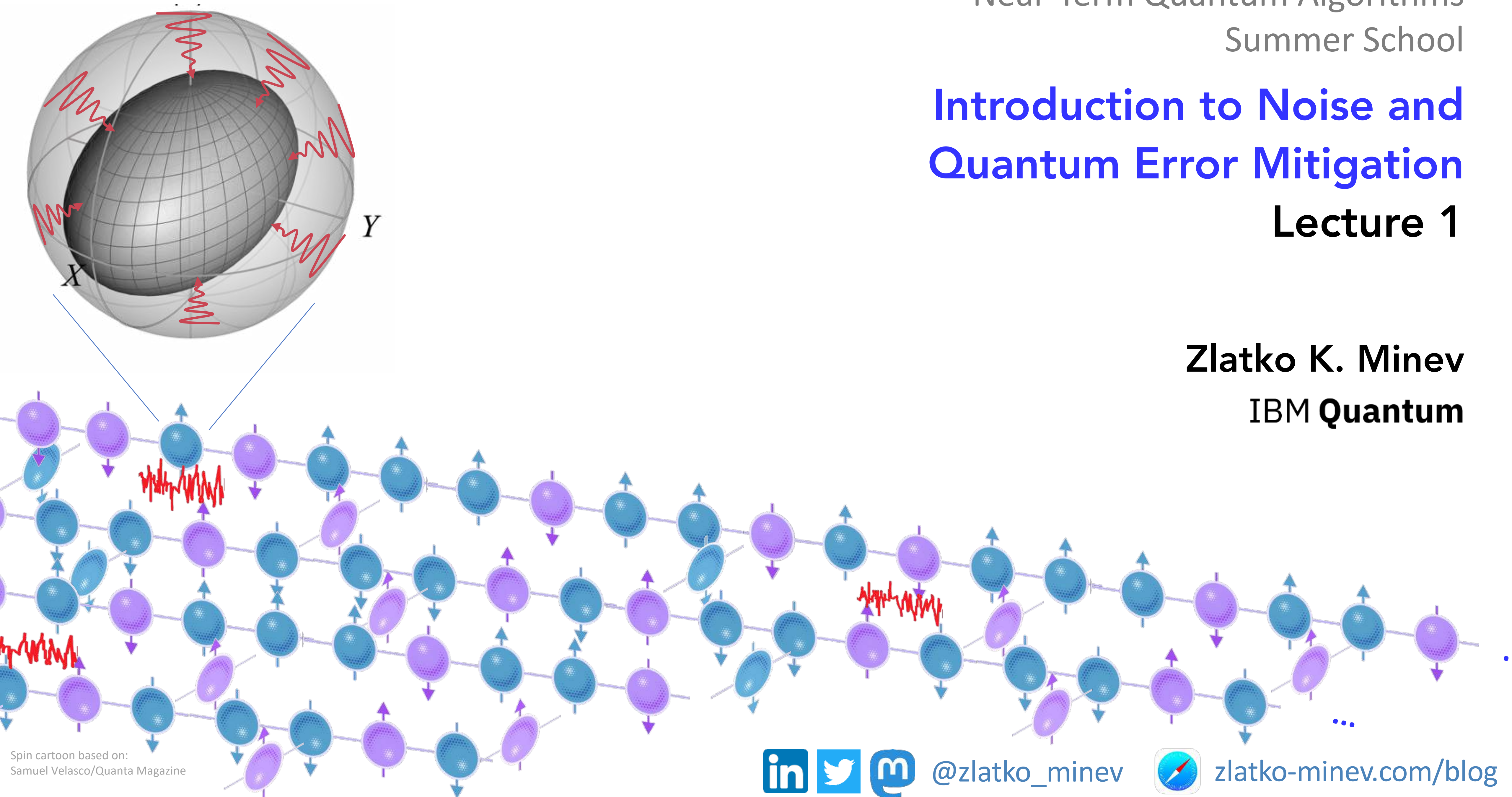
Near-Term Quantum Algorithms
Summer School

Introduction to Noise and Quantum Error Mitigation

Lecture 1



Spin cartoon based on:
Samuel Velasco/Quanta Magazine

Zlatko K. Minev
IBM Quantum

@zlatko_minev zlatko-minev.com/blog

Have you used

a quantum computer?

Image: Freepik

Quantum Noise and Error Mitigation

Lecture 1

Big picture

Quantum computers status

Why error mitigation?
- Noise in quantum computers
- Overview of error mitigation

Mitigation fundamentals

Probabilistic error cancelation (PEC)
- Introduction
- One qubit example

Next lecture

Learning noise
State-of-art mitigation experiments
Hardware
Outlook

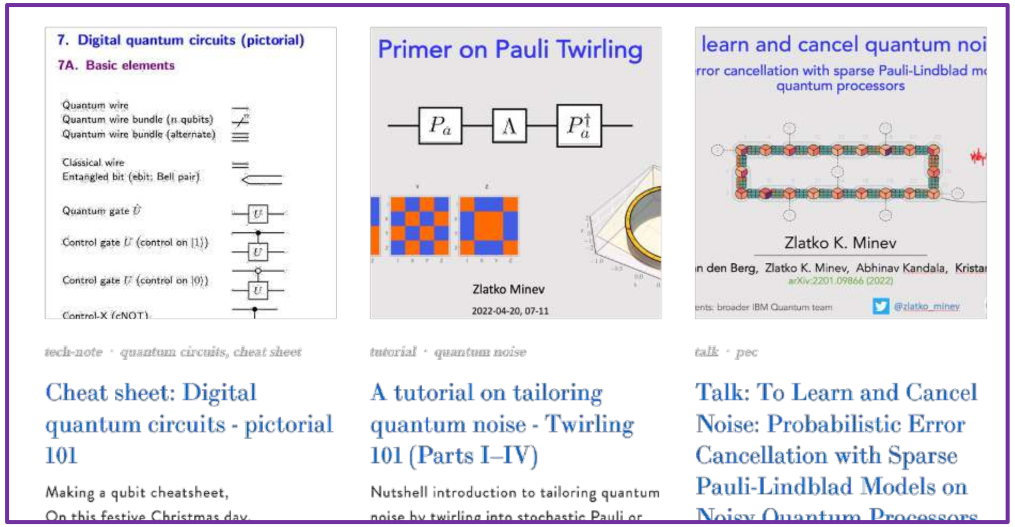
road based on: freepik; Monster image by jcomp on Freepik

Where?

Slides from lecture

See school slack

Will also post here zlatko-minev.com/education



Related

Lectures qiskit.org/learn

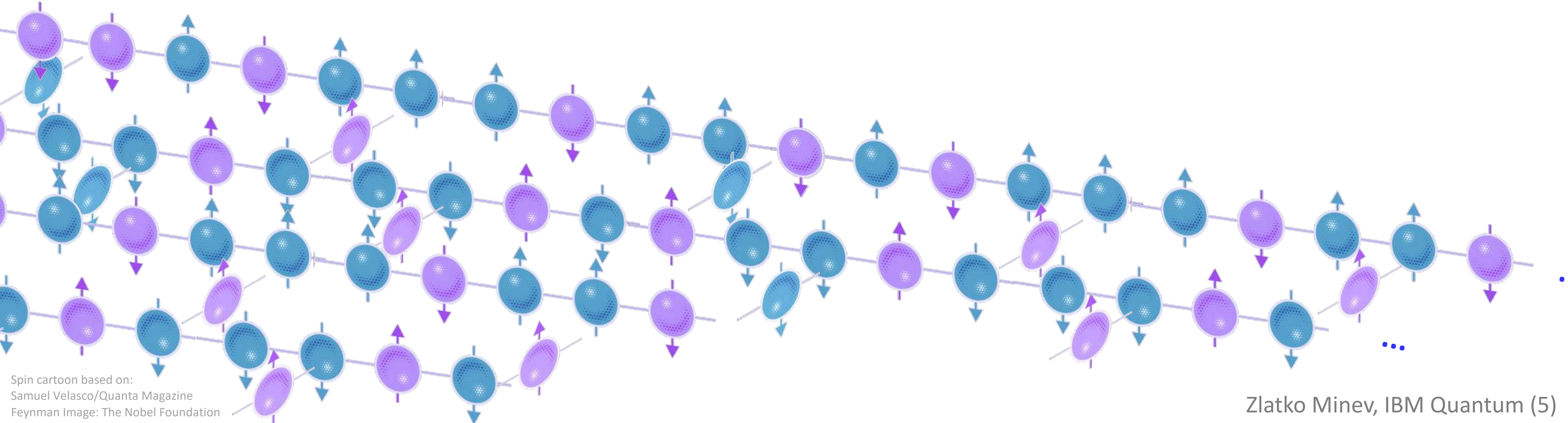
Tutorials zlatko-minev.com/blog

(Twirling, Measurements, Walsh-Hadamard, …)

Weekly seminars

qiskit.org/events/seminar-series

How is it going for quantum computers?



Spin cartoon based on:
Samuel Velasco/Quanta Magazine
Feynman Image: The Nobel Foundation

This year 2023

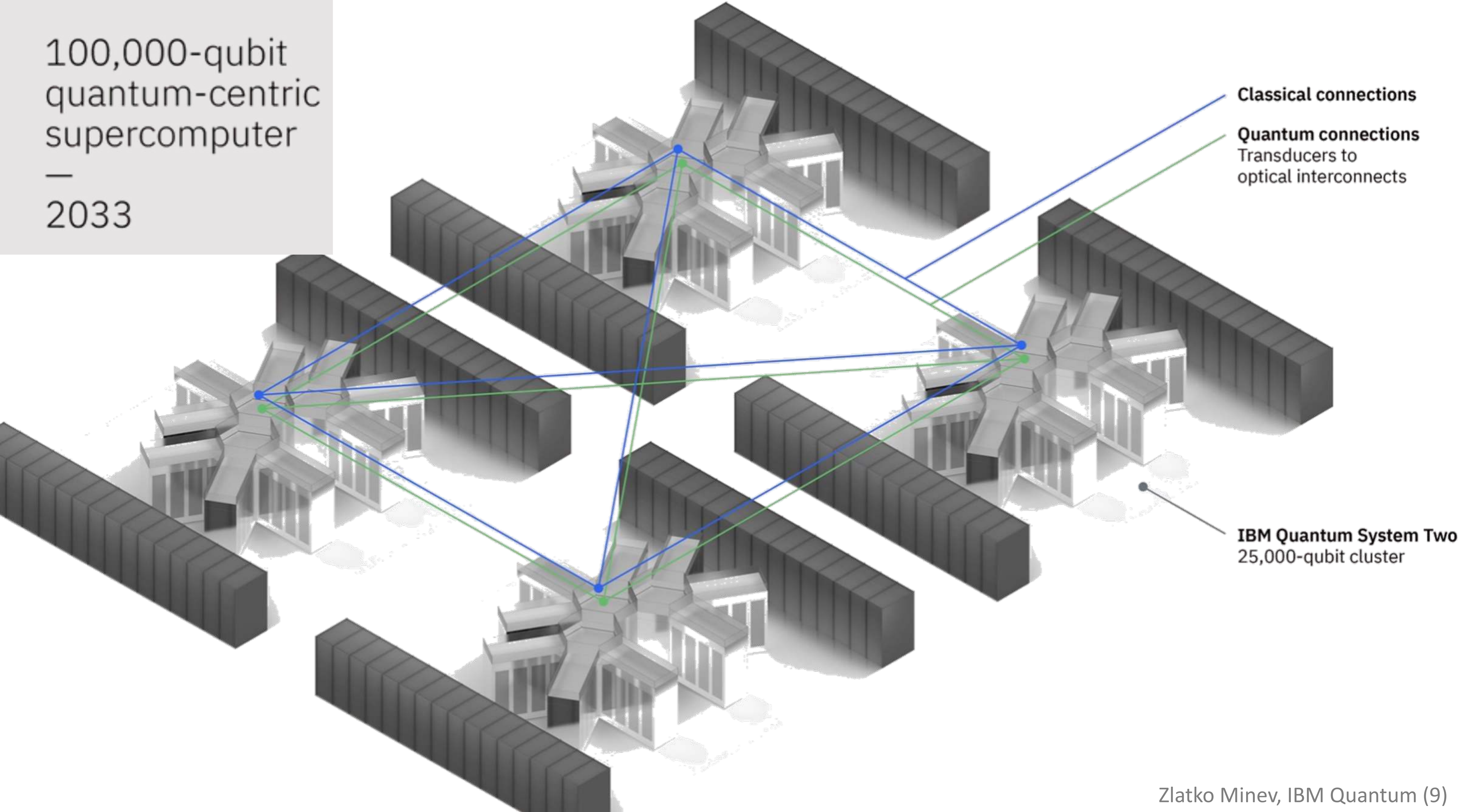
A 127-qubit quantum computer installed in the the lobby cafeteria of a research building dutifully executing jobs almost all the time.

at Cleveland Clinic

Credit: Connie Zhou for IBM

Image: Freepik

100,000-qubit quantum-centric supercomputer
—
2033



Back to today …

Example: Quantum simulation

Example task:

Simulate the out-of-equilibrium quantum dynamics of a 2D spin chain lattice to find the evolution of the global and local magnetization.

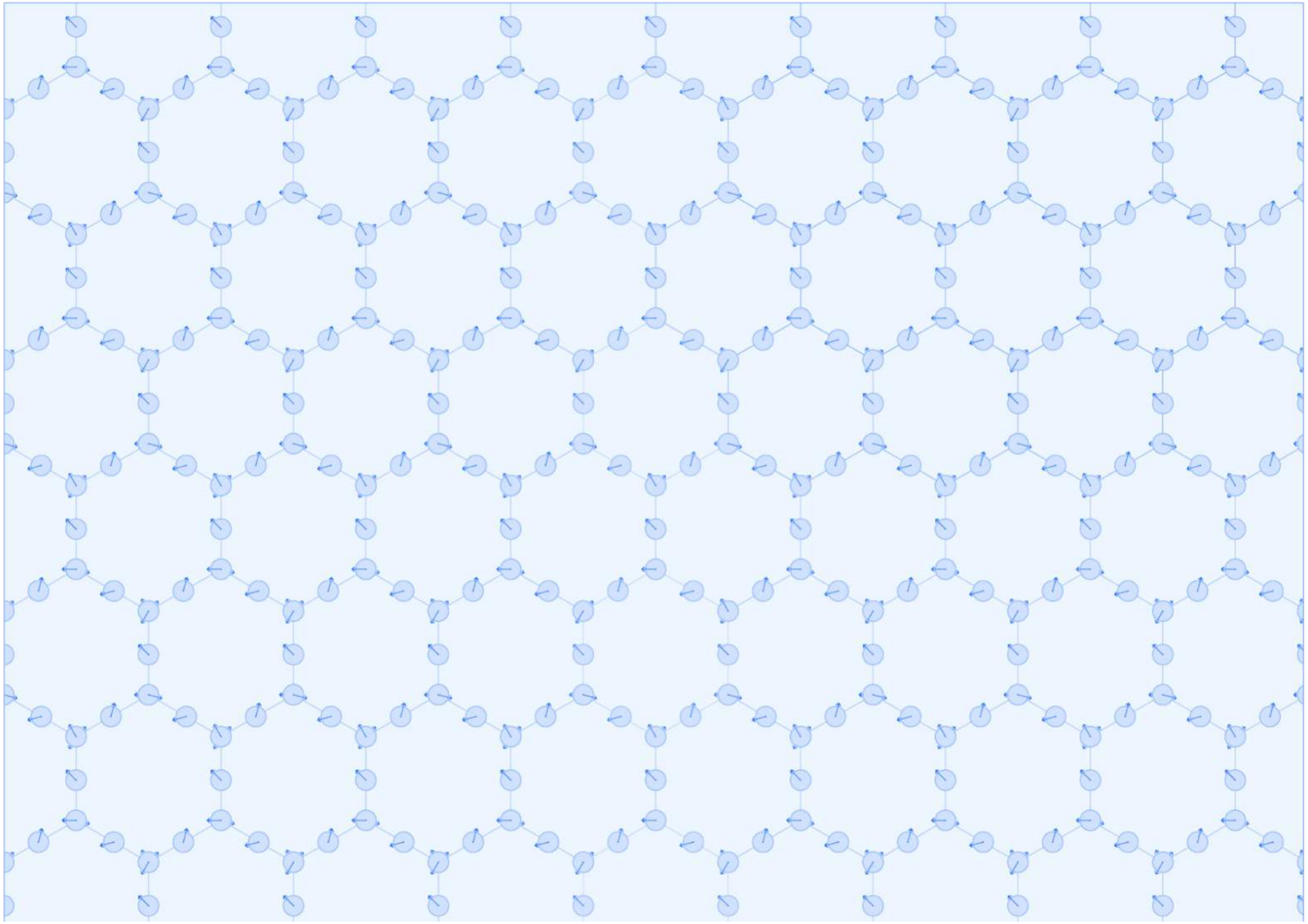


Image: IBM Quantum

Quantum simulation on a quantum computer

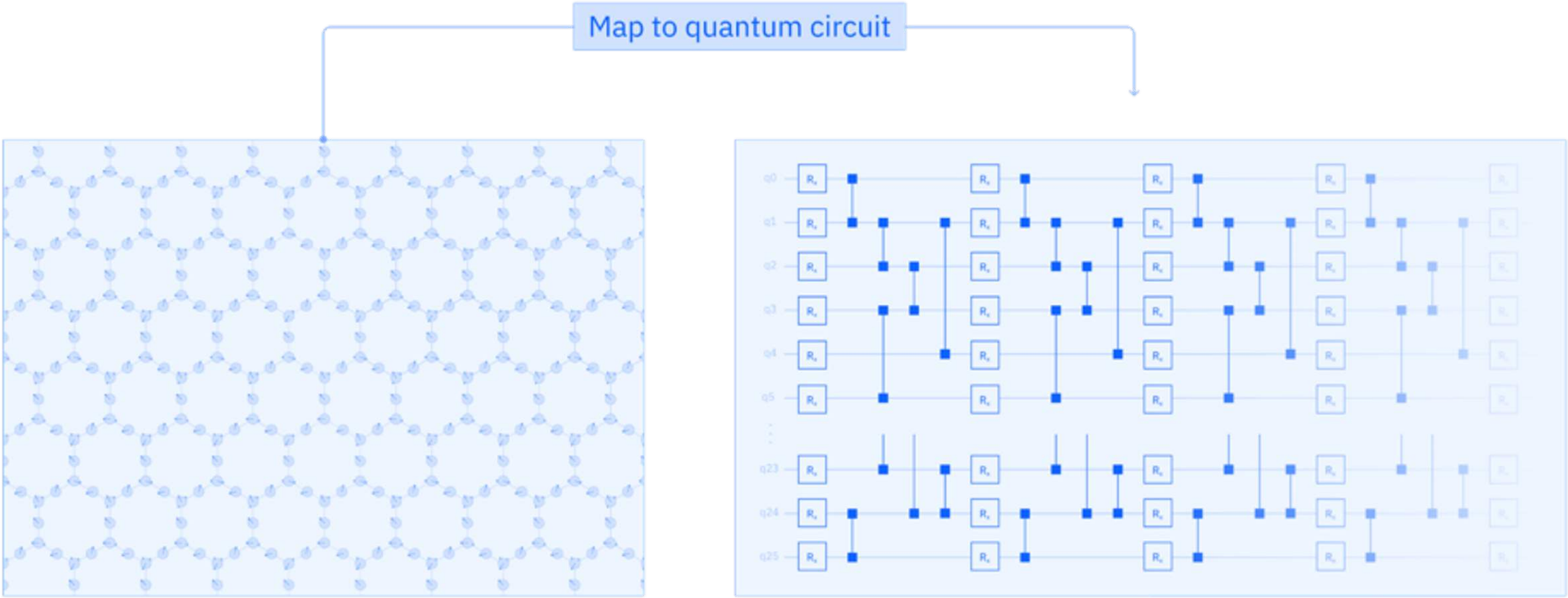


Image: IBM Quantum

Quantum simulation on a quantum computer

Execute on a real quantum computer device and obtain results as classical data

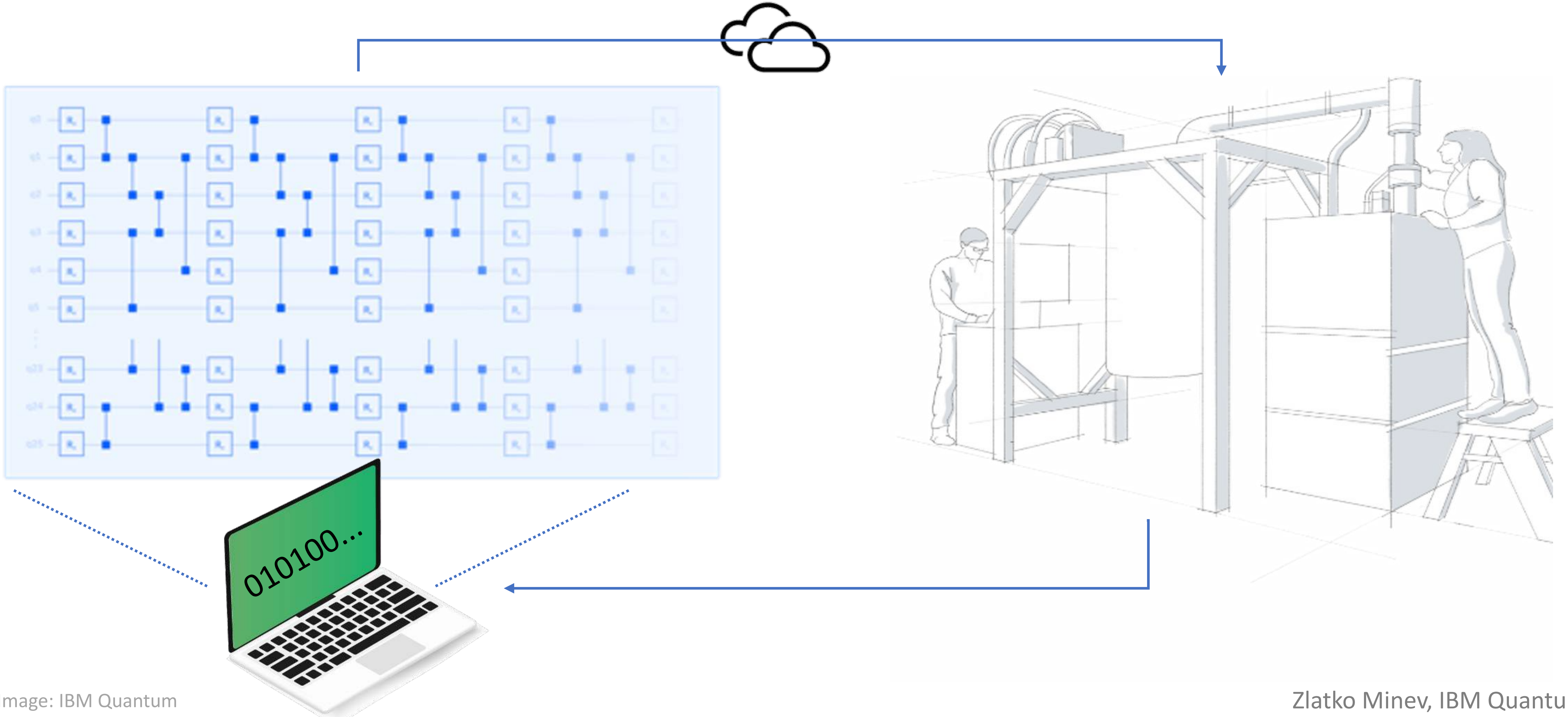


Image: IBM Quantum

Biggest challenge?

