{L}(\rho) = \sum_{k\in\mathcal{K}} \lambda_k (P_k \rho P_k - \rho)$$

Highlight: Ewout van den Berg

Magic

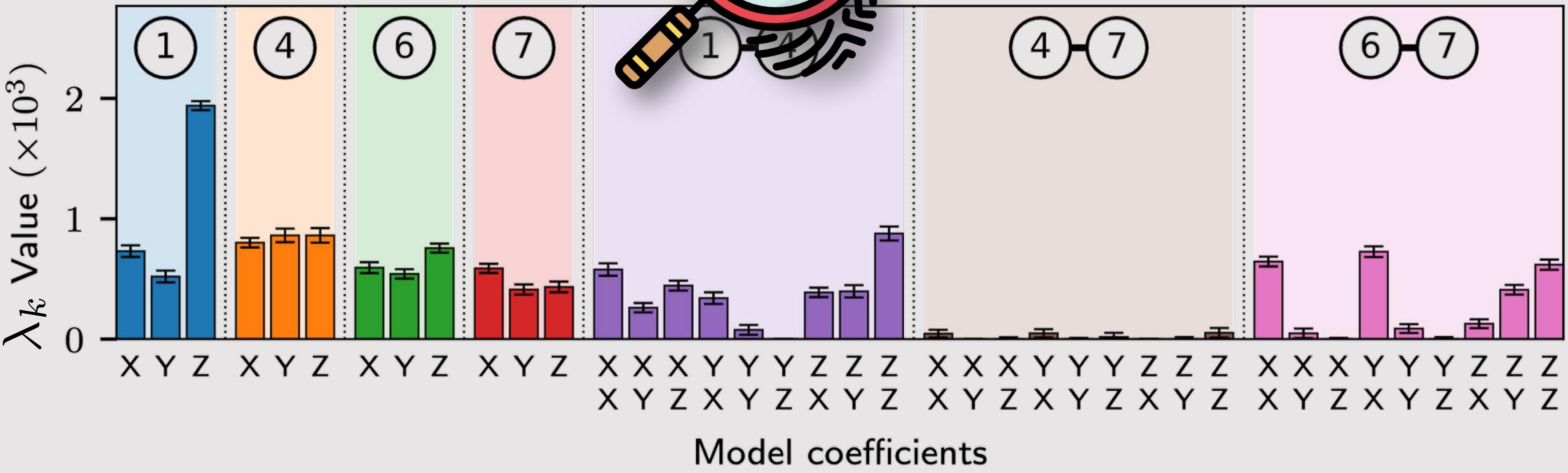
icon: Eucalyp

Fingerprint of the noise: sparse Lindblad tomogram



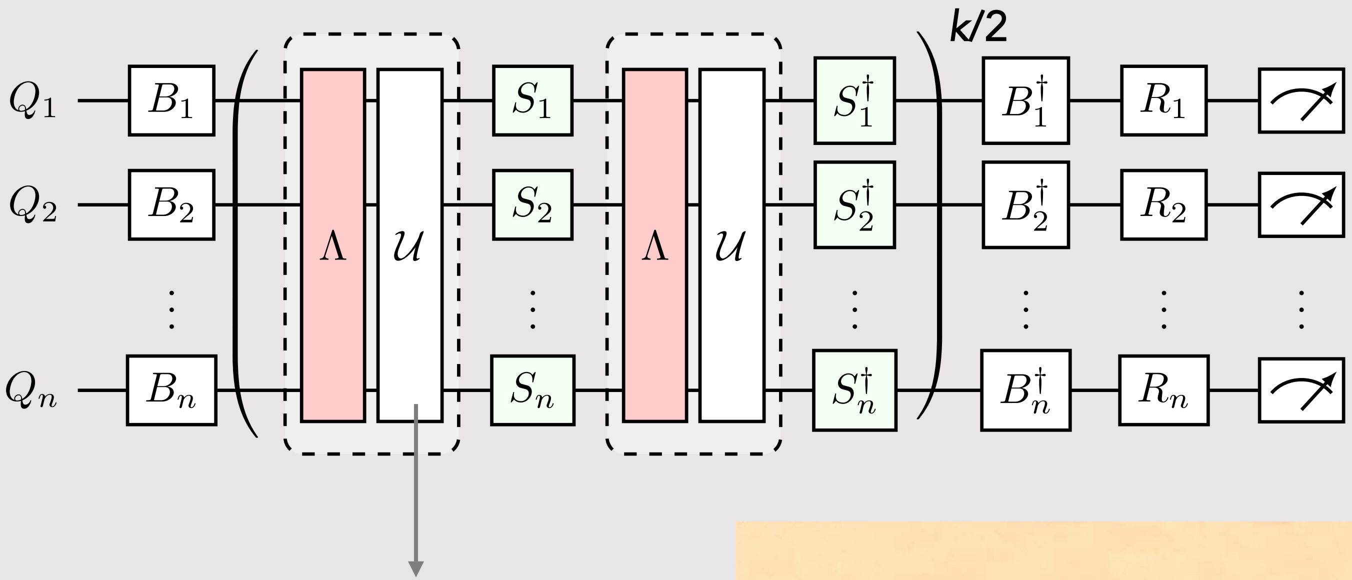
$$\Lambda(\rho) = \exp[\mathcal{L}](\rho)$$

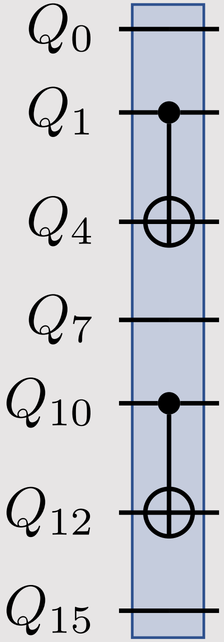
$$\mathcal{L}(\rho) = \sum_{k \in \mathcal{K}} \lambda_k \left(P_k \rho P_k^\dagger - \rho \right)$$

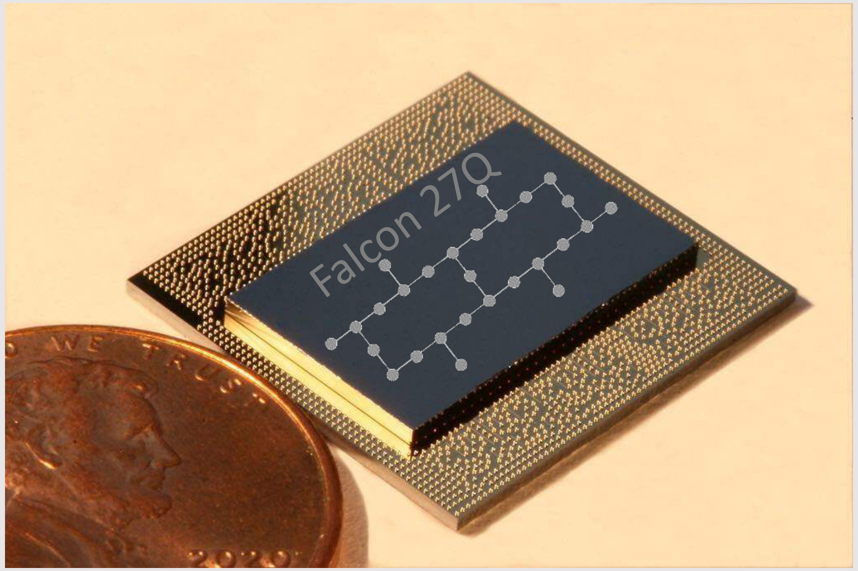


Fingerprint image: freepik

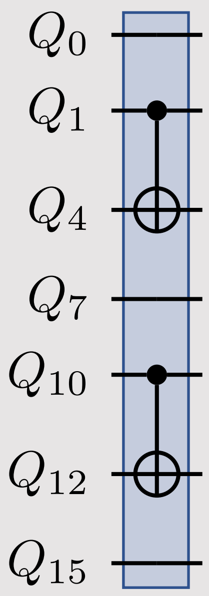
Experiment on a larger, mixed layer



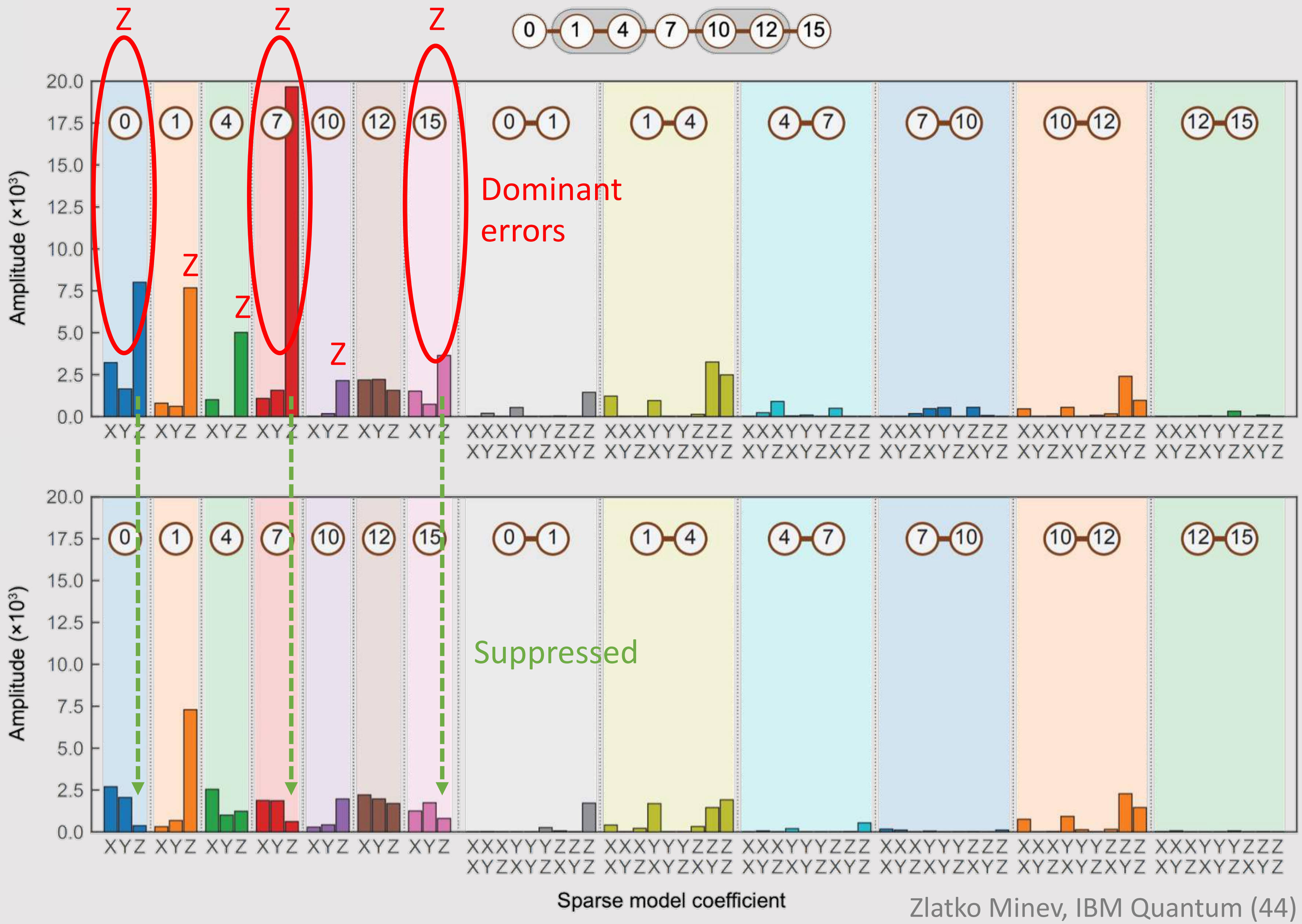




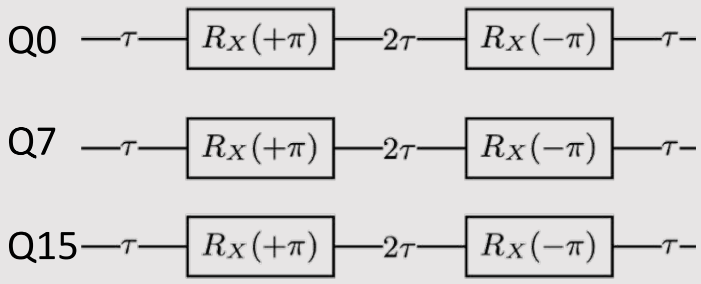
Diagnosing errors with the Lindblad tomogram



