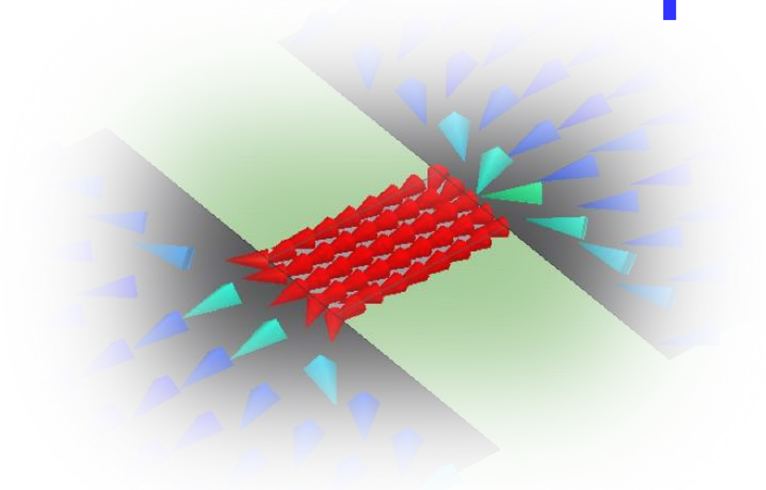
Energy-participation approach to Josephson circuit quantization



arXiv:1902.10355 (Ch. 4)
Manuscript in preparation
Github

Zlatko K. Minev*

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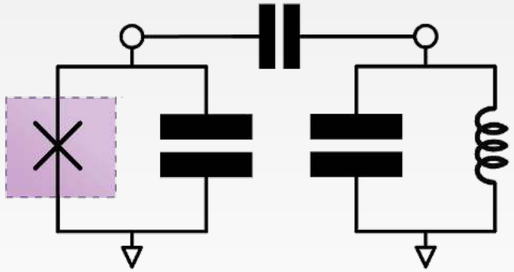
Acknowledgements: S.M. Girvin, R.J. Schoelkopf, A. Blais, S. Nigg, H. Paik, F. Solgun, RSL, Qulab, …

Yale Institute for Nanoscience and Quantum Engineering

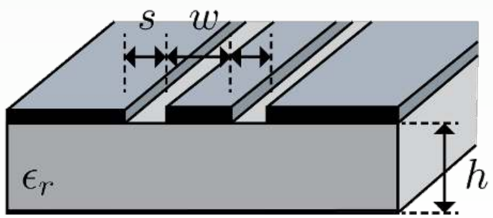
How to design a superconducting quantum circuit / computer?

Circuit quantization

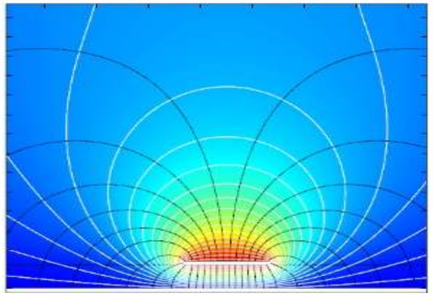
Lumped-element approximation



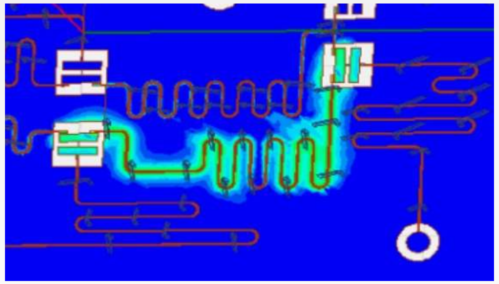
analytical



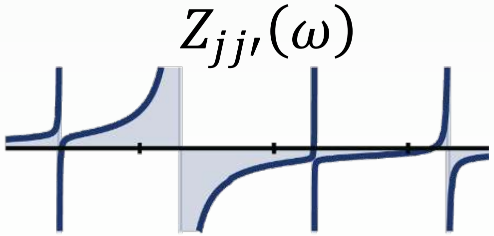
FE/boundary



Full Maxwell equations



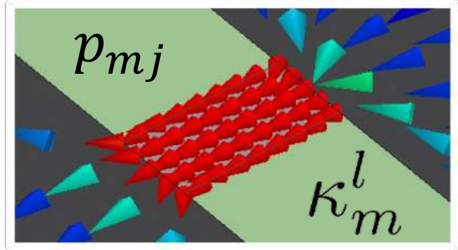
impedance*



eigen and driven

energy†

Image: Bronn *et al.*, QST (2018)



eigen

Captures more information, complexity, accuracy

* Nigg, Paik, *et al.* (2012); Bourassa *et al.* (2012); Solgun *et al.* (2014, 2015, 2017) …

† Minev *et al.*, arXiv:1902.10355, in prep. (2019)

A unified framework to handle all these questions.

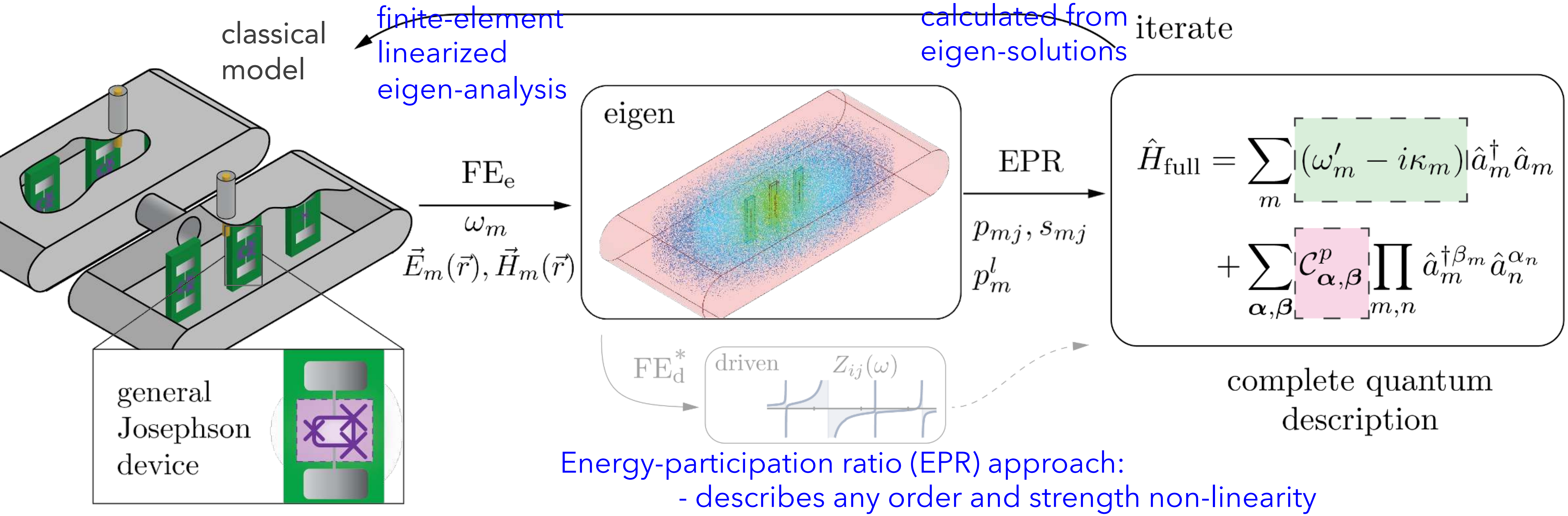
The solution reduces to asking:

Where is the energy?

What *fraction of the energy* of the mode is stored in the non-linear/dissipative element?

$$0 \leq p, p^l \leq 1$$

Overview of energy approach



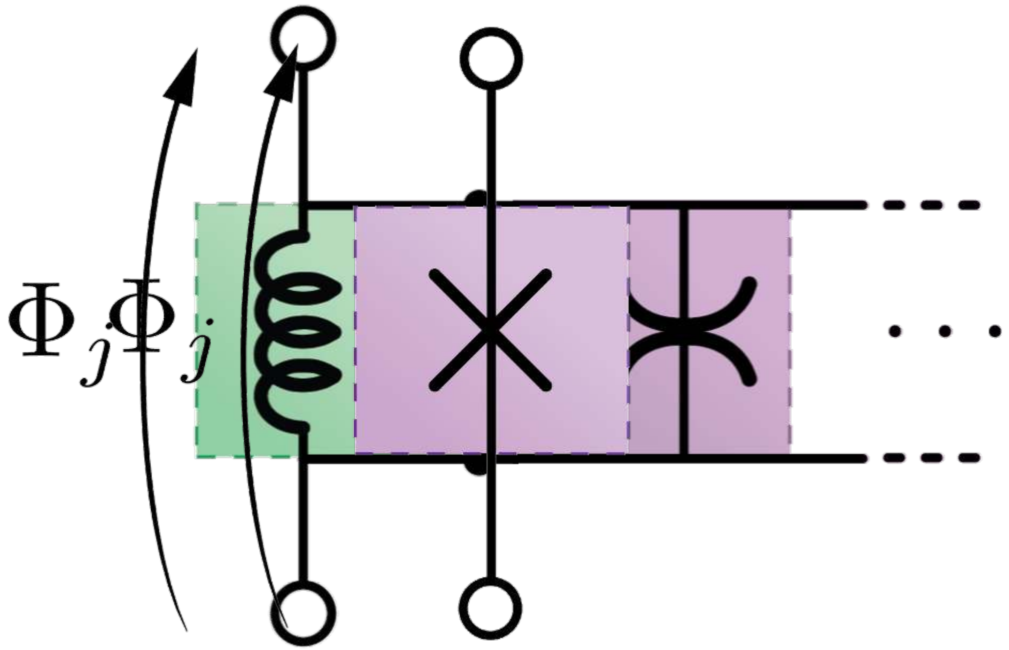
Energy-participation ratio (EPR) approach:

- describes any order and strength non-linearity
- describes arbitrary (composite) non-linear inductive devices
- first-principle derivation
- zero approximations (aside from truncation of modes)
- fully automated in python (github.com/zlatko-minev)

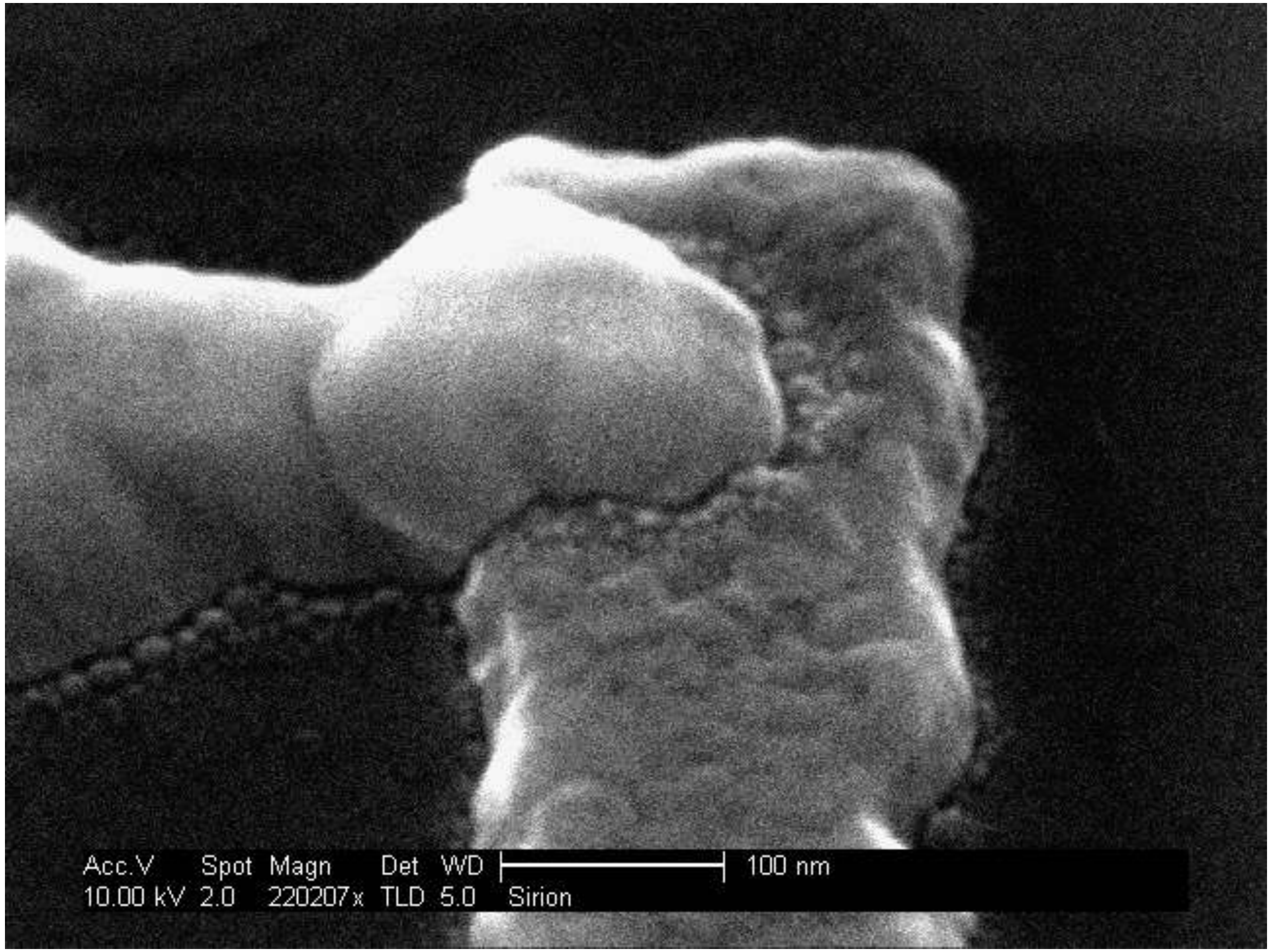
Practical limits: Fock and mode basis truncation due to computing power

* Nigg, Paik, *et al.*, PRL (2012), Bourassa *et al.* (2012), Solgun *et al.* (2014, 2015, 2017), …

Non-linear element: Josephson tunnel junction



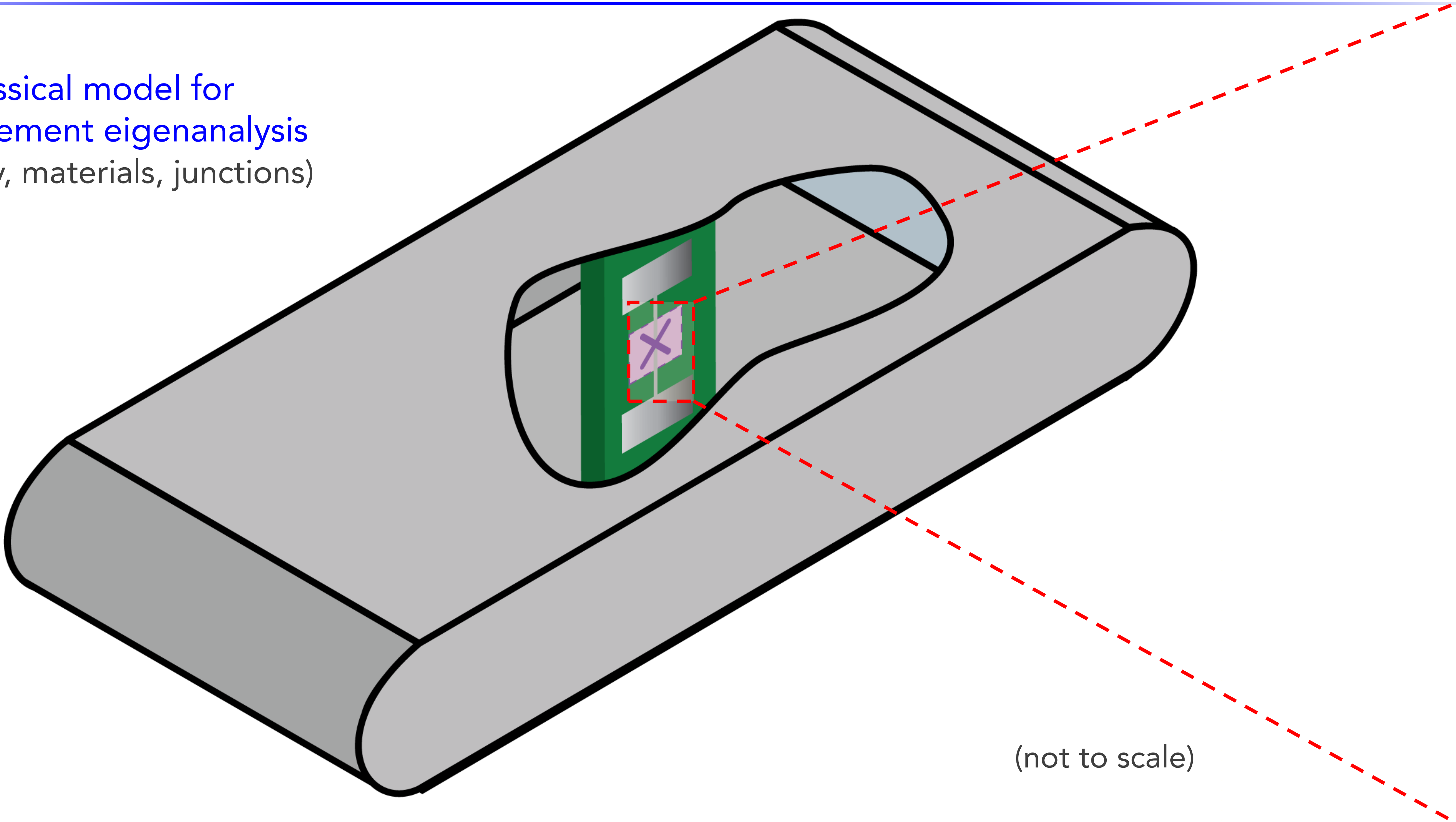
$$\mathcal{E}_j(\Phi_j) = \mathcal{E}_j^{\mathrm{lin}}(\Phi_j) + \mathcal{E}_j^{\mathrm{nl}}(\Phi_j)$$



SEM image: L. Frunzio

Transmon qubit coupled to cavity

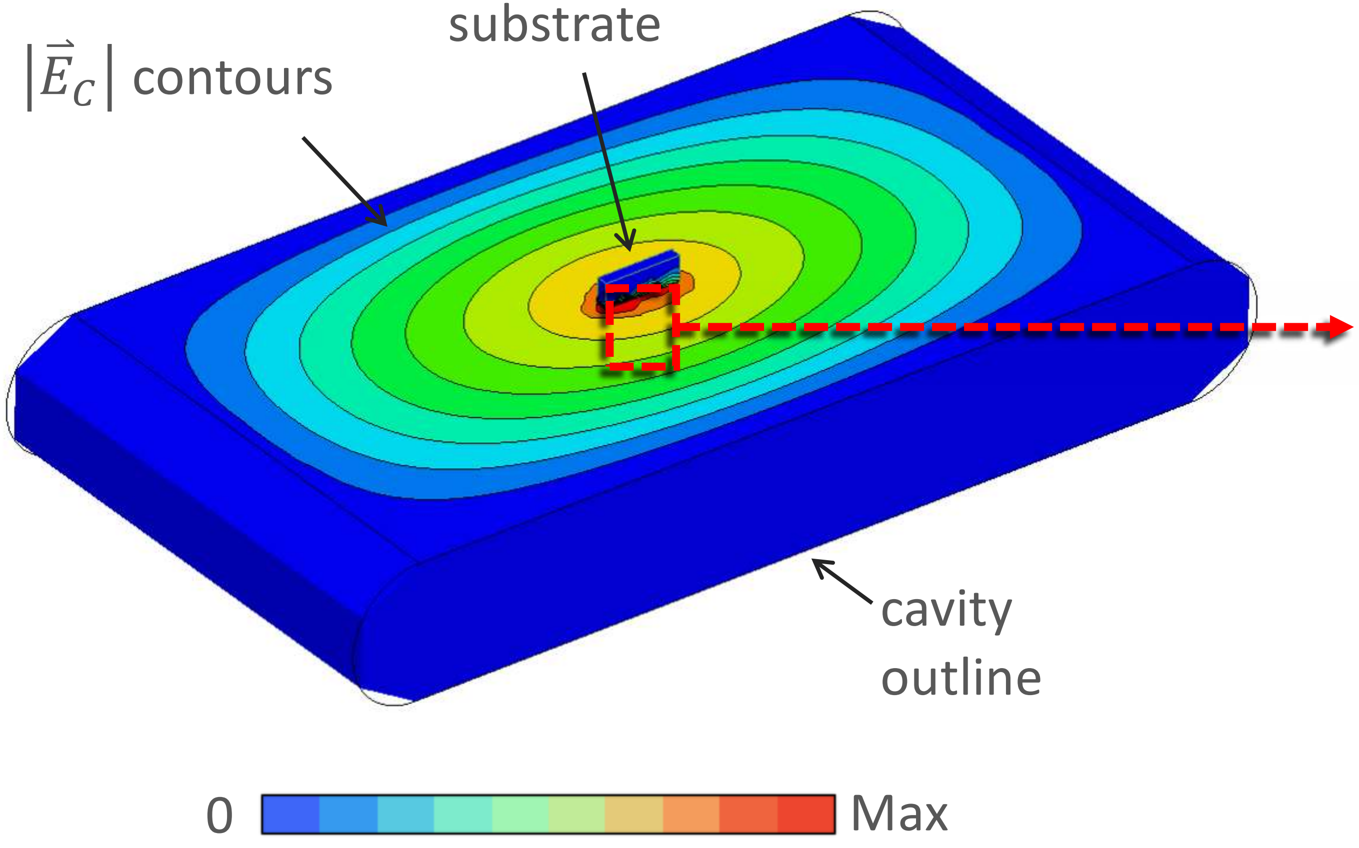
Classical model for
finite-element eigenanalysis
(geometry, materials, junctions)



(not to scale)

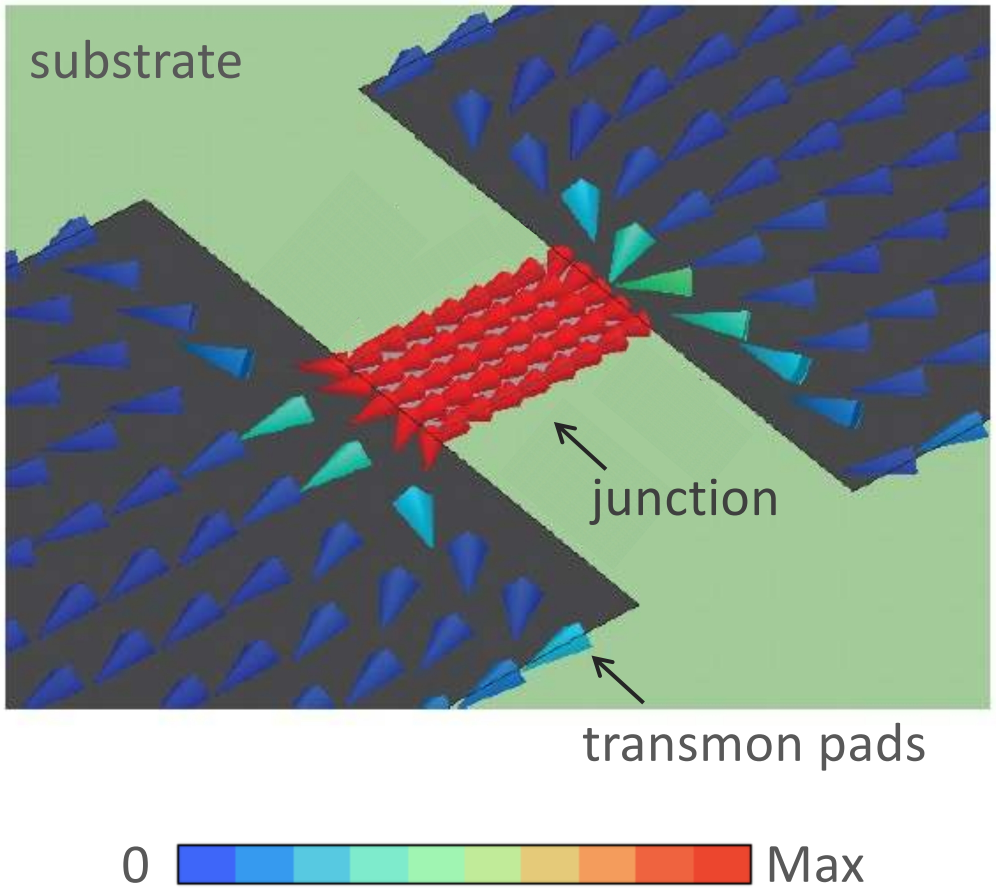
$\mathcal{H}_{\text{lin}}$ eigen modes

Cavity mode (7.0 GHz)



E-field magnitude

Qubit mode (linearized, 5 GHz)



Current-density magnitude

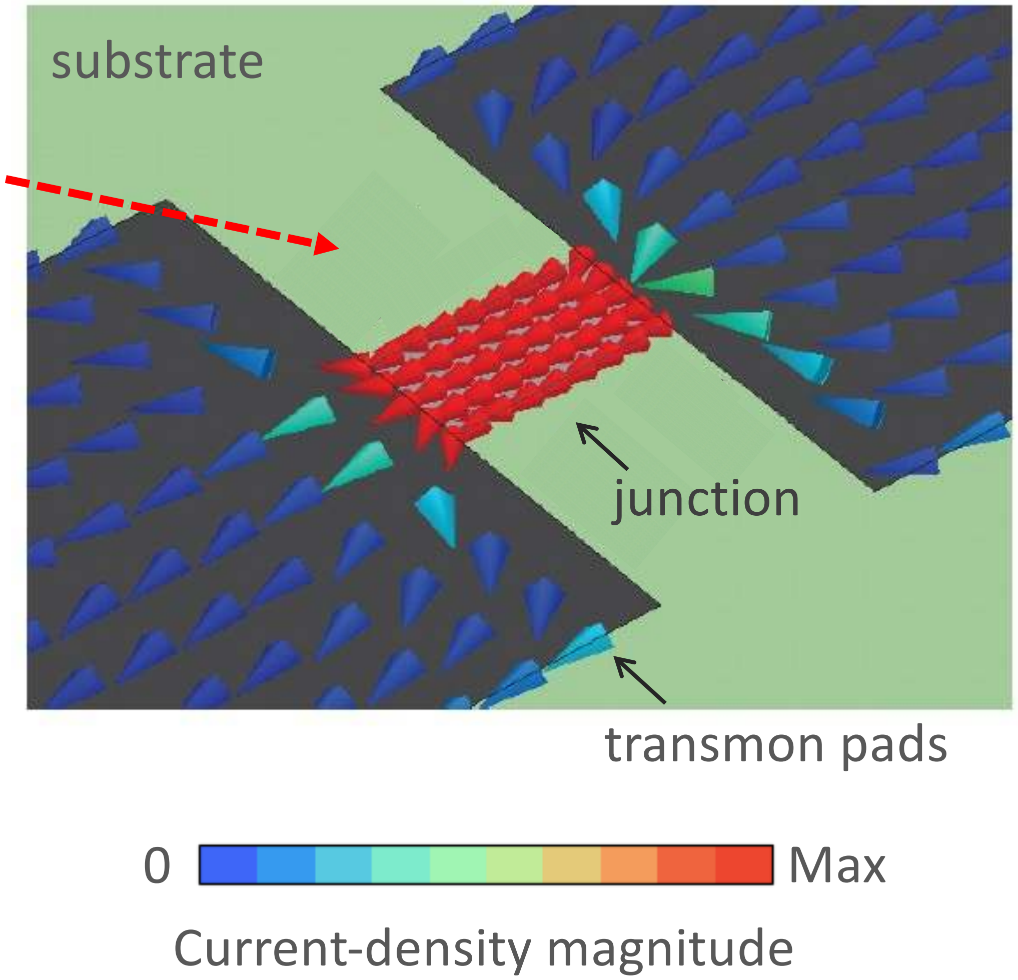
* Minev *et al.*, in prep. (2019)   Impedances: Nigg *et al.*, PRL (2012); Bourassa *et al.* (2012); Solgun *et al.* (2014, 2015, 2017) …

Energy participation of the junction

$$p_m = \frac{\text{Energy stored in junction}}{\text{Inductive energy stored in mode } m}$$

$$p_m = \frac{\frac{1}{2} L_J I_{mJ}^2}{\mathcal{E}_{ind,m}}$$

Qubit mode (linearized, 5 GHz)

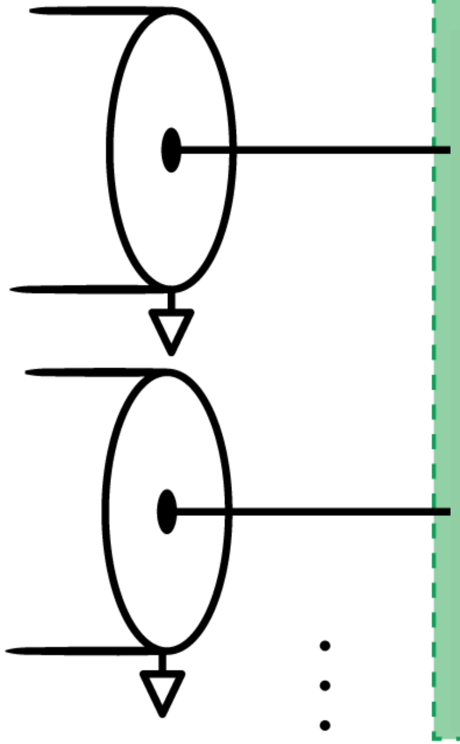


* Minev *et al.*, in prep. (2019)

Decomposition of a general circuit

$$\hat{H}_{\text{full}}$$

κ_m^l



$$\hat{H}_{\text{lin}}$$

$$\sum_{m=1}^{M} \hbar\omega_m \hat{a}_m^\dagger \hat{a}_m$$

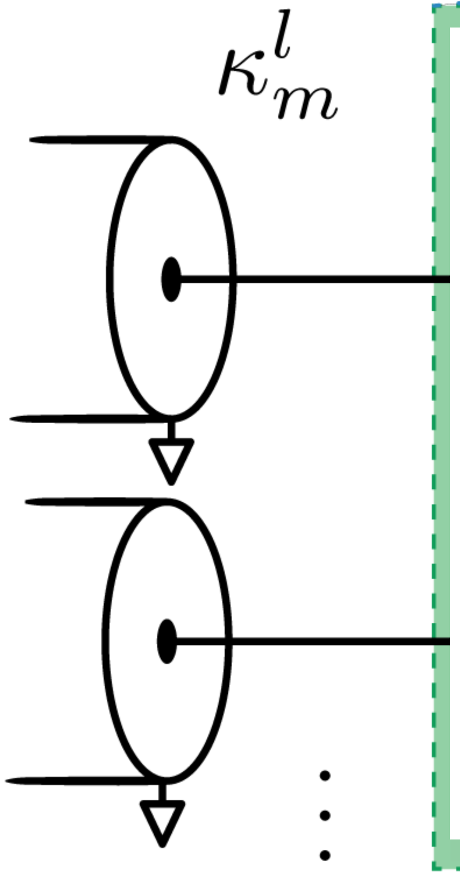
$$\hat{H}_{\text{nl}}$$

$$\hat{H}_{\text{nl}}(\boxed{\hat{\phi}_j})$$

second quantization in eigen basis of linearized circuit

Decomposition of a general circuit

$\hat{H}_{\text{full}}$



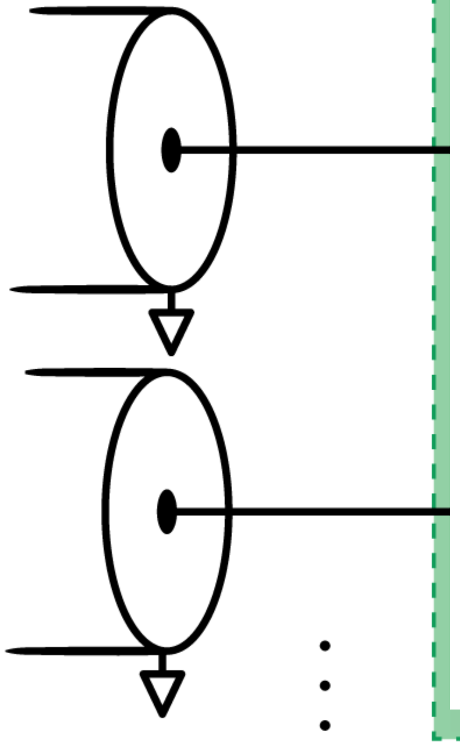
What is zero-point fluctuation of the phase of junction *j* due to mode *m*?

$$\boxed{\hat{\phi}_j} = \sum_{m=1}^{M} \boxed{\phi_{mj}} \left(\hat{a}_m^\dagger + \hat{a}_m\right)$$

Decomposition of a general circuit

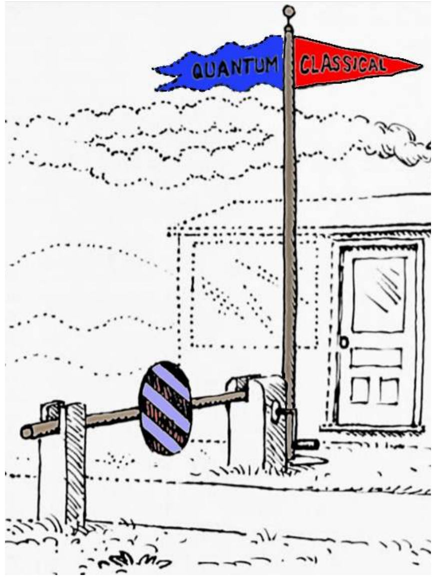
$\hat{H}_{\text{full}}$

κ_m^l



What fraction of the energy of mode *m* is stored in junction *j*?

