

# A detailed portrait of quantum many-body dynamics

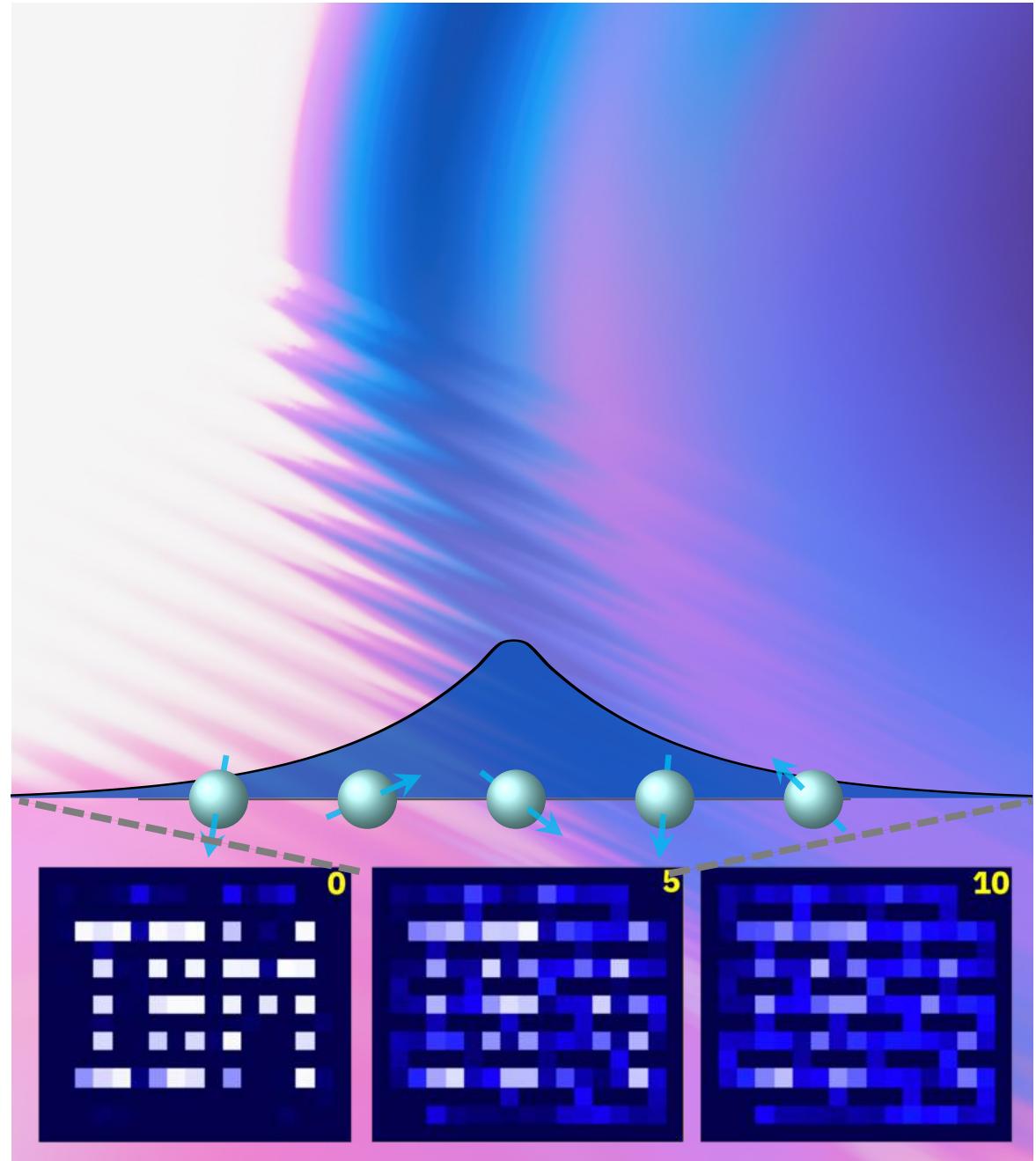
Zlatko K. Minev  
IBM Quantum

Oles Shtanko\*, Derek Wang\*, Haimeng Zhang, Nikhil Harle, Alireza Seif, Ramis Movassagh, Zlatko K. Minev  
arXiv:2307.07552

Acknowledgements: A. Deshpande, O. Dial, A. Eddins, D. Egger, B. Fuller, J. Garrison, D. Hahn, A. Kandala, W. Kirby, D. Layden, H. Liao, D. Luitz, S. Majumder, A. Mezzacapo, T. Prosen, J. Raftery, D. Sels, K. Temme, M. Tepaske, K. Wei, the IBM Quantum team, and many friends and colleagues



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# A detailed portrait of quantum many-body dynamics

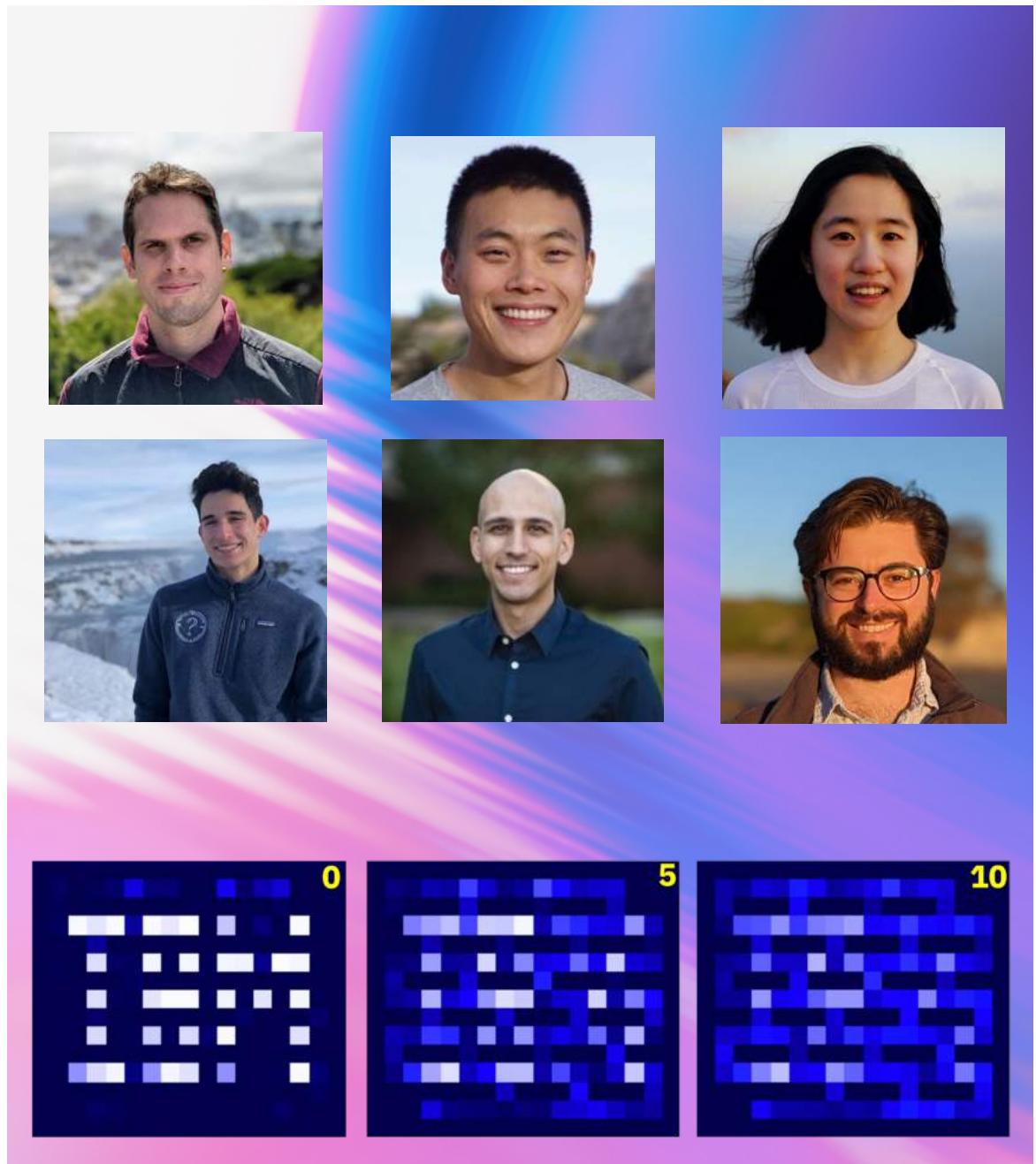
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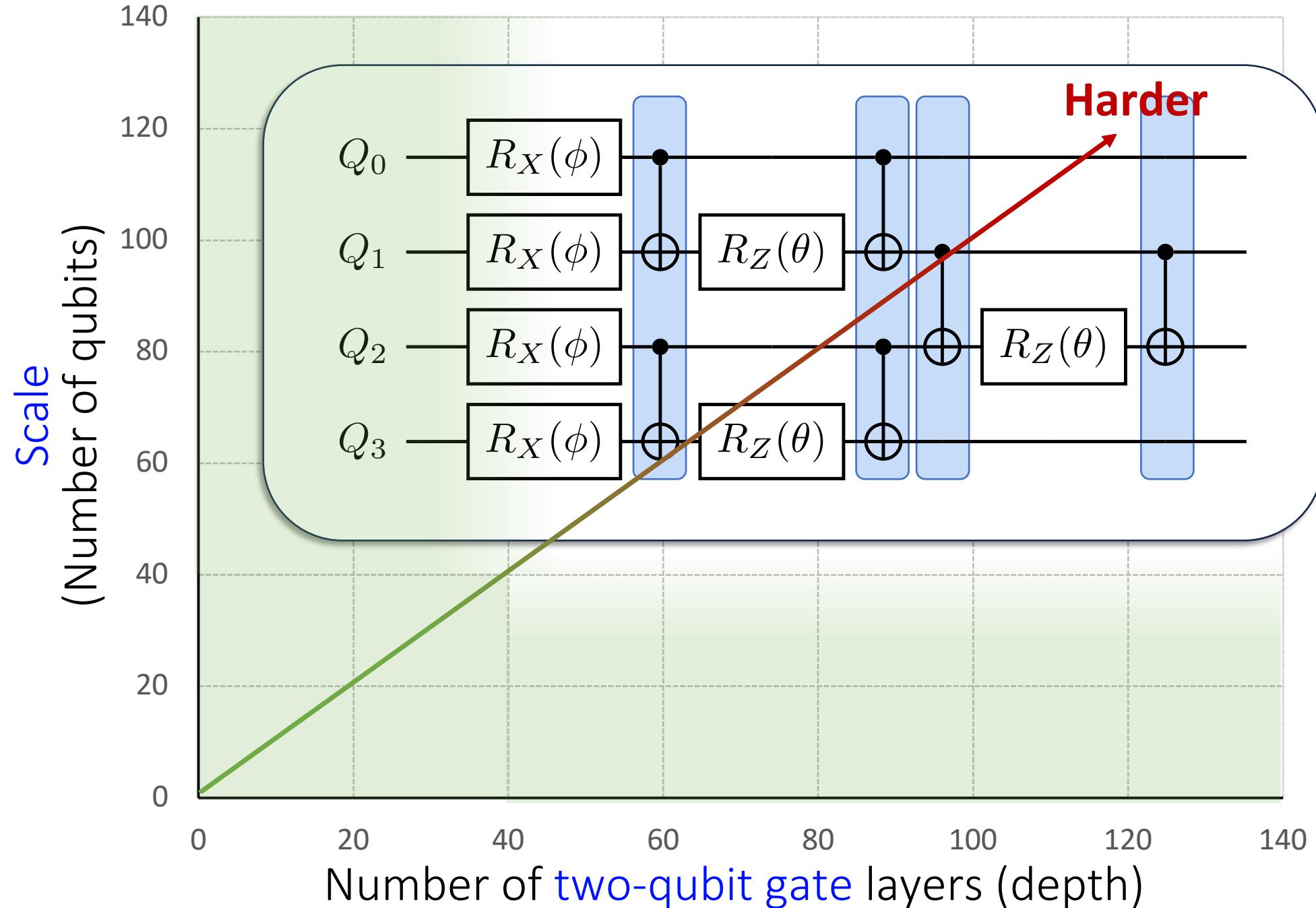
Acknowledgements: A. Deshpande, O. Dial, A. Eddins, D. Egger, B. Fuller, J. Garrison, D. Hahn, A. Kandala, W. Kirby, D. Layden, H. Liao, D. Luitz, S. Majumder, A. Mezzacapo, T. Prosen, J. Raftery, D. Sels, K. Temme, M. Tepaske, K. Wei, the IBM Quantum team, and many friends and colleagues



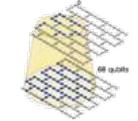
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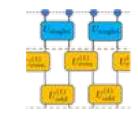
# Quantum dynamics with large-scale superconducting quantum computers



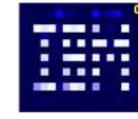
# Some early utility-scale experiments



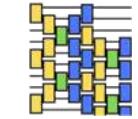
[0] Kim, Eddins, ..., Temme, Kandala.  
Nature 618, 500–505 (2023)



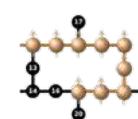
[1] Yu, Zhao, Wei.  
arXiv: 2207.09994 (2022)



[2] Shtanko, Wang, Zhang, Harle, Seif,  
Movassagh, Minev.  
arXiv: 2307.07552 (2023)



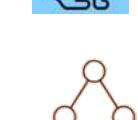
[3] Farrell, Illa, Ciavarella, Savage.  
arXiv: 2308.04481 (2023)



[4] Bäumer, Tripathi, .... Seif,  
Minev. APS Y45.4  
arXiv: 2308.13065 (2023)

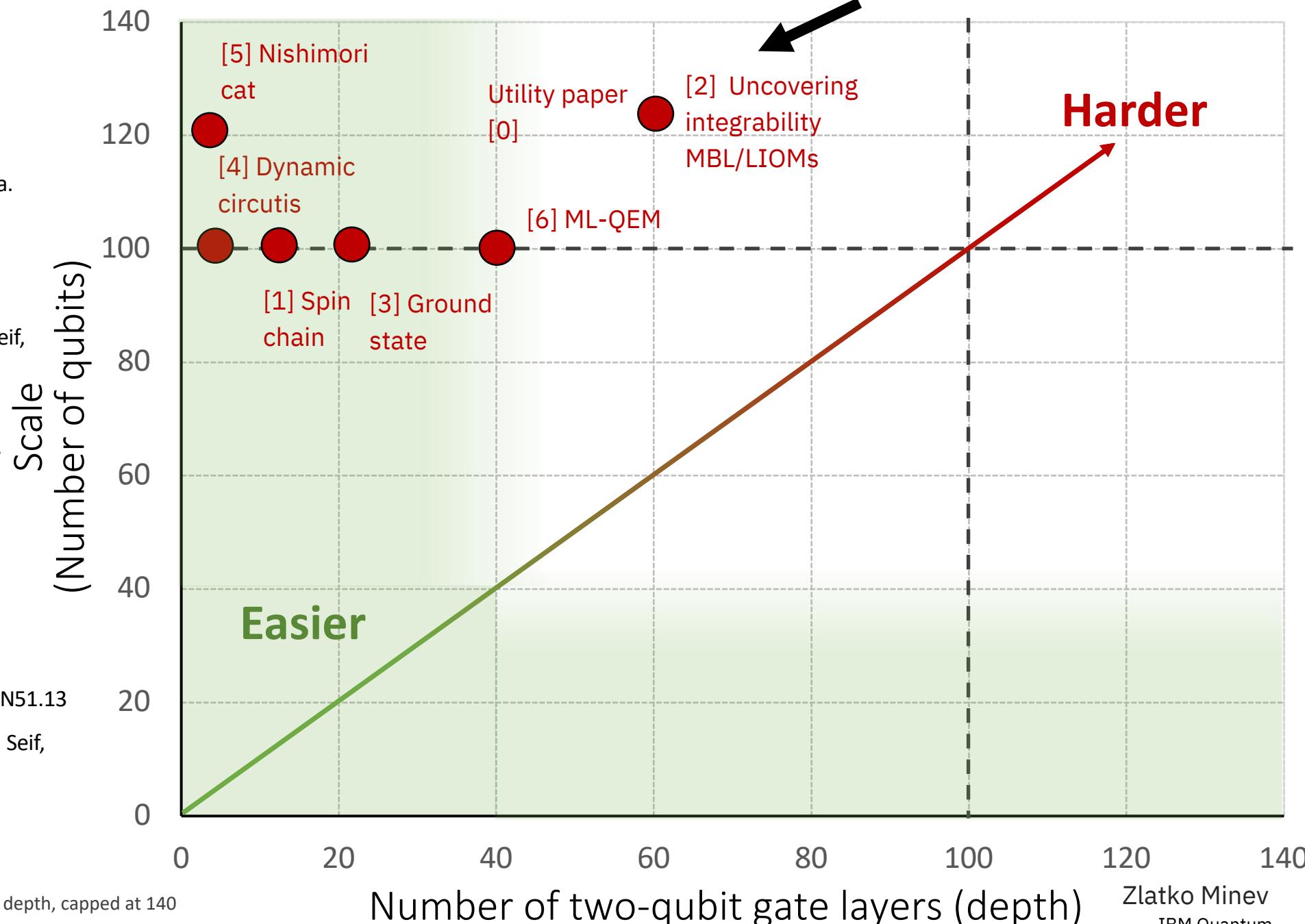


[5] Chen, Zhu, Verresen, Seif,  
Baümer, ... Trebst, Kandala.  
arXiv: 2309.02863 (2023) APS N51.13



[6] Liao, Wang, Situdikov, Salcedo, Seif,  
Minev.  
arXiv: 2308.13065 (2023)

...

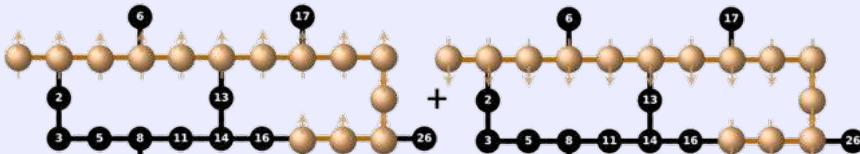


\* signal stops or decays more than 50% beyond this depth, capped at 140

\* note: quantum advantage with shallow circuits, Bravyi, Gosset, Konig, and related

# Check out APS24

## Y45.4: Efficient Long-Range Entanglement using Dynamic Circuits (invited)

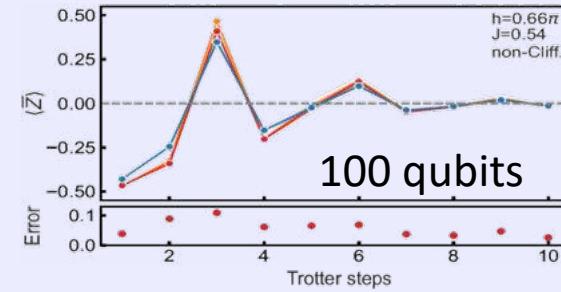


Gate teleportation over 100+ qubits

arXiv:2308.13065 (2023)

Bäumer, Tripathi, Wang, Rall, Chen, Majumder, Seif, Minev

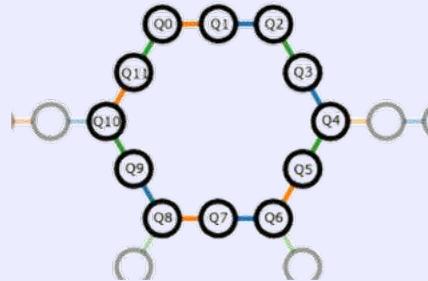
## D52.3: Machine Learning for Practical Quantum Error Mitigation (ML-QEM)



arXiv:2308.13065 (2023)

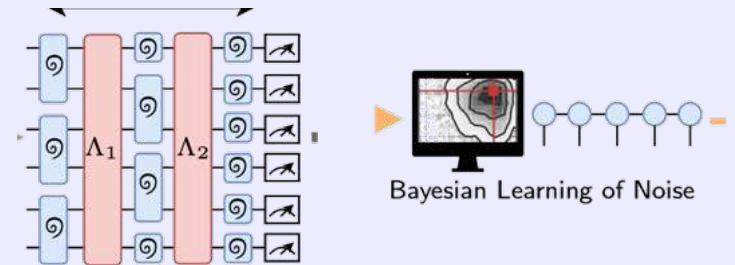
Liao, Wang, Situdikov, Salcedo, Seif, Minev

## Y51.5: Learning about quantum noise at scale



Seif, Liao, Majumder, Chen, Wang, Bäumer, Malekakhlagh, Javadi-Abhari, Jiang, Minev

## D50.9 Demonstration of Robust and Efficient Quantum Property Learning with Shallow Shadows



arXiv:2402.17911 (2024)  
Hu, Gu, Majumder, Ren, Zhang, Wang, Minev, You, Seif, Yelin

Can quantum computers be useful for  
many-body physics in the next 3-5 years?

# Uncovering the dynamics of many-body systems

## Many-body quantum systems and their dynamics

- fundamental and technological
- but generically difficult to simulate and understand

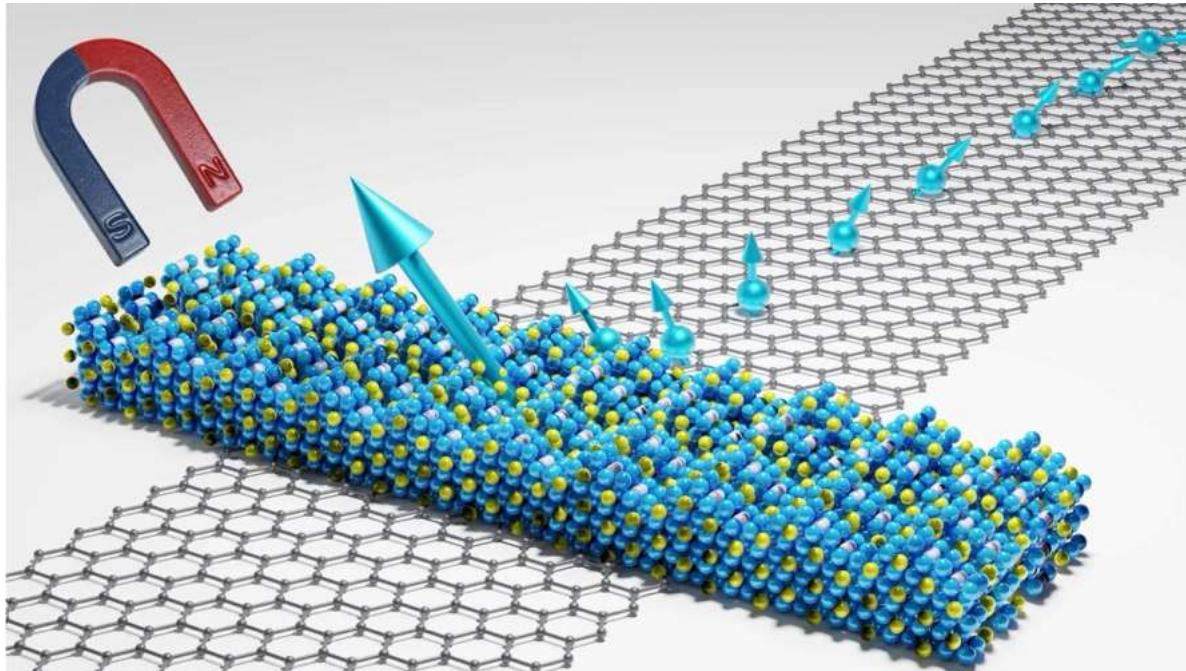


Image: [Chalmers](#)

## Symmetries, conservation laws, and integrability

- can unravel intricacies of these complex systems
- but generically difficult to discover

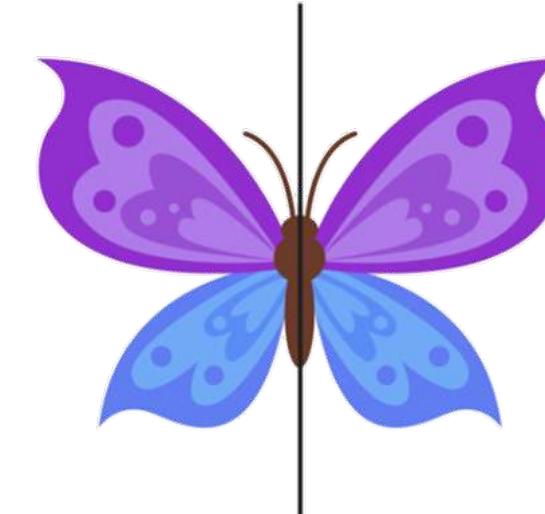


Image: [SuperSimple](#)

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# Classical Physics: Integrals of Motion

Before



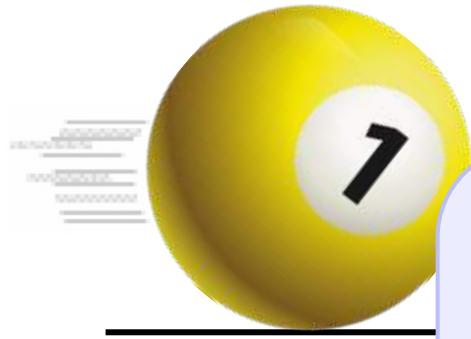
Integrals of motion:  
Total Linear Momentum  
Total Kinetic Energy

After



# Classical Physics: Integrals of Motion

Before



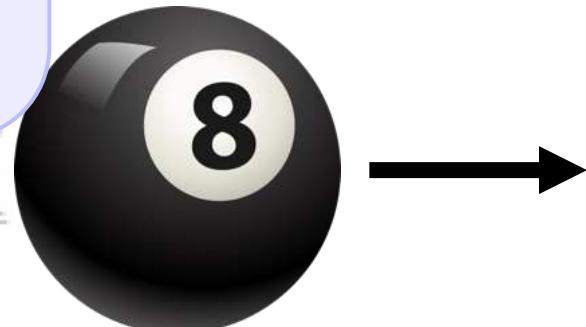
If you can find enough  
*Integrals of Motion (IOM)*,

Integrals of

Total Linear Momentum

Total Kinetic Energy

After



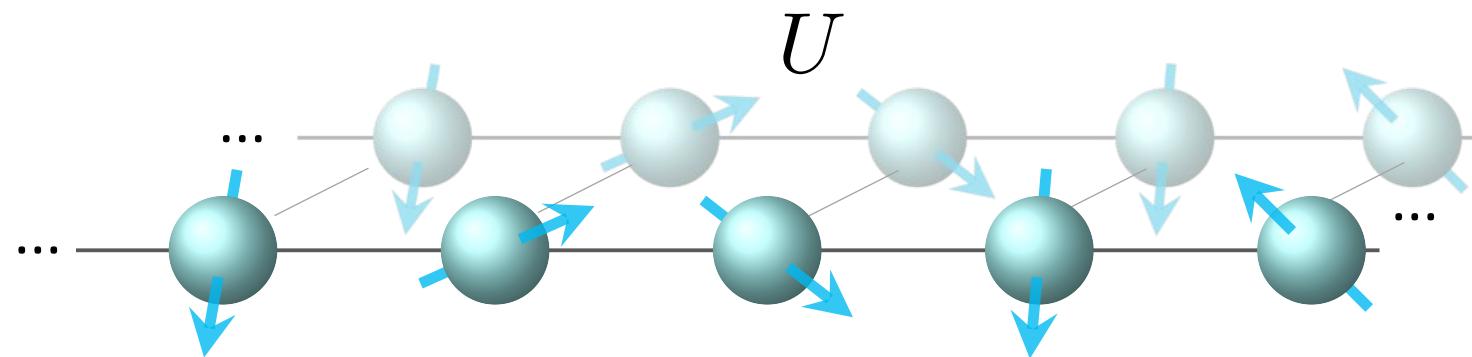
# Quantum Physics: Integral of motion

**Integral of motion  $L$**

$$[U, L] = 0 \quad [H, L] = 0$$

$$\langle L \rangle = \text{const}$$

$$L = \sum_{\mu=1}^{4^n - 1} a_\mu P_\mu$$



# Integrals of motion (IOM): toy example

Integral of motion  $L$

$$[U, L] = 0 \quad [H, L] = 0$$

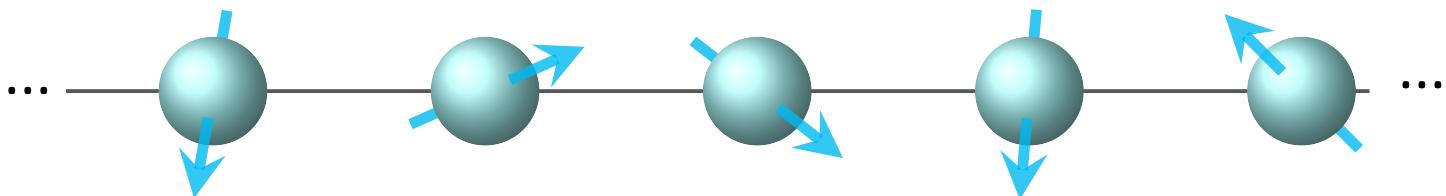
$$\langle L \rangle = \text{const}$$

$$L = \sum_{\mu=1}^{4^n - 1} a_\mu P_\mu$$

$n$  integrals of motion (IOM)

can label eigenstates\*

$$H = \sum_{i=0}^{n-1} c_i Z_i + \sum_{i \neq j} c_{ij} Z_i Z_j$$



$$[H, Z_0] = 0$$

$$[H, Z_1] = 0$$

$$[H, Z_{n-2}] = 0$$

$$[H, Z_{n-1}] = 0$$

$$L_0 = Z_0, L_1 = Z_1, \dots$$

$$|l_0 l_1 \cdots l_{n-1}\rangle = |l_0\rangle_{L_0} \otimes |l_1\rangle_{L_1} \otimes \cdots \otimes |l_{n-1}\rangle_{L_{n-1}}$$

For this trivial toy model easy,  
but generically very hard / intractable! +

+ N. Shiraishi and K. Matsumoto,  
Undecidability in quantum thermalization,  
Nature Comm. 12, 5084 (2021).

\* Say if we construct  $n$  orthogonal IOMs with eigenvalues  $\pm 1$ .

Energy is the only constant of motion in a non-integrable system (time independent).  
In general, an **integrable system** has constants of motion other than the energy.