Please do share

hardware development

decoherence

loss

stability

heat

algo development

error correction overheads

Noise (Errors)

high error rates

scalability

Biggest challenge?

engineering

modularization

importance of N in NISQ

hype

need CS/EE talent

material quality

gravity

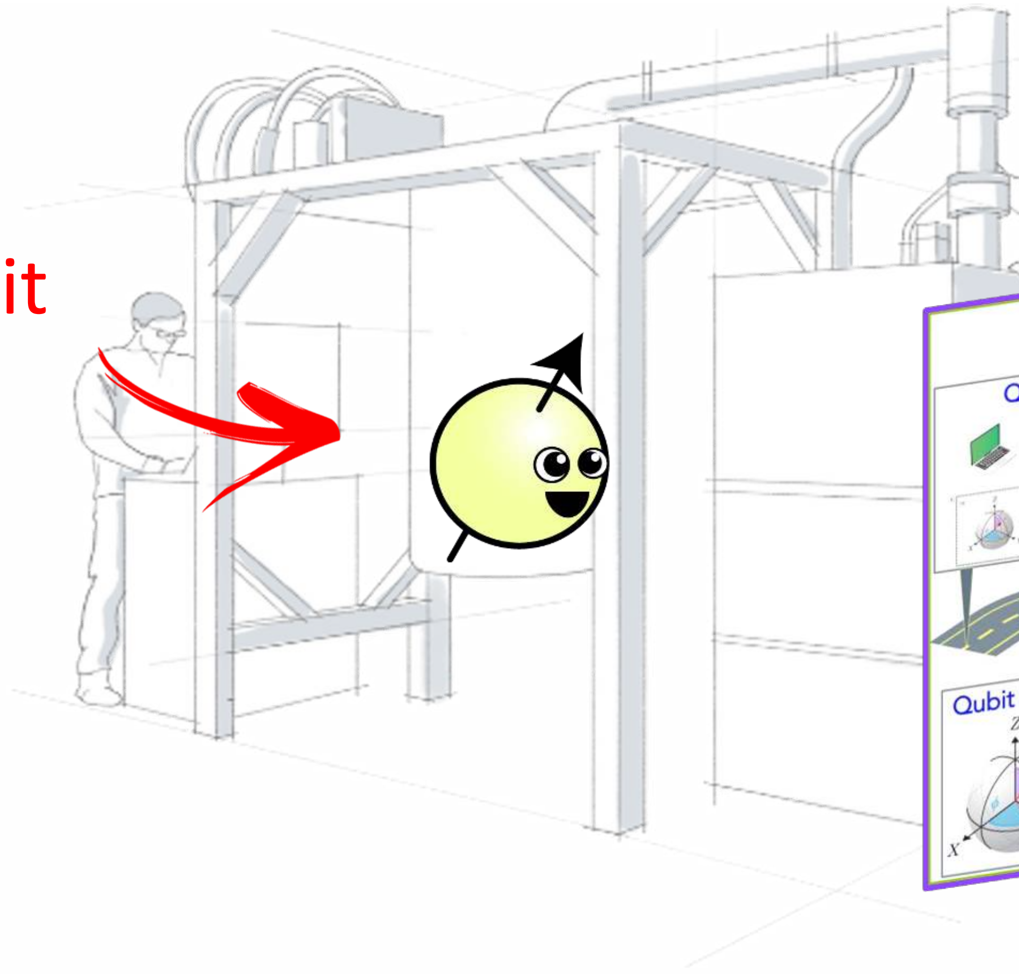
expectations

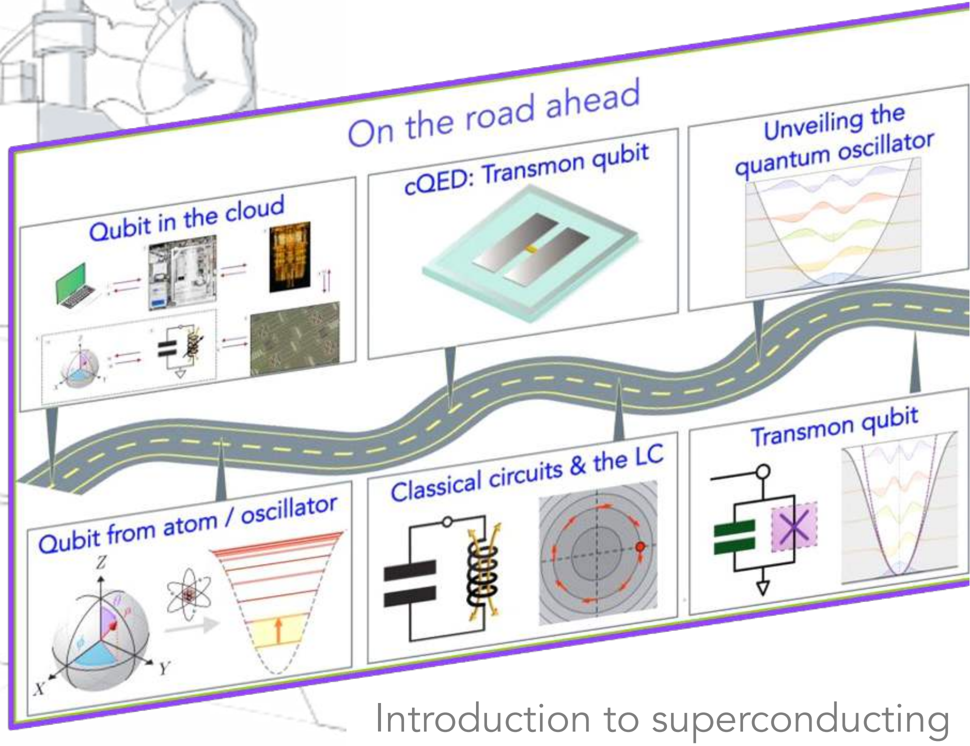
Biggest challenge

Noise
(Errors)

Hello World with a real experiment!

A qubit

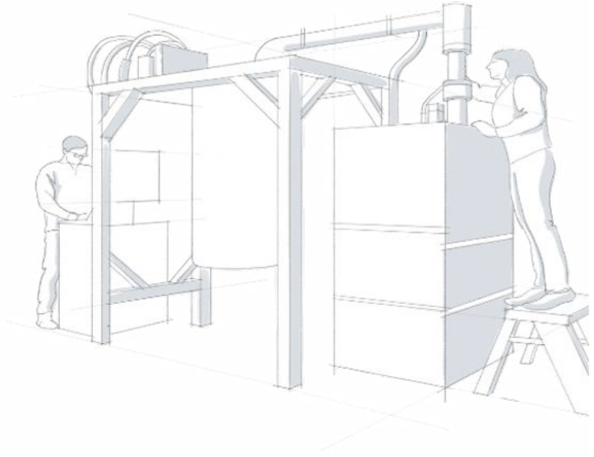




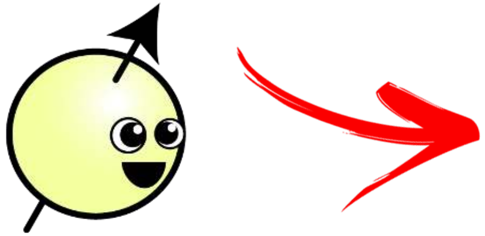
Introduction to superconducting qubits (cQED)
Lecs. 16-21 Minev
QGSS 2020 at qiskit.org/learn

image: qiskit

Hello World! building blocks



A qubit



$|1\rangle$

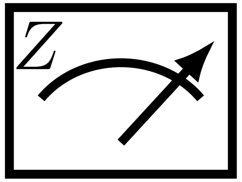
$|0\rangle$

Computational basis states

Operations: qubit gate

$\boxed{X}$

Measurements: qubit observable

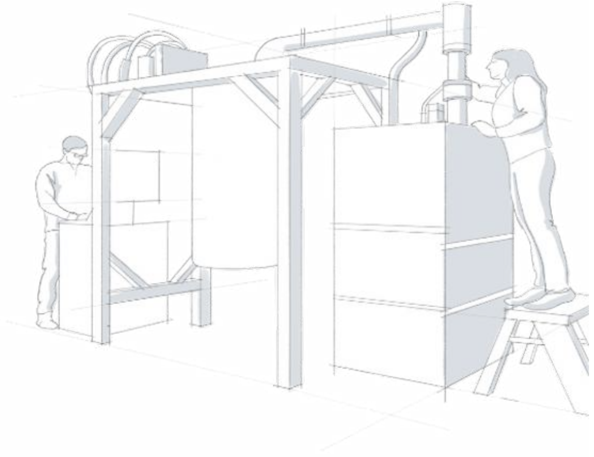


refresher:

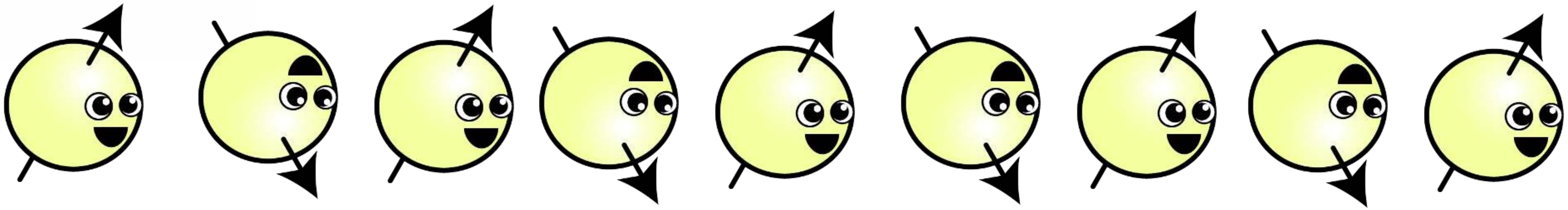
$$X\,|0\rangle = |1\rangle \qquad Z\,|0\rangle = +1\,|0\rangle$$

$$X\,|1\rangle = |0\rangle \qquad Z\,|1\rangle = -1\,|1\rangle$$

Hello World! Even-odd algo: qubit flipper



Task: Classify or report if a classical positive integer *d* is even or odd.



flip spin *d* times, measure polarization

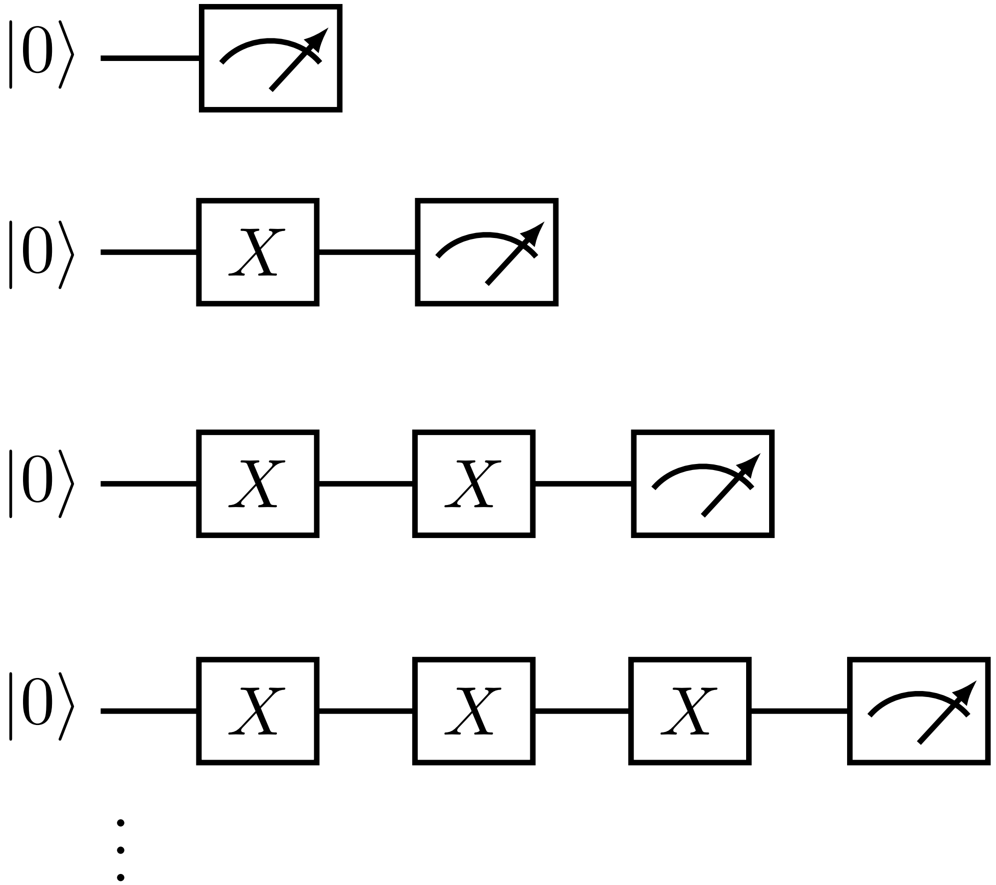
refresher:

$$X|0\rangle = |1\rangle \qquad Z|0\rangle = +1|0\rangle$$

$$X|1\rangle = |0\rangle \qquad Z|1\rangle = -1|1\rangle$$

Hello World! qubit flipper quantum circuits

depth

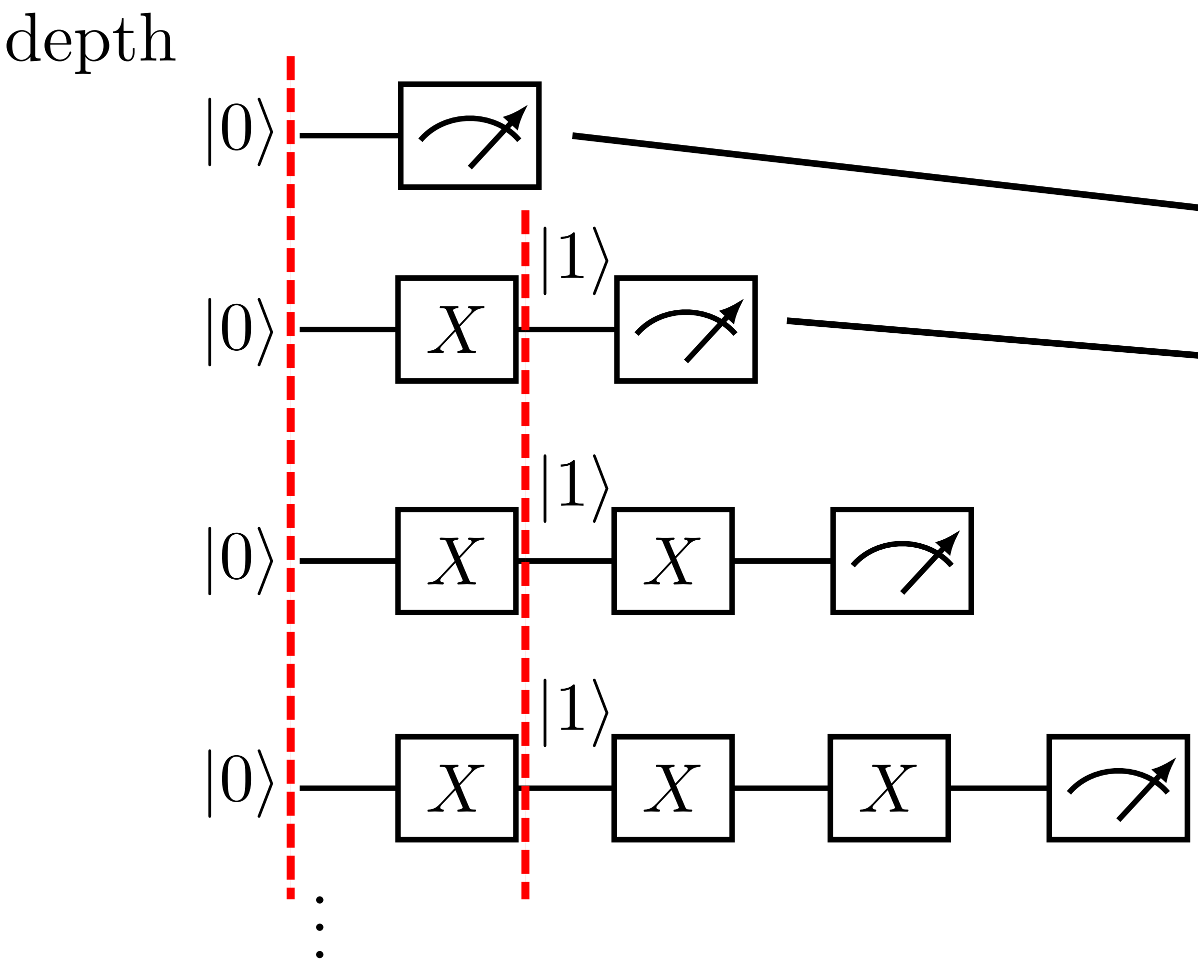


refresher:

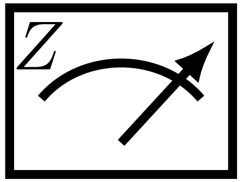
$$X|0\rangle = |1\rangle \qquad Z|0\rangle = +1|0\rangle$$

$$X|1\rangle = |0\rangle \qquad Z|1\rangle = -1|1\rangle$$

Hello World! “debugger” step through



Measurement



$$\langle 0|Z|0\rangle = +1$$

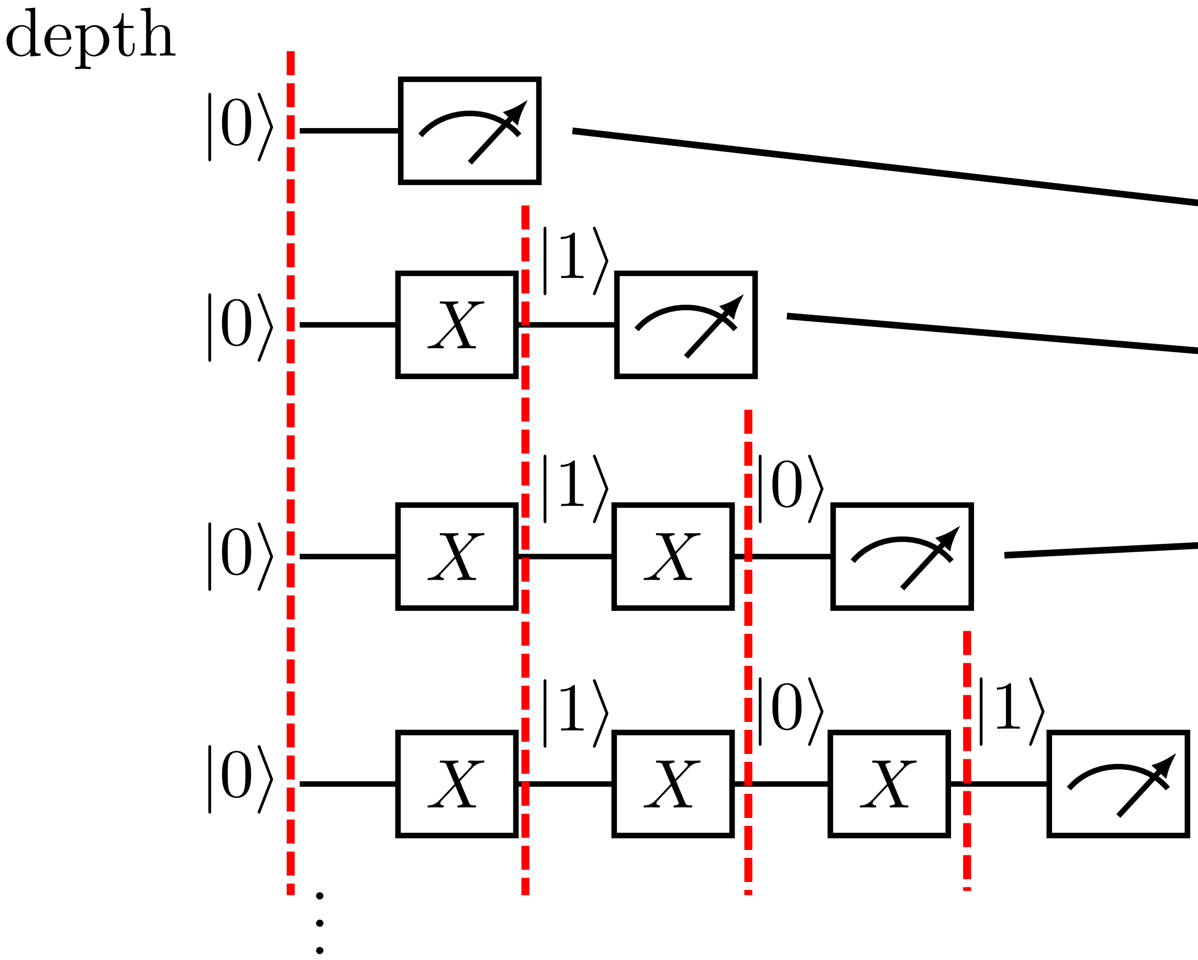
$$\langle 1|Z|1\rangle = -1$$

refresher:

$$X\,|0\rangle = |1\rangle \qquad Z\,|0\rangle = +1\,|0\rangle$$

$$X\,|1\rangle = |0\rangle \qquad Z\,|1\rangle = -1\,|1\rangle$$

Hello World! “debugger” step through



Measurement

$$\langle 0|Z|0\rangle = +1$$

$$\langle 1|Z|1\rangle = -1$$

$$\langle 0|Z|0\rangle = +1$$

$$\langle Z\rangle = (-1)^d$$

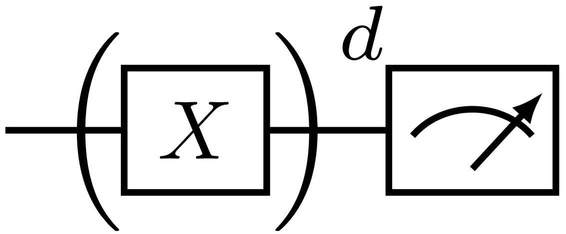
where d is the circuit depth

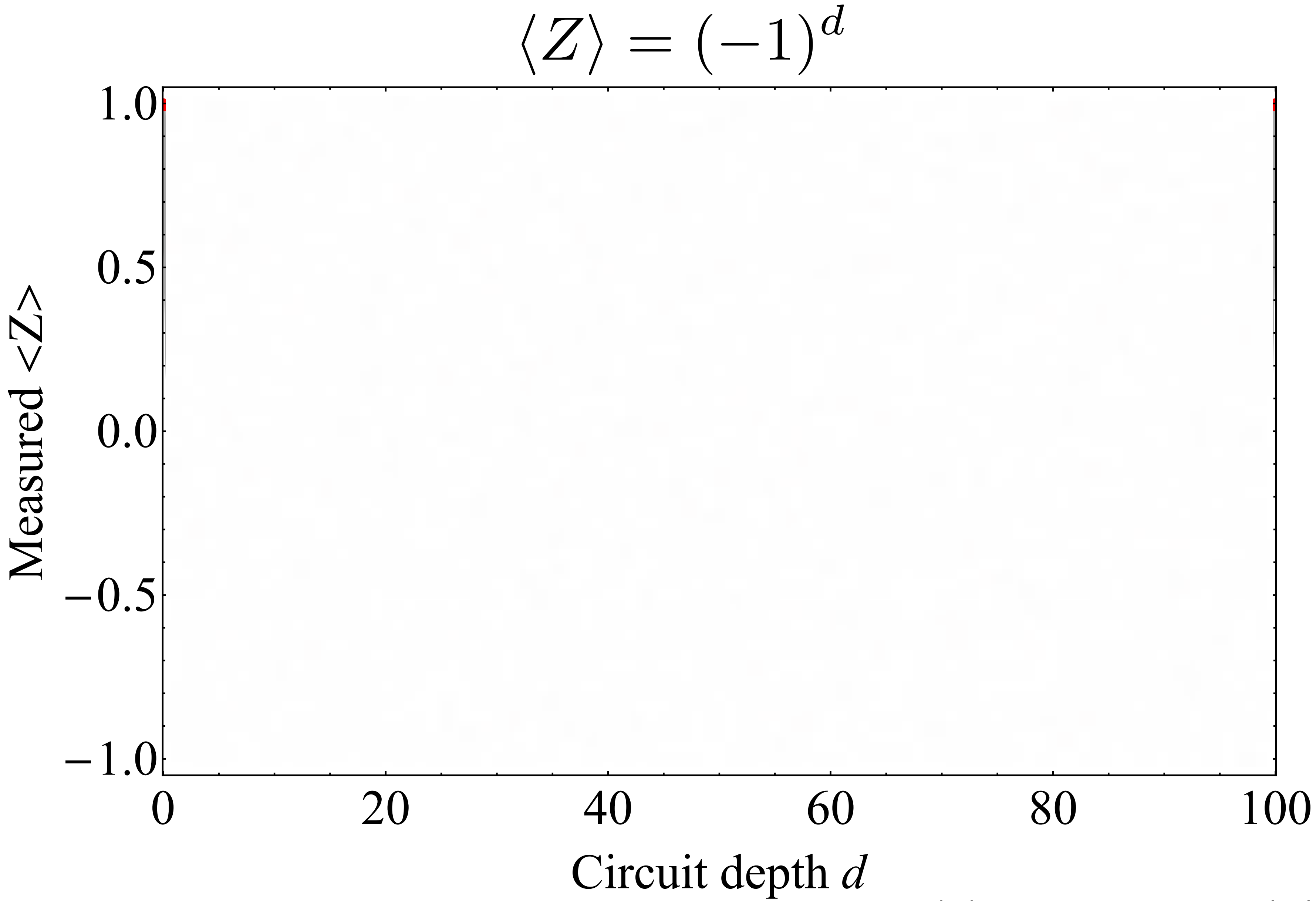
refresher:

$$X|0\rangle = |1\rangle \qquad Z|0\rangle = +1|0\rangle$$

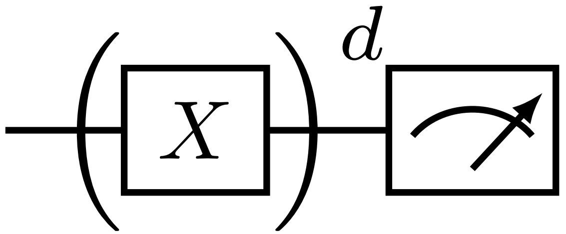
$$X|1\rangle = |0\rangle \qquad Z|1\rangle = -1|1\rangle$$

Hello World! Ideal expectation results

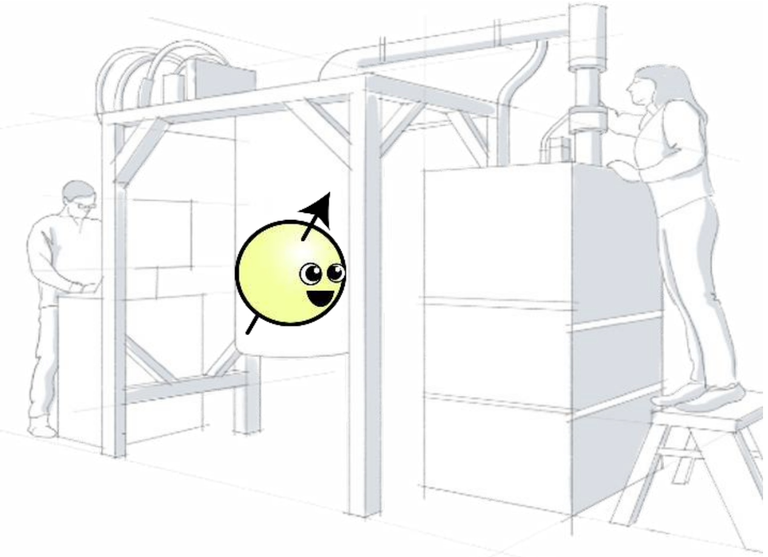




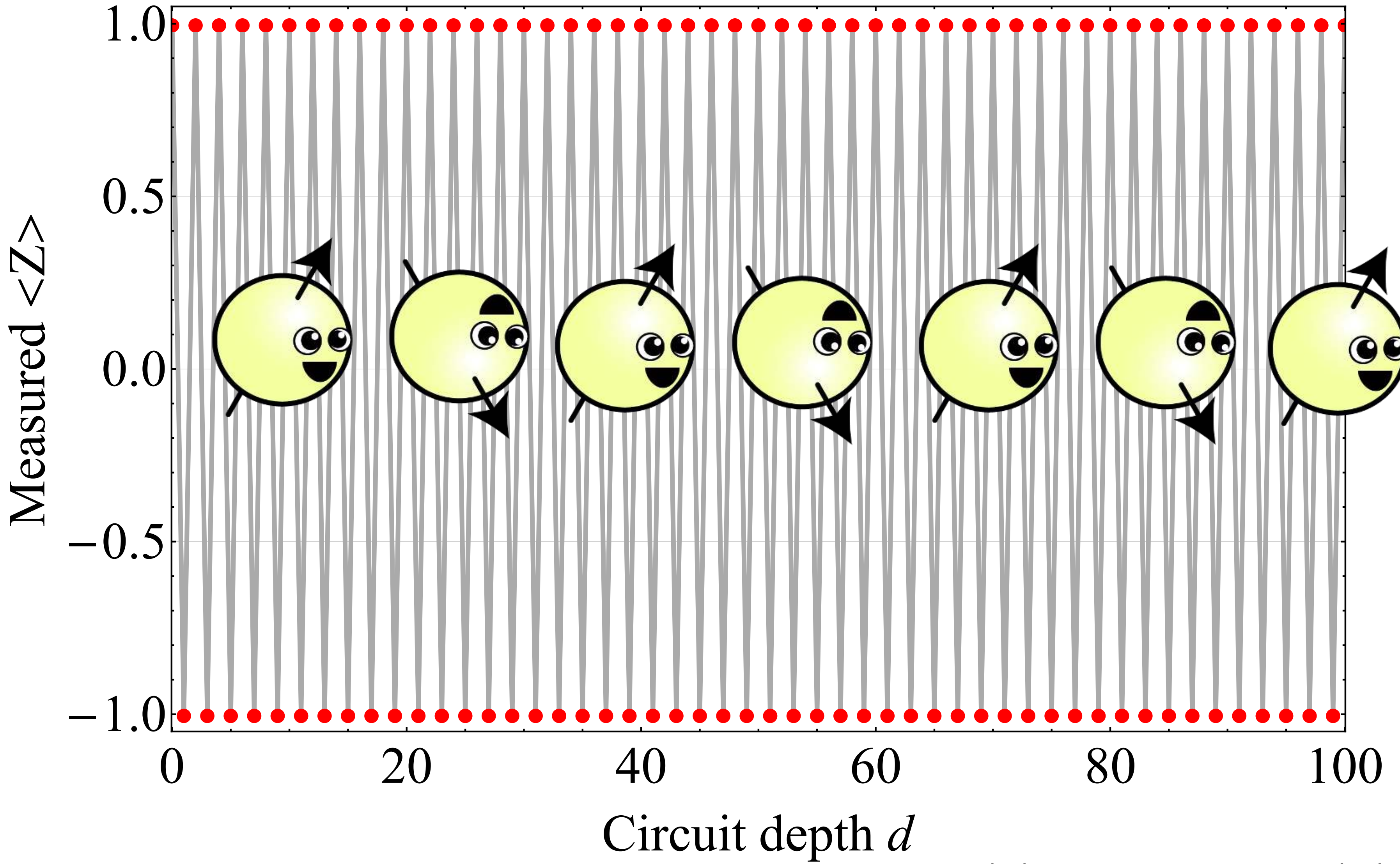
Hello World! Ideal expectation results



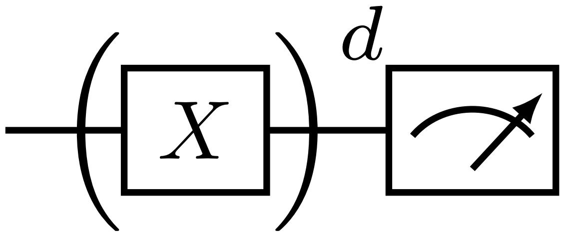
Let's run on a real device!

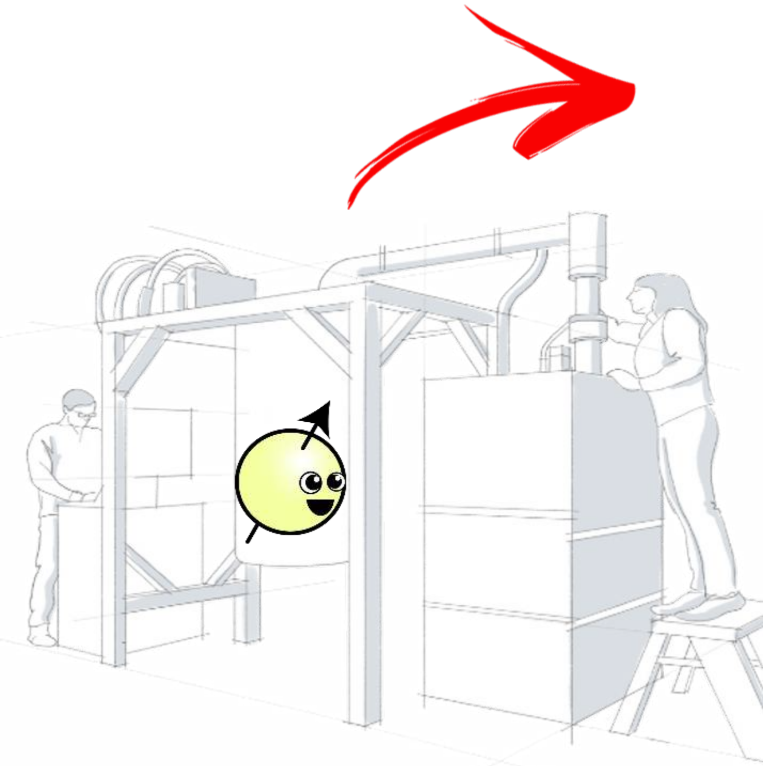


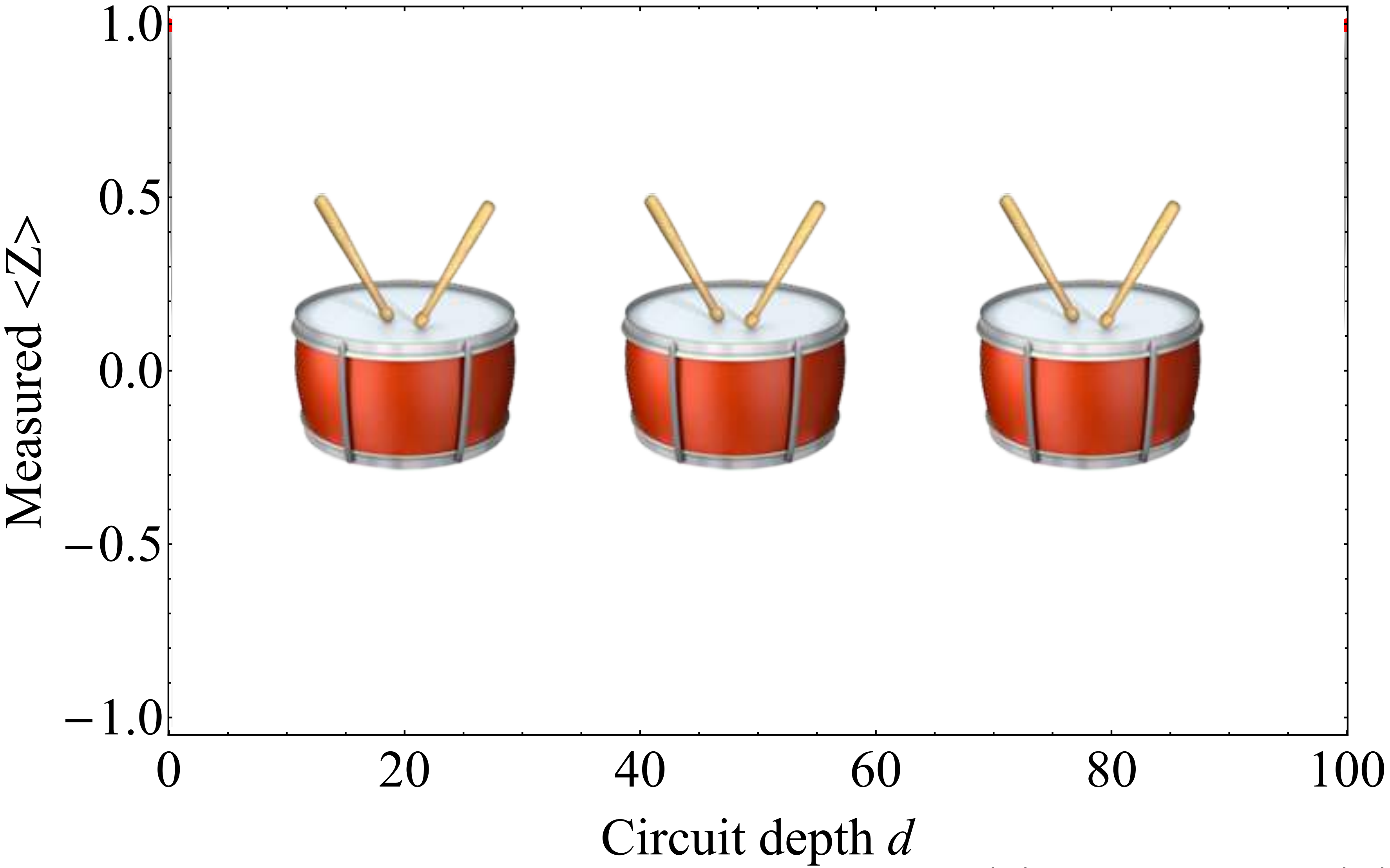
$\langle Z \rangle = (-1)^d$



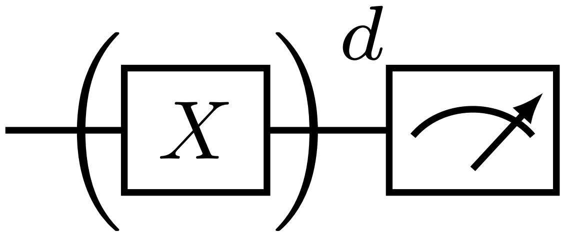
Hello World! Running ....