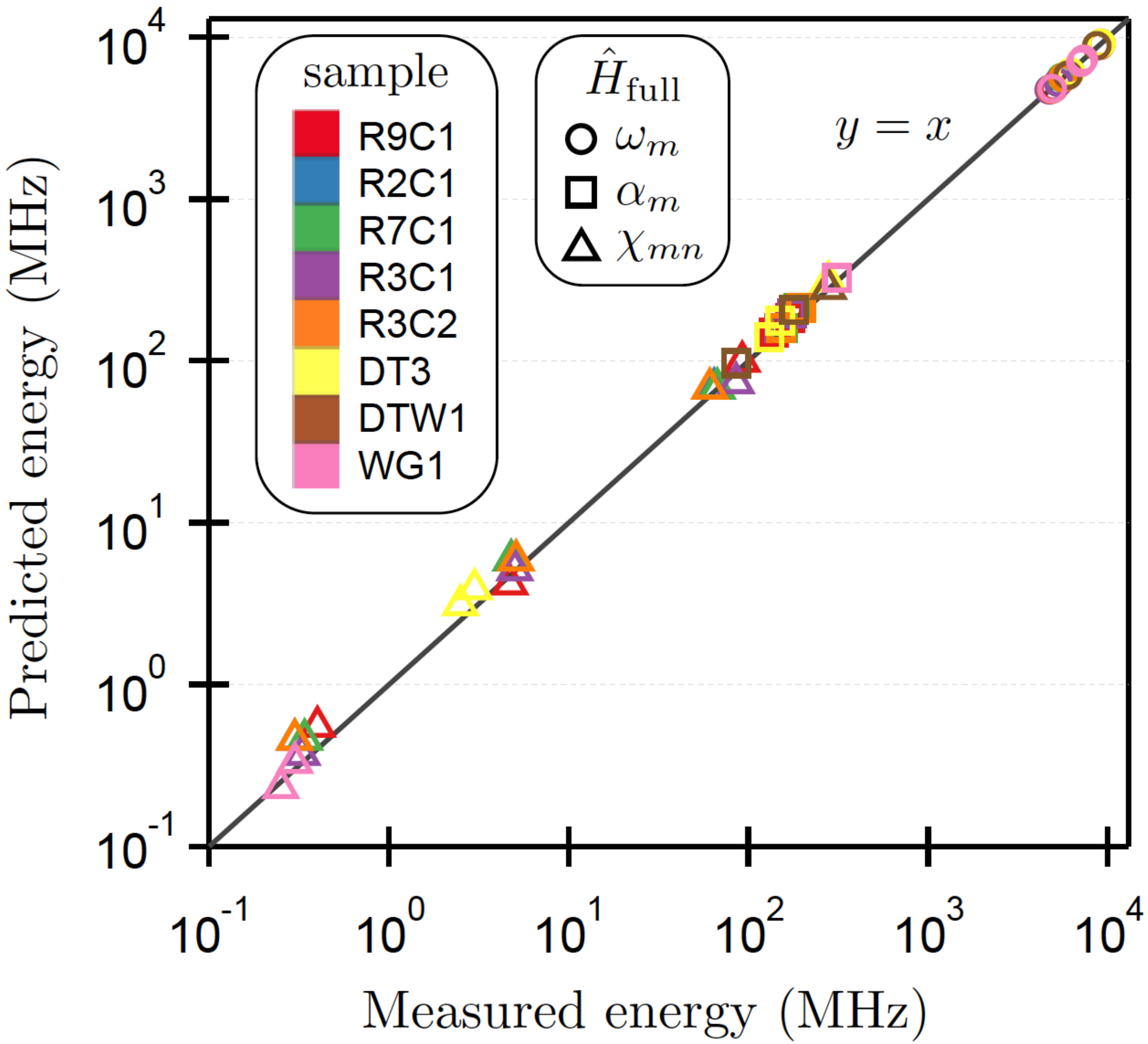
$$\boxed{\frac{1}{\hbar}\phi_{mj}^2 = p_{mj}\frac{\omega_m}{2E_j}}$$

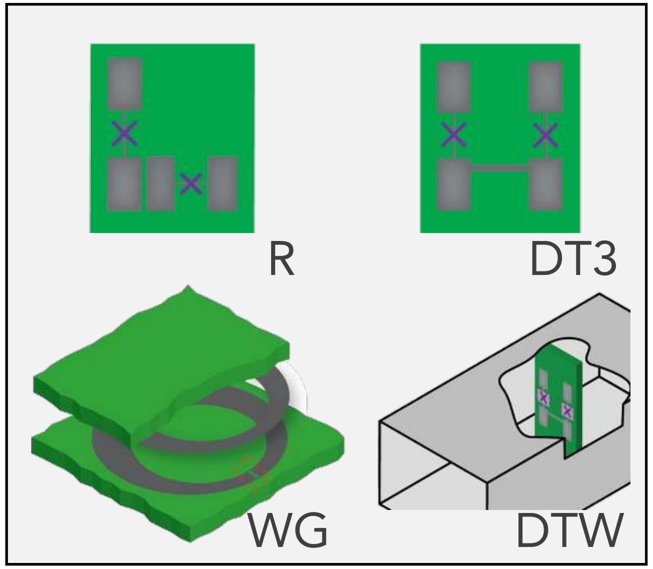


for j>1, root requires sign bit $s_{mj} = \pm 1$

Drawing: Zurek, Physics Today (1991)

Theory vs. experiment: agreement over 5 orders of magnitude





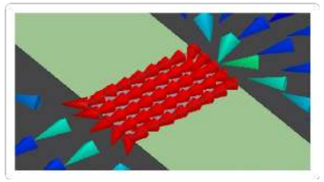
R: Minev *et al.* (2018)
WG: Minev *et al.* (2013, 2016)
DT3, DTW: Minev *et al.* (2019)

Energy-Participation-Ratio (EPR) Approach

Simple and unified analysis of dissipation and Hamiltonian

single-simulation efficient

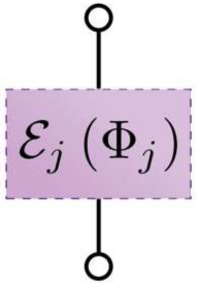
obviate driven simulations and notion of $Z_{ij}(\omega)$



General

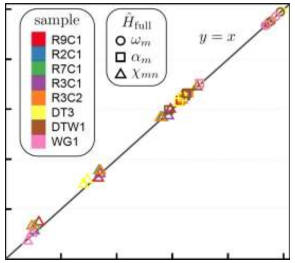
handle arbitrary architectures and non-linear devices

e.g., nanowires, …



Accuracy

experimental ten to percent level over 5 orders in $\widehat{H}_{\mathrm{full}}$



Suited for automated, robust analysis of large systems

fully automated and tested (see GitHub)

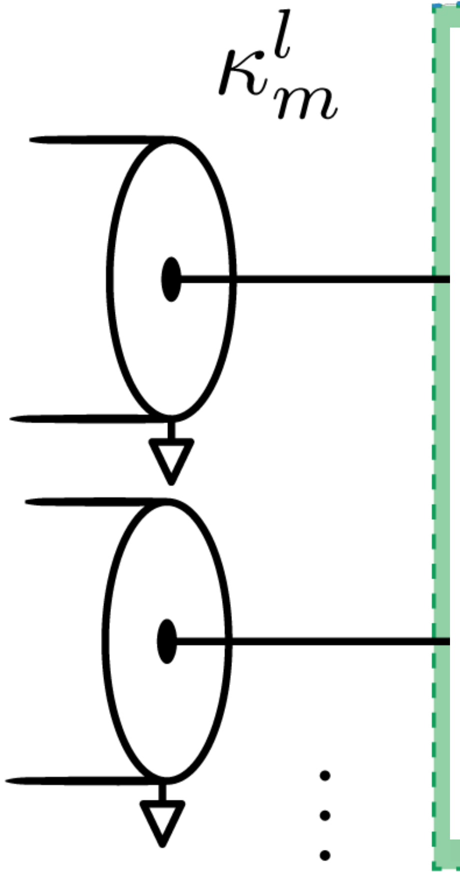
arXiv:1902.10355 (Ch. 4); to appear soon; see also E28.7

Application to J and CR gate, …

EXTRA SLIDES

Fundamental constraints with EPR

$$\hat{H}_{\text{full}}$$



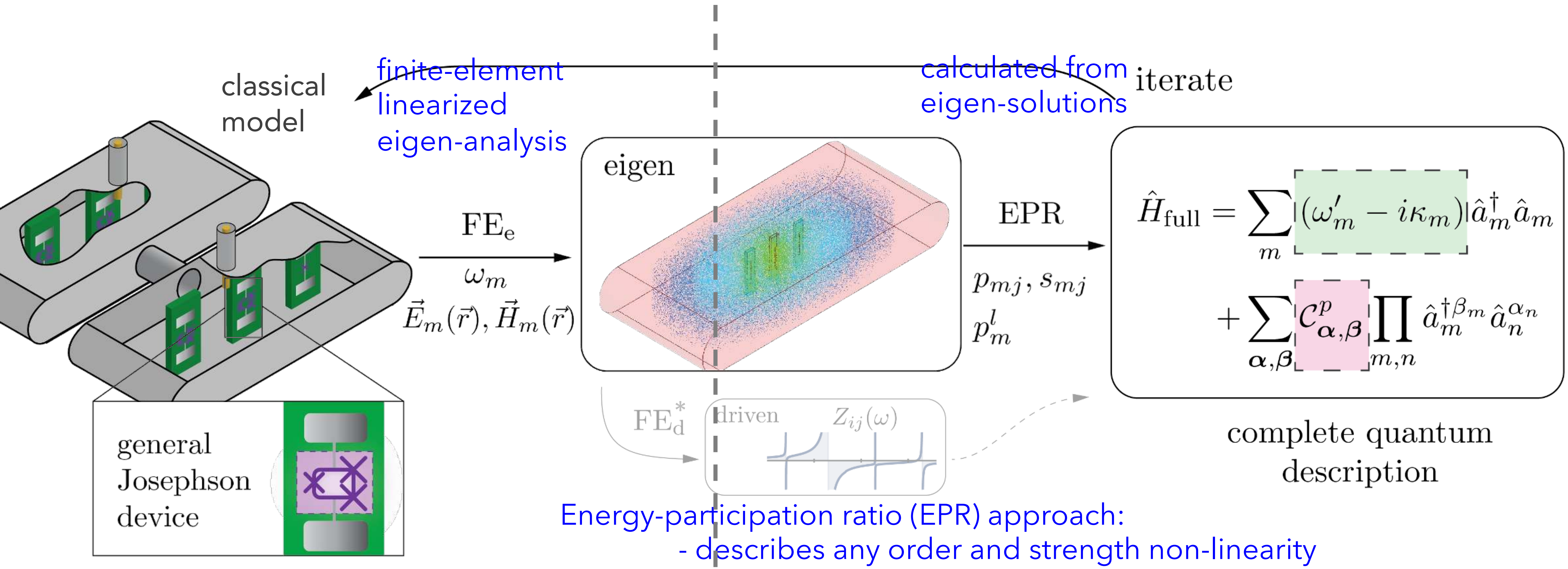
What circuits/parameters can and cannot be realized?

$$\sum_{m=1}^{M} p_{mj} = 1 \qquad 0 \leq \sum_{j=1}^{J} p_{mj} \leq 1$$

...

Convergence checks

Overview of energy approach



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Practical limits: Fock and mode basis truncation due to computing power

* Nigg, Paik, *et al.*, PRL (2012), Bourassa *et al.* (2012), Solgun *et al.* (2014, 2015, 2017), …

What fraction of the energy of the mode is ‘present’ in the dissipative element?

$$0 \leq p^l \leq 1$$

Non-linear interactions with EPR

Energy participation overlap

$$\chi_{mn} = \sum_{j=1}^{J} \frac{\hbar \omega_m \omega_n}{4 E_j} p_{mj} p_{nj}$$

(only variable subject to significant variation)

$$\chi_{mn} \hat{a}_m^\dagger \hat{a}_m \hat{a}_n^\dagger \hat{a}_n$$

Outline

The problem

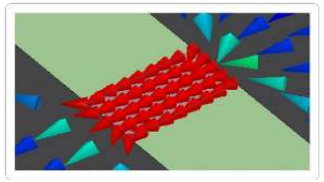
Energy-participation approach

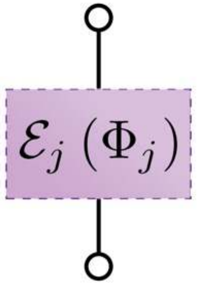
overview

simple example

CR gate

advantages & general formulation

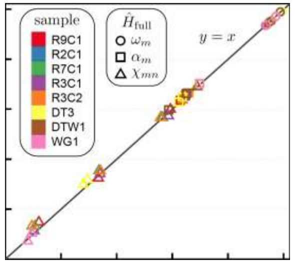


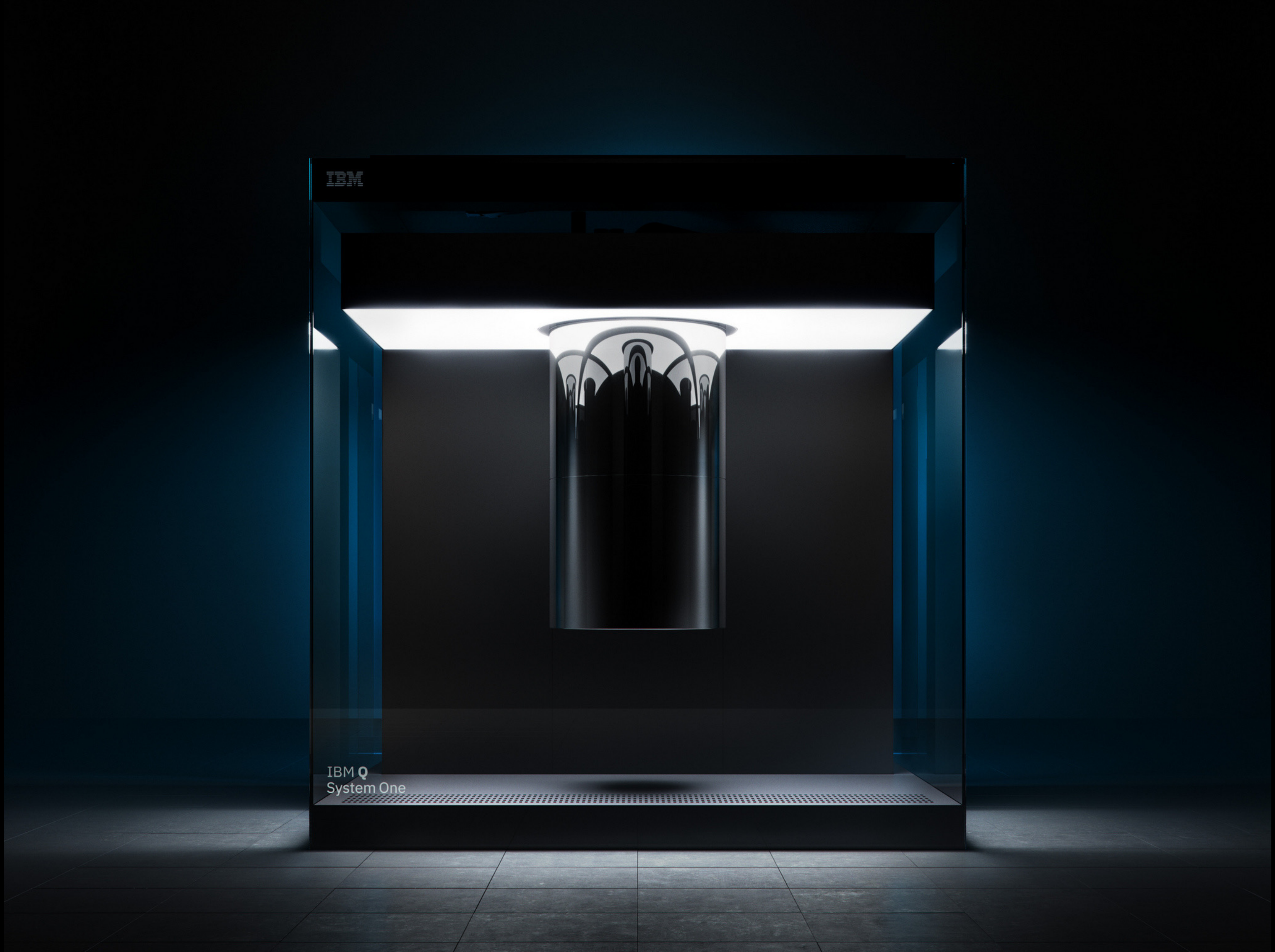


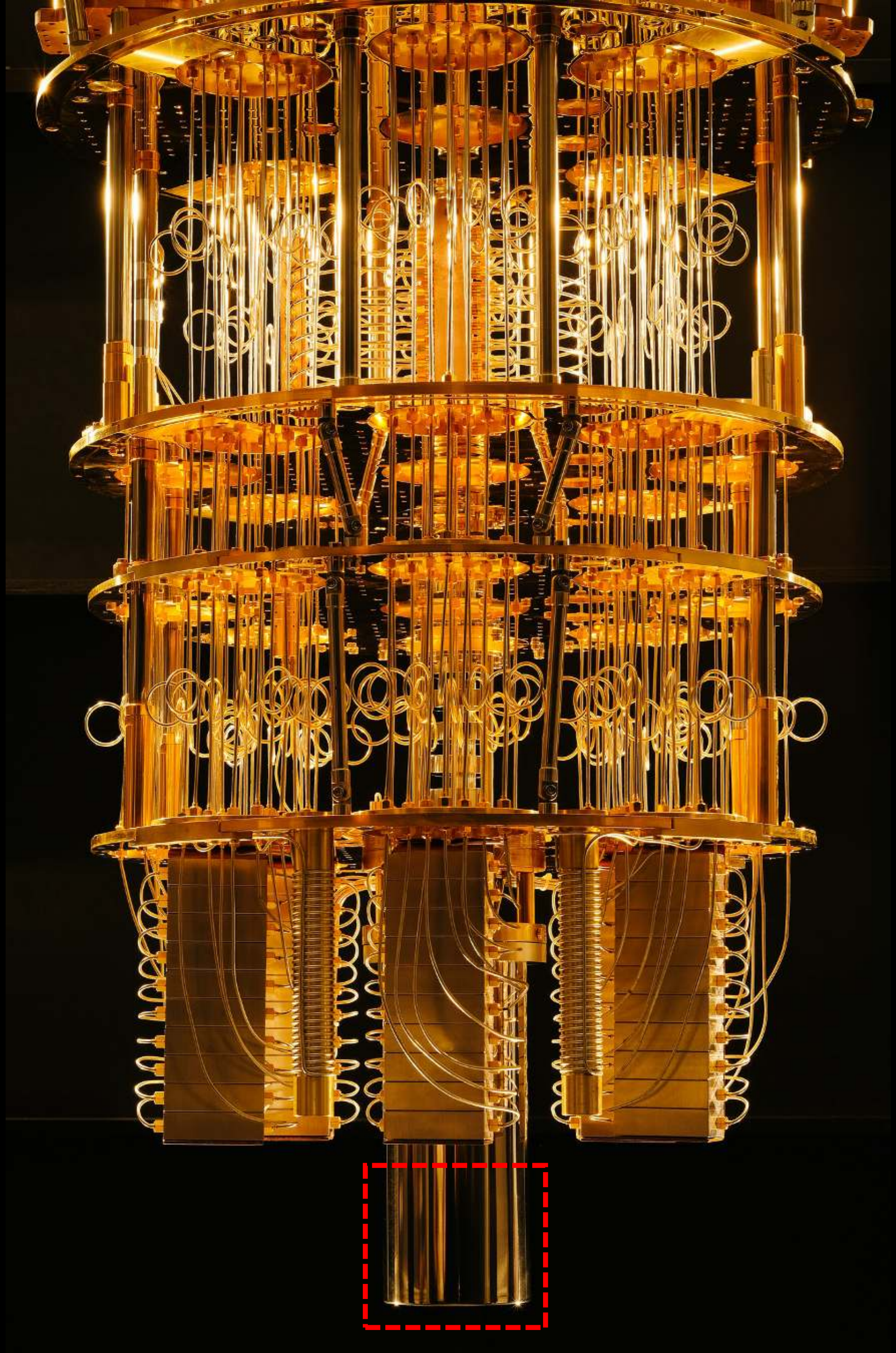
Experimental results

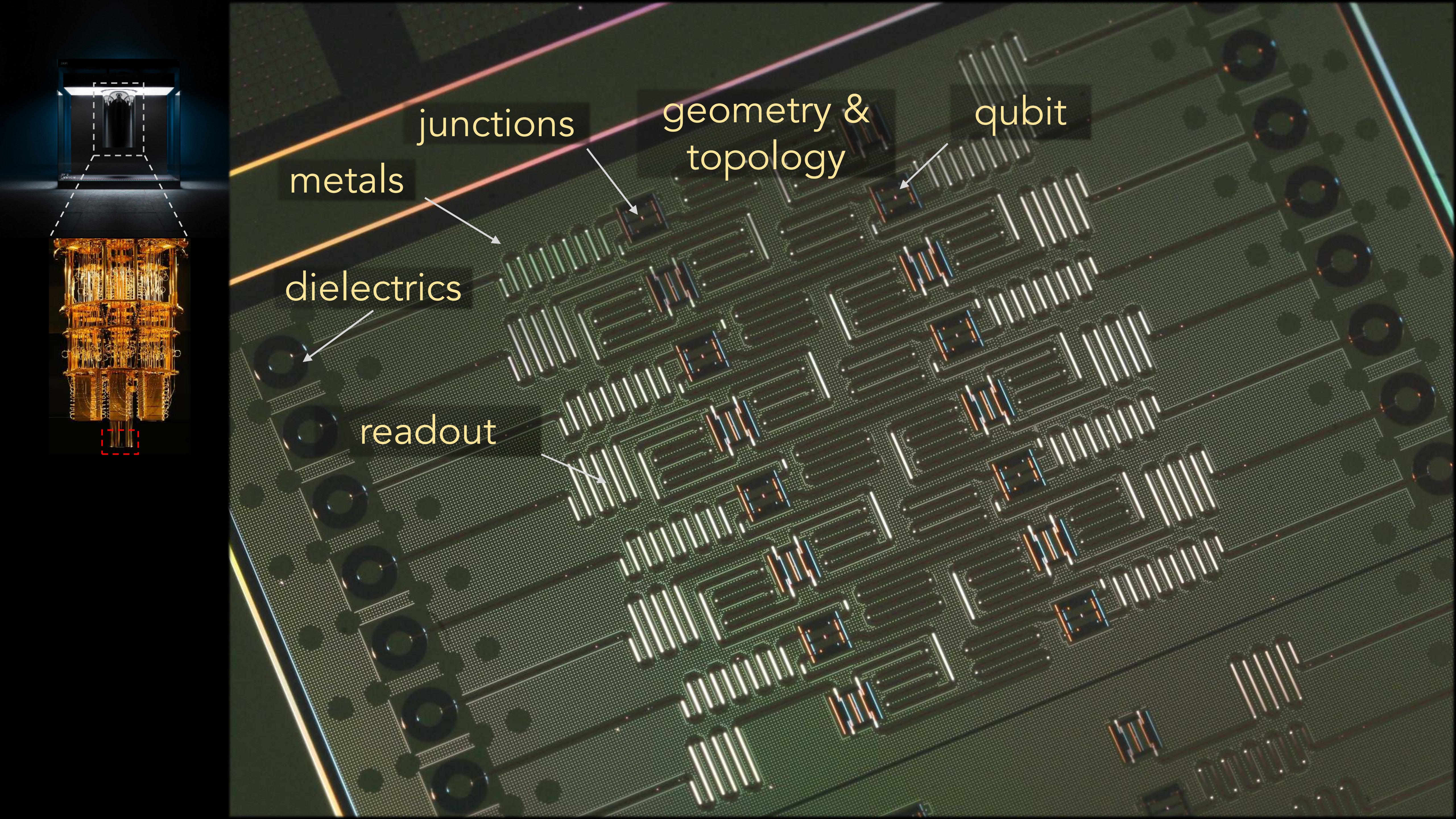
samples: multi-qubit, 2.5D flip-chip, 3D, waveguides, …