# Local integral of motion (LIOM)

**Integral of motion  $L$**

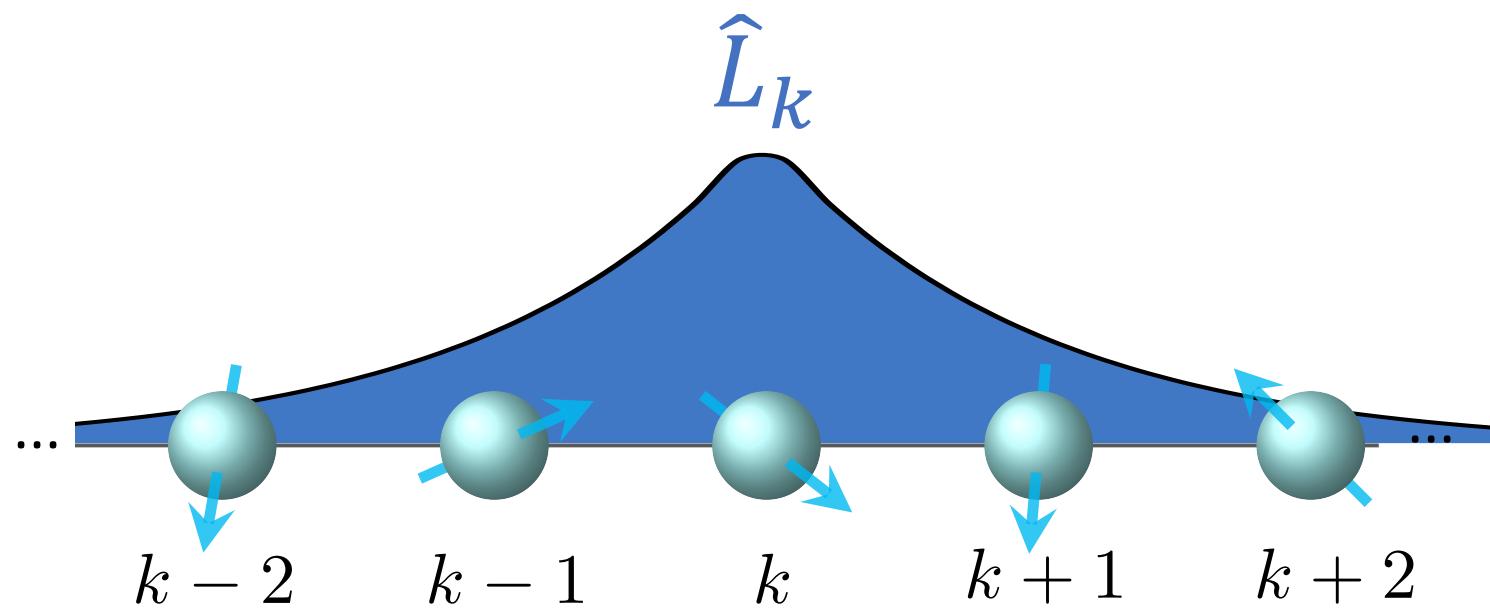
$$[U, L] = 0 \quad [H, L] = 0$$

$$\langle L \rangle = \text{const}$$

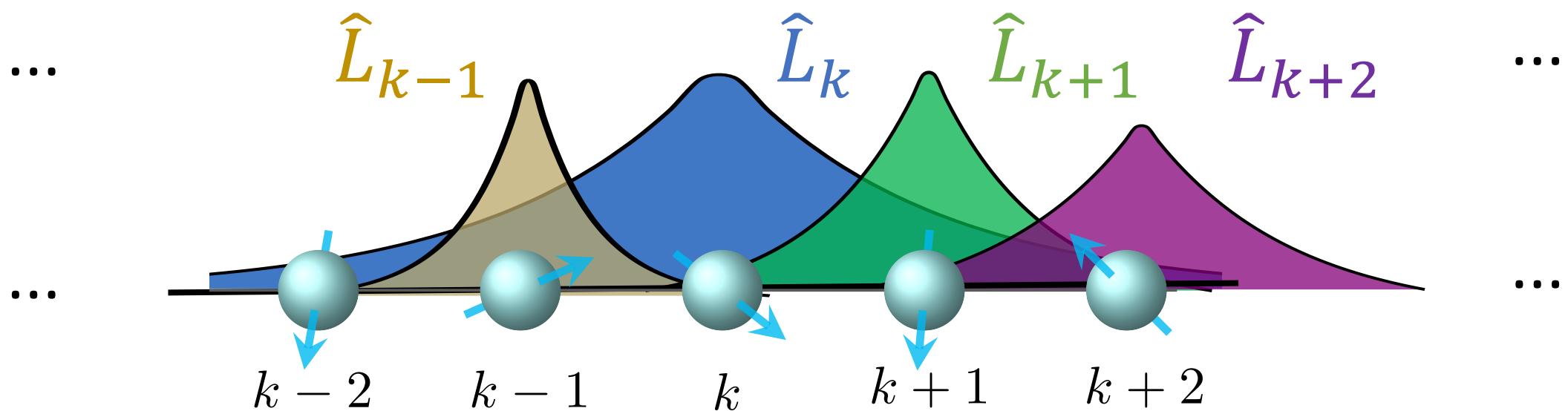
**Local integral of motion (LIOM)  $L$**

$$L_k = \sum_{\mu \in \mathcal{N}(k)} a_\mu P_\mu$$

local neighborhood



# Local integrals of motion (LIOM) & Integrability



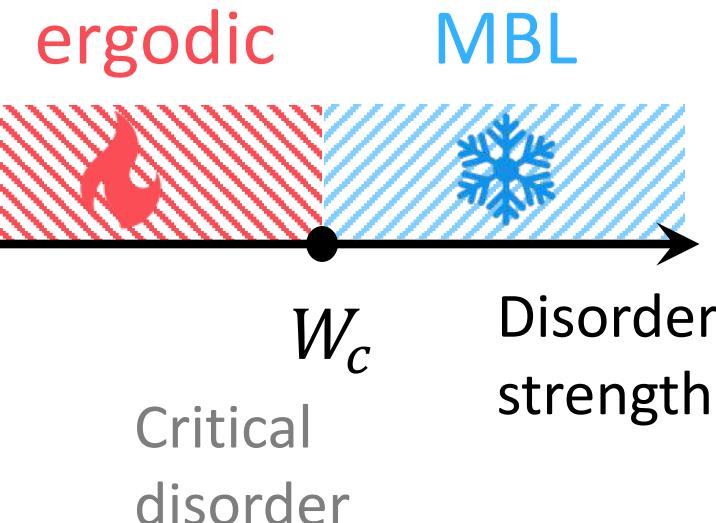
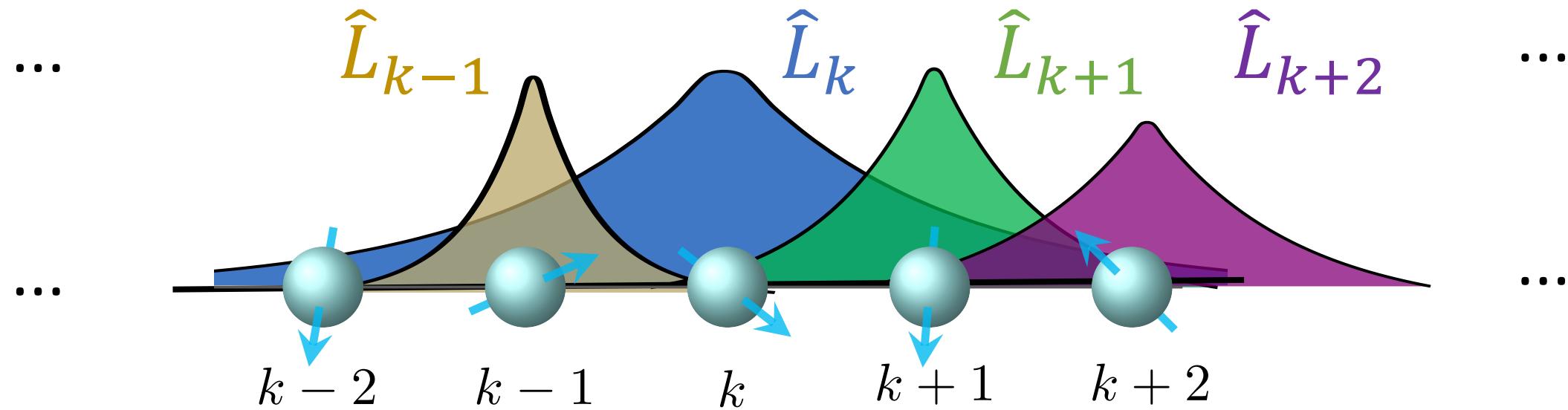
## Quantum integrable

if a system is described by an extensive set of local integrals of motion (LIOMs).

S. Caux and J. Mossel, J. Stat. Mech.-Theory E. (2011).

Quantum integrability is tricky to define formally, see Caux and Mossel, for relationship to Yang-Baxter.

# Example connection to disorder and many-body localization (MBL)



**Many-body localized (MBL) systems** suspected to have an extensive set of local integrals of motion (LIOMs)  $\{ L_k \}$ . Hypothesis: In the MBL regime, the original Hamiltonian can be understood as

$$\hat{H} = \sum_k \epsilon_k \hat{L}_k + \sum_{k < j} J_{kj}^{(2)} \hat{L}_k \hat{L}_j + \sum_{i < j < k} J_{ijk}^{(3)} \hat{L}_i \hat{L}_j \hat{L}_k + \dots$$

MBL: Basko, Aleiner, Altshuler, Ann Phys (2006), Pal and Huse, RRB (2010), Serbyn, Papic, Abanin, PRL (2013), Huse, Nandkishore, Oganesyan PRB (2014) ... See also recent work by Dries Sels

Zlatko Minev

# New to LIOMs and many-body localization?

Boulder School for Condensed Matter and Materials Physics  
Non-Equilibrium Quantum Dynamics

July 3-28, 2023

A. Chandran, M. Fisher, V. Khemani, S. Vijay, L. Radzhovskiy  
(Organizers)

[www.boulderschool.yale.edu/2023/boulder-school-2023](http://www.boulderschool.yale.edu/2023/boulder-school-2023)

Lots of video lectures here on YouTube.  
Also here lectures on quantum error mitigation by Z. Minev.

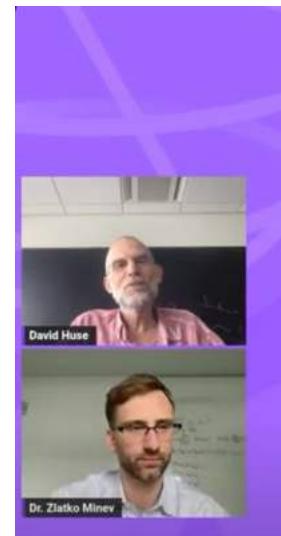
## Reviews

### ***Many-body localization, thermalization, and entanglement***

Dmitry A. Abanin, Ehud Altman, Immanuel Bloch, and  
Maksym Serbyn Rev. Mod. Phys. 91, 021001

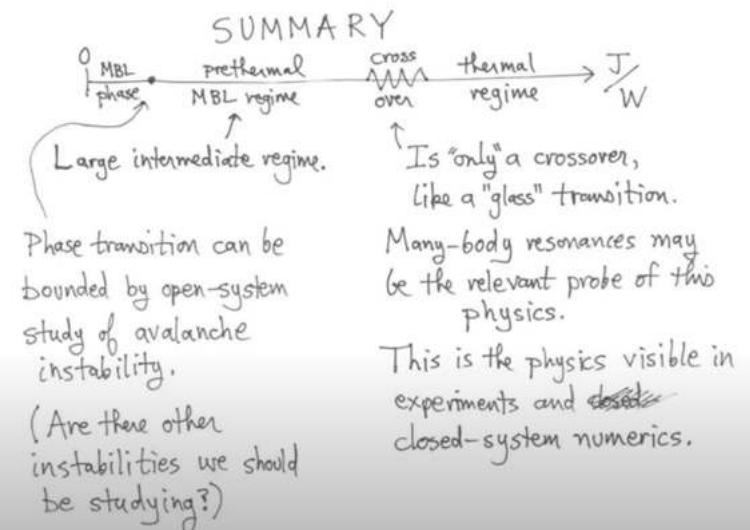
...

Qiskit Quantum Seminar  
(YouTube)



▶ ▶ 🔍 1:08:12 / 1:16:52

“Many-Body Localization” talk by David Huse  
<https://qiskit.org/events/seminar-series>



# Experiments related to many-body localization

- M. Schreiber, S. S. Hodgman, P. Bordia, H. P. Lüschen, M. H. Fischer, R. Vosk, E. Altman, U. Schneider, and I. Bloch, Observation of many-body localization of interacting fermions in a quasirandom optical lattice, *Science* 349, 842 (2015).
- J.-y. Choi, S. Hild, J. Zeiher, P. Schauß, A. Rubio-Abadal, T. Yefsah, V. Khemani, D. A. Huse, I. Bloch, and C. Gross, Exploring the many-body localization transition in two dimensions, *Science* 352, 1547 (2016).
- J. Smith, A. Lee, P. Richerme, B. Neyenhuis, P. W. Hess, P. Hauke, M. Heyl, D. A. Huse, and C. Monroe, Many-body localization in a quantum simulator with programmable random disorder, *Nat. Phys.* 12, 907 (2016).
- P. Roushan, C. Neill, J. Tangpanitanon, V. M. Bastidas, A. Megrant, R. Barends, Y. Chen, Z. Chen, B. Chiaro, A. Dunsworth, C. Neill, J. T. Preskill, J. M. Martinis, and R. B. Blakestad, Spectroscopic signatures of localization with interacting photons in superconducting qubits, *Science* 358, 1175 (2017).
- P. Bordia, H. Lüschen, U. Schneider, M. Knap, and I. Bloch, Periodically driving a many-body localized quantum system, *Nat. Phys.* 13, 460 (2017).
- P. Bordia, H. Lüschen, S. Scherg, S. Gopalakrishnan, M. Knap, U. Schneider, and I. Bloch, Probing slow relaxation and many-body localization in two-dimensional quasiperiodic systems, *Phys. Rev. X* 7, 041047 (2017).
- M. Rispoli, A. Lukin, R. Schittko, S. Kim, M. E. Tai, J. Leonard, and M. Greiner, Quantum critical behaviour at the many-body localization transition, *Nature* 573, 385 (2019).
- A. Lukin, M. Rispoli, R. Schittko, M. E. Tai, A. M. Kaufman, S. Choi, V. Khemani, J. Leonard, and M. Greiner, Probing entanglement in a many-body-localized system, *Science* 364, 256 (2019).
- Q. Guo, C. Cheng, Z.-H. Sun, Z. Song, H. Li, Z. Wang, W. Ren, H. Dong, D. Zheng, Y.-R. Zhang, R. Mondaini, H. Fan, and H. Wang, Observation of energy-resolved many-body localization, *Nat. Phys.* 17, 234 (2021).
- X. Mi, M. Ippoliti, C. Quintana, A. Greene, Z. Chen, J. Gross, F. Arute, K. Arya, J. Atalaya, R. Babbush, et al., Time-crystalline eigenstate order on a quantum processor, *Nature* 601, 531 (2022).
- J. Leonard, S. Kim, M. Rispoli, A. Lukin, R. Schittko, J. Kwan, E. Demler, D. Sels, and M. Greiner, Probing the onset of quantum avalanches in a many-body localized system, *Nat. Phys.* 19, 481 (2023).
- ... ....



**Pedram Roushan**



**Peter Schauß**

Zlatko Minev  
IBM Quantum

# Local integrals of motion (LIOMs): Example connection to ergodicity breaking and many-body localization

Prototypical phenomenon:

**prethermalization and many-body localization (MBL)**

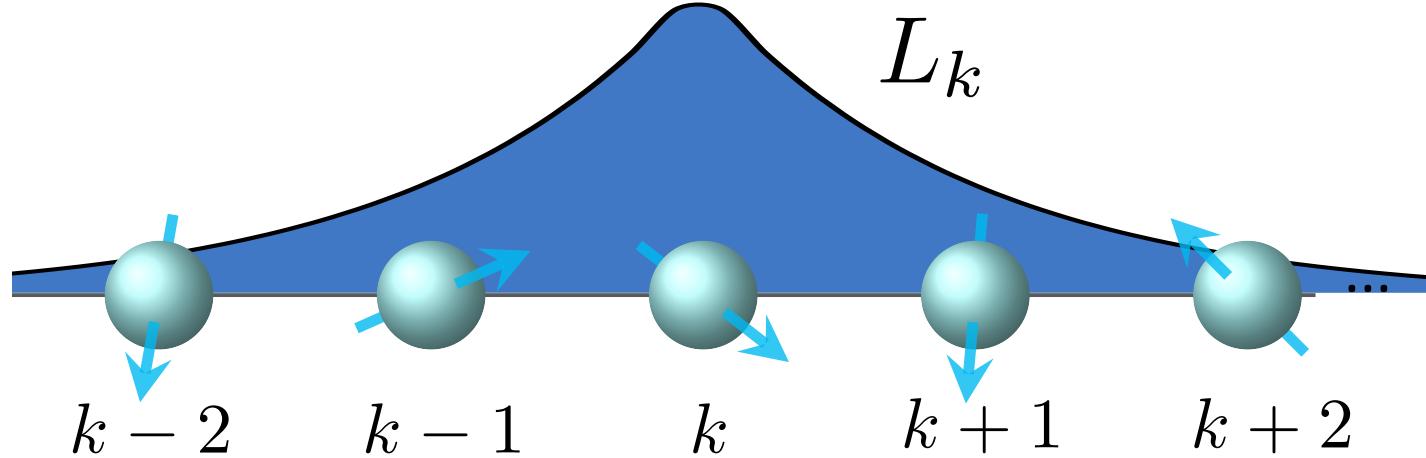
Goes back to Anderson and **disordered** systems,  
but this was single non-interacting particles



*"for their fundamental theoretical investigations of the electronic structure of magnetic and disordered systems"*

Existence of MBL phase is under debate.

Most of community agree that MBL exists in 1D.

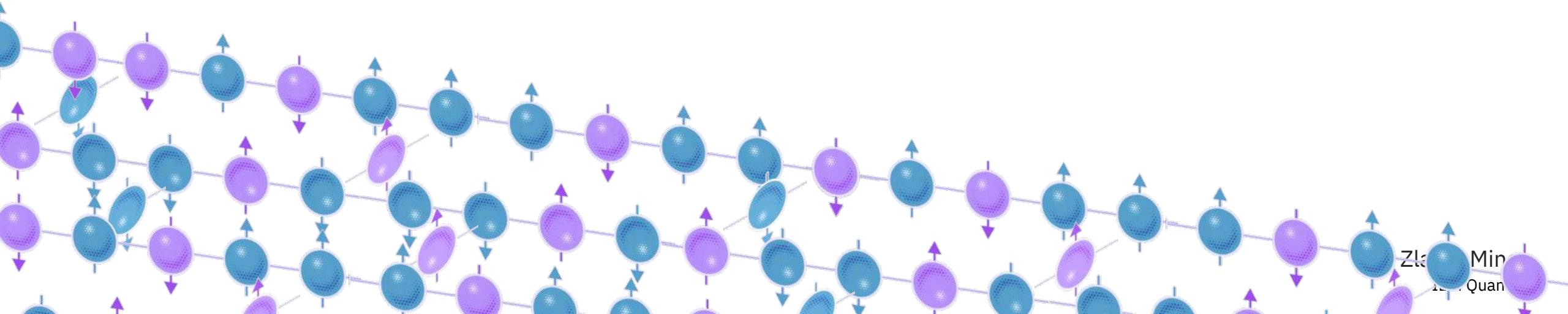


- **MBL systems** have extensive set of local integrals of motion (LIOMs).
- **Prethermal systems (non-MBL)** can have approximate LIOMs
$$[e^{-iHt}, L_k] \approx 0$$
- We uncover LIOMs in 1D and approximate LIOMs in 2D in 104 and 124 qubit lattices using a digital quantum computer

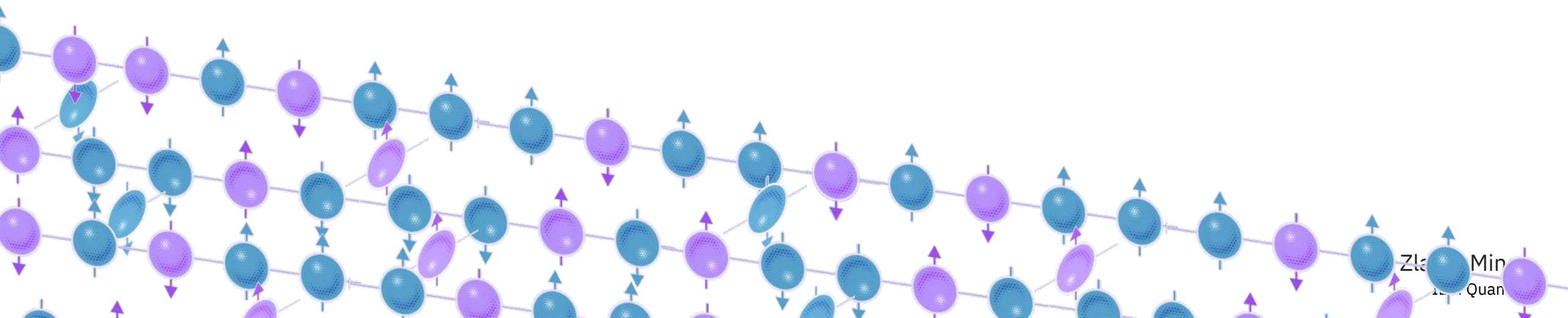
**Tricky to find**

- Beyond finding ground states
- We will be interested in dynamics and the full Hilbert space and spectrum

Can we uncover all integrals of motion  $\{L\}$   
of a large, disordered many-body system  
using a quantum computer?



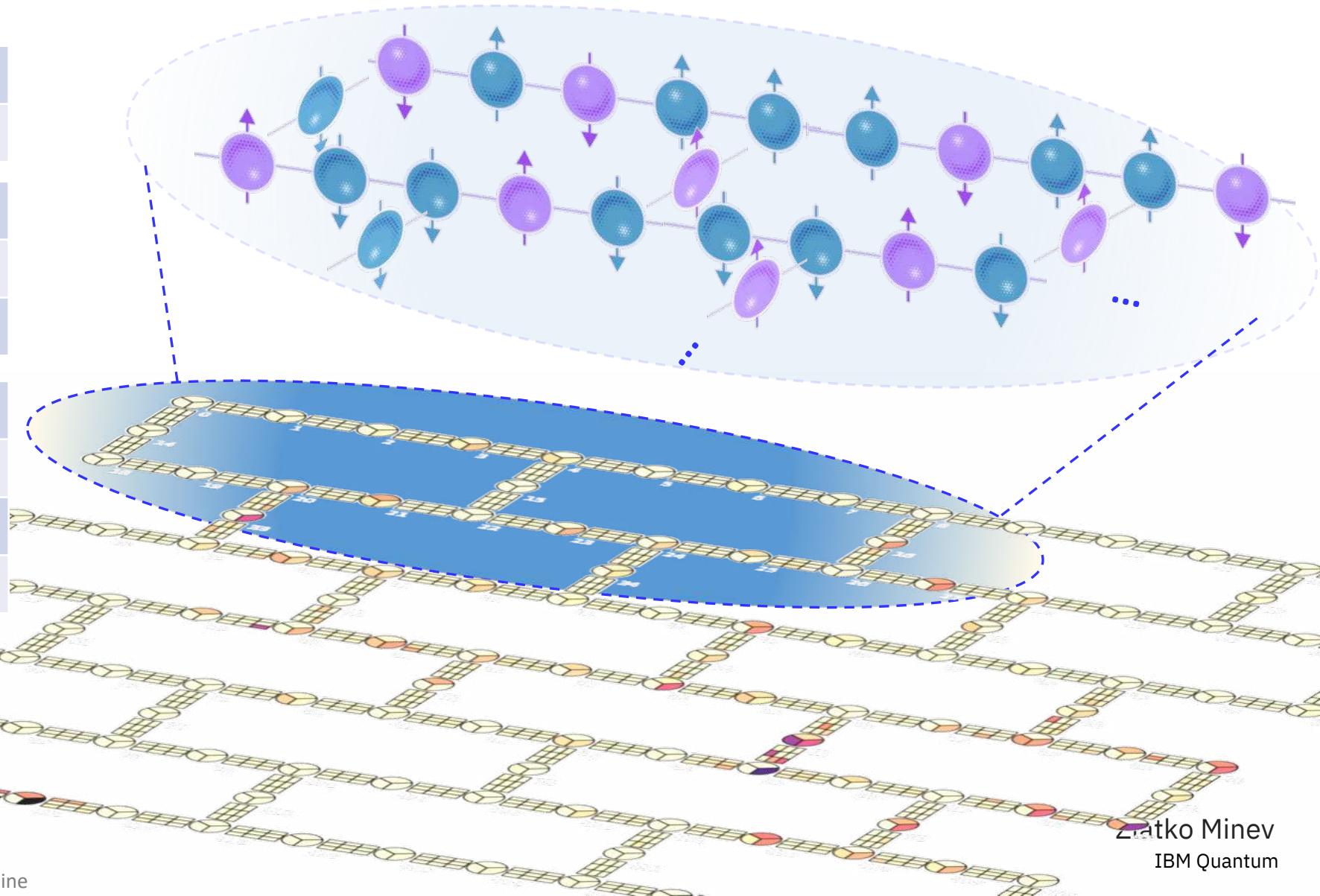
# Preview



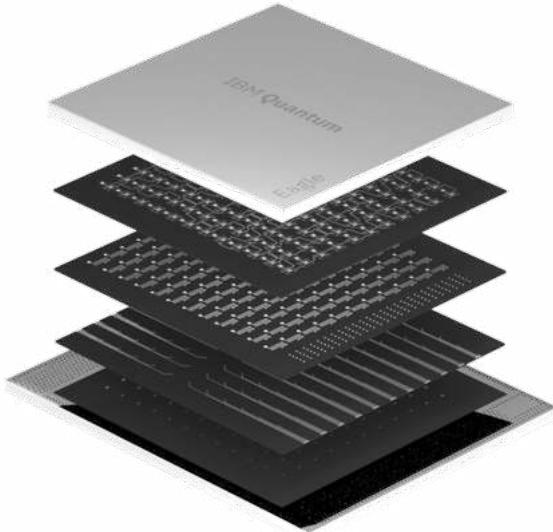
# QSim on 100Q+: 2D interacting many-body Floquet system

## Experimental setting

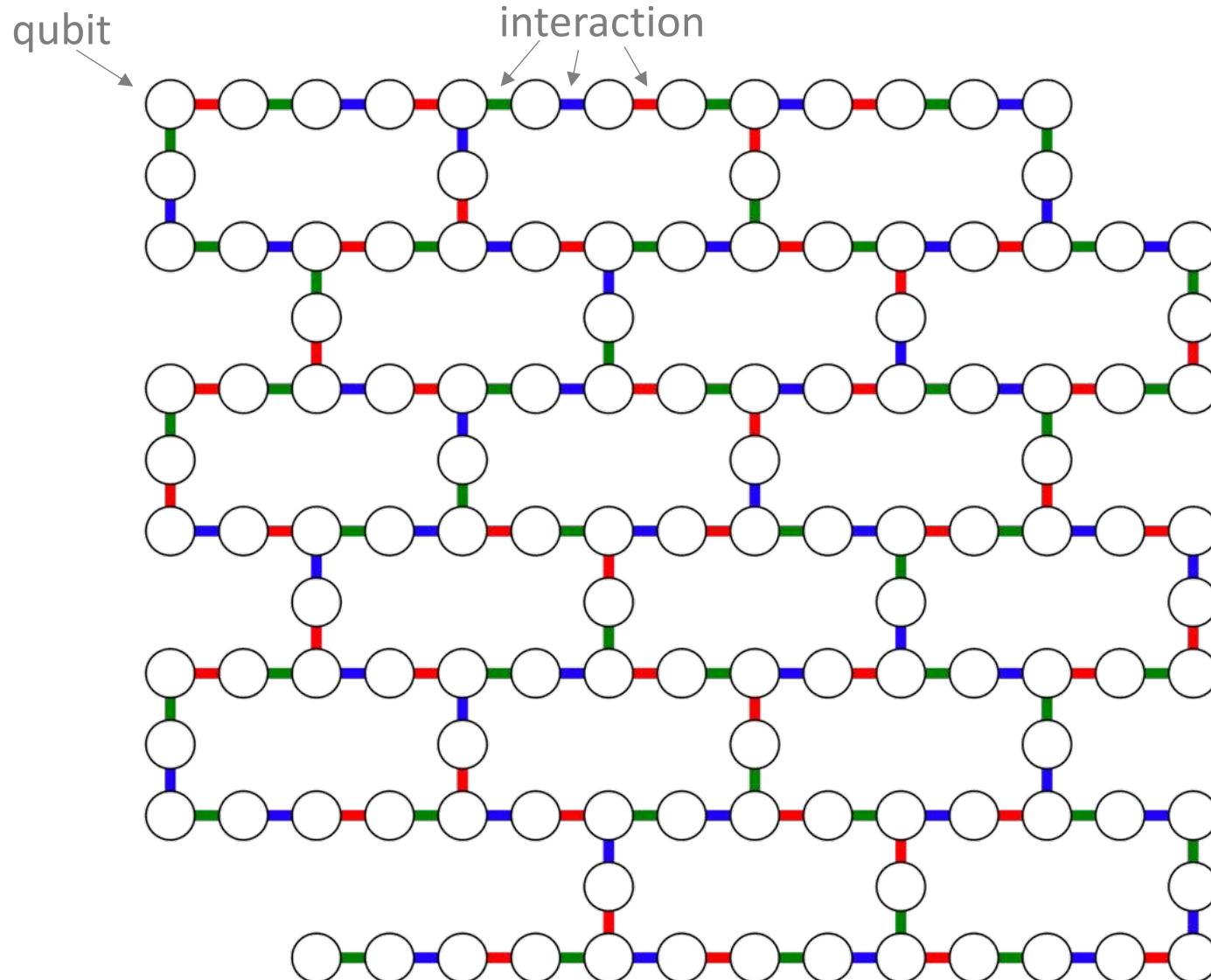
Number of qubits	<b>124Q</b>
Connectivity	<b>2D h-hex</b>
Depth in 2Q layers	<b>60</b>
Floquet steps	<b>20</b>
Total num. of cX gates	<b>2,641</b>
Circuits (entire paper)	<b><math>3.5 \times 10^5</math></b>
Shots (entire paper)	<b><math>5.3 \times 10^8</math></b>
QPU runtime	<b>80 hours</b>
Environment	<b>Cloud</b>