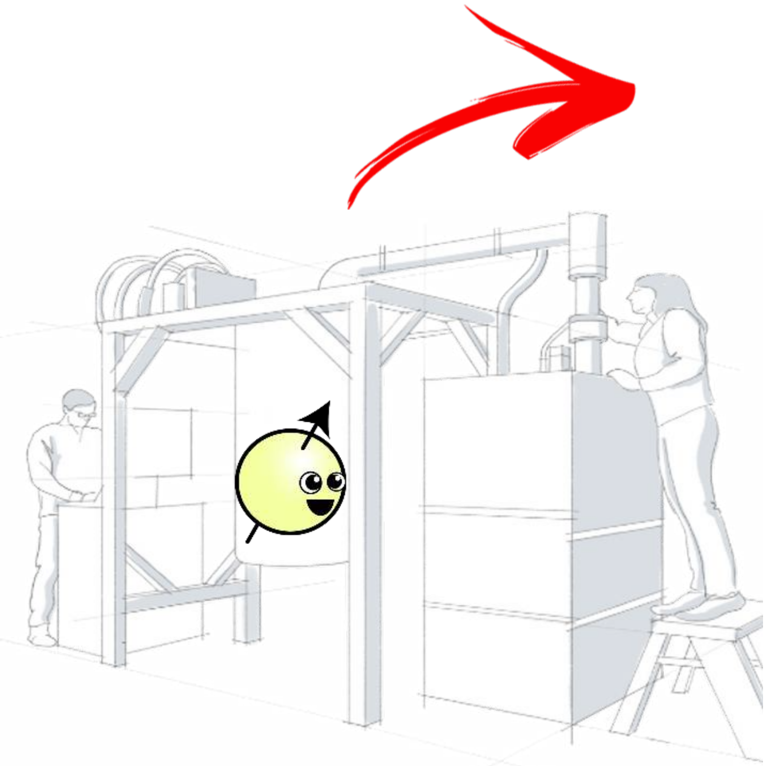


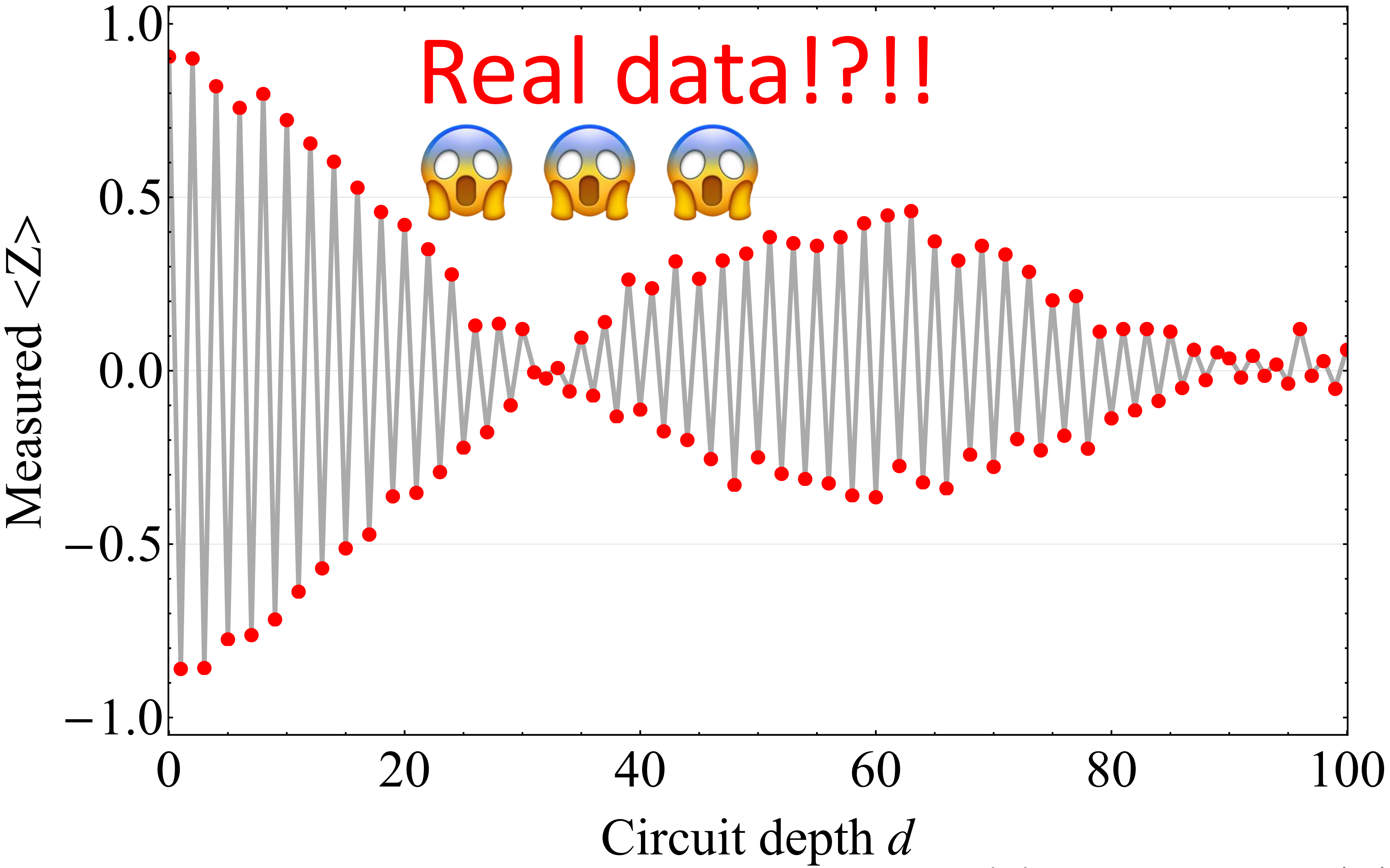




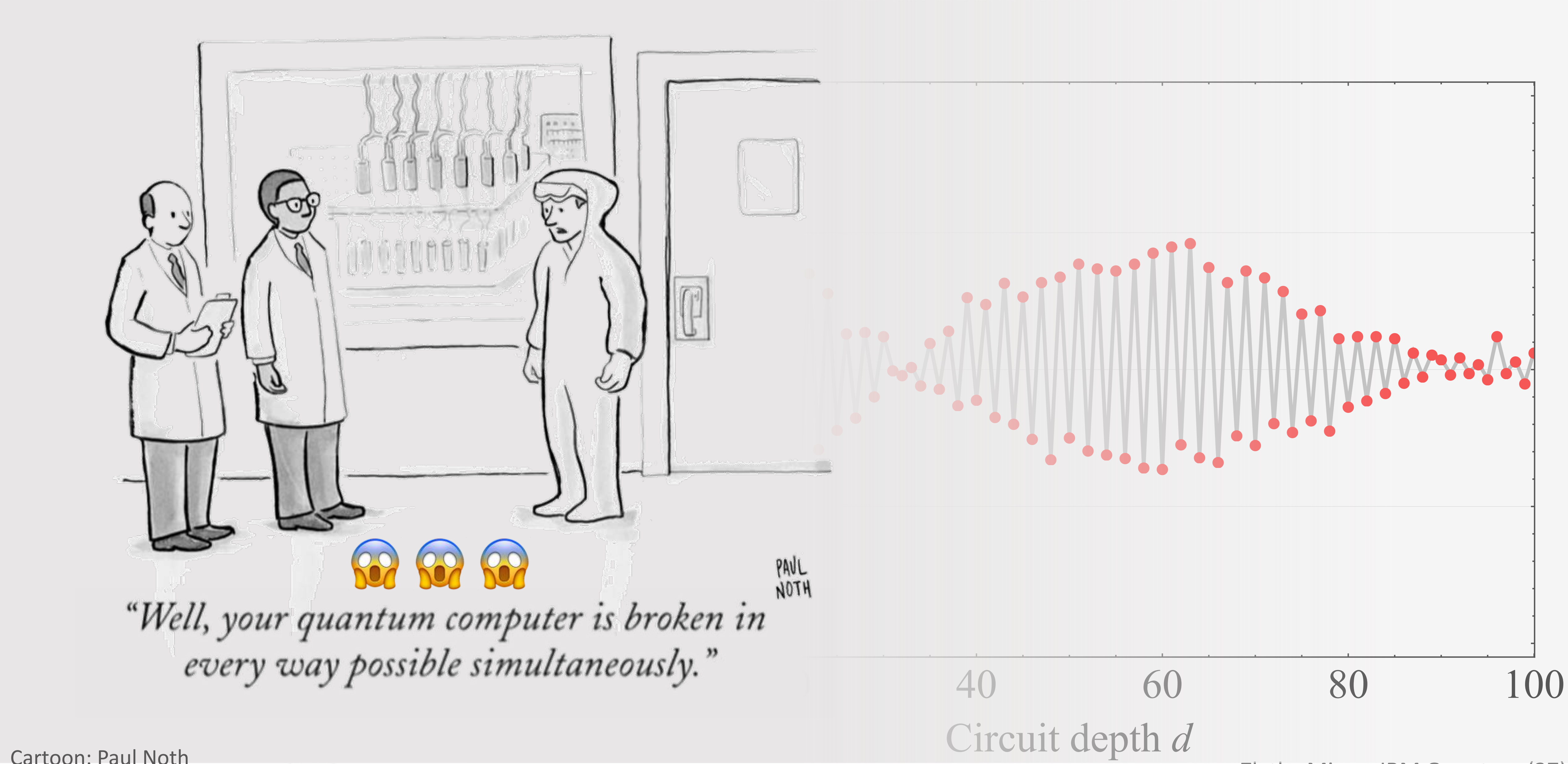
Hello World! Real expectation results



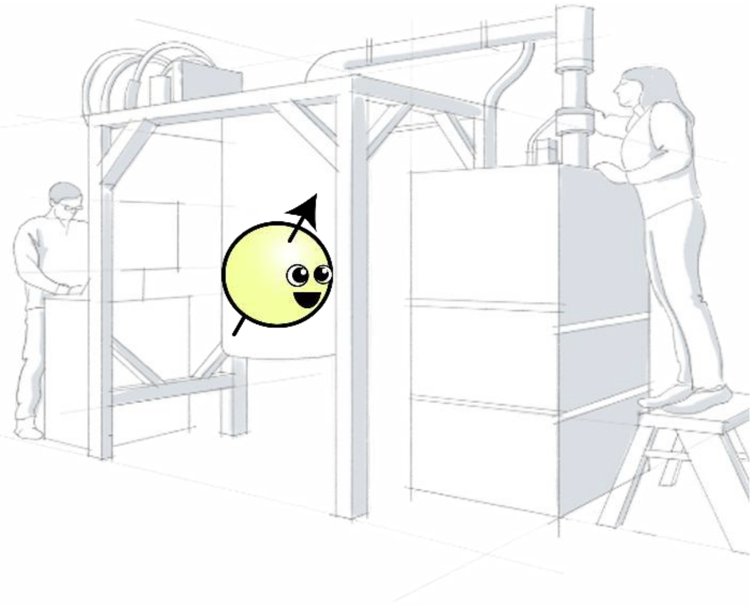




Real & noisy quantum processors: Why study noise?



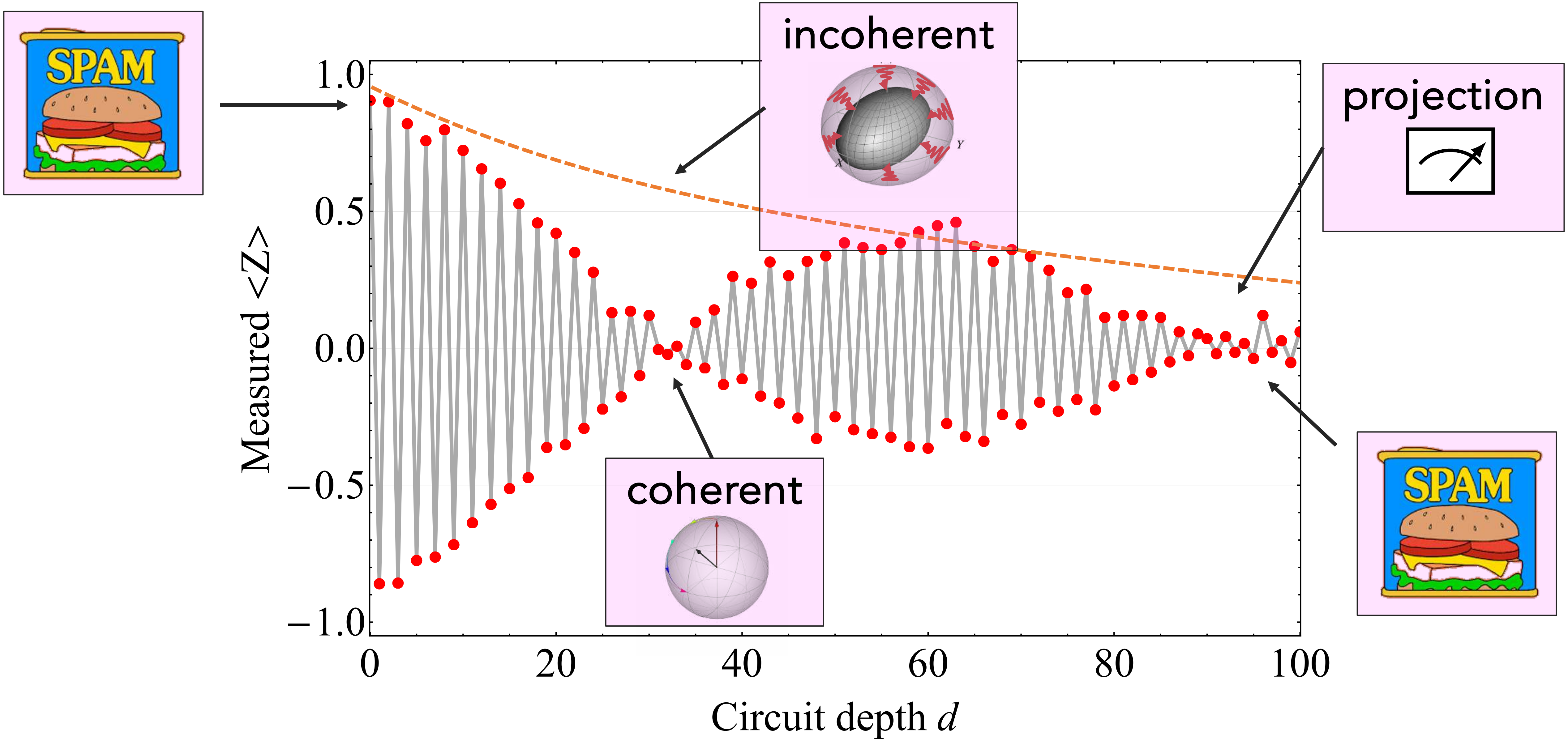
Cartoon: Paul Noth



"Quantum phenomena
do *not* occur in a Hilbert space,
they occur in a laboratory."

Asher Peres

Elements of 😱 noise



How to deal with errors due to noise?

Monitor
Error occurs
Error detect

Quantum error correction

Shor, PRA (1995), ...

Monitor
Error anticipated
Tell signal detected

Catch and reverse

Minev, Nature (2019), ...