precision: theory vs. experiment

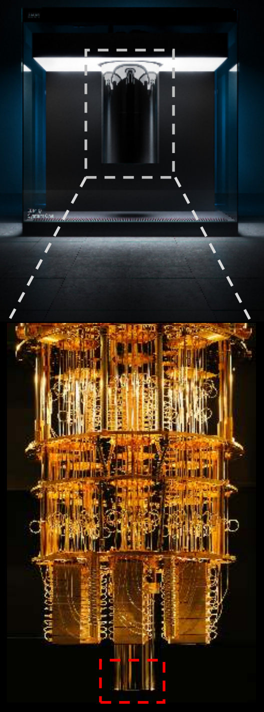


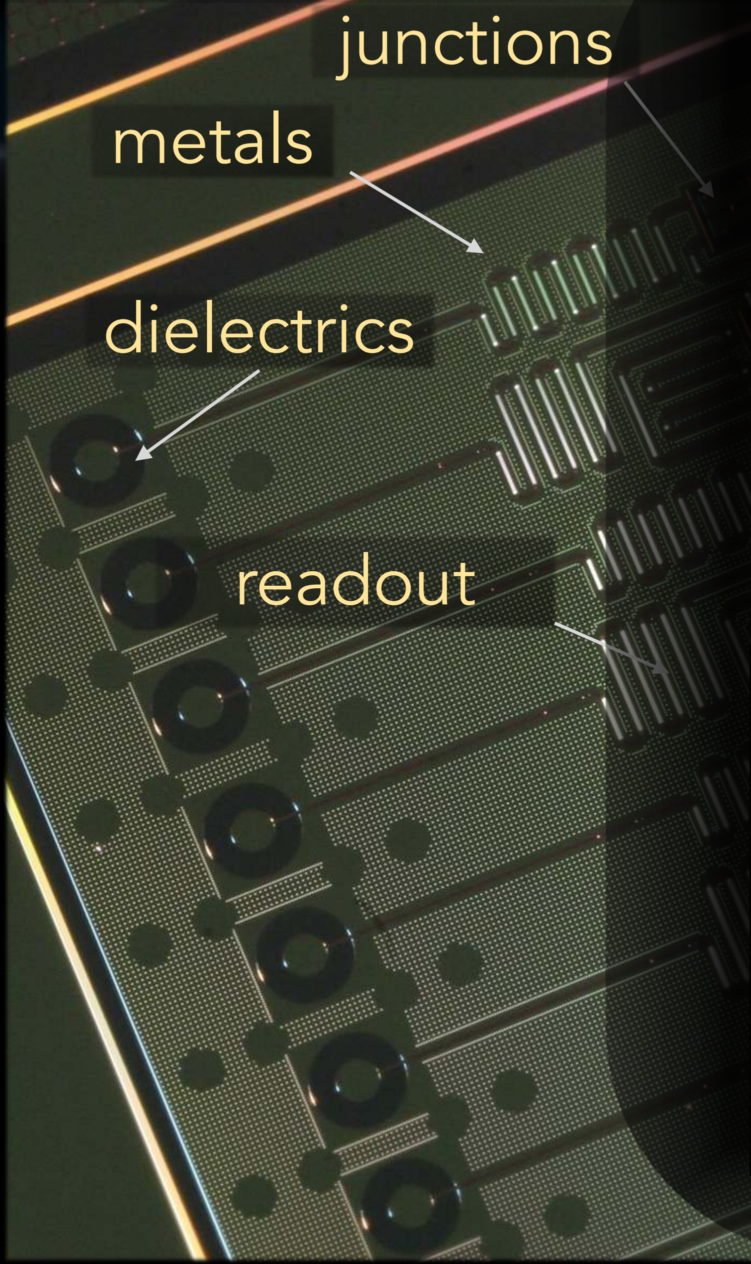

IBM
IBM Q
System One



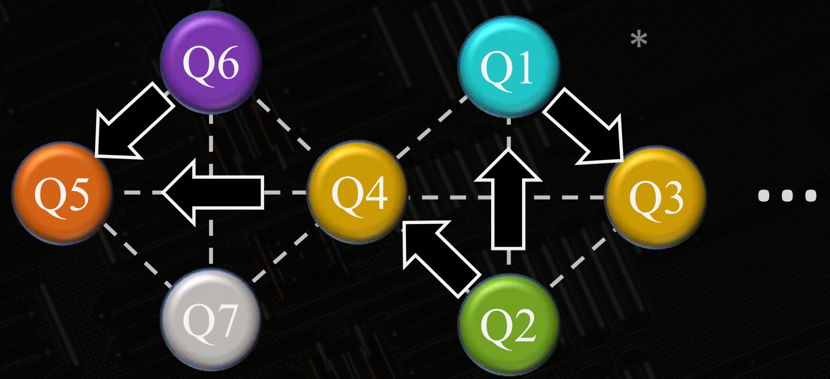

junctions
geometry &
topology
qubit
metals
dielectrics
readout

From chip to algorithm

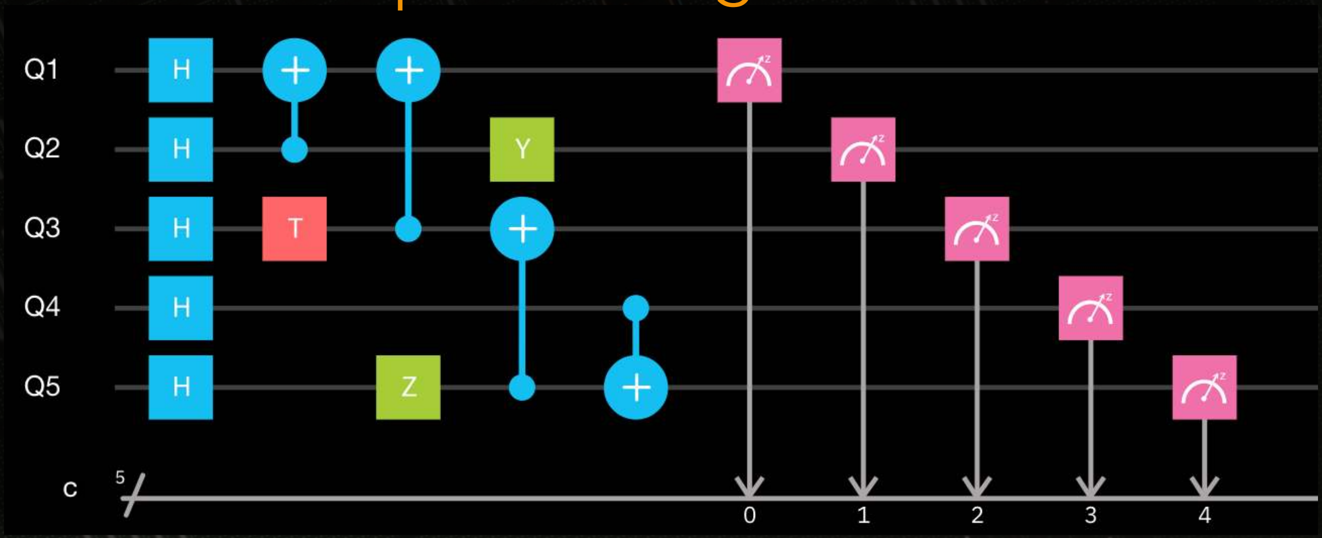




qubit connectivity graph*

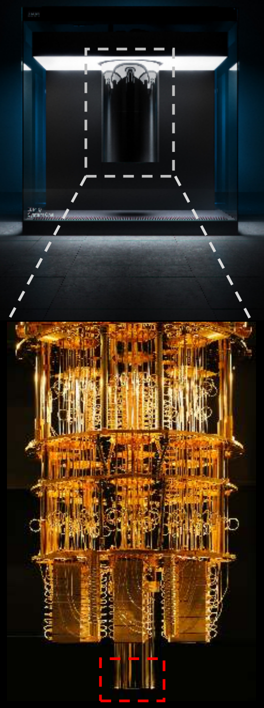


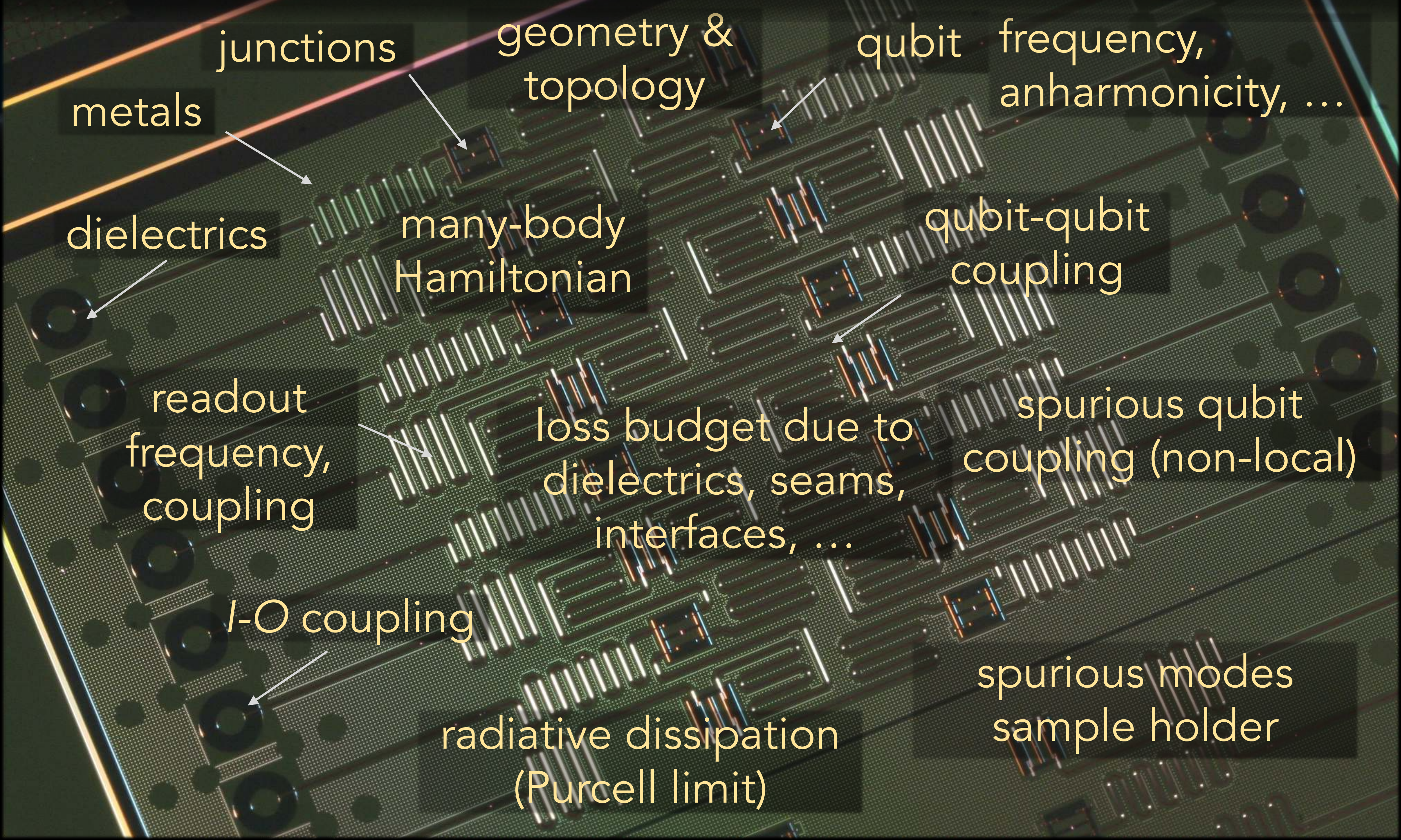
quantum algorithm+

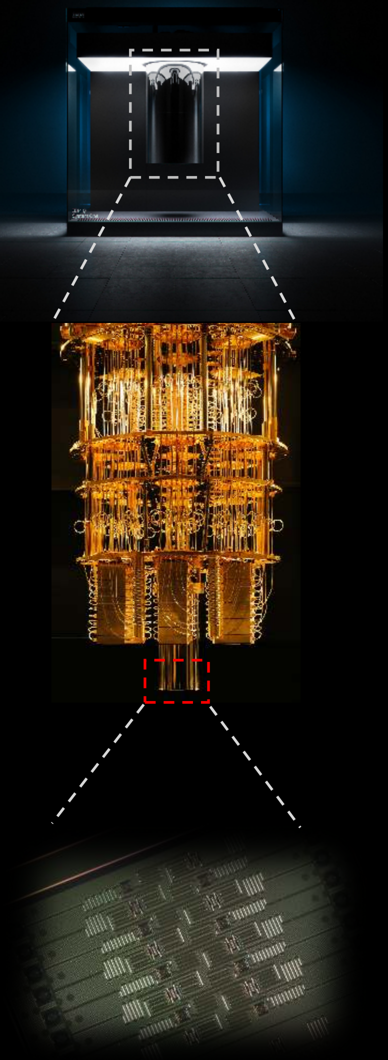


Images adapted from: * Kandala *et al.*, Nature (2017) + IBM quantum experience

What needs to be designed?



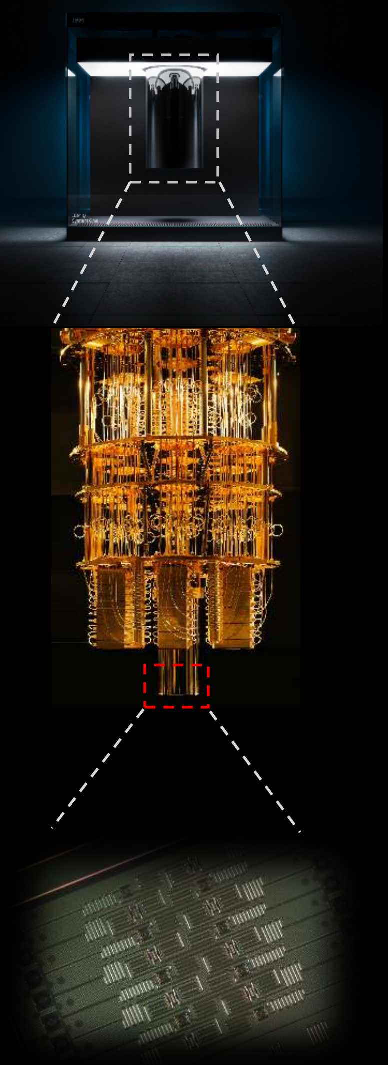




A unified framework to handle all these questions.

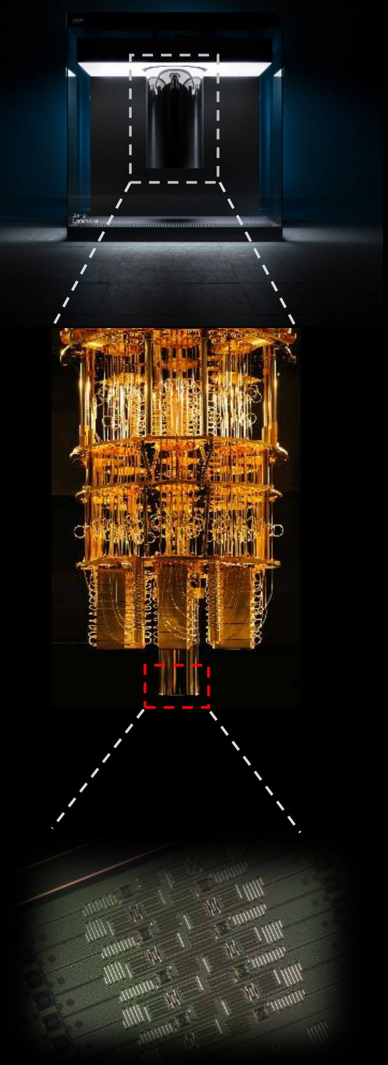
The solution reduced to:

Where is the energy?



Solution reduced to:

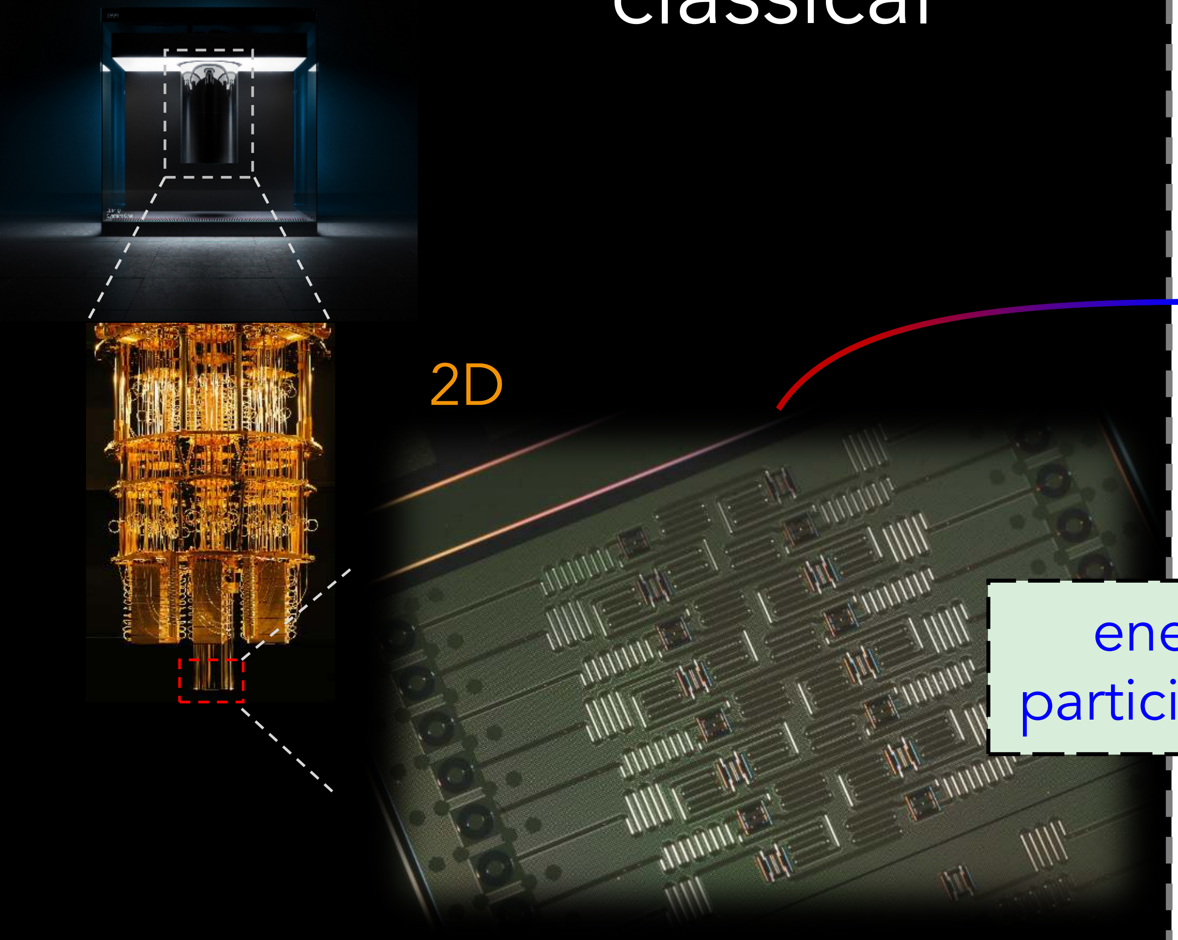
What fraction of the energy of the mode is stored in the junction?



Solution reduced to:

What fraction of the energy of the mode is stored in the dissipative element?

classical



energy participation

geometry, circuits, materials,
electromagnetic fields,
...

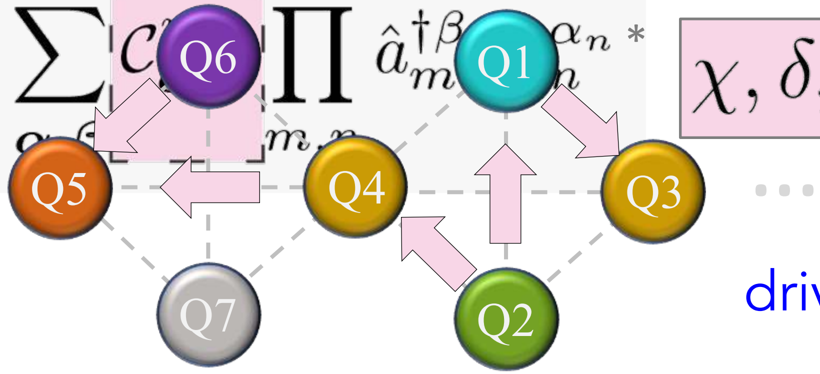
quantum

bosonic modes coupled
to environment

$$\hat{H}_{\text{full}} = \sum_m \boxed{(\omega'_m - i\kappa_m)} \hat{a}^{\dagger}_m \hat{a}_m$$

$$+ \sum c \prod \hat{a}^{\dagger\beta}_{m} \ldots {}^{\alpha_n}_{n} \, {}^*$$

interactions

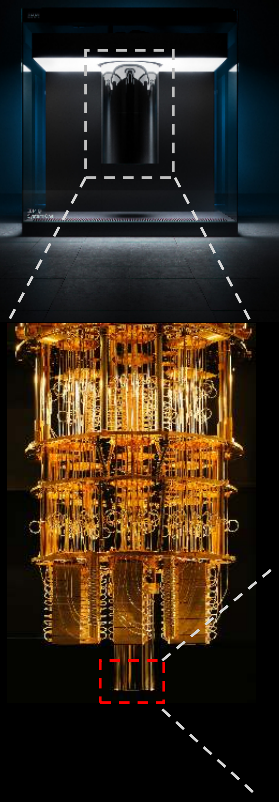


drive terms

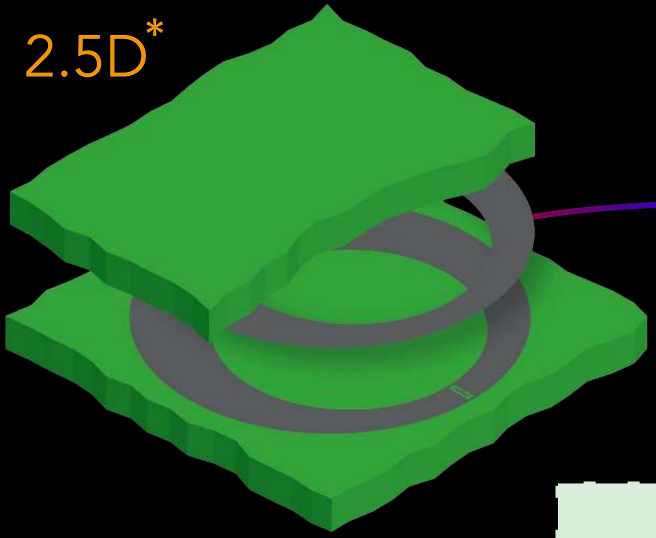
$\chi, \delta, \alpha, \ldots$

classical

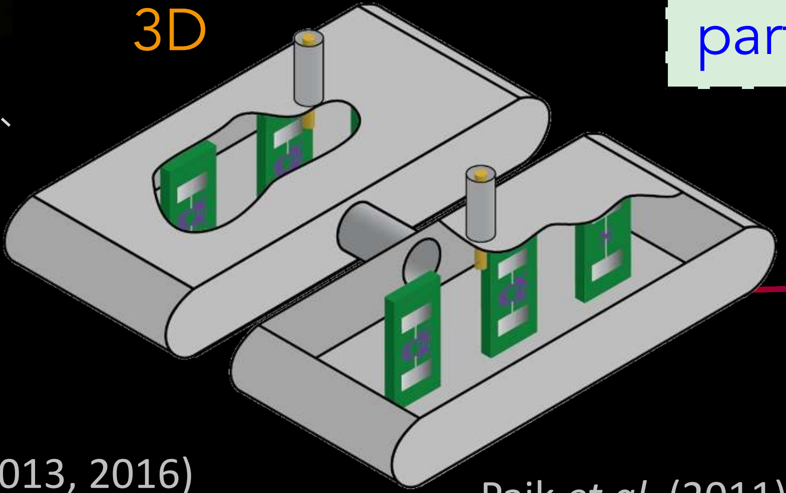
quantum



2.5D*



3D



bosonic modes coupled to environment

$$\hat{H}_{\mathrm{full}} = \sum_m \boxed{(\omega'_m - i\kappa_m)} \hat{a}_m^\dagger \hat{a}_m$$

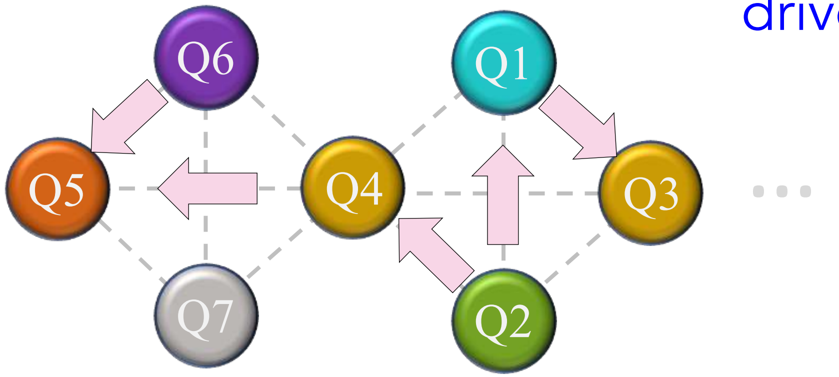
interactions

$$+ \sum_{\boldsymbol{\alpha},\boldsymbol{\beta}} \boxed{\mathcal{C}^p_{\boldsymbol{\alpha},\boldsymbol{\beta}}} \prod_{m,n} \hat{a}_m^{\dagger\beta_m} \hat{a}_n^{\alpha_n}$$

$\chi, \delta, \alpha, \ldots$

energy participation

drive terms

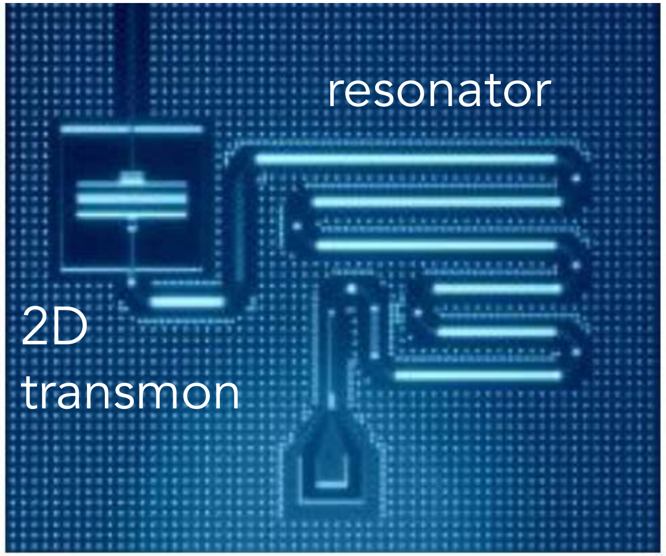


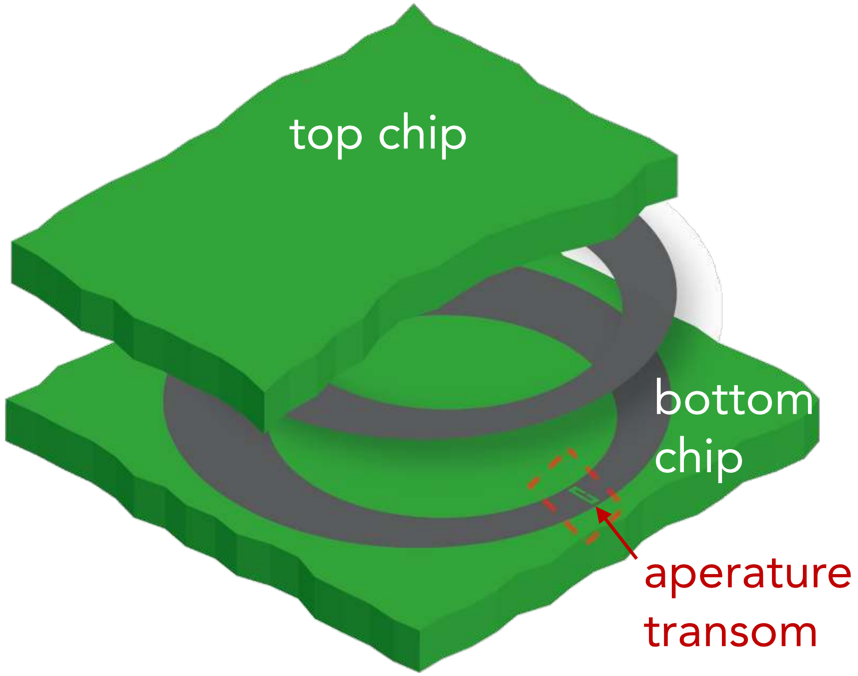
* Minev *et al.* (2013, 2016)
For flip chip, also see:
Li *et al.* (2010),
Gambetta *et al.* (2017), Rosenberg *et al.* (2017), …

Paik *et al.* (2011) ,
…

A simple example:

transmon qubit coupled to resonator





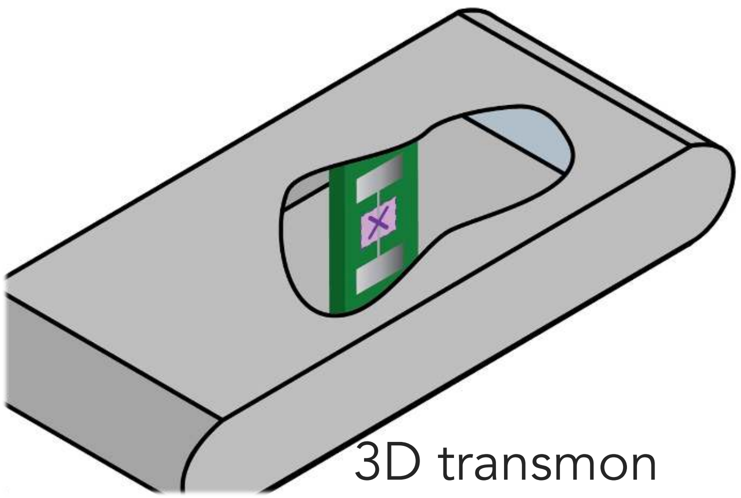
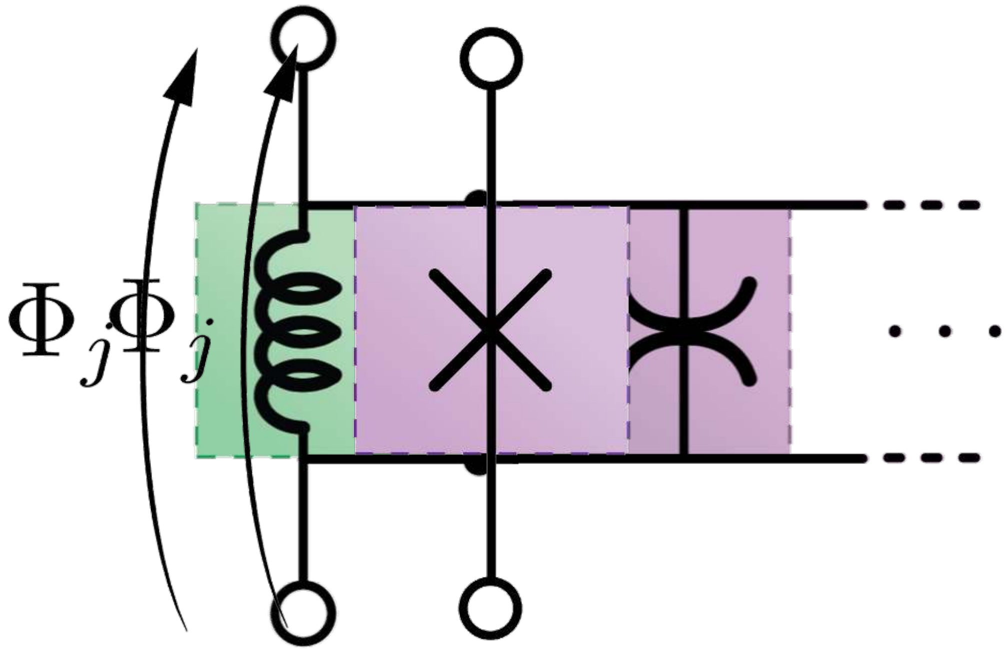