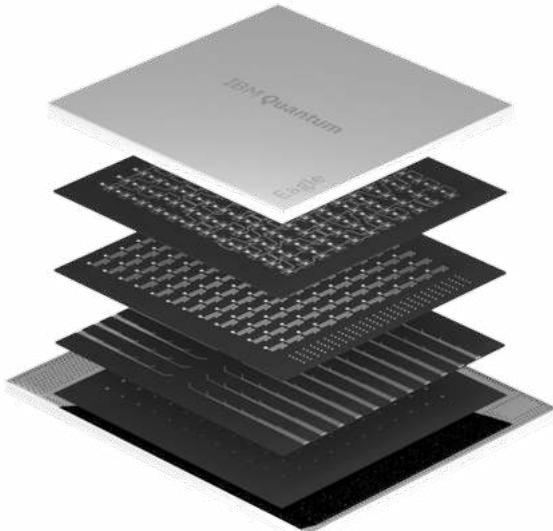
# Interaction map and device layers



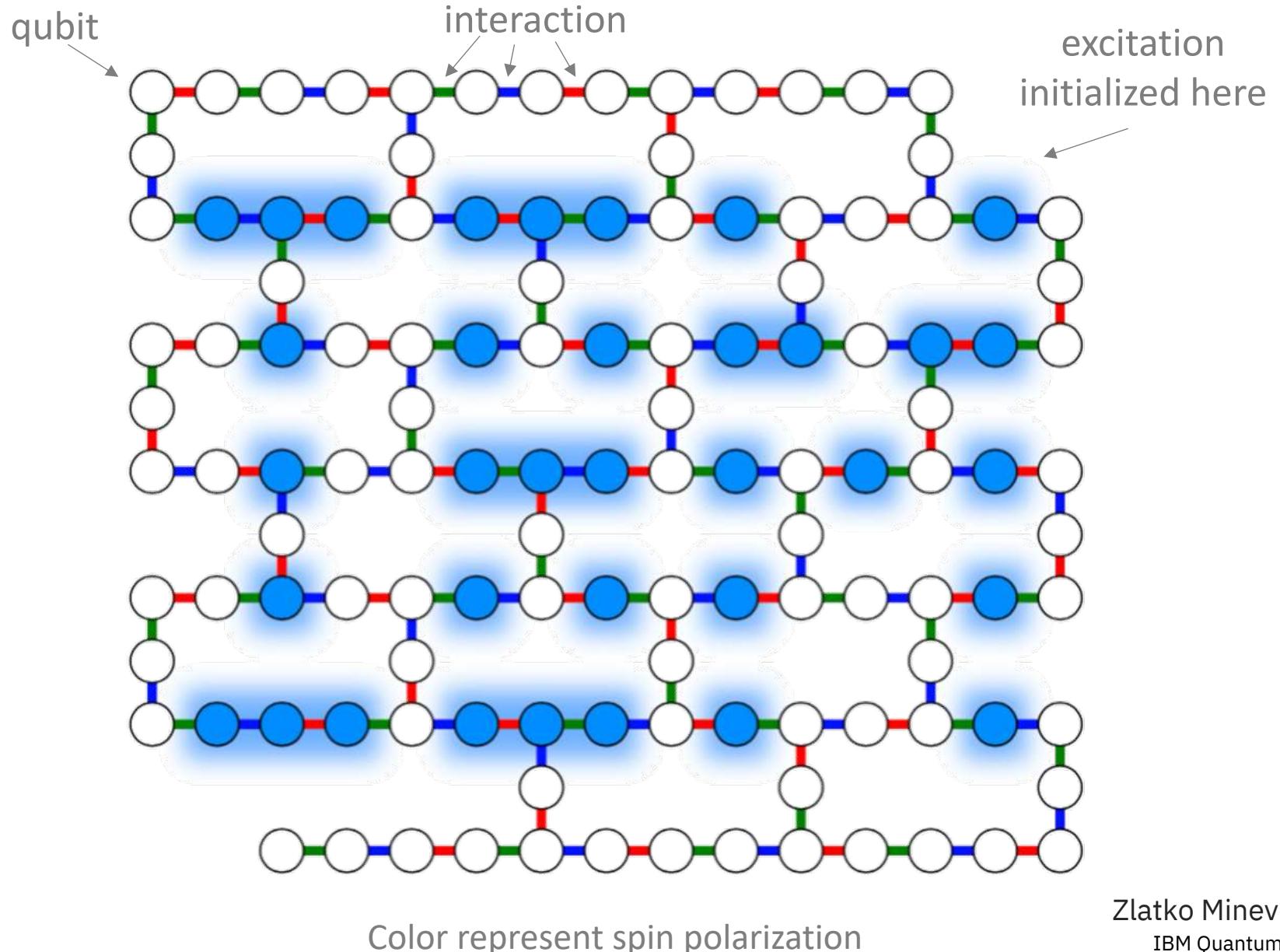
Number of qubits	124Q
Connectivity	2D h-hex
Depth in cX gates	60
Floquet steps	20
Total number of cXs	2,641



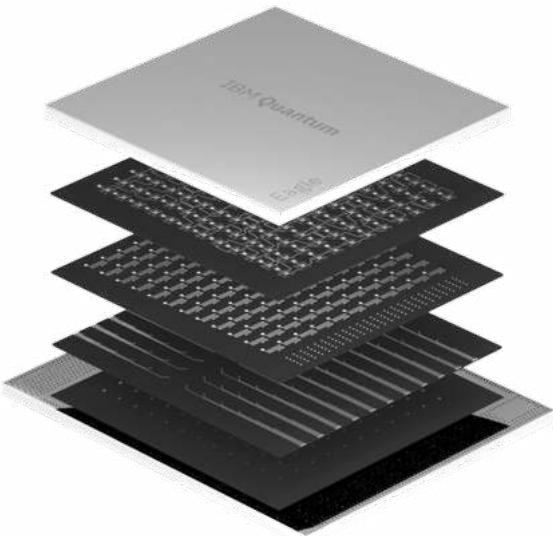
# Initialize lattice in fun states



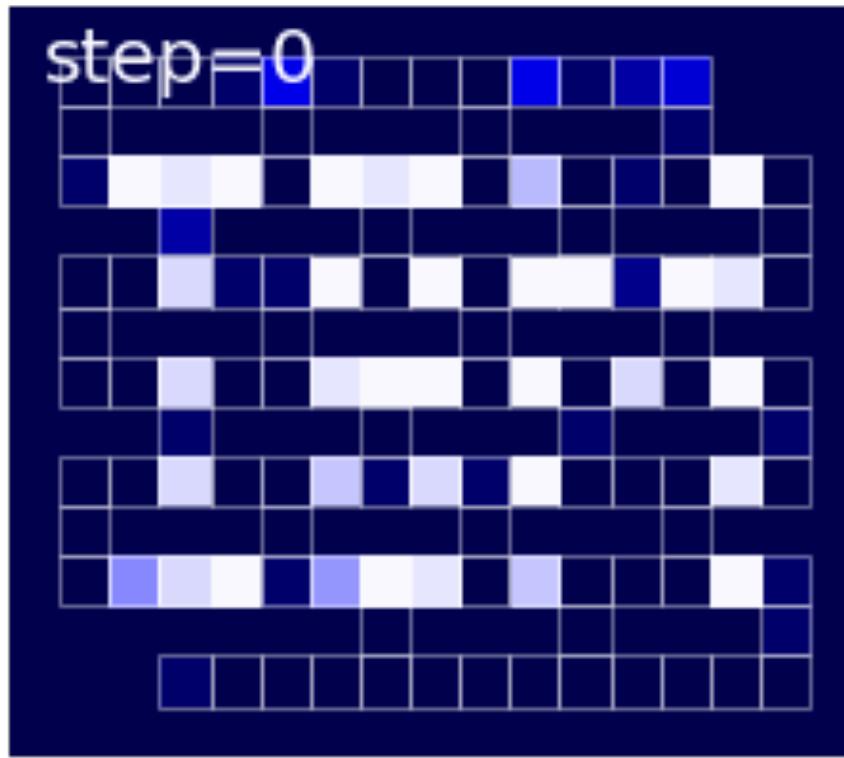
Number of qubits	124Q
Connectivity	2D h-hex
Depth in cX gates	60
Floquet steps	20
Total number of cXs	2,641



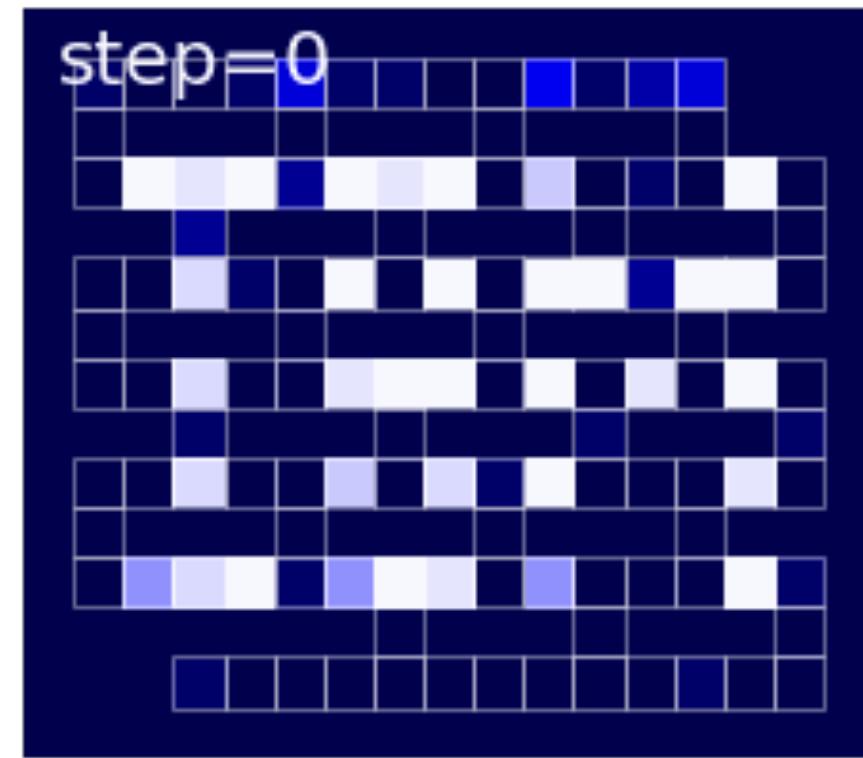
# Quantum dynamics in different regimes



Number of qubits	124Q
Connectivity	2D h-hex
Depth in cX gates	60
Floquet steps	20
Total number of cXs	2,641

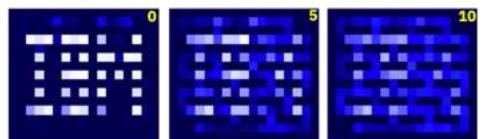


Thermalizing regime

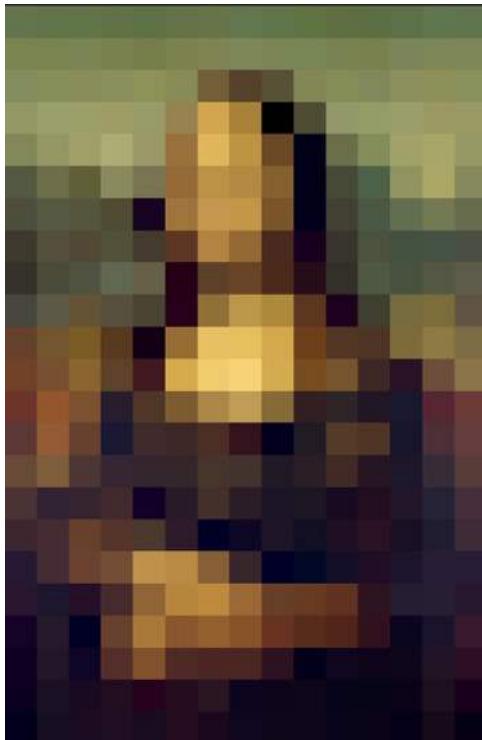


Prethermal regime

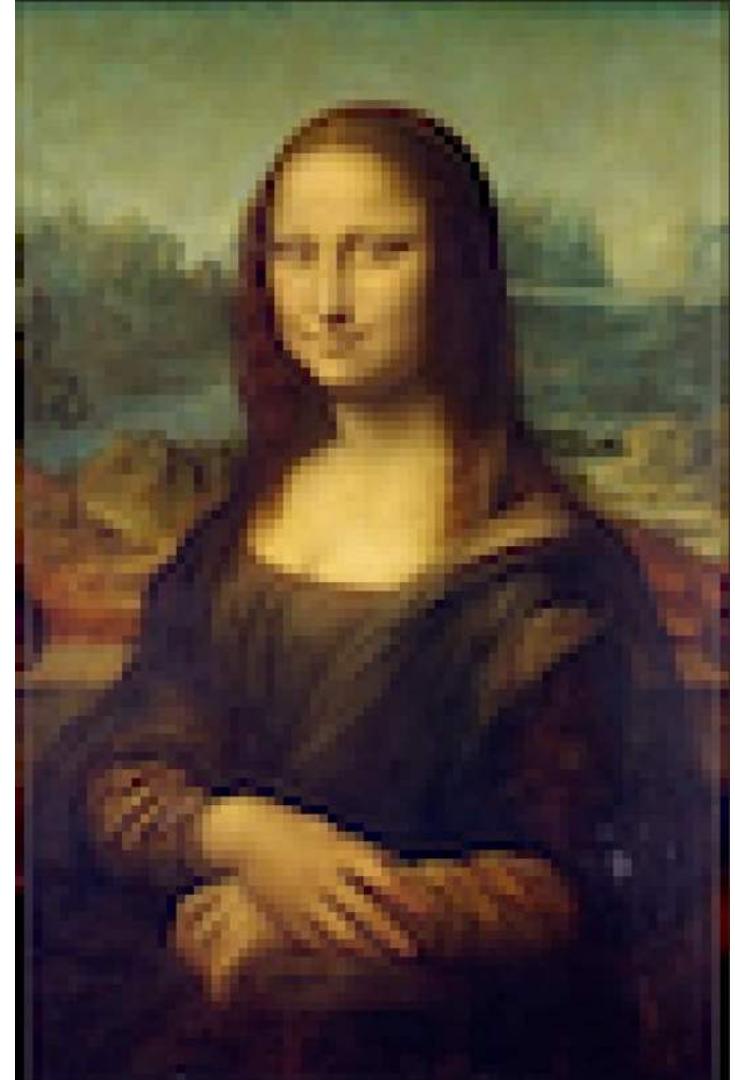
# A more detailed portrait



Spin  
imbalance

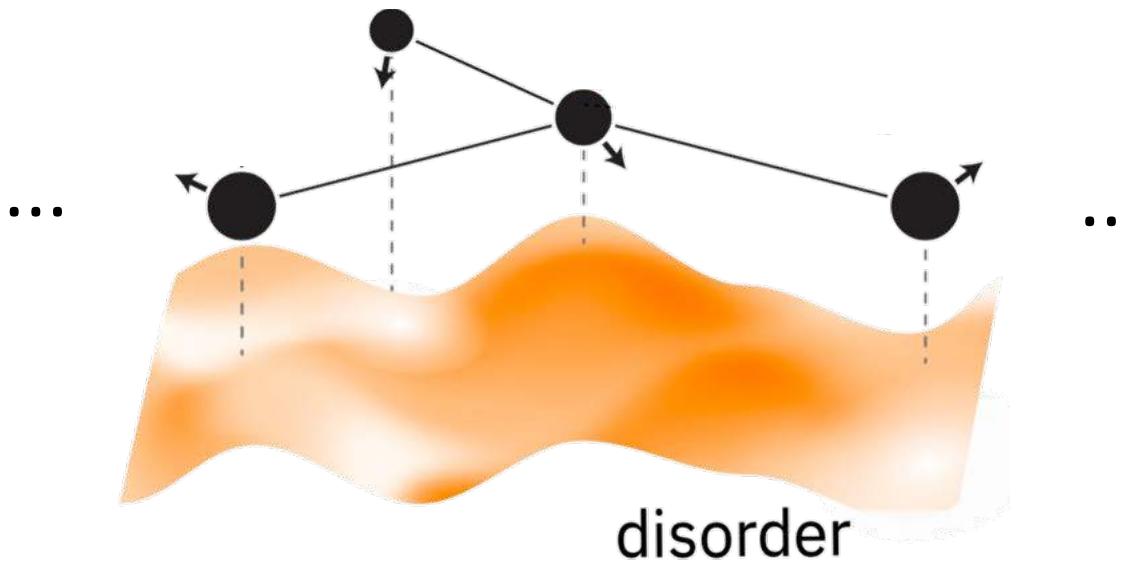


One-particle density  
matrix (OPDM)

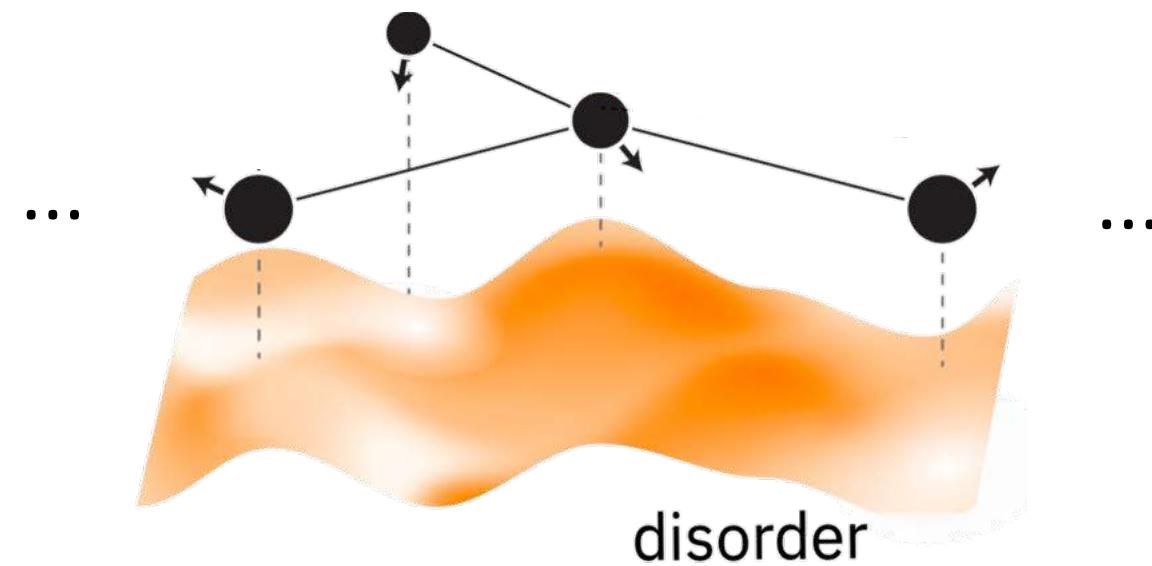
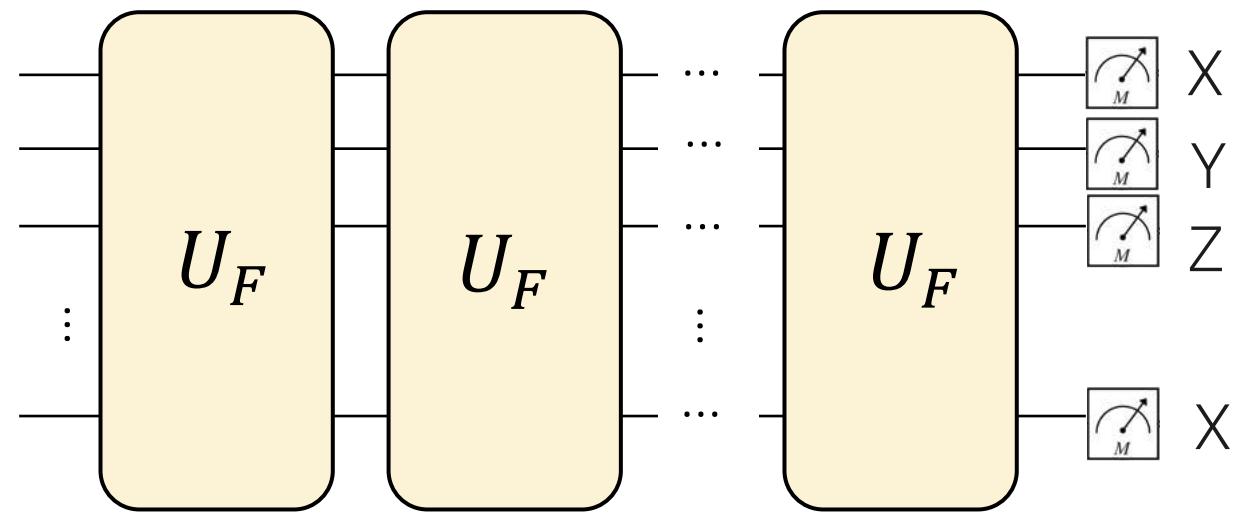


Local integrals of motion  
(LIOMs)

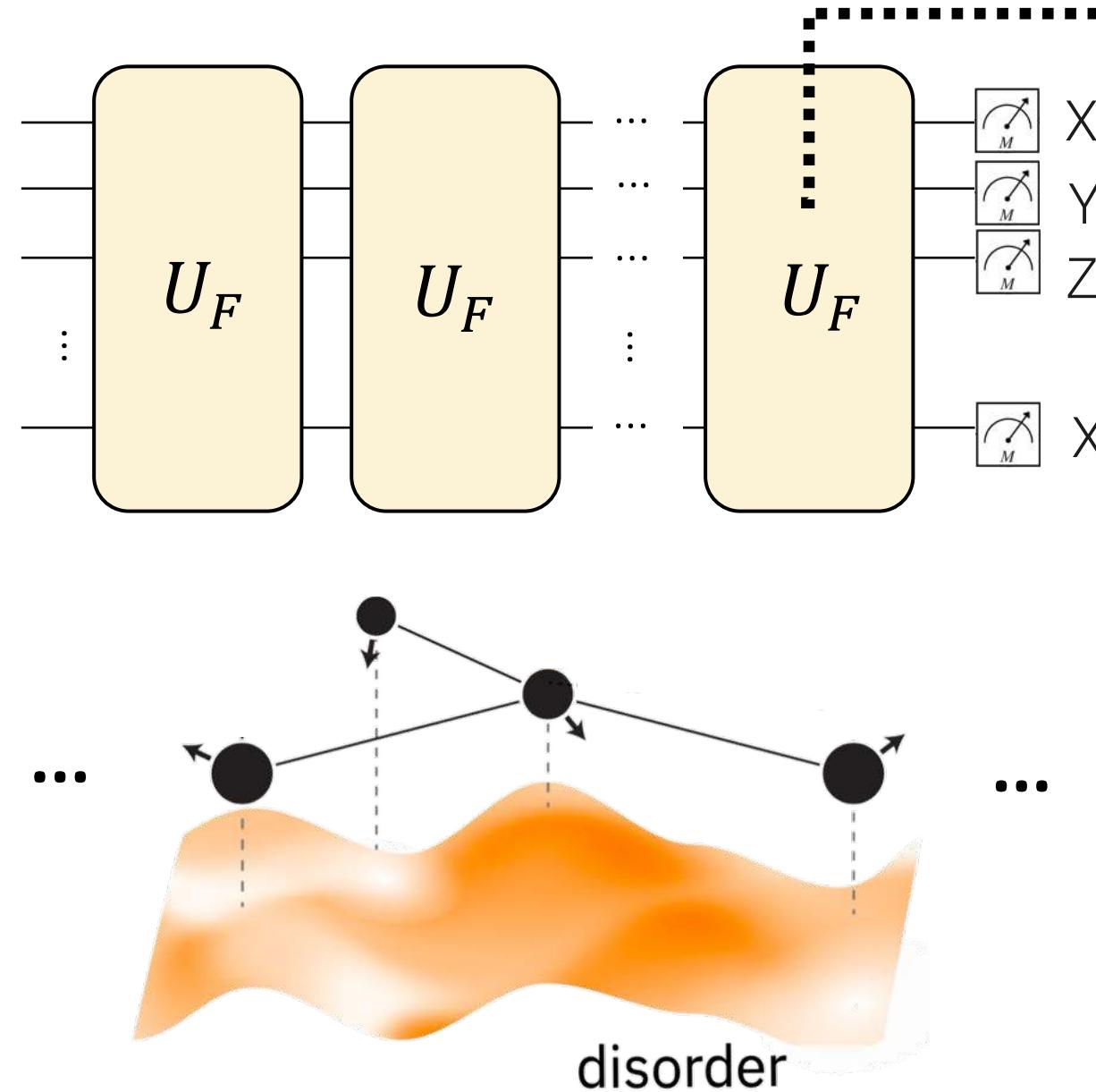
# Spin lattice system over some potential



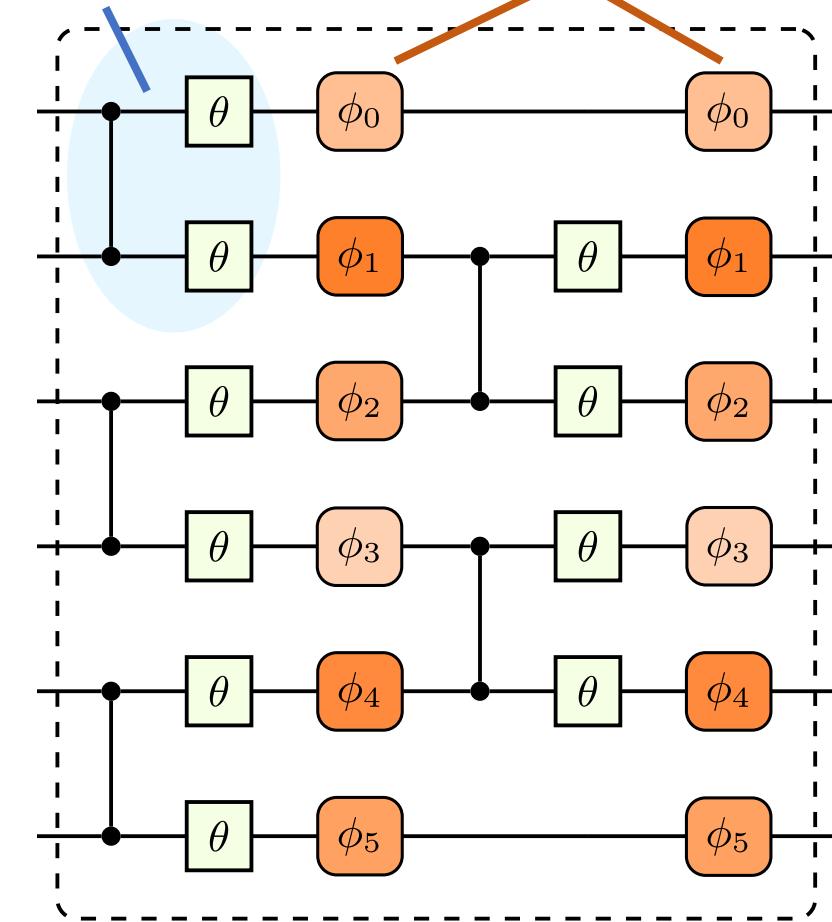
# Floquet circuit evolution



# Circuit model



Kinetic term  
+ Interactions      Spatial disorder



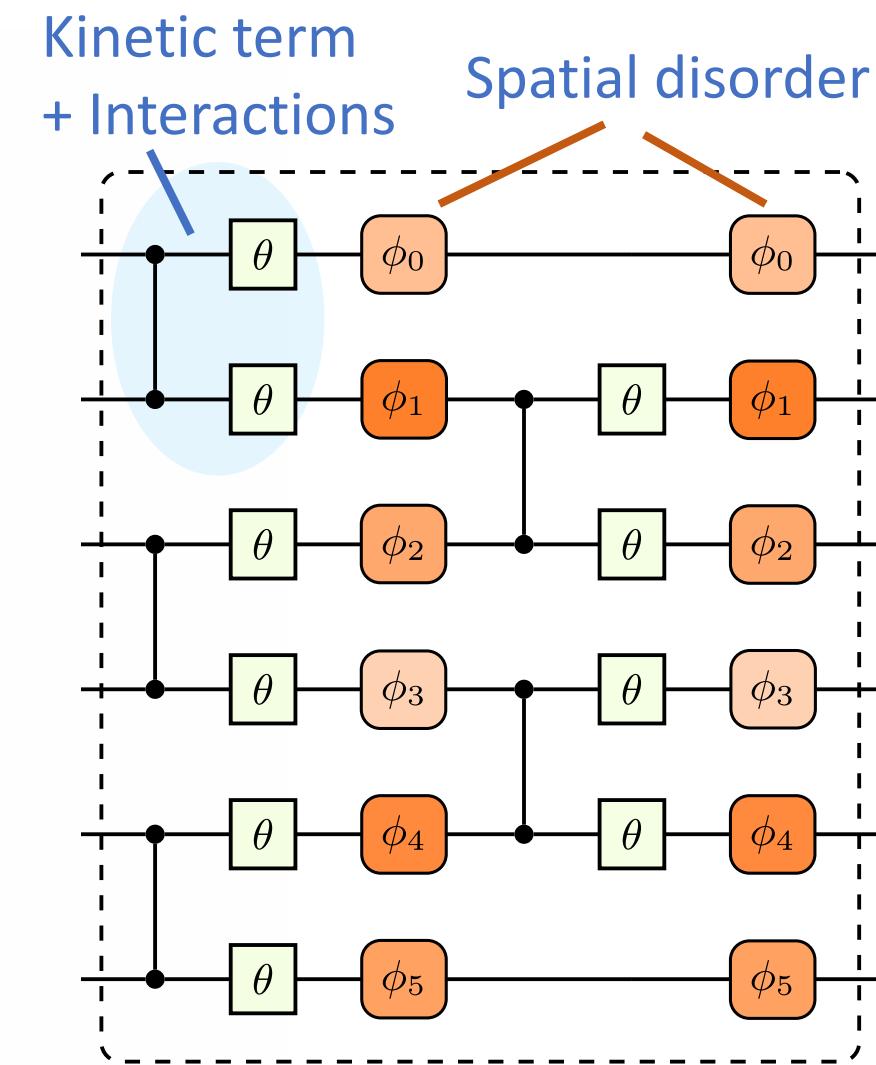
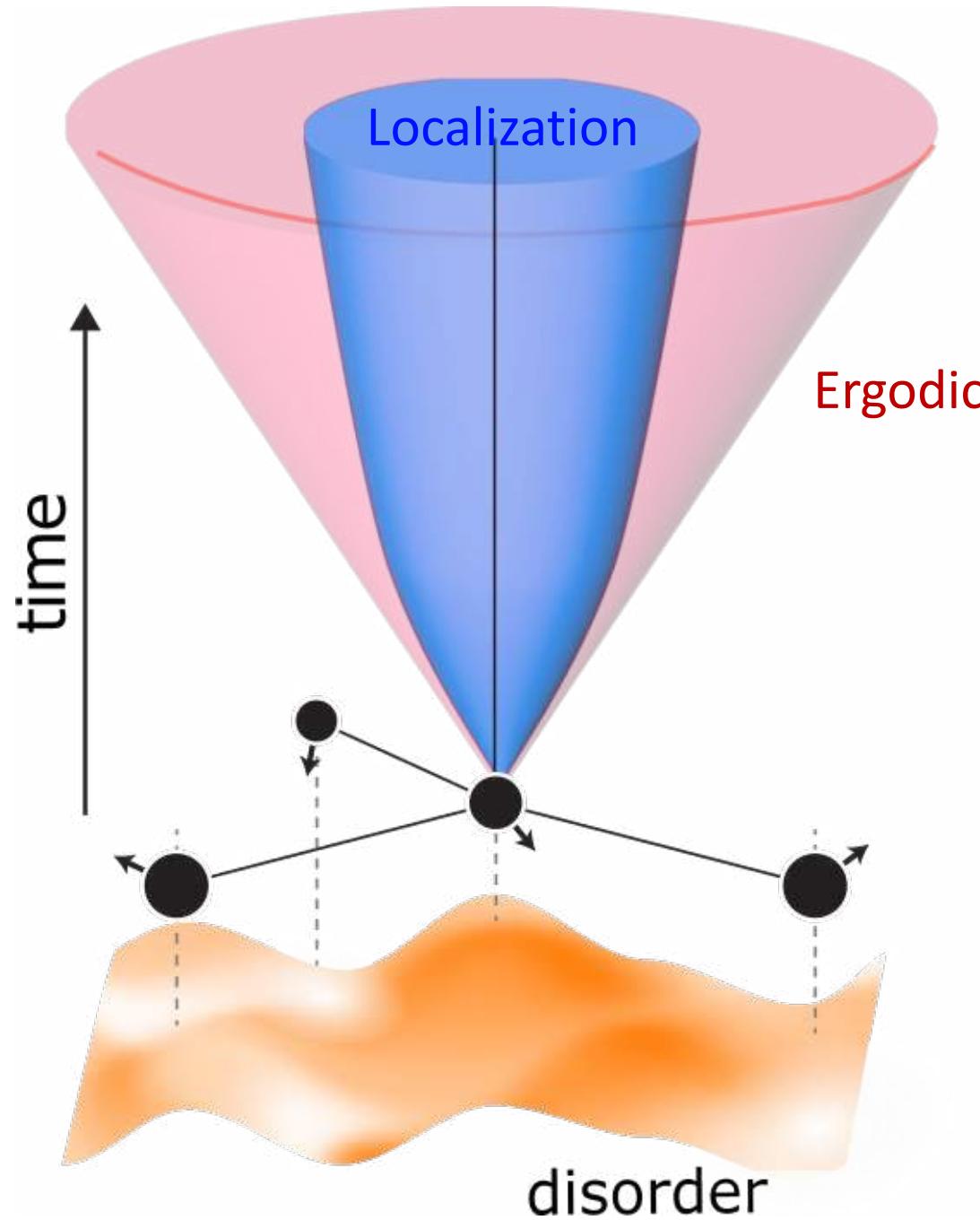
$$P(\phi_k) = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi_k} \end{pmatrix}$$