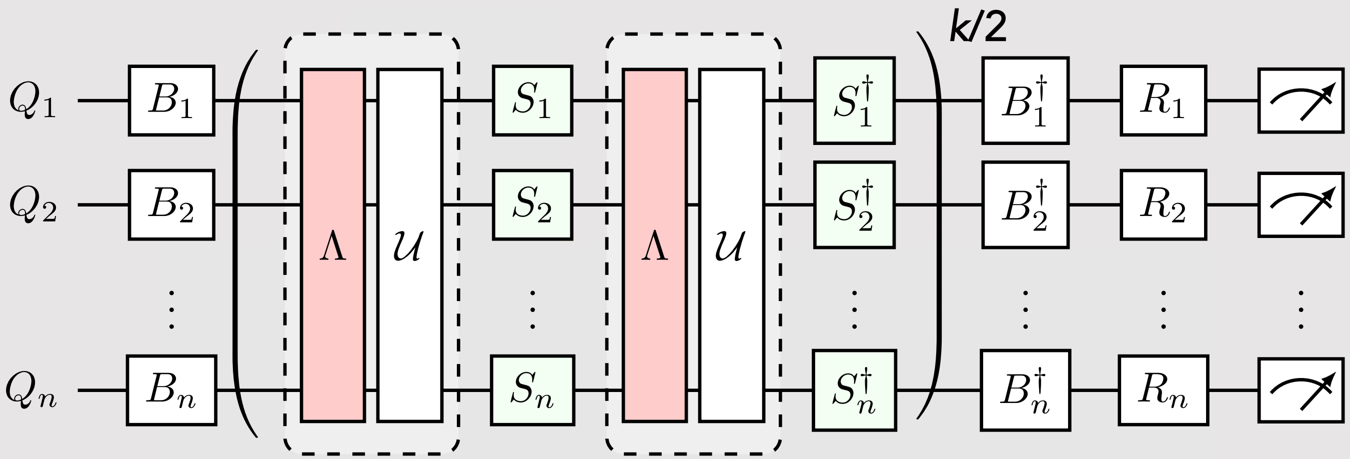
Without dynamical decoupling (DD)

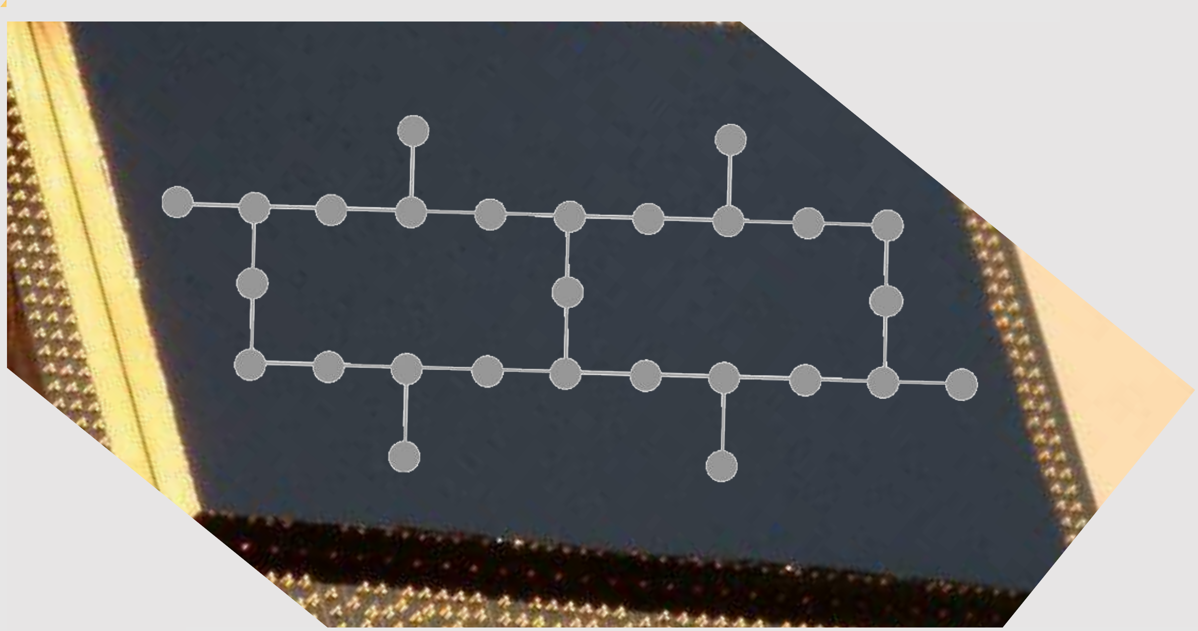


With DD on idle qubits

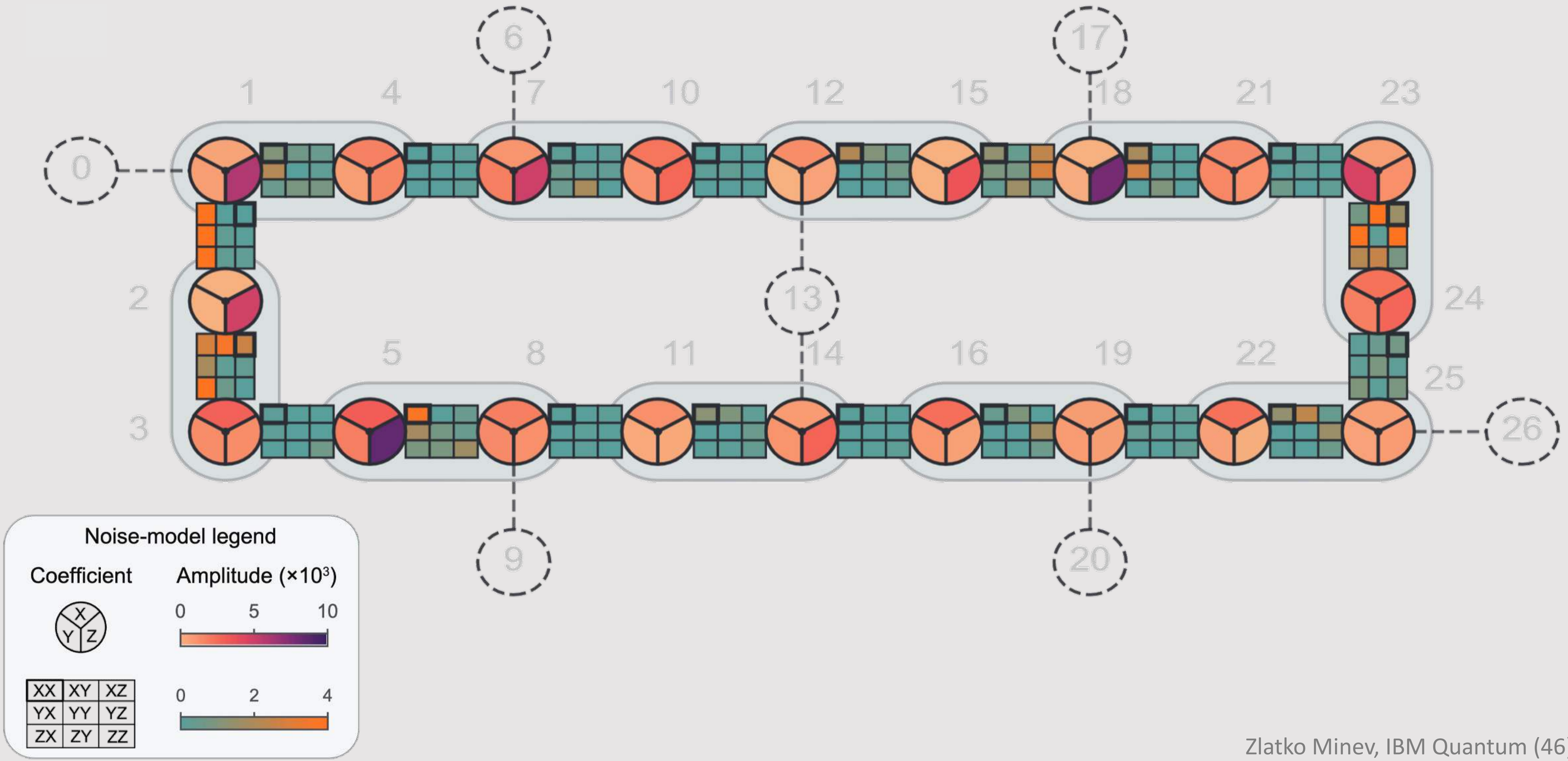


Experiment on a 20Q layer





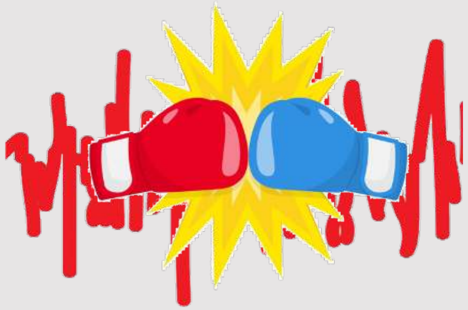
20 qubit layer tomogram



Outline

Idea

Learn



Cancel

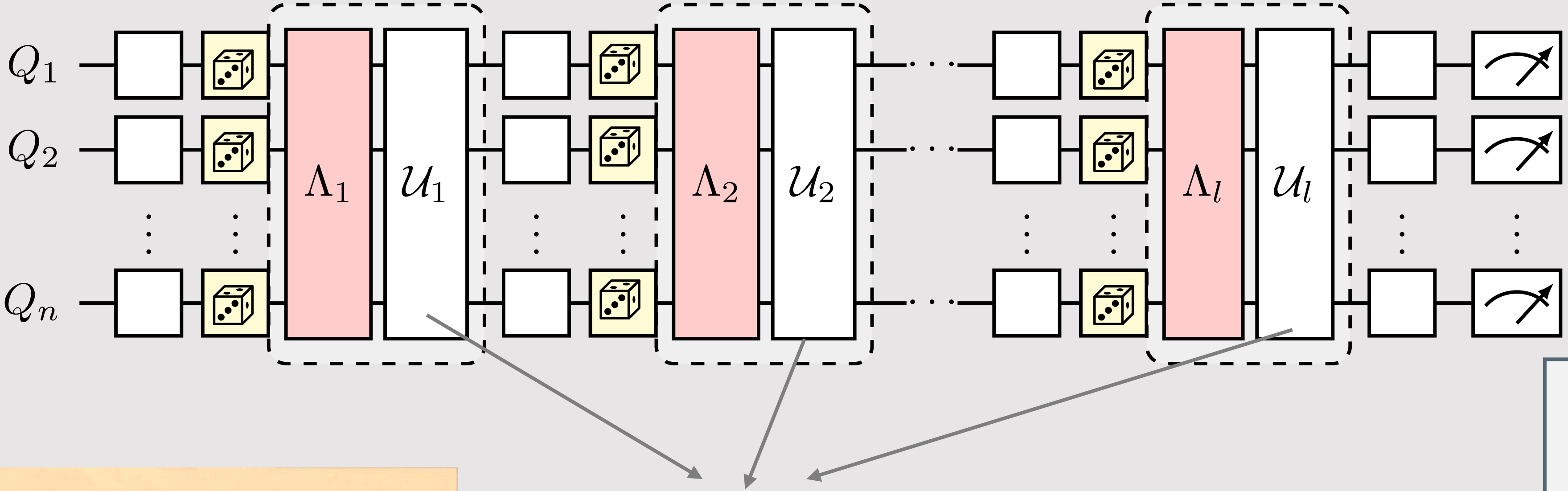
(realization)

