AI VIETNAM
Free AI Course 2020

Linear Algebra and Applications

Quang-Vinh Dinh
Hong-Phuc Nguyen

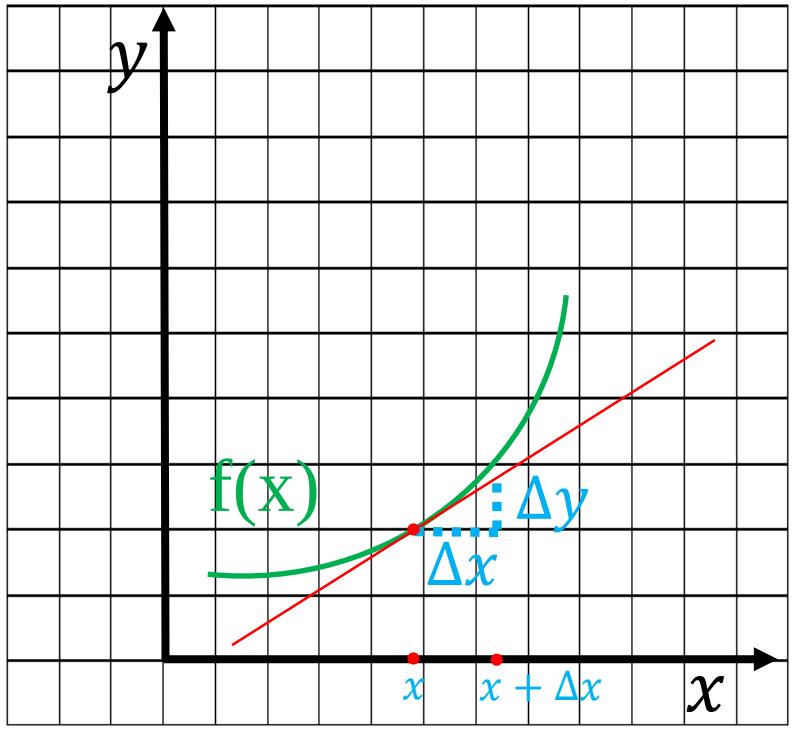
Year 2020

Outline

- Derivative/Gradient
- Interpolation
- Vector and Matrix
- Cosine Similarity
- Integral

Derivative and Applications

Đạo hàm cho hàm liên tục

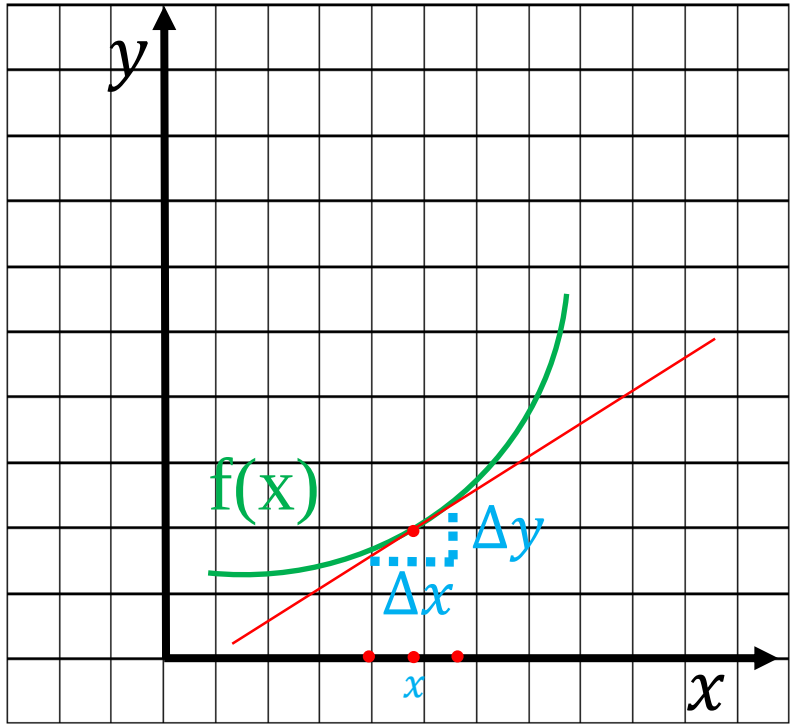


$$\text{Đạo hàm} = \frac{\textit{Thay đổi theo } y}{\textit{Thay đổi theo } x} = \frac{\Delta y}{\Delta x}$$

$$\frac{d}{dx} f(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Δx cần tiến về 0 để đường tiếp tuyến tiến về hàm f(x) trong vùng lân cận tại x

Á p dụng cho hàm rời rạc



$$\text{Đạo hàm} = \frac{Thay\ đổi\ theo\ y}{Thay\ đổi\ theo\ x} = \frac{\Delta y}{\Delta x}$$

$$\frac{d}{dx} f(x) = \lim_{\Delta x \to 0} \frac{f\left(x + \frac{\Delta x}{2}\right) - f\left(x - \frac{\Delta x}{2}\right)}{\Delta x}$$

32	30	45	36	160	156	155	170

x

$\Delta x = 2$

$$\frac{d}{dx} f(x) = \frac{f(x+1) - f(x-1)}{2} = \frac{156 - 36}{2} = 60$$

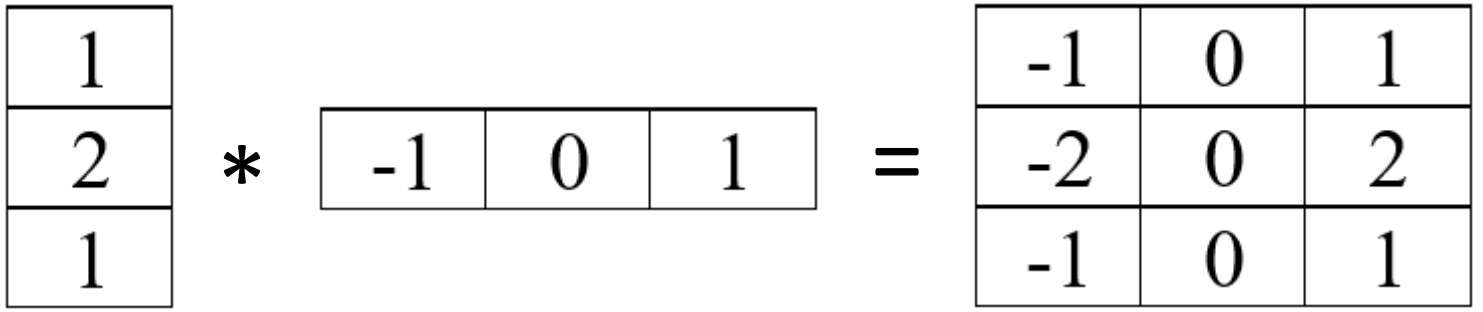
➡

-1	0	1

x-derivative filter

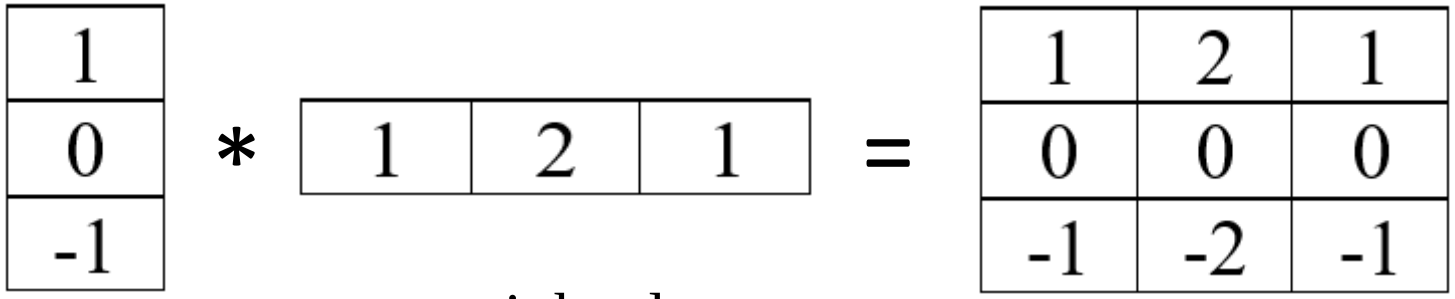
Derivative and Applications

Tính đạo hàm trung bình theo hướng x



weighted average | x-derivative | Sobel for x direction

Tính đạo hàm trung bình theo hướng y



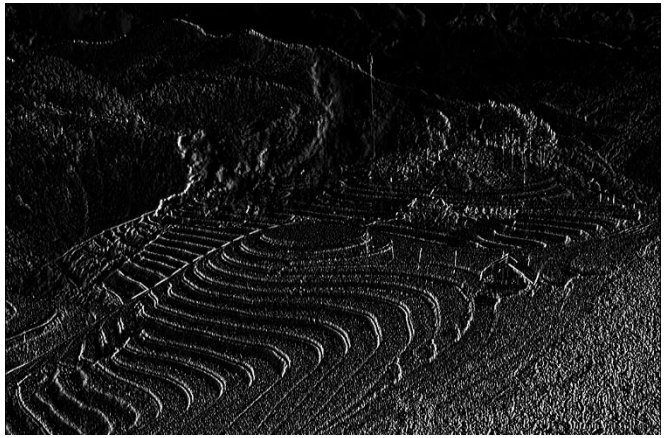
y-derivative | weighted average | Sobel for y direction

Derivative and Applications

Ứng dụng đạo hàm cho edge detection

Edge detection →

Sobel-X



$$G_x = \begin{bmatrix} -1 & 0 & +1 \\ -2 & 0 & +2 \\ -1 & 0 & +1 \end{bmatrix} * I$$

$$G_y = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ +1 & +2 & +1 \end{bmatrix} * I$$

Sobel-Y

Derivative and Applications

❖ Code

```python
import numpy as np
import cv2

# read image and convert to grayscale
img1 = cv2.imread('vn.jpeg', 0)

# compute sobel-x
sobelx = cv2.Sobel(img1, cv2.CV_64F,1,0)

# compute sobel-y
sobely = cv2.Sobel(img1, cv2.CV_64F,0,1)

# save results
cv2.imwrite('vn_edge_x.jpg', sobelx)
cv2.imwrite('vn_edge_y.jpg', sobely)
cv2.imwrite('vn_grayscale.jpg', img1)
```

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- Derivative/Gradient
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- Vector and Matrix
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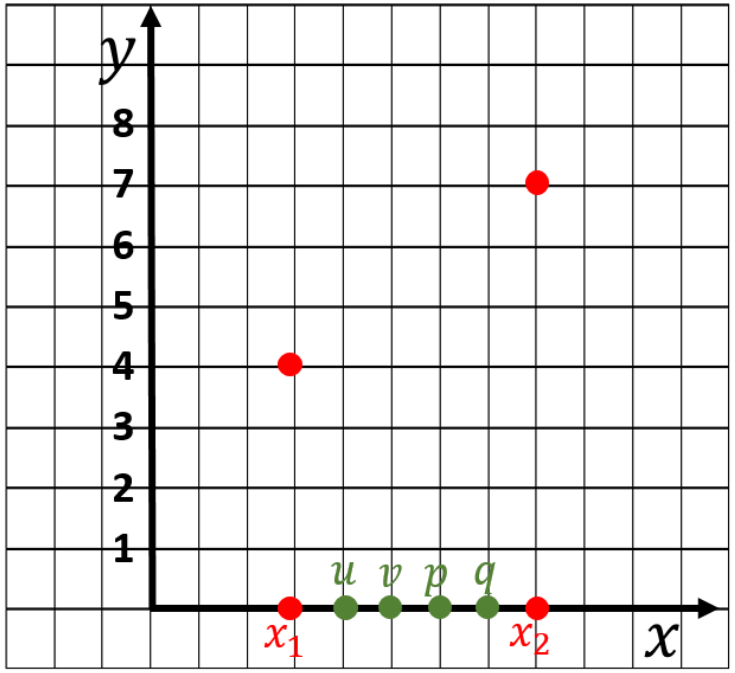
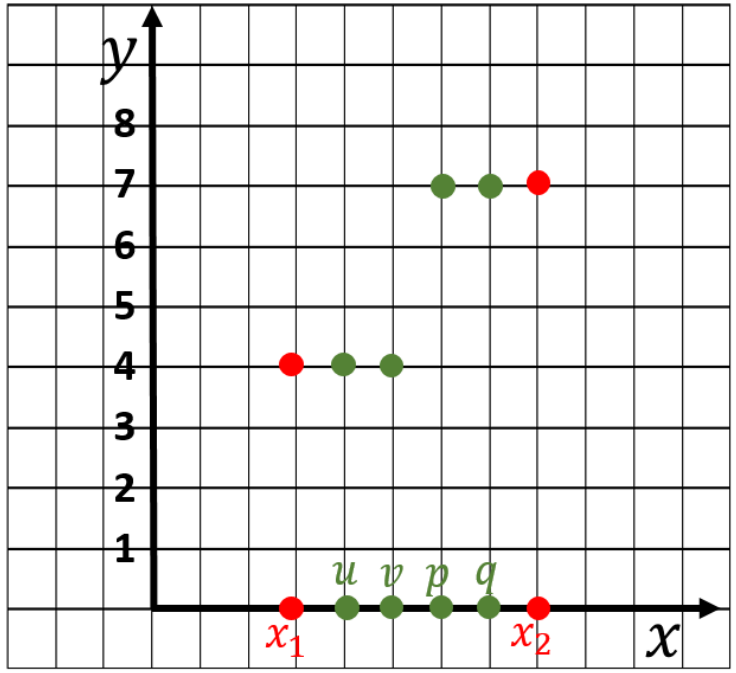
- Integral