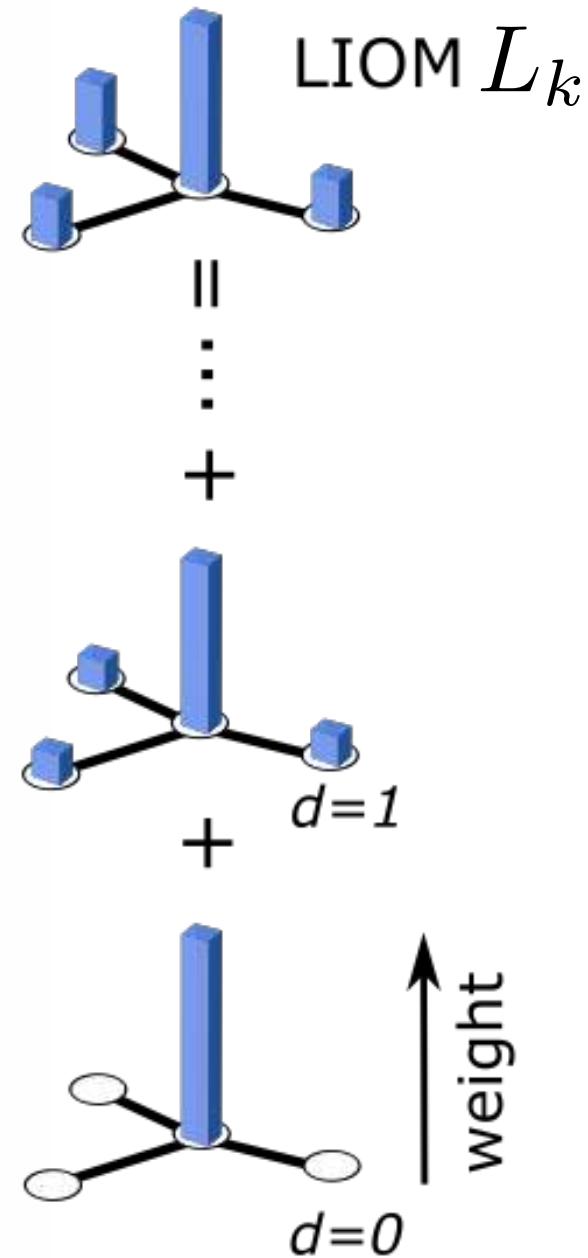
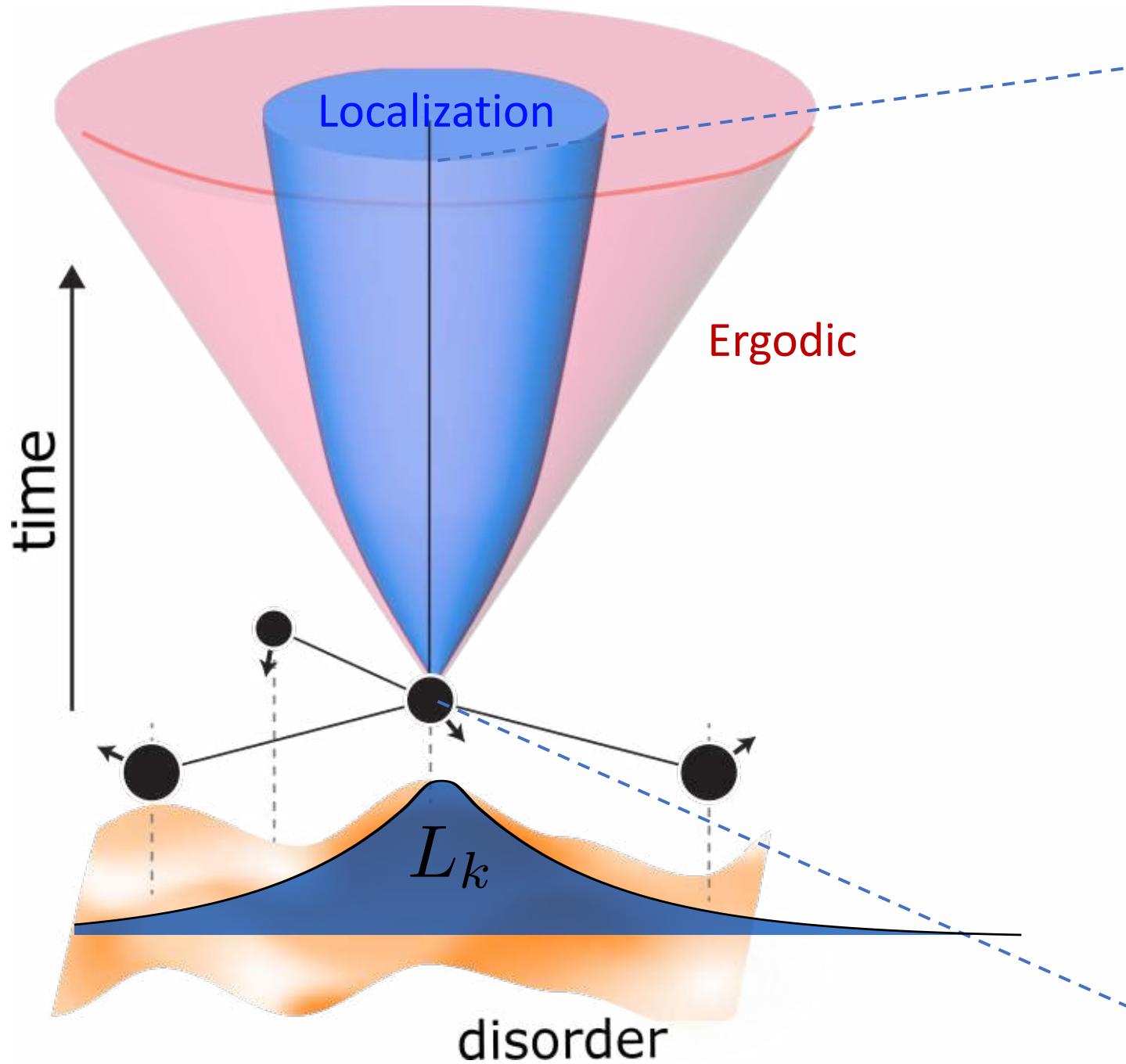
$$\phi_k \in [-\pi, \pi]$$

Uniformly sample  
disorder

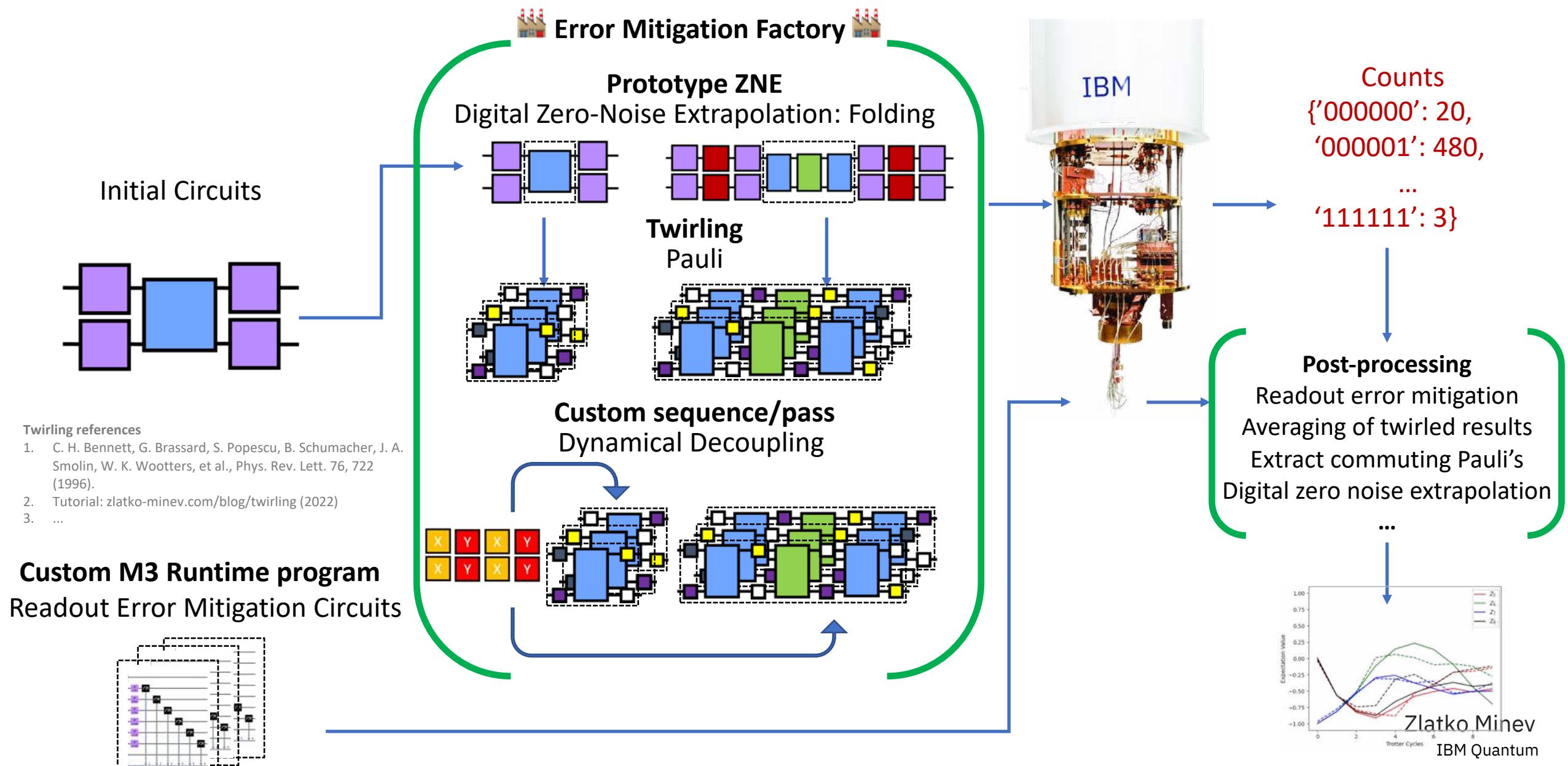
$$U(\theta) = \begin{pmatrix} \cos \theta/2 & \sin \theta/2 \\ \sin \theta/2 & -\cos \theta/2 \end{pmatrix}$$

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# A composite error mitigation strategy



# Benchmarks of quantum hardware and model

**Spin imbalance experiments**

Memory of initial state

**Classical numerics in 1D**

Spectral and time analysis

**Benchmarks**

**1D**

**2D**

**LIOMs**

**1D**

**2D**

**One-particle density matrices (OPDM)  
experiments**

Natural orbitals and occupations

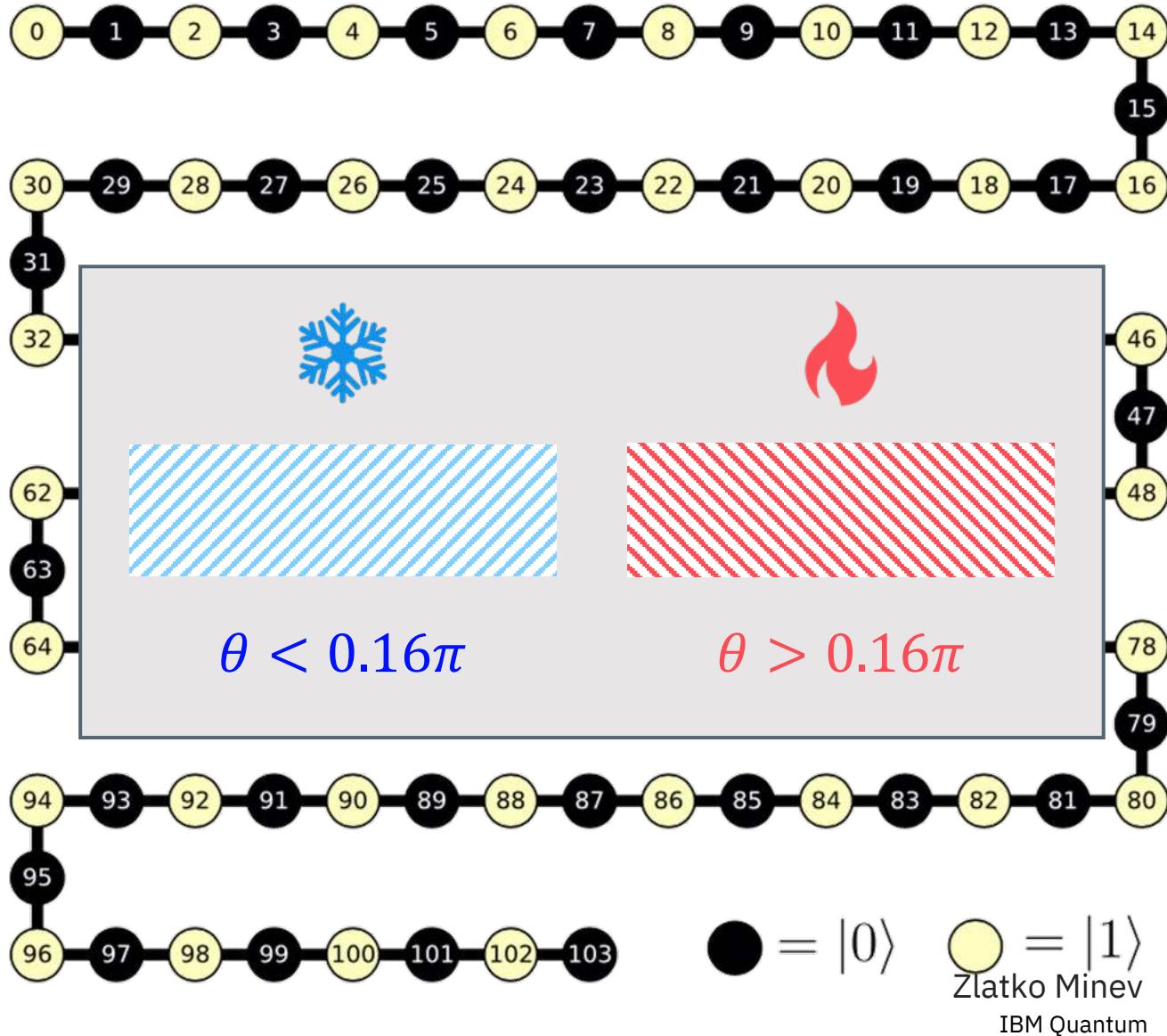
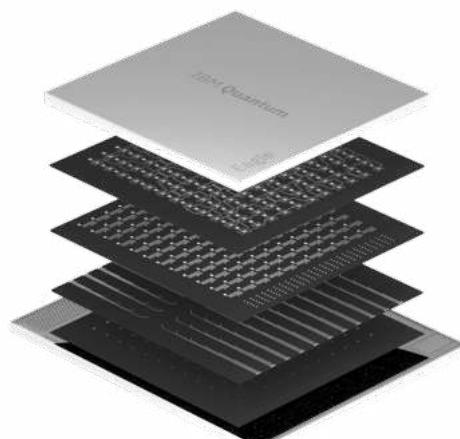
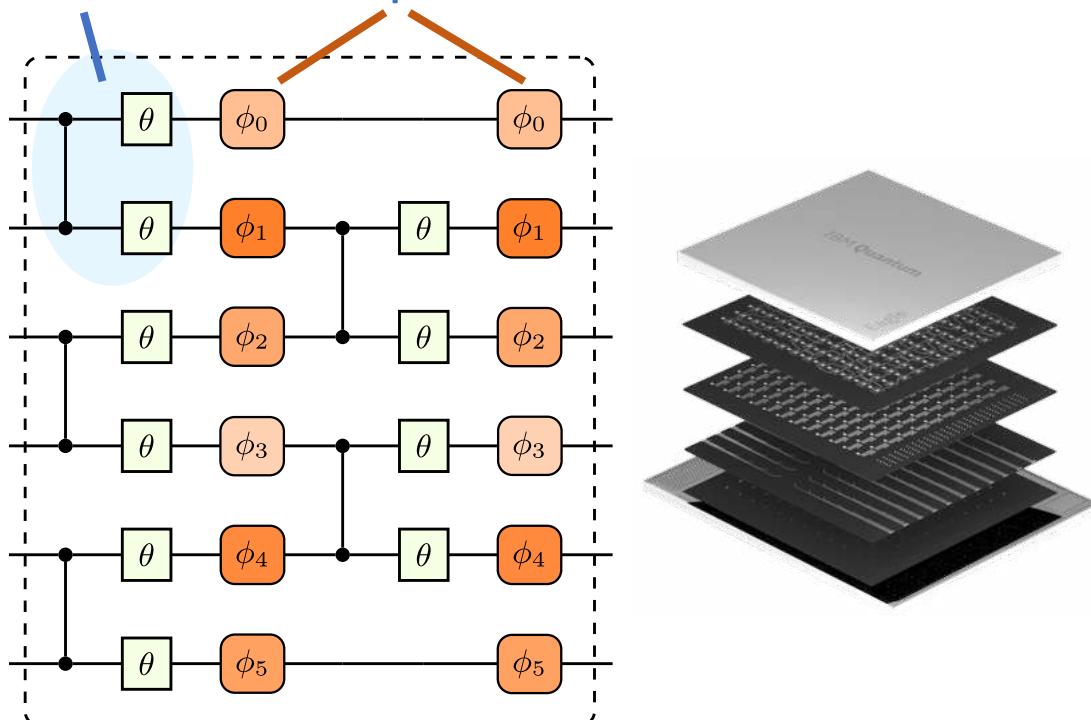
Scaling with system size: cross-overs

...

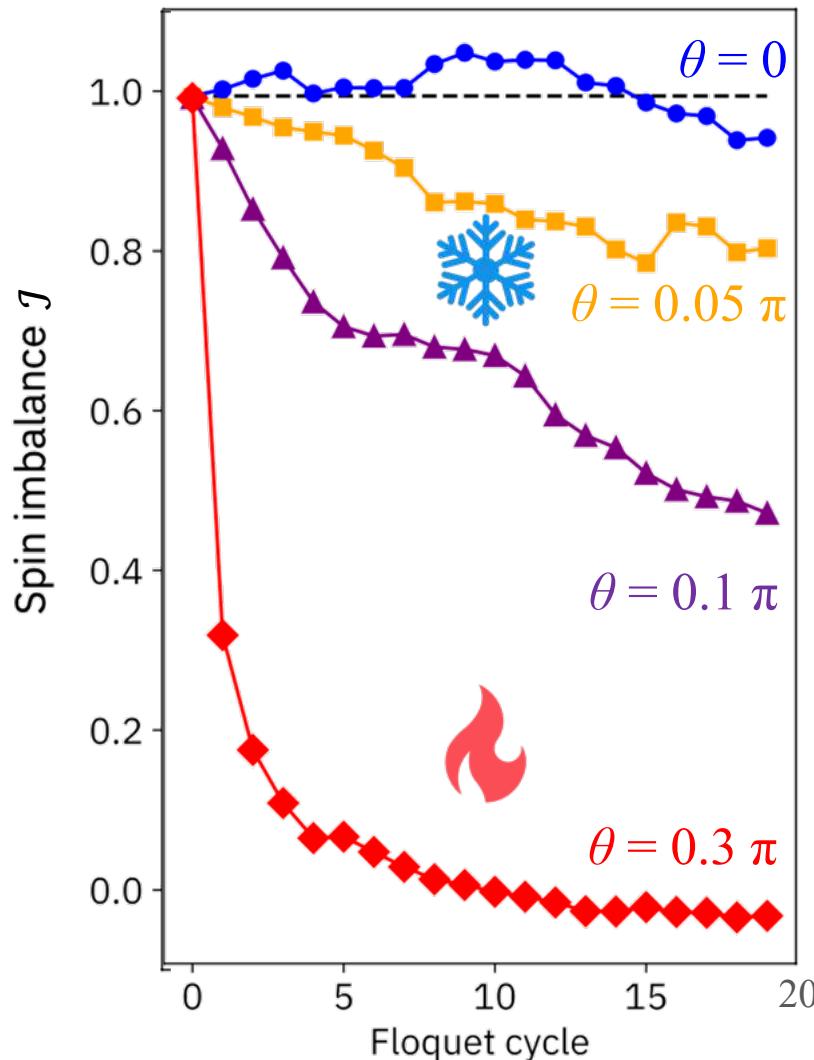
# Initial antiferromagnetic state with spin imbalance

$$|\psi_0\rangle = |1\rangle|0\rangle|1\rangle|0\rangle|1\rangle|0\rangle \dots |1\rangle|0\rangle|1\rangle|0\rangle|1\rangle$$

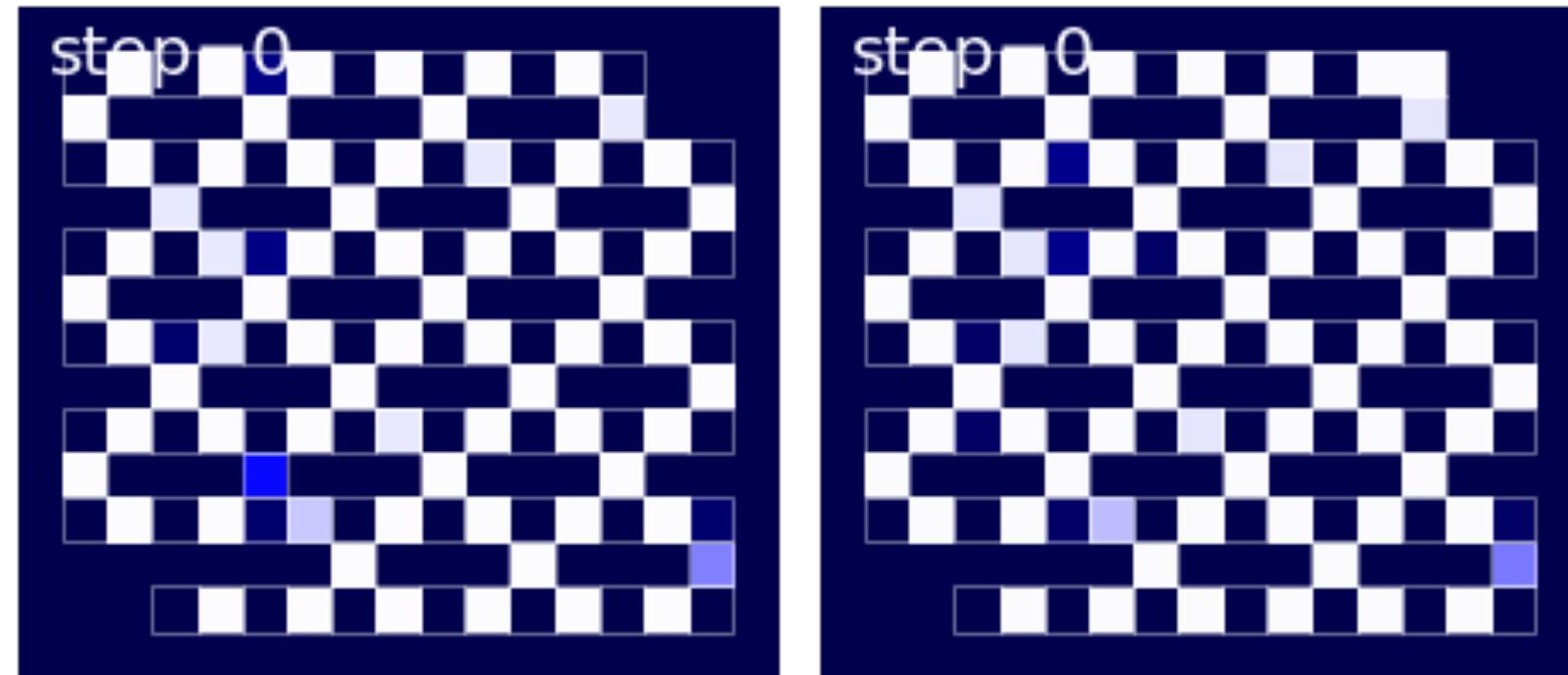
Kinetic term      Spatial disorder



# Spin imbalance for antiferromagnetic ordering



$$J = \frac{n_1 - n_0}{n_1 + n_0}$$



Thermalizing regime

$$\theta = 0.3\pi$$

Prethermal regime

$$\theta = 0.1\pi$$

Color represent spin polarization

# More benchmarks ...

## One-particle density matrices (OPDM) experiments

Natural orbitals and occupations

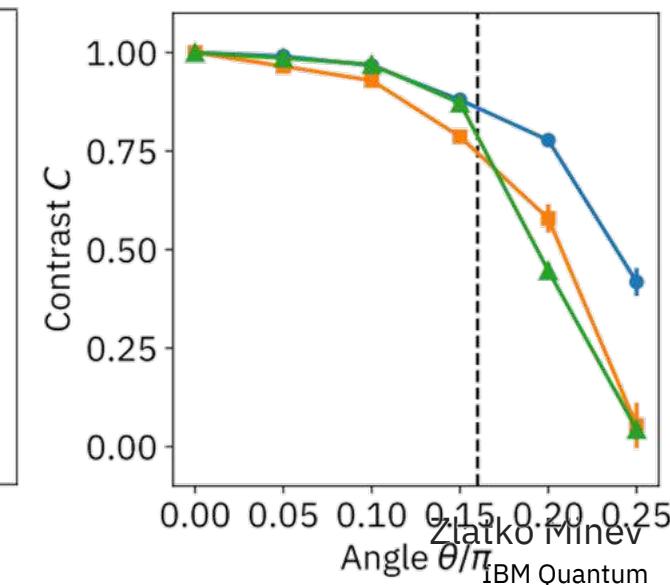
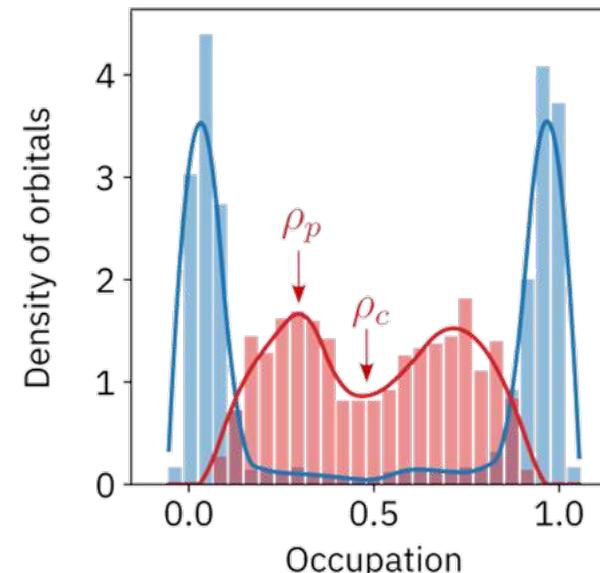
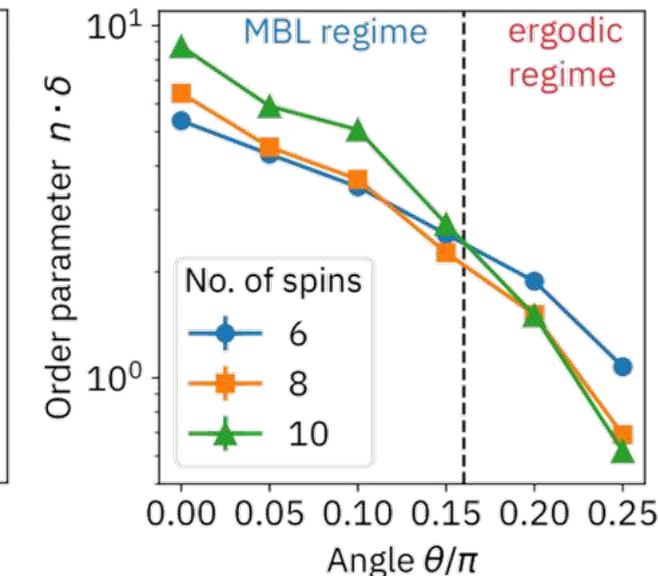
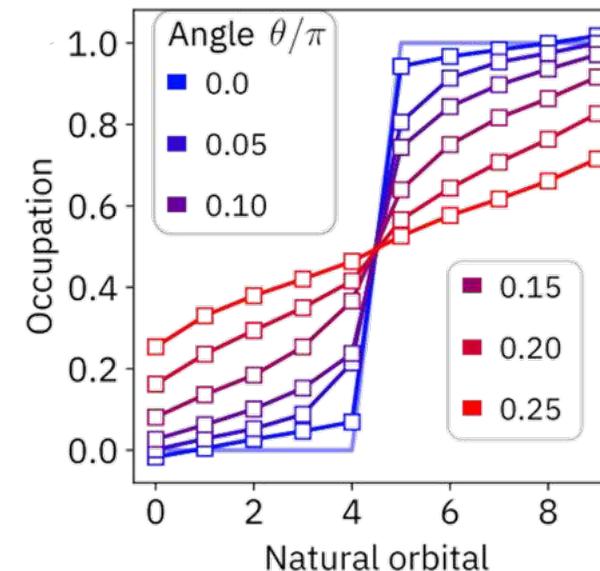
Scaling with system size: cross-overs

...