PEC mitigation:

inverting noise to obtain expectation values

Icon: Flaticon

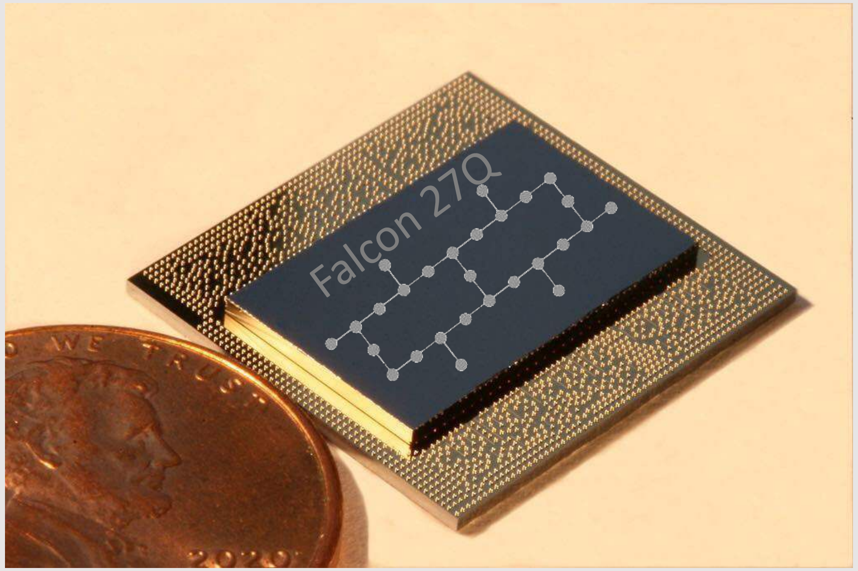
Simple example: mitigation of repeated cNOT

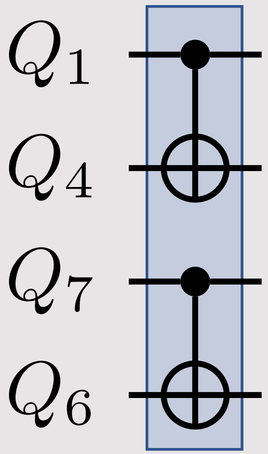


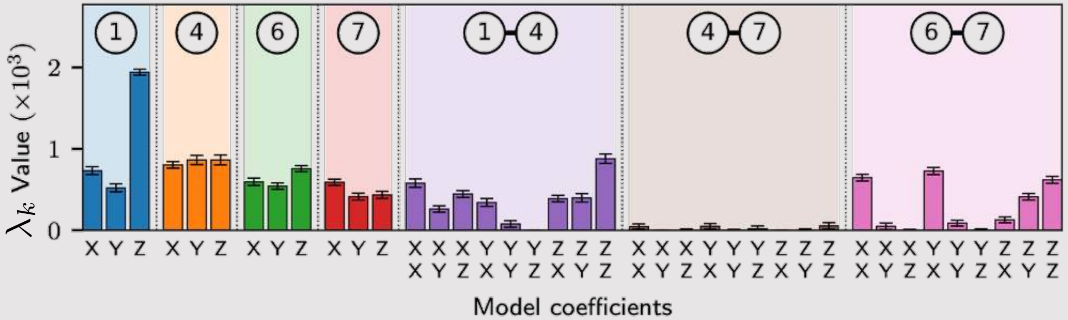
$$\mathrm{CNOT}^{2n} = I$$

<II> = <IZ> = 1

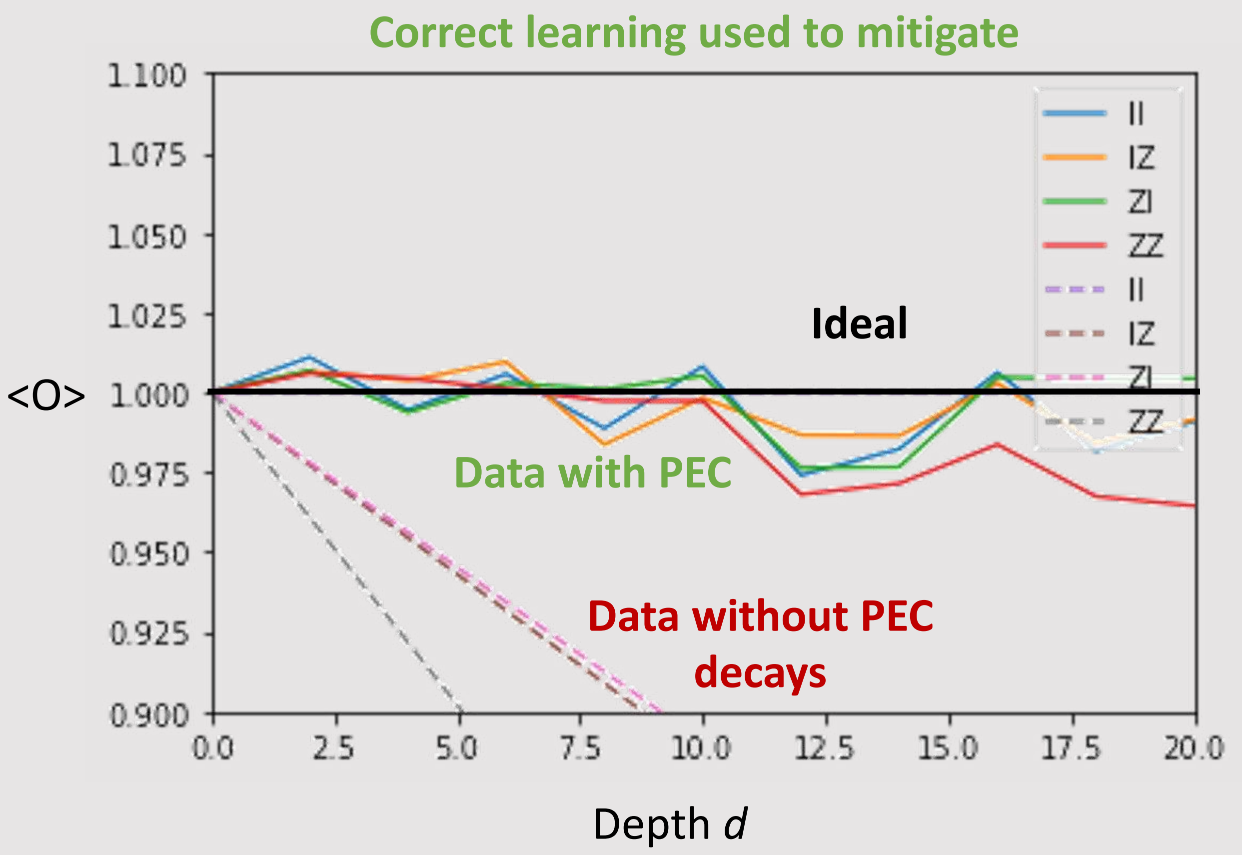
<ZI> = <ZZ> = 1

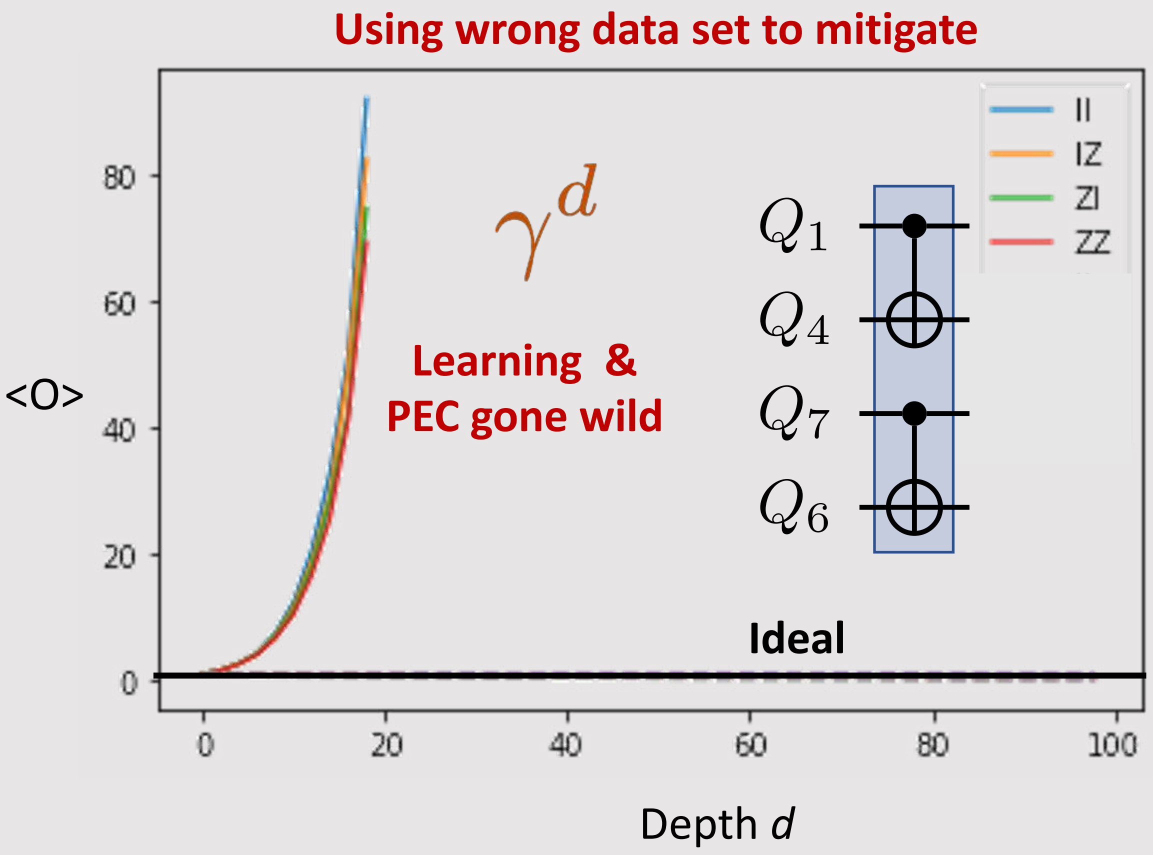




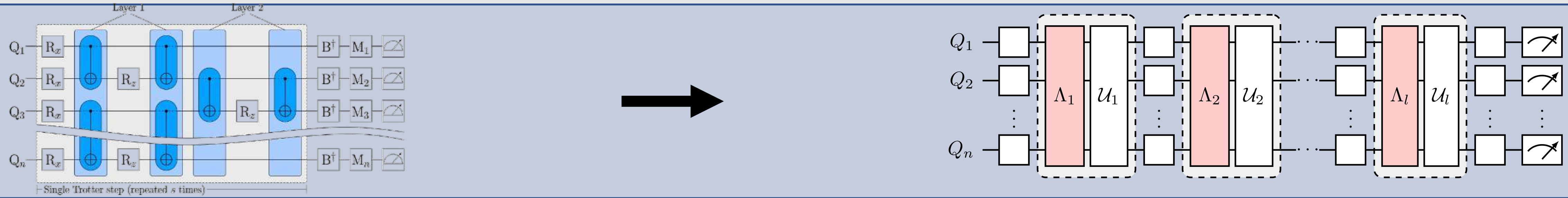


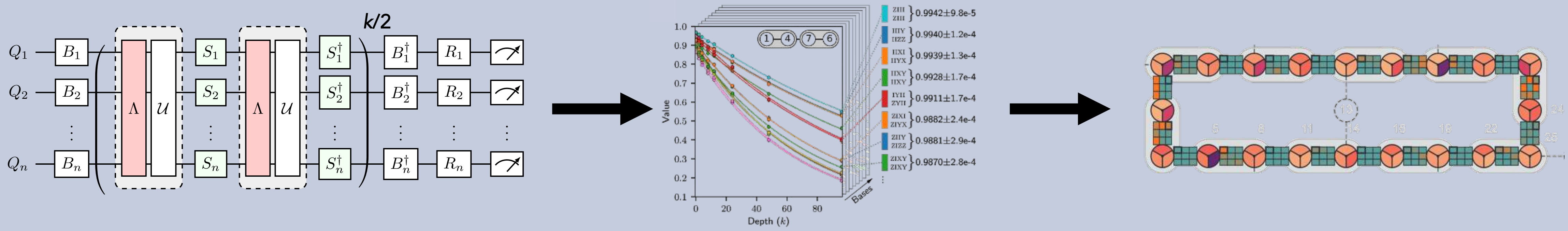
Simple example: mitigation of repeated cNOT