Interpolation

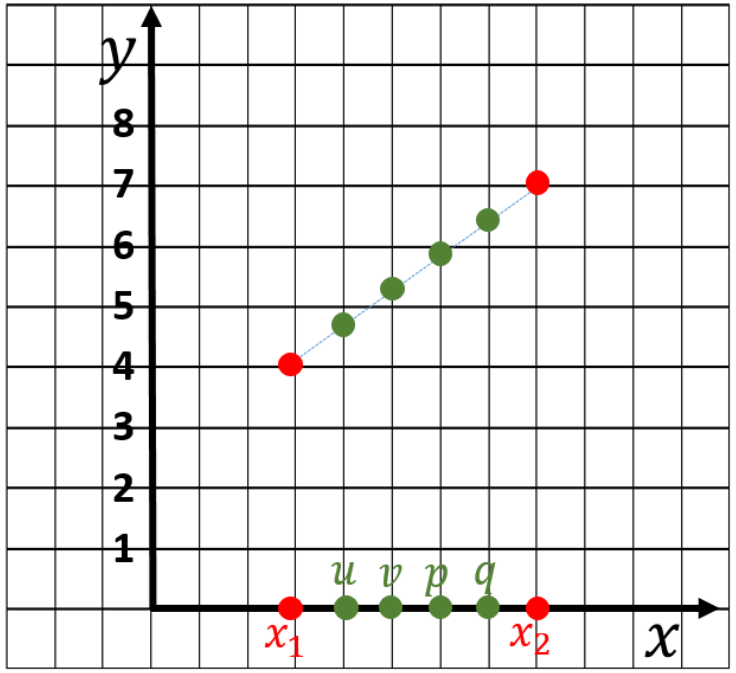
❖ Two simplest techniques



Tìm giá trị cho các vị trí u, v, p và q



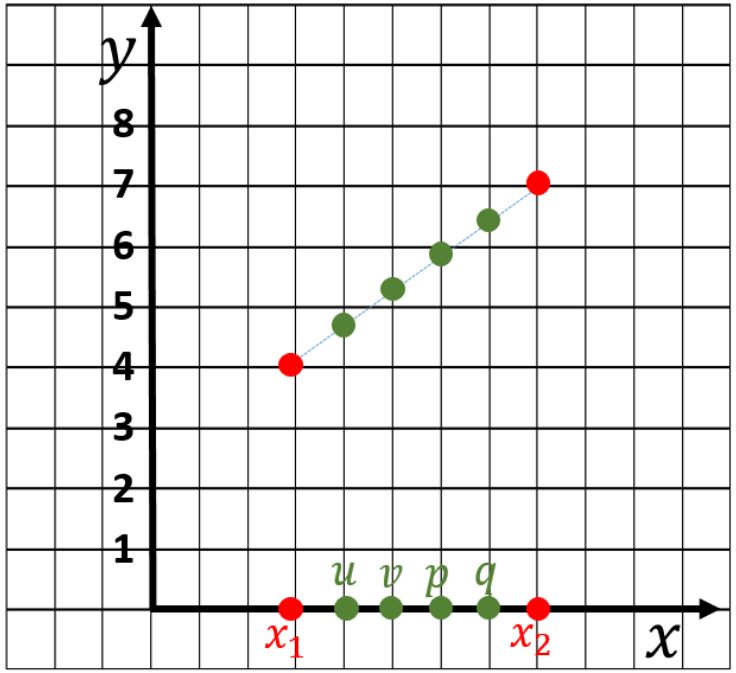
Nearest neighbor: Tính khoảng cách đến x_1 và x_2, và lấy giá trị của x gần hơn



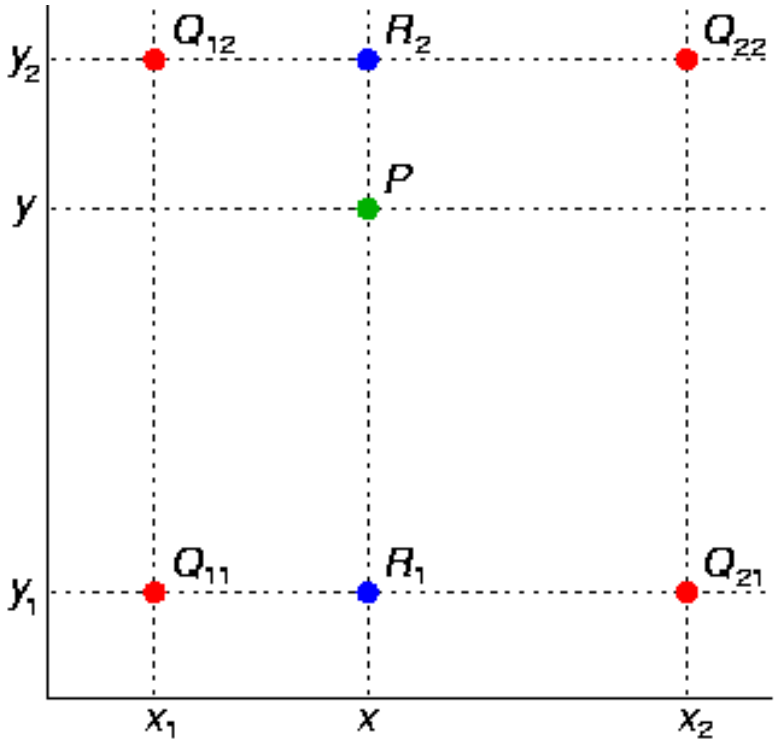
Nội suy theo hàm tuyến tính

Interpolation

❖ Linear and bilinear interpolation



Nội suy theo hàm tuyến tính



http://en.wikipedia.org/wiki/Bilinear_interpolation

Interpolation

❖ Image upsampling

Phóng to ảnh (upsampling/super-resolution) có thể giải quyết bằng phép nội suy; tìm giá trị cho các điểm ảnh mới ở ảnh phóng to, dựa vào thông tin từ ảnh gốc.

Ảnh gốc

Ảnh phóng to dùng nearest neighbor

Ảnh phóng to dùng hàm tuyến tính

Interpolation

❖ Image upsampling

Ảnh gốc

Ảnh phóng to dùng hàm tuyến tính

Advanced Method

Interpolation

❖ Code

```python
import numpy as np
import cv2

# read a color image
img = cv2.imread('image.jpg')

# get meta-data of the image
height, width, channels = img.shape

# new dimension
new_dim = (width*4, height*4)

# upsampling the image
resize_nearest  = cv2.resize(img, new_dim, interpolation=cv2.INTER_NEAREST)
resize_bilinear = cv2.resize(img, new_dim, interpolation=cv2.INTER_LINEAR)

# save results
cv2.imwrite('resize_nearest.jpg', resize_nearest)
cv2.imwrite('resize_bilinear.jpg', resize_bilinear)
```

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Vector & Matrix

Vector

n is a natural number

$\mathcal{R}$ is a set of real numbers

$\vec{v}$ has a length of n and contain real numbers

$\vec{v} \in \mathcal{R}^n$

$$\vec{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \in \begin{bmatrix} \mathcal{R} \\ \mathcal{R} \\ \mathcal{R} \end{bmatrix} = \mathcal{R}^3$$

Matrix

Matrix A has the shape of rectangle

Has m rows and n columns

Use capital letter

$A \in \mathcal{R}^{m \times n}$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix} \in \begin{bmatrix} \mathcal{R} & \mathcal{R} \\ \mathcal{R} & \mathcal{R} \\ \mathcal{R} & \mathcal{R} \end{bmatrix} = \mathcal{R}^{3 \times 2}$$

Vector Operations

Addition

$$\vec{v} = \begin{bmatrix} v_1 \\ \dots \\ v_3 \end{bmatrix} \qquad \vec{u} = \begin{bmatrix} u_1 \\ \dots \\ u_3 \end{bmatrix}$$

$$\vec{v} + \vec{u} = \begin{bmatrix} v_1 \\ \dots \\ v_3 \end{bmatrix} + \begin{bmatrix} u_1 \\ \dots \\ u_3 \end{bmatrix} = \begin{bmatrix} v_1 + u_1 \\ \dots \\ v_3 + u_3 \end{bmatrix}$$

```python
def add_vectors(vector1, vector2):
    '''
    Add corrosponding elements between two vectors
    vector1 and vector2 are with list type
    output is a vector (list)
    '''

    return [v1+v2 for v1, v2 in zip(vector1, vector2)]

# Test case
vector1 = [1, 2, 3]
vector2 = [4, 5, 6]

output = add_vectors(vector1, vector2)
print(output)
```

```
[5, 7, 9]
```

Vector Operations

Subtraction

$$\vec{v} = \begin{bmatrix} v_1 \\ \dots \\ v_n \end{bmatrix} \qquad \vec{u} = \begin{bmatrix} u_1 \\ \dots \\ u_n \end{bmatrix}$$

$$\vec{v} + \vec{u} = \begin{bmatrix} v_1 \\ \dots \\ v_n \end{bmatrix} - \begin{bmatrix} u_1 \\ \dots \\ u_n \end{bmatrix} = \begin{bmatrix} v_1 - u_1 \\ \dots \\ v_n - u_n \end{bmatrix}$$

Multiply with a number

$$\alpha\vec{u} = \alpha \begin{bmatrix} u_1 \\ \dots \\ u_n \end{bmatrix} = \begin{bmatrix} \alpha u_1 \\ \dots \\ \alpha u_n \end{bmatrix}$$

Length of a vector

$$\|\vec{u}\| = \sqrt{u_1^2 + \cdots + u_n^2}$$

Matrix Operations

$$A = \begin{bmatrix} a_{11} & \dots & a_{1n} \\ \dots & \dots & \dots \\ a_{m1} & \dots & a_{mn} \end{bmatrix} \quad B = \begin{bmatrix} b_{11} & \dots & b_{1n} \\ \dots & \dots & \dots \\ b_{m1} & \dots & b_{mn} \end{bmatrix}$$

Addition

$$A + B = \begin{bmatrix} (a_{11} + b_{11}) & \dots & (a_{1n} + b_{1n}) \\ \dots & \dots & \dots \\ (a_{m1} + b_{m1}) & \dots & (a_{mn} + b_{mn}) \end{bmatrix}$$

Subtraction

$$A - B = \begin{bmatrix} (a_{11} - b_{11}) & \dots & (a_{1n} - b_{1n}) \\ \dots & \dots & \dots \\ (a_{m1} - b_{m1}) & \dots & (a_{mn} - b_{mn}) \end{bmatrix}$$

Matrix Operations

Multiplication

$$\mathrm{A} = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \cdots & \cdots & \cdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix} \quad \mathrm{B} = \begin{bmatrix} b_{11} & \cdots & b_{1k} \\ \cdots & \cdots & \cdots \\ b_{n1} & \cdots & b_{nk} \end{bmatrix}$$

$$\mathrm{A} \in \mathcal{R}^{m \times n} \qquad \mathrm{B} \in \mathcal{R}^{n \times k}$$

$$\mathrm{C} = \mathrm{AB}$$

$$c_{11} = \sum_{l=1}^{n} a_{il} b_{lj}$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \\ a_{31}b_{11} + a_{32}b_{21} & a_{31}b_{12} + a_{32}b_{22} \end{bmatrix}$$

Matrix Operations

Multiplication

$$\mathrm{A} = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \cdots & \cdots & \cdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix} \qquad \vec{x} = \begin{bmatrix} x_1 \\ \cdots \\ x_n \end{bmatrix}$$

$$\mathrm{A} \in \mathcal{R}^{m \times n}$$

$$\mathrm{C} = \mathrm{A}\vec{x}$$

$$c_{11} = \sum_{l=1}^{n} a_{il} x_l$$

Example

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} a_{11}x_1 + a_{12}x_2 \\ a_{21}x_1 + a_{22}x_2 \\ a_{31}x_1 + a_{32}x_2 \end{bmatrix}$$