No monitor
Error occurs
Error undetected

Error mitigation

... subject of today

Quantum simulation on a noisy quantum computer

Execute on a real quantum computer device and obtain results as classical data

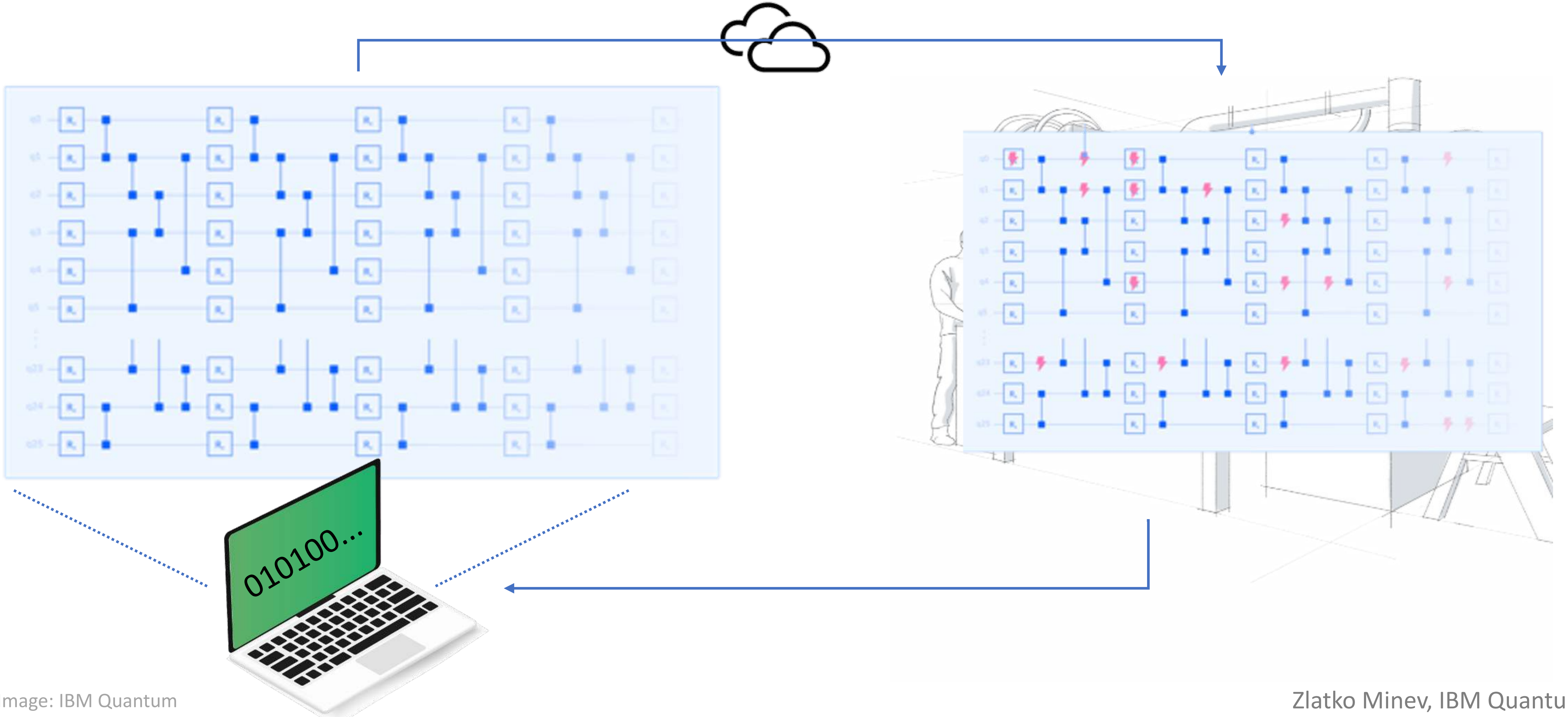


Image: IBM Quantum

Quantum error mitigation overview

Execute on a real quantum computer device and obtain results as classical data

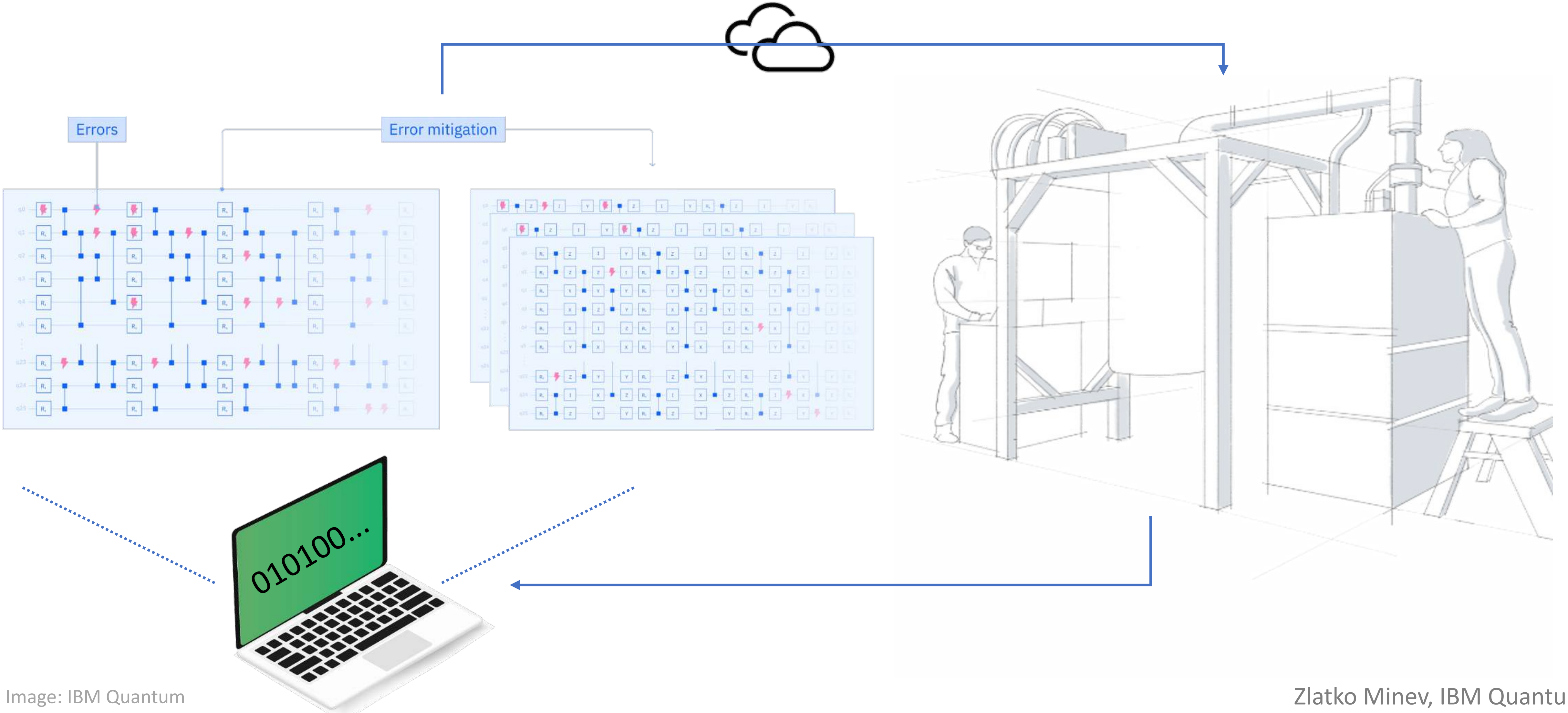
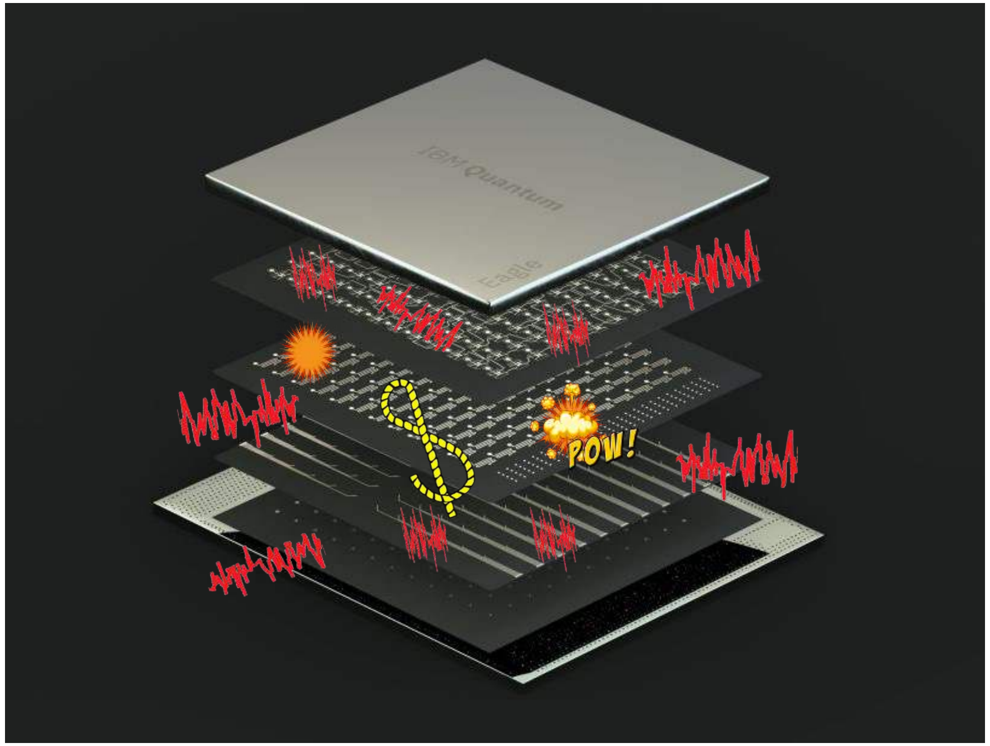


Image: IBM Quantum

Error mitigation and error correction



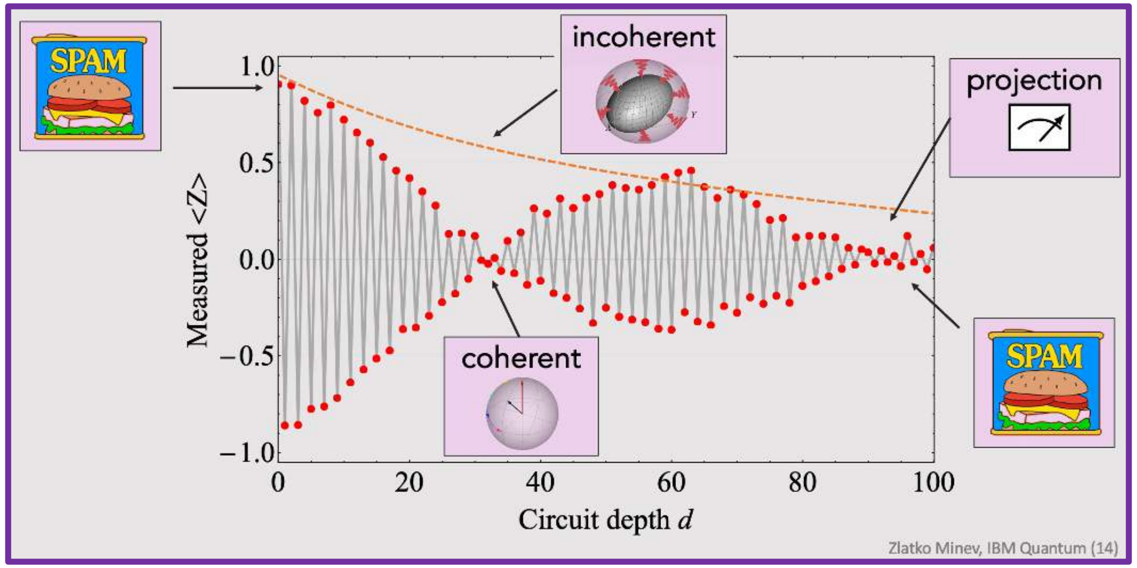
Error mitigation: working with what you have

- **benefit** suppress errors on classical results (expectation values)
- **q-cost** no extra qubits or hardware resources needed
- **c-cost** trades classical resources (post-processing) for lower error
- **limitation** bad asymptotic scaling: high number of samples & circuits

Error correction: protecting quantum information

- **benefit** suppress & correct errors to arbitrarily small level
- **q-cost** very large qubit and hardware overhead
- **c-cost** decoding and encoding can be classically costly
- **challenge** requires fault-tolerant operations and readout

Error mitigation landscape



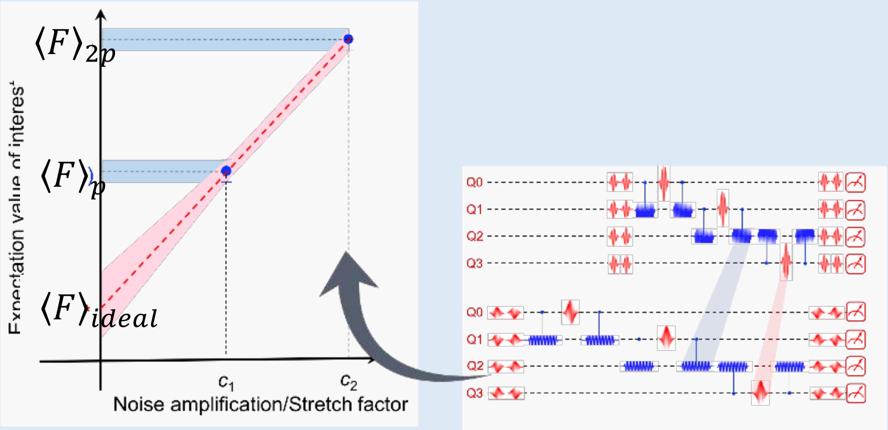
more speed

more information, accuracy

Photos courtesy of Y. Liu

Error mitigation landscape

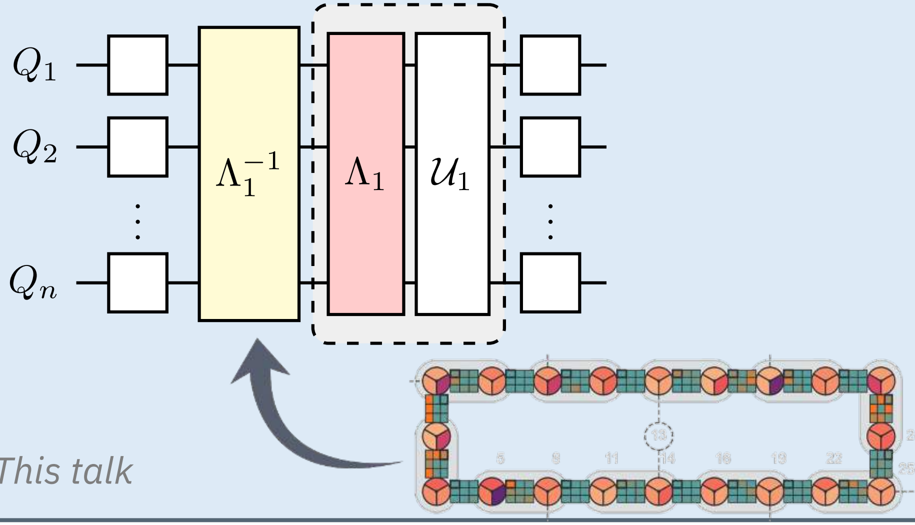
Zero-noise extrapolation (ZNE)



Nature 567, 491 (2019)

…

Probabilistic error cancelation (PEC)



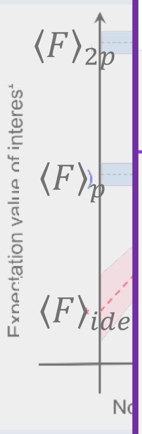
This talk

more speed

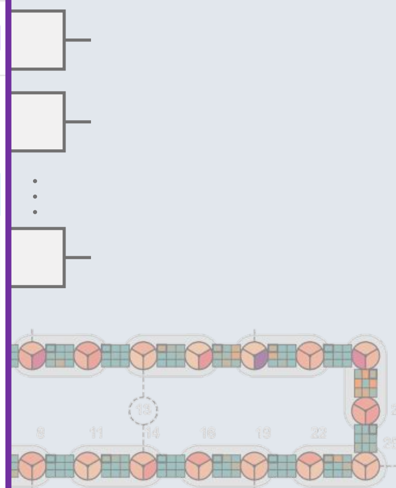
more information, accuracy

Error mitigation landscape

Zero-noise extrapolation (ZNE)



Probabilistic error cancelation (PEC)



Qiskit IBM Qiskit

Overview Learn Community

English

Search Qiskit Runtime IBM

Get started

- Overview
- Getting Started
- backend.run vs. Qiskit Runtime
- Introduction to primitives

Tutorials

- Get started with Estimator
- Get started with error suppression and error mitigation
- VQE with Estimator
- CHSH with Estimator
- Get started with Sampler

Qiskit Runtime IBM Client documentation > Error supression and error mitigation with Qiskit Runtime

NOTE

This page was generated from docs/tutorials/Error-Suppression-and-Error-Mitigation.ipynb.

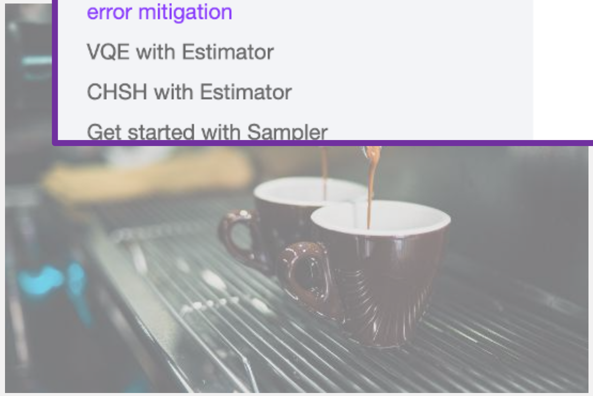
Error supression and error mitigation with Qiskit Runtime

[1]:

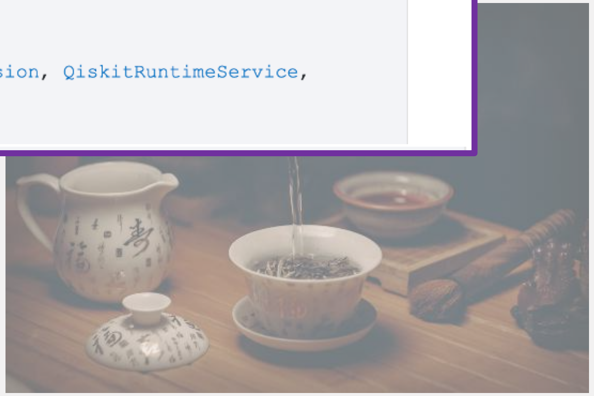
```
import datetime
import numpy as np
import matplotlib as mpl
import matplotlib.pyplot as plt
plt.rcParams.update({"text.usetex": True})
plt.rcParams["figure.figsize"] = (6,4)
mpl.rcParams["figure.dpi"] = 200

from qiskit_ibm_runtime import Estimator, Session, QiskitRuntimeService,
Options
from qiskit.quantum_info import SparsePauliOp
```

more speed

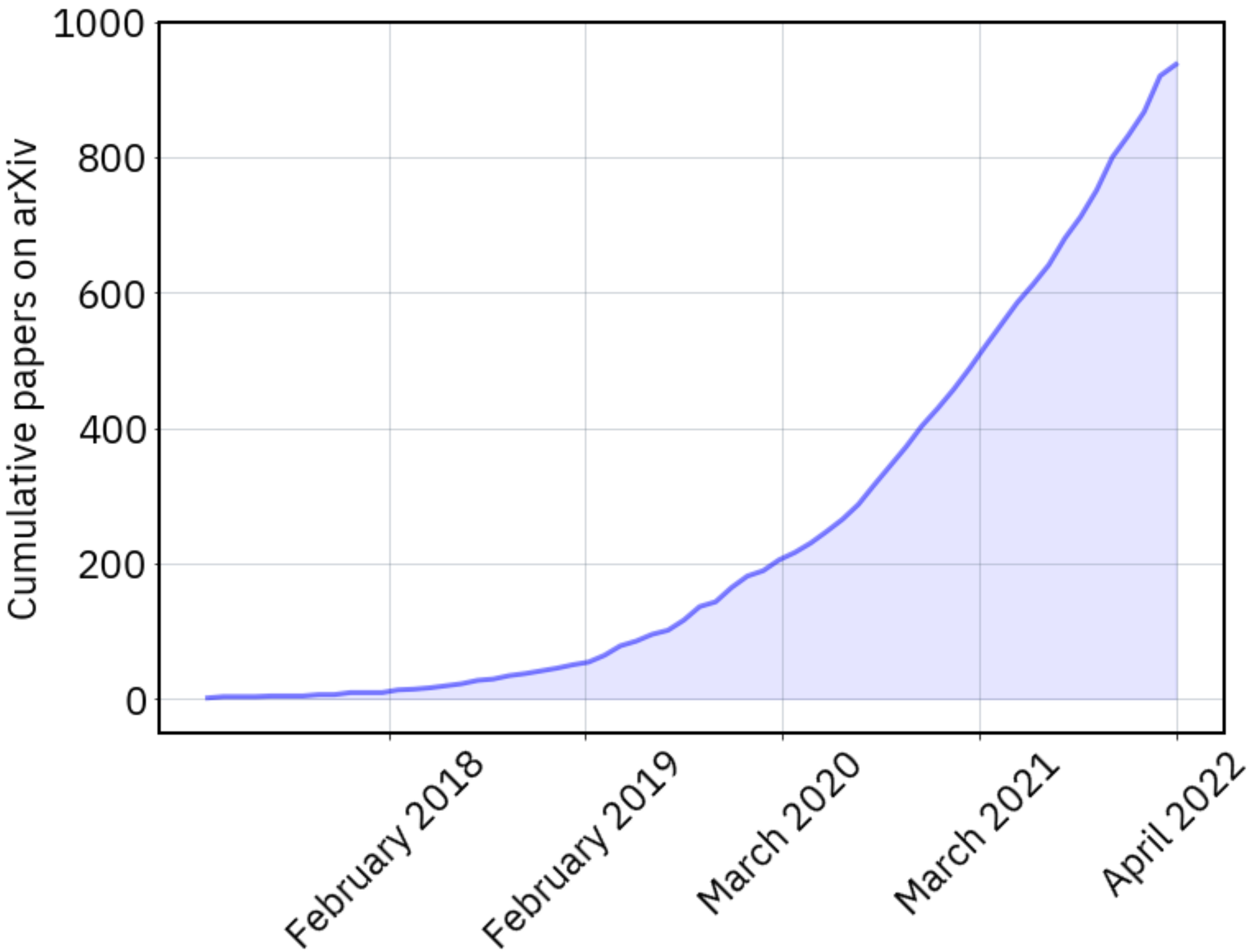


more information, accuracy



Adoption of error mitigation

Papers involving error mitigation over time



Examples

ARTICLE
https://doi.org/10.1038/s41467-020-14376-z OPEN
Error-mitigated quantum gates exceeding physical fidelities in a trapped-ion system
Shuaining Zhang[1], Yao Lu[1], Kuan Zhang[1,2], Wentao Chen[1], Ying Li[3*], Jing-Ning Zhang[1,4*] & Kihwan Kim[1*]

Matrix product channel: Variation...
to mitigate noise and reduce errors...
Sergey Filippov,* Boris Sokolov, Ma...
Borrelli, Daniel Cavalcanti, Sabrina...
Algorithmiq Ltd, Kanavakat...

Article | Published: 08 May 2023
Probabilistic error cancellation with sparse Pauli–Lindblad models on noisy quantum processors
Ewout van den Berg, Zlatko K. Minev, Abhinav Kandala & Kristan Temme
Nature Physics (2023) | Cite this article

Quantum Error Mitigation
Zhenyu Cai,[1,2,*] Ryan Babbush,[3] ...
McClean,[3] and Thomas E. O'Brie...
[1] Department of Materials, Universit...
[2] Quantum Motion, 9 Sterling Way, ...
[3] Google Quantum AI, Venice, Califo...
[4] NTT Computer and Data Science ...
[5] Graduate School of China Academy...
(Dated: July 3, 2023)

npj | quantum information
ARTICLE OPEN
Fundamental limits of quantum error mitigation
Ryuji Takagi[1], Suguru Endo[2], Shintaro Minagawa[3] and Mile Gu[1,4]

Single-shot error mitiga...
...wout van den Berg, Sergey B...
Dmitri Maslo...
...ntum, IBM T.J. Watson Resea...
Dece...

PHYSICAL REVIEW LETTERS 127, 200505 (2021)
Error Mitigation for Universal Gates on Encoded Qubits
Christophe Piveteau
IBM Quantum, IBM Research—Zurich, 8803 Rüschlikon, Switzerland

Model-free readout-error mitigation for quantum expectation values
Ewout van den Berg, Zlatko K. Minev, and Kristan Temme
Phys. Rev. A **105**, 032620 – Published 30 March 2022

Is science with noisy devices of broad interest today?

Some of these ideas covered in other lectures at the school

Deep dive:
Probabilistic error cancellation (PEC)
To learn and cancel quantum noise

Got Slides?