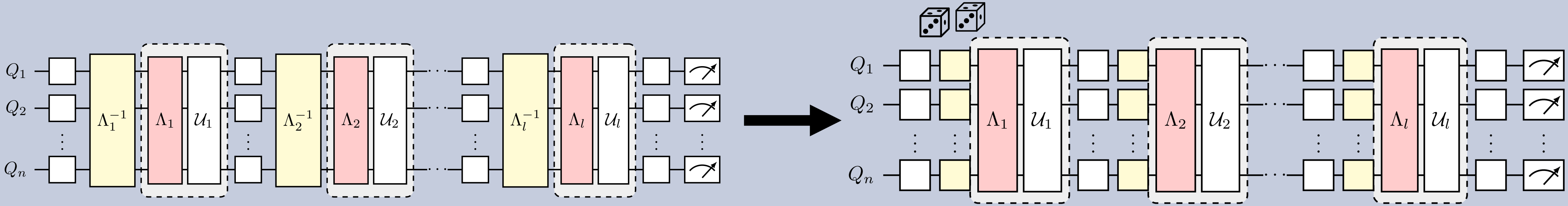




Protocol overview







Ising model Trotterized time evolution

$$\vec{M} := \sum_n \left(\langle X \rangle_n , \langle Y \rangle_n , \langle Z \rangle_n \right) / N$$

Layer 1

Layer 2

$\gamma_1 = 1.0309 \pm 8.40 \cdot 10^{-5}$ $\gamma_2 = 1.0384 \pm 2.20 \cdot 10^{-4}$

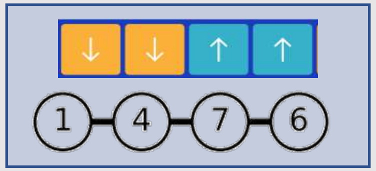
Observables for *N* = 4 (then 10)

XIII	YIII	ZIII
IXII	IYII	IZII
IIXI	IIYI	IIZI
IIIX	IIIY	IIIZ

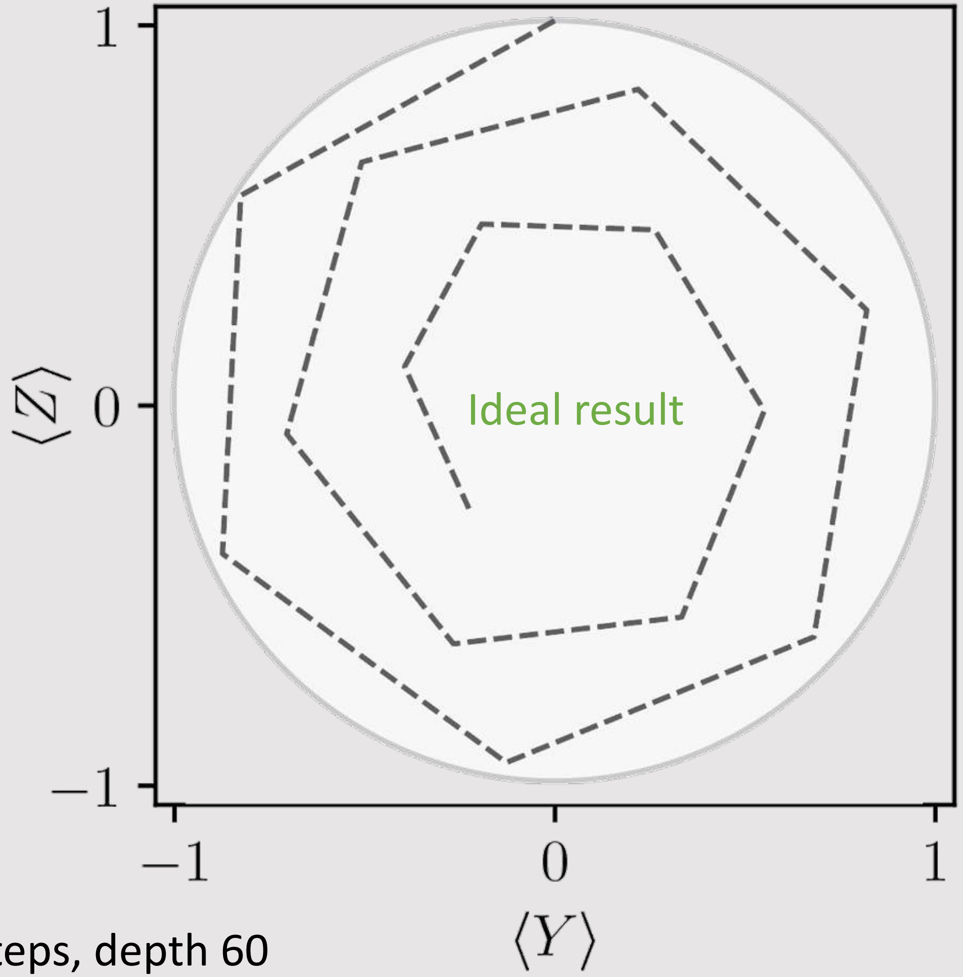
$h = 1, J = \quad 0.15, \delta t = 1/4$

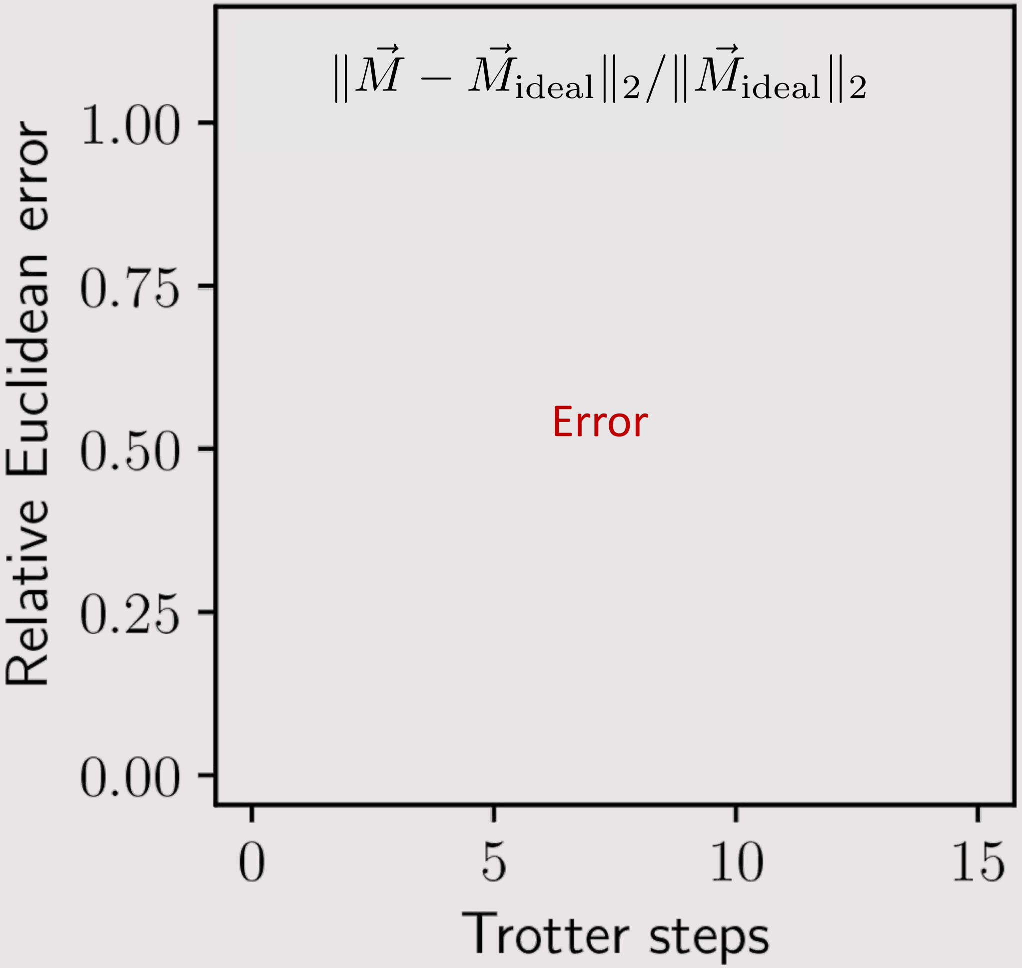
Top image: Samuel Velasco/Quanta Magazine

Ideal Ising model evolution



$$\vec{M} := \sum_n \left(\langle X \rangle_n , \langle Y \rangle_n , \langle Z \rangle_n \right) / N$$