Image: Gambetta, Chow, and Steffen (2017)   Minev *et al.* (2013, 2016)   Paik *et al.* (2011), …

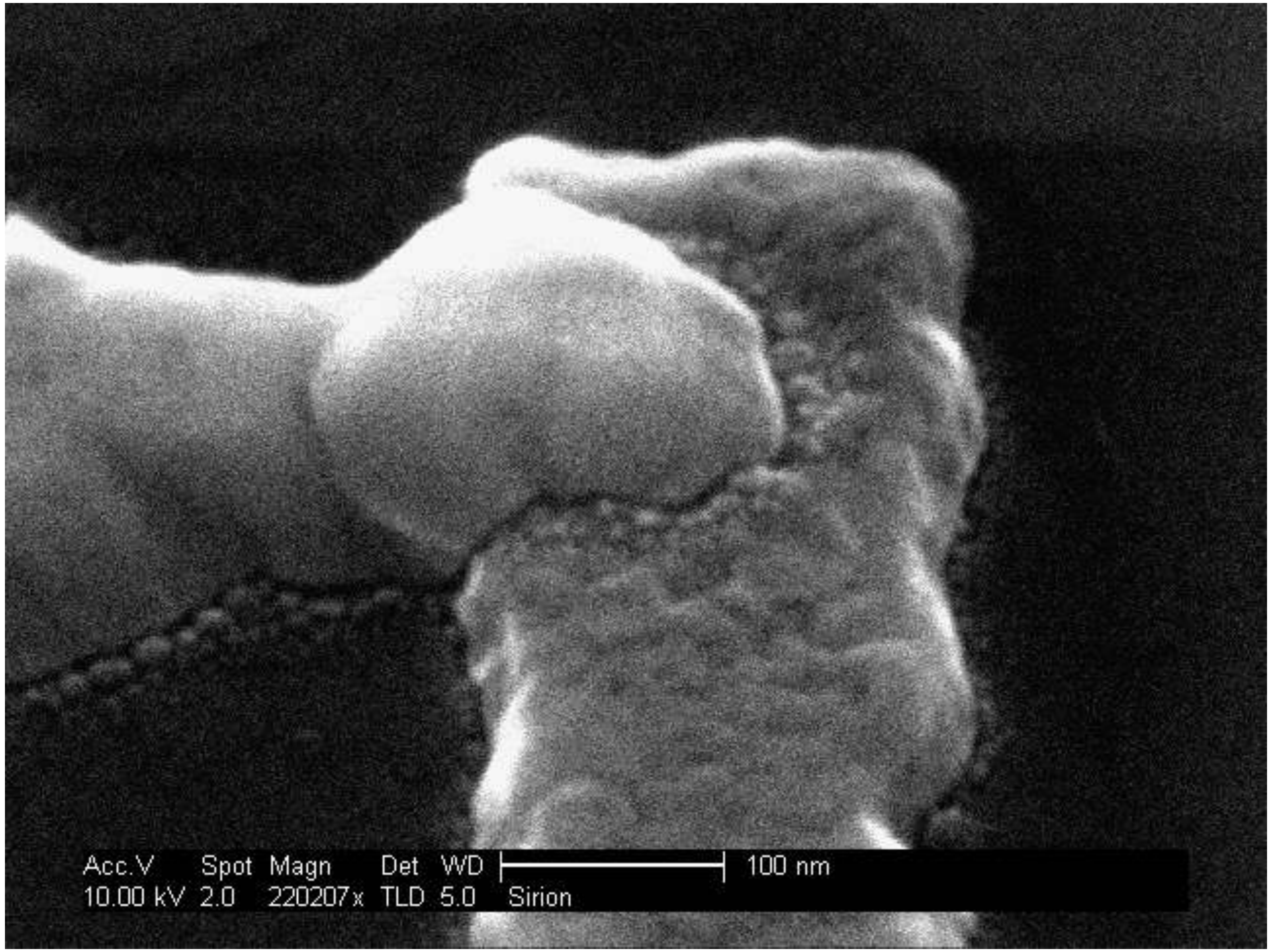
Non-linear element: Josephson tunnel junction



$$\mathcal{E}_j\left(\Phi_j\right) = E_j\left(1 - \cos\left(\Phi_j/\phi_0\right)\right)$$

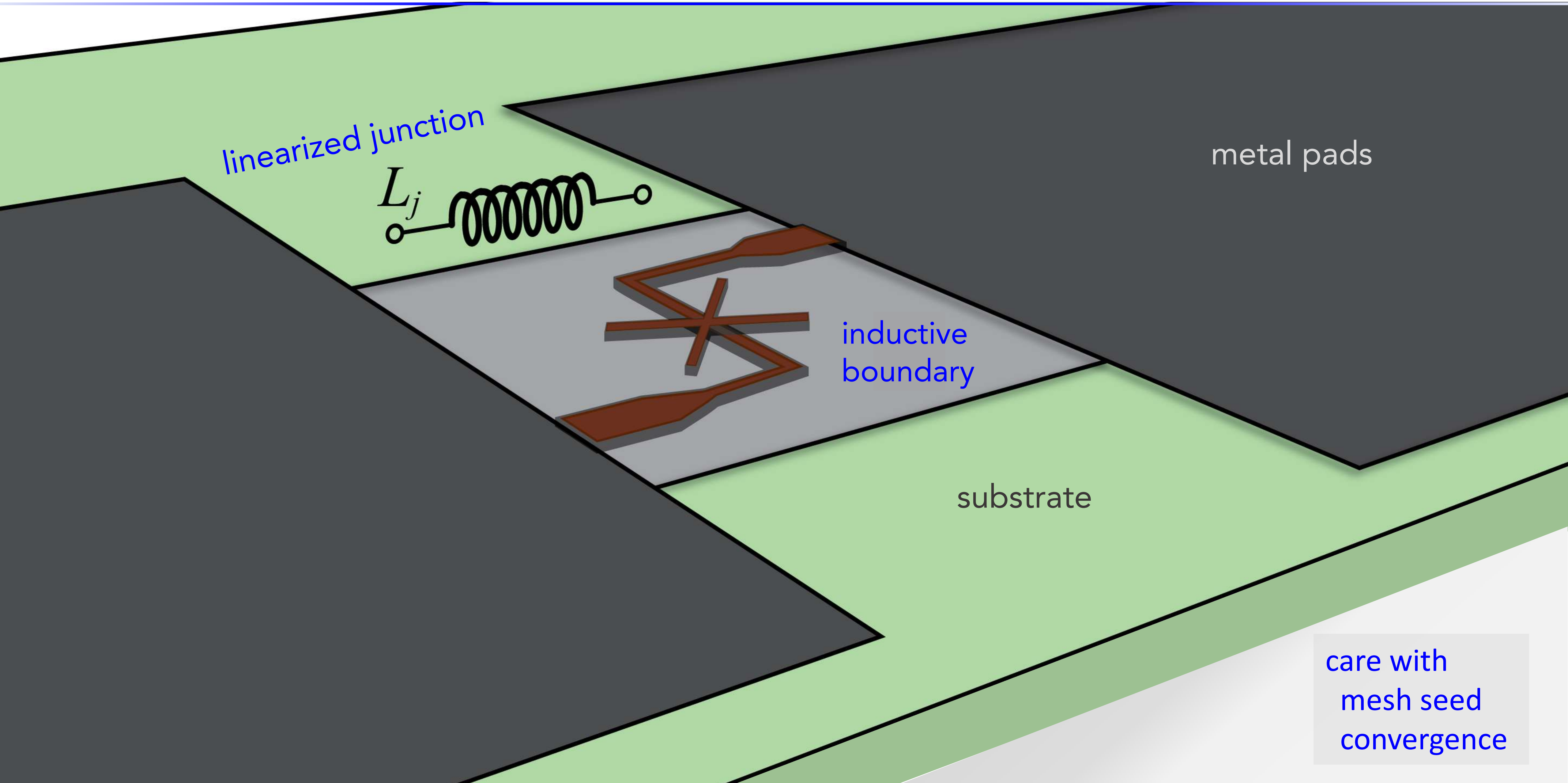
$$\phi_0 \equiv \hbar/2e\ \Phi_j) + \mathcal{E}_j^{\mathrm{nl}}\left(\Phi_j\right)$$

$$= \frac{E_j}{2}\left(\frac{\Phi_j}{\phi_0}\right)^2 - \frac{E_j}{4!}\left(\frac{\Phi_j}{\phi_0}\right)^4 + \mathcal{O}\left(\Phi_j^6\right)$$



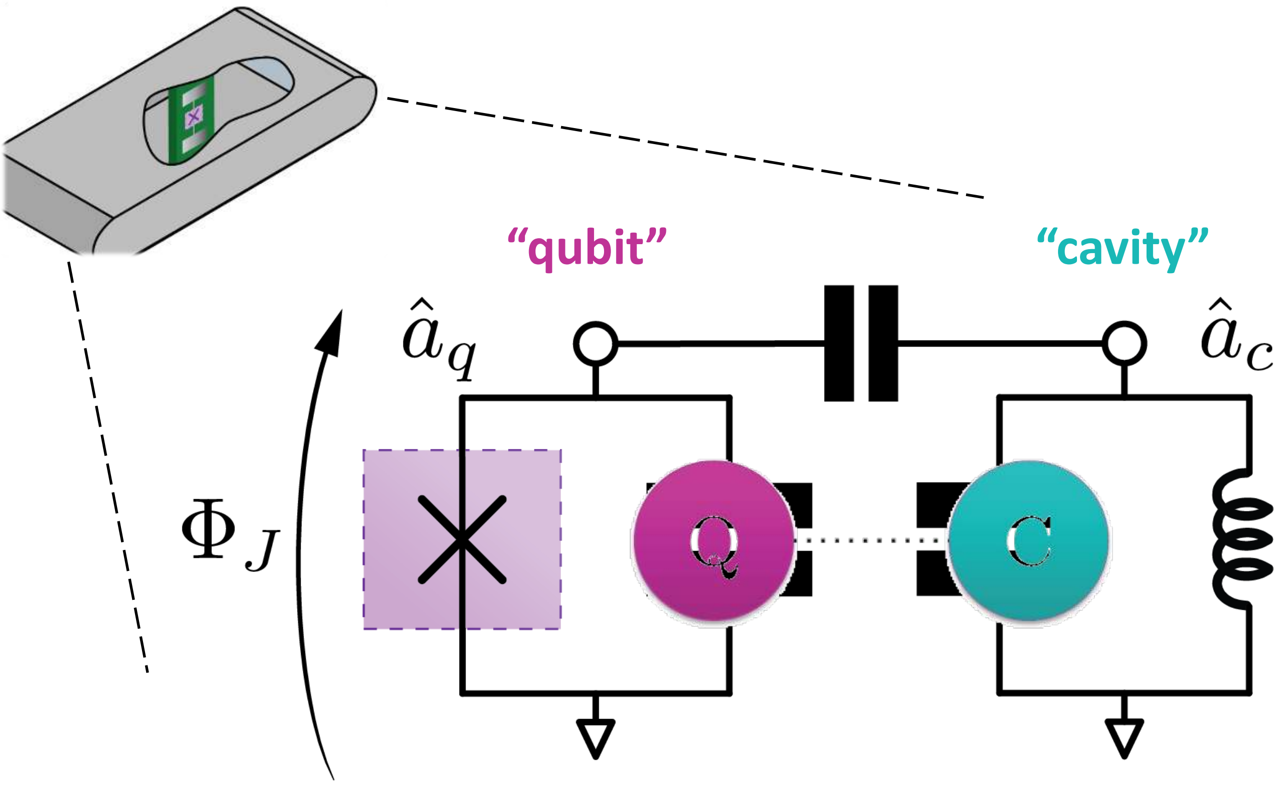
SEM image: L. Frunzio

Finite-element model of linearized Junction



care with mesh seed convergence

Explanation of quantization



(only conceptual picture)

$$\boxed{\hbar\omega_c \hat{a}_c^\dagger \hat{a}_c + \hbar\omega_q \hat{a}_q^\dagger \hat{a}_q}$$

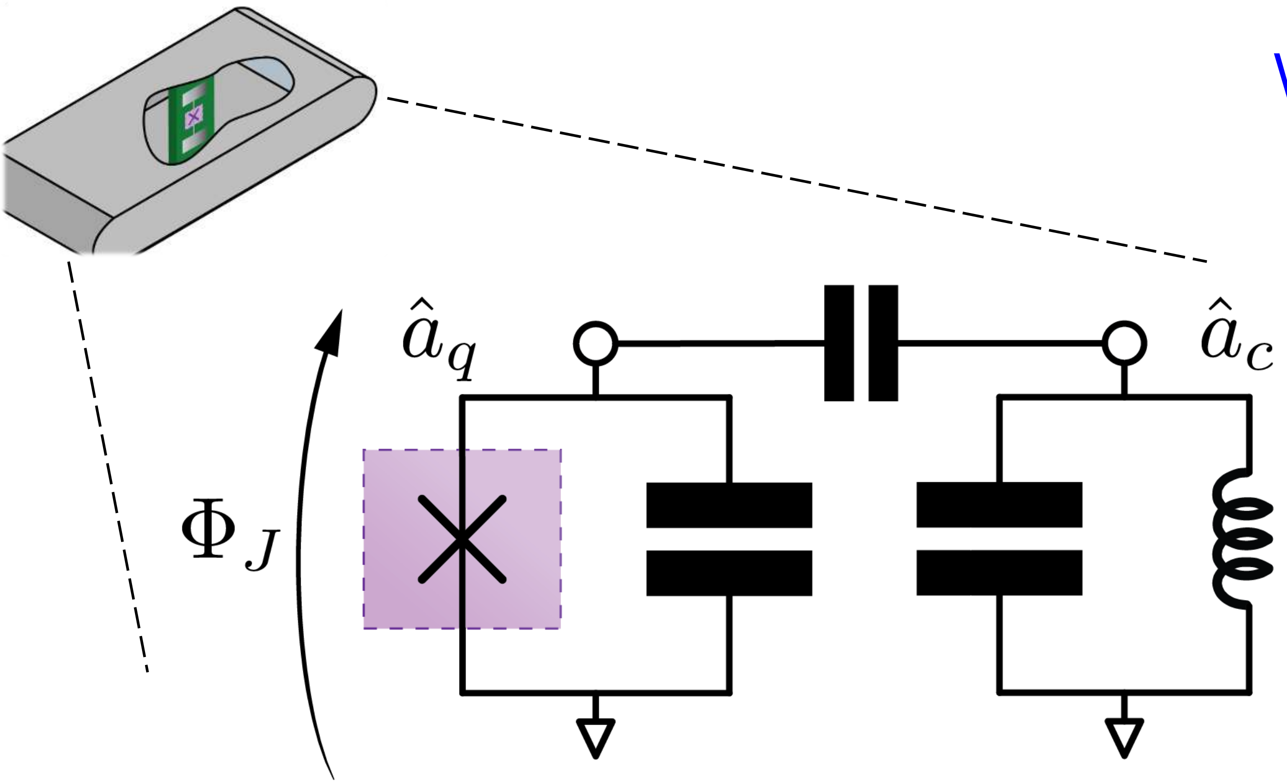
$$\hat{H}_{\text{full}} = \boxed{\hat{H}_{\text{lin}}} + \boxed{\hat{H}_{\text{nl}}}$$

$$\boxed{-E_J\left(\cos\left(\hat{\phi}_J\right) + \hat{\phi}_J^2/2\right)}$$

$$\boxed{\hat{\phi}_J = \phi_c^{\text{ZPF}}\left(\hat{a}_c^\dagger + \hat{a}_c\right) + \phi_q^{\text{ZPF}}\left(\hat{a}_q^\dagger + \hat{a}_q\right)}$$

second quantization in eigen basis of linearized circuit

Energy-participation ratio (EPR) and the quantum bridge

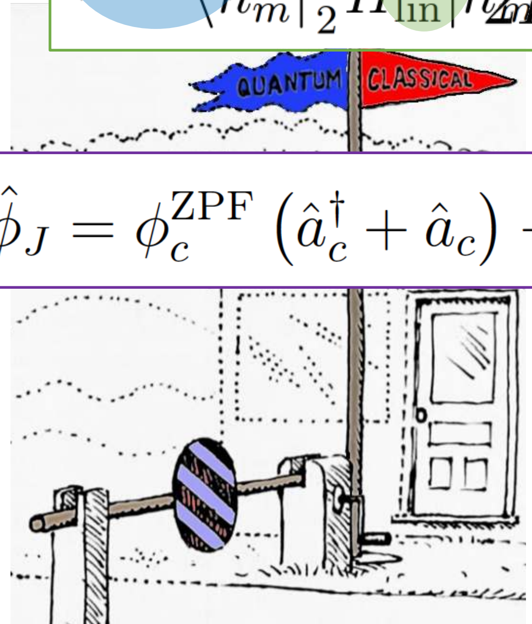


What fraction of the energy of a mode *m* is stored in the junction?

$$p_m = \frac{\text{Energy stored in junction}}{\text{Inductive energy stored in mode } m}$$

$$\frac{\langle n_m | :\frac{1}{2} E_J \phi_J^2: | n_m \rangle}{\langle n_m | \frac{1}{2} \hat{H}_{\text{lin}} | n_m \rangle}$$

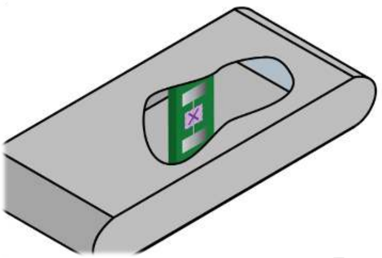
$$\left(\phi_m^{\text{ZPF}}\right)^2 = p_m \frac{\hbar \omega_m}{2 E_J}$$



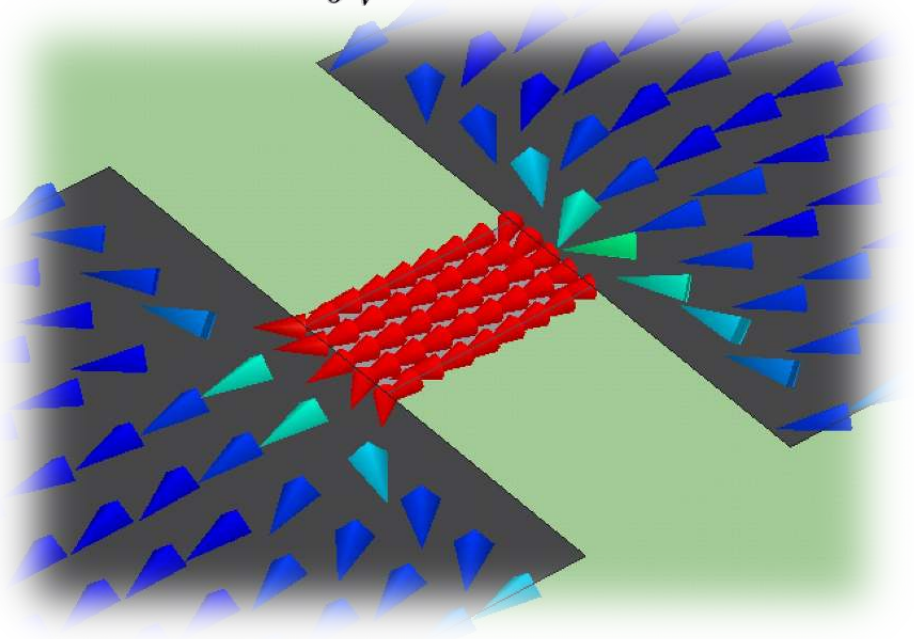
$$\hat{\phi}_J = \phi_c^{\text{ZPF}} \left(\hat{a}_c^\dagger + \hat{a}_c\right) + \phi_q^{\text{ZPF}} \left(\hat{a}_q^\dagger + \hat{a}_q\right)$$

Drawing:
Zurek, Physics Today (1991)

Calculation of the EPR from the classical solution of $\mathcal{H}_{\mathrm{lin}}$



$$\mathcal{E}_{\mathrm{mag}} = \frac{1}{4}\mathrm{Re}\int_V \mathrm{d}v \vec{H}^*_{\mathrm{max}} \overleftrightarrow{\mu} \vec{H}_{\mathrm{max}}$$



What fraction of the energy of a mode m is stored in the junction?

$$p_m = \frac{\text{Energy stored in junction}}{\text{Inductive energy stored in mode } m}$$

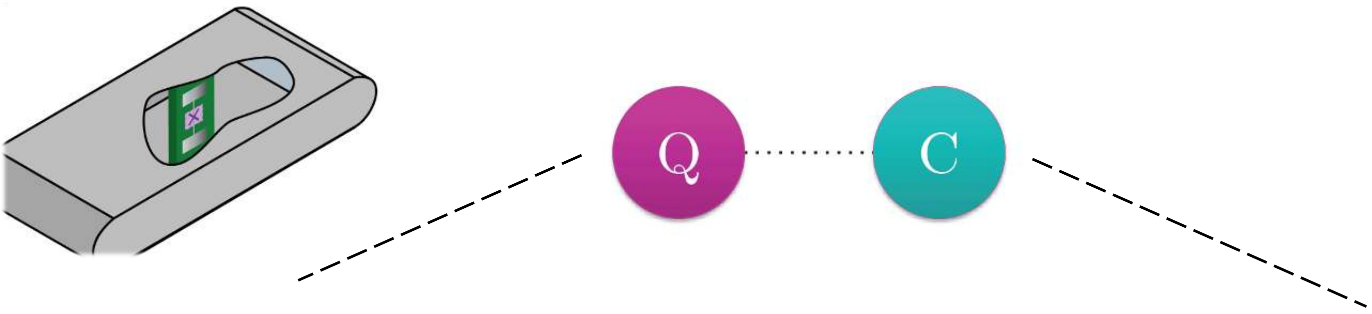
$$= \frac{\frac{1}{2} L_j I_{mj}^2}{\mathcal{E}_{ind,m}}$$

Resolution of the energy

$$\mathcal{E}_{\mathrm{mag}} + \mathcal{E}_{\mathrm{kin}} = \mathcal{E}_{\mathrm{ind}} = \mathcal{E}_{\mathrm{cap}} = \mathcal{E}_{\mathrm{elec}}$$

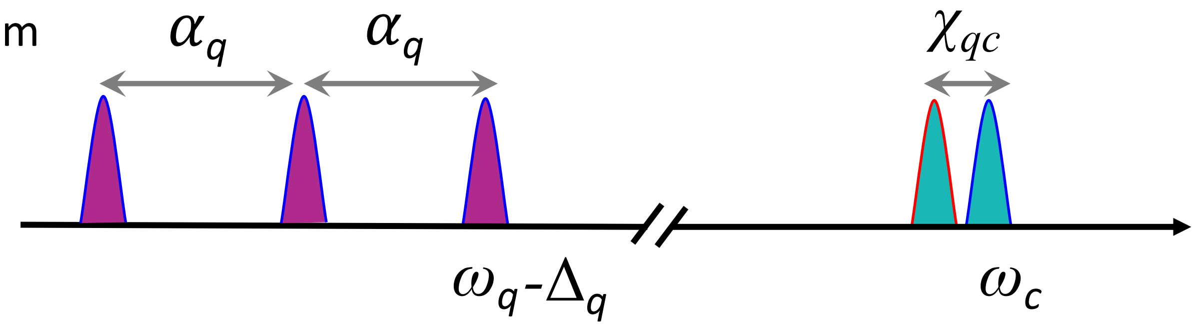
$$p_m = \frac{\mathcal{E}_{\mathrm{elec}} - \mathcal{E}_{\mathrm{mag}}}{\mathcal{E}_{\mathrm{elec}}}$$

Quantum Hamiltonian from the EPR



$$\hat{H}_{\text{eff}} = \boxed{(\omega_q - \Delta_q)\hat{n}_q} + \boxed{\omega_c \hat{n}_c} - \boxed{\chi_{qc}\hat{n}_q\hat{n}_c}$$
$$\boxed{-\frac{1}{2}\alpha_q\hat{n}_q\left(\hat{n}_q - \hat{1}\right)} - \boxed{\frac{1}{2}\alpha_c\hat{n}_c\left(\hat{n}_c - \hat{1}\right)}\,,$$

Transition spectrum



for simplicity, showing up to $\mathcal{O}(\varphi^6)$ in RWA

Qubit/cavity anharmonicity

$$\alpha_{q/c} = p_{q/c}^2 \frac{\hbar\omega_{q/c}^2}{8E_J}$$

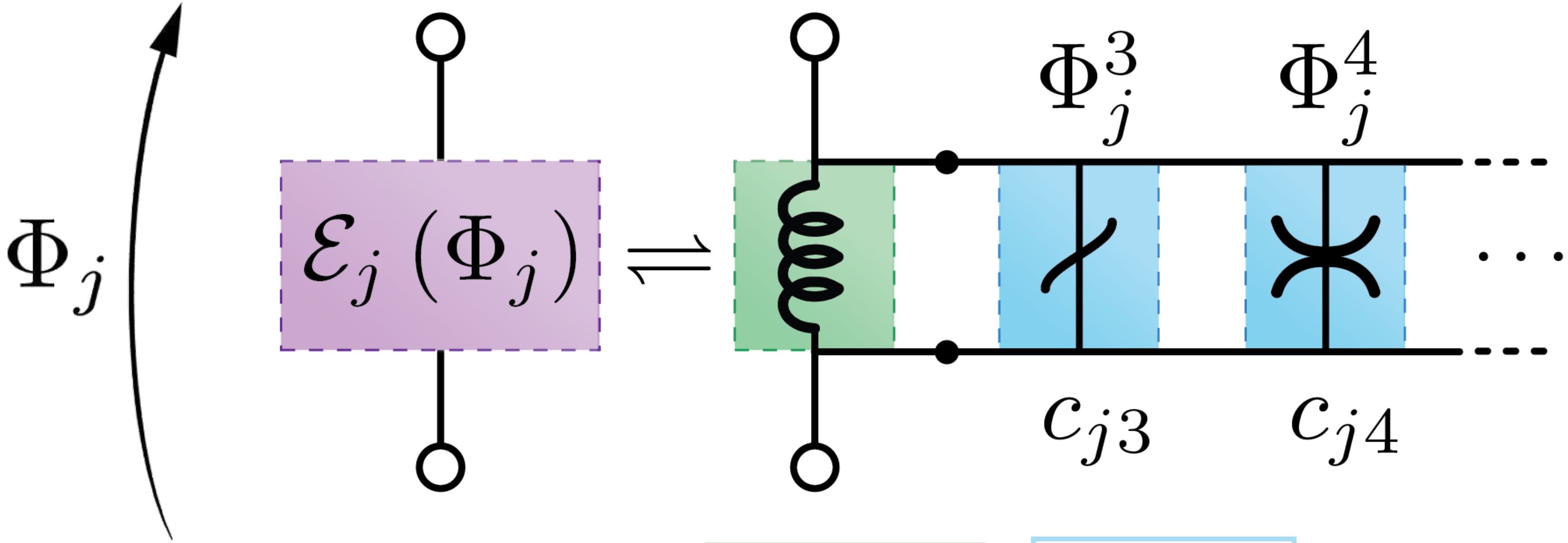
Qubit-cavity dispersive shifty

$$\chi_{qc} = p_q p_c \frac{\hbar\omega_q\omega_c}{4E_J}$$

Qubit Lamb shift

$$\Delta_q = \alpha_q - \tfrac{1}{2}\chi_{qc}$$

General Josephson device

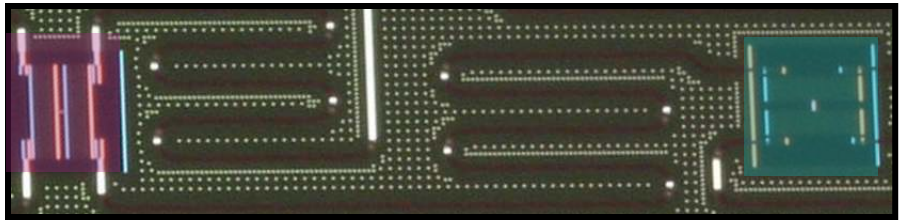


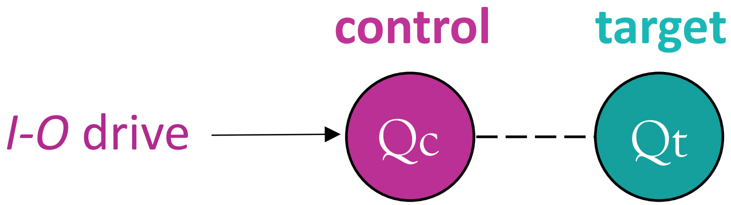
$$\equiv \boxed{\mathcal{E}_j^{\mathrm{lin}}(\Phi_j)} + \boxed{\mathcal{E}_j^{\mathrm{nl}}(\Phi_j)}$$