SCAN ME

Ewout van den Berg, Zlatko K. Minev, Abhinav Kandala, Kristan Temme
Nature Physics (2023)

Cancel quantum noise

Icon: Flaticon

High-level message

Learn
accurate, efficient, scalable

Cancel
noise with noise,
practical

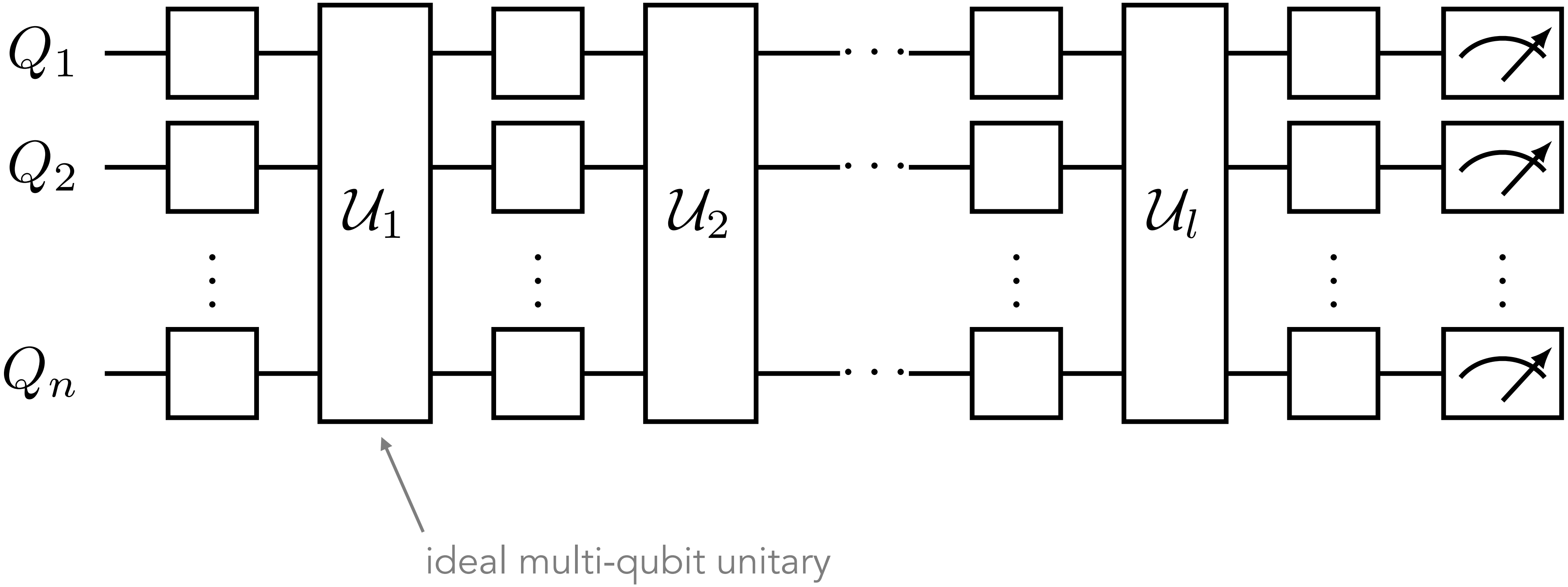
Cost
more noise more cost

Icon: Flaticon

Idea

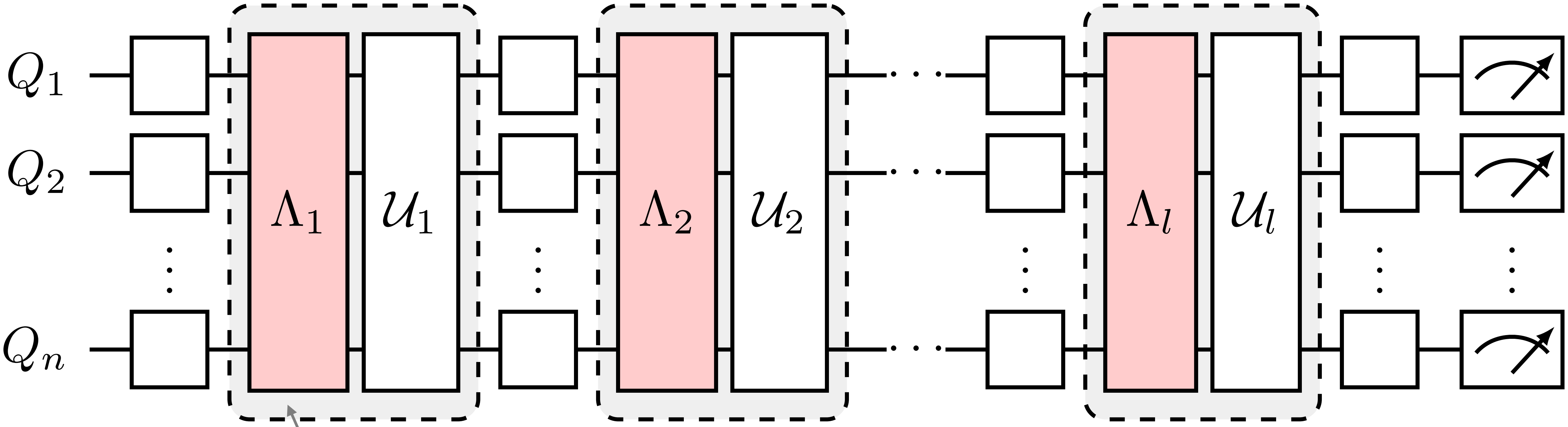
Icon: Flaticon

Ideal (noise-free) quantum circuit



A circuit can be decomposed into a layer construction
Example: Trotterization of Ising model simulation

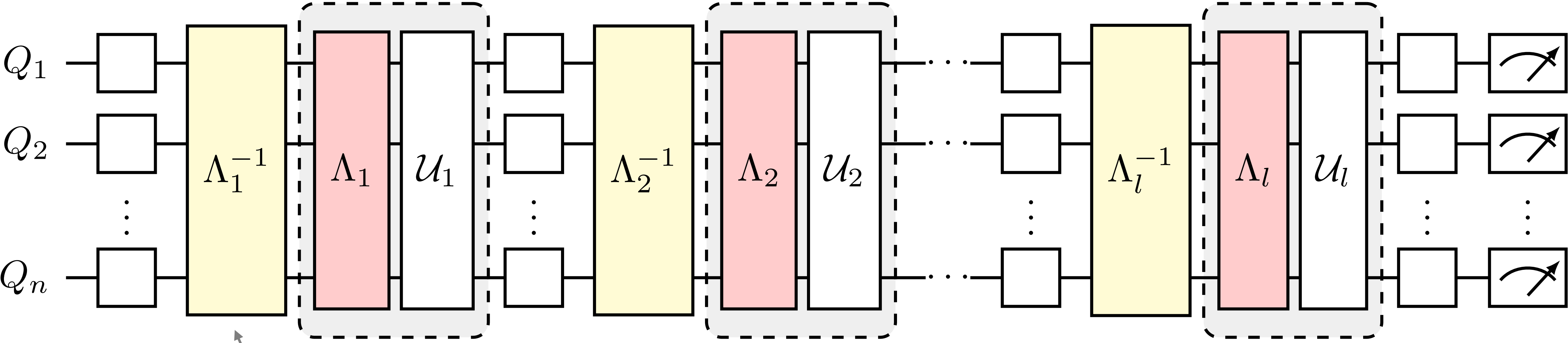
Real (noisy) quantum circuit



multi-qubit noise channel
inseparable from gate

completely positive and trace preserving (CPTP)
representable by a 4^n x 4^n matrix

Why not invert noise?



inverse operation of noise channel

unphysical

would need to know lost information due to noise

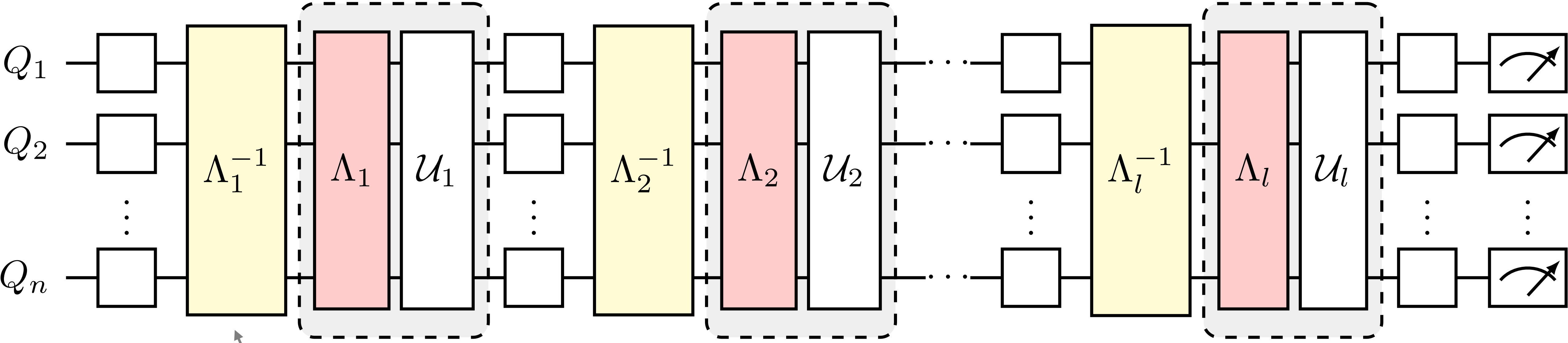
non CPTP map

has negative eigenvalues

...

Not possible?

Probabilistic error cancelation

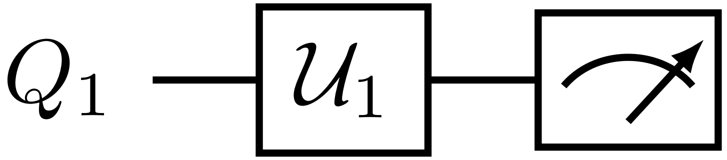


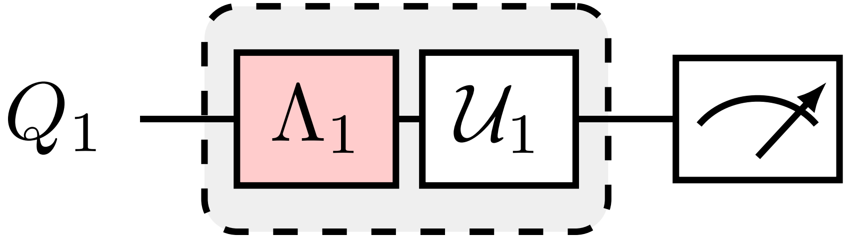
inverse operation of noise channel
implement on average

K. Temme, S. Bravyi, and J. M. Gambetta
PRL 119, 180509 (2017)

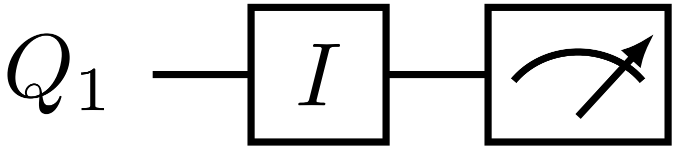
See also S. Endo, S. Benjamin, and Y. Li
Phys. Rev. X 8, 031027 (2018)

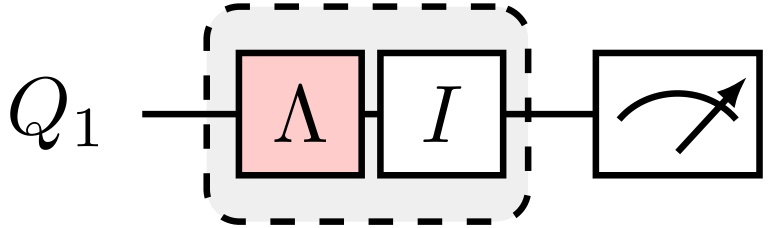
Toy model





Toy model: noise unraveling into quantum trajectories



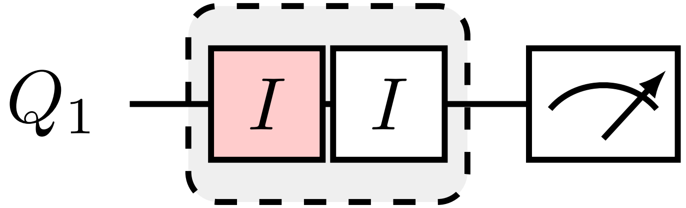


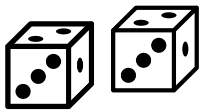
bit-flip noise

=

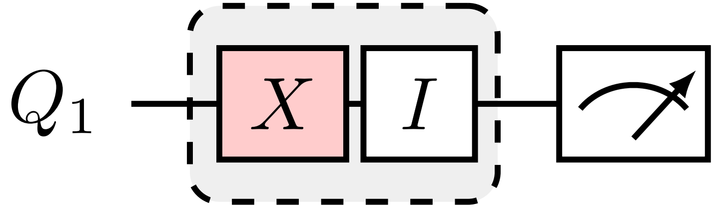
unraveling
(quantum trajectories)

probability 1-p



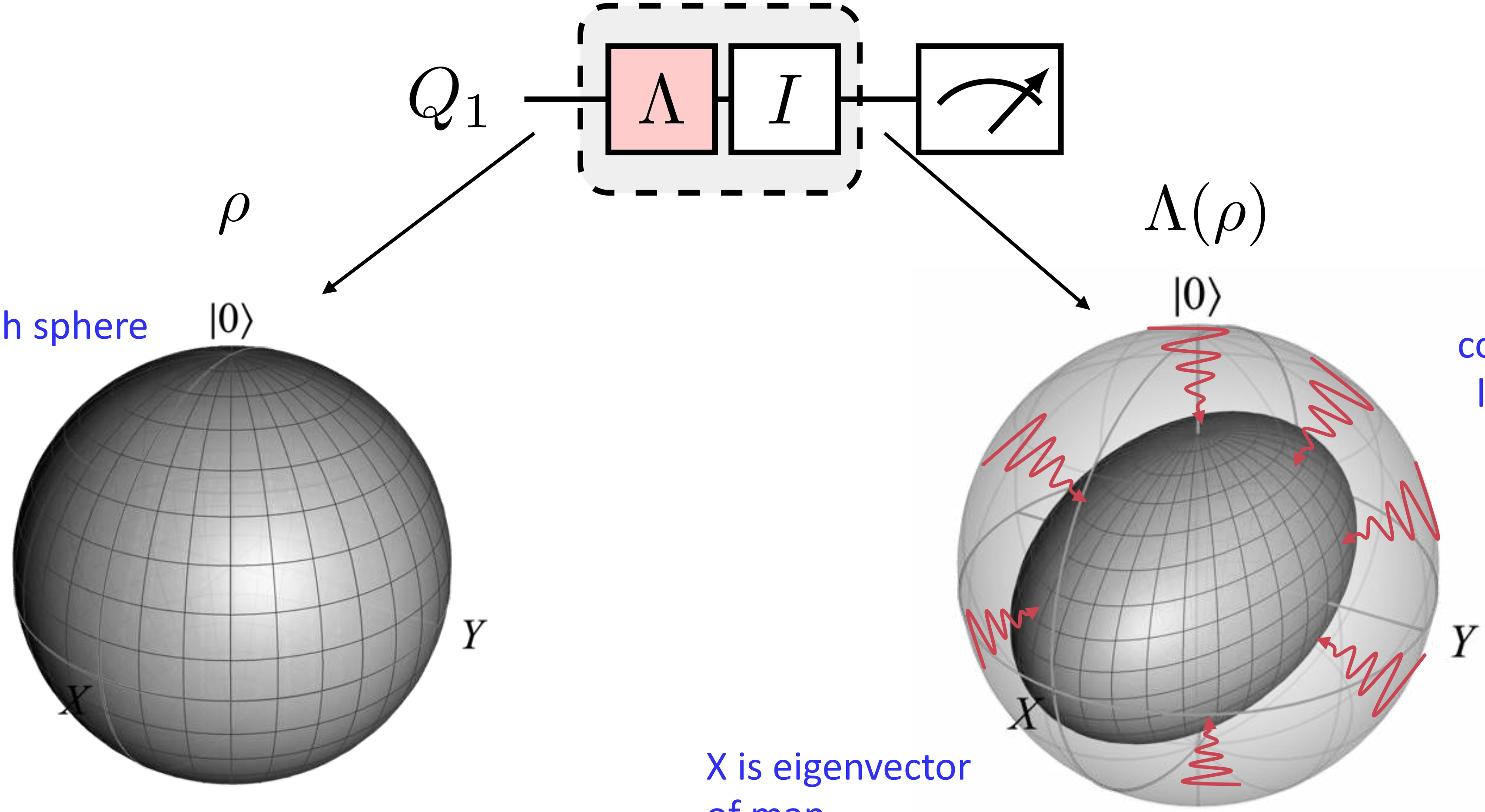


probability p



$$\Lambda\left(\rho\right) = (1-p)I\rho I + pX\rho X$$

Toy model: noise unraveling into quantum trajectories

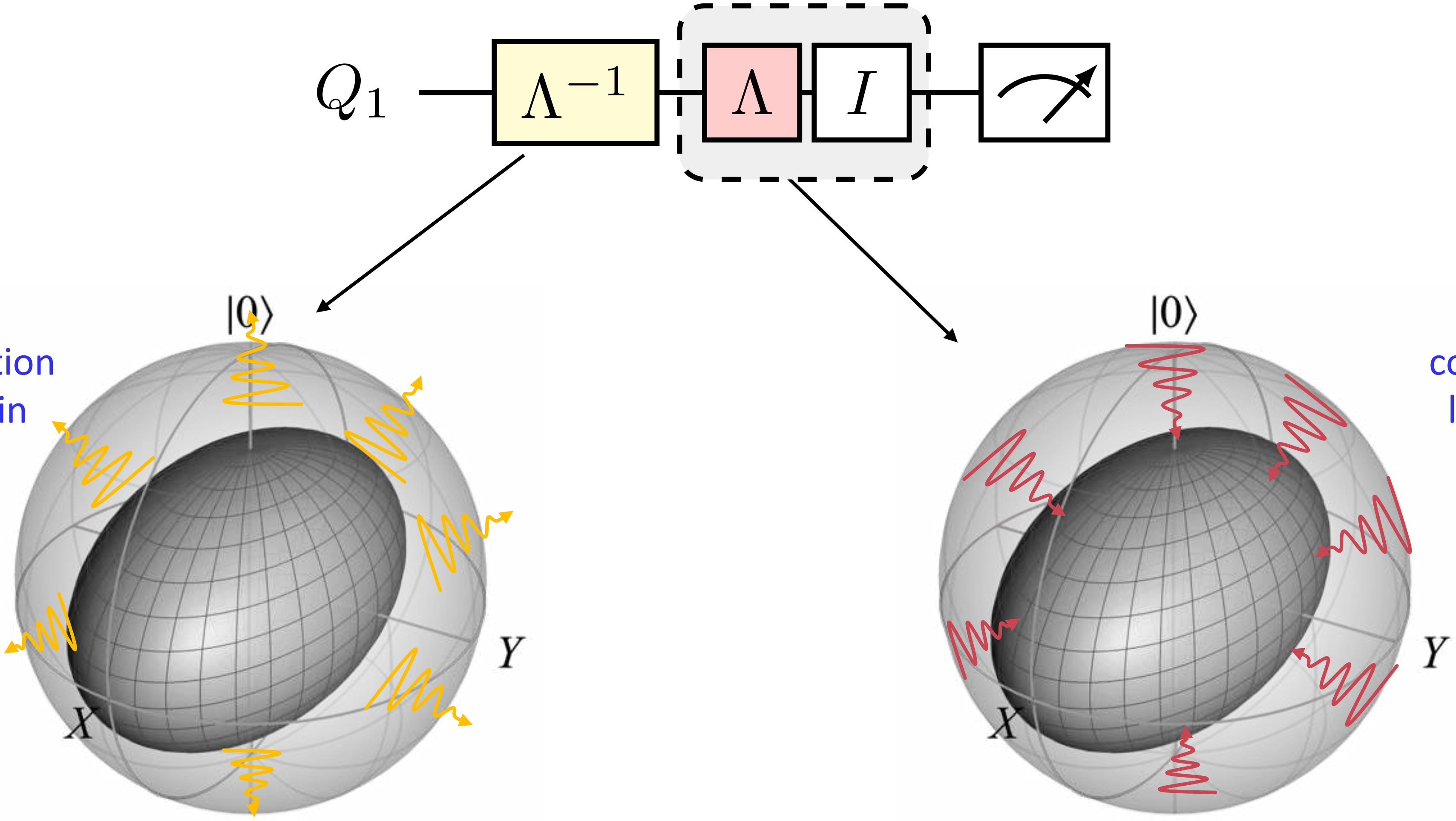


Bloch sphere

contraction
loss of information
reduced purity
eigenvalues > 0

X is eigenvector of map

Inverse of noise map is not physical



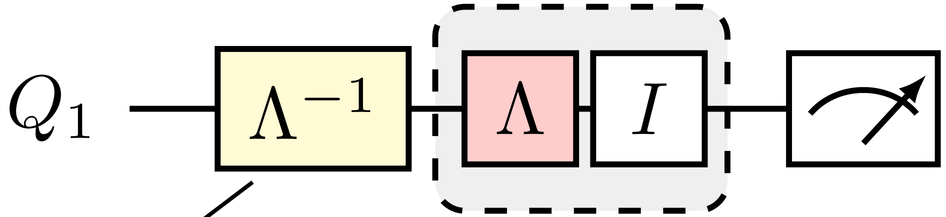
dilation
information gain
increase purity
eigenvalues < 0