Matrix Operations

Multiplication

$$A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \cdots & \cdots & \cdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix} \quad B = \begin{bmatrix} b_{11} & \cdots & b_{1k} \\ \cdots & \cdots & \cdots \\ b_{n1} & \cdots & b_{nk} \end{bmatrix}$$

$$A \in \mathcal{R}^{m \times n} \qquad B \in \mathcal{R}^{n \times k}$$

$$C = AB$$

$$c_{11} = \sum_{l=1}^{n} a_{il} b_{lj}$$

```python
def matrix_multiplication(matrix1, matrix2):
    '''
    This function does the multiplication between two matrices.
    #columns of matrix1 == #rows of matrix2
    '''
    matrix1_nrows = len(matrix1)
    matrix1_ncols = len(matrix1[0])

    matrix2_nrows = len(matrix2)
    matrix2_ncols = len(matrix2[0])

    # tạo matrix kết quả
    result = [[0]*matrix2_ncols for i in range(matrix1_nrows)]

    for i in range(matrix1_nrows):
        for j in range(matrix2_ncols):
            for k in range(matrix2_nrows):
                result[i][j] += matrix1[i][k] * matrix2[k][j]

    return result

# test case
# 3x3 matrix
matrix1 = [[1, 2, 3],
           [4, 5, 6],
           [7, 8, 9]]

# 3x4 matrix
matrix2 = [[1, 1, 2, 1],
           [1, 2, 1, 1],
           [1, 1, 1, 2]]

result = matrix_multiplication(matrix1, matrix2)
print(result[0])
print(result[1])
print(result[2])
```

```
[6, 8, 7, 9]
[15, 20, 19, 21]
[24, 32, 31, 33]
```

Matrix Operations

Transpose

$$\mathrm{A} = \begin{bmatrix} a_{11} & \dots & a_{1n} \\ \dots & \dots & \dots \\ a_{m1} & \dots & a_{mn} \end{bmatrix}$$

$$\mathrm{A}^T = \begin{bmatrix} a_{11} & \dots & a_{m1} \\ \dots & \dots & \dots \\ a_{1n} & \dots & a_{mn} \end{bmatrix}$$

Example

$$\begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \end{bmatrix}^T = \begin{bmatrix} \alpha_1 & \beta_1 \\ \alpha_2 & \beta_2 \\ \alpha_3 & \beta_3 \end{bmatrix}$$

Matrix and Vector

Phép nhân giữa ma trận và vector

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad \boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad \boldsymbol{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$A\boldsymbol{x} = b$$

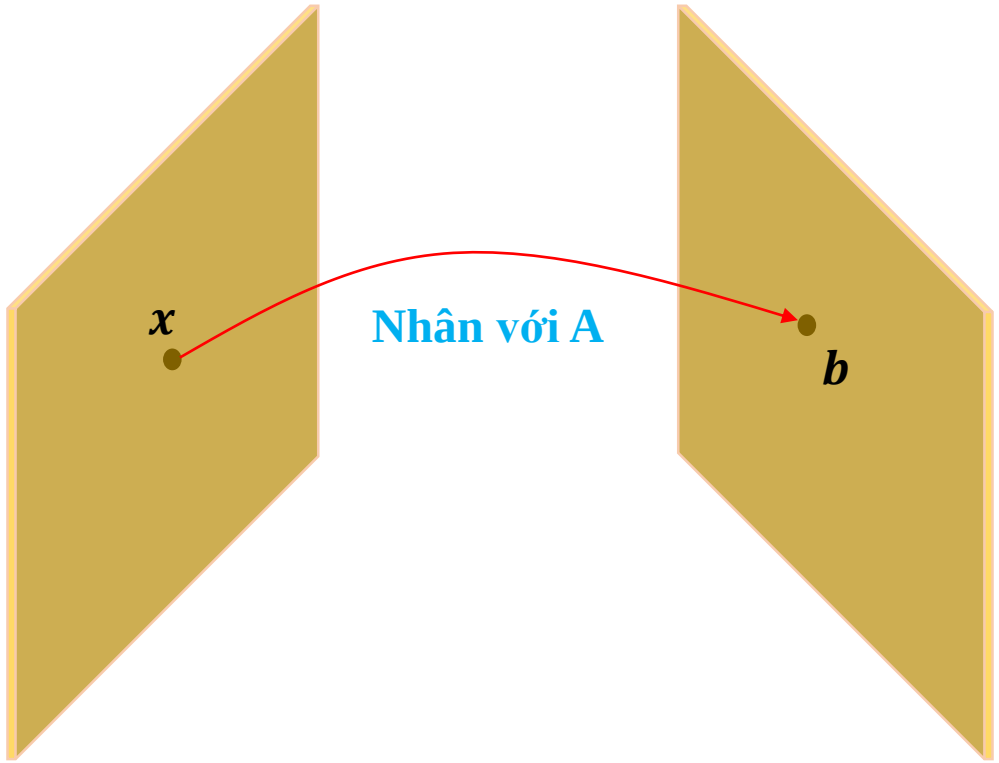
$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} a_{11}x_1 + a_{12}x_2 \\ a_{21}x_1 + a_{22}x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

Ma trận A biến đổi/dịch chuyển x sang b

Giá trị từng phần tử của b là tổ hợp tuyến tính của tất cả các phần tử của x

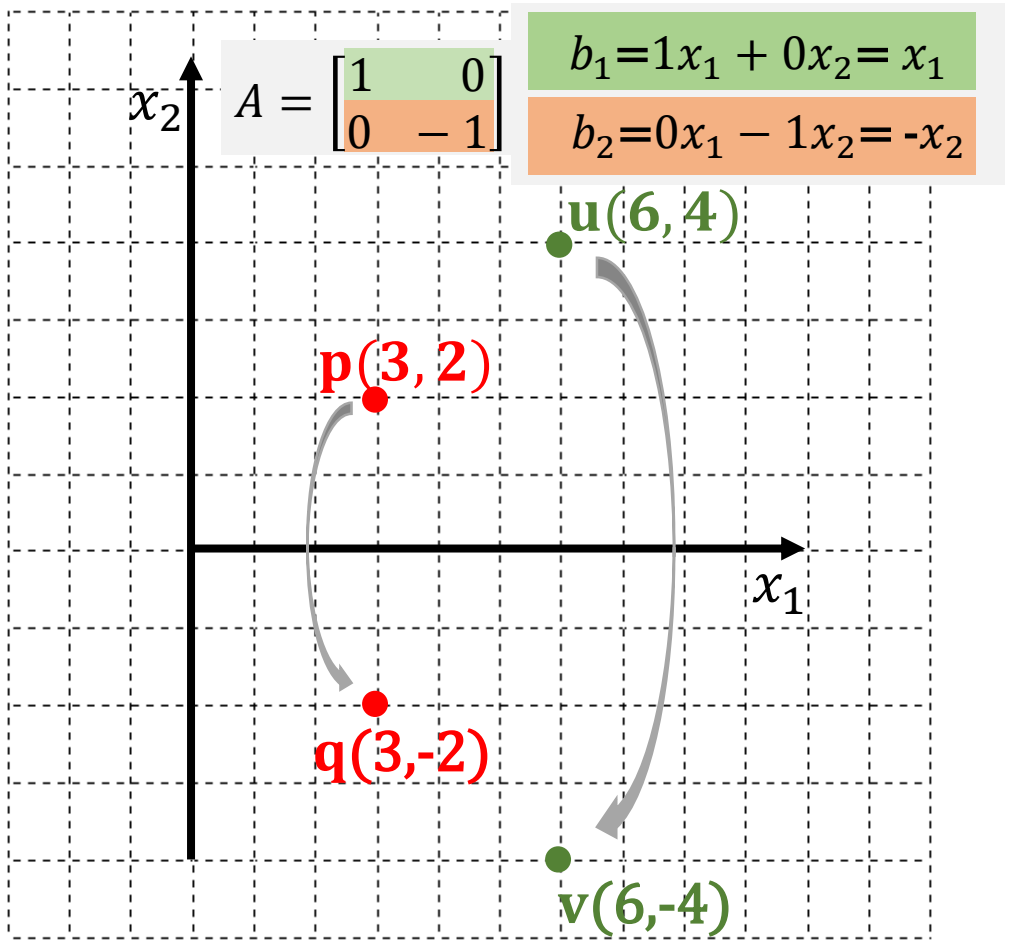
$$b_1 = a_{11}x_1 + a_{12}x_2$$

$$b_2 = a_{21}x_1 + a_{22}x_2$$



Matrix and Vector

Ma trận A dịch chuyển điểm x đối xứng qua trục x_1



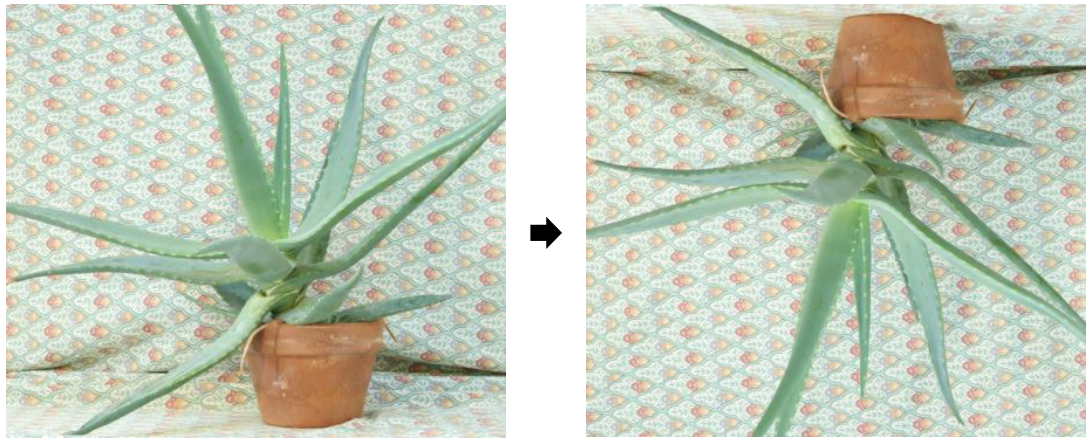
Ứng dụng lật ảnh đối xứng qua trục ngang

Giá trị màu (red, green, blue) của điểm **p**

Các thuộc tính của pixel p(x_1, x_2, r, g, b)

Tọa độ điểm **p**

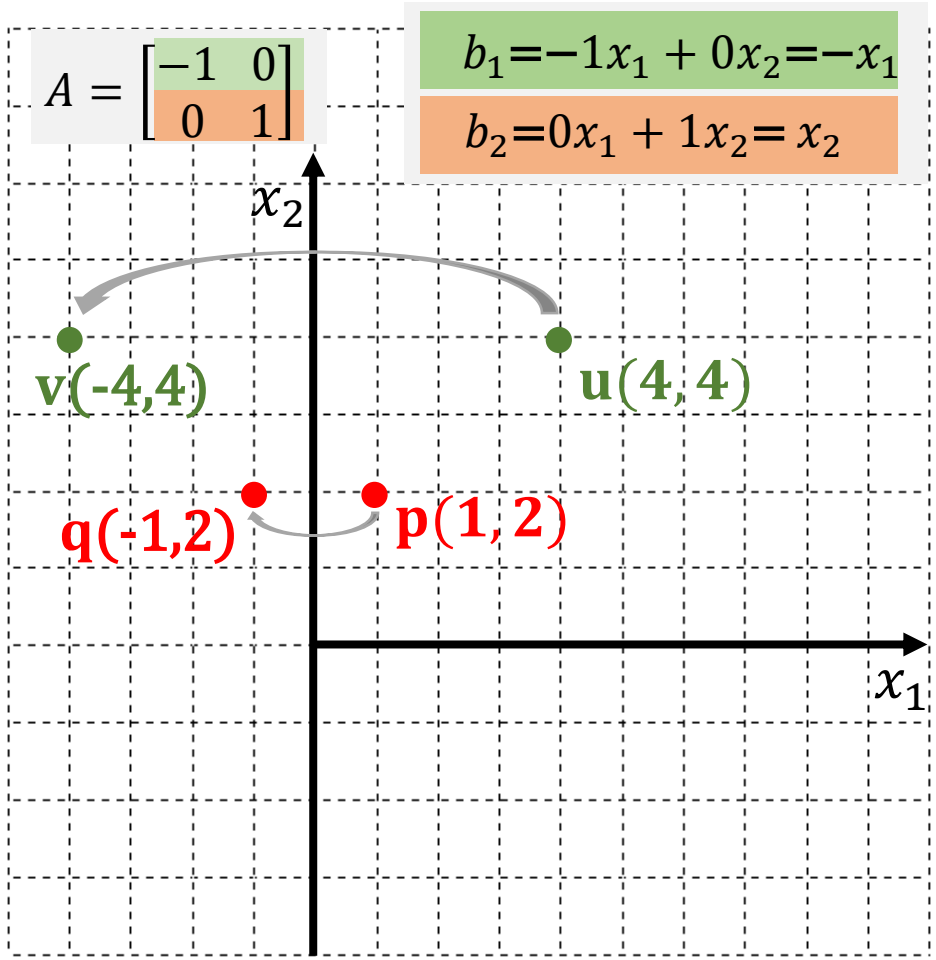
Dịch chuyển pixel (x_1, x_2) **theo** $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$