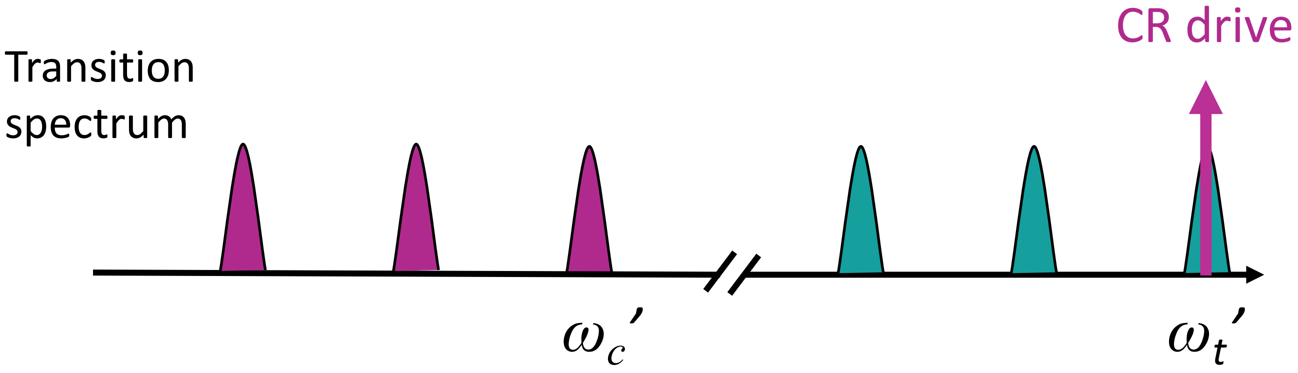
$$\boxed{E_j \sum_{p=3}^{\infty} c_{jp} \left(\frac{\Phi_j}{\phi_0}\right)^p}$$

series with respect to operational equilibrium point

Cross-resonance (CR) entangling gate

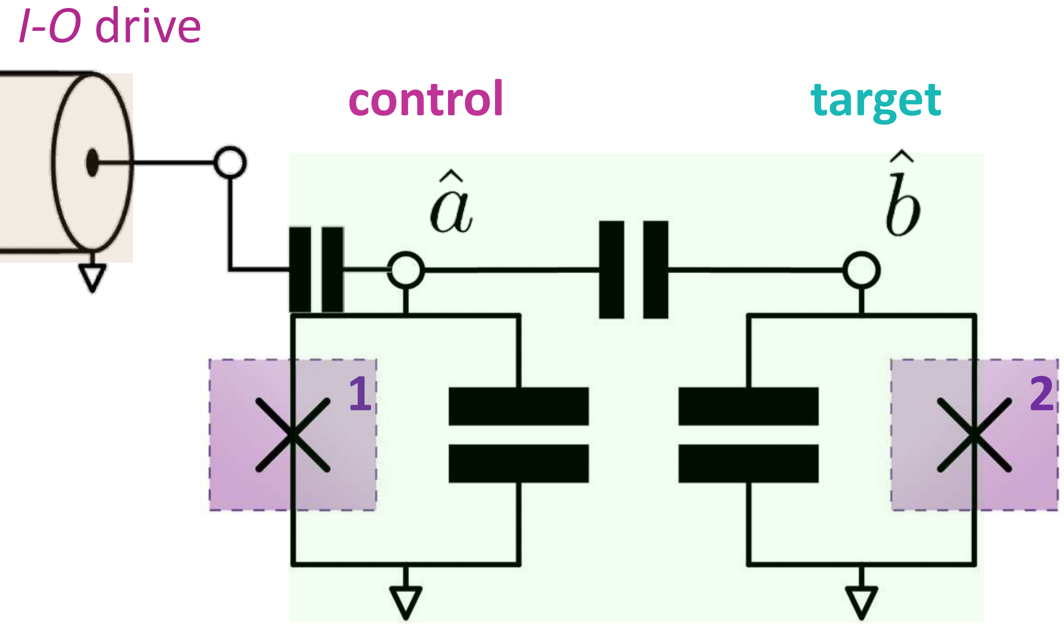






dispersive regime; χ_{qc} small

Cross-resonance (CR) entangling gate



$$\hat{H} = \omega_a \hat{a}^\dagger \hat{a} + \omega_a \hat{b}^\dagger \hat{b} \quad \text{Eigen modes}$$

$$- \sum_{j=1}^{2} \frac{E_j}{4!} \left(\phi_{aj} \left(\hat{a} + \hat{a}^\dagger \right) + \phi_{bj} \left(\hat{b} + \hat{b}^\dagger \right) \right)^4 + \ldots \quad \text{Non-linear}$$

$$- i\epsilon_a(t) \left(\hat{a}^\dagger - \hat{a} \right) - i\epsilon_b(t) \left(\hat{b}^\dagger - \hat{b} \right) . \quad \text{Drives}$$

$$-\frac{E_j}{4!} \phi_a^3 \phi_b \left[\left(\hat{a}^\dagger \hat{a} - \frac{1}{2} \right) \left(\xi \hat{b} + \xi^* \hat{b}^\dagger \right) \right]$$

$$\hat{H}_{\mathrm{CR}} = \frac{1}{2} \omega_{zx} \hat{\sigma}_z \hat{\sigma}_x$$

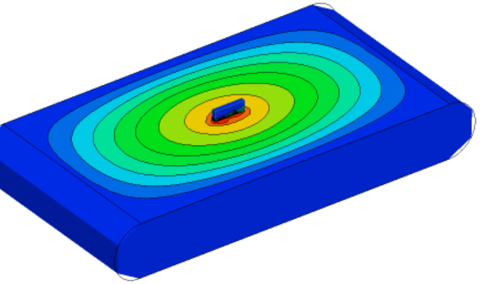
$$\omega_{zx} \approx \sqrt{p_{b1}} \times \frac{1}{2} \,\mathrm{GHz}$$

Participation matrix

$$\begin{pmatrix} p_{a1} & p_{a2} \\ p_{b1} & p_{b2} \end{pmatrix} \approx \begin{pmatrix} 0.92 & 2 \times 10^{-4} \\ 2 \times 10^{-4} & 0.92 \end{pmatrix}$$

dispersive regime; χ_{qc} small

Dissipation budget and the energy-participation ratio (EPR)



Quality of mode m

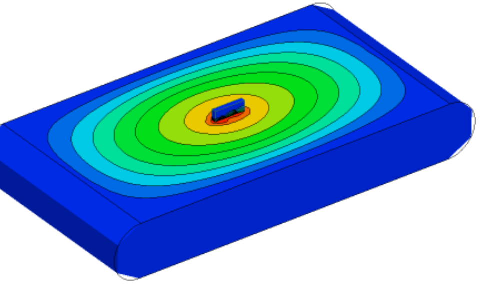
EPR of lossy element l in mode m

Quality of lossy element l

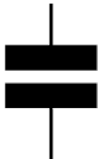
$$Q_m^{-1} = \sum_l p_m^l Q_l^{-1}$$

Pozar, Microwave engineering (1990); Gao, Thesis (2008), Zmuidzinas, ARCMP (2012); …
Geerlings, Thesis (2013); Minev *et al.* (2013); Brecht *et al.*, APL (2015); Wang *et al.,* APL (2015); Dial SST *et al.* (2016); …

Lossy energy-participation ratios (EPRs)



Capacitive



bulk

$$p^l_{\text{cap}} = \frac{1}{\mathcal{E}_{\text{elec}}}\frac{1}{4}\text{Re}\int_{V_l} \mathrm{d}v\, \vec{E}^*_{\max} \overleftrightarrow{\epsilon}\, \vec{E}_{\max}$$

surface

$$p^l_{\text{cap,surf}} = \frac{1}{\mathcal{E}_{\text{elec}}}\frac{t_l\epsilon_l}{4}\text{Re}\int_{\text{surf}_l} \mathrm{d}s \left|\vec{E}_{\max}\right|^2$$

Inductive

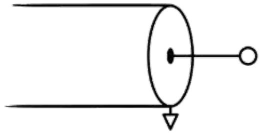
surface

$$p^l_{\text{ind,surf}} = \frac{1}{\mathcal{E}_{\text{mag}}}\frac{\lambda_0\mu_l}{4}\text{Re}\int_{\text{surf}_l} \mathrm{d}s \left|\vec{H}_{\max,\parallel}\right|^2$$

seam

$$p^l_{\text{ind,seam}} = \frac{1}{\mathcal{E}_{\text{mag}}}\frac{\lambda_0 t_l\mu_l}{4}\text{Re}\int_{\text{seam}_l} \mathrm{d}l \left|\vec{H}_{\max,\perp}\right|^2$$

Radiative

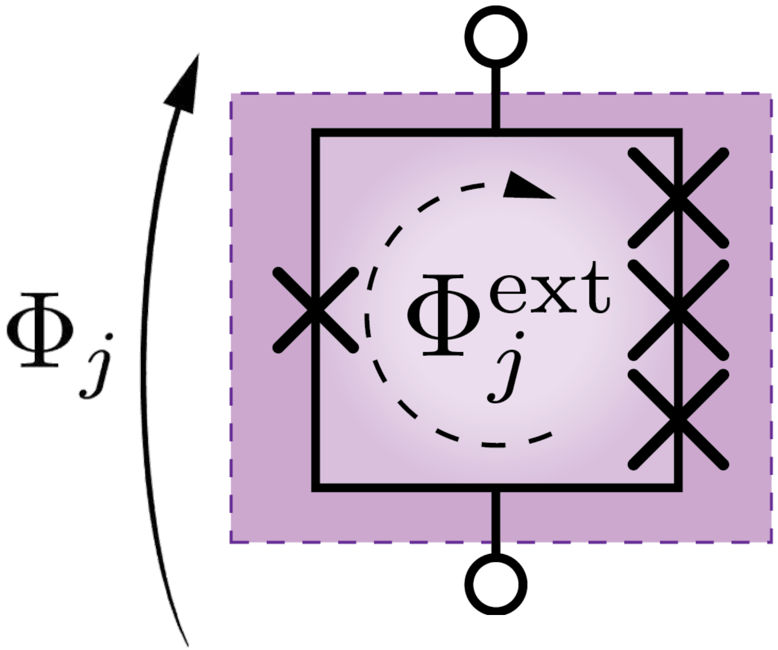


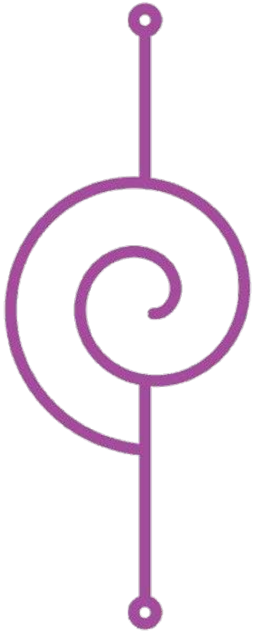
$$Q_{mp} = \frac{\omega_m \mathcal{E}_m(0)}{\frac{1}{2}RI^2_{mp}}$$

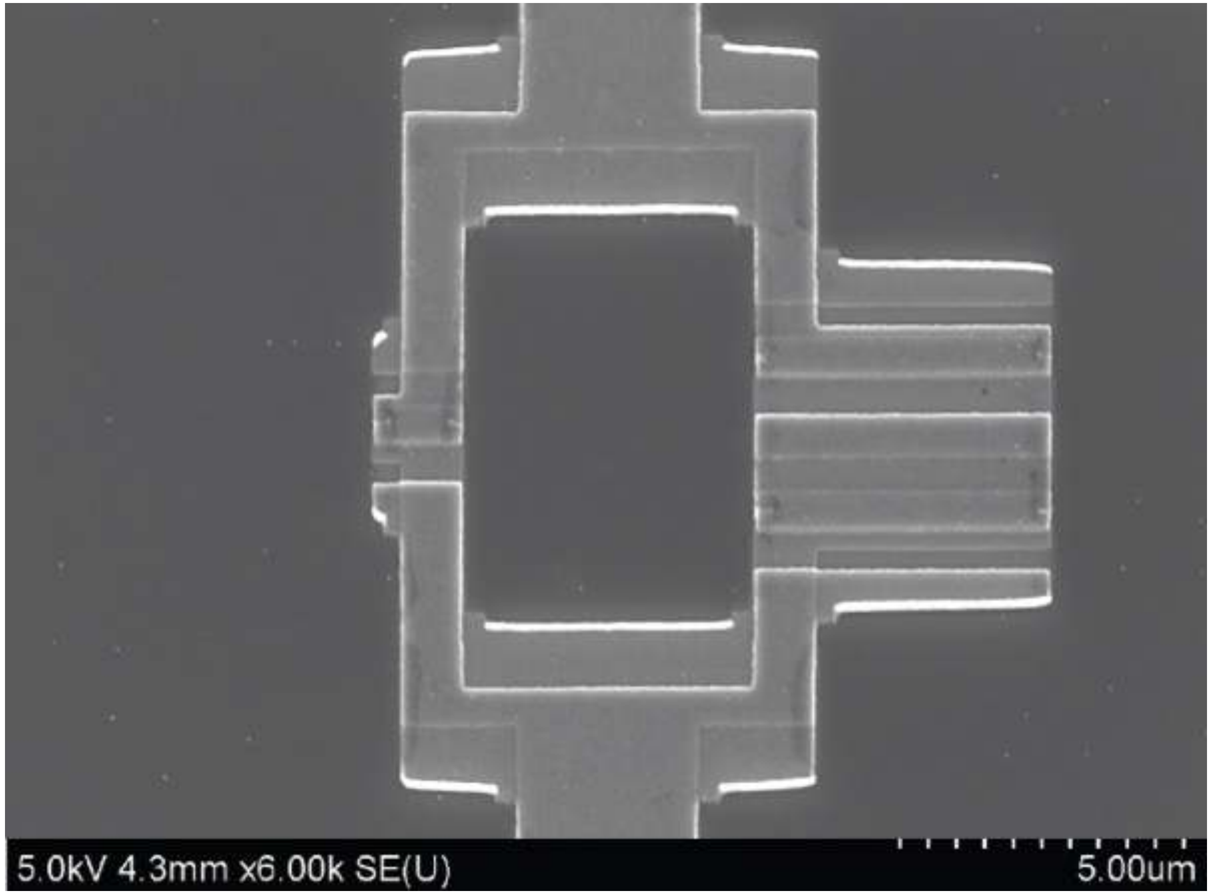
Pozar, Microwave engineering (1990); Gao, Thesis (2008), Zmuidzinas, ARCMP (2012); …
Geerlings, Thesis (2013); Minev *et al.* (2013); Brecht *et al.*, APL (2015); Wang *et al.*, APL (2015); Dial SST *et al.* (2016); …

General circuits and non-linear devices

Composite Josephson device: example





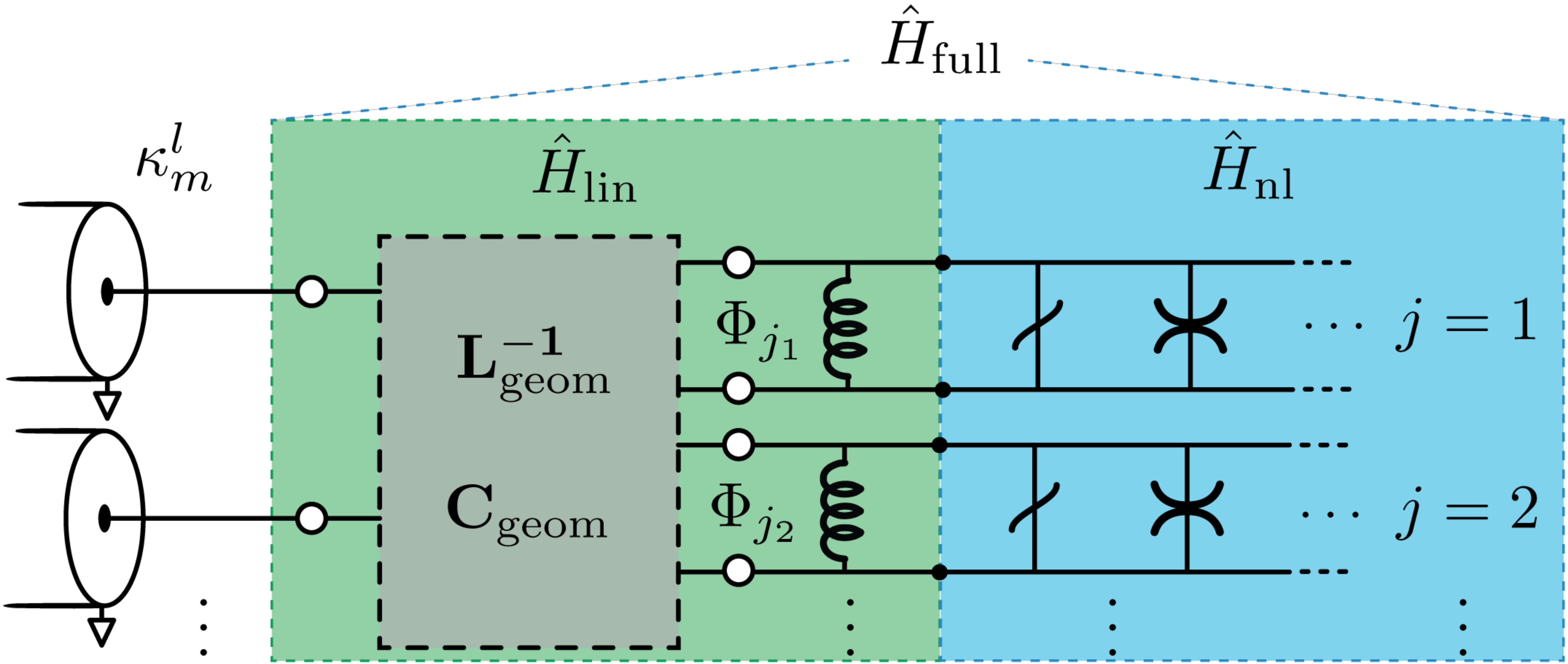


Energy function

$$\mathcal{E}_j\left(\Phi_j;\Phi_j^{\mathrm{ext}}\right)$$

SEM image: Frattini & Sivak; see APL (2017)

Decomposition of the general circuit



Non-linear interaction with EPR

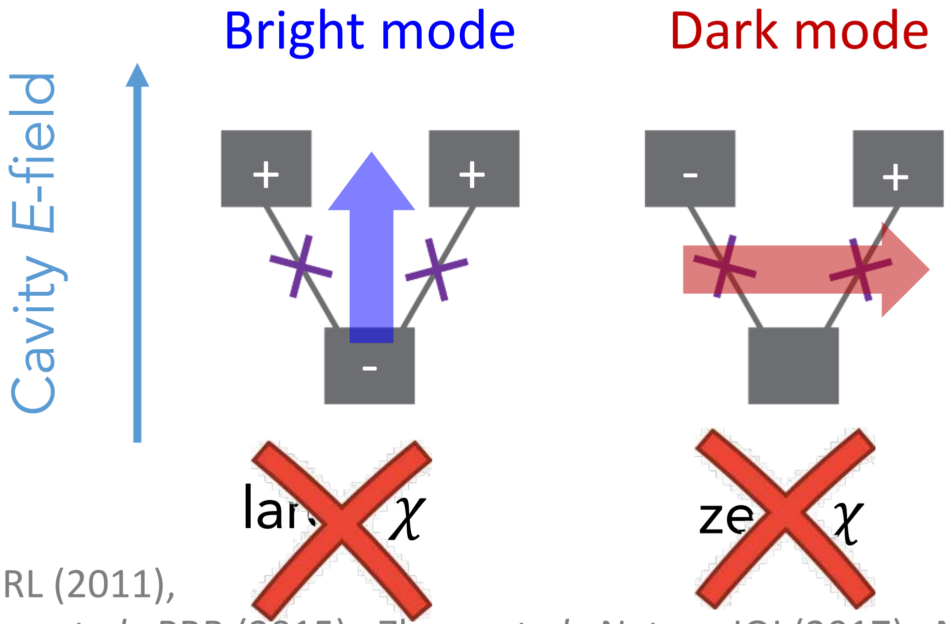
Non-linear interactions in terms of energy overlap

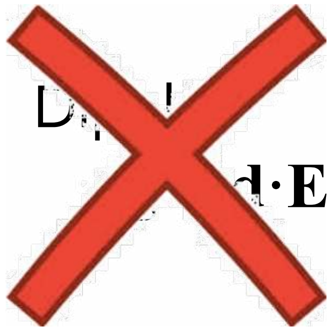
$$\chi_{mn} = \sum_{j=1}^{J} \frac{\hbar \omega_m \omega_n}{4E_j} p_{mj} p_{nj}$$

Energy participation overlap

(only variable subject to significant variation)

Intuition with dipole and EPR





same p_{mj} for both χs are ***equal***

For device, see Gambetta *et al.*, PRL (2011), Srinivasan *et al.*, PRL (2011), Dumur *et al.*, PRB (2015), Zhang *et al.*, Nature JQI (2017), Minev *et al.*, arXiv:1803.00545 (2018)

Physical insights with EPR

Non-linear interactions in terms of energy overlap

$$\chi_{mn} = \sum_{j=1}^{J} \frac{\hbar\omega_m\omega_n}{4E_j} p_{mj}p_{nj}$$

Energy participation overlap

(only variable subject to significant variation)

Fundamental constraints

What circuits/parameters can and cannot be realized?

$$\sum_{m=1}^{M} p_{mj} = 1 \qquad 0 \leq \sum_{j=1}^{J} p_{mj} \leq 1 \qquad \sum_{m=1}^{M} s_{mj}s_{mj'}\sqrt{p_{mj}p_{mj'}} = 0$$

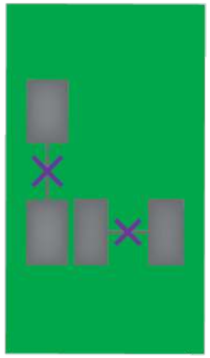
Arbitrary strength non-linear interaction can be calculated

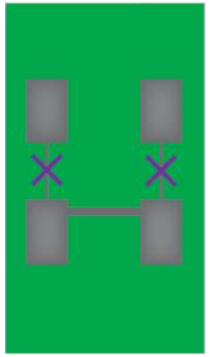
Experimental results

2Q, 1C theory vs. experiment

Legend

measured
predicted
error (%)



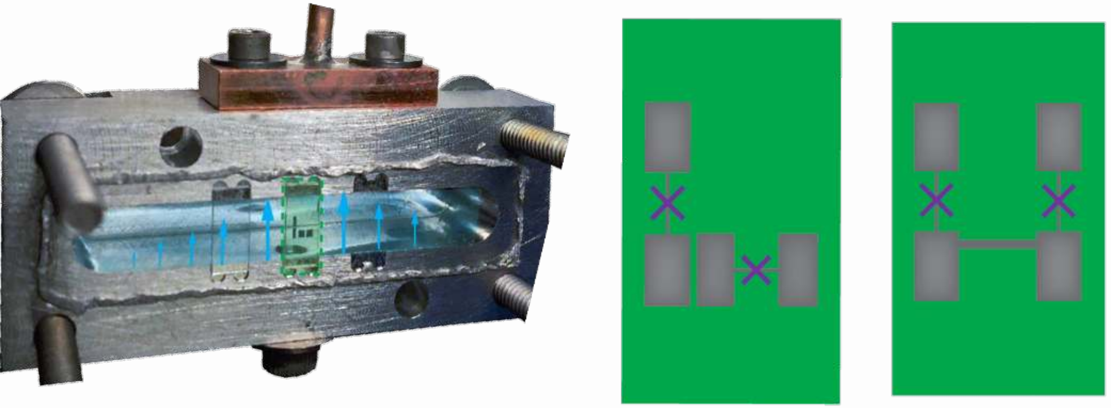


Device	Frequency (MHz) $\omega_D/2\pi$	$\omega_B/2\pi$	$\omega_C/2\pi$	Anharmonicity (MHz) $\alpha_D/2\pi$	$\alpha_B/2\pi$	Cross-Kerr (MHz) $\chi_{DB}/2\pi$	$\chi_{BC}/2\pi$	$\chi_{DC}/2\pi$	*I-O* coupling Q_C
R9C1	4951	5664	9158	138	170	92.	4.7	0.4	5.20×10^3
	4866	5691	9154	150	185	99.	4.2	0.55	7.40×10^3
	-1.7%	0.5%	-0.04%	8%	8%	7%	-12%	27%	29%
R2C1	4823	5567	8947	150	192	64.5	4.8	0.3	4.97×10^3
	4770	5640	8950	161	211	67.7	5.88	0.46	5.44×10^3
	-1.1%	1.3%	0.03%	6.8%	9%	4.7%	18%	35%	9 %
R7C1	4726	5475	8999	156	189	67.	4.8	0.34	2.68×10^3
	4770	5640	8950	161	211	67.7	5.88	0.46	3.07×10^3
	0.9%	2.9%	-0.55%	3.1%	10%	1%	18%	26%	13%
R3C2	4845	5620	8979	152	195	61	5.1	0.3	2.11×10^3
	4770	5640	8950	161	211	67.7	5.88	0.46	1.78×10^3
	-1.5%	0.4%	-0.3%	5.6%	7.6%	9.9%	13%	35%	-19%
R3C1	4688	5300	9003	148	174	85	5.	0.33	2.43×10^3
	4745	5265	8922	159	198	73	5.1	0.37	5.65×10^3
	1.2%	-0.7%	-0.9%	6.9%	12.1%	-16%	2%	9%	57%
DT3	6160	7110	9170	130	150	278	3.	2.5	9.17×10^3
	6100	7141	9155	140	177	312	3.9	3.1	7.33×10^3
	-1.0%	0.4%	-0.15%	7%	15%	11%	23%	19%	-25%

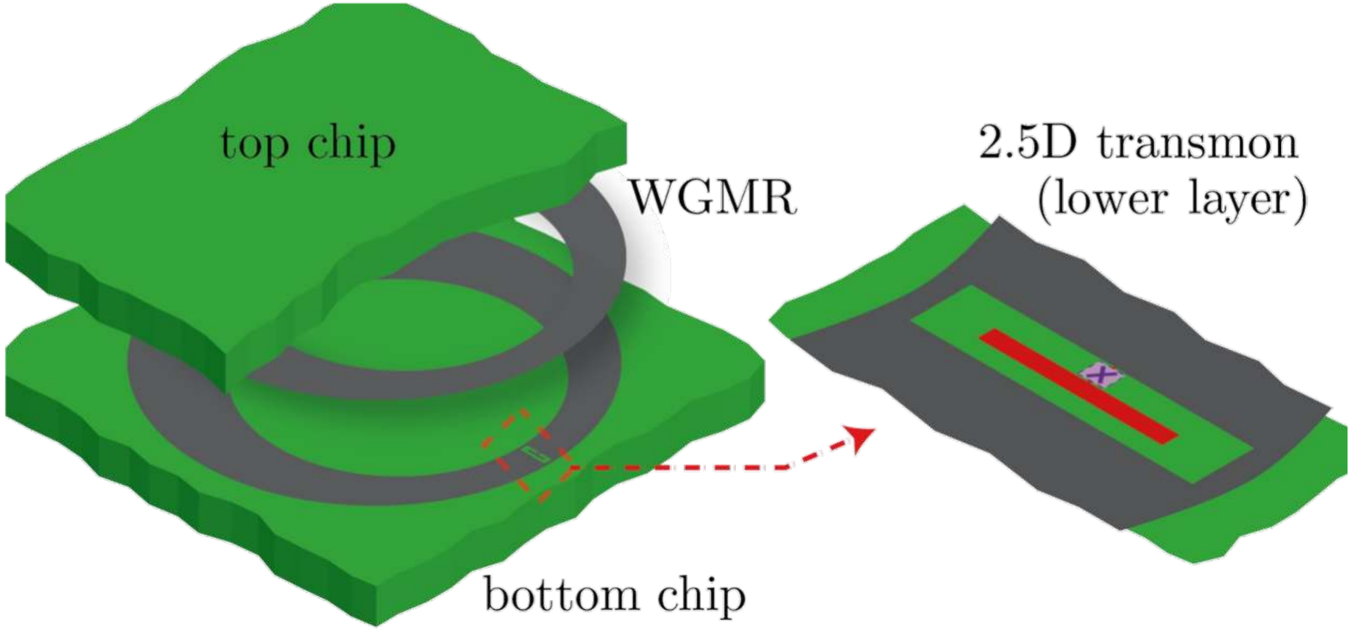
Aside from coarse adjustment on L_J, no adjustable parameters