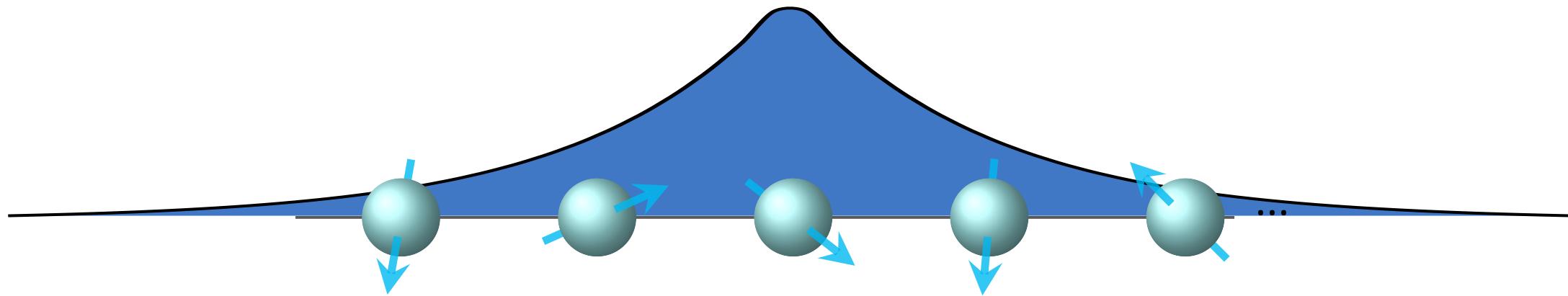
## Classical numerics in 1D

### Spectral and time analysis

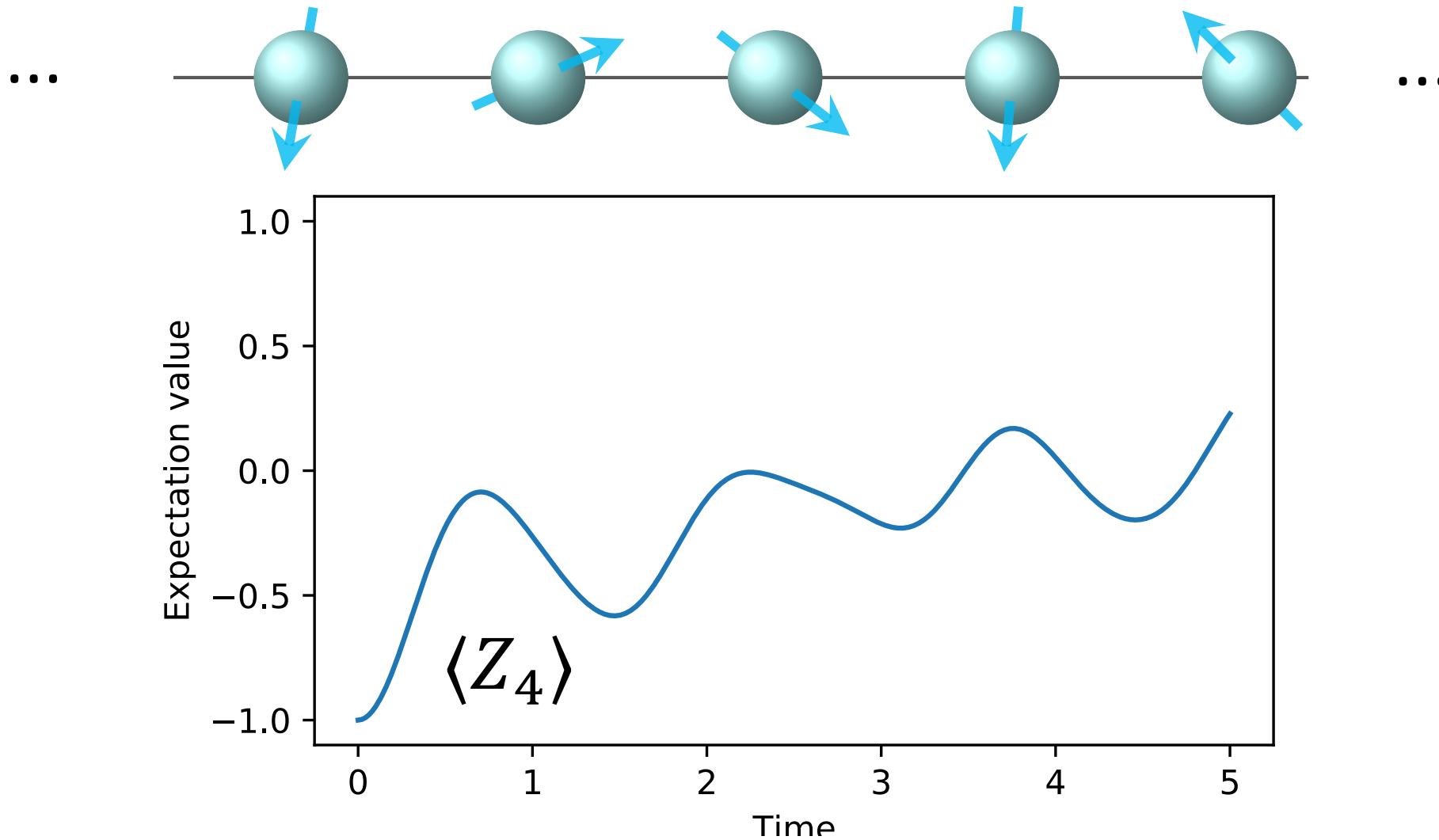
...



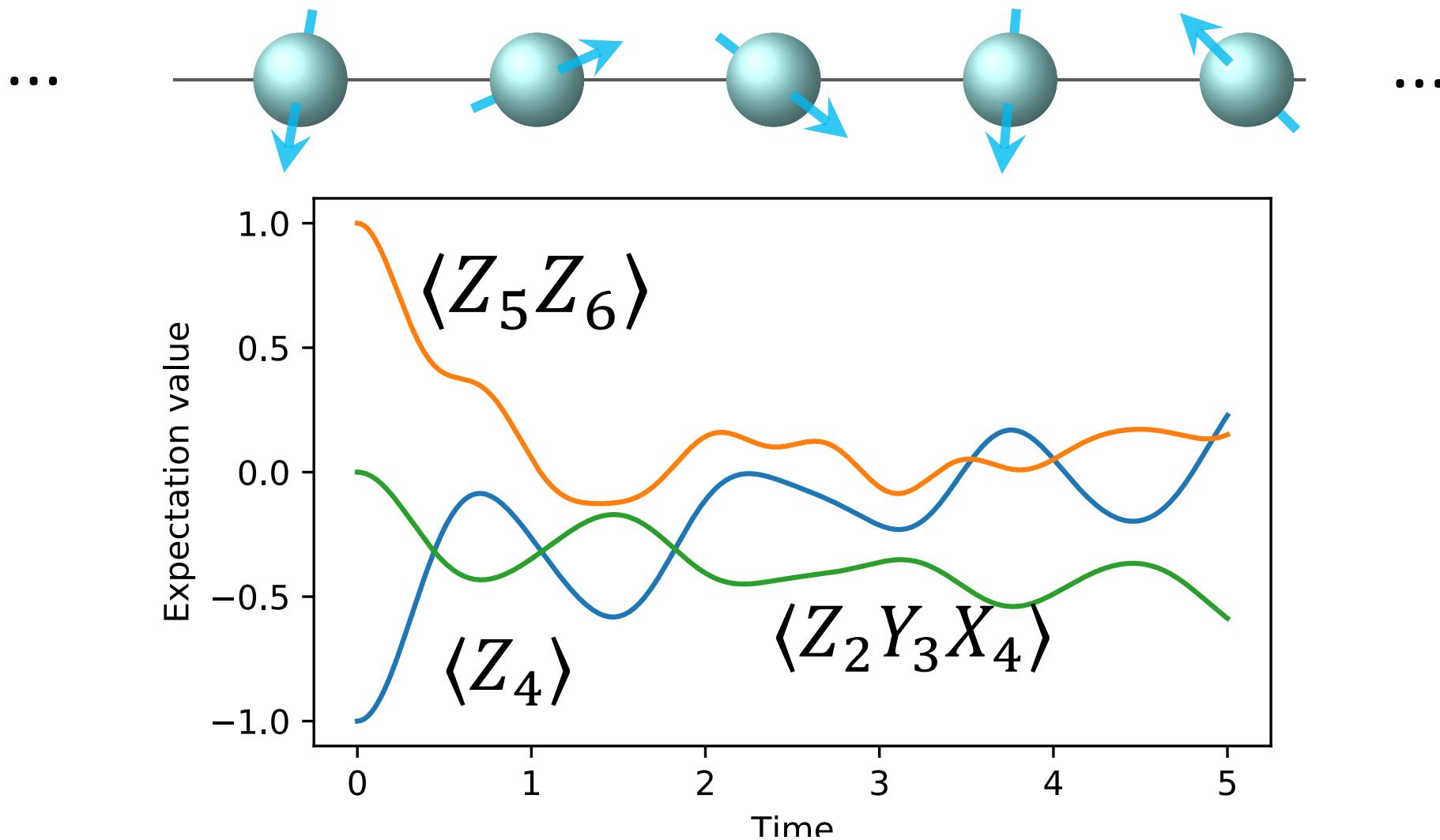
# Local integrals of motion (LIOMs)



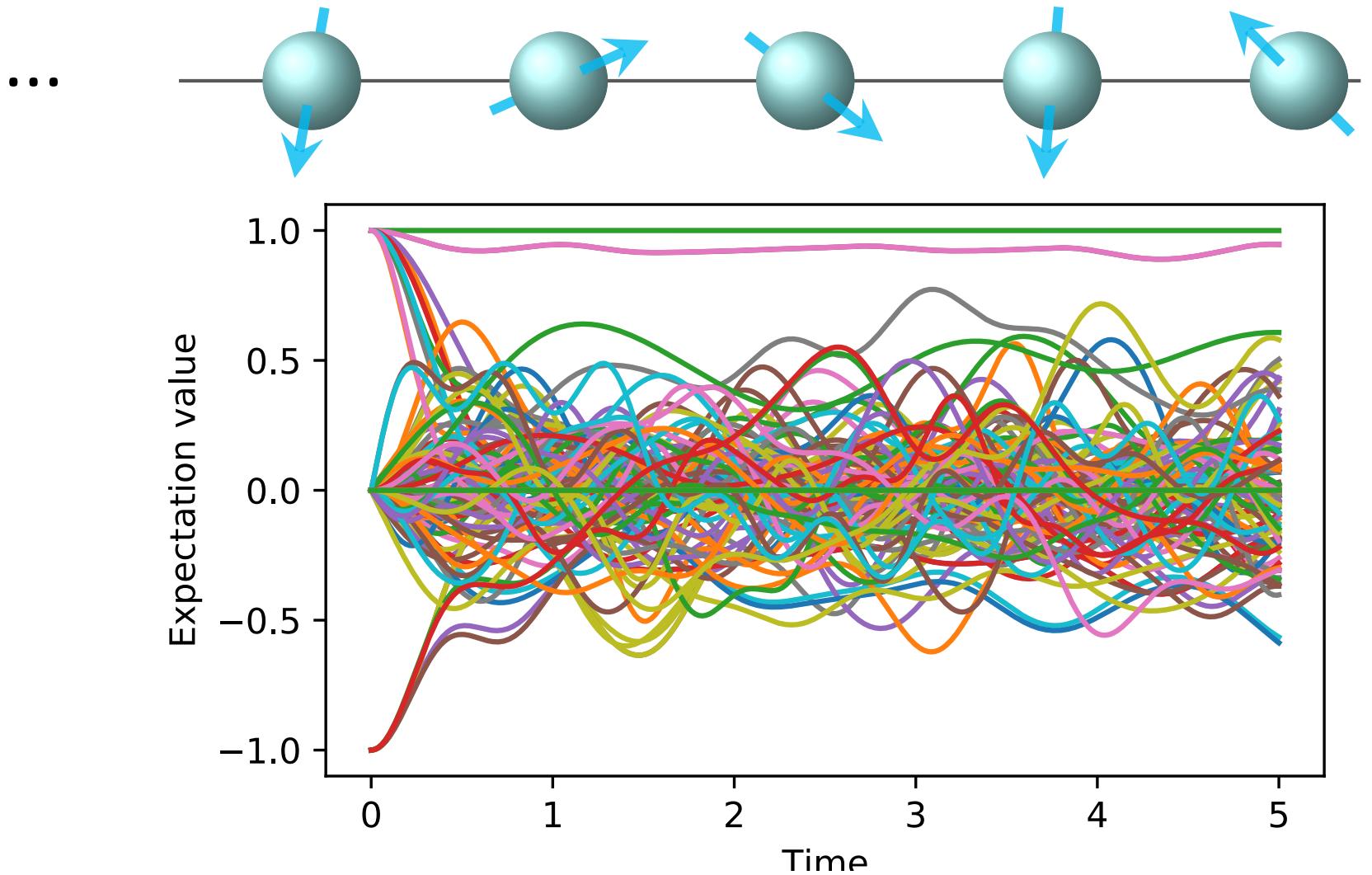
# Concept: Consider some spin chain



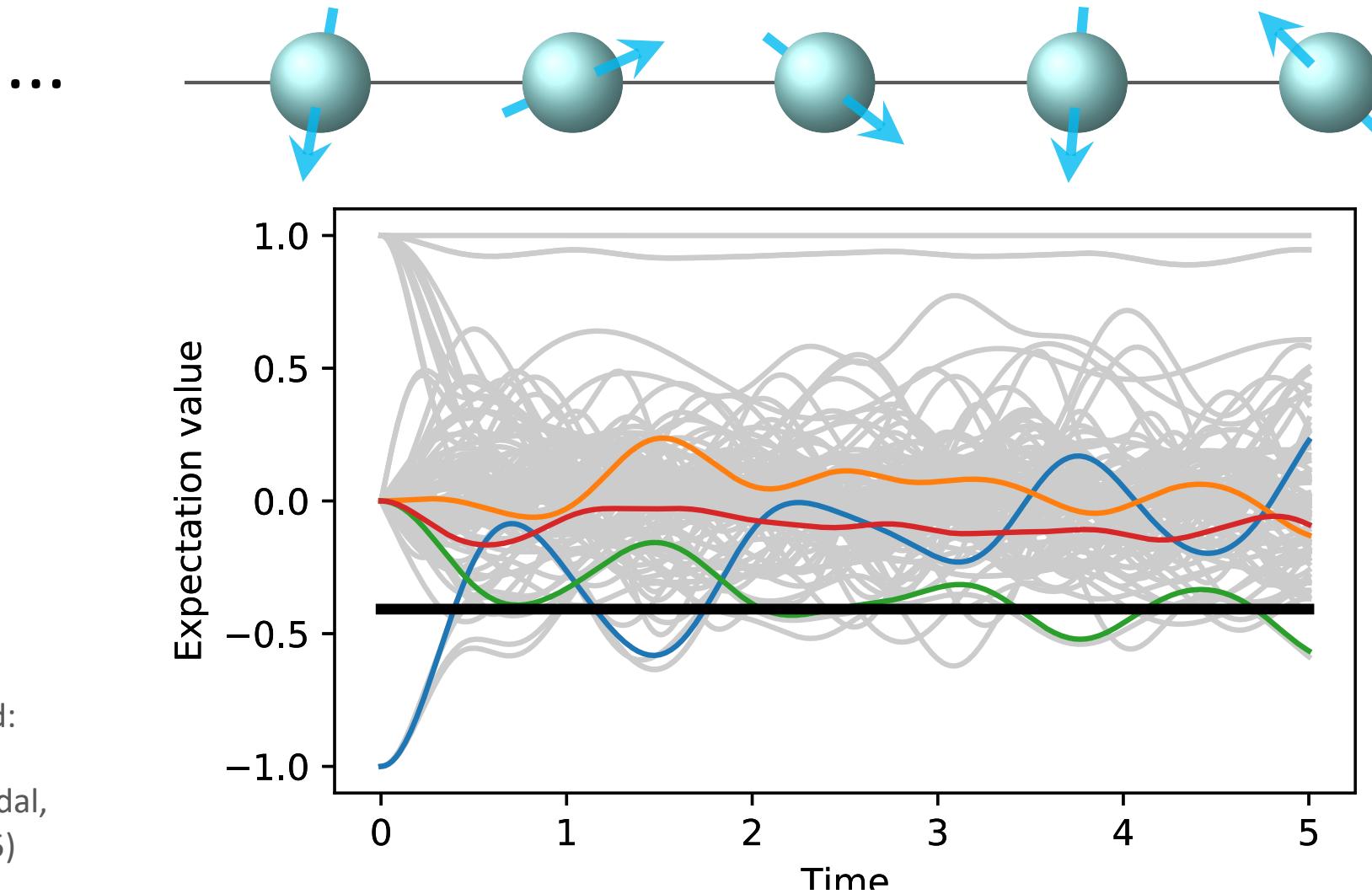
# Measure Pauli observables over time



# Measure many of them



# Find constant of motion over observation timescale



Repeat for different initial states and find constant of motion over entire data set

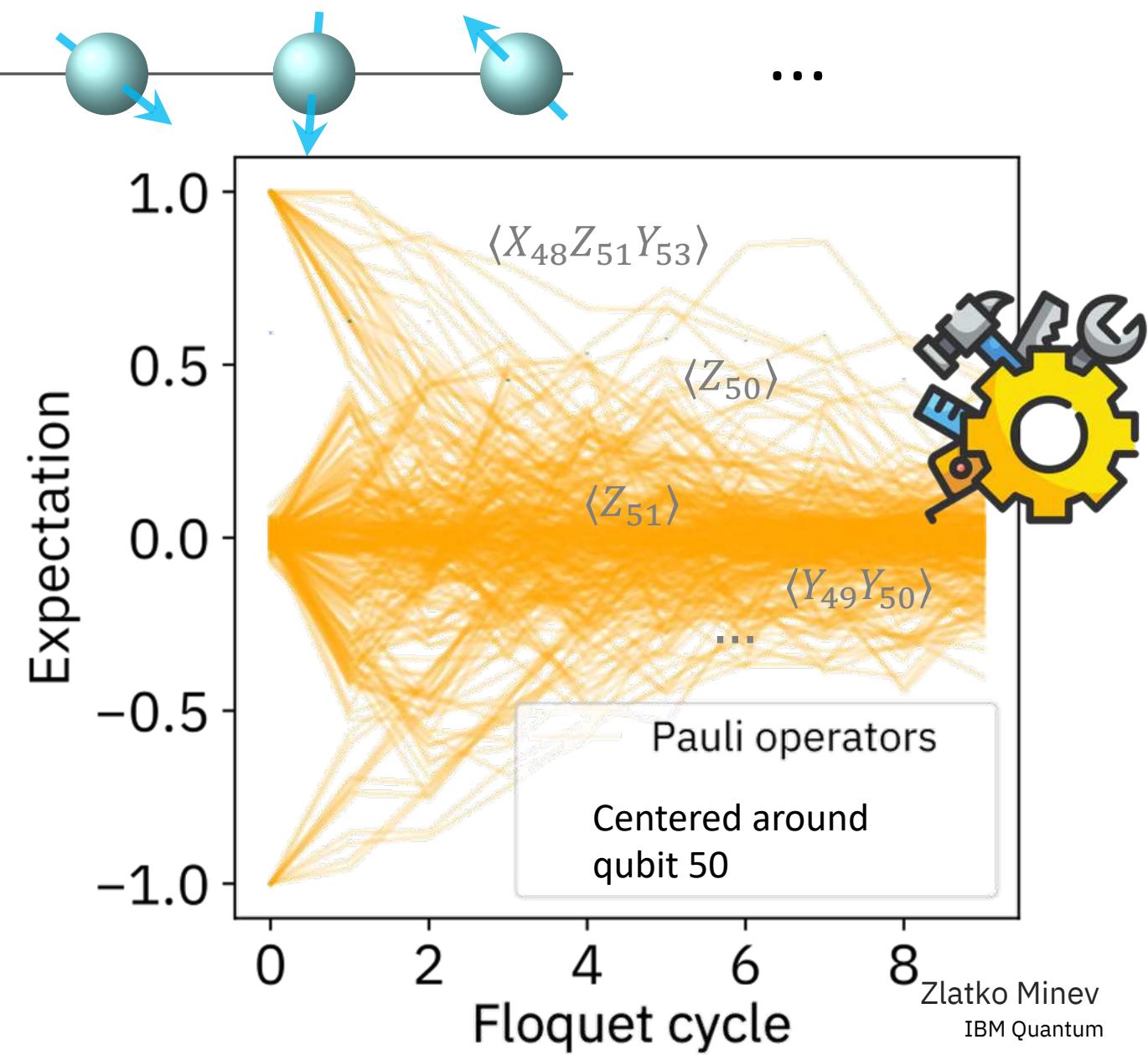
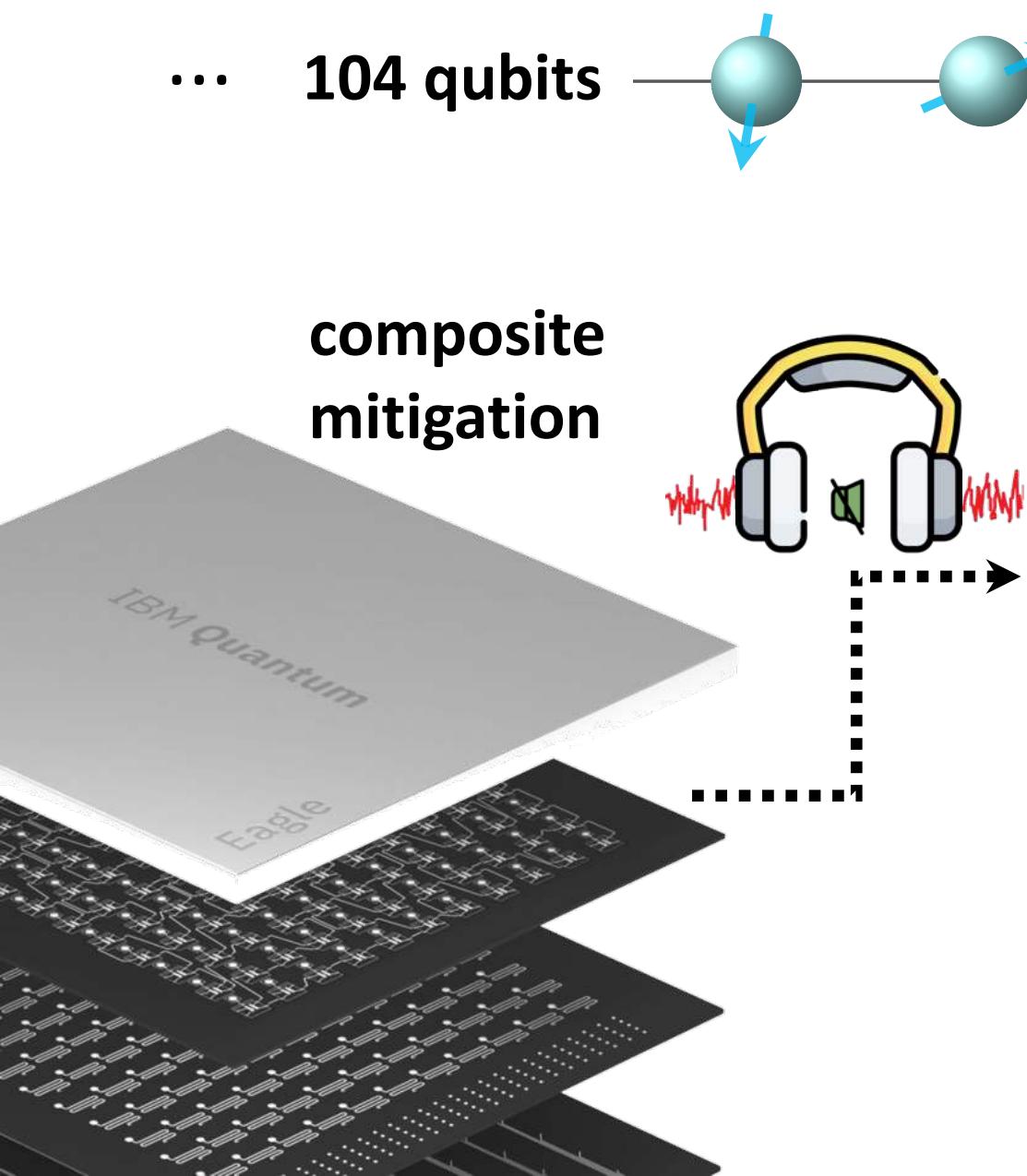
See closely related:

Chandran, Kim, Vidal,  
Abanin, PRB (2015)

Mierzejewski, Prelovsek,  
Prosen, PRL (2015)

$$a\langle Z_2 \rangle + b\langle X_1 Y_2 \rangle + c\langle Y_2 X_3 \rangle + d\langle X_1 Z_2 X_3 \rangle = \text{const}$$

# In hardware experiment: Example data



# Example uncovered LIOM from experiment

Integral of motion  $L$

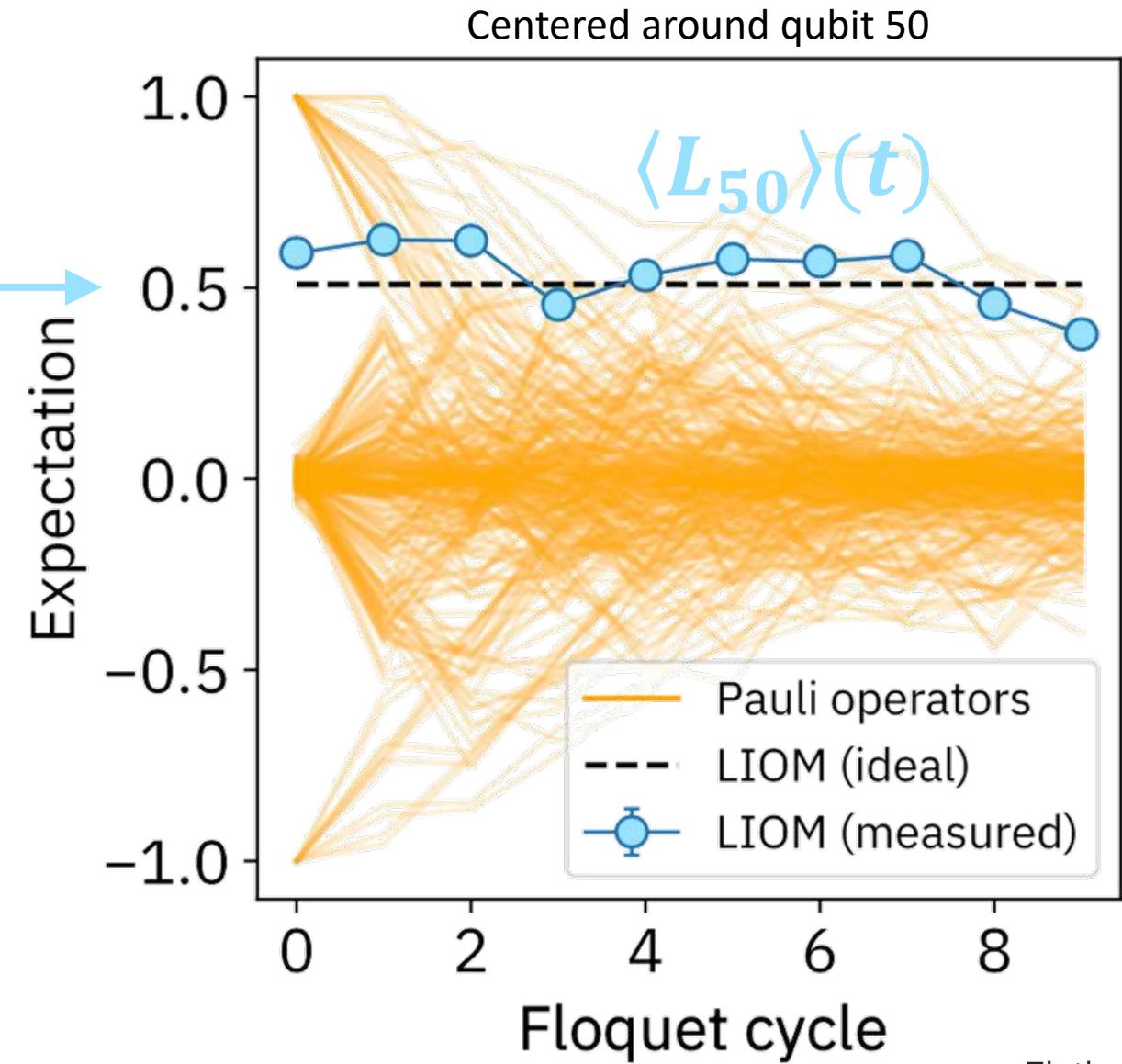
$$[U, L] = 0 \quad [H, L] = 0$$

$$\langle L \rangle = \text{const}$$

Local integral of motion (LIOM)  $L$

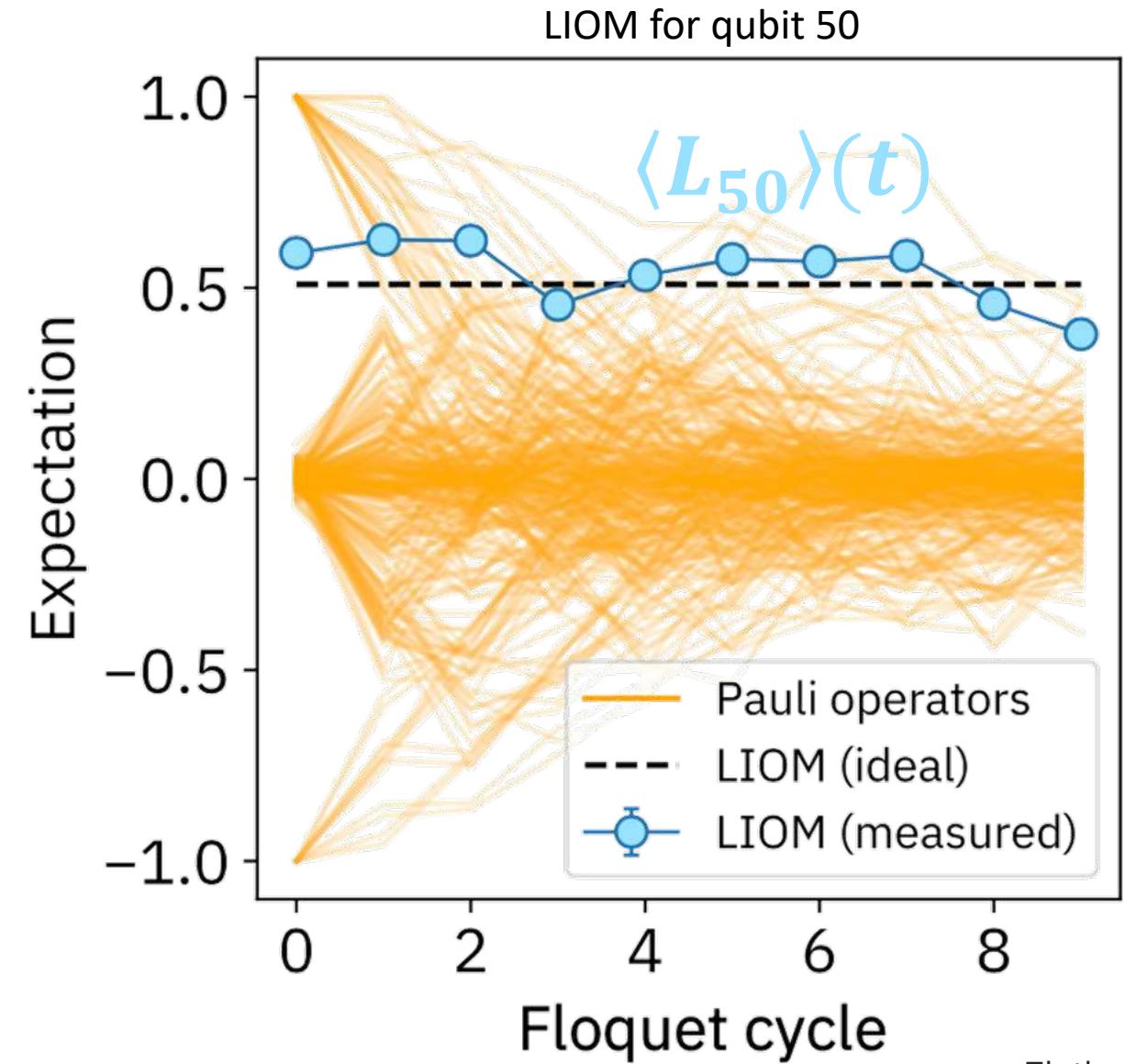
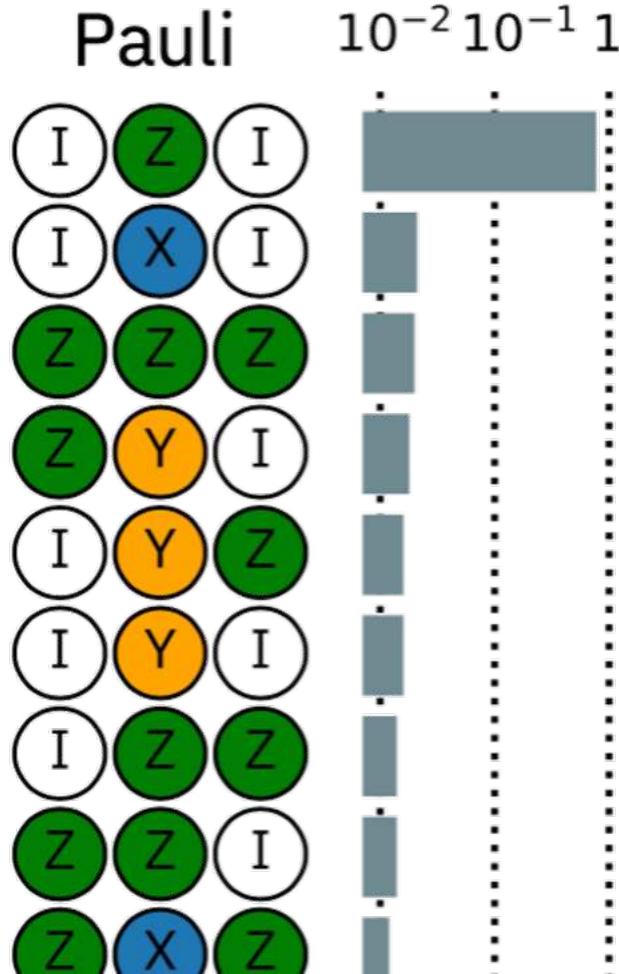
$$L_k = \sum_{\mu \in \mathcal{N}(k)} a_\mu P_\mu$$