Ảnh gốc ➡ **Ảnh kết quả**

Matrix and Vector

Ma trận A dịch chuyển điểm x đối xứng qua trục x_2



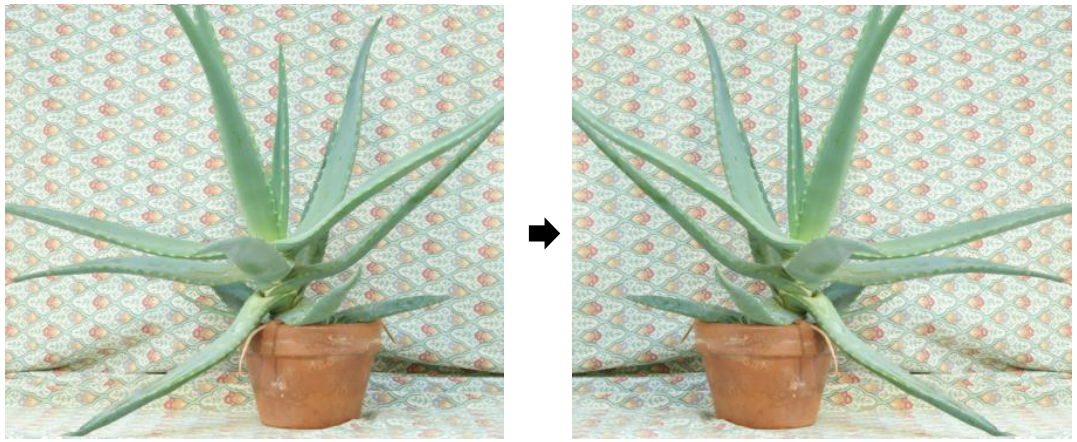
Ứng dụng lật ảnh đối xứng qua trục đứng

Giá trị màu (red, green, blue) của điểm **p**

Các thuộc tính của pixel p(x_1, x_2, r, g, b)

Tọa độ điểm **p**

Dịch chuyển pixel (x_1, x_2) **theo** $A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$



Ảnh gốc | Ảnh kết quả

Background Subtraction

Background

coming image

Background Subtraction

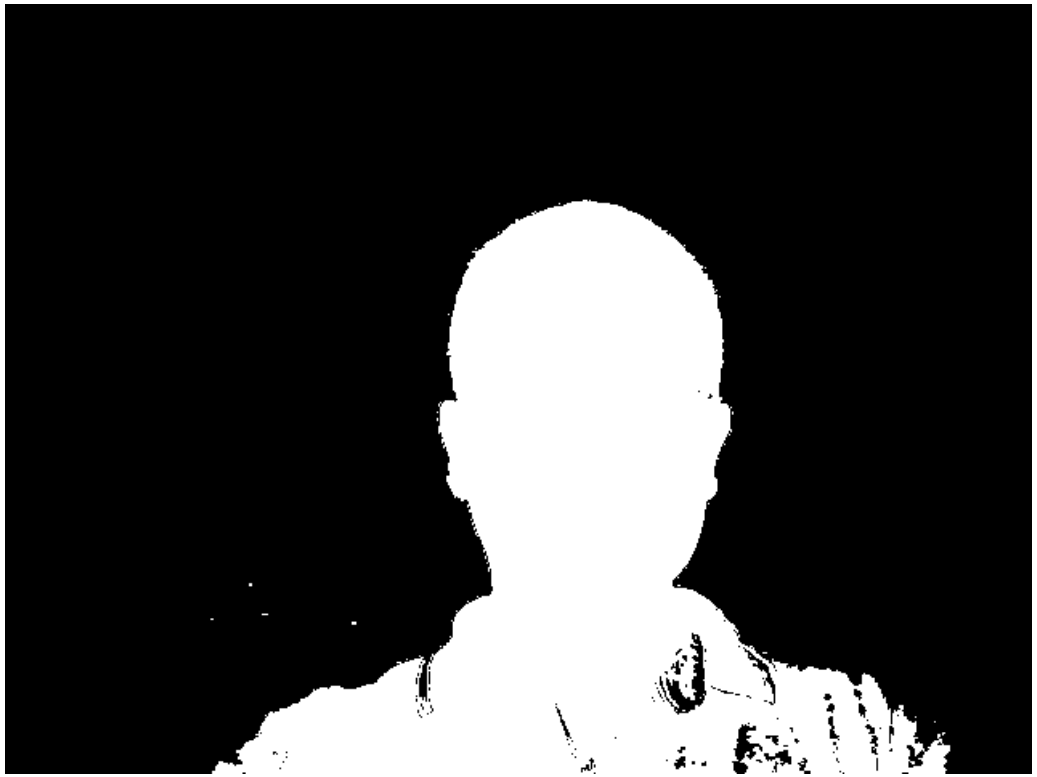
Background

Coming image

Background Subtraction

Background Subtraction

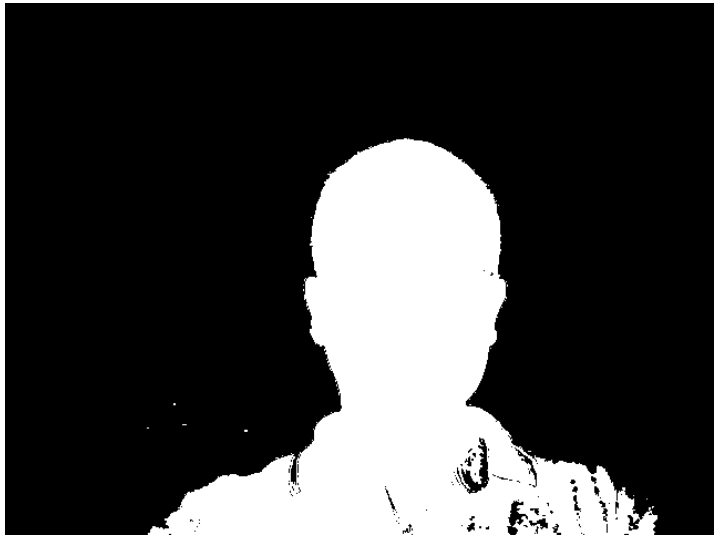
Subtracted Background



Mask

Background Subtraction

Background

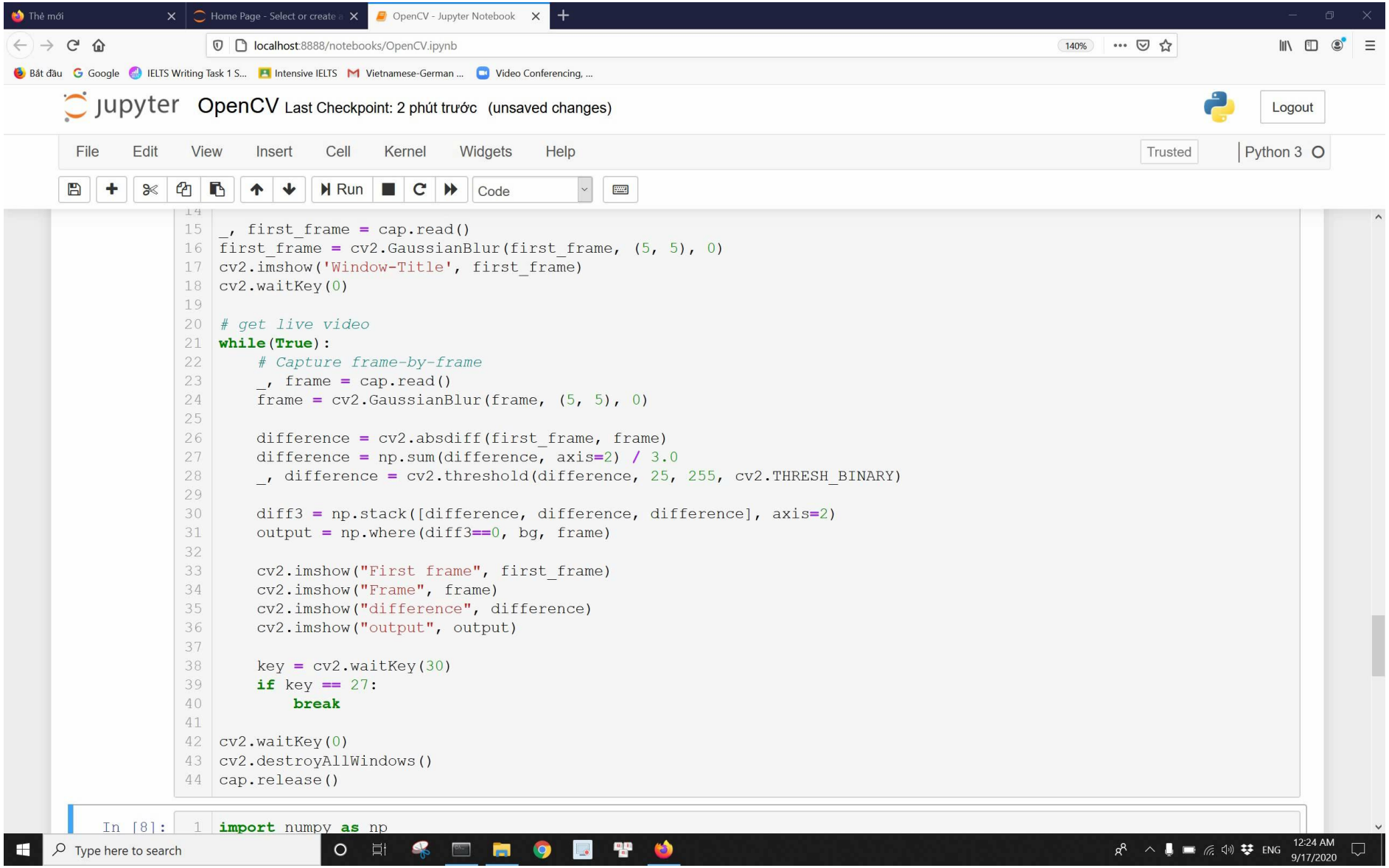


Mask

Coming image

FakeBackground

Background Subtraction



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Vector Operations

Dot product

$$\vec{v} = \begin{bmatrix} v_1 \\ \dots \\ v_n \end{bmatrix} \qquad \vec{u} = \begin{bmatrix} u_1 \\ \dots \\ u_n \end{bmatrix}$$

$$\vec{v} \cdot \vec{u} = v_1 \times u_1 + \dots + v_n \times u_n$$

```python
def dot_product(vector1, vector2):
    '''
    Compute dot product between two vectors
    Output is a floating-point number
    '''

    return sum([v1*v2 for v1, v2 in zip(vector1, vector2)])

# test case
vector1 = [1, 2, 3]
vector2 = [2, 3, 4]

ouptut = dot_product(vector1, vector2)
print(ouptut)
```

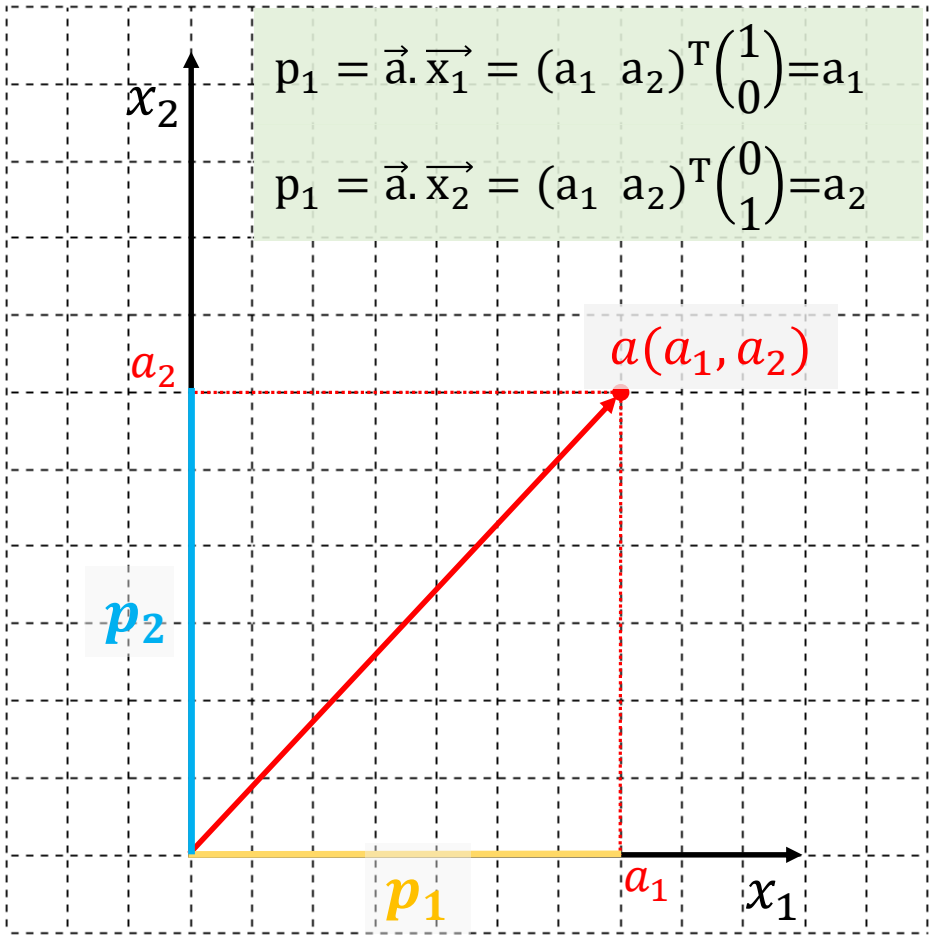
```
20
```

Dot Product

$\overrightarrow{x_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\overrightarrow{x_2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$p_1 = \vec{a}.\overrightarrow{x_1} = (a_1\ a_2)^T\begin{pmatrix} 1 \\ 0 \end{pmatrix} = a_1$