Measured architectures and devices

Three-dimensional (3D) *

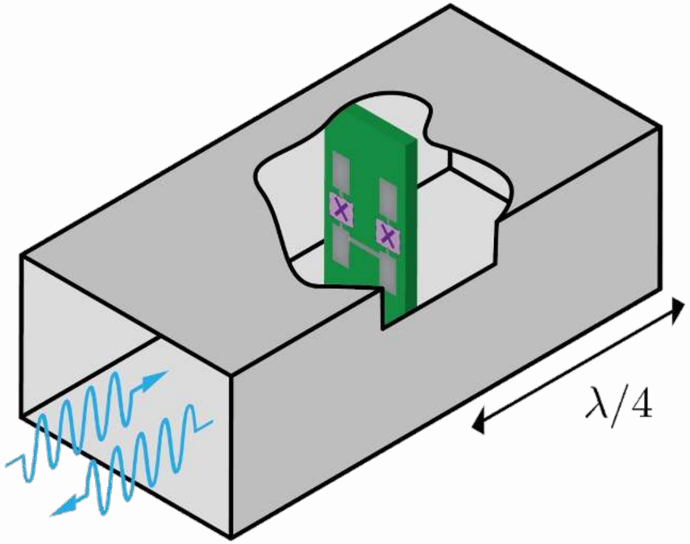


Multilayer (2.5D) cQED



Minev *et al.*, APL (2013)
Minev *et al.*, WO/2016/138395 (2015)
Minev *et al.*, Phys. Rev. App. (2016)

Waveguide



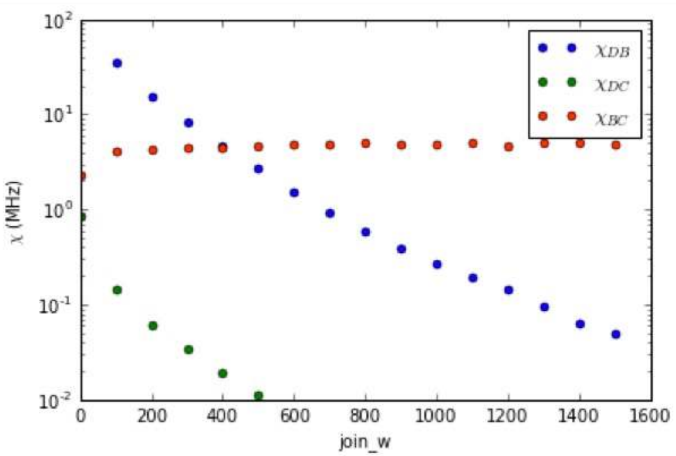
* Minev *et al.*, arXiv:1803.00545 (2018); Related: Gambetta *et al.*, PRL (2011), Srinivasan *et al.*, PRL (2011), Dumur *et al.*, PRB (2015), Zhang *et al.*, Nature JQI (2017) …

How to use the EPR approach

- Fully automated
  - control of HFSS
  - calculations
- Full numerical treatment
- Convergence checks

See my open source project on github

Used at Yale, ENS Paris, IBM, Berkeley, Saclay, and others



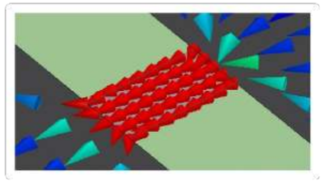

```
from pyEPR import *

# 1.  Project and design. Open link to HFSS controls.
project_info = Project_Info('c:/sims',
                            project_name = 'two_qubit_one_cavity',
                            design_name  = 'Alice_Bob'
                            )

# 2a. Junctions. Specify junctions in HFSS model
project_info.junctions['jAlice'] = {'Lj_variable':'LJAlice', 'rect':'qubitA
project_info.junctions['jBob']   = {'Lj_variable':'LJBob',   'rect':'qubitB

# 2b. Dissipative elements.
project_info.dissipative.dielectrics_bulk    = ['si_substrate']
project_info.dissipative.dielectric_surfaces = ['interface']

# 3.  Run analysis
epr_hfss = pyEPR_HFSS(project_info)
epr_hfss.do_EPR_analysis()

# 4.  Hamiltonian analysis
epr      = pyEPR_Analysis(epr_hfss.data_filename)#
epr.analyze_all_variations(cos_trunc = 8, fock_trunc = 7)
epr.plot_Hresults()
```

Conclusions / future directions

Simple and unified dissipation and Hamiltonian analysis

single-simulation efficient

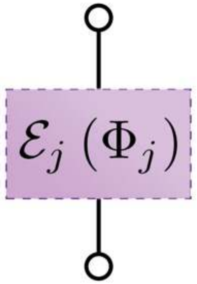
obviate driven simulations and notion of $Z_{ij}(\omega)$



General

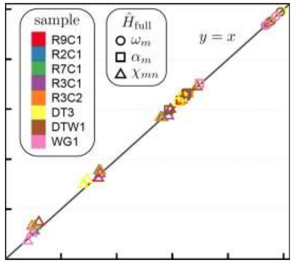
handle arbitrary architectures and non-linear devices

e.g., nanowires, …



Accuracy

experimental ten to percent level over 5 orders in $\widehat{H}_{\mathrm{full}}$



Suited for automated, robust analysis of large systems

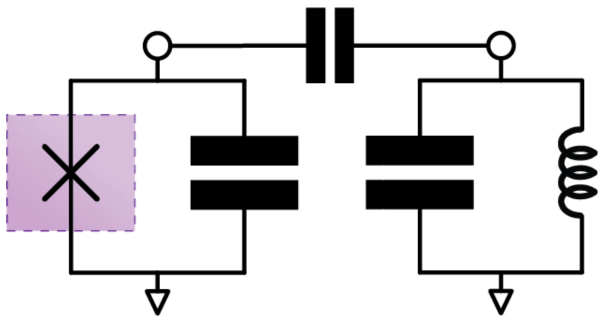
fully automated and tested

***arXiv*:very-soon;   Minev, Ph.D. Thesis, Ch. 4 (2018)**

Application to *J* and CR gate, …

Where did g and J(ay) go?

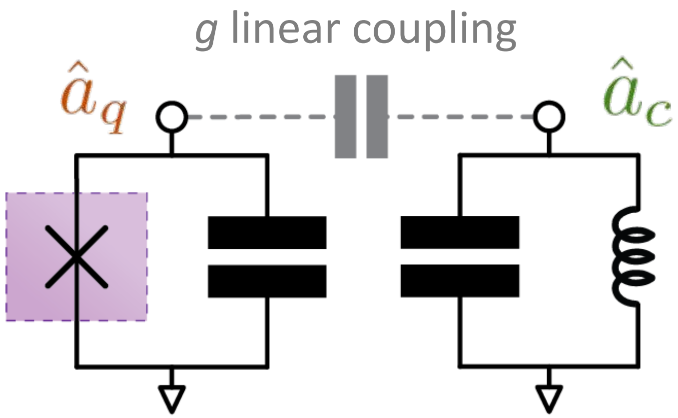
Full



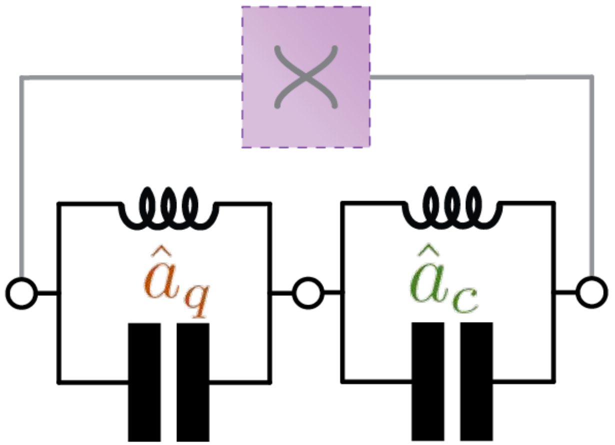
Classical simulation contains g coupling and diagonalizes it away;
i.e., the linearized modes returned by the simulation are the linear-hybridized (g-dressed) modes.

Essentially work in a more efficient basis for second quantization;
i.e., what basis are the ladder operators $\hat{a}_q$, $\hat{a}_c$ in?

g basis



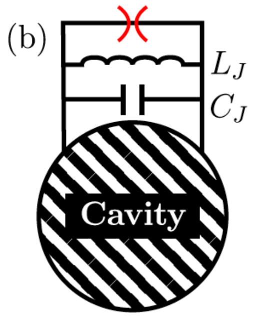
eigen basis

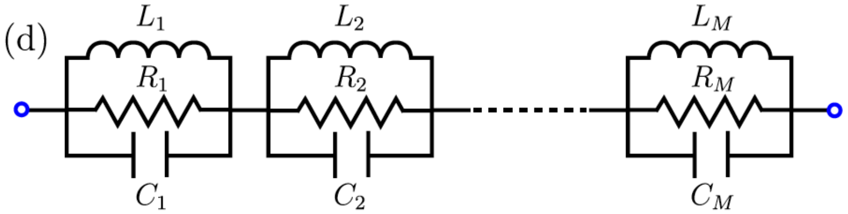


purely non-linear and quantum coupling

Relation to impedance

Single junction case





$$\phi_m^{\mathrm{ZPF}} = \sqrt{\frac{\hbar Z_a^{\mathrm{eff}}}{2}} = \sqrt{\frac{\hbar}{\mathrm{Im}\omega_a Y'(\omega_a)}}$$

$$L_m = p_m L_J/2$$
$$C_m^{-1} = \omega_m^2 L_m$$

Multi-junction case

$$\hat{\phi}_l^{(k)} = \sum_{p=1}^{M} \frac{Z_{lk}(\omega_p)}{Z_{kk}(\omega_p)} \sqrt{\frac{\hbar}{2} Z_{kp}^{\mathrm{eff}}}(a_p + a_p^\dagger),$$

where $Z_{kp}^{\mathrm{eff}} = 2/[\omega_p \mathrm{Im} Y_k'(\omega_p)]$

* Minev *et al.*, in prep. (2018) Impedances: Nigg *et al.*, PRL (2012); Bourassa *et al.* (2012); Solgun *et al.* (2014, 2015, 2017) …

Arbitrary non-linear terms

$$\hat{H}_{\text{full}} = \sum_m \boxed{(\omega'_m - i\kappa_m)}\,\hat{a}^\dagger_m \hat{a}_m + \sum_{\boldsymbol{\alpha},\boldsymbol{\beta}} \boxed{\mathcal{C}^p_{\boldsymbol{\alpha},\boldsymbol{\beta}}} \prod_{m,n} \hat{a}^{\dagger\beta_m}_m \hat{a}^{\alpha_n}_n$$

$$\boxed{\frac{p!}{\boldsymbol{\alpha}!\boldsymbol{\beta}!k!2^k} \sum_{j=1}^{J} E_j c_{jp} \phi^{\boldsymbol{\alpha}}_{mj} \phi^{\boldsymbol{\beta}}_{mj} \phi^{2k}_{j,\text{tot}}}$$

where $\boldsymbol{\alpha} \equiv (\alpha_1, \ldots, \alpha_M)$ and $\boldsymbol{\beta} \equiv (\beta_1, \ldots, \beta_M)$

$k \equiv \frac{1}{2}\left(p - (|\boldsymbol{\alpha}| + |\boldsymbol{\beta}|)\right)$

EPR matrix

Definition

$$\mathbf{P} \equiv \begin{pmatrix} p_{11} & \cdots & p_{1J} \\ \vdots & \ddots & \vdots \\ p_{M1} & \cdots & p_{MJ} \end{pmatrix} ,$$

Non-linear interaction amplitude matrices

$$\begin{aligned} \mathbf{X} &= \tfrac{\hbar}{4}\mathbf{\Omega}\mathbf{P}\mathbf{E}_{\mathbf{J}}^{-1}\mathbf{P}^{\intercal}\mathbf{\Omega} && \text{Kerr matrix}\,, \\ \boldsymbol{\alpha} &= \tfrac{1}{2}\mathrm{diag}(\mathbf{X}) && \text{Anharmonicity}\,, \\ \boldsymbol{\Delta} &= \tfrac{1}{2}\mathbf{X}\mathbf{1}_M && \text{Lamb shift}\,, \end{aligned}$$

where

$$\mathbf{E}_j^{-1} \equiv \begin{pmatrix} E_1^{-1} & & \\ & \ddots & \\ & & E_J^{-1} \end{pmatrix} \qquad \mathbf{\Omega} = \begin{pmatrix} \omega_1 & & \\ & \ddots & \\ & & \omega_M \end{pmatrix}$$

Generalization to arbitrary order

$$\boldsymbol{\chi}_p \equiv \begin{pmatrix} \chi_{11}^{(p)} & \cdots & \chi_{1M}^{(p)} \\ \vdots & \ddots & \vdots \\ \chi_{M1}^{(p)} & \cdots & \chi_{MM}^{(p)} \end{pmatrix}$$

$$\boxed{\boldsymbol{\chi}_p = \hbar c_p \left(\mathbf{\Omega}\mathbf{P}\right) \mathbf{E}_j^{-1} \phi_{\sigma,tot}^{p-4} \left(\mathbf{\Omega}\mathbf{P}\right)^{\mathrm{T}} ,}$$

$$\boldsymbol{\chi} \equiv \sum_{p=4,6,\ldots} \boldsymbol{\chi}_p \qquad \boldsymbol{\alpha}_p = \frac{1}{2}\mathrm{diag}\,\boldsymbol{\chi}_p \qquad \boldsymbol{\Delta}_p = c_p \left(\mathbf{\Omega}\mathbf{P}\right) \phi_{\sigma,tot}^{p-2} \mathbf{1}_M$$

Field calculations

Hamiltonian

$$\boxed{p_m = \frac{\mathcal{E}_{\mathrm{elec}} - \mathcal{E}_{\mathrm{mag}}}{\mathcal{E}_{\mathrm{elec}}} \,.}$$

energies

$$\mathcal{E}_{\mathrm{elec}} = \frac{1}{4}\mathrm{Re}\int_V \mathrm{d}v \vec{E}^*_{\mathrm{max}} \overleftrightarrow{\epsilon} \vec{E}_{\mathrm{max}} \,,$$
$$\mathcal{E}_{\mathrm{mag}} = \frac{1}{4}\mathrm{Re}\int_V \mathrm{d}v \vec{H}^*_{\mathrm{max}} \overleftrightarrow{\mu} \vec{H}_{\mathrm{max}} \,,$$

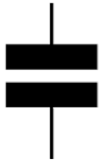
where

$$\vec{E}\,(x,y,z,t) = \mathrm{Re}\,\vec{E}_{\mathrm{max}}\,(x,y,z)\, e^{i\omega_m t}$$

Generalized to multi-junction

Dissipation budget

Capacitive



bulk

surface

$$p^l_{\mathrm{cap}} = \frac{1}{\mathcal{E}_{\mathrm{elec}}}\frac{1}{4}\mathrm{Re}\int_{V_l} \mathrm{d}v \vec{E}^*_{\mathrm{max}} \overleftrightarrow{\epsilon} \vec{E}_{\mathrm{max}}$$

$$p^l_{\mathrm{cap,surf}} = \frac{1}{\mathcal{E}_{\mathrm{elec}}}\frac{t_l \epsilon_l}{4}\mathrm{Re}\int_{\mathrm{surf}_l} \mathrm{d}s \left|\vec{E}_{\mathrm{max}}\right|^2$$

Inductive

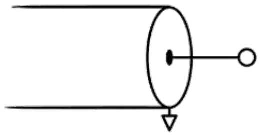
surface

seam

$$p^l_{\mathrm{ind,surf}} = \frac{1}{\mathcal{E}_{\mathrm{mag}}}\frac{\lambda_0 \mu_l}{4}\mathrm{Re}\int_{\mathrm{surf}_l} \mathrm{d}s \left|\vec{H}_{\mathrm{max},\parallel}\right|^2$$

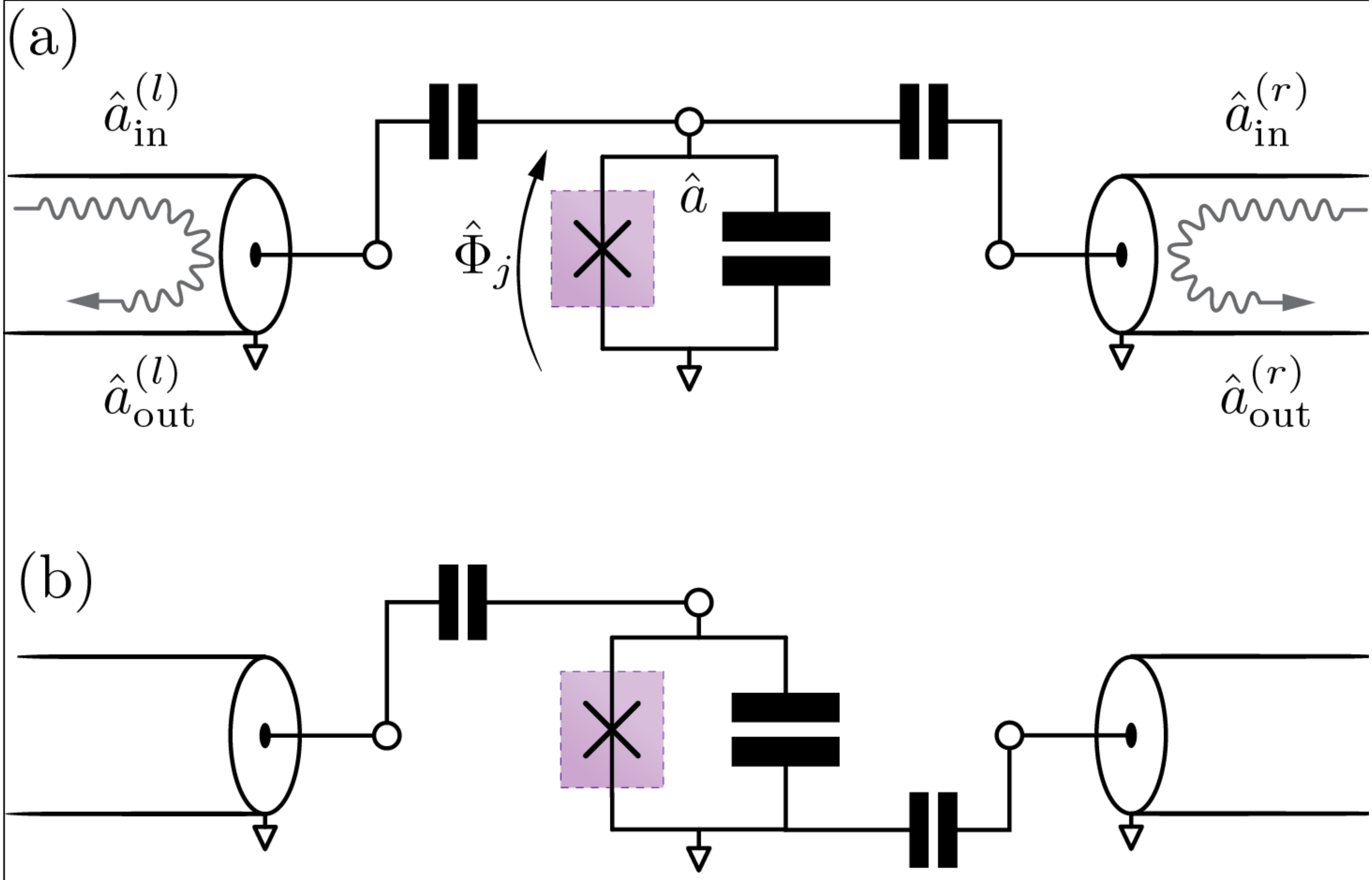
$$p^l_{\mathrm{ind,seam}} = \frac{1}{\mathcal{E}_{\mathrm{mag}}}\frac{\lambda_0 t_l \mu_l}{4}\mathrm{Re}\int_{\mathrm{seam}_l} \mathrm{d}l \left|\vec{H}_{\mathrm{max},\perp}\right|^2$$

Radiative



$$Q_{mp} = \frac{\omega_m \mathcal{E}_m(0)}{\frac{1}{2} R I_{mp}^2}$$

Pozar, Microwave engineering (1990); Gao, Thesis (2008)
Zmuidzinas, ARCMP (2012); Geerlings, Thesis (2013);
Brecht *et al.*, APL (2015); Wang *et al.*, APL (2015); Dial SST *et al.* (2016); …



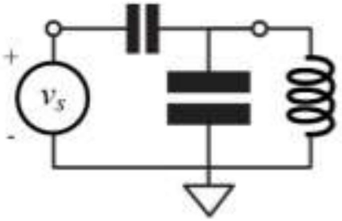
(a)
$\hat{a}_{\mathrm{in}}^{(l)}$
$\hat{a}_{\mathrm{in}}^{(r)}$
$\hat{a}$
$\hat{\Phi}_j$
$\hat{a}_{\mathrm{out}}^{(l)}$
$\hat{a}_{\mathrm{out}}^{(r)}$
(b)

Device	Frequency (MHz)		Anharmonicity (MHz)		Cross-Kerr (MHz)
	$\omega_{\mathrm{D}}/2\pi$	$\omega_{\mathrm{B}}/2\pi$	$\alpha_{\mathrm{D}}/2\pi$	$\alpha_{\mathrm{B}}/2\pi$	$\chi_{\mathrm{DB}}/2\pi$
DTW1	6010	8670	85	180	278
	5824	8878	97	206	281
	-3.2%	2.3%	12%	13%	1.1%

Table III. Two-qubit, one-waveguide devices. Summary of measured and calculated Hamiltonian parameters for the device described in Sec. #TODO. Indices D and B denote the dark and bright modes respectively. For each device, the first (second) row summarizes the measured, m, (calculated, c) values. The third row summarizes the agreement, $(c-m)/c$.

Device	Frequency (MHz)			Anharmonicity (MHz)	Cross-Kerr (MHz)		*I-O* coupling
	$\omega_{\mathrm{Q}}/2\pi$	$\omega_{\mathrm{S}}/2\pi$	$\omega_{\mathrm{C}}/2\pi$	$\alpha_{\mathrm{D}}/2\pi$	$\chi_{\mathrm{QS}}/2\pi$	$\chi_{\mathrm{QC}}/2\pi$	Q_{C}
WG1	4890	7070	7267	310	0.25	0.30	20×10^3
	4820	7020	7340	325	0.29	0.33	16×10^3
	-1.4%	-0.7%	1.0%	5.1%	13%	9%	-22%

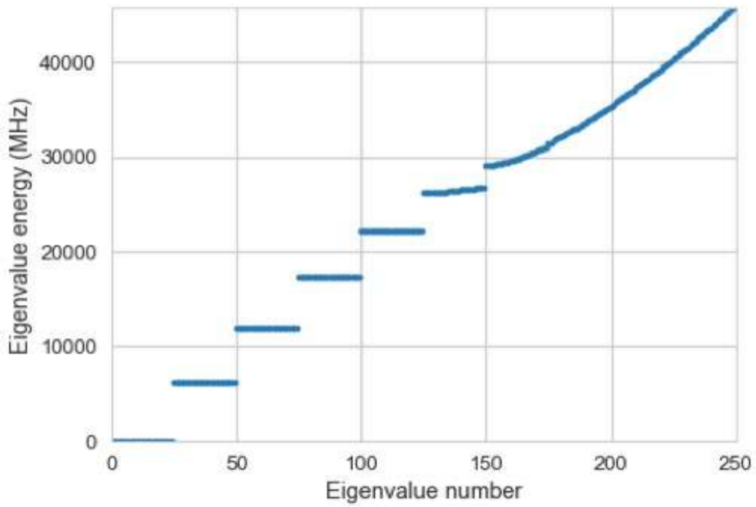
Table II. Flip-chip (2.5D), one-qubit, one-storage-cavity, one-readout-cavity devices. Summary of measured and calculated Hamiltonian and input-output (*I-O*) coupling parameters for the device described in Sec. #TODO. Indices Q, S, C denote the qubit, storage, and readout cavity modes respectively. The input-output quality factor to the readout cavity is denoted Q_{C}. For each device, the first (second) row summarizes the measured, m, (calculated, c) values. The third row summarizes the agreement, $(c-m)/c$.

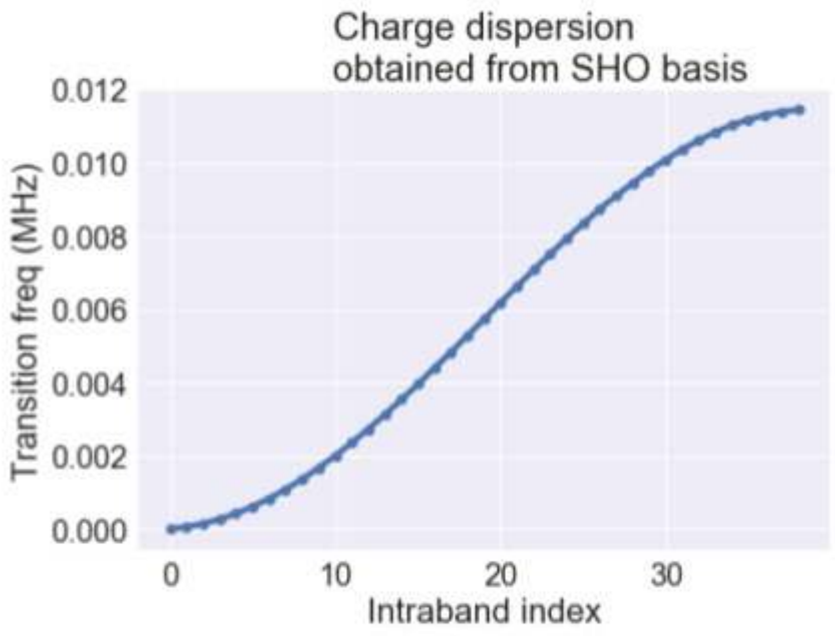


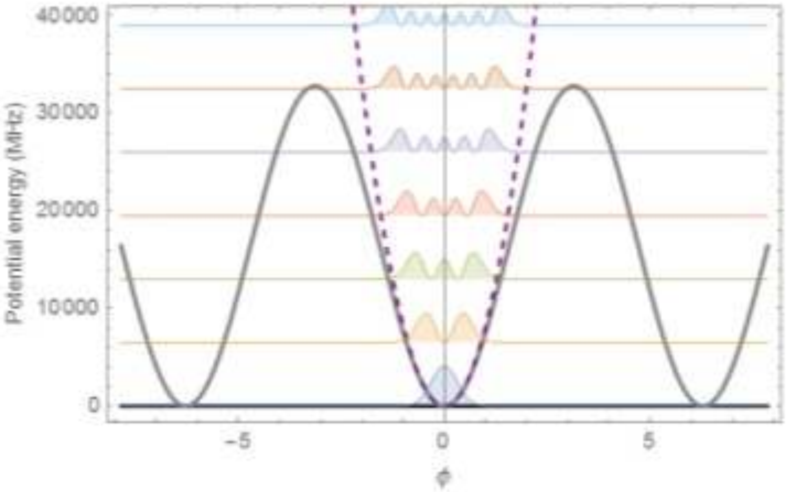
Charge dispersion with EPR

ω_0 = 6.50 GHz
$\alpha^{(4)}$ = 322.00 MHz
ϕ_{ZPF} = 0.45

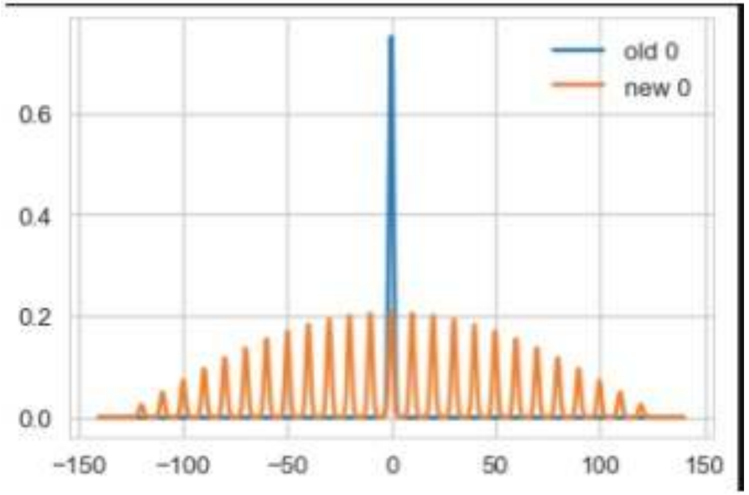
E_J = 16.40GHz
E_C = 0.32 GHz (E_J/E_C=50.90)
Dispersion =-12.90 KHz



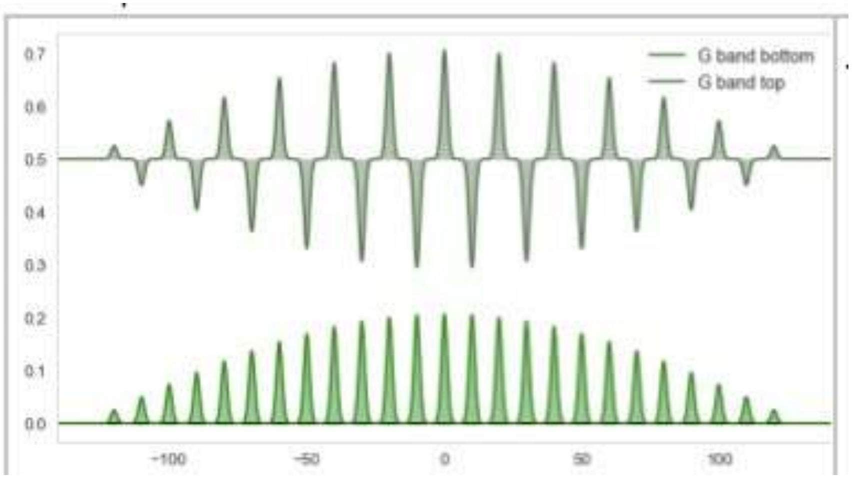




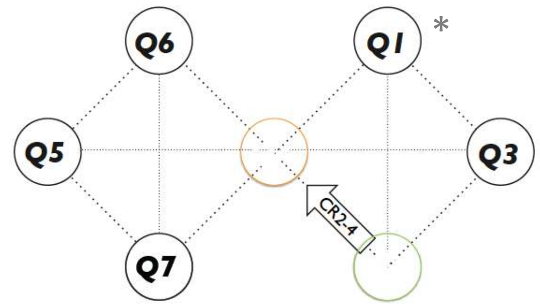
|psi|^2

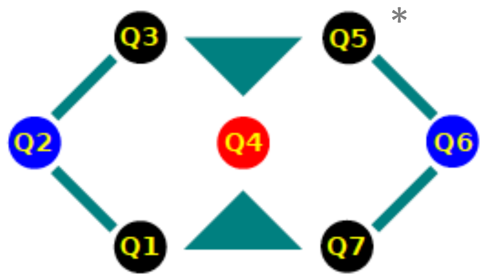


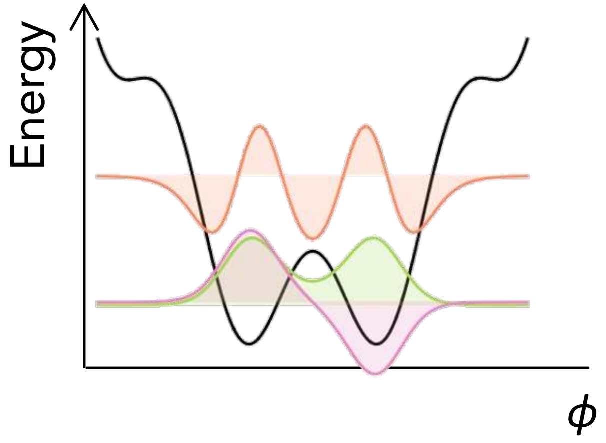
\phi



Is it possible to find optimal distributed circuit given $\mathcal{H}$ and $\mathcal{D}$?