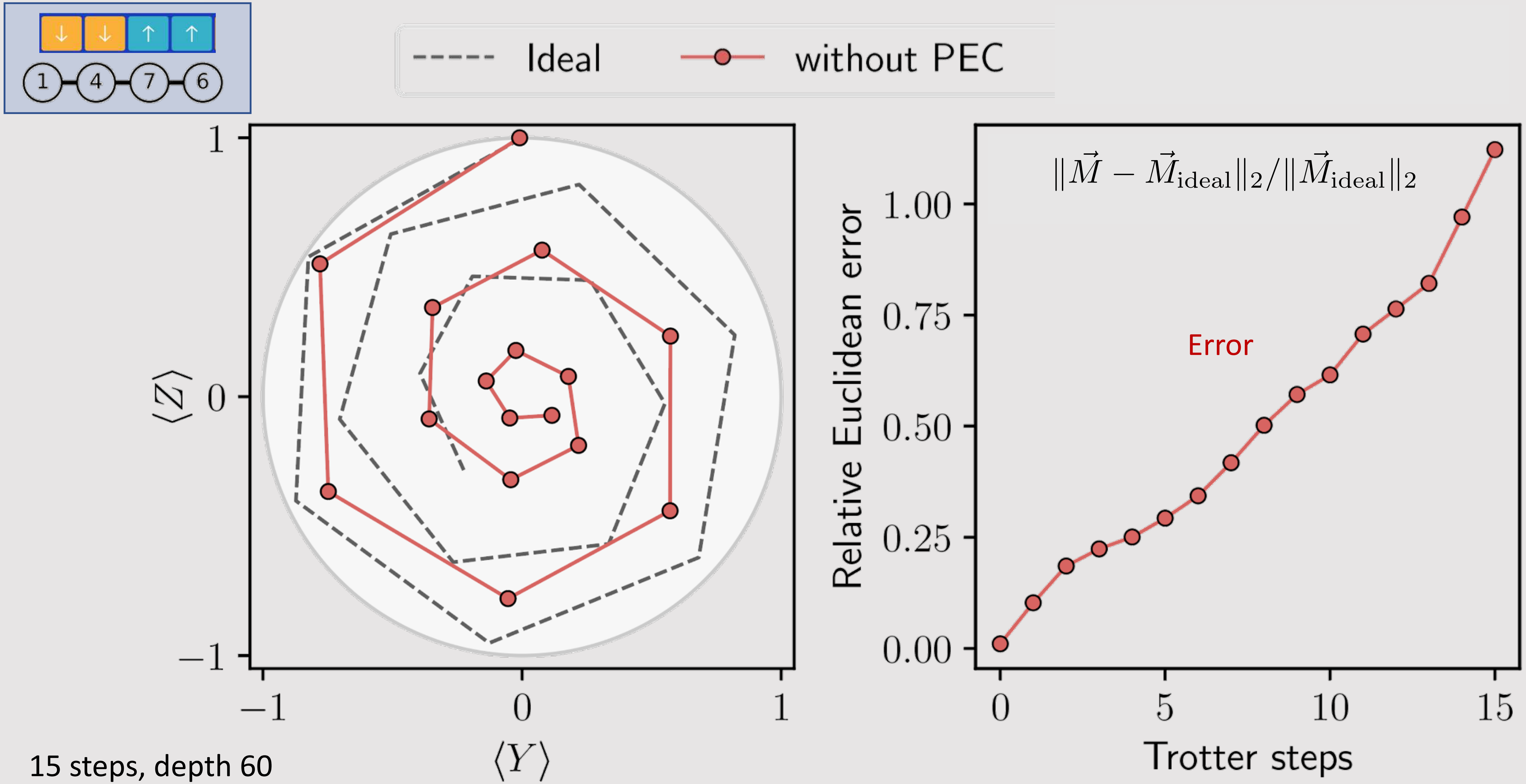




4Q, 15 steps, depth 60

$h = 1, J = -0.15, \delta t = 1/4$

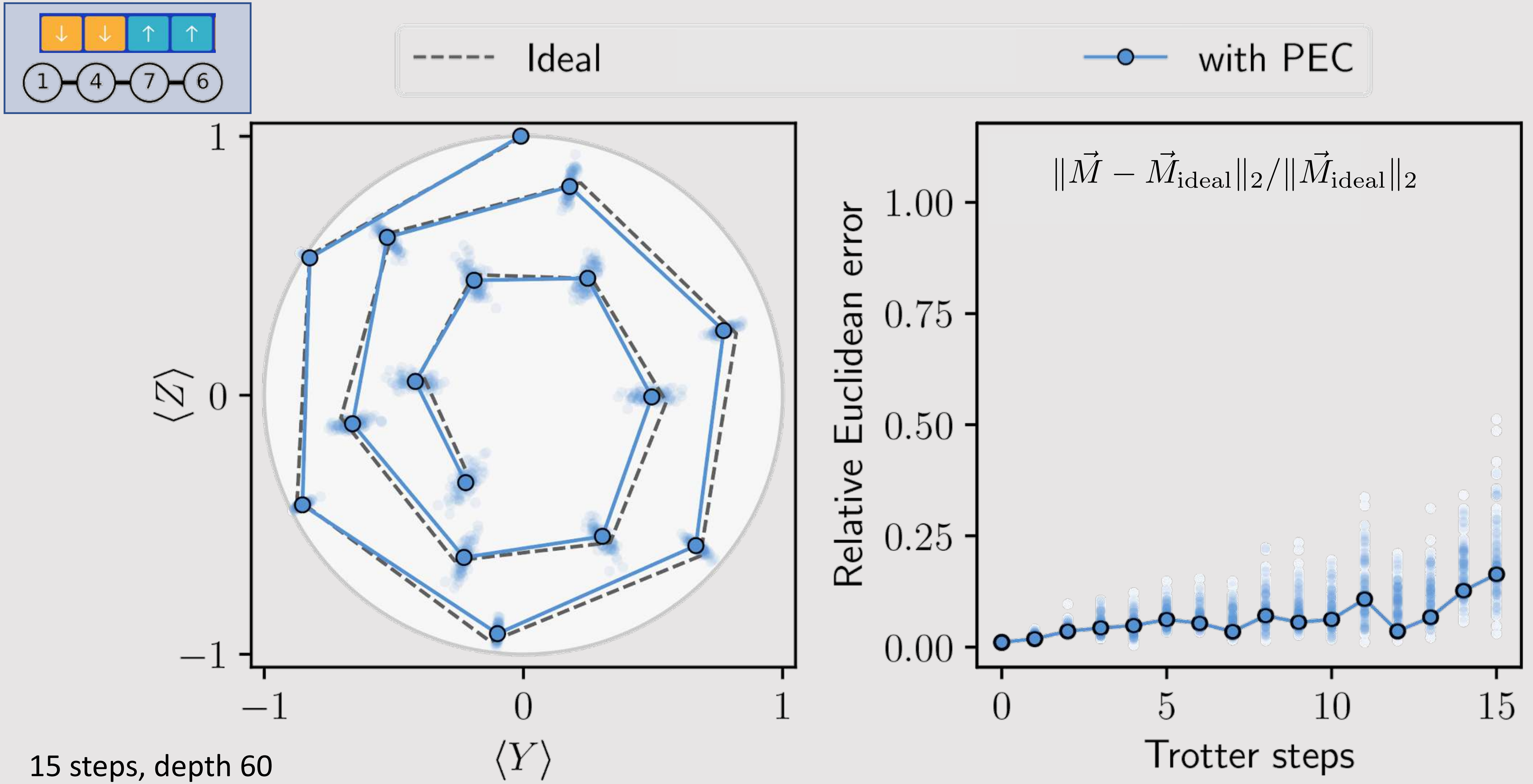
Without PEC: but with DD & twirl readout mitigation