local neighborhood

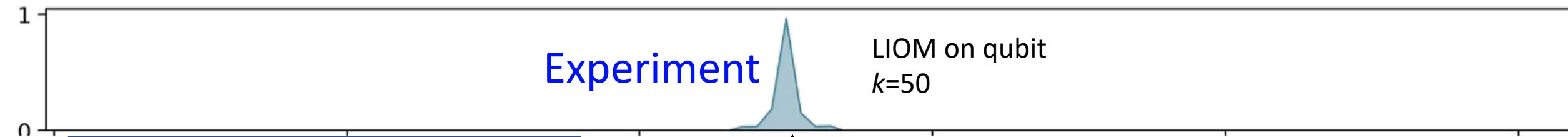


# Operator decomposition of the LIOM

$$L_k = \sum_{\mu \in \mathcal{N}(k)} a_\mu P_\mu$$

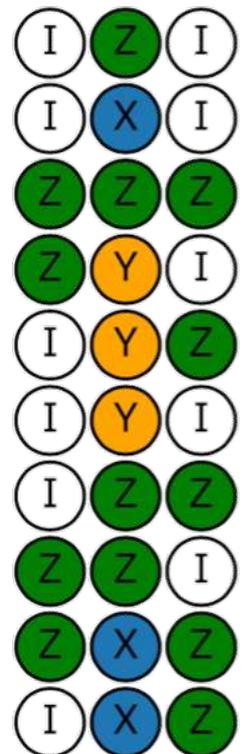


# Operator density for LIOMs $\hat{L}_k$ in the 1D lattice

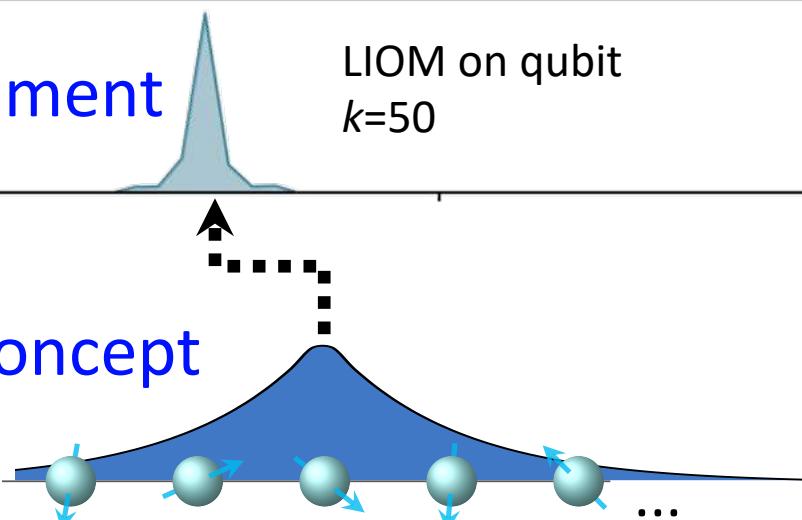


$$L_k = \sum_{\mu \in \mathcal{N}(k)} a_\mu P_\mu$$

Pauli  $10^{-2} 10^{-1} 1$



Concept



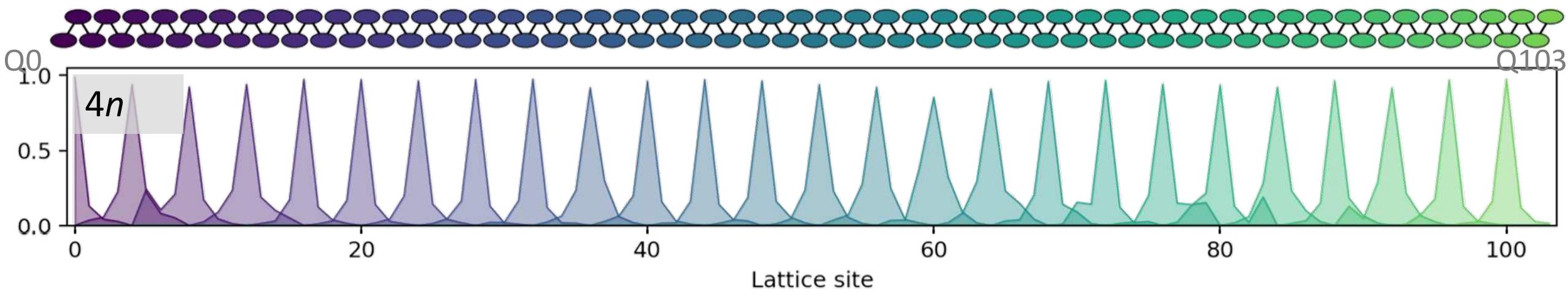
**Total density over  
all Pauli weights**  
 $W(x)$

$$W(x) = \sqrt{\sum_p W(x, p)^2}$$

$$W(x, p) = \sqrt{\sum_{\mu \sim N(k, p)} a_\mu^2 w_{x\mu}}$$

Operator density for LIOMs  $\hat{L}_k$  in the 1D lattice

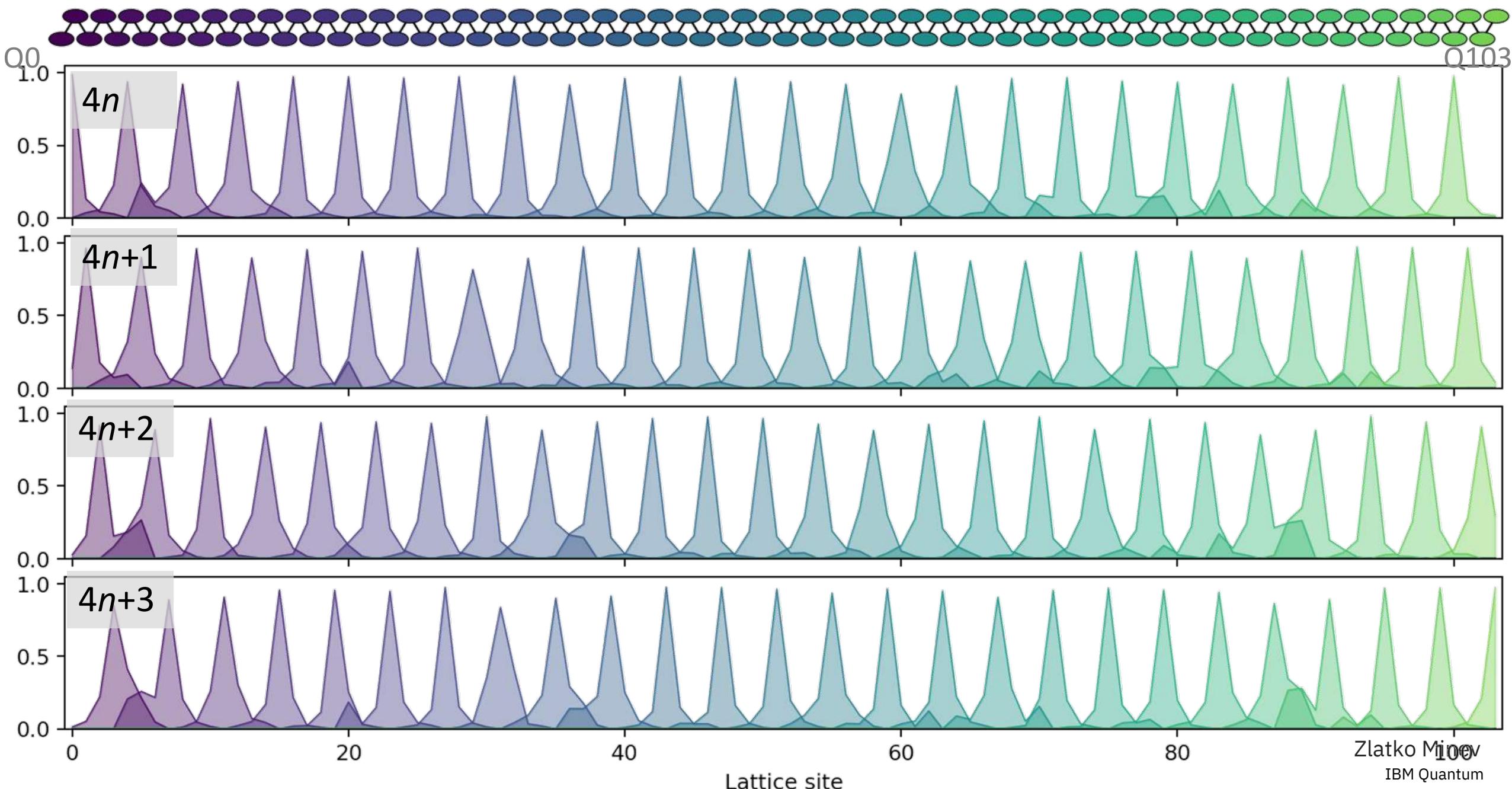
All the 1D LIOMs



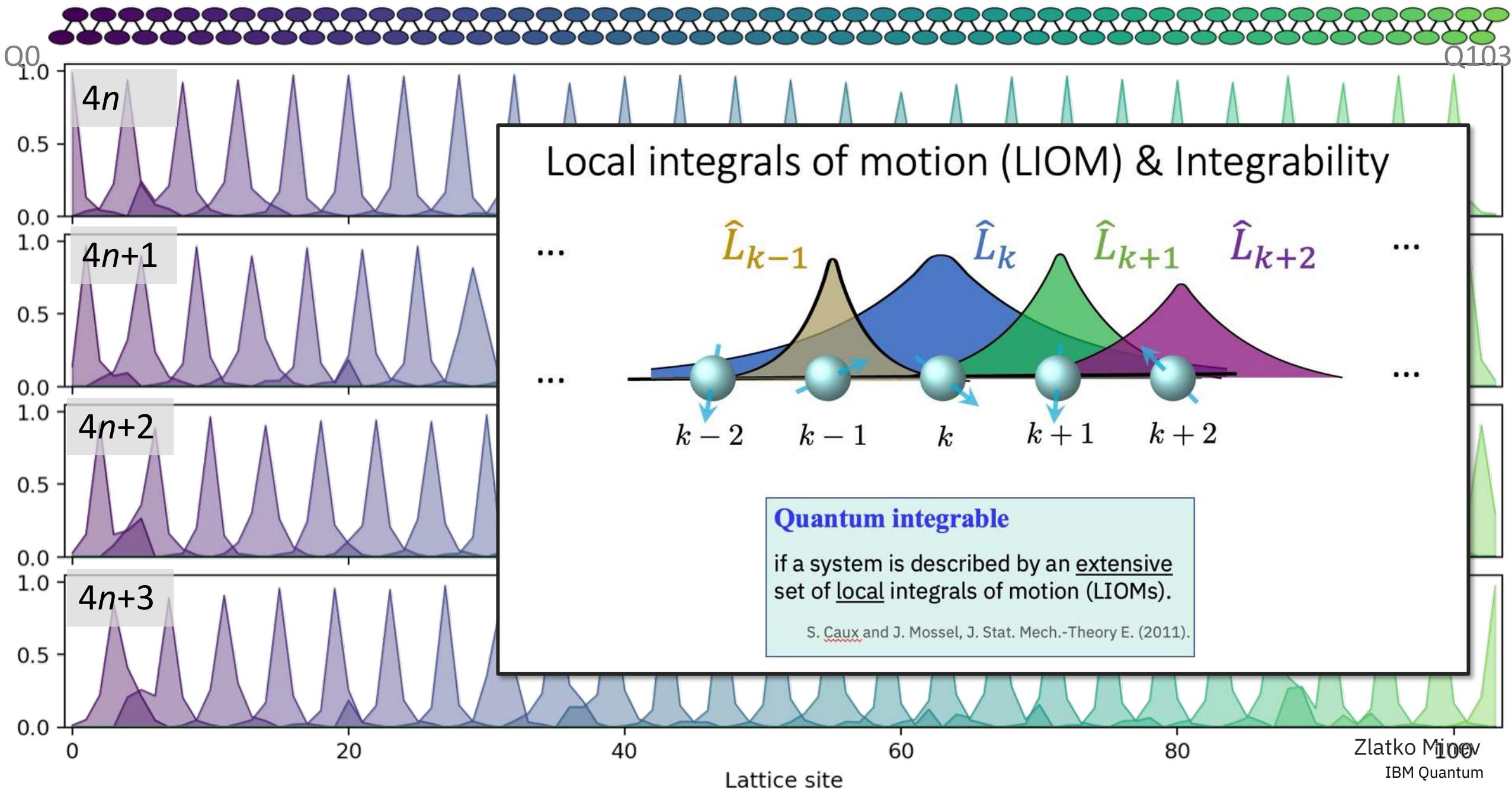
Every 4<sup>th</sup> LIOM

# All LIOMs in 1D

Operator density for LIOMs  $\hat{L}_k$  in the 1D lattice



# Integrable system on the observation timescale

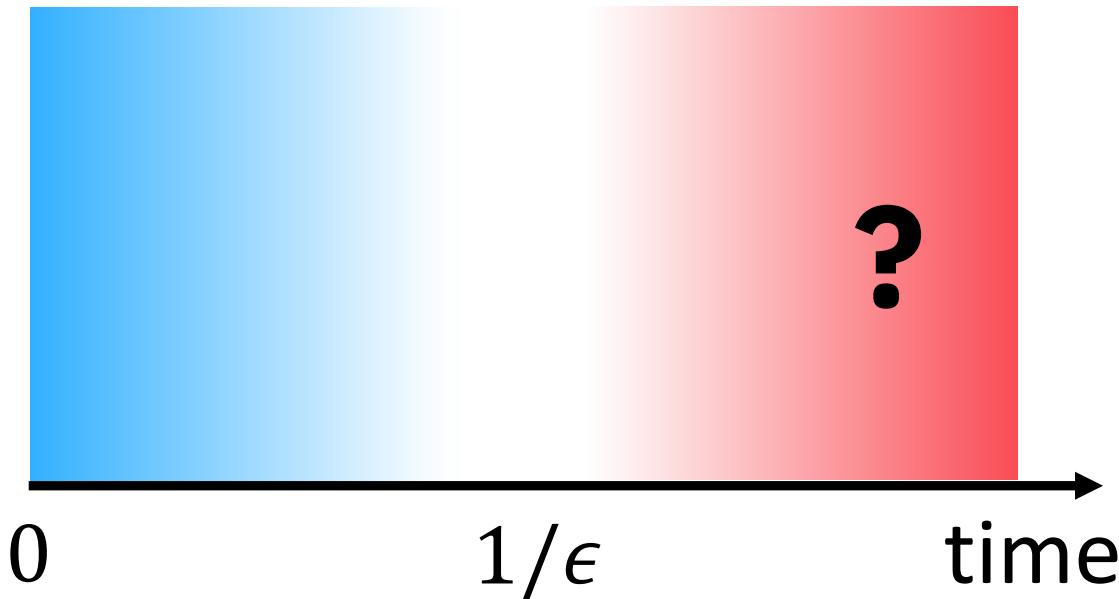


# Caution: Ultimate integrability

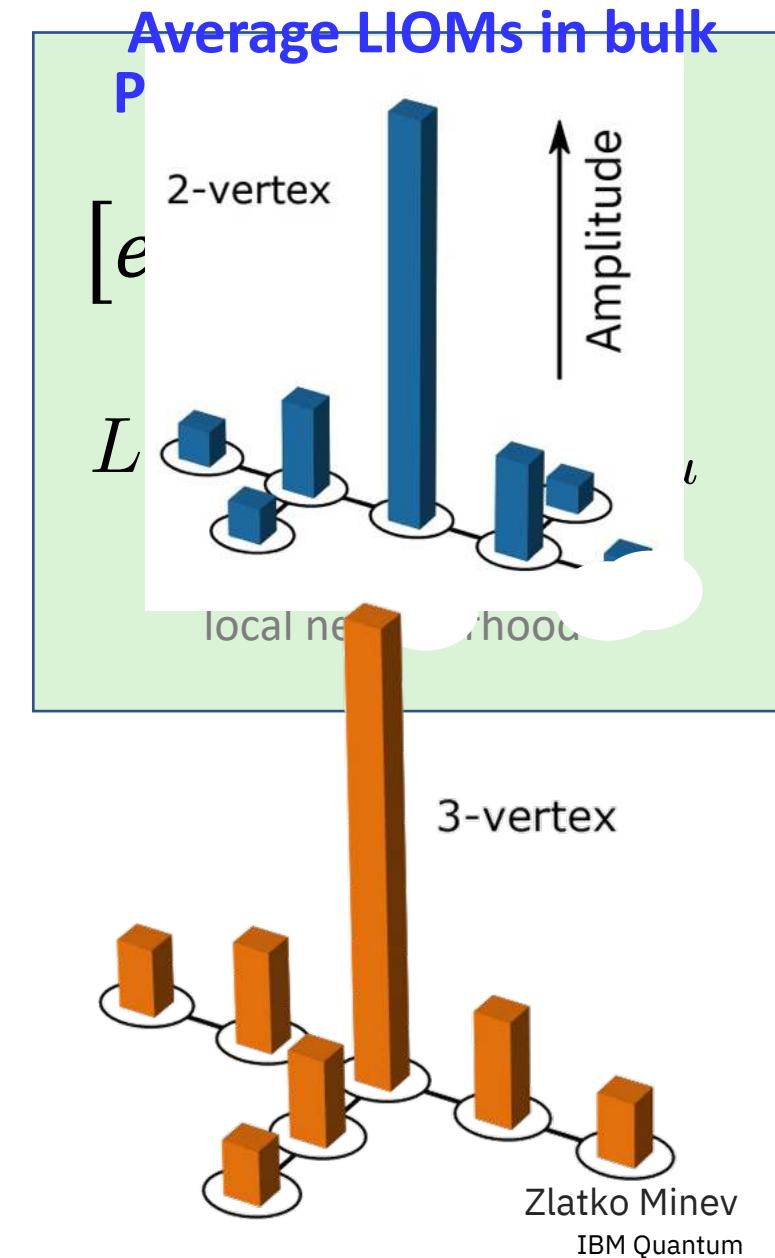
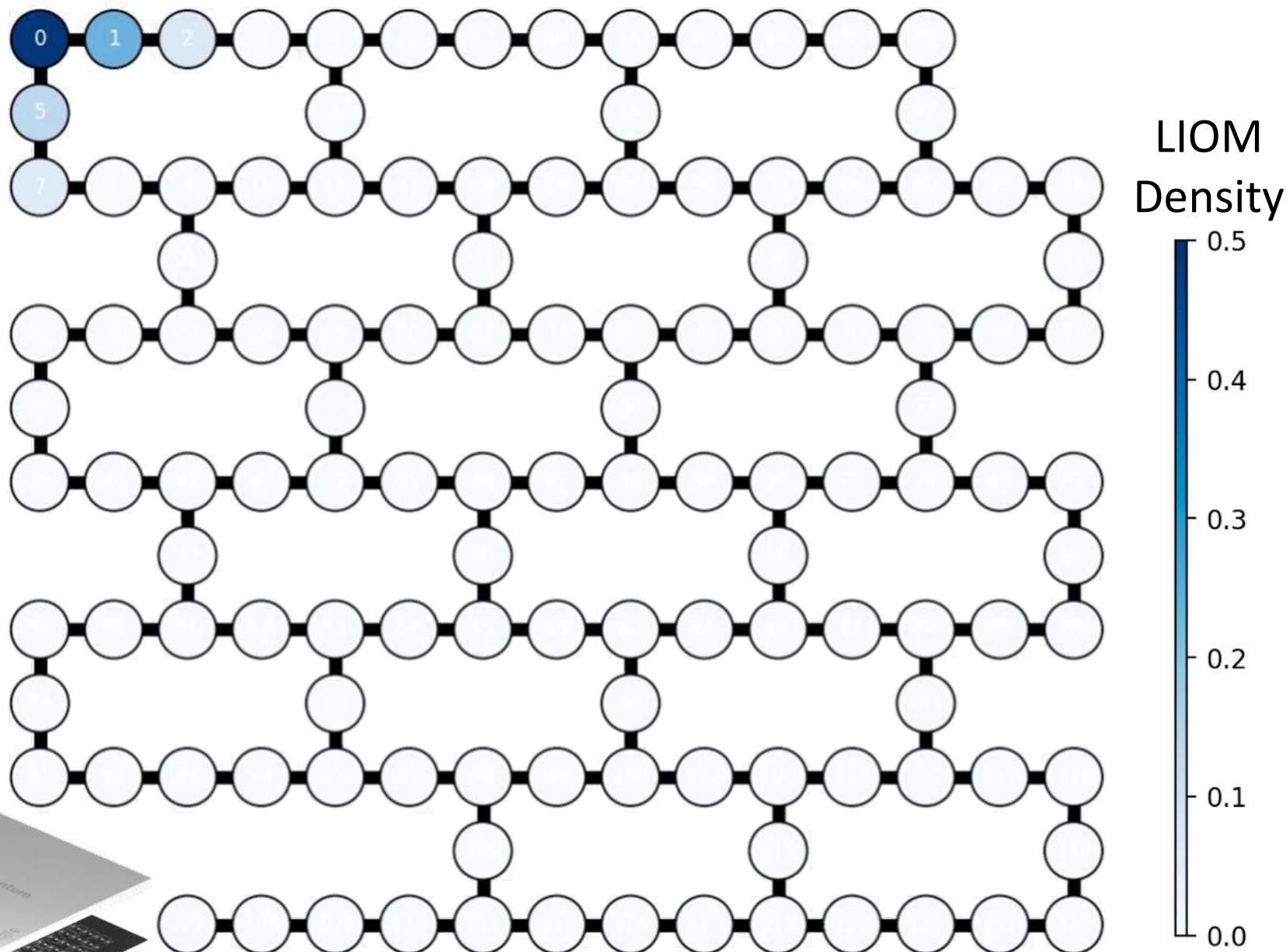
Derived LIOMs are conserved over the observation timescale.  
In the long-time limit, they are always understood as approximate.  
The ultimate integrability of a system is an undecidable problem in general.

Shiraishi, Matsumoto, *Nat Comm* (2021)

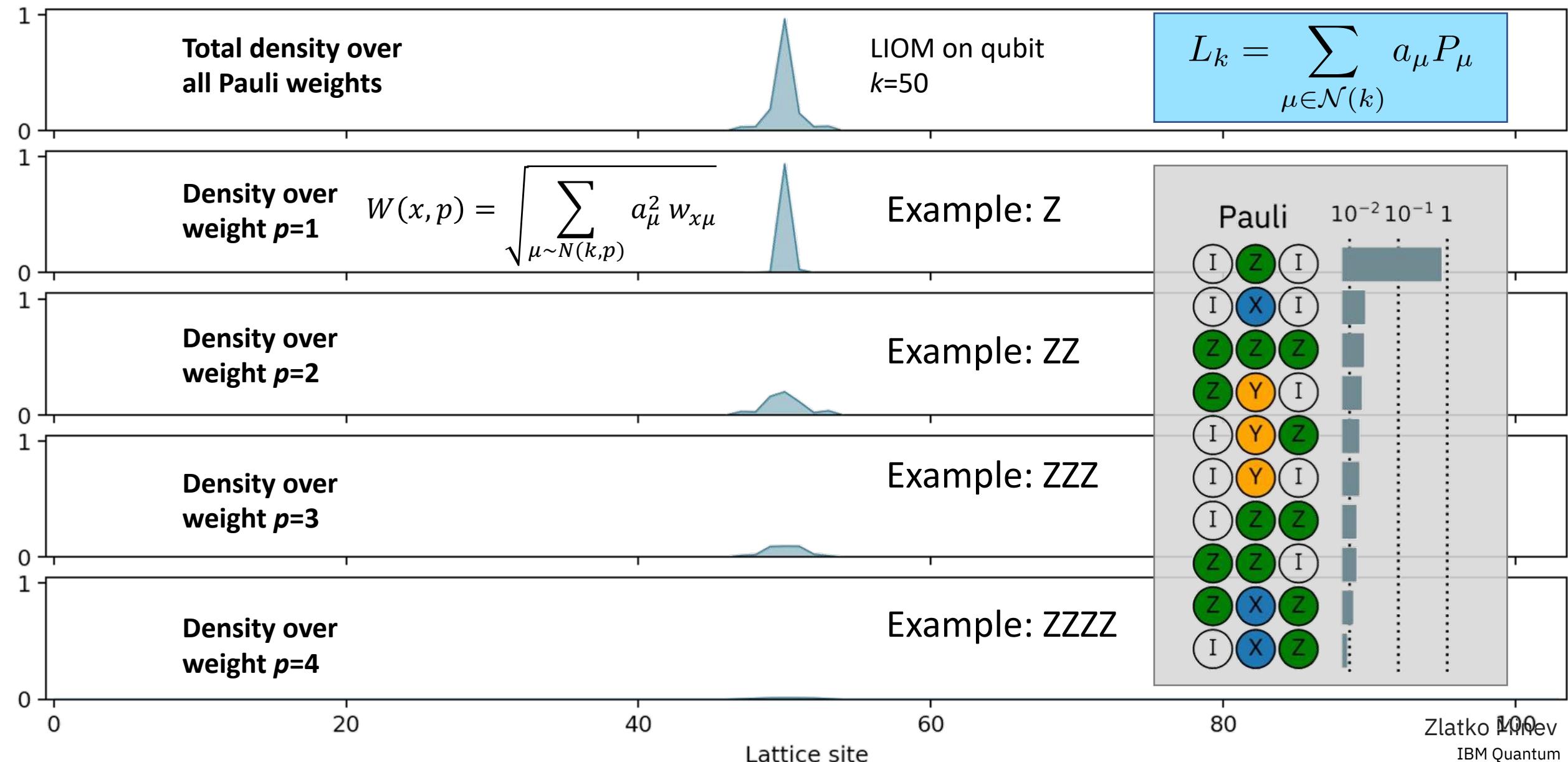
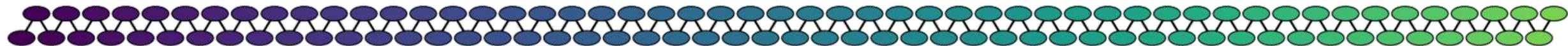
$$[U_F, L] = O(\epsilon)$$