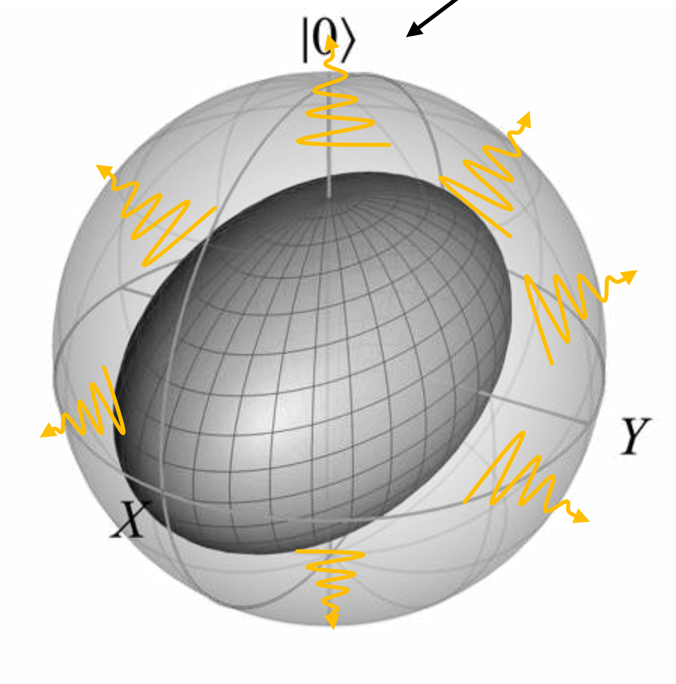
$\Lambda^{-1}(\rho) = ?$

contraction
loss of information
reduced purity
eigenvalues > 0

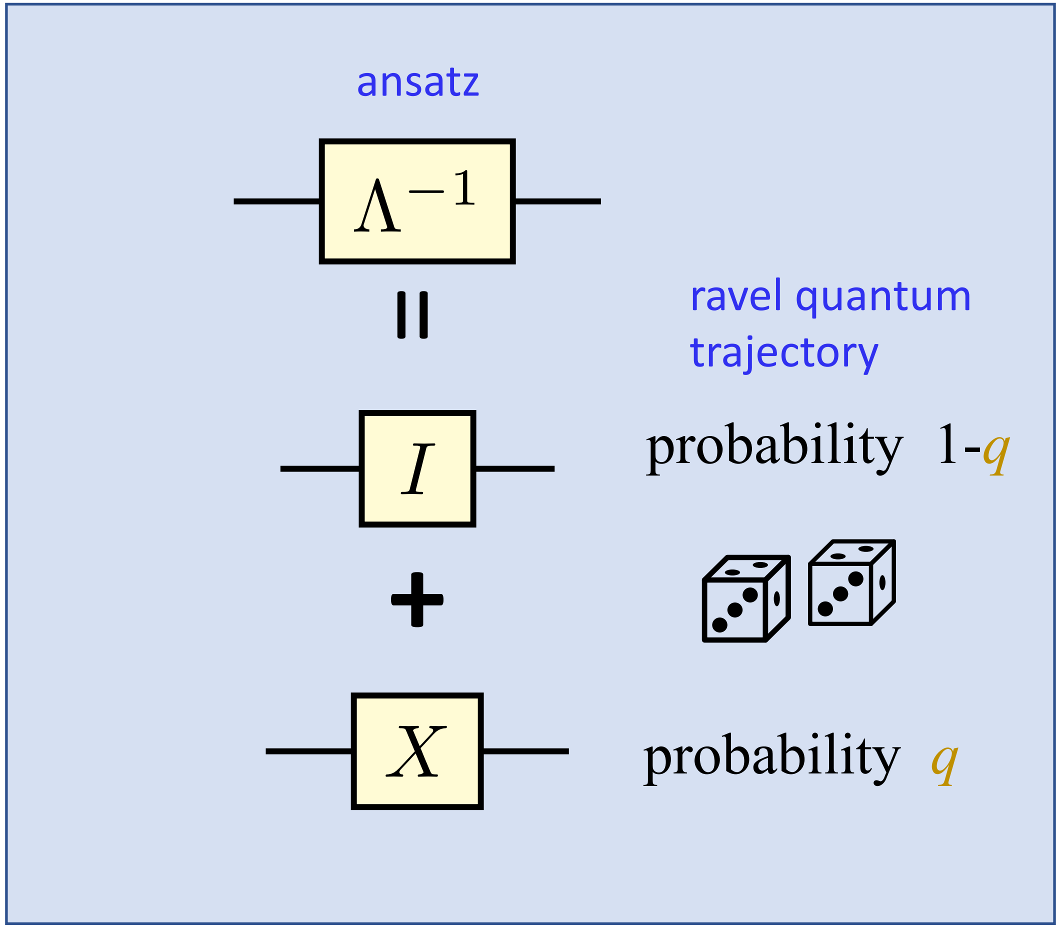
$\Lambda(\rho)$

Inverse of noise map is not physical



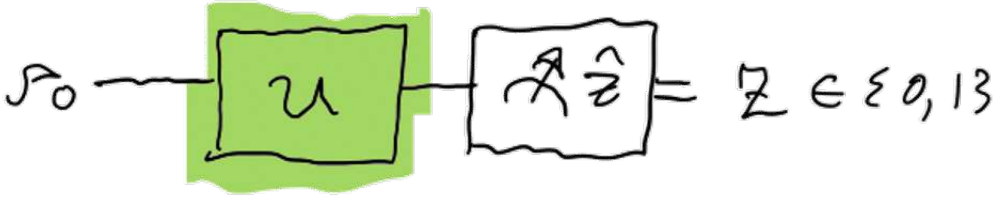


$$\Lambda^{-1}(\rho) = ?$$



Blackboard derivation

Setup



Details on notation:

Quantum register alphabet $\Sigma = \{0,1\}$

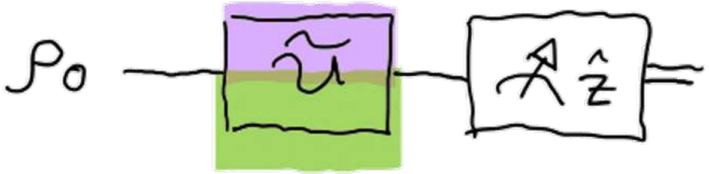
Hilbert space $\mathcal{H} = \mathbb{C}^{\Sigma}$

Initial state $\rho_0 \in D(\mathcal{H}) \subset L(\mathcal{H})$

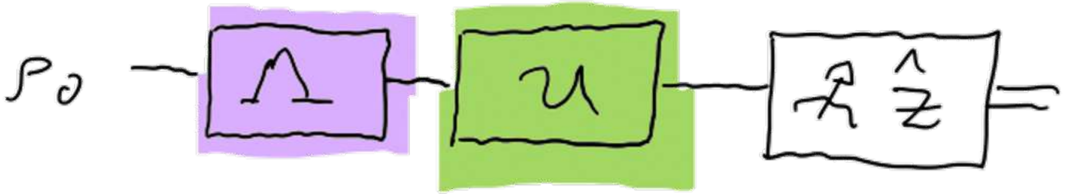
Ideal unitary $U \in U(\mathcal{H}) \subset L(\mathcal{H})$

Ideal u-channel $\mathcal{U}(\rho) = U \rho U^{\dagger}$

$\mathcal{U} \in C(\mathcal{H}) \subset L(L(\mathcal{H}))$



Noisy gate / circuit $\tilde{\mathcal{U}} \in L(L(\mathcal{H}))$



Decompose noisy gate $\tilde{\mathcal{U}} = \mathcal{U}\Lambda$

Blackboard derivation

Simple Example

Keeping it simple and illustrative, lets do a simple case

let $U = I$

$\mathcal{U}\cdot = I \cdot I$

For the noise, lets play with the simplest bit-flip channel

$$n(\rho) = \underbrace{(1-p)}_{\text{prob of no error}} I \rho I + \underbrace{p}_{\text{prob of a bit flip error}} X \rho X$$

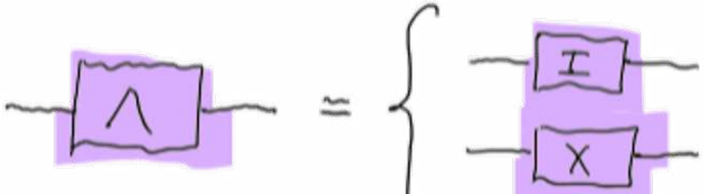
$$\Lambda_\rho = (1-p)\,\mathcal{I}_\rho + p\,\mathcal{X}_\rho$$

Equivalent superoperator channel representation

$\mathcal{X}_\rho = X \rho X$

$\mathcal{I}_\rho = I \rho I = \rho$

Equivalent trajectory unraveling

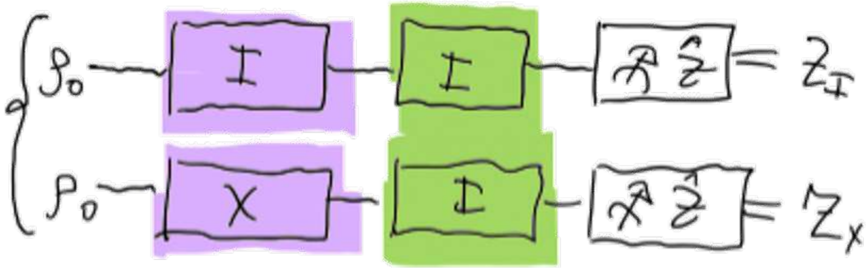


: Prob $1-p$

: Prob p

Blackboard derivation

Our circuit then is equivalent to either



with prob $1-p$

with prob p

Simple Example

Keeping it simple and illustrative, let's do a simple case

Let $U = I$

$\mathcal{U} = I \cdot I$

For the noise, let's play with the simplest bit-flip channel

$$\Lambda(\rho) = \underbrace{(1-p)}_{\text{prob of no error}} I \rho I + \underbrace{p}_{\text{prob of a bit flip error}} X \rho X$$

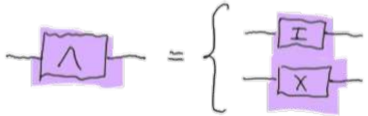
$$\Lambda_p = (1-p)\mathcal{I}_p + p\,\mathcal{X}_p$$

Equivalent superoperator channel representation

$\mathcal{X}_\rho = X \rho X$

$\mathcal{I}_\rho = I \rho I = \rho$

Equivalent trajectory unraveling



: Prob $1-p$

: Prob p

The ideal expectation value is

$$Z_{ideal} = \langle \hat{Z} \rangle = \mathrm{Tr}(Z \mathcal{I} \rho_0) = \mathrm{Tr}(Z \rho_0) = \rho_Z$$

When the channel introduces an error however,

$$\mathbb{E}[Z_X] = \mathrm{Tr}(Z X \rho X) = \mathrm{Tr}(X Z X \rho)$$

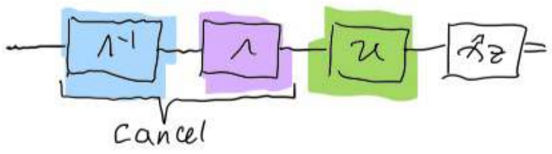
$$= \mathrm{Tr}(-Z \rho)$$

$$= -\rho_Z$$

Blackboard derivation

Noise Inverse

To undo the noise, we'd like to introduce the inverse noise



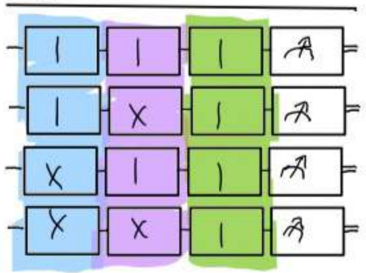
$\Lambda^{-1}\Lambda = \Lambda\Lambda^{-1} = I$

Taking the ansatz $\Lambda^{-1}(p) = (1-r)\, I\cdot I + r(X\cdot X)$

we see 4 cases of unraveling

inverse	noise	no error	prob
I	I	✓	$(1-r)(1-p)$
I	X	x	$(1-r)p$
X	I	x	$r(1-p)$
X	X	✓	rp

Circuit



ideally, we what to interfere trajectories so that the no-error on will coherently add to unity probably, and the ones with an error will cancel.

$\therefore$ Ⓐ $(1-r)(1-p) + r\cdot p = 1$ $\wedge$ Ⓑ $(1-r)p + r(1-p) = 0$

$1 - r - p + 2rp = 1$ $\qquad p + r - 2rp = 0$

$r + p - 2rp = 0$ $\qquad$ same condition

$\Rightarrow \quad r(1-2p) = -p$

$$\therefore \quad \boxed{r = \frac{-p}{1-2p}}$$

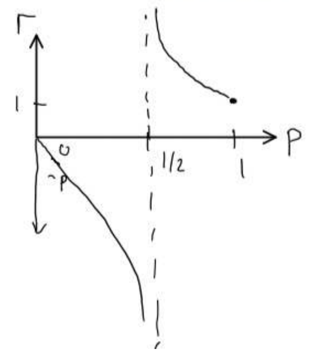
Recall p is a probability $0 \le p \le 1$,

$p = 0 \Rightarrow r = 0$ $\qquad$ no noise, no need to do anything

$p = 1 \Rightarrow r = 1$ $\qquad$ deterministic bit-flip, requires deterministic bit flip as well for Λ^{-1}

$p = 1/2 \Rightarrow r = \infty$ $\qquad$ singular value, since at $p=1/2$, we fully scramble the state

$p \ll 1 \Rightarrow r \simeq -p$



Blackboard derivation

Note that we could equivalently have used the algebraic condition and solved for r

$$\Lambda\left(\Lambda^{-1}(\rho)\right) = \mathcal{I}(\rho) = \rho \qquad \text{solve for } r$$

$$= \Lambda\left[(1-r)\rho + r X\rho X\right]$$

$$= \underbrace{(1-p)(1-r)\rho + pr\,\cancel{XX}\rho\cancel{XX}}_{\text{no error}} + \underbrace{(1-p)r X\rho X + (1-r)p X\rho X}_{\text{error}}$$

$$= \overbrace{\left[(1-p)(1-r) + pr\right]\rho}^{\text{no error}} + \overbrace{\left[(1-p)r + (1-r)p\right] X\rho X}^{\text{error}}$$

Same conditions as above! solution $r = \frac{-p}{1-2p}$

Blackboard derivation

How to implement? Quasi - Probability

$$\Lambda^{-1} = (1-r) I \rho I + r X \rho X \qquad r = \frac{-p}{1-2p}$$

$$= \left[\frac{|1-r|}{|1-r| + |r|} \operatorname{sgn}(1-r) I \rho I + \frac{|r|}{|1-r|+|r|} \operatorname{sgn}(r) X \rho X \right] (|1-r| + |r|)$$

$$= \gamma \left[s_I P_I I \rho I + s_X P_X X \rho X \right]$$

with

$$\gamma = |1-r| + |r|$$

$$P_I = \frac{|1-r|}{\gamma} \qquad s_I = \operatorname{sgn}(1-r)$$

$$P_X = \frac{|r|}{\gamma} \qquad s_X = \operatorname{sgn}(r)$$

valid prob distribution

$$0 \leq P_I, P_X \leq 1 \quad \text{and} \quad |P_I| + |P_X| = 1$$

Blackboard

How to sample?

$$\langle Z \rangle = \mathrm{Tr}\left(Z \mathcal{I} \Lambda \Lambda^{-1} \rho_0 \right)$$

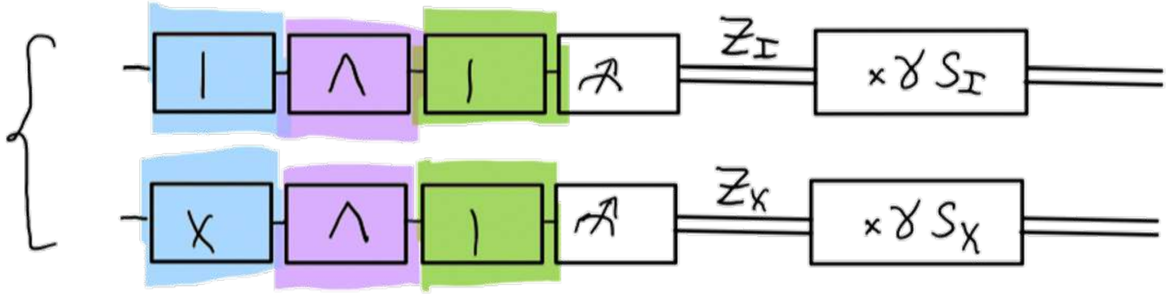
$$= \mathrm{Tr}\left(Z \tilde{\mathcal{I}} \left[\gamma s_I p_I \rho_0 + \gamma s_X p_X X \rho_0 X \right] \right)$$

$$= \gamma s_I p_I \mathrm{Tr}\left(Z \tilde{\mathcal{I}} \rho_0 \right) + \gamma s_X p_X \mathrm{Tr}\left(Z \tilde{\mathcal{I}} X \rho_0 X \right)$$

$$= \gamma \left[s_I p_I \underbrace{\langle Z \rangle_I}_{\text{quantum circuit exp. val}} + s_X p_X \langle Z \rangle_X \right]$$

we can find

Equivalent interportation : sample prob



$\gamma s_I Z_I$: p_I

$\gamma s_X Z_X$: p_X

Blackboard

Estimator $\qquad E_{mitg} = \gamma s_I Z_I + \gamma s_X Z_X$

$$\mathbb{E}[E_{mitg}] = \langle \hat{Z} \rangle_{ideal}$$

$$\mathbb{V}[E_{mitg}] = \mathbb{V}[\gamma s_I Z_I] + \mathbb{V}[\gamma s_X Z_X]$$

$$= \gamma^2 \mathbb{V}[Z_I] + \gamma^2 \mathbb{V}[Z_X]$$

$$= \gamma^2 (2 \sigma^2_{ideal})$$

$$\sigma^2_{ideal} = \mathbb{V}[Z_I] = 4q(1-q)$$

$$q = 1 - 2p_Z = \left\langle \frac{1-\hat{Z}}{2} \right\rangle$$

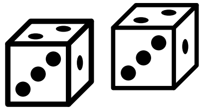
Since the X just flip $Z \to -Z$ of ρ, it follows that the variance is the same, since symmetric

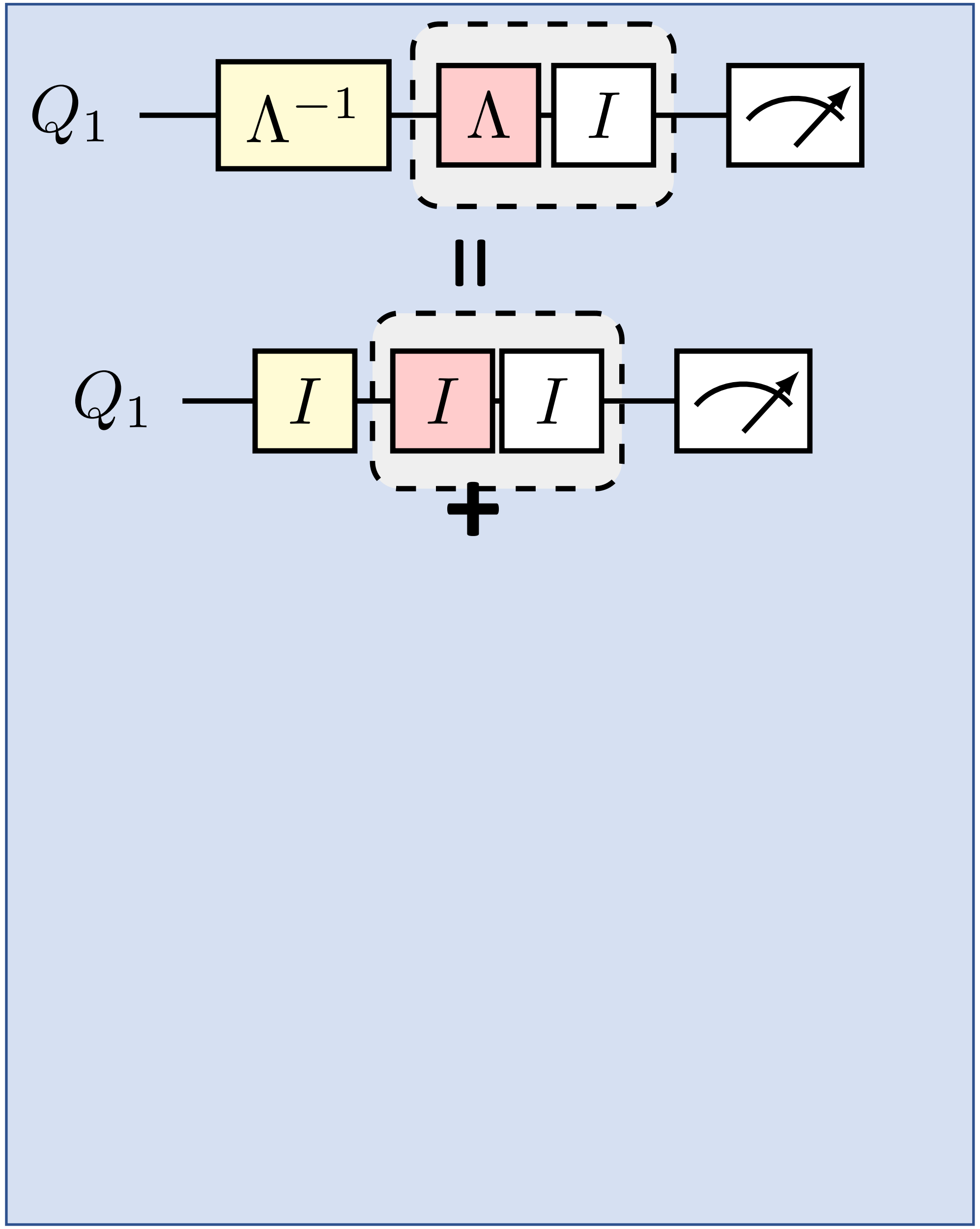
Raveling quantum trajectories to undo noise