15 steps, depth 60

$h = 1, J = -0.15, \delta t = 1/4$

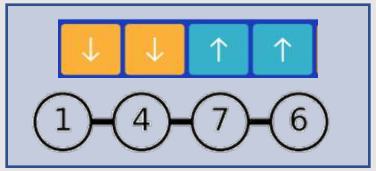
With PEC

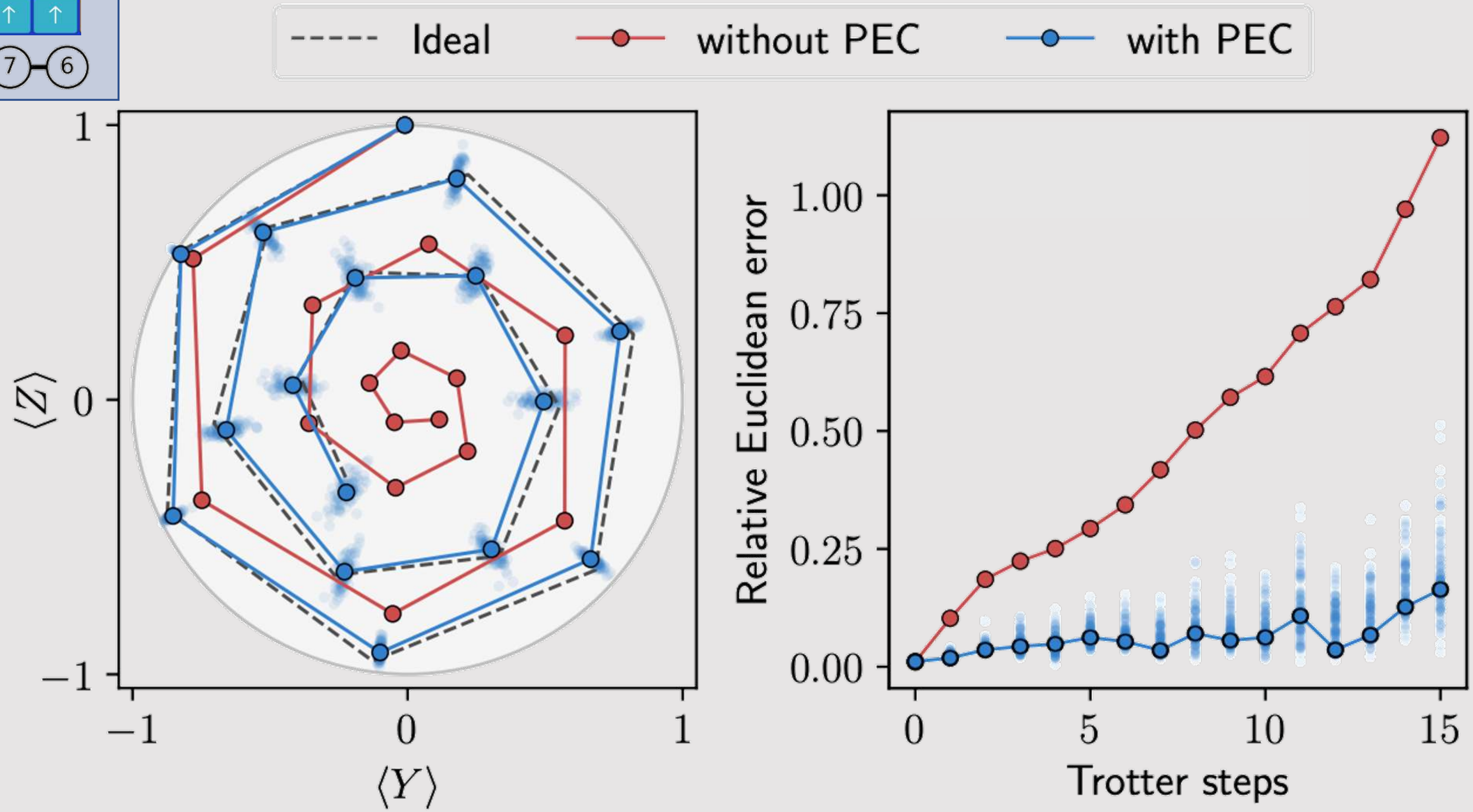


15 steps, depth 60

$h = 1, J = -0.15, \delta t = 1/4$

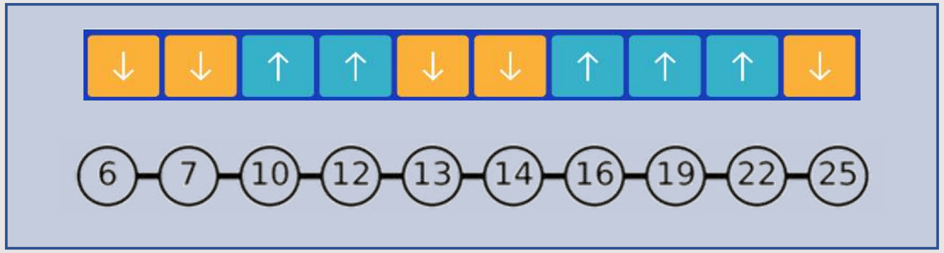
With vs. without PEC

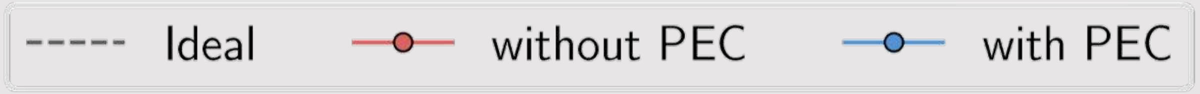


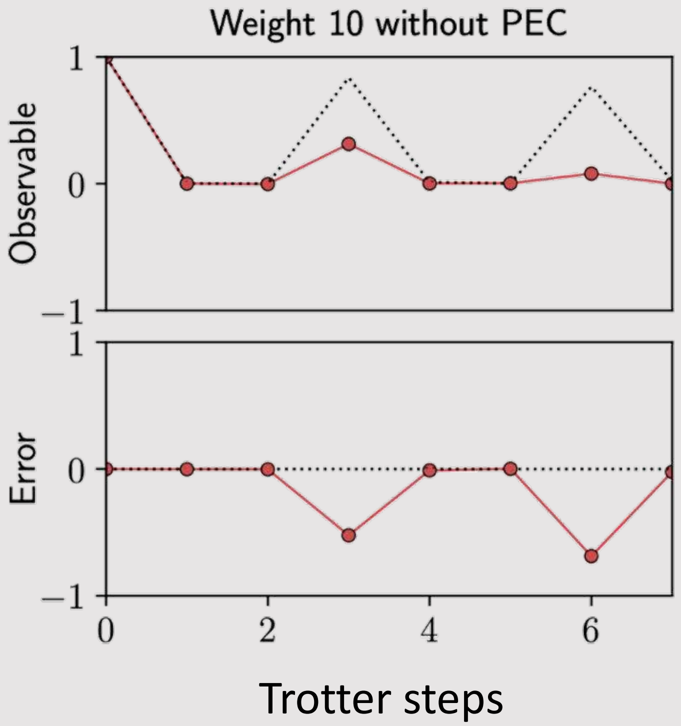


$h = 1, J = -0.15, \delta t = 1/4$

With vs without PEC: 10 qubit high-weight observables



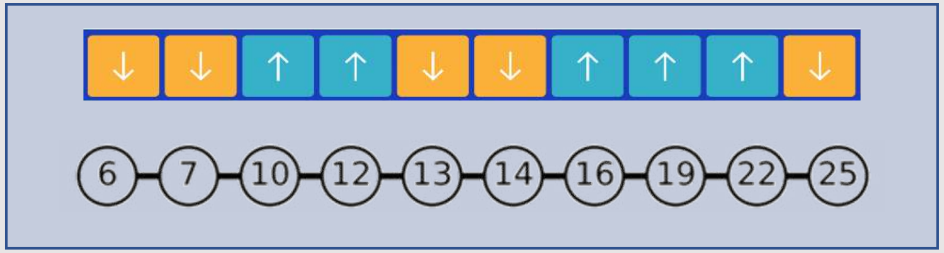


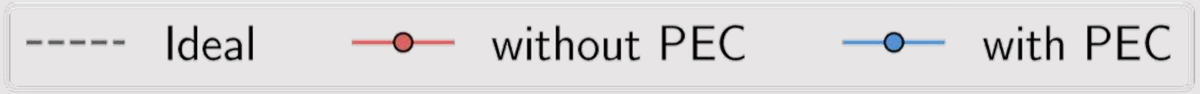


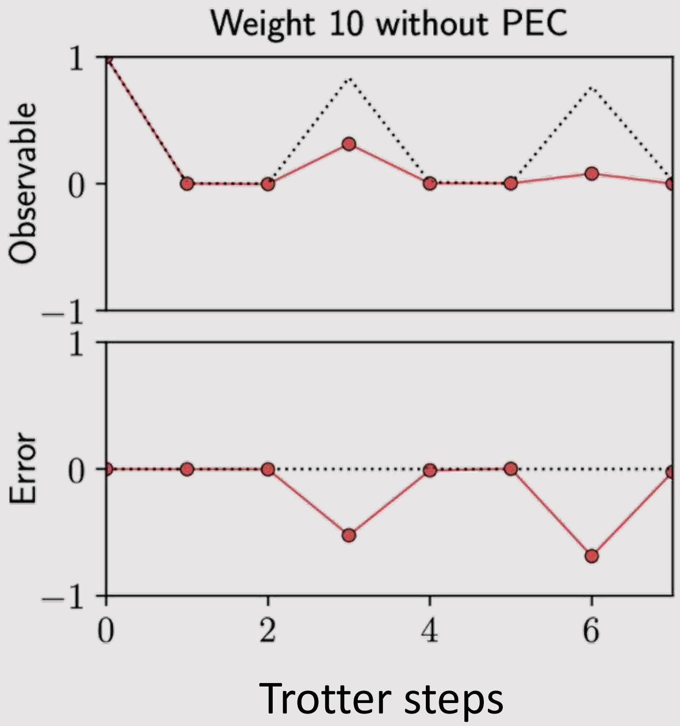
ZZZZZZZZZZ

$h = 1, J = -0.5236, \delta t = 1/4$

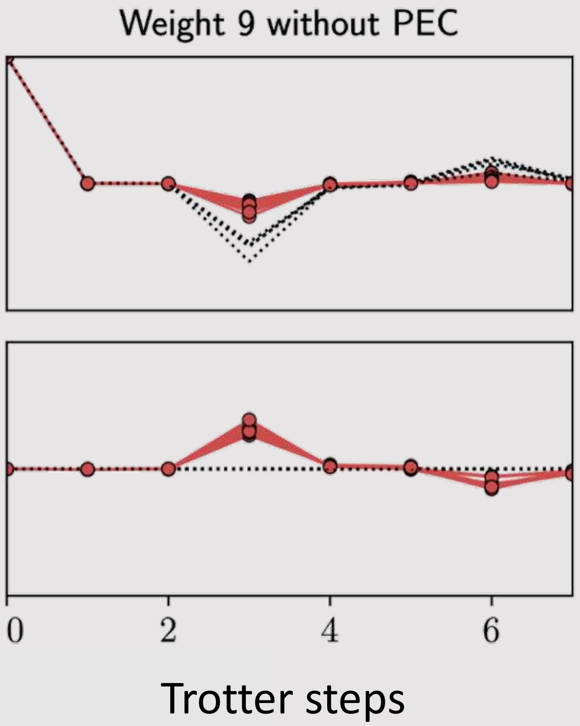
With vs without PEC: 10 qubit high-weight observables







ZZZZZZZZZZ



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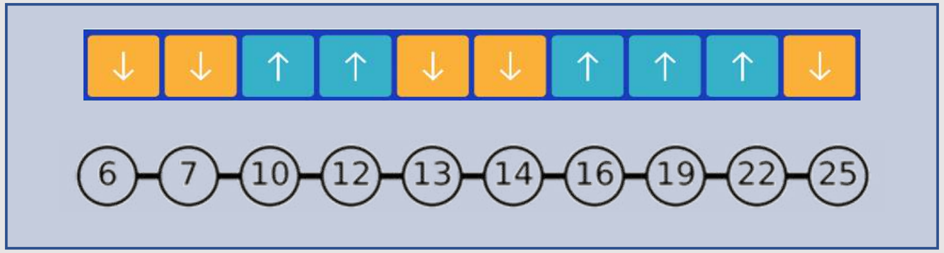
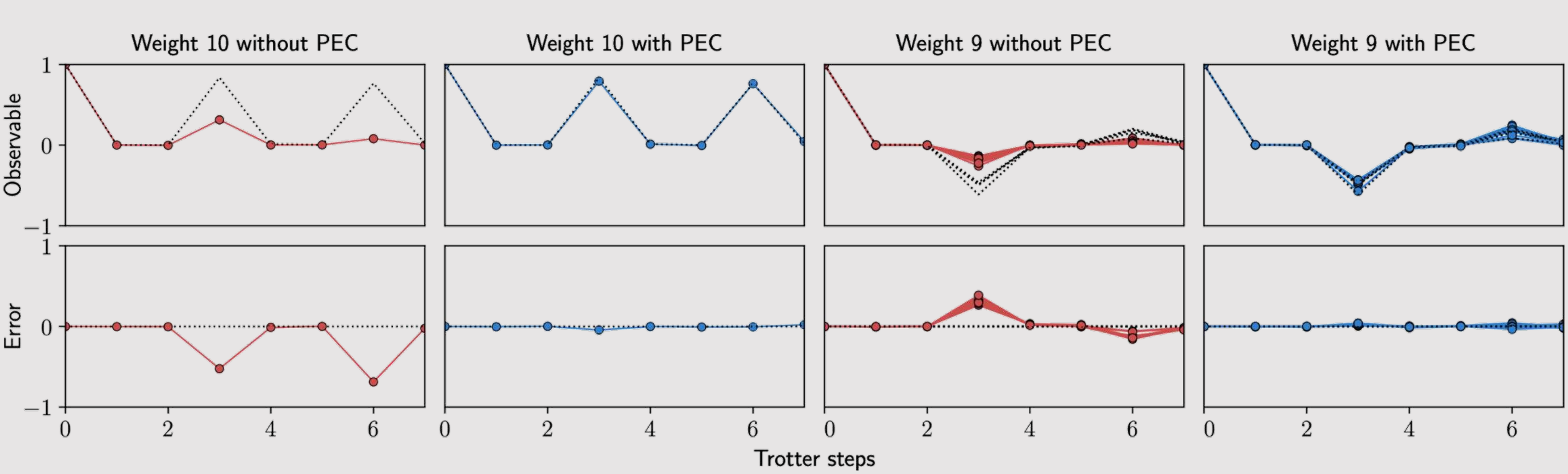
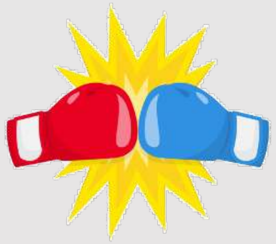
$h = 1, J = -0.5236, \delta t = 1/4$