$p_1 = \vec{a}.\overrightarrow{x_2} = (a_1\ a_2)^T\begin{pmatrix} 0 \\ 1 \end{pmatrix} = a_2$

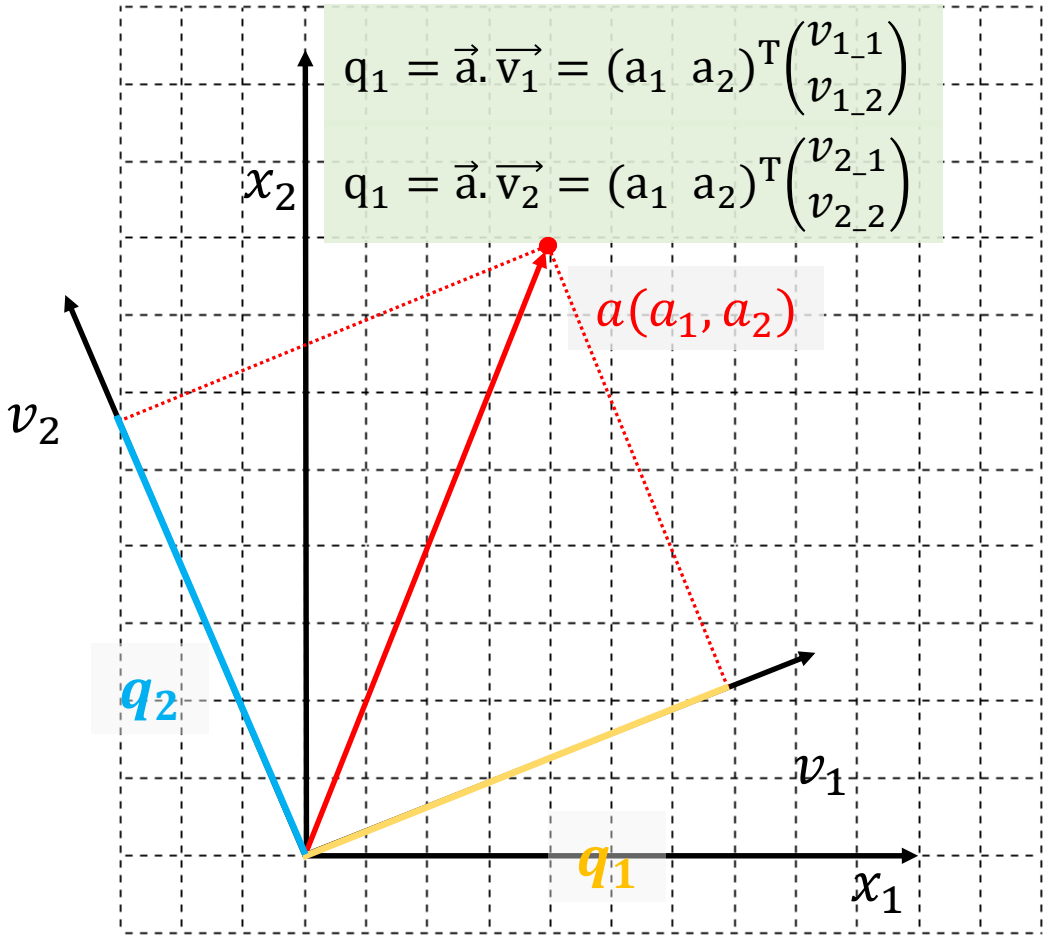


Tìm độ dài hình chiếu của $\vec{a}$ lên $\overrightarrow{x_1}$ và $\overrightarrow{x_2}$

$\overrightarrow{v_1} = \begin{pmatrix} v_{1\_1} \\ v_{1\_2} \end{pmatrix}$ $\overrightarrow{v_2} = \begin{pmatrix} v_{2\_1} \\ v_{2\_2} \end{pmatrix}$

$q_1 = \vec{a}.\overrightarrow{v_1} = (a_1\ a_2)^T\begin{pmatrix} v_{1\_1} \\ v_{1\_2} \end{pmatrix}$

$q_1 = \vec{a}.\overrightarrow{v_2} = (a_1\ a_2)^T\begin{pmatrix} v_{2\_1} \\ v_{2\_2} \end{pmatrix}$



Tìm độ dài hình chiếu của $\vec{a}$ lên $\overrightarrow{v_1}$ và $\overrightarrow{v_2}$

Dot Product

Matrix decomposition

Mục đích: Đưa ma trận Q về dạng UΣ sao cho các vector (cột) trong U có độ dài bằng 1.

Công thức

$$\begin{pmatrix} a & c \\ b & d \end{pmatrix} = \begin{pmatrix} \frac{a}{\sqrt{a^2+b^2}} & \frac{c}{\sqrt{c^2+d^2}} \\ \frac{b}{\sqrt{a^2+b^2}} & \frac{d}{\sqrt{c^2+d^2}} \end{pmatrix} \begin{pmatrix} \sqrt{a^2+b^2} & 0 \\ 0 & \sqrt{c^2+d^2} \end{pmatrix}$$

Ví dụ

$$\begin{pmatrix} 3 & 2 \\ 4 & 0 \end{pmatrix} = \begin{pmatrix} \frac{3}{\sqrt{3^2+4^2}} & \frac{2}{\sqrt{2^2+0}} \\ \frac{4}{\sqrt{3^2+4^2}} & 0 \end{pmatrix} \begin{pmatrix} \sqrt{3^2+4^2} & 0 \\ 0 & \sqrt{2^2+0} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{3}{5} & 1 \\ \frac{4}{5} & 0 \end{pmatrix} \begin{pmatrix} 5 & 0 \\ 0 & 2 \end{pmatrix}$$

Cách tính tương tự khi Q có kích thước khác

Dot Product

Singular Value Decomposition

Tìm độ dài hình chiếu của $\vec{a}$ lên $\overrightarrow{v_1}$ và $\overrightarrow{v_2}$

$$q_1 = \vec{a}.\overrightarrow{v_1} = (a_1 \ \ a_2)^T \begin{pmatrix} v_{1\_1} \\ v_{1\_2} \end{pmatrix}$$

$$q_1 = \vec{a}.\overrightarrow{v_2} = (a_1 \ \ a_2)^T \begin{pmatrix} v_{2\_1} \\ v_{2\_2} \end{pmatrix}$$

Viết lại dạng ma trận cho ngắn gọn

$$\vec{a}.V = (a_1 \ \ a_2)^T \begin{pmatrix} v_{1\_1} & v_{2\_1} \\ v_{1\_2} & v_{2\_2} \end{pmatrix}$$

$$= (q_1 \ \ q_2)$$

Nếu có thêm $\vec{b}$

$$\underset{A}{\begin{pmatrix} a_1 & a_2 \\ b_1 & b_2 \end{pmatrix}^T} \underset{V}{\begin{pmatrix} v_{1\_1} & v_{2\_1} \\ v_{1\_2} & v_{2\_2} \end{pmatrix}} = \underset{Q}{\begin{pmatrix} q_{1a} & q_{2a} \\ q_{1b} & q_{2b} \end{pmatrix}}$$

Ma trận các điểm | Ma trận các trục | Ma trận độ dài hình chiếu

Giả sử V là ma trận trực giao

$$AV = Q$$

$$A = QV^{-1} = QV^T \quad \text{chuyển về V}$$

$$A = QV^{-1} = U\Sigma V^T \quad \text{tách Q}$$

Tổng quát

$$A = U\Sigma V^T$$

A là ma trận $n \times d$, có n điểm và mỗi điểm có d phần tử

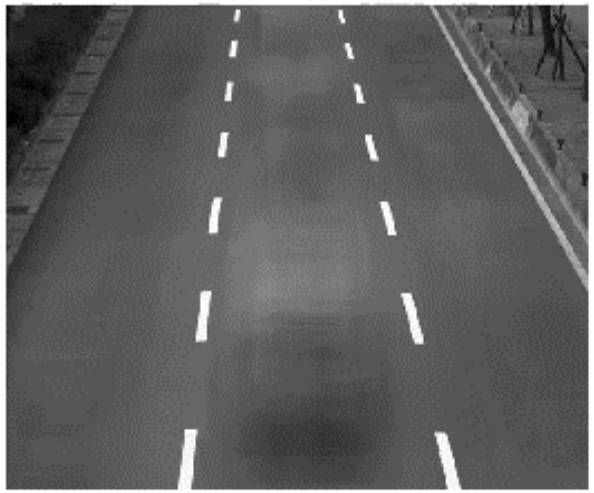
U là ma trận $n \times n$ chứa độ dài hình chiếu, trong đó các vector cột có chiều dài bằng 1

Σ là ma trận đường chéo $(n \times d)$, xác định độ dài của các vector

V là ma trận $d \times d$ chứa các trục

Dot Product

Ứng dụng SVD cho background removal



Vector Operations

Hadamard product

$$\vec{v} = \begin{bmatrix} v_1 \\ \dots \\ v_n \end{bmatrix} \qquad \vec{u} = \begin{bmatrix} u_1 \\ \dots \\ u_n \end{bmatrix}$$

$$\vec{v} \odot \vec{u} = \begin{bmatrix} v_1 \\ \dots \\ v_n \end{bmatrix} \odot \begin{bmatrix} u_1 \\ \dots \\ u_n \end{bmatrix} = \begin{bmatrix} v_1 \times u_1 \\ \dots \\ v_n \times u_n \end{bmatrix}$$

```python
def Hadamard_product(vector1, vector2):
    '''
    Compute Hadamard product between two vectors
    Output is a vector
    '''
    return [v1*v2 for v1, v2 in zip(vector1, vector2)]

# test case
vector1 = [1, 2]
vector2 = [3, 4]

output = Hadamard_product(vector1, vector2)
print(output)
```

```
[3, 8]
```

Cosine similarity

Cosine similarity (cs) được dùng để đo mức độ giống nhau/tương đồng giữa hai vector

Gọi $\vec{x}$ và $\vec{y}$ là hai vector, cs được tính như sau

$$\text{cs}(\vec{x},\vec{y}) = \frac{\vec{x}.\vec{y}}{\|\vec{x}\|\|\vec{y}\|} = \frac{\sum_1^n x_i y_i}{\sqrt{\sum_1^n x_i^2}\sqrt{\sum_1^n y_i^2}}$$

Tính chất 1: $\text{cs}(\vec{x},\vec{y}) = \text{cs}(a\vec{x}, b\vec{y})$

$$\text{cs}(a\vec{x}, b\vec{y}) = \frac{a\vec{x}.b\vec{y}}{\|a\vec{x}\|\|b\vec{y}\|} = \frac{\sum_1^n a x_i b y_i}{\sqrt{\sum_1^n a^2 x_i^2}\sqrt{\sum_1^n b^2 y_i^2}}$$

$$= \frac{ab\sum_1^n x_i y_i}{\sqrt{a^2\sum_1^n x_i^2}\sqrt{b^2\sum_1^n y_i^2}}$$

$$= \frac{\sum_1^n x_i y_i}{\sqrt{\sum_1^n x_i^2}\sqrt{\sum_1^n y_i^2}} = \text{cs}(\vec{x},\vec{y})$$