No error

probability

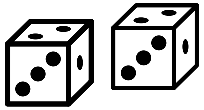
(1-q) (1-p)

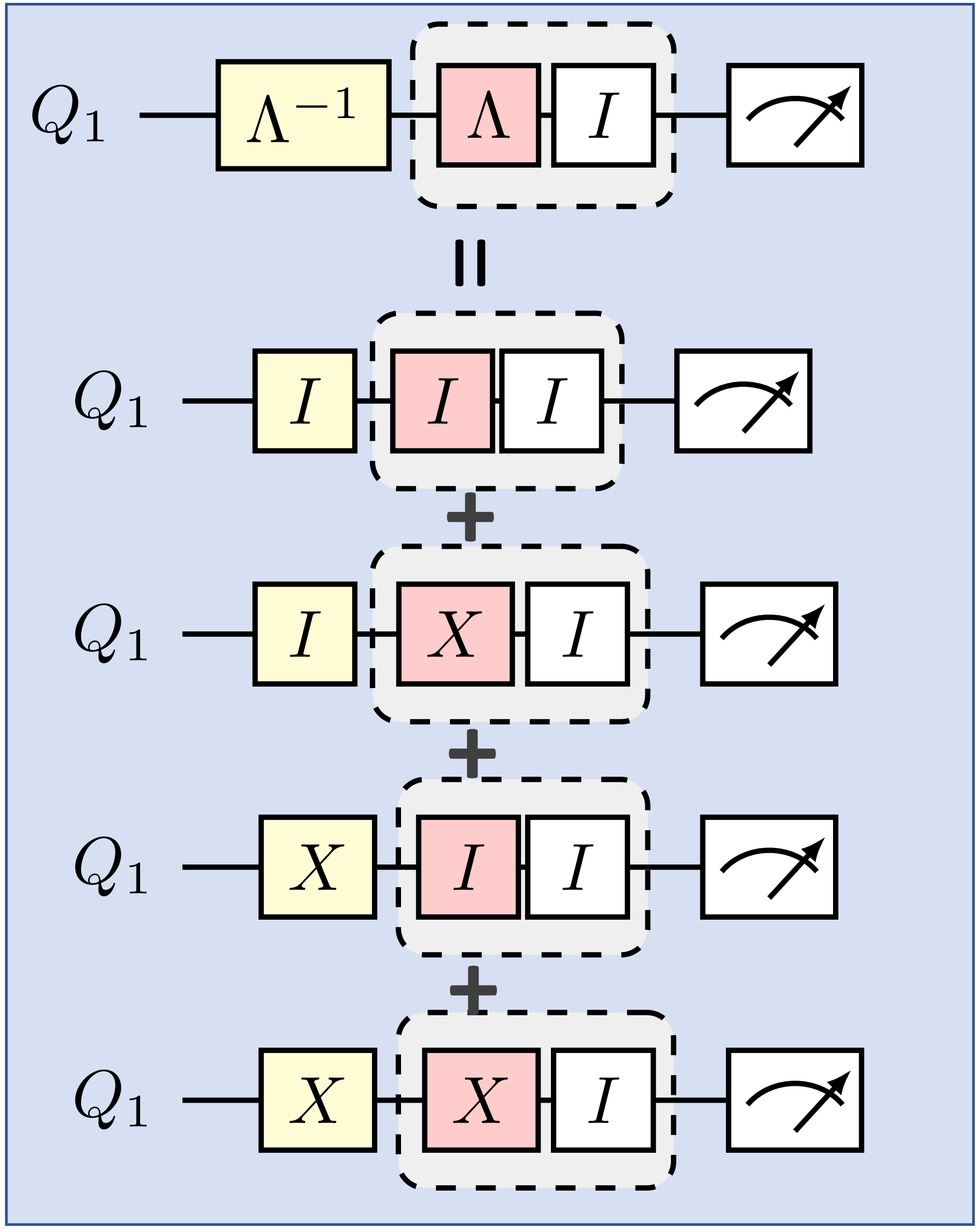




Raveling quantum trajectories to undo noise

	probability
No error	(1-q) (1-p)
ERROR!	(1-q) p
ERROR!	q (1-p)
Error CANCELED!	q p

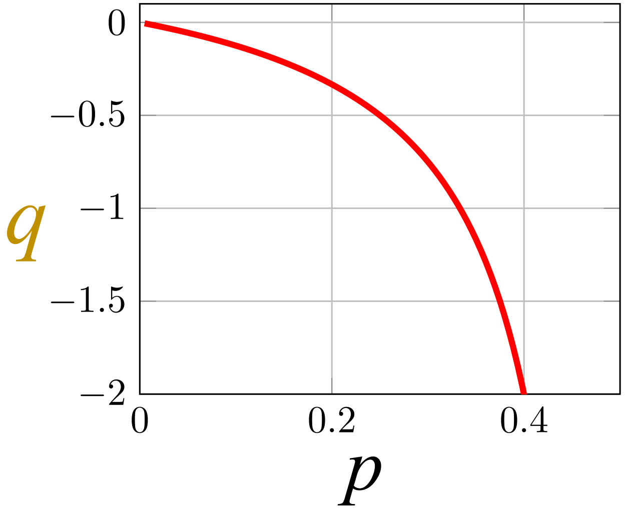




Solution to noise free!

$$q = \frac{-p}{1-2p}$$

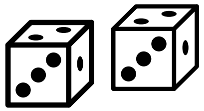
Sign & scale:
quasi-probability



How to implement?

probability

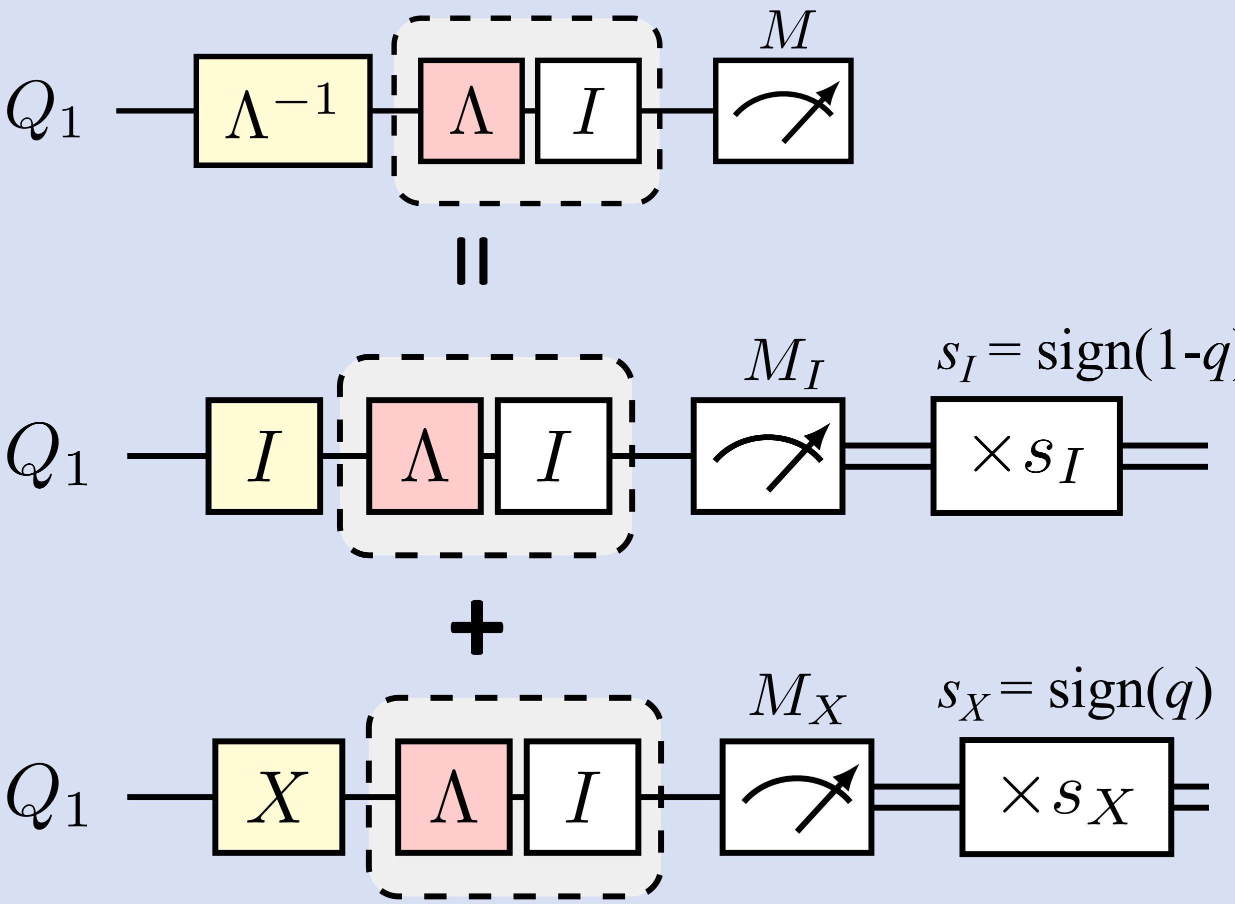
$P_I = |1\text{-}q|/\gamma$



$P_X = |q|/\gamma$

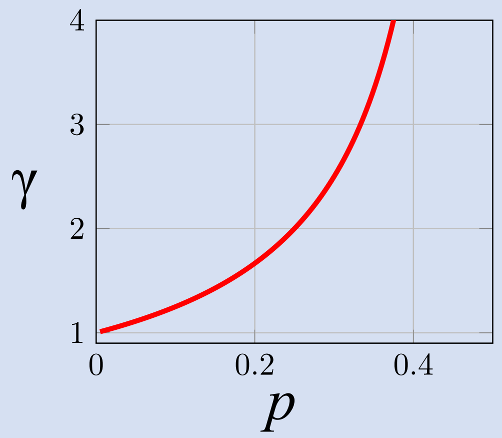
$P_I + P_X = 1$

random circuits
(quantum trajectories)



sampling overhead

$\gamma = |1\text{-}q| + |q|$



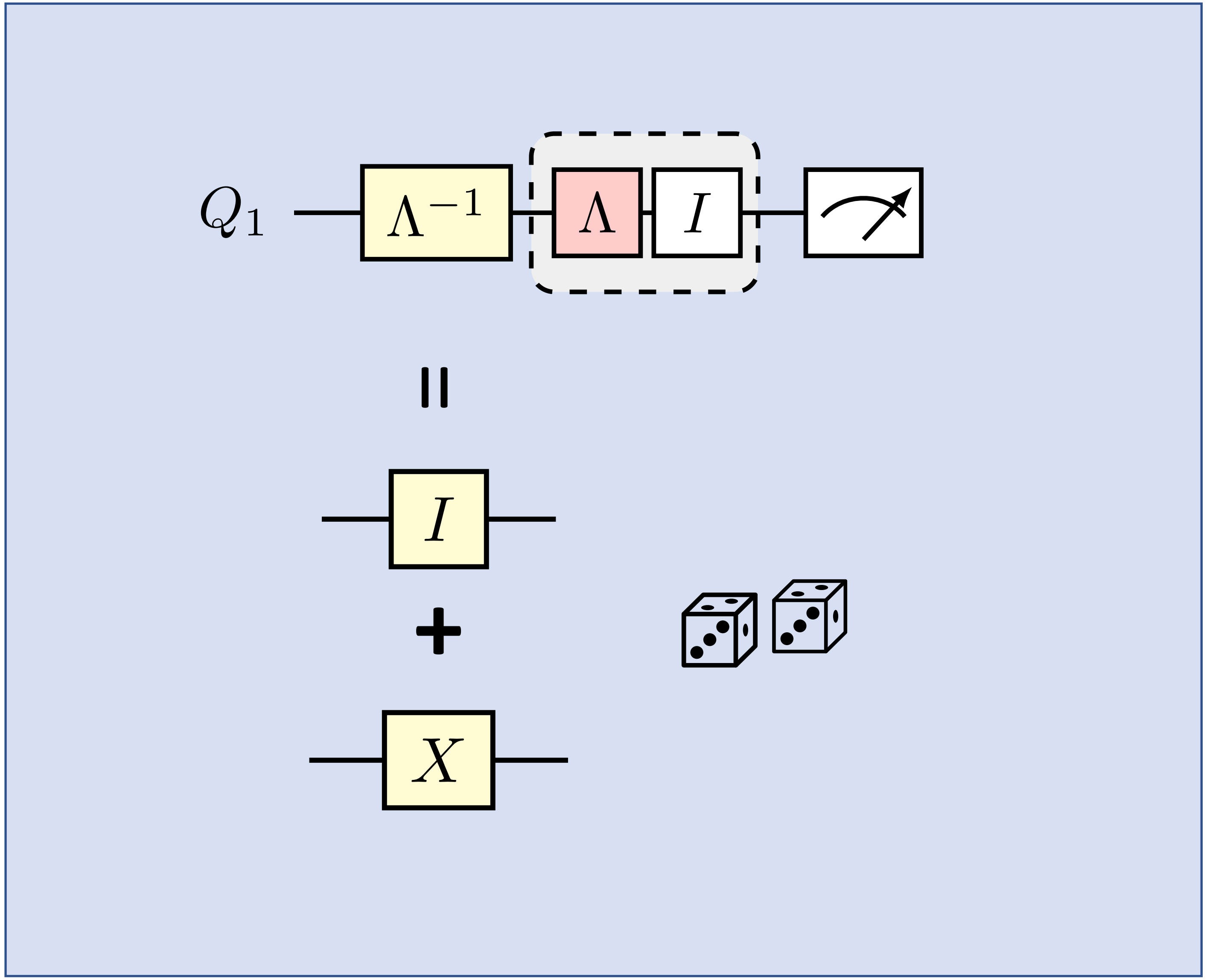
mitigated expectation

$$\langle M \rangle = \gamma\left(s_I P_I M_I + s_X P_X M_X\right)$$

Gain: Bias-free estimate!

Cost: Variance

Canceling noise with noise



Canceling noise with noise: Drunkard's classical random walk analogy

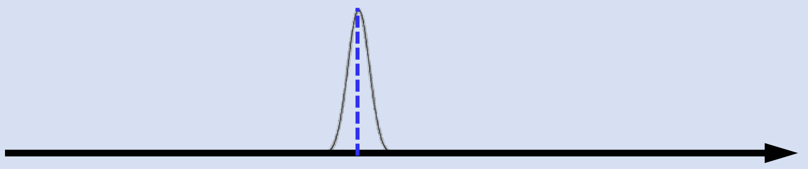
Random step

P(1 step left) = ½-p

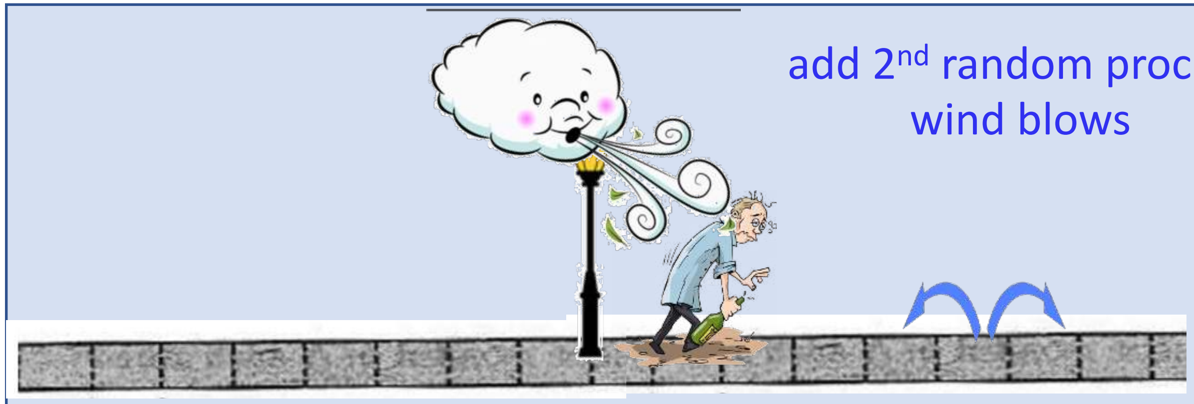
P(1 step right) = ½+p

Distribution of random walk

$t = 0$



$t > 0$



add 2[nd] random process
wind blows

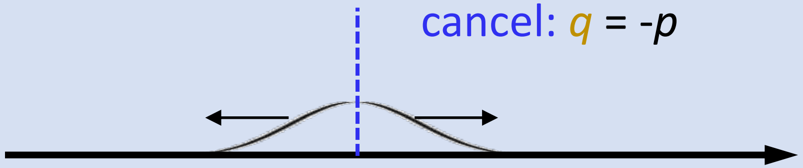
P(1 step left) = ½+q

P(1 step right) = ½-q

Distribution of random walk with wind

cancel: $q = -p$

$t > 0$



Gain: Bias-free estimate!

Cost: Variance

Quantum Noise and Error Mitigation

Lecture 1

Big picture

Quantum computers status

Why error mitigation?

Noise in quantum computers

Overview of error mitigation

Mitigation fundamentals

Probabilistic error cancelation (PEC)

Introduction

One qubit example

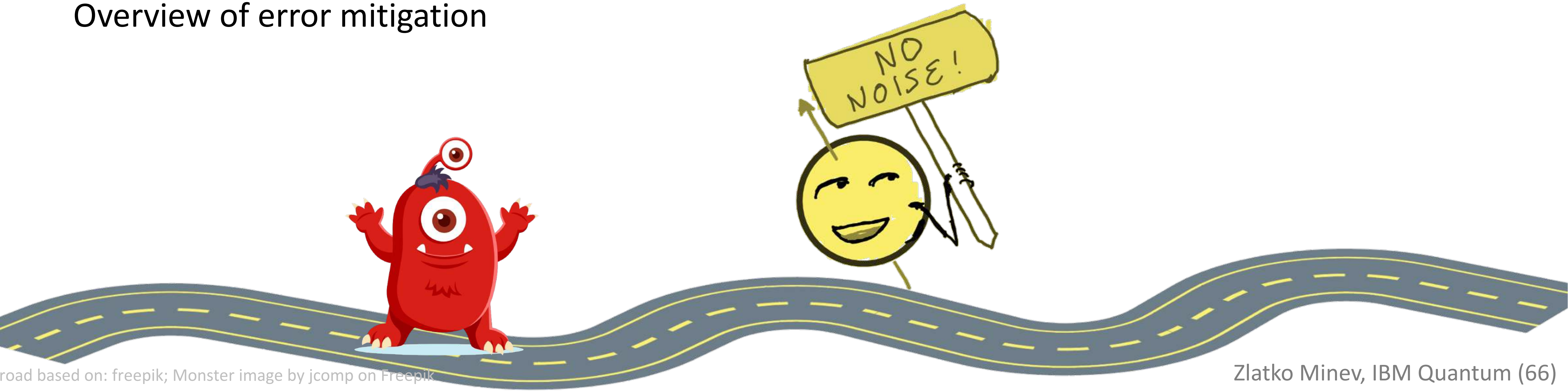
Next lecture

Learning noise

State-of-art mitigation experiments

Hardware

Outlook



road based on: freepik; Monster image by jcomp on Freepik