With vs without PEC: 10 qubit high-weight observables



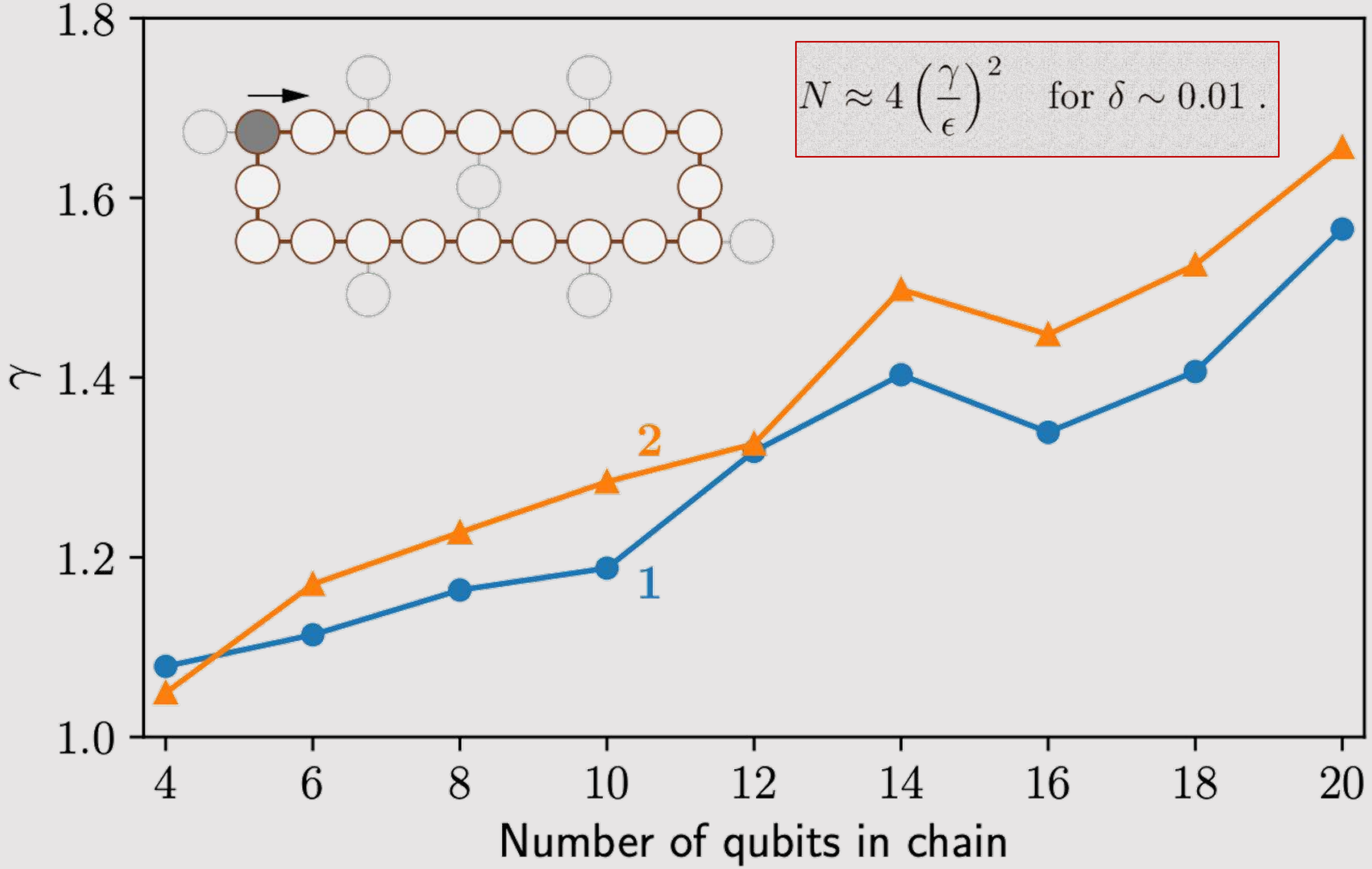




$h = 1, J = -0.5236, \delta t = 1/4$

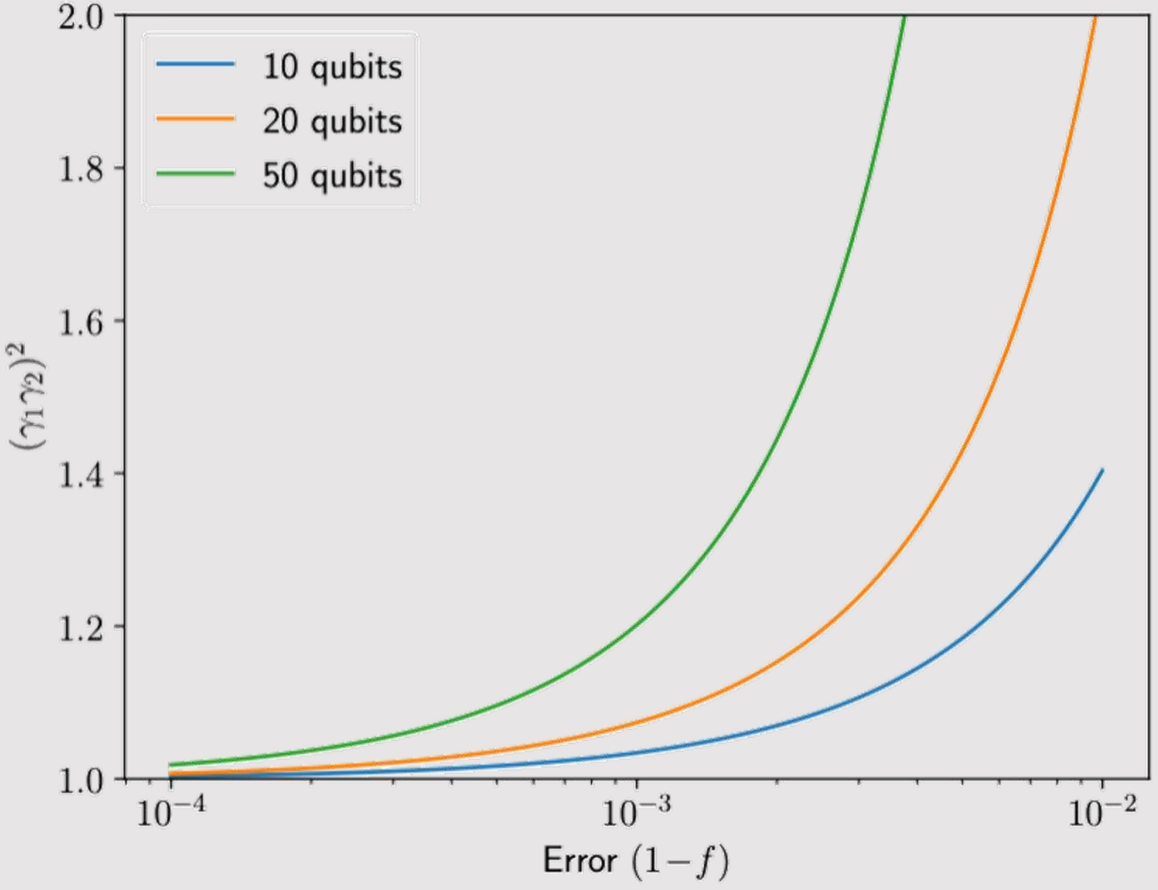
Error budget and scaling

Mitigation sampling overhead

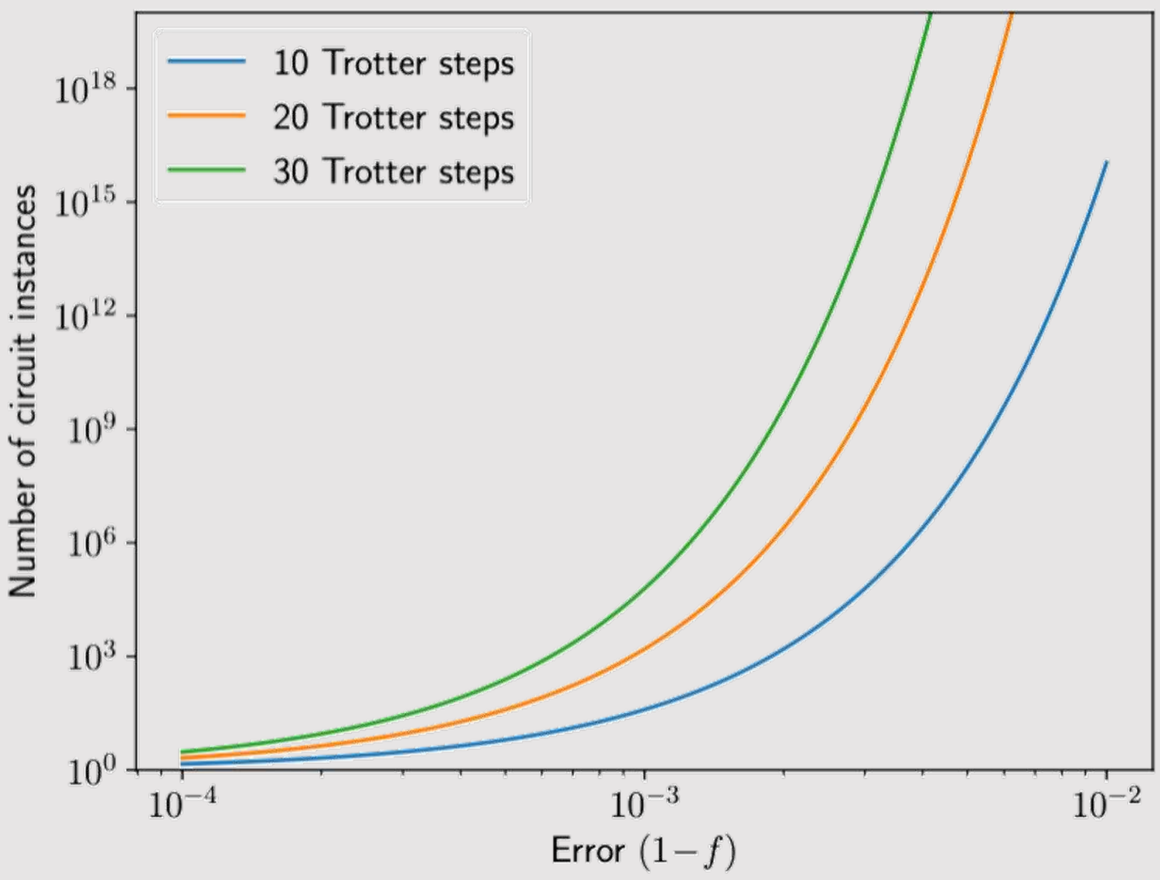


Outlook

Runtime & lower gate errors!



50 qubits



CLOPS: A. Wack, et al., Quality, speed, and scale: three key attributes to measure the performance of near-term quantum computers (2021).

Path to quantum computing

<u>Noise-free</u> estimators can be obtained from noisy quantum computers TODAY, at a runtime cost that is exponential in number of qubits *n* and circuit depth *d*

$$\text{Runtime} = (1/\delta) \times d \times (\gamma)^{n*d} \text{ seconds}$$

d is the depth of the quantum circuit

δ is a measure of how many circuit layer operations that can be done per second (increase by pushing speed)

γ is a measure of the collective quantum noise (increasing quality brings it closer to 1)

n is the number of operational qubits (increase by pushing scale)

IBM Quantum

Thank you!

Lindbladian learning is accurate, efficient, and scalable

Powerful characterization and benchmarking tool

Enable the study and mitigation of noise in quantum processors at a new scale

Ewout van den Berg, Zlatko K. Minev, Abhinav Kandala, Kristan Temme
arXiv:2201.09866 (2022)

Thanks to discussion with many folks on the broader IBM team.