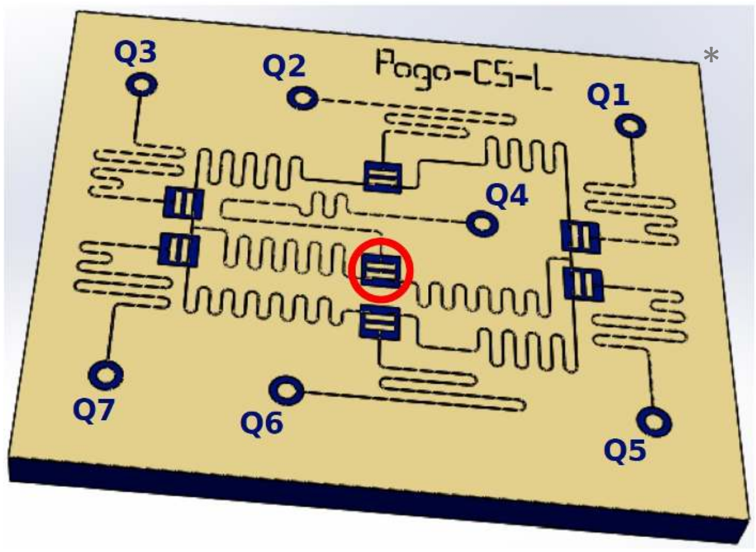






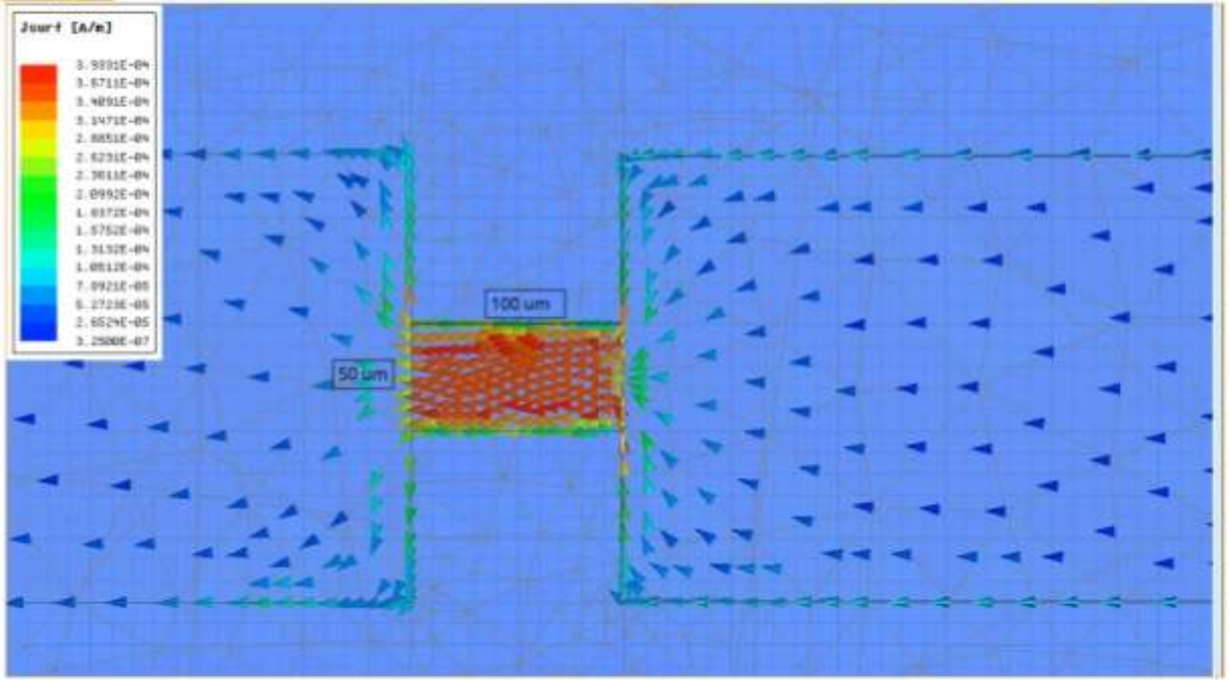
$$\hat{a}^\dagger \hat{b}^2 \, \xi(t)$$

...

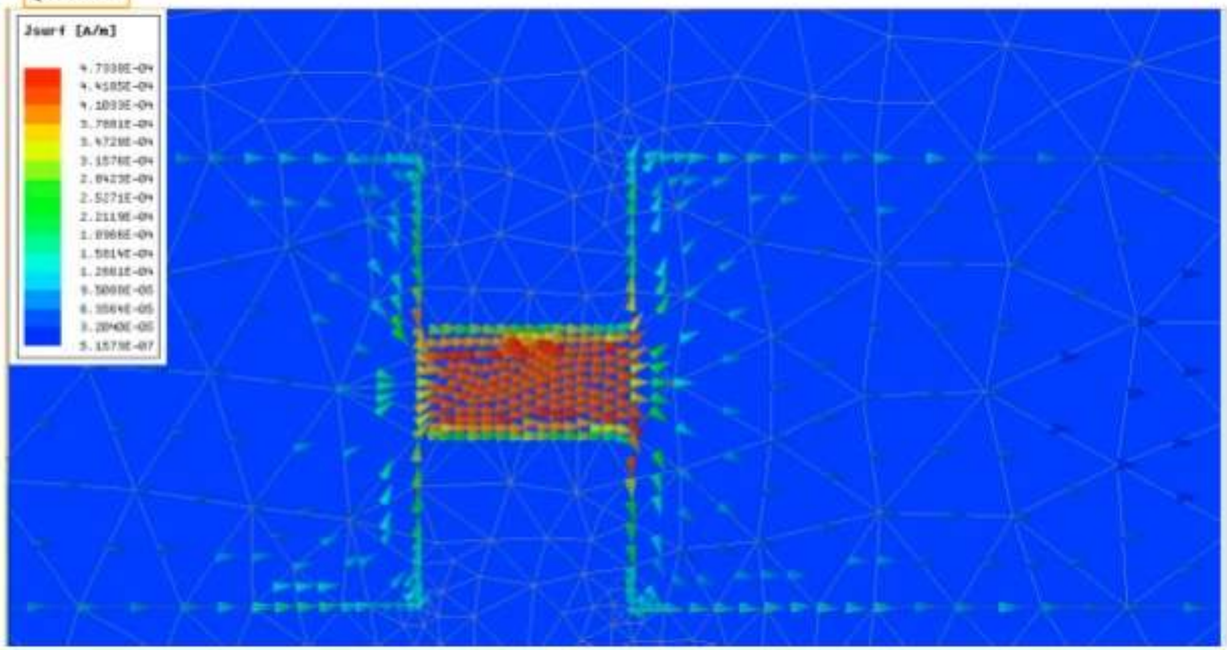


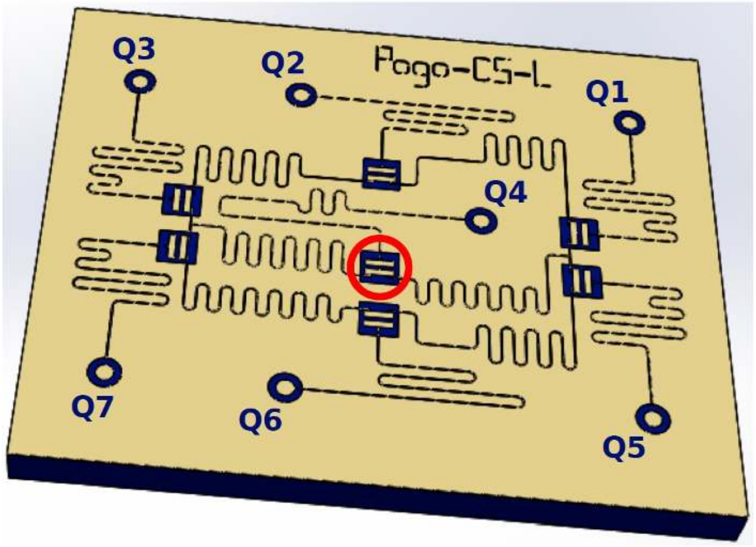
Images: Kandala *et al.*, Nature (2017), Bronn *et al.*, QST (2018)

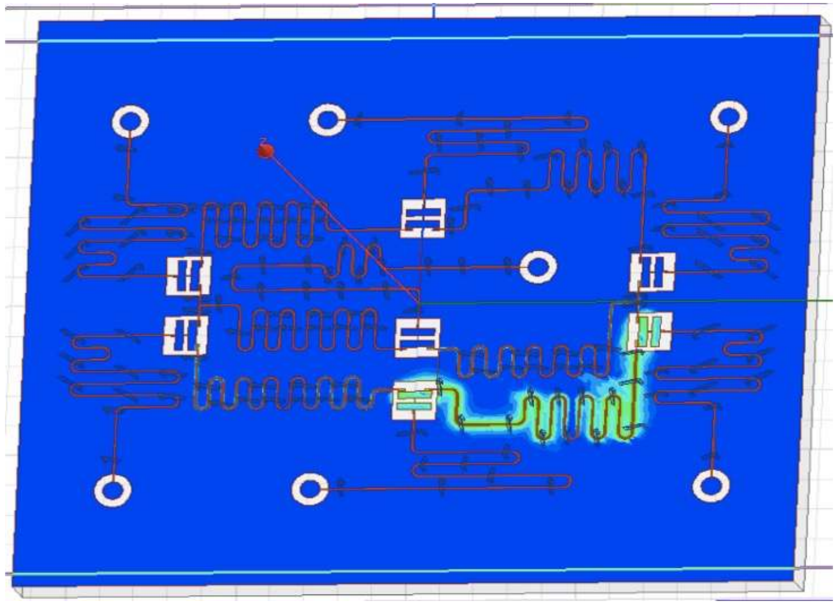
Cavity phase



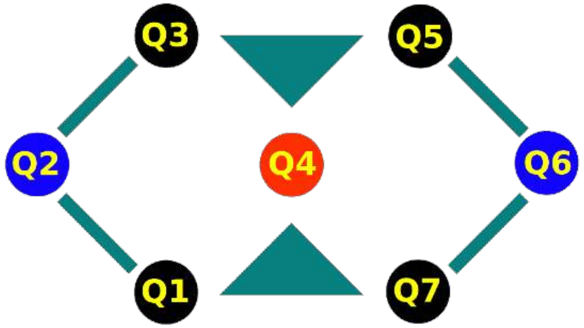
Qubit mode

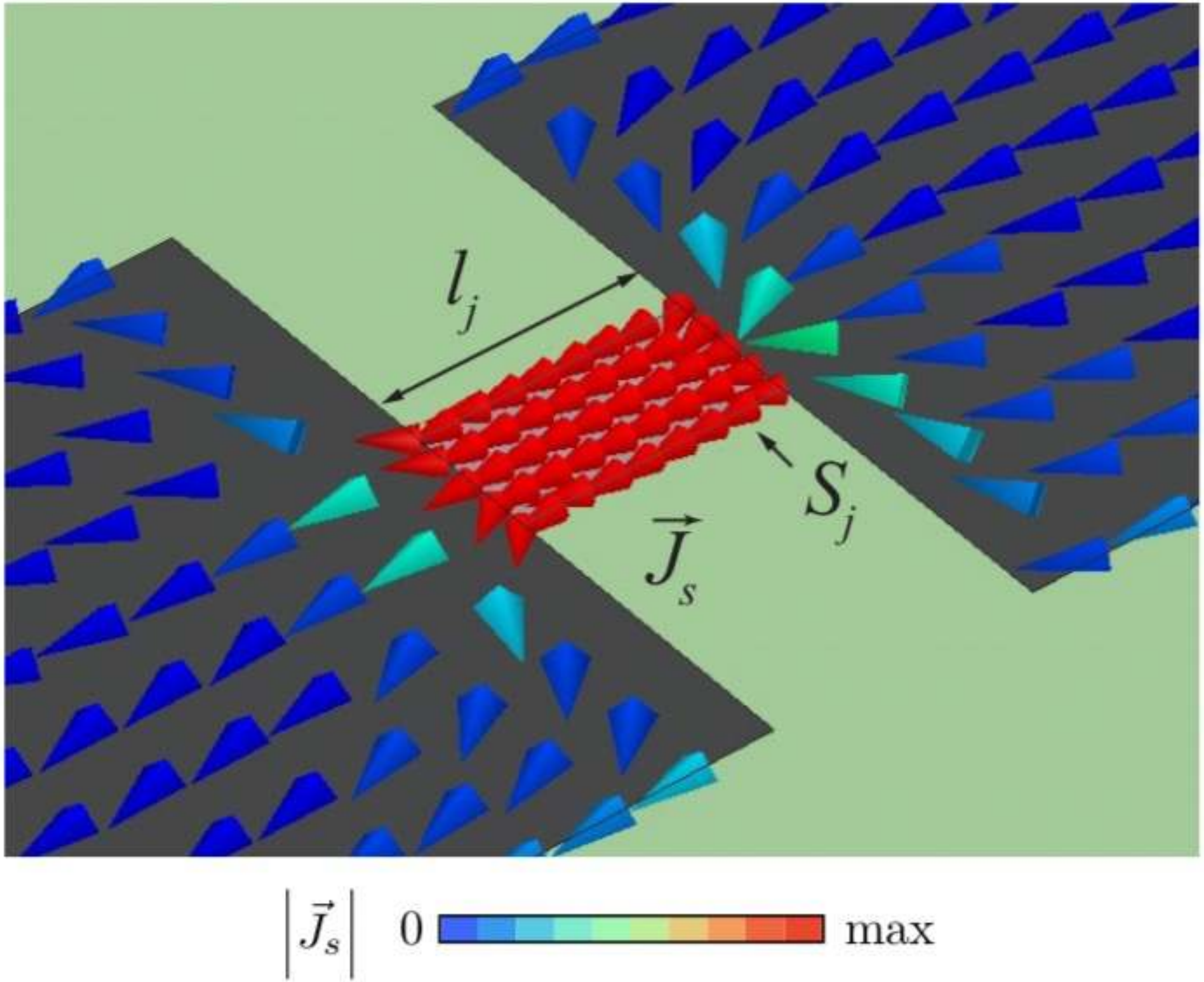






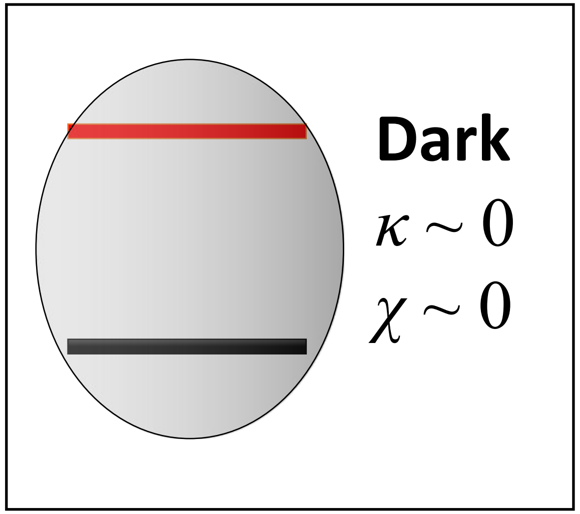
* Bronn, N.T. *et al.*, QST (2018)

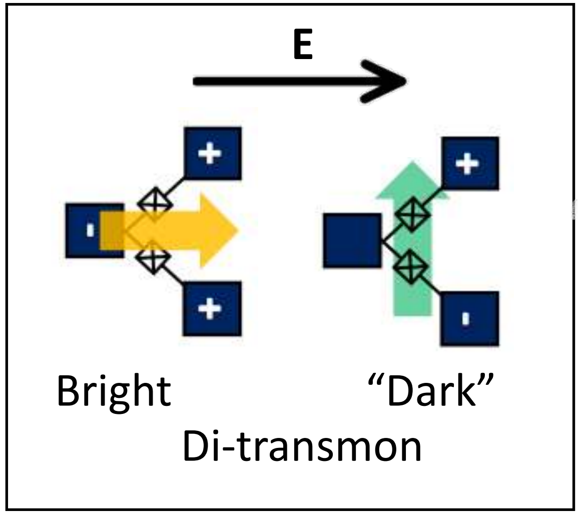


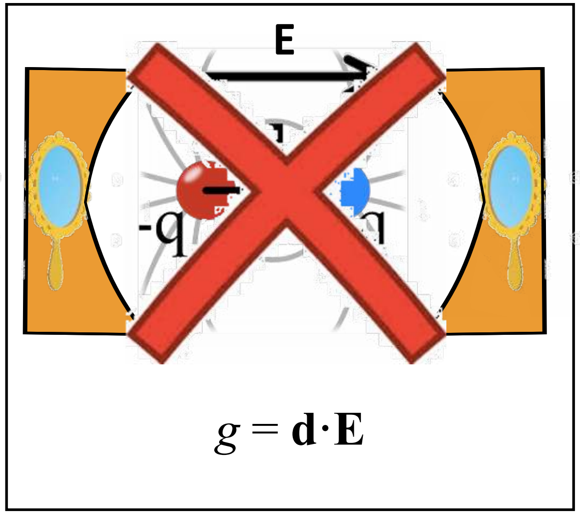


l_j
S_j
$\vec{J}_s$
$\left|\vec{J}_s\right|$ 0 max

Step 2: Zero-χ qubit, but large-χ bright level







Goal: Dark mode

$\kappa_r \sim 0$

$\chi_B \sim \chi_D$ (not 0!)

κ is sort of given by dipole coupling, but χ is absolutely <u>not</u>!

$$\chi_{mn} = \sum_{j=1}^{J} \frac{\hbar^2 \omega_m \omega_n}{4E_j} p_{mj} p_{nj}$$

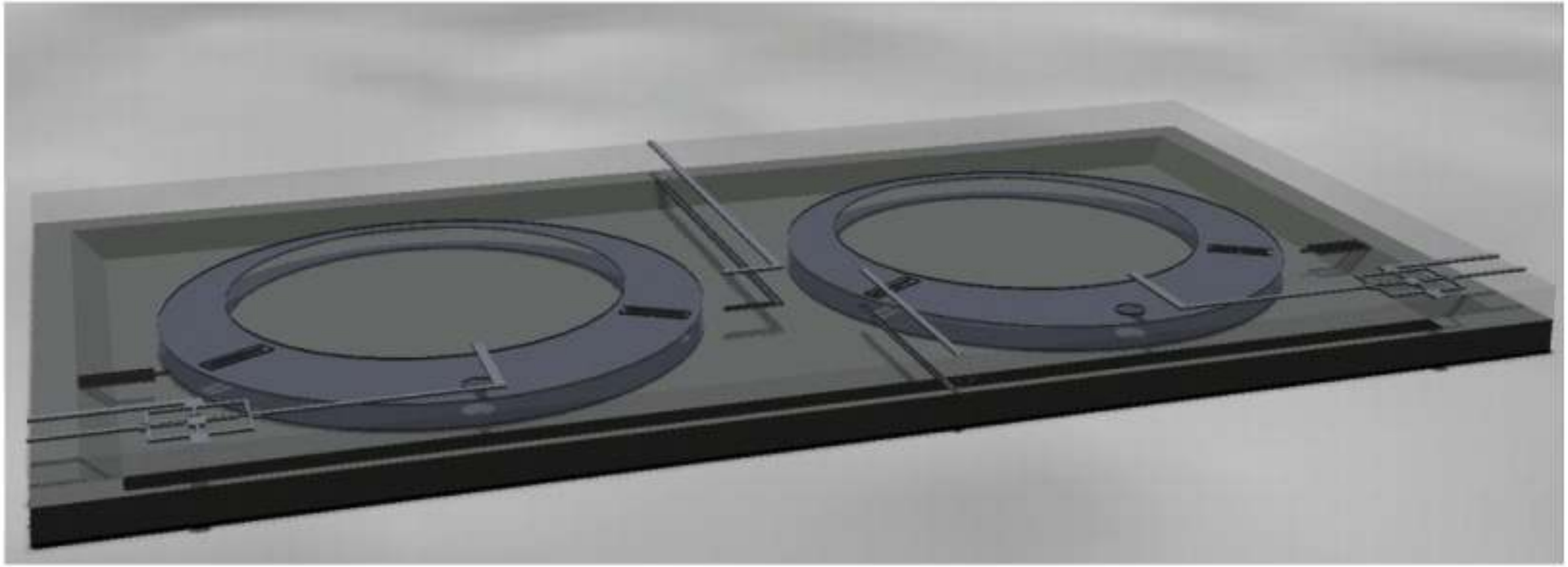
Mode

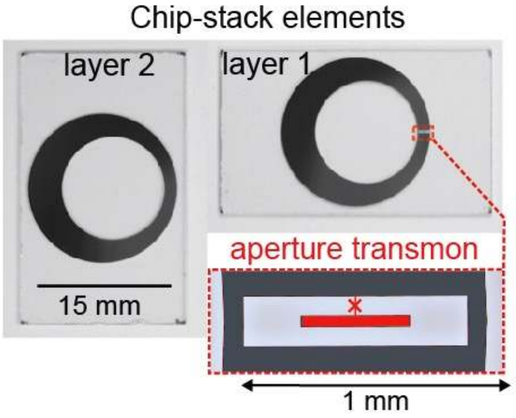
Junction

Fixed

Only thing to change

Energy participation overlap




Chip-stack elements
layer 2
layer 1
15 mm
aperture transmon
1 mm

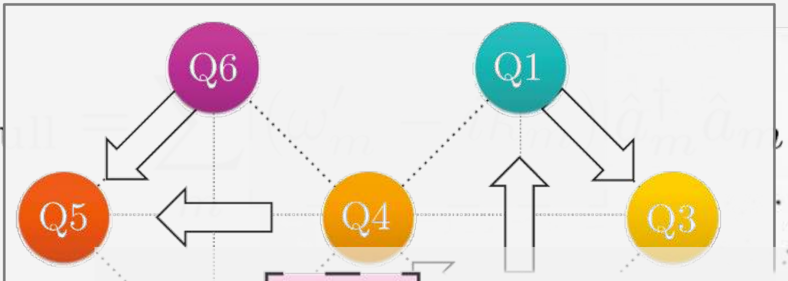
reduced to

What fraction of the energy of a mode is stored in the junction?

Quantum-classical bridge: Quantization

bosonic modes coupled to environment

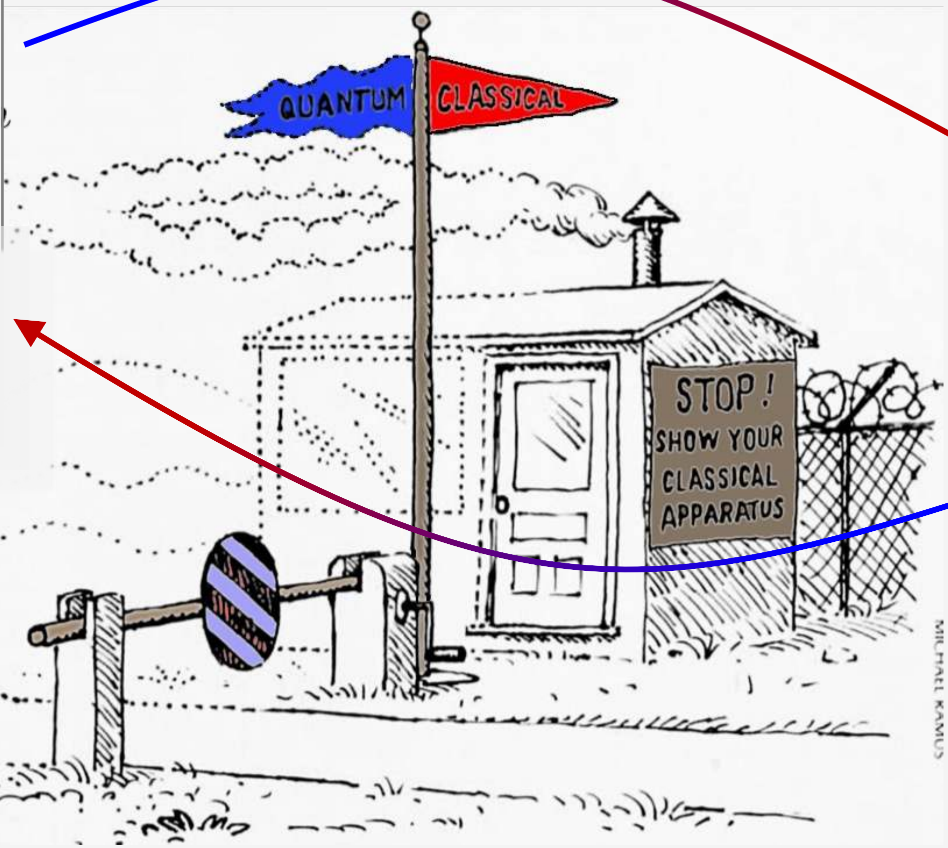
quantum system



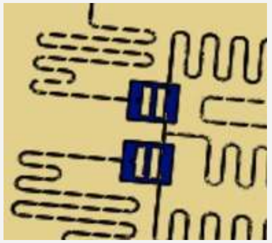
interactions

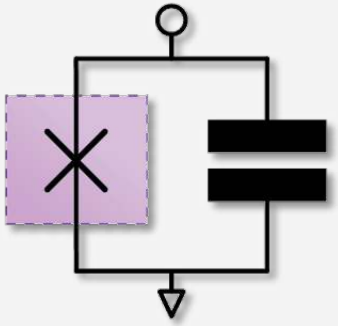
$$+\sum_{\boldsymbol{\alpha},\boldsymbol{\beta}} \boxed{\mathcal{C}^{p}_{\boldsymbol{\alpha},\boldsymbol{\beta}}} \prod_{m,n} \hat{a}_m^{\dagger\beta_m} \hat{a}_n^{\alpha_n}$$