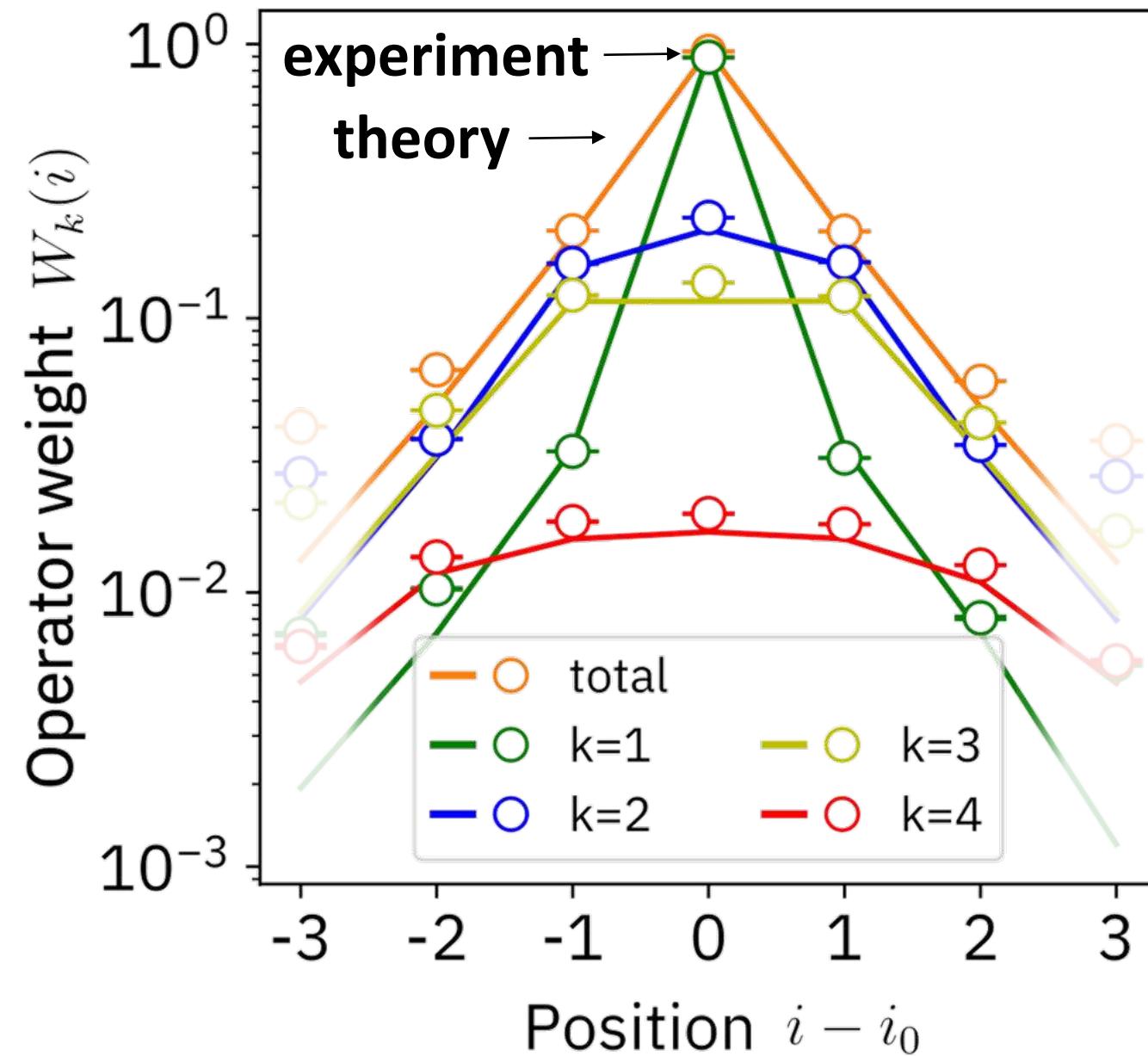
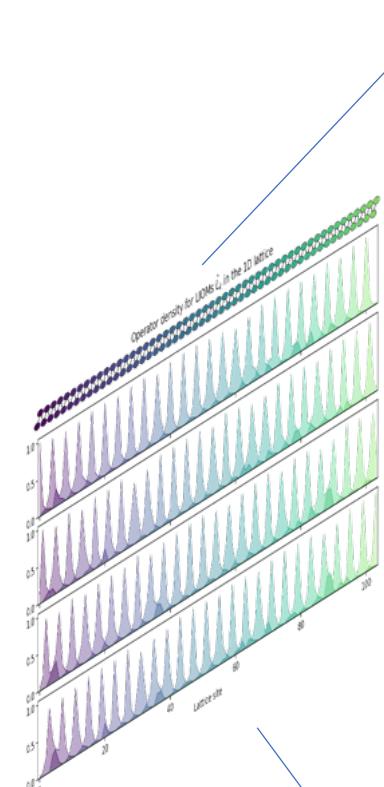
# 2D: All 124 pre-thermal LIOMs $L_k$



# Operator density for LIOMs $\hat{L}_k$ in the 1D lattice



# Average LIOM decomposed by weight: experiment vs. theory

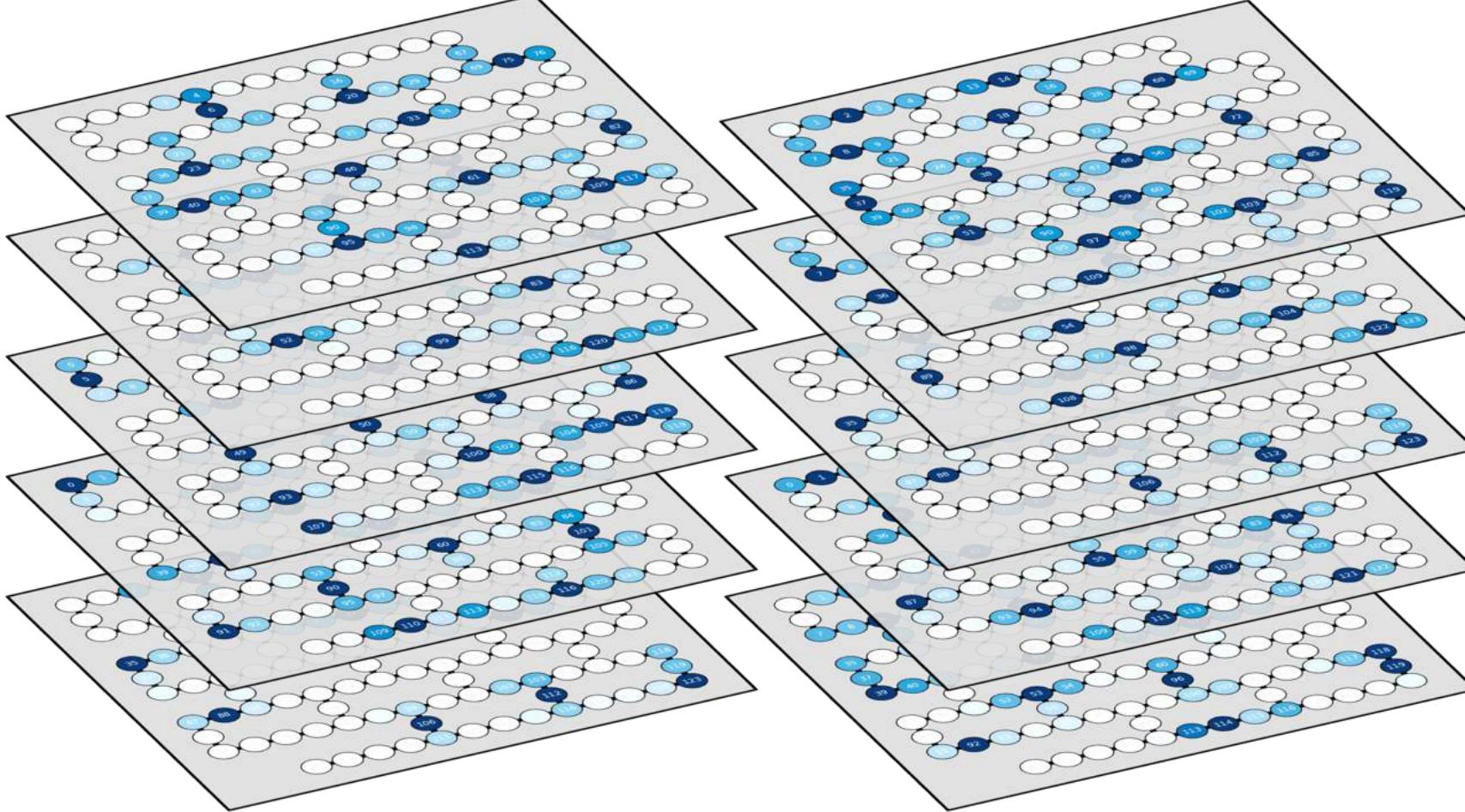
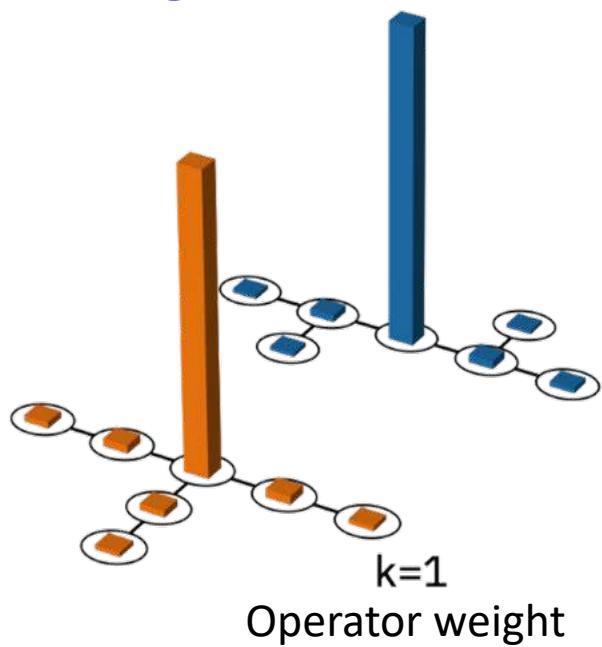


$$L_k = \sum_{\mu \in \mathcal{N}(k)} a_\mu P_\mu$$

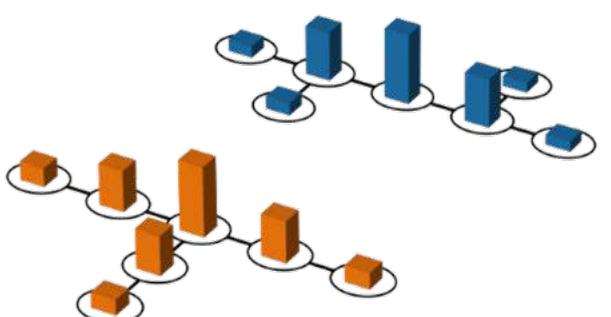
total  
weight 1  
weight 2  
weight 3  
weight 4

# Decomposition by operator weight

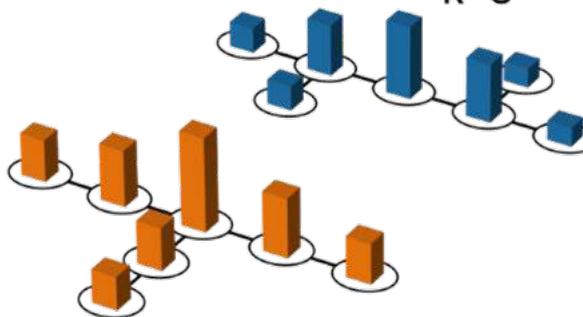
Average LIOM density  
for a given operator  
weight



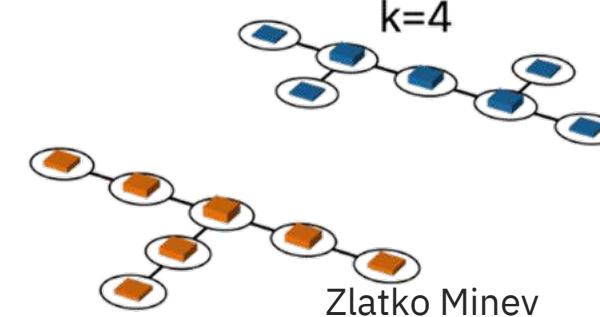
k=2



k=3

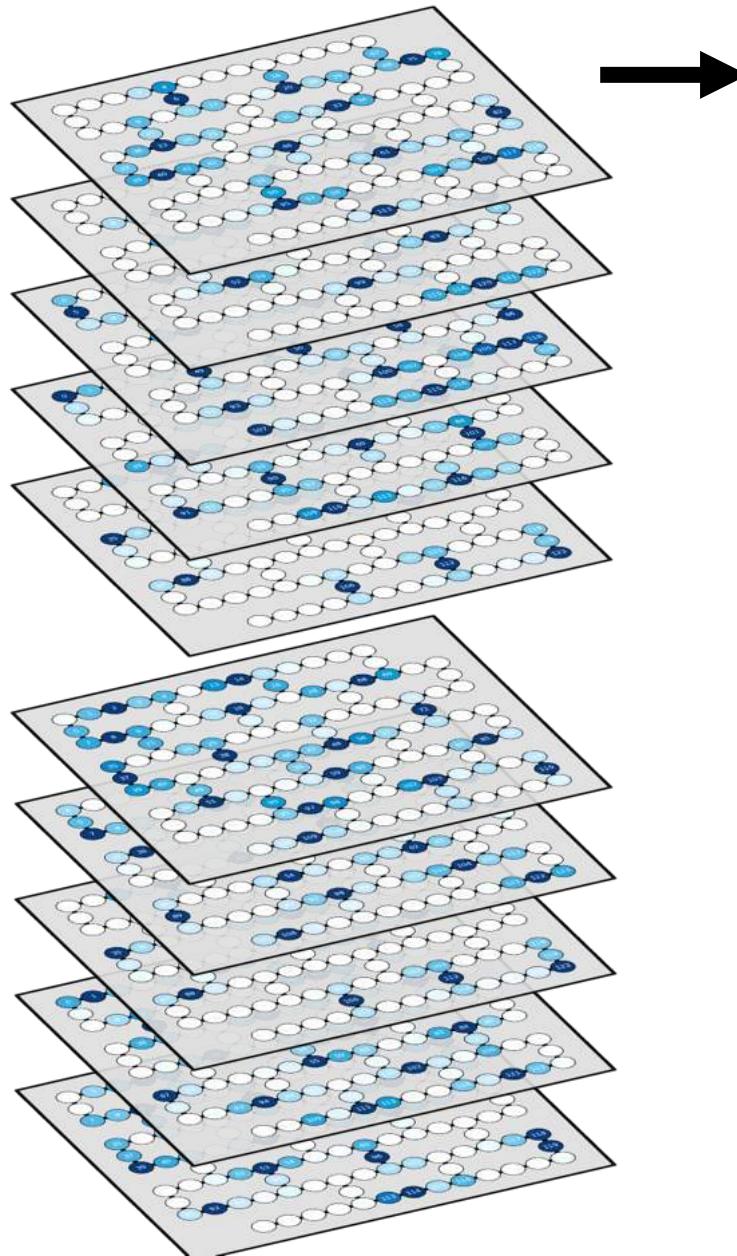


k=4

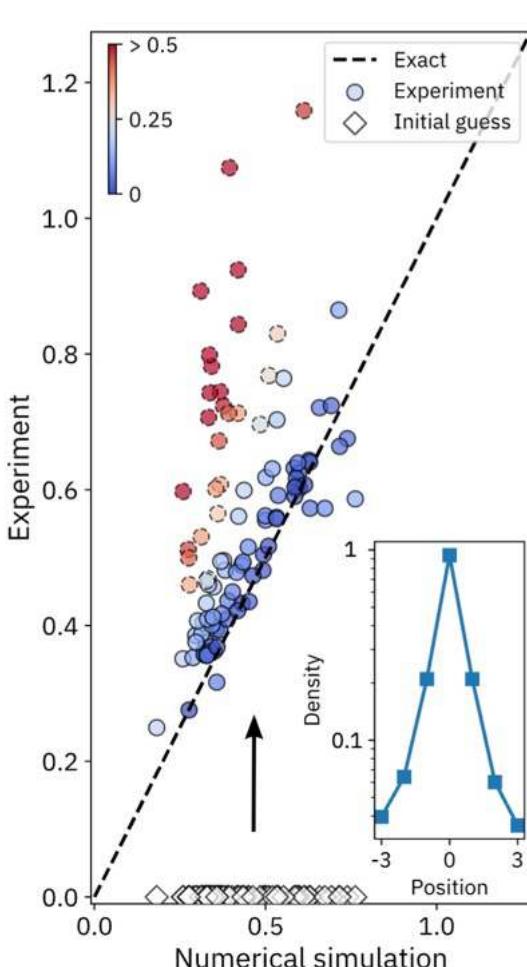


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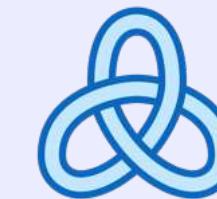
# Example applications of the LIOMs and next steps



## Site-based localization lengths



**Melting** of the LIOMs  
across the phase /  
regime transition

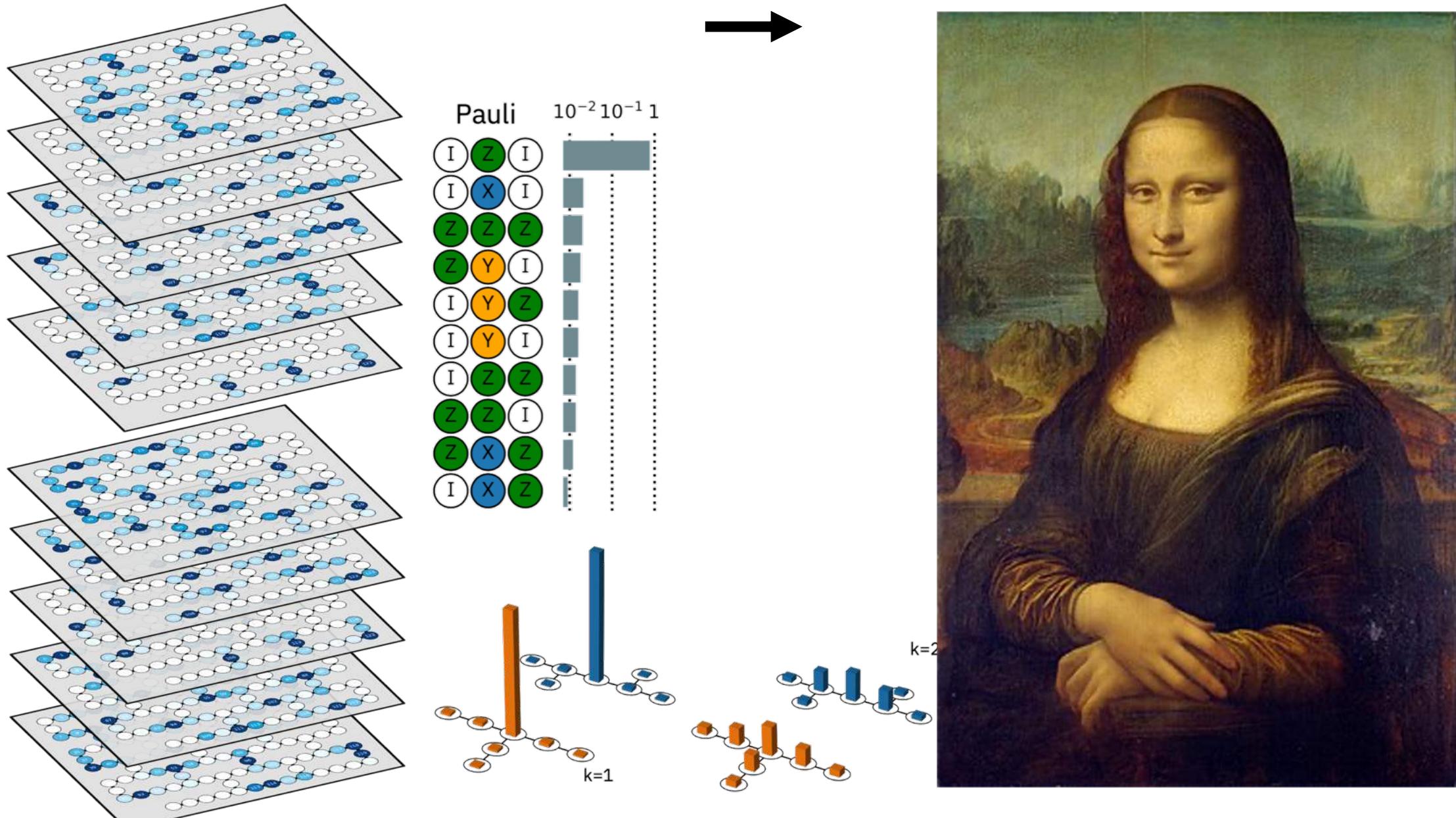


**Topological** setting  
LIOMs and melting



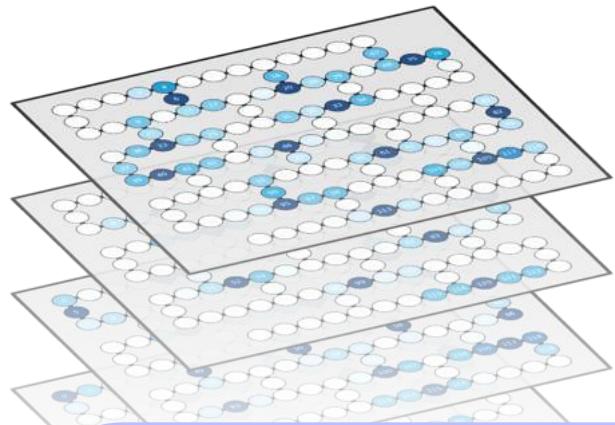
**Recover & track**  
locally scrambled  
information

# Operational LIOM description on observation timescales



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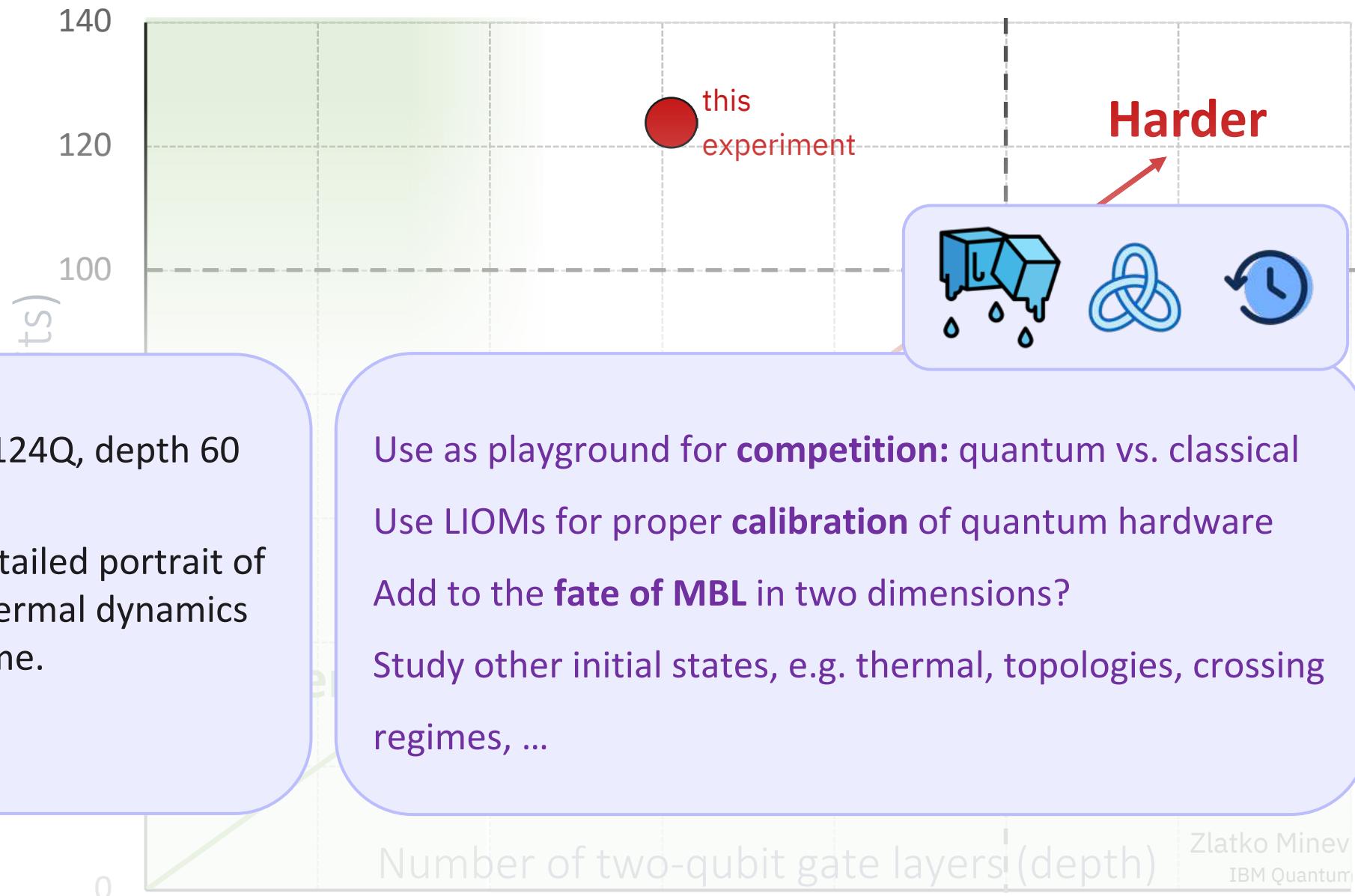
# Conclusions and future



Many-body dynamics with 124Q, depth 60 error-mitigated circuits

Operationally restored a detailed portrait of a system's localized / prethermal dynamics in a new experimental regime.

Scalable in  $n$  protocol. ...



# A detailed portrait of quantum many-body dynamics

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IBM Quantum

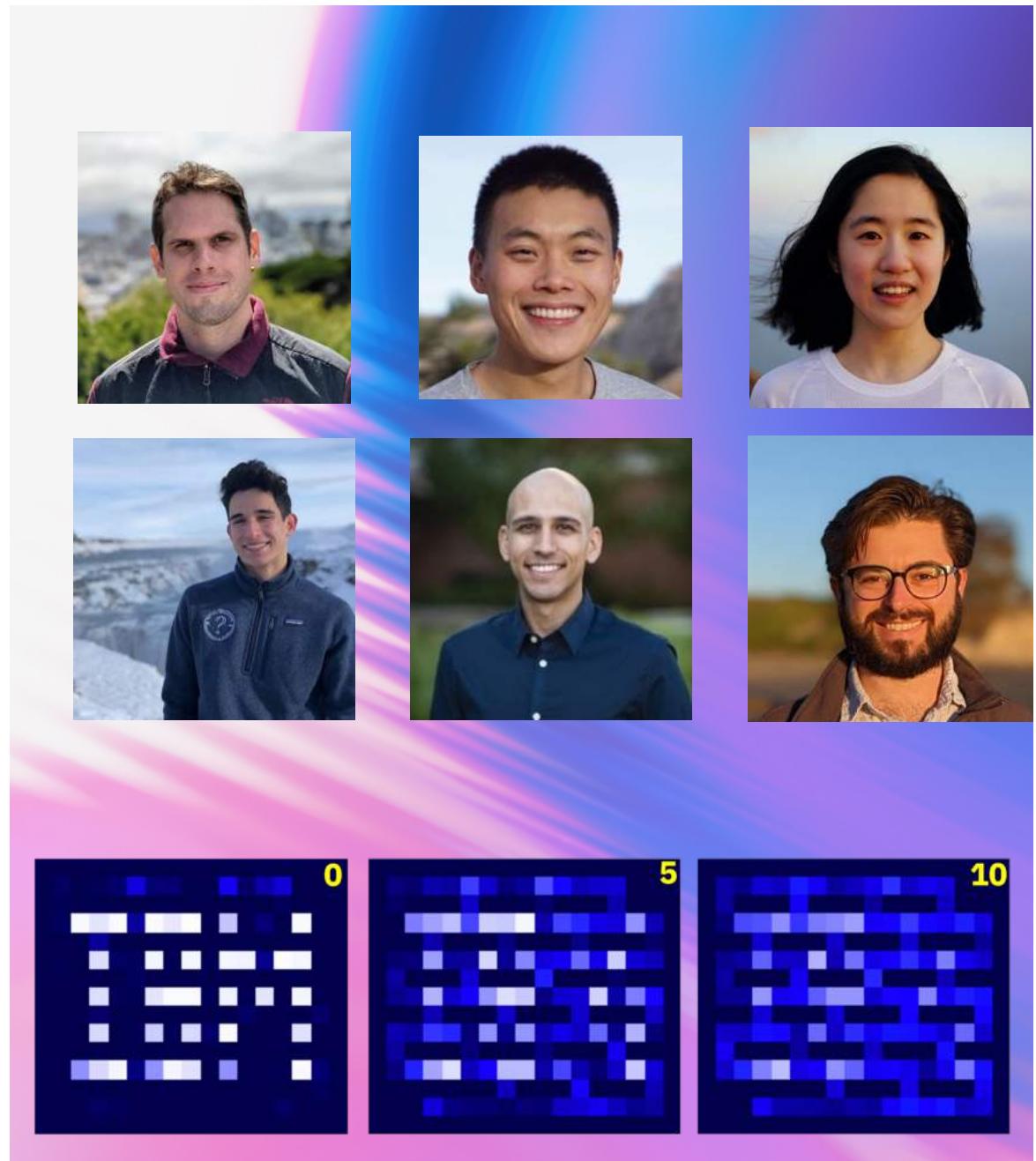
Oles Shtanko\*, Derek Wang\*, Haimeng Zhang, Nikhil Harle, Alireza Seif, Ramis Movassagh, Zlatko K. Minev  
arXiv:2307.07552

Acknowledgements: A. Deshpande, O. Dial, A. Eddins, D. Egger, B. Fuller, J. Garrison, D. Hahn, A. Kandala, W. Kirby, D. Layden, H. Liao, D. Luitz, S. Majumder, A. Mezzacapo, T. Prosen, J. Raftery, D. Sels, K. Temme, M. Tepaske, K. Wei, the IBM Quantum team, and many friends and colleagues



Got Slides?