Tính chất 2: $\text{cs}(\vec{x},\vec{y}) \neq \text{cs}(\vec{x}+c, \vec{y}+d)$

Ví dụ:

$$\vec{x} = [4,2,1,2]^T$$

$$\vec{y} = [1,2,2,0]^T$$

$$\vec{u} = 2\vec{x} = [8,4,2,4]^T$$

$$\vec{v} = 3\vec{y} = [3,6,6,0]^T$$

$$\text{cs}(\vec{x},\vec{y}) = \frac{4*1+2*2+1*2+2*0}{\sqrt{4^2+2^2+1^2+2^2}\sqrt{1^2+2^2+2^2+0}}$$

$$= \frac{10}{\sqrt{25}\sqrt{9}} = \frac{10}{15} = 0.67$$

$$\text{cs}(\vec{u},\vec{v}) = \frac{8*3+4*6+2*6+4*0}{\sqrt{8^2+4^2+2^2+4^2}\sqrt{3^2+6^2+6^2+0}}$$

$$= \frac{60}{\sqrt{100}\sqrt{81}} = \frac{60}{90} = 0.67$$

$$= \text{cs}(\vec{x},\vec{y})$$

Cosine similarity

❖ Code

$$cs(\vec{x}, \vec{y}) = \frac{\vec{x}.\vec{y}}{\|\vec{x}\|\|\vec{y}\|} = \frac{\sum_1^n x_i y_i}{\sqrt{\sum_1^n x_i^2}\sqrt{\sum_1^n y_i^2}}$$

```python
import math

def cosine_similarity(vector1, vector2):
    '''
    Compute dot product between two vectors
    Output is a floating-point number
    '''

    sumxy = sum([v1*v2 for v1, v2 in zip(vector1, vector2)])
    sumxx = sum([v1*v2 for v1, v2 in zip(vector1, vector1)])
    sumyy = sum([v1*v2 for v1, v2 in zip(vector2, vector2)])

    return sumxy/math.sqrt(sumxx*sumyy)

# test case
vector1 = [5, 3, 2, 7]
vector2 = [2, 9, 4, 1]

output = cosine_similarity(vector1,vector2)
print(output)
```

```
0.552005787925351
```

Stereo Matching

Stereo Matching / Binocular Depth Estimation

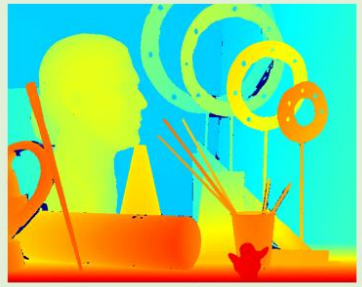
Hình từ camera trái

Hình từ camera phải

Stereo camera

Thuật toán stereo matching

Hình chiều sâu cho hình trái

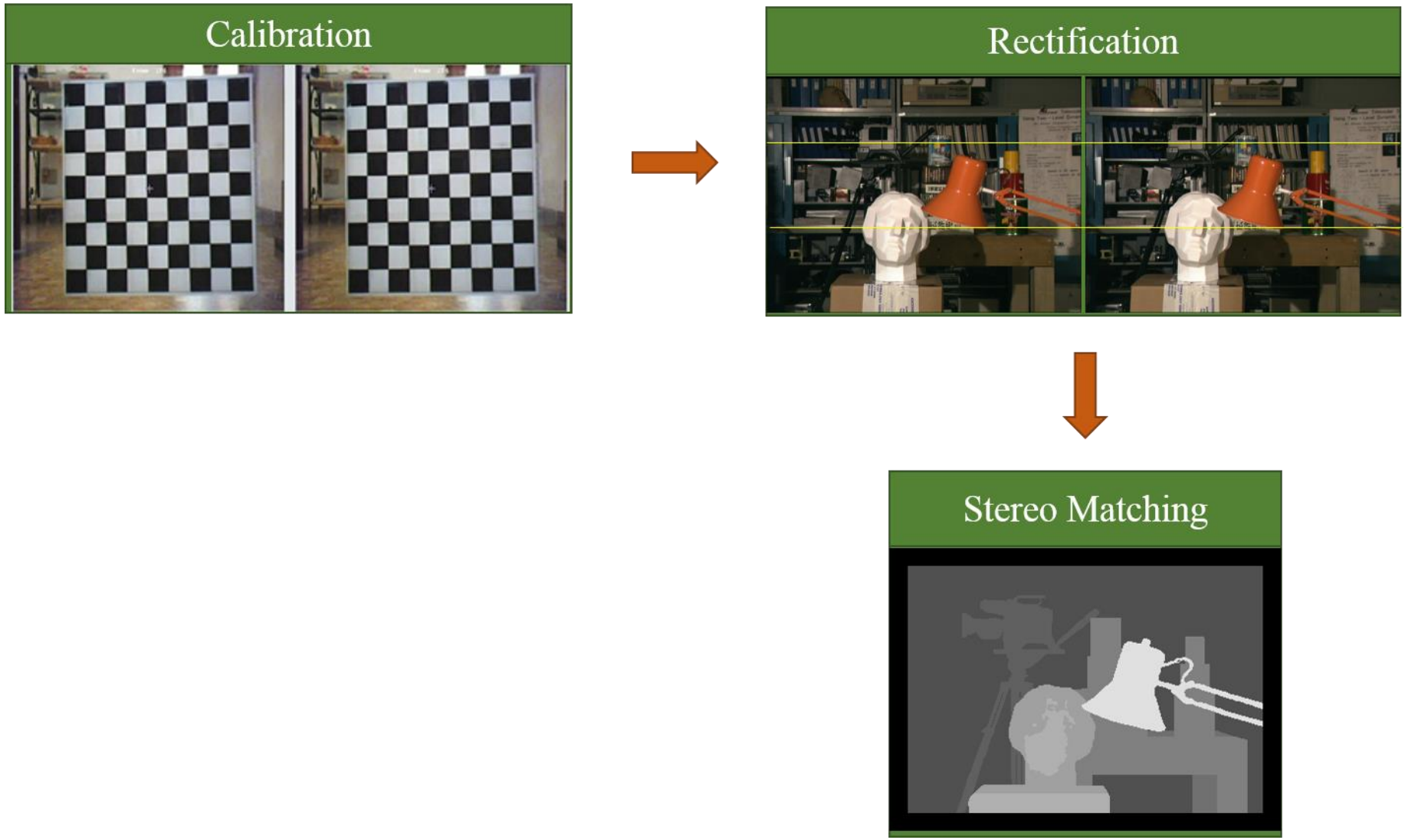


Hình chiều sâu cho hình phải

Ứng dụng

- Đo khoảng các tới các object trong hình
- Xây dựng mô hình 3D
- Thông tin cho các ứng dụng khác: detection, tracking, …

Stereo Matching



Stereo Matching

❖ **Left and right images of a stereo pair**

Left Image

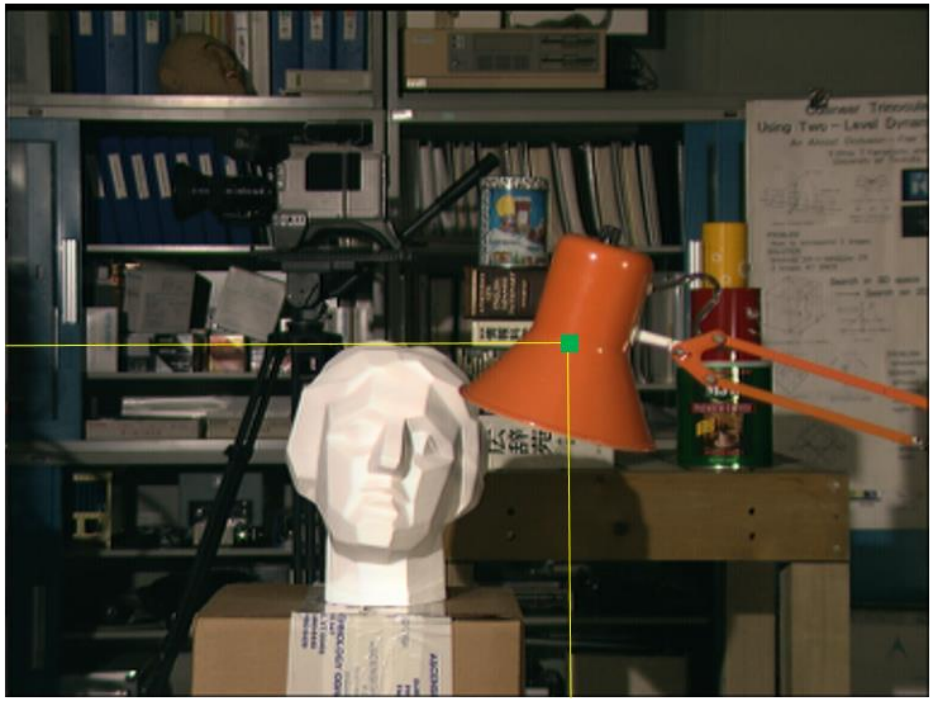
Right Image

Stereo Matching

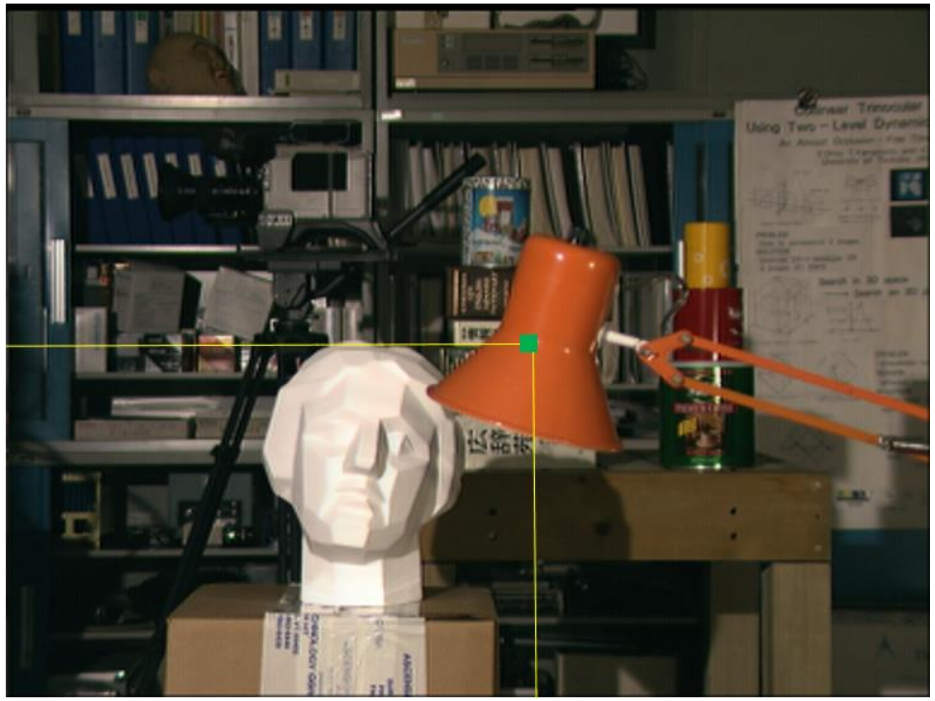
p = (234, 140)

q = (220, 140)