$$\boxed{\chi, \delta, \alpha, \ldots}$$



electromagnetic fields, circuits

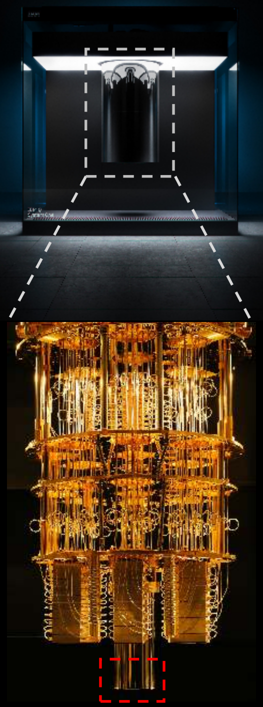


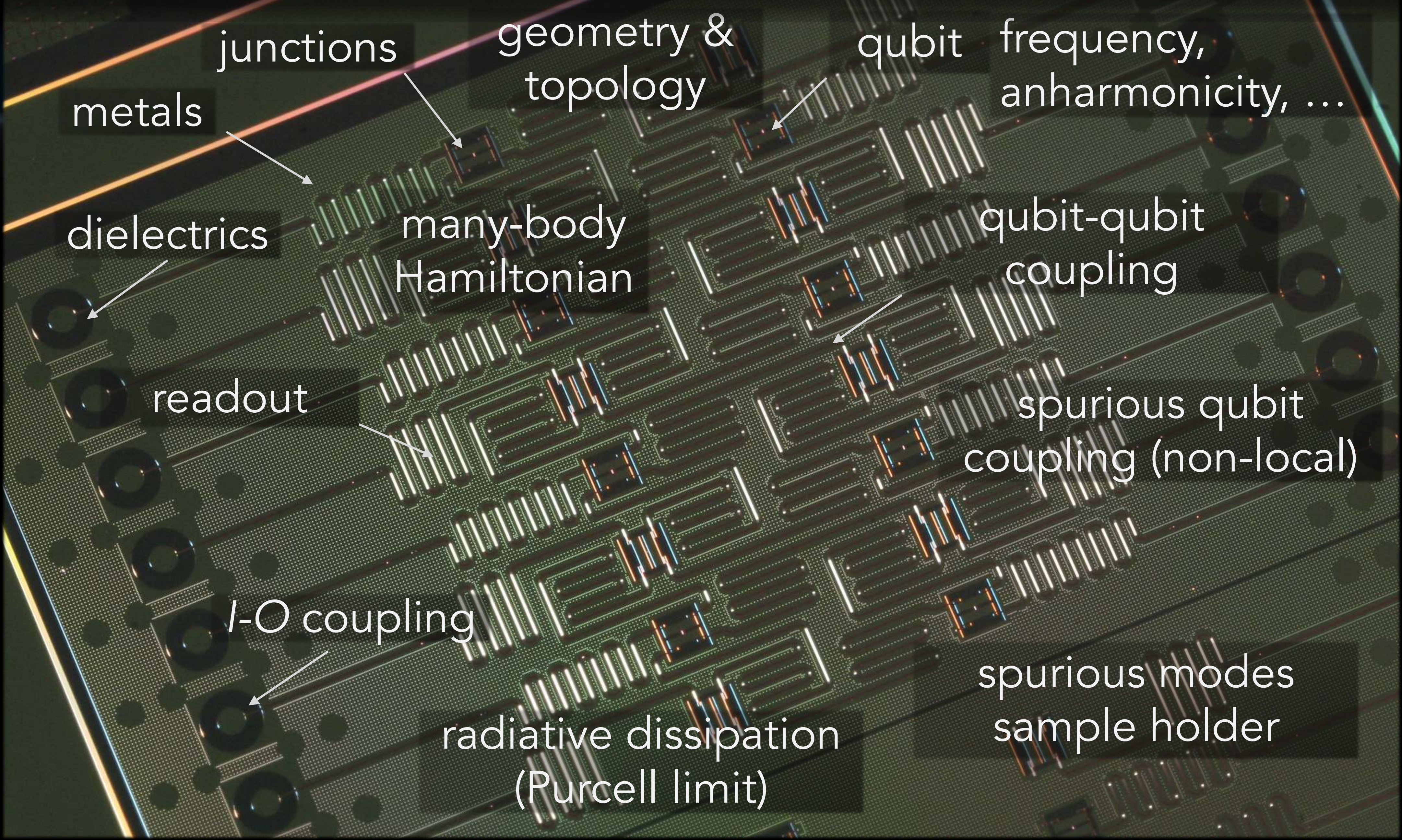


geometry, metals, dielectrics, …

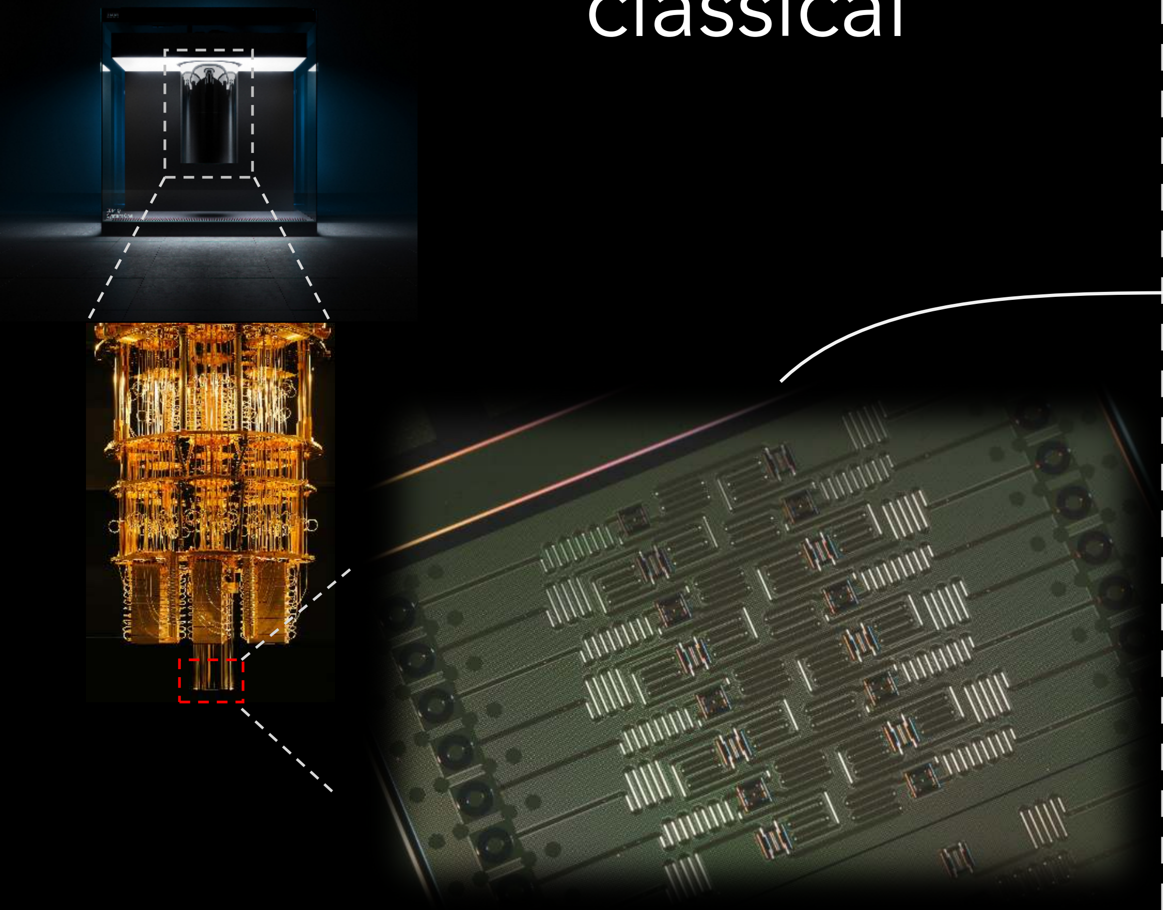
Line drawing: Zurek, Physics Today (1991); Image: Kandala *et al.*, Nature (2017), Bronn *et al.*, QST (2018)

To design …

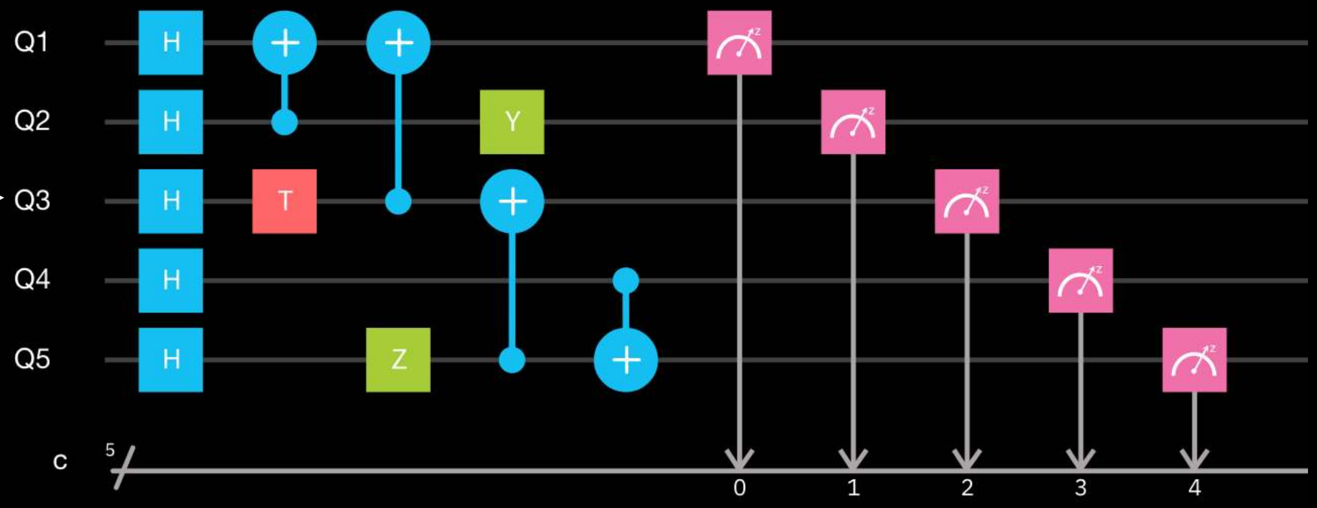


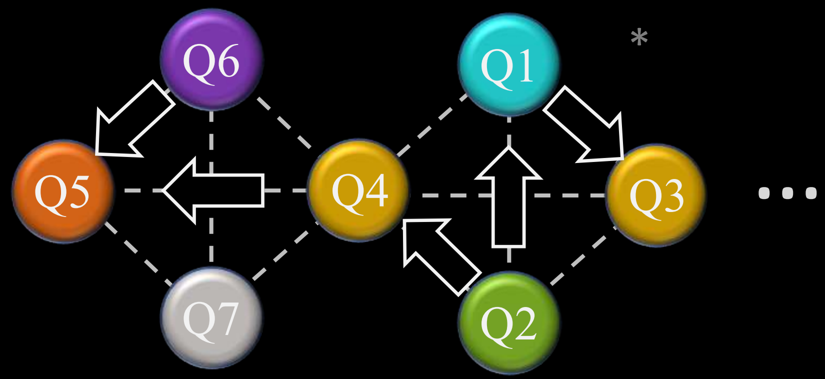

junctions
geometry & topology
qubit
frequency, anharmonicity, …
metals
dielectrics
many-body Hamiltonian
qubit-qubit coupling
readout
spurious qubit coupling (non-local)
I-O coupling
radiative dissipation (Purcell limit)
spurious modes sample holder

classical



quantum





* IBM quantum experience

Review of ideal CR

Ideal

$$\hat{H}_{\mathrm{CR}} = \frac{1}{2}\omega_{zx}\hat{\sigma}_z\hat{\sigma}_x \ ,$$

$$\boxed{\tau_{\mathrm{CNOT}} = \frac{\pi}{2\omega_{zx}} \ ,}$$

Unitary

$$\hat{U}_{\mathrm{CR}}(\tau) = \cos\left(\frac{\omega_{zx}\tau}{2}\right)\hat{\sigma}_0\hat{\sigma}_0 - i\sin\left(\frac{\omega_{zx}\tau}{2}\right)\hat{\sigma}_z\hat{\sigma}_x$$

CNOT

$$\mathrm{CNOT} = \begin{array}{c} \\ gg \\ ge \\ eg \\ ee \end{array} \begin{pmatrix} gg & ge & eg & ee \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\mathrm{CNOT} = [ZI]^{-1/2}[ZX]^{1/2}[IX]^{-1/2}$$

On target

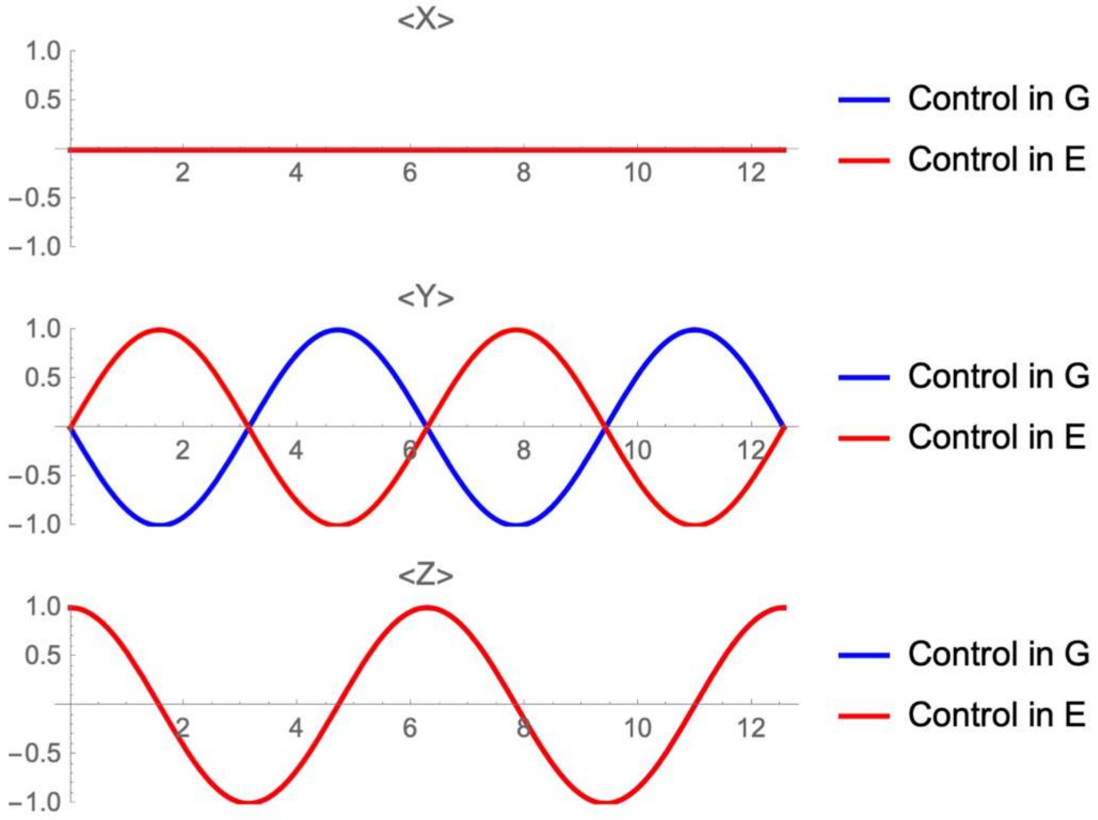


Figure 3. Ideal CR gate, ala Sheldon calibration.

Review of ideal CR

Ideal

$$\hat{H}_{\text{CR}} = \frac{1}{2}\omega_{zx}\hat{\sigma}_z\hat{\sigma}_x \,,$$

$$\boxed{\tau_{\text{CNOT}} = \frac{\pi}{2\omega_{zx}} \,,}$$

Unitary

$$\hat{U}_{\text{CR}}(\tau) = \cos\left(\frac{\omega_{zx}\tau}{2}\right)\hat{\sigma}_0\hat{\sigma}_0 - i\sin\left(\frac{\omega_{zx}\tau}{2}\right)\hat{\sigma}_z\hat{\sigma}_x$$

Spurious IX term

$$\hat{H}_{\text{CR}} = \frac{1}{2}\omega_{zx}\hat{\sigma}_z\hat{\sigma}_x + \frac{1}{2}\omega_{ix}\hat{\sigma}_0\hat{\sigma}_x$$

$$\begin{aligned}\hat{U}_{\text{CR}}(\tau) = {}& \cos\left(\frac{\tau\omega_{\text{ix}}}{2}\right)\cos\left(\frac{\tau\omega_{\text{zx}}}{2}\right)\hat{I}\hat{I} - i\cos\left(\frac{\tau\omega_{\text{ix}}}{2}\right)\sin\left(\frac{\tau\omega_{\text{zx}}}{2}\right)\hat{Z}\hat{X} \\ & - i\sin\left(\frac{\tau\omega_{\text{ix}}}{2}\right)\cos\left(\frac{\tau\omega_{\text{zx}}}{2}\right)\hat{I}\hat{X} - \sin\left(\frac{\tau\omega_{\text{ix}}}{2}\right)\sin\left(\frac{\tau\omega_{\text{zx}}}{2}\right)\hat{Z}\hat{I}\,,\end{aligned}$$

$$\frac{\omega_{zx}+\omega_{ix},}{2} \text{ and } \frac{\omega_{zx}-\omega_{ix}}{2}$$

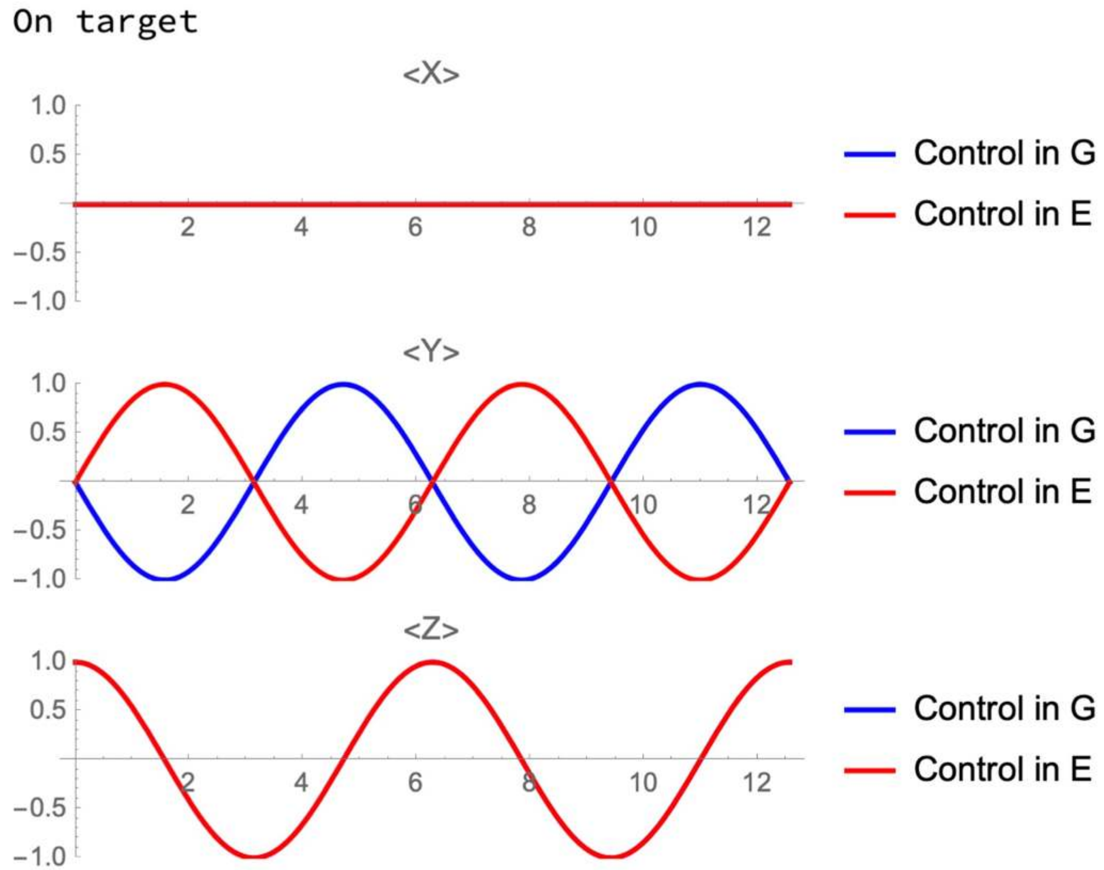
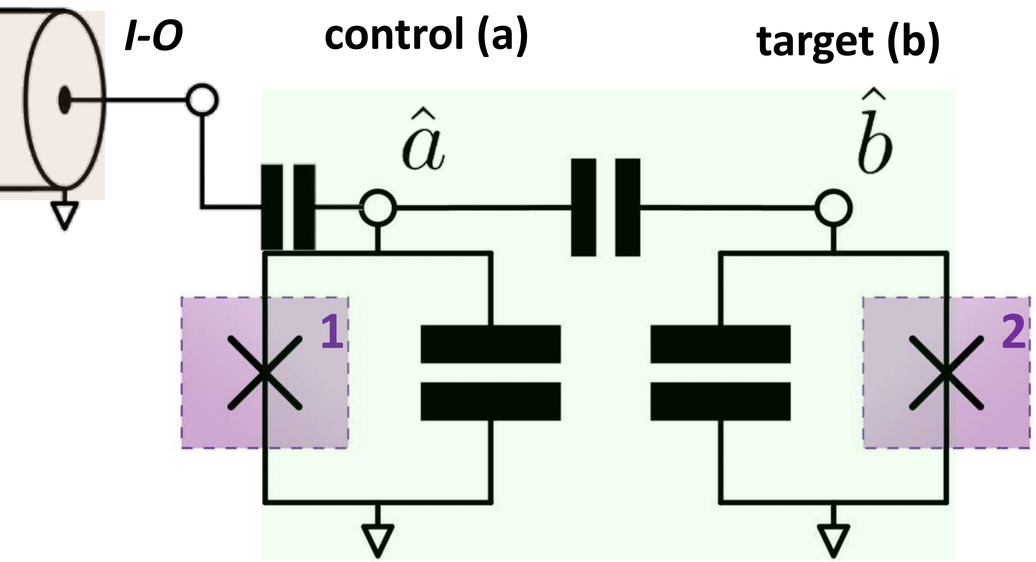


Figure 3. Ideal CR gate, ala Sheldon calibration.

Direct coupler: complete description



$$\hat{H} = \boxed{\omega_a \hat{a}^\dagger \hat{a} + \omega_a \hat{b}^\dagger \hat{b}} \quad \text{Eigen modes}$$

$$- \sum_{j=1}^{2} \frac{E_j}{4!} \left(\phi_{aj} \left(\hat{a} + \hat{a}^\dagger \right) + \phi_{bj} \left(\hat{b} + \hat{b}^\dagger \right) \right)^4 + \ldots \quad \text{Non-linear}$$

$$- i\epsilon_a(t) \left(\hat{a}^\dagger - \hat{a} \right) - i\epsilon_b(t) \left(\hat{b}^\dagger - \hat{b} \right) . \quad \text{Drives}$$

Regime of interest

$$\begin{pmatrix} p_{a1} & p_{a2} \\ p_{b1} & p_{b2} \end{pmatrix} \approx \begin{pmatrix} 0.92 & 2 \times 10^{-4} \\ 2 \times 10^{-4} & 0.92 \end{pmatrix}$$

$$\phi_{mj}^2 = p_{mj} \frac{\hbar \omega_m}{2E_j}$$

χs that are small, $\approx$200 KHz,

First approximations (removed later):

Ignore target junction? ($E_2 \rightarrow 0$)

YES, for ZX, no for IX.

Ignore target direct drive? ($\varepsilon_b \rightarrow 0$)

YES, for ZX, no for IX.

second quantization in eigen basis of linearized circuit

Direct coupler: control junction, no CX talk

Coupled System Hamiltonian (drive-free)

$$\hat{H}_{4,\mathrm{RWA1}} = H_a + H_b + H_{ab} = (\omega_a - \Delta_a)\,\hat{a}^\dagger\hat{a} - \frac{\alpha_a}{2}\hat{a}^{\dagger 2}\hat{a}^2$$

Control single-mode

$$+ (\omega_a - \Delta_b)\,\hat{b}^\dagger\hat{b} - \frac{\alpha_b}{2}\hat{b}^{\dagger 2}\hat{b}^2$$

Target single-mode

$$- \chi_{ab} a^\dagger a b^\dagger b\,,$$

Control-target

amplitudes

$$\Delta_{a/b} = \alpha_{a/b} + \tfrac{1}{2}\chi_{ab}\,. \quad \alpha_m = \frac{1}{2}\sum_{j=1}^{J=2} E_j \phi_{mj}^4 \qquad \chi_{ab} = \frac{1}{2}\sum_{j=1}^{J=2} E_j \phi_{aj}^2 \phi_{bj}^2$$

{p=4,RWA1}

Important corrections to {p=4,RWA2} and {p=6, RWA1}

Driven

$$\xi(t) = \frac{\epsilon}{\Delta_{ad}} e^{i\omega_b t}$$

$$\hat{\phi}_{j,\mathrm{disp}} = \phi_a \left(\hat{a} e^{-i\omega_a t} + \hat{a}^\dagger e^{i\omega_a t} + \xi(t) + \xi^*(t)\right) + \phi_b \left(\hat{b} e^{-i\omega_b t} + \hat{b}^\dagger e^{i\omega_b t}\right)$$

CR Gate: weakly anharmonic system

{p=4, RWA1}

$$
\begin{aligned}
\hat{H}_{\mathrm{CR},p=4,\mathrm{RWA1}} = & \boxed{-\frac{E_j}{4!}24\phi_a^3\phi_b\left[\left(\hat{a}^\dagger\hat{a}-\frac{1}{2}\right)\left(\xi\hat{b}+\xi^*\hat{b}^\dagger\right)\right]} && \text{CR Gate Term (ZX)!} && \text{(II.4)}\\
& -\frac{E_j}{4!}24\left|\xi\right|^2\left[\phi_a^4\hat{a}^\dagger\hat{a}+\phi_a^2\phi_b^2\hat{b}^\dagger\hat{b}\right] && \text{Stark shifts } (Z) \\
& -\frac{E_j}{4!}12\xi\left(\left|\xi\right|^2\phi_a^3\phi_b+\phi_a\phi_b^3\right)\hat{b}+\mathrm{H.c.} && \text{Spurious target}_b \text{ X Y drive term (X,Y)!} \\
& -\frac{E_j}{4!}\left(6\xi^2\phi_a^2\phi_b^2\right)\hat{b}^{\dagger 2}+\mathrm{H.c.} && \text{Spurious target}_b \text{ two-photon term} \\
& -\frac{E_j}{4!}\left(12\xi\phi_a\phi_b^2\right)\hat{b}^\dagger\left(\hat{b}^\dagger\hat{b}\right)+\mathrm{H.c.} && \text{Spurious target}_b \text{ three-photon term}
\end{aligned}
$$

Two-level manifold

$$\left(\hat{a}^\dagger\hat{a}-\frac{1}{2}\hat{I}\right)\to-\frac{1}{2}\hat{\sigma}_z\hat{\sigma}_0\,,\qquad\left(\hat{b}^\dagger+\hat{b}\right)\to\hat{\sigma}_0\hat{\sigma}_x\,,\quad\left(\hat{a}^\dagger\hat{a}\right)\to\frac{1}{2}\left(\hat{\sigma}_0-\hat{\sigma}_z\right)\hat{\sigma}_0\,,$$

, $\omega_{zx}=\frac{J}{2\Delta}$, see Rigetti (2010),

CR Gate: weakly anharmonic system

{p=4, RWA1, TLS}

$$\begin{aligned}\hat{H} &= \frac{1}{2}\omega_{zx}\hat{\sigma}_z\hat{\sigma}_x && \text{CR Gate Term (ZX)}\\ &+ \frac{1}{2}\omega_{ix}\hat{\sigma}_0\hat{\sigma}_x && \text{Spurious target } (IX)\\ &+ \frac{1}{2}\omega_{zi}\left(\hat{1}-\hat{\sigma}_z\hat{\sigma}_0\right) + \frac{1}{2}\omega_{iz}\left(\hat{1}-\hat{\sigma}_0\hat{\sigma}_z\right) && \text{Stark shifts } (Z)\end{aligned}$$

For control junction:

$$\boxed{\omega_{zx} = -s_a s_b \frac{\sqrt{\hbar^4\omega_a^3\omega_b}}{4E_j}\sqrt{p_a^3 p_b}\xi} \approx \sqrt{p_b} \times \frac{1}{2}\,\mathrm{GHz}\,.$$

for multiple junctions just sum this expression over j.

$\phi_b \ll \phi_a$,

$$\omega_{zx} = E_j\phi_a^3\phi_b\xi \qquad \omega_{zi} = -E_j\phi_a^4\left|\xi\right|^2 = -2\alpha_a\left|\xi\right|^2$$

$$\omega_{ix} \approx -E_j\phi_a^3\phi_b\xi\left|\xi\right|^2 \qquad \omega_{iz} = -E_j\phi_a^2\phi_b^2\left|\xi\right|^2 = -\chi_{ab}\left|\xi\right|^2$$

, $\omega_{zx} = \frac{J}{2\Delta}$, see Rigetti (2010),

CR Gate: weakly anharmonic system

{p=4, RWA1, TLS}

ϕa = 0.467873

p_a = 0.92

ωa = $2\pi \times 6000.$ MHz

E_J = $2\pi \times 12608.$ MHz,

$\alpha_{p=4,RWA1}$ = $2\pi \times 302.$ MHz

$\alpha_{p=4,RWA2}$ = $2\pi \times 65.$ MHz = 65. MHz

$\alpha_{p=6,RWA1}$ = $2\pi \times -33.$ MHz

α_{total} = $2\pi \times 334.$ MHz

Stark shift $(\xi+\xi^*)$ ξ=1: $2\pi \times \{604., 264., 56.\}$ MHz

----

$E_J = \begin{pmatrix} 12\,610. \\ 13\,660. \end{pmatrix}$ MHz

$\phi_{mj} = \begin{pmatrix} 0.468 & -0.007 \\ 0.007 & 0.442 \end{pmatrix}$

$\alpha_m^{(p=4,RWA1)} = \begin{pmatrix} 302. \\ 261. \end{pmatrix}$ MHz

$\chi_{mm'}^{(p=4,RWA1)} = \begin{pmatrix} 604.20 & 0.24 \\ 0.24 & 521.10 \end{pmatrix}$ MHz Diagonal is 2x anharr

(MHz)	$\xi_b = 0$	$\xi_b = 0.1$	Significant
$\omega_{xz}/2\pi$	8.76	8.78	N
$\omega_{iz}/2\pi$	-0.25	-3.9	so-so
$\omega_{ix}/2\pi$	-0.47	-27	Y
$\omega_{zi}/2\pi$	-604	-606	N

For $\xi = 1$. For standard params I used above too.

Conclusions / future directions

Direct coupling in EPR basis (M=2, J=2)

All qubit parameters & driven shifts

{p=4,RWA2} and {p=6,RWA1}

CR rate rate and spurious terms in {p=4,RWA1}

Numerically verified (understood which terms matter)

Direct coupling

{p=4,RWA2}. And {p=6, RWA1}

anharmonic basis

idea: back out J

Bus coupling (M=3, J=1)

{p=4,RWA1}

eliminate bus

{p=4,RWA2}. And {p=6, RWA1}

anharmonic basis