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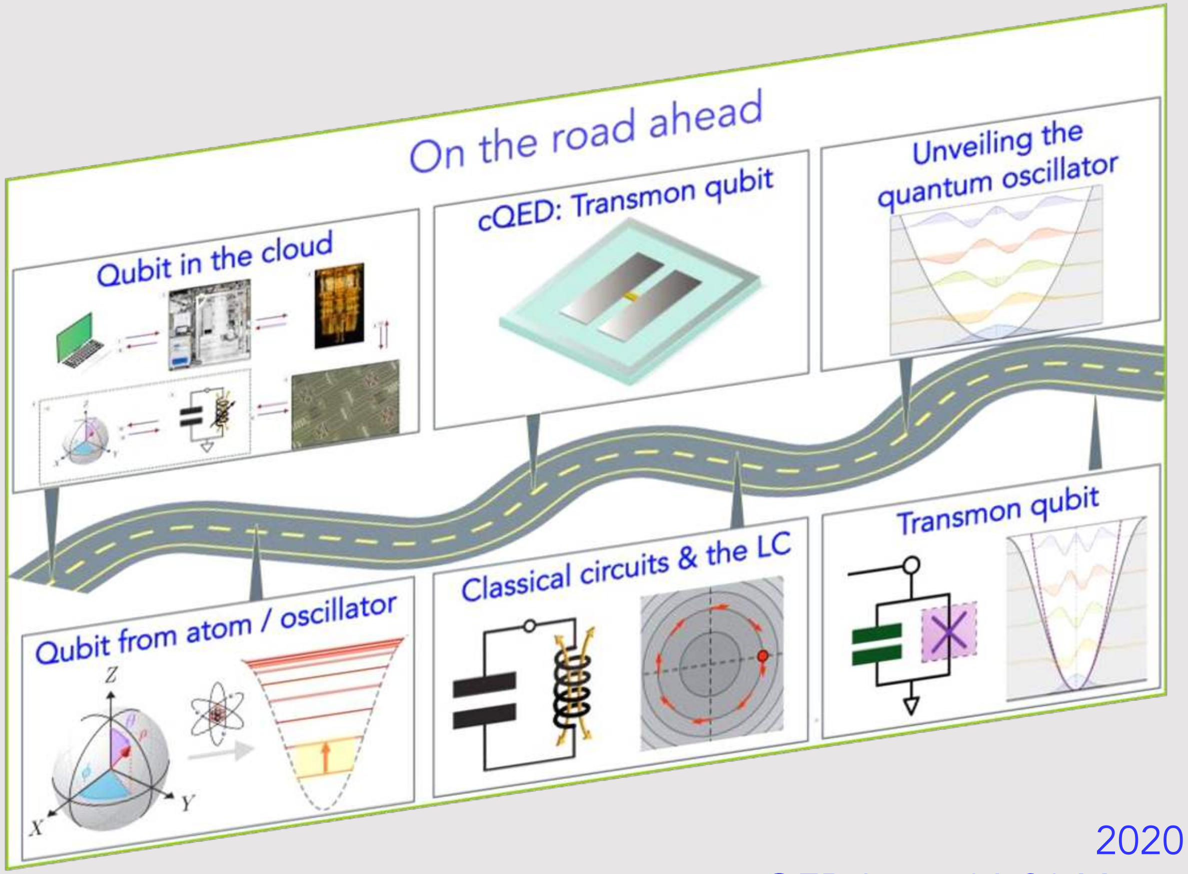
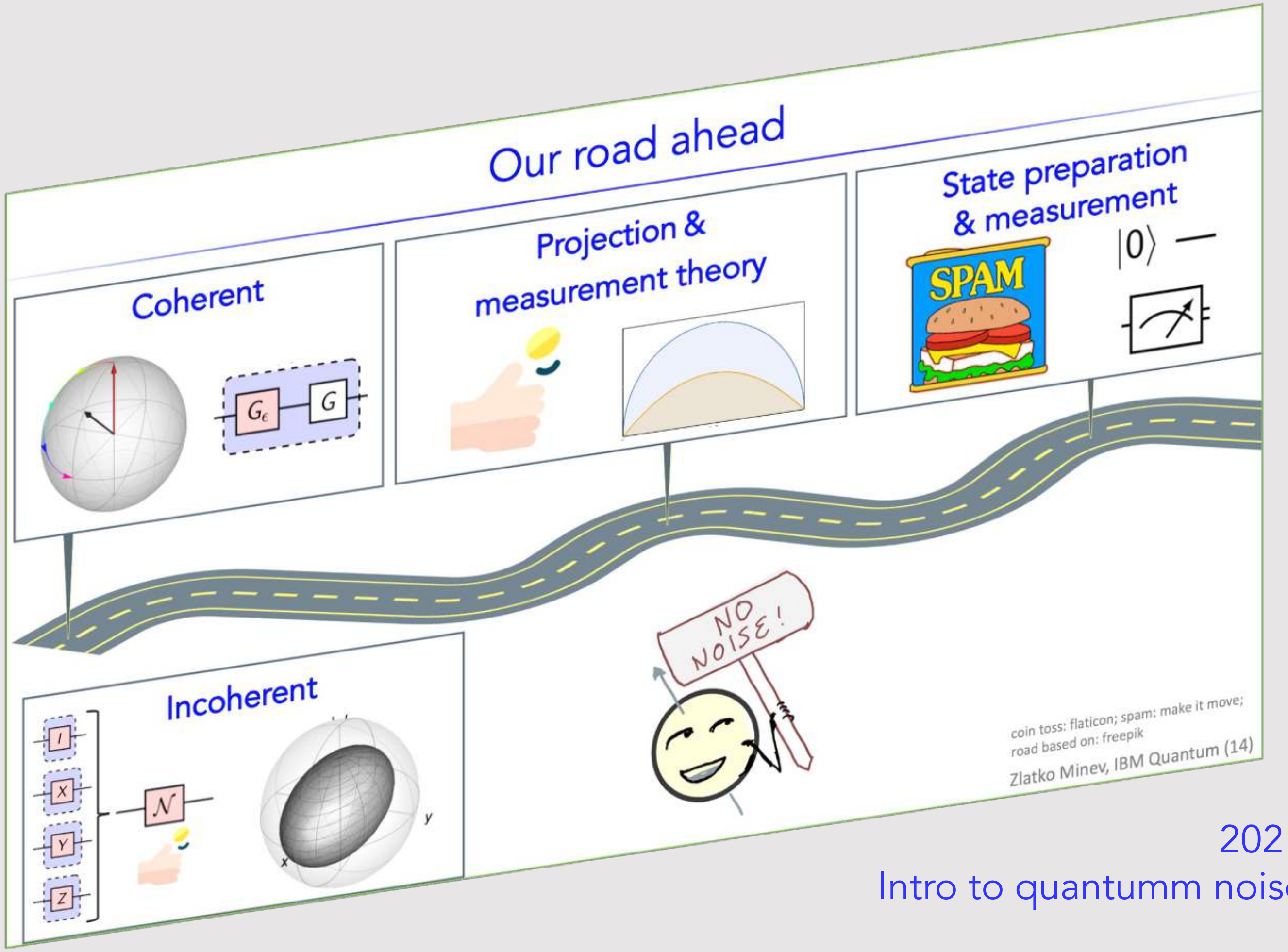
Global Summer Schools QGSS



2020
cQED Lecs. 16-21 Minev



2021
Intro to quantumm noise

2022 coming up soon!

Tech Note T23: Probabilistic error cancellation: paper notes

Summary for key notation of *Probabilistic error cancellation with sparse Pauli-Lindblad models on noisy quantum processors* [Ref. van den Berg et al. (2020)]

Symbol	Description	Value
	Circuit constants & indices	
n	number of qubits in circuit	$n = 1, 2, 3, \ldots$
l	number of layers in circuit	$l = 1, 2, 3, \ldots$
i	layer index within circuit	$i = 1, \ldots, l$
	Pauli-Lindblad model inputs	
k	model coefficient index, corresponds to a Pauli	$k = 1, 2, 3, \ldots$ or equiv. n-qubit Pauli string
b	fidelity vector indices	$b = 1, 2, \ldots$ or equiv. n-qubit Pauli string
$\mathcal{K}$	set of Pauli fidelity support indices for the sparse model $\mathcal{L}$ of Λ. Small set	$k \in \mathcal{K}$
$\mathcal{B}$	set of benchmark Paulis (Pauli fidelity support indices of Λ). Can be all of them, big set	$b \in \mathcal{B}$
N	number of error-mitigated circuit instances	$N = 1, 2, 3, \ldots$
	Pauli-Lindblad model variables	
γ	sampling overhead	$\gamma \geq 1$, $\gamma = \exp\left(\sum_{k\in\mathcal{K}} 2\lambda_k\right)$
$\gamma_i, \gamma(l)$	sampling overhead for the i-th layer and a total of l layers	$\gamma(l) = \Pi_{i=1}^{l} \gamma_i$
λ_k	k-th model coefficient	$\lambda_k \geq 0$
w_k	noise model weight in Λ factoring	$w_k := \frac{1}{2}\left(1 + e^{-2\lambda_k}\right)$
f_b	Pauli Λ fidelity of b-th Pauli index	$f_b := \frac{1}{2^n} \operatorname{Tr}\left(P_b^\dagger \Lambda(P_b)\right)$
f	vector of Pauli fidelities of Λ	$f = \{f_b\}_{b\in\mathcal{B}}$
$\hat{f}$	fidelity estimates for a set of	
$M(\mathcal{B},\mathcal{K})$	binary matrix with entries $M_{b,k} = \langle b, k\rangle_{sp}$ where the symplectic product is 0 is the Paulis commute and 1 if they do not	$\log(f) = -2M(\mathcal{B},\mathcal{K})\lambda$ (element-wise log) (used to fit with $\lambda \geq 0$)
	Quantum operators and superoperators	
P_k	Pauli operator, indexed by k	$P_k \in \{I, X, Y, Z\}^{\otimes n}$
$U, \mathcal{U}, \tilde{\mathcal{U}}_i$	unitary ideal gate; tilde: noisy i-th layer unitary	
Λ, Λ_i	noise channel	$\Lambda(\rho) = \exp[\mathcal{L}](\rho) = \Pi_{k\in\mathcal{K}}\left(w_k \cdot + (1-w_k) P_k \cdot P_k^\dagger\right)\rho$
Λ^{-1}	inverse noise map	$\Lambda^{-1}(\rho) = \exp[-\mathcal{L}](\rho) = \gamma \Pi_{k\in\mathcal{K}}\left(w_k \cdot - (1-w_k) P_k \cdot P_k^\dagger\right)\rho$
$\mathcal{L}(\rho)$	Lindblad generator	$\mathcal{L}(\rho) = \sum_{k\in\mathcal{K}} \lambda_k (P_k \rho P_k - \rho)$
$\left\langle \hat{A}_N \right\rangle$	average error mitigated estimate of $\left\langle \hat{A} \right\rangle$ for some operator $\hat{A}$ using N circuit instances	

23.1 Common questions

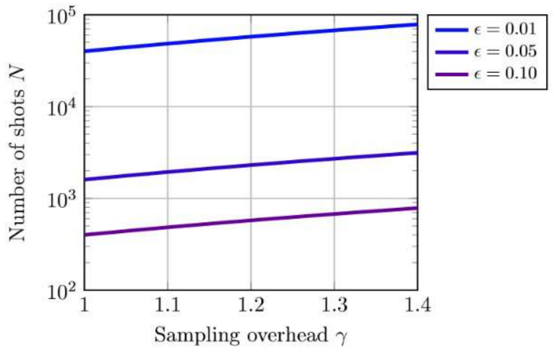
How many random circuits to run? For a maximum error bound ϵ between the the ideal and mitigated expectation values $\left|\left\langle \hat{A} \right\rangle_{\text{ideal}} - \left\langle \hat{A}_N \right\rangle\right| \leq \epsilon$ satisfied with a probability $1-\delta$ (assuming a weak enough noise, $C^{l\tau} \approx 1$), we can solve for the number of error-mitigated, random circuit instances N in van den Berg et al. (2020)

$$\epsilon = \gamma \frac{\sqrt{2\log(2/\delta)}}{\sqrt{N}},$$

$$\therefore \quad N = 2\log(2/\delta)\left(\frac{\gamma}{\epsilon}\right)^2,$$

$$N \approx 4\left(\frac{\gamma}{\epsilon}\right)^2 \quad \text{for } \delta \sim 0.01\,.$$

For probability $\delta = 2\%$, $\log(2/\delta) = 2$, a small overhead; for $\delta = 0.01$, it is 2.3, and for $\delta = 0.001$, it is 3.3.



Bibliography

E. van den Berg, Z. K. Minev, and K. Temme, arXiv (2020), ISSN 23318422, 2012.09738, URL http://arxiv.org/abs/2012.09738. Nature Physics **16**, 233 (2020), ISSN 1745-2473, URL https://doi.org/10.1038/s41567-020-0847-3http://www.nature.com/articles/s41567-020-0847-3.

Caveat emptor These pages are a work in progress, inevitably imperfect, incomplete, and surely enriched with typos and unannounced inaccuracies. Sources credited in Bibliography to the best of my ability, though certain omissions certainly remain.

Paper and talk notation summary: https://www.zlatko-minev.com/blog/pec-notation