y_p=140



y_q=140



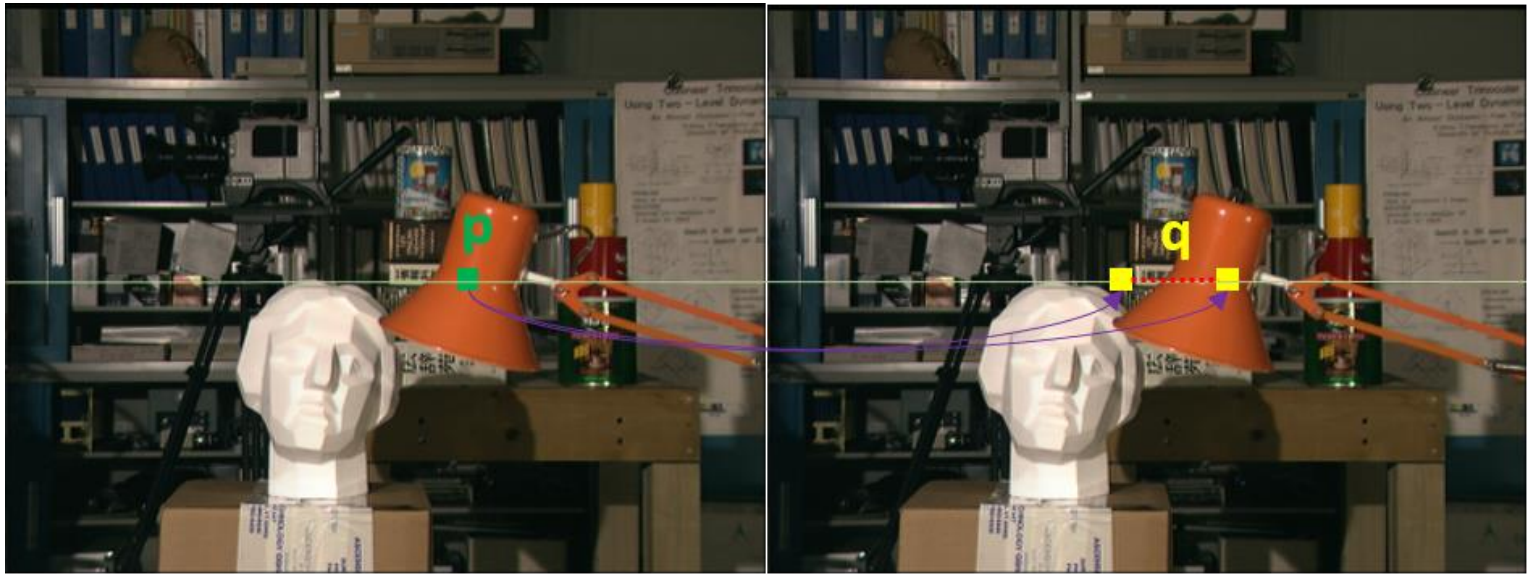
x_p = 234

x_q = 220

disparity_q = x_p – x_q = 14

Stereo Matching

❖ Method 1



Left Image Right Image

$$d_p = \arg\min_{d \in D}(C(p,q))$$

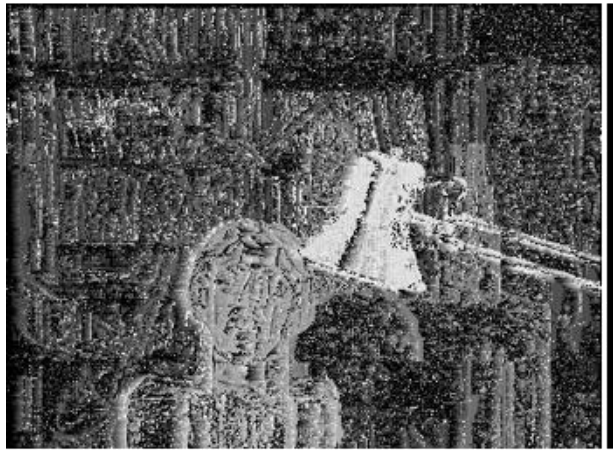
$$where \;\; C(p,q) = (L(p) - R(q))^2$$

$$and \;\; q = (x_p - d, y_p)$$

Stereo Matching

Left Image

Right Image

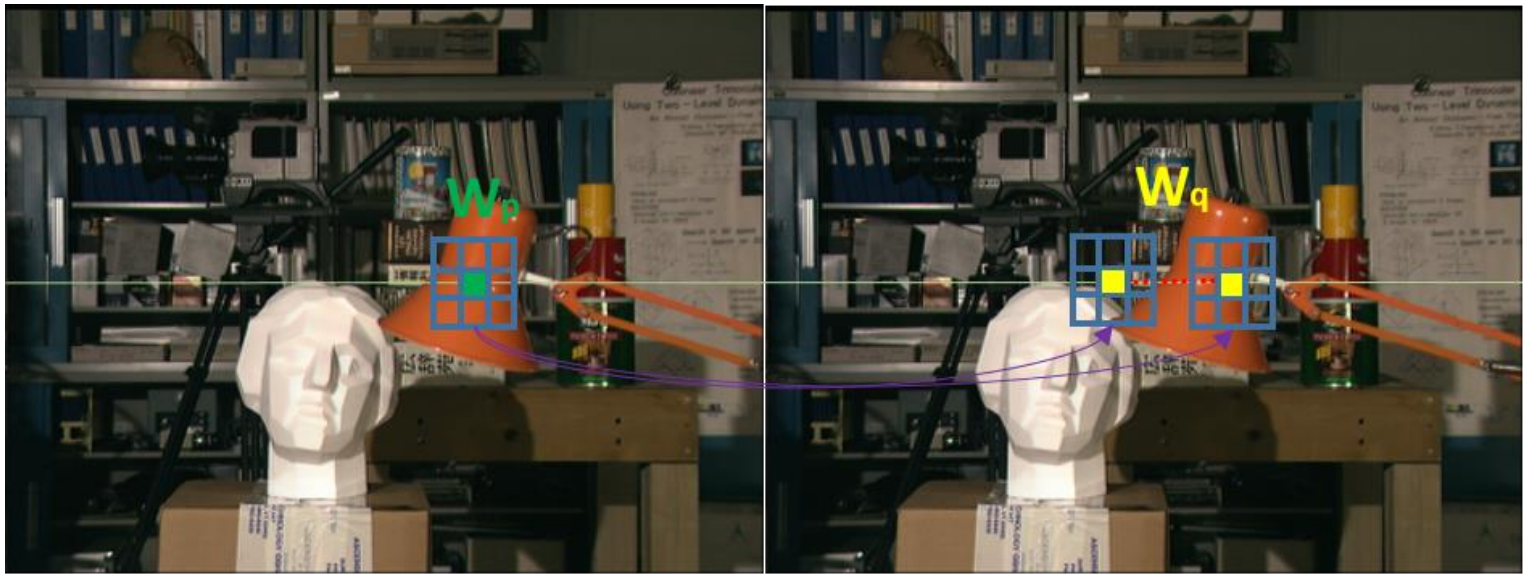


Disparity Map

Ground Truth

Stereo Matching

❖ Method 2



Left Image Right Image

$$d_p = \arg\min_{d \in D}(C(p,q))$$

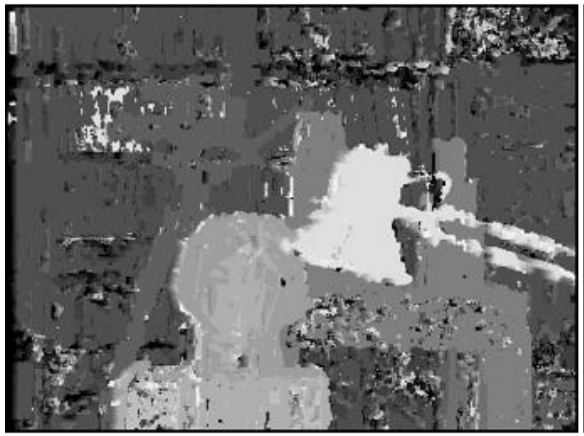
$$where \;\; C(p,q) = \sum_{(u,v)\in(W_p,W_q)} (L(u) - R(v))^2$$

$$and \;\; q = (x_p - d, y_p)$$

Stereo Matching

Left Image

Right Image



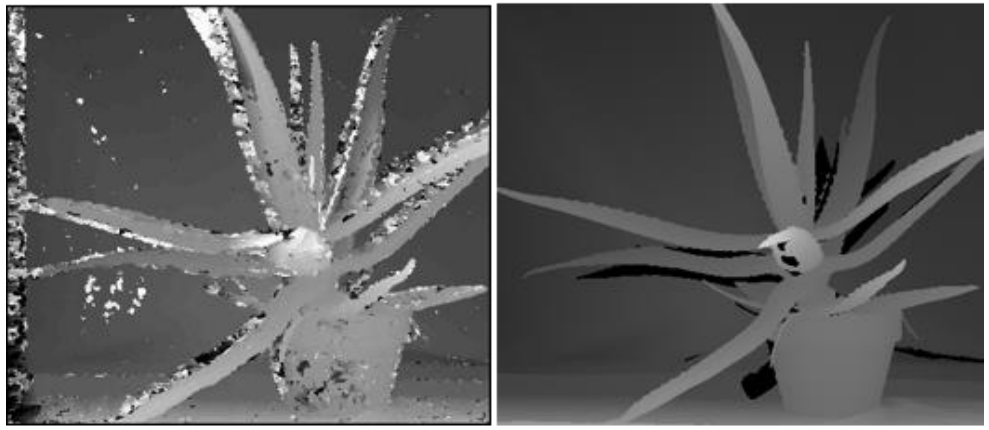
Disparity Map

Ground Truth

Stereo Matching

❖ Aloe stereo pair

Left Image Right Image



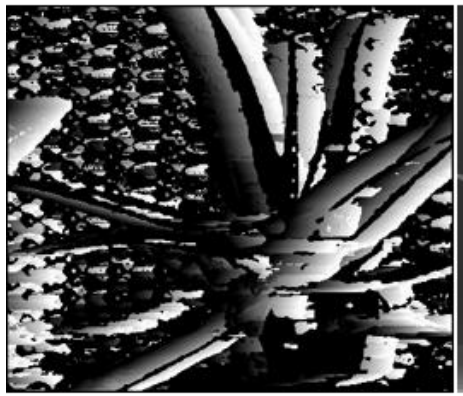
Disparity Map Ground Truth

Stereo Matching

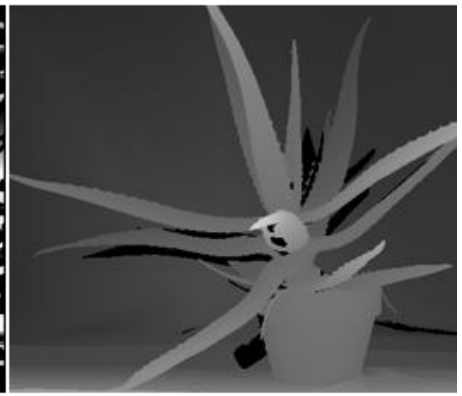
❖ Aloe stereo pair

Left Image

Right Image



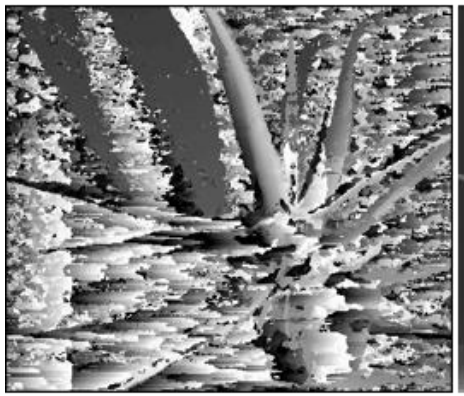
Disparity Map



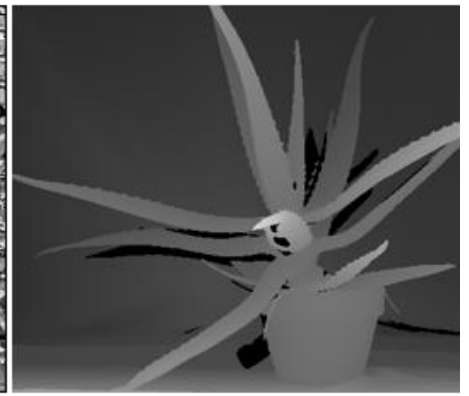
Ground Truth

Left Image

Right Image



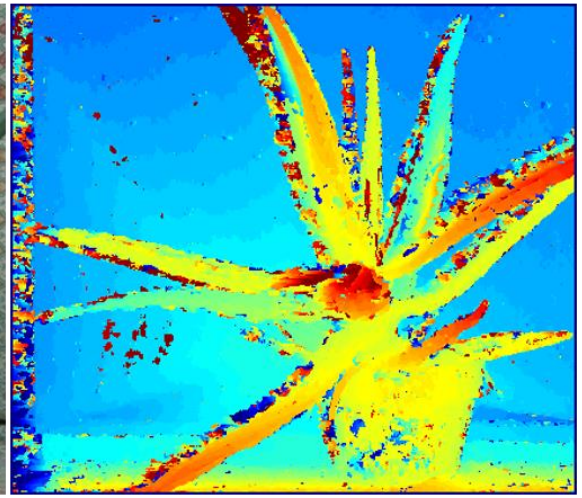
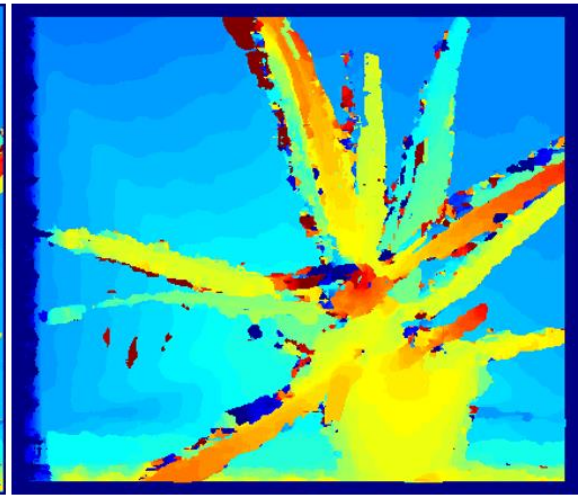
Disparity Map