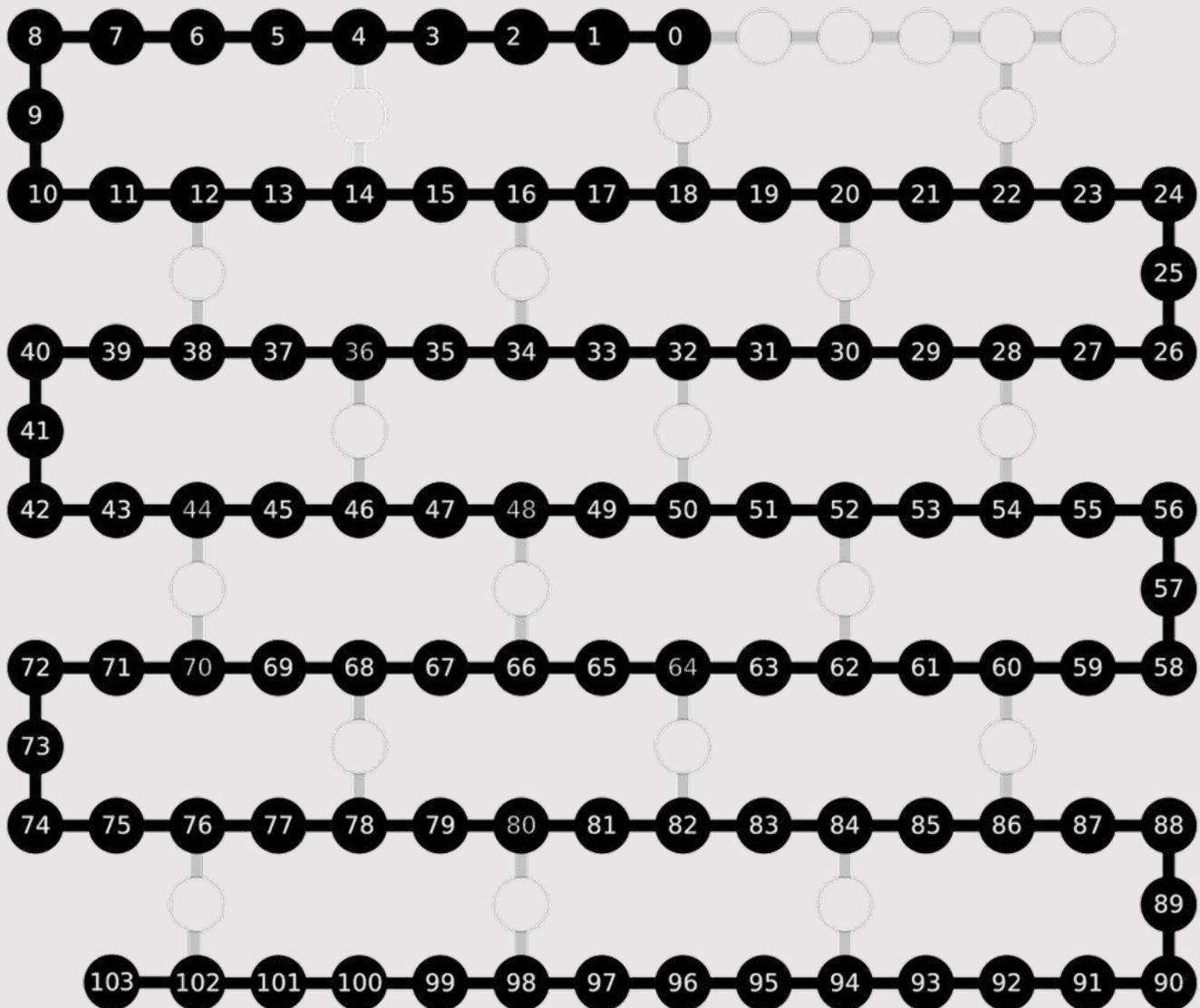
[zlatko-minev.com/blog](http://zlatko-minev.com/blog) @zlatko\_minev



Bonus content:

Taken out from 60 min version

# 1D spin chain



# Basis of protocol for measuring LIOMs

**Given:** Unitary operator  $U_F$

**Goal:** Find an operator that commutes with the unitary  $[U_F, L] = 0$

**Solution.** Follow the steps:

1. Start with a suitable local operator  $L_0$
2. Evaluate the operator

$$L \propto L_0 + U_F L_0 U_F^\dagger + U_F^2 L_0 U_F^{2\dagger} + \dots + U_F^D L_0 U_F^{D\dagger}$$

In the limit  $D \gg 1$ ,

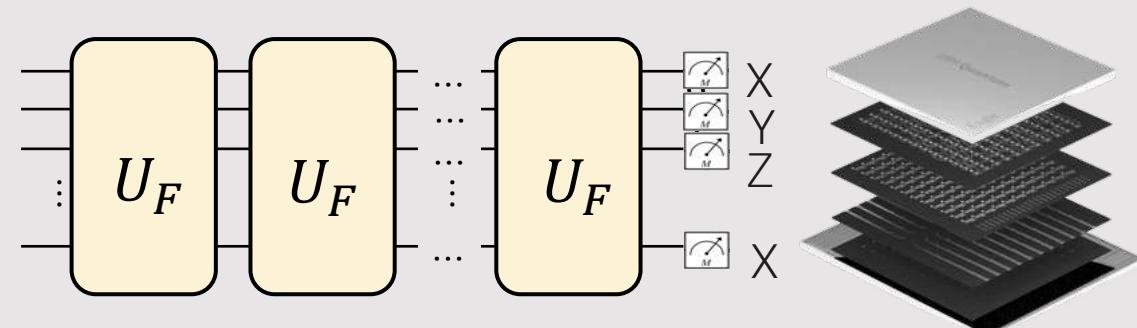
$$[U_F, L] \sim O(D^{-1})$$

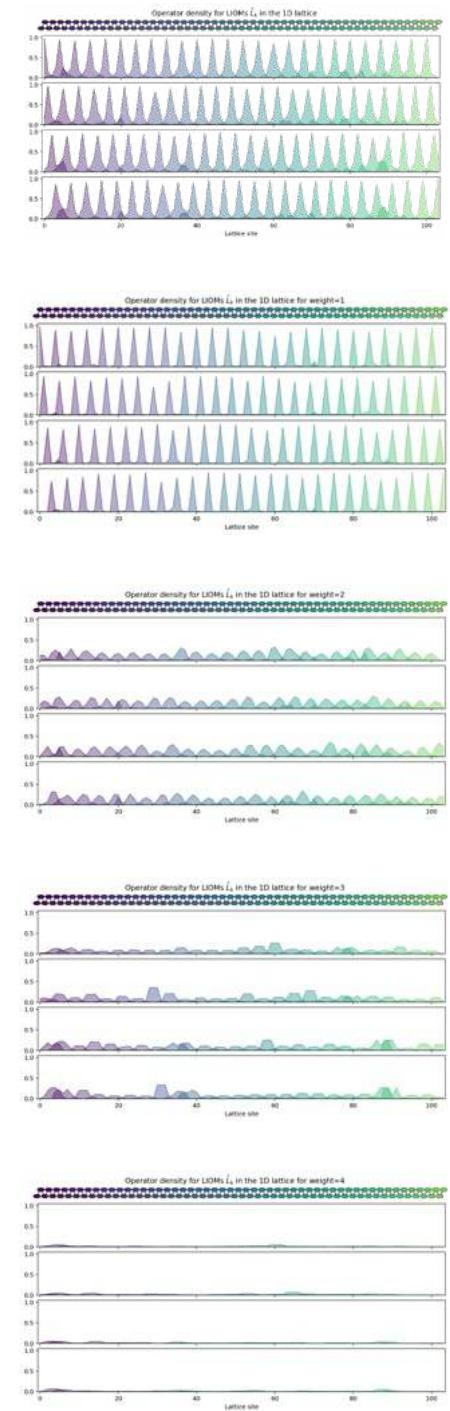
Pauli decomposition of LIOM:

$$L = \sum_{\mu=1}^{4^n-1} a_\mu P_\mu$$

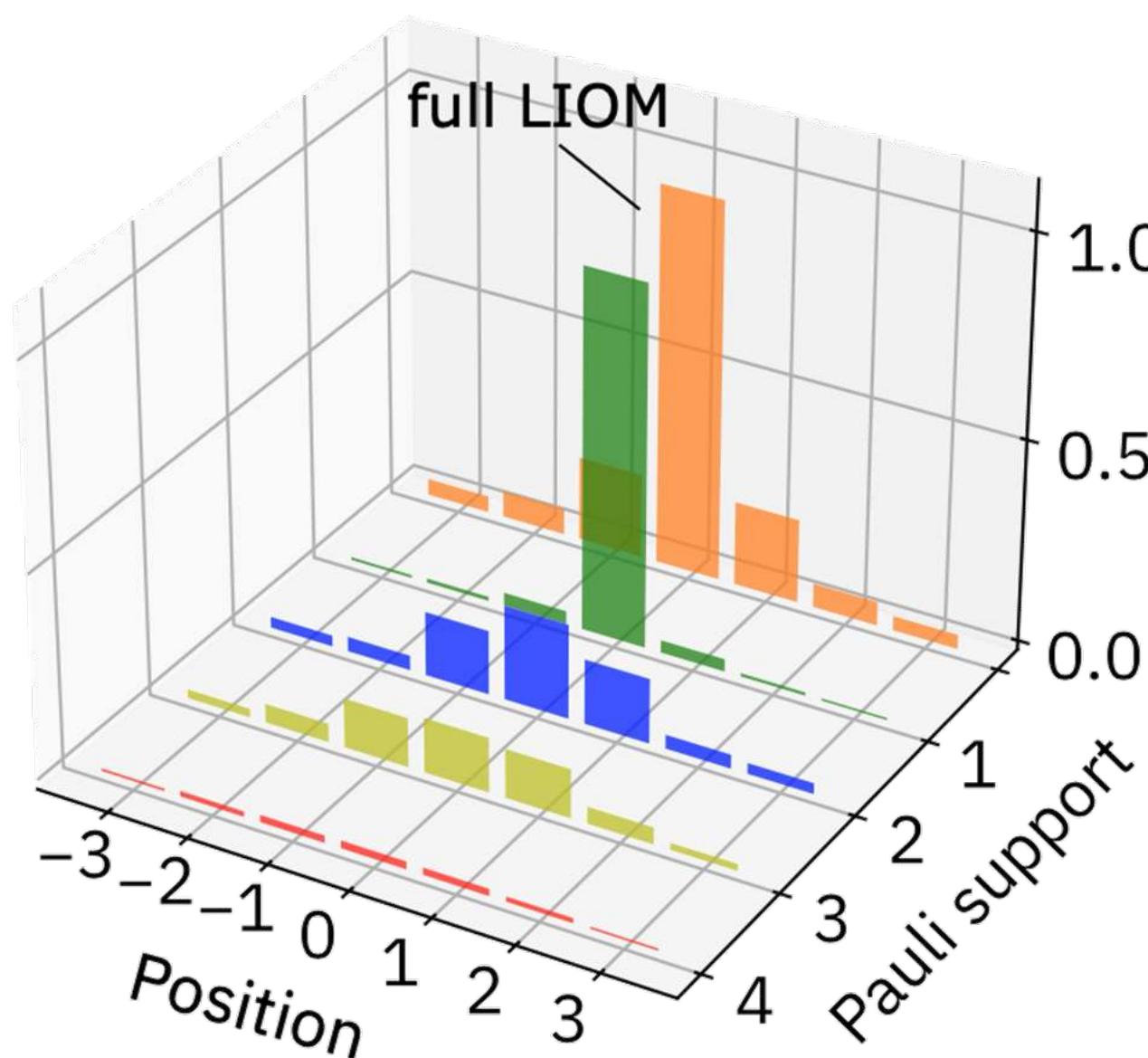
Coefficients are

$$a_\mu = \frac{1}{2^n} \frac{1}{D+1} \sum_{d=0}^D \text{Tr}(L_0 U^{d\dagger} P_\mu U^d)$$





# Average LIOM decomposed by weight



$$L_k = \sum_{\mu \in \mathcal{N}(k)} a_\mu P_\mu$$

total  
weight 1  
weight 2  
weight 3  
weight 4

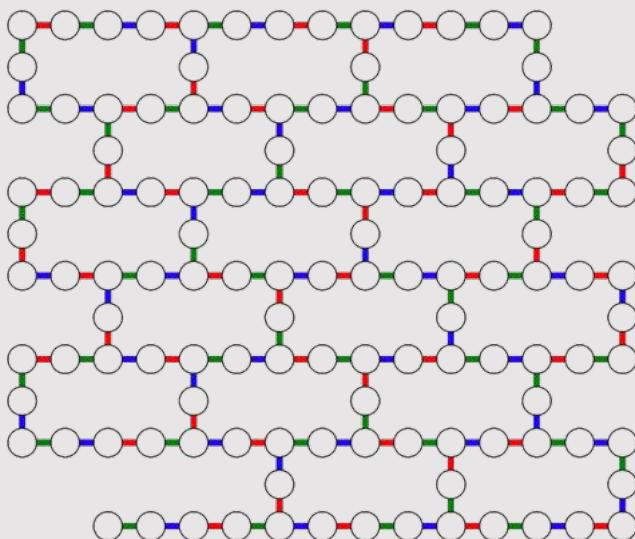
# 124 LIOMs in 2D

## Prethermal LIOMS

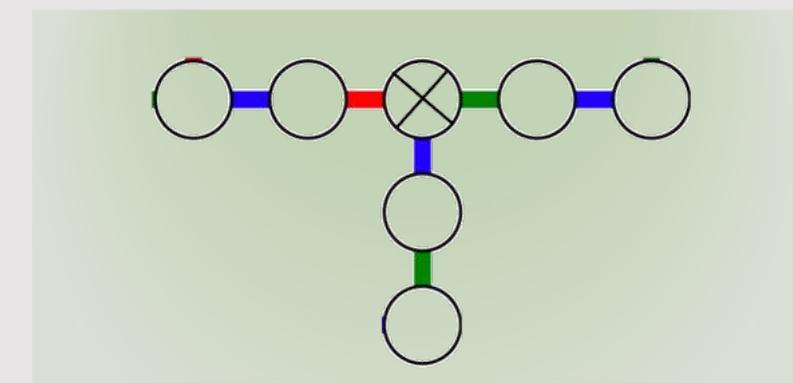
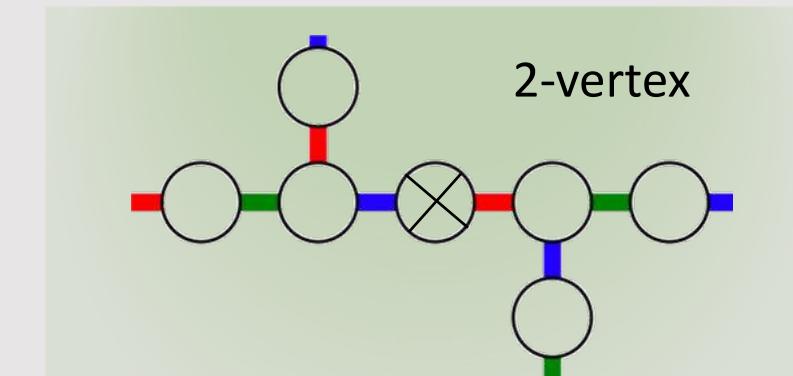
$$[e^{-iHt}, L_k] \approx 0$$

$$L_k = \sum_{\mu \in \mathcal{N}(k)} a_\mu P_\mu$$

local neighborhood

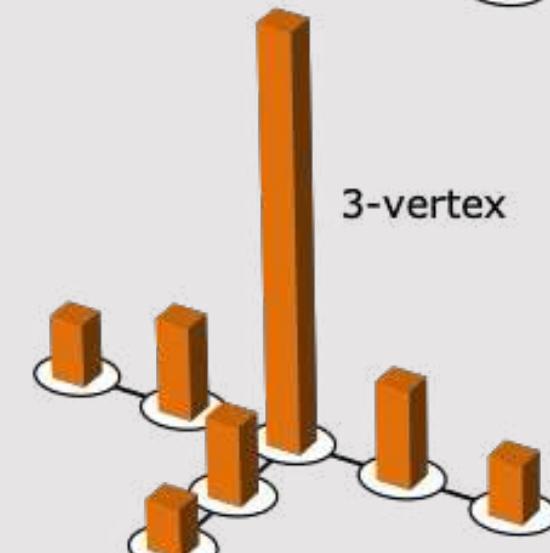
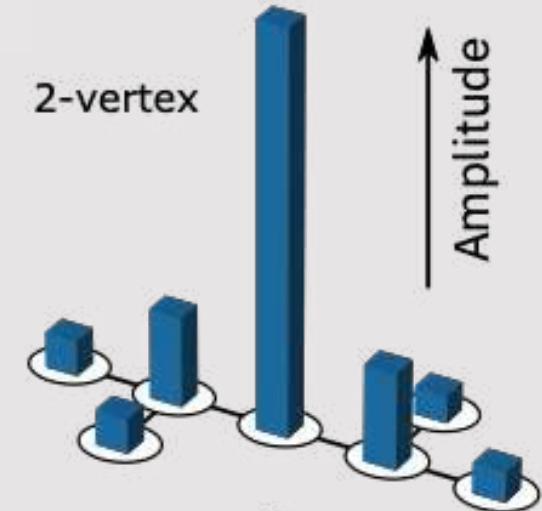


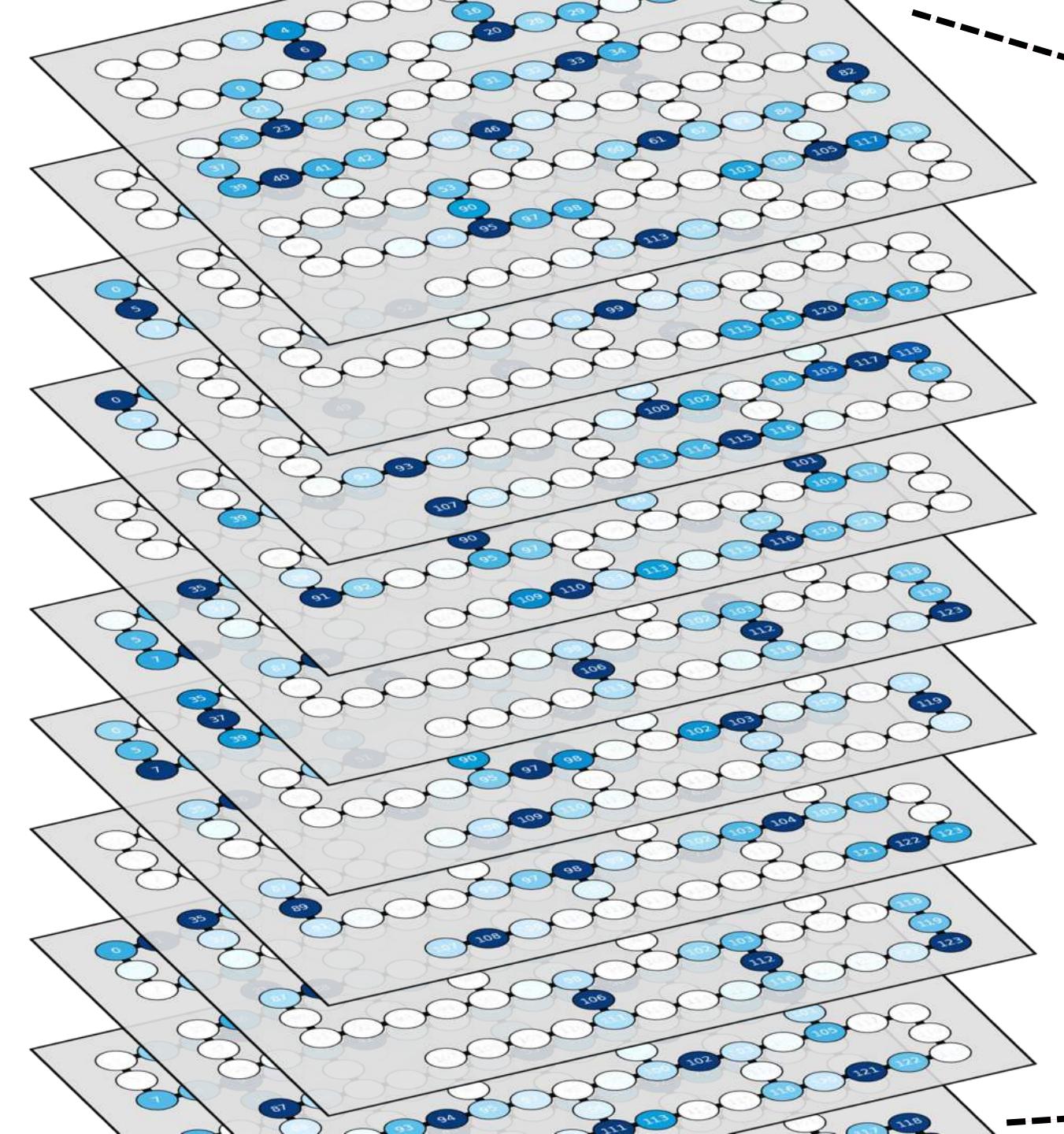
## LIOM topologies



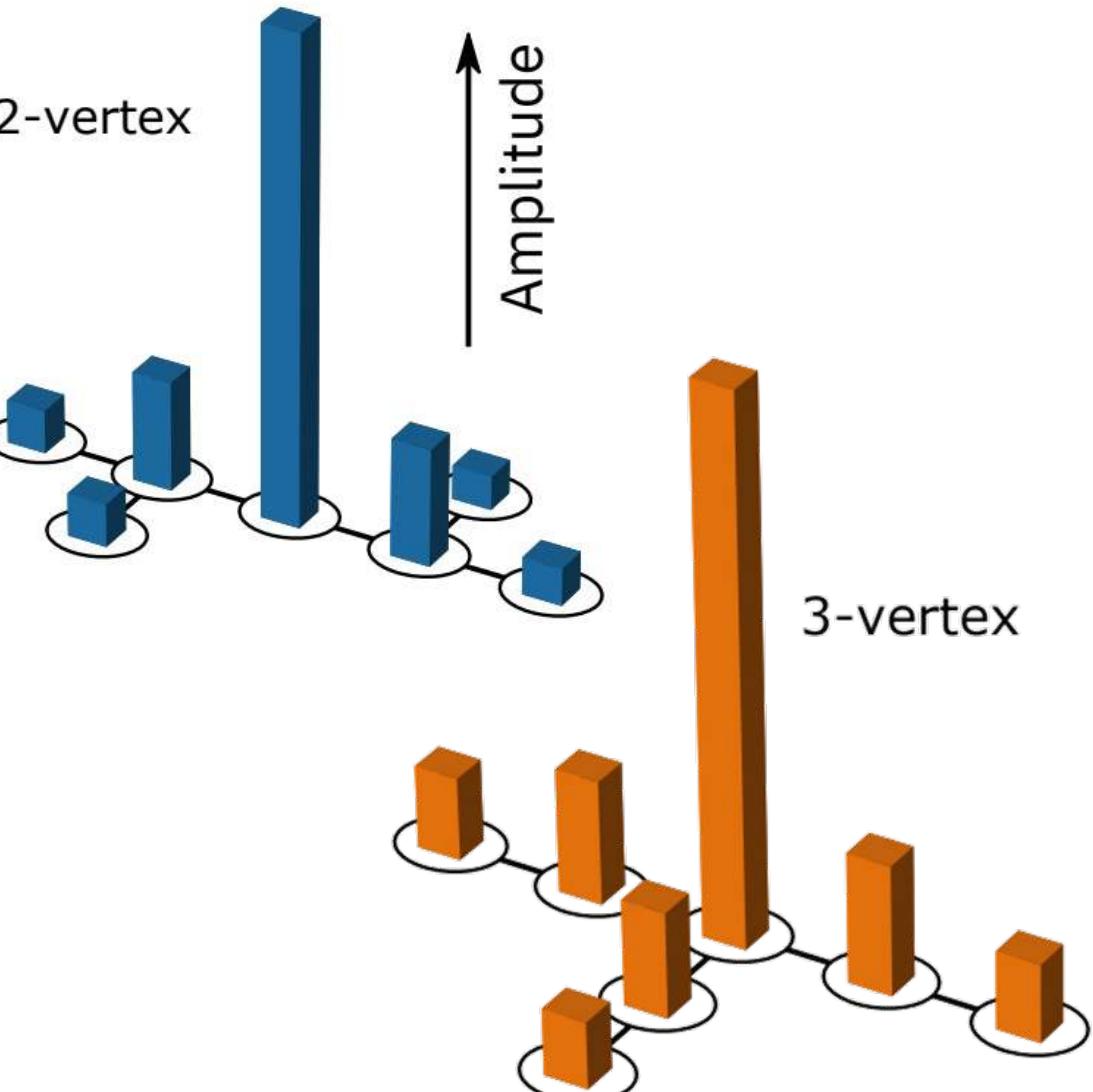
⊗ LIOM center  
(position of initial guess)

## Bulk LIOM average



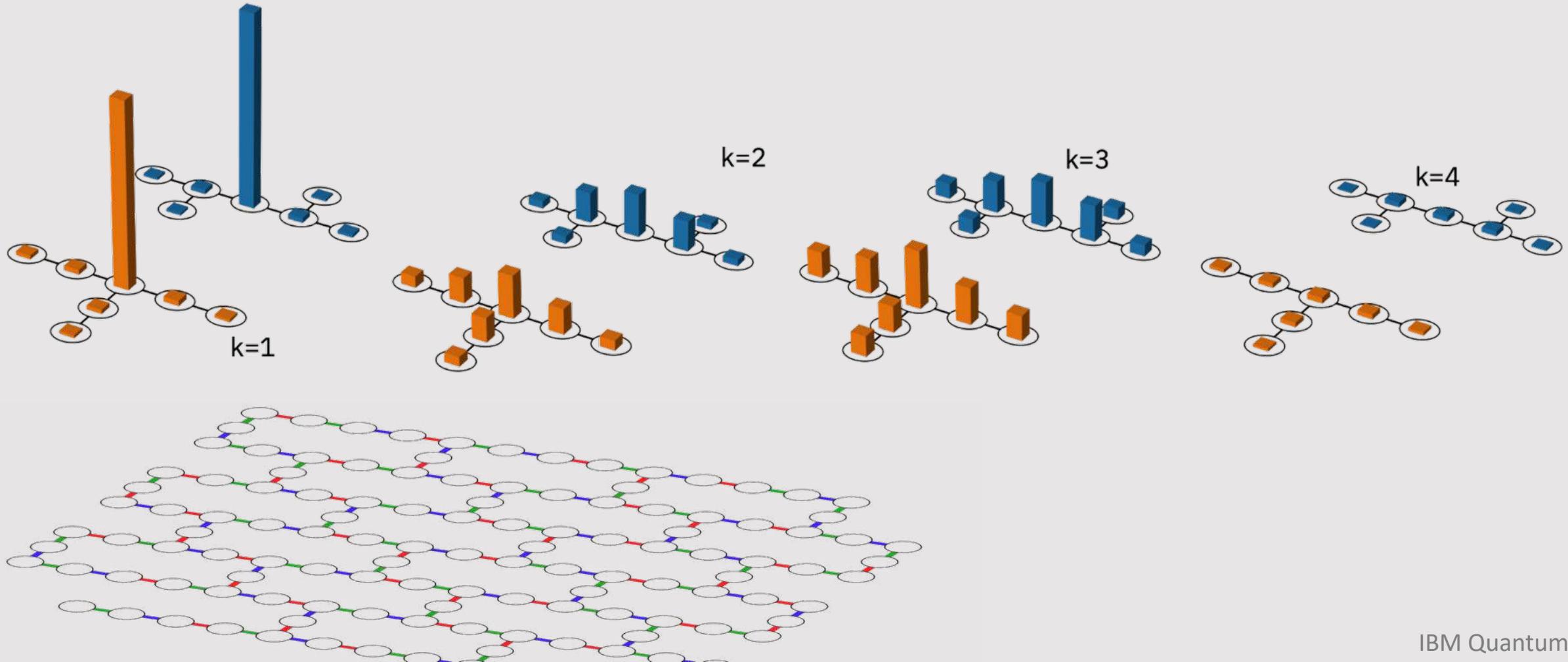


Average pre-thermal  
LIOM in bulk



# LIOMs in 2D

Decomposition of the average LIOM into contributions from  $k$ -local Pauli operators



# Density definition

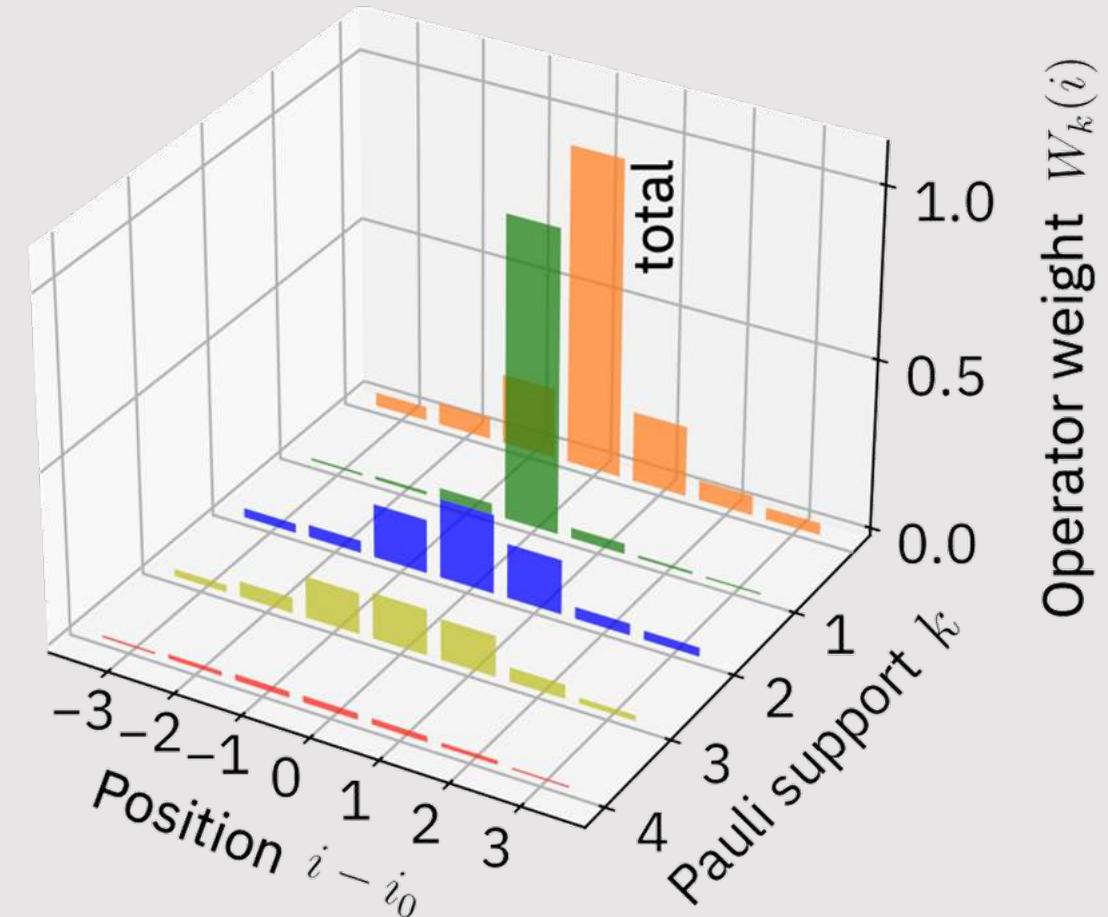
We visualize LIOMs using the density function

$$W^2(x, p) = \sum_{\mu \in Q_p} a_\mu^2 w_{x\mu}$$

Position  
Operator weight  
Locality  
set of  $p$ -local Pauli operators

Weight of Pauli relative to site  $x$

$\left\{ \begin{array}{l} 0, \text{ if acts as identity} \\ 1/S, \text{ if acts as Pauli matrix, where } S \text{ is the Pauli weight} \end{array} \right.$



We compare to a unitary simulation of the experiment (truncated subsystem)