Ground Truth

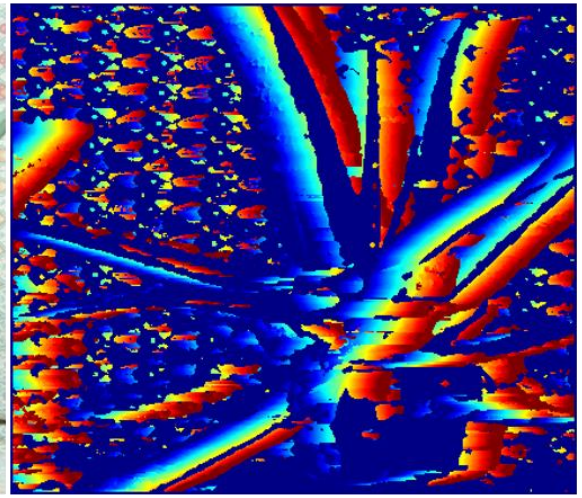
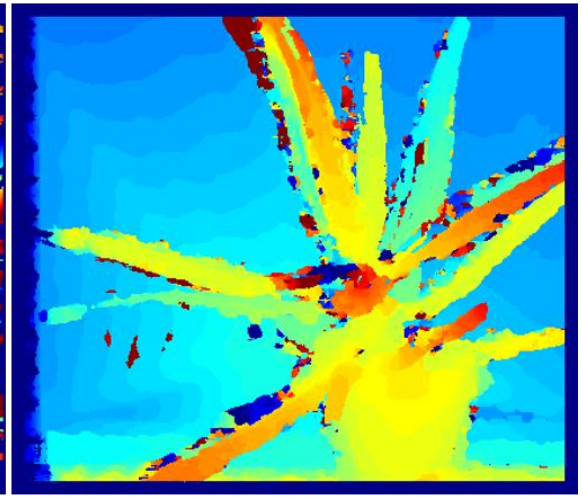
Ứng đụng tìm chiều sâu (depth estimation)



Ảnh stereo có cùng độ sáng | Absolute difference | Cosine Similarity



Ảnh stereo khác độ sáng | Absolute difference | Cosine Similarity

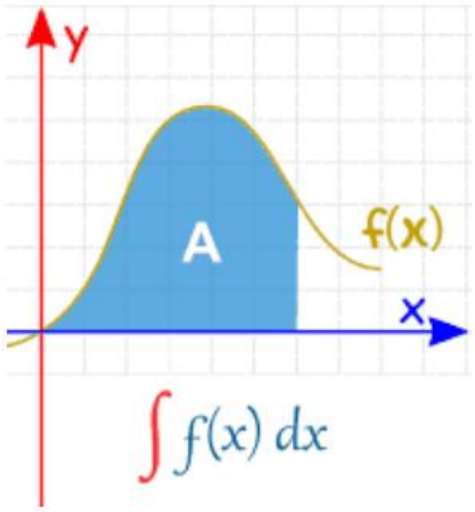
Cosine similarity hoạt động ổn định khi ảnh stereo thay đổi độ sáng

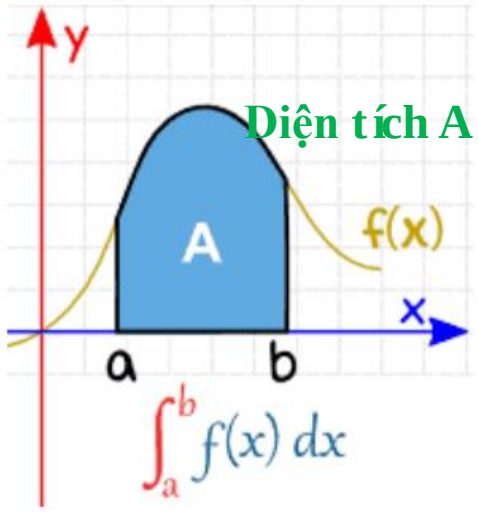
Outline

- Derivative/Gradient
- Interpolation
- Vector and Matrix
- Cosine Similarity
- Integral

Integral

Công thức





https://www.mathsisfun.com/calculus/integration-introduction.html

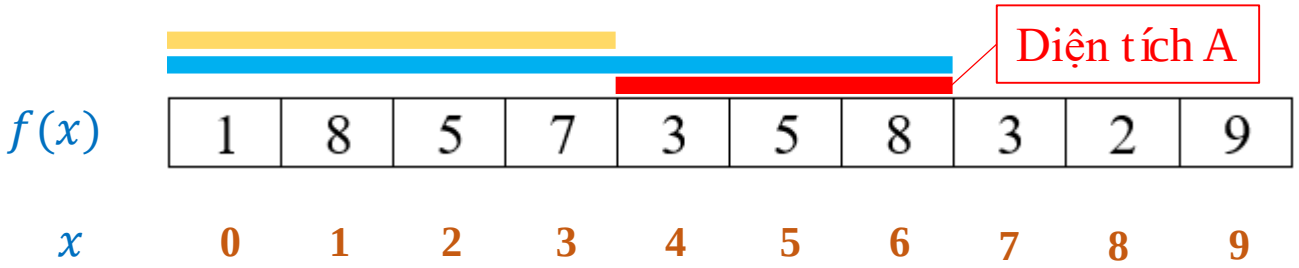
$$F(a) = \int_{-\infty}^{a} f(a)$$

$$F(b) = \int_{-\infty}^{b} f(b)$$

Diện tích A

$$A = F(b) - F(a)$$

Á p dụng cho hàm rời rạc (1D)



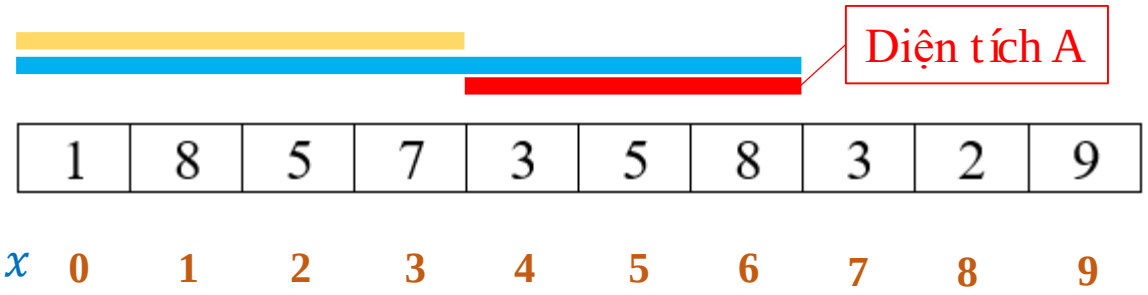
$$F(3) = \sum_{x \le 3} f(x) = f(0) + f(1) + f(2) + f(3)$$
$$= 1 + 8 + 5 + 7 = 21$$

$$F(6) = \sum_{x \le 6} f(x) = 1 + 8 + 5 + 7 + 3 + 5 + 8 = 37$$

$$A = F(6) - F(3) = \sum_{4 \le x \le 6} f(x) = 3 + 5 + 8 = 16$$

Integral

Á p dụng cho hàm rời rạc (1D)



$$F(3) = \sum_{x \le 3} f(x) = f(0) + f(1) + f(2) + f(3)$$
$$= 1 + 8 + 5 + 7 = 21$$

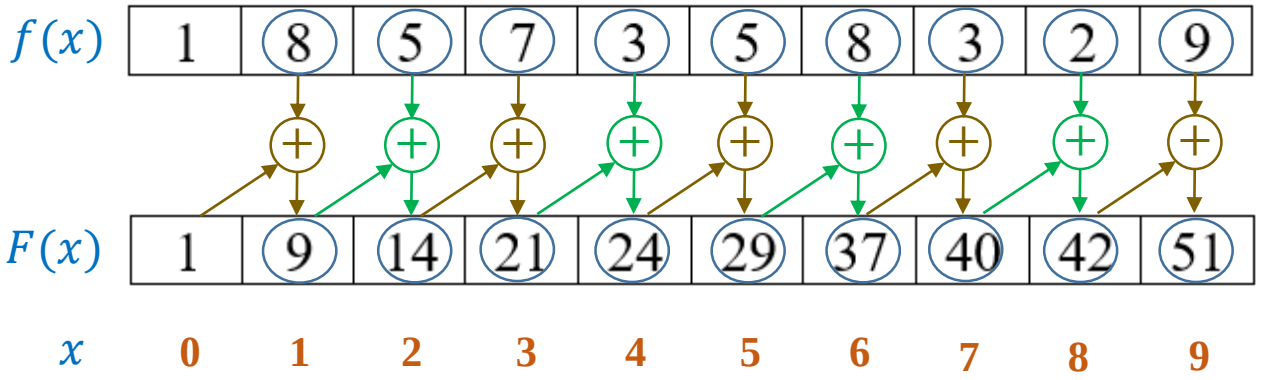
$$F(6) = \sum_{x \le 6} f(x) = 1 + 8 + 5 + 7 + 3 + 5 + 8 = 37$$

$$A = F(6) - F(3) = \sum_{4 \le x \le 6} f(x) = 3 + 5 + 8 = 16$$

Tính chất

$$F(x) = f(x) + F(x - 1)$$
$$F(7) = f(7) + F(6) = 3 + 37 = 40$$

Xây dựng integral array dùng tính chất $F(x) = f(x) + F(x - 1)$



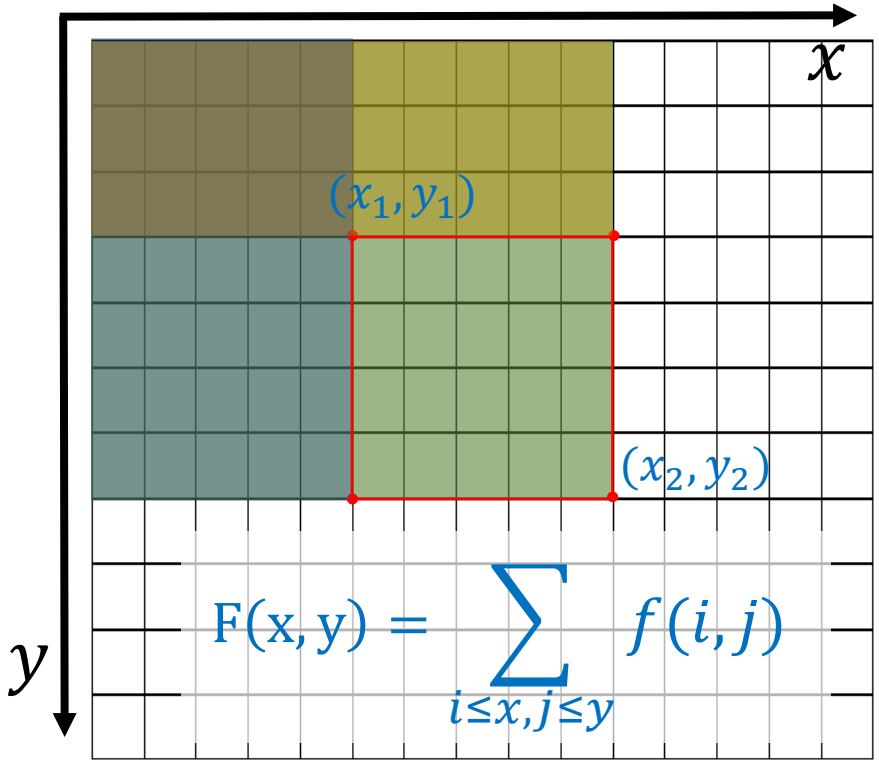
Tính tổng với độ phức tạp ~ O(1)

$$\sum_{a \le x \le b} f(x) = F(b) - F(a - 1)$$

$$\sum_{4 \le x \le 6} f(x) = F(6) - F(3) = 37 - 21 = 16$$

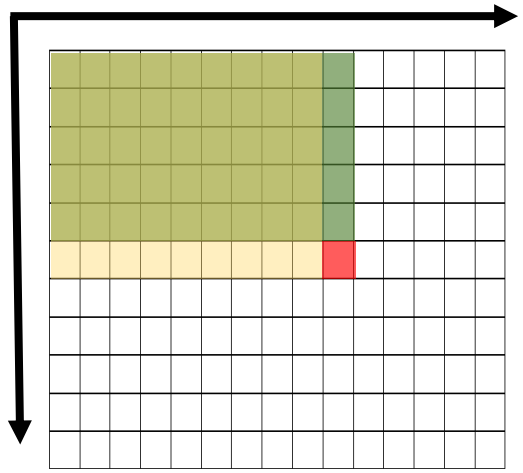
Integral

Á p dụng cho hàm rời rạc (2D)



Xây dựng integral array với tính chất

$$F(x,y) = f(x,y) + F(x-1,y) + F(x,y-1) - F(x-1,y-1)$$



Diện tích A

$$A = \sum_{\substack{x_1 \le i \le x_2 \\ y_1 \le j \le y_2}} f(i,j) = F(x_2,y_2) - F(x_2-1,y_2) - F(x_2,y_2-1) + F(x_2-1,y_2-1)$$

Ứng dụng cho computer vision

Tính tổng

$$cs(\vec{x},\vec{y}) = \frac{\vec{x}.\vec{y}}{\|\vec{x}\|\|\vec{y}\|} = \frac{\sum_1^n x_i y_i}{\sqrt{\sum_1^n x_i^2}\sqrt{\sum_1^n y_i^2}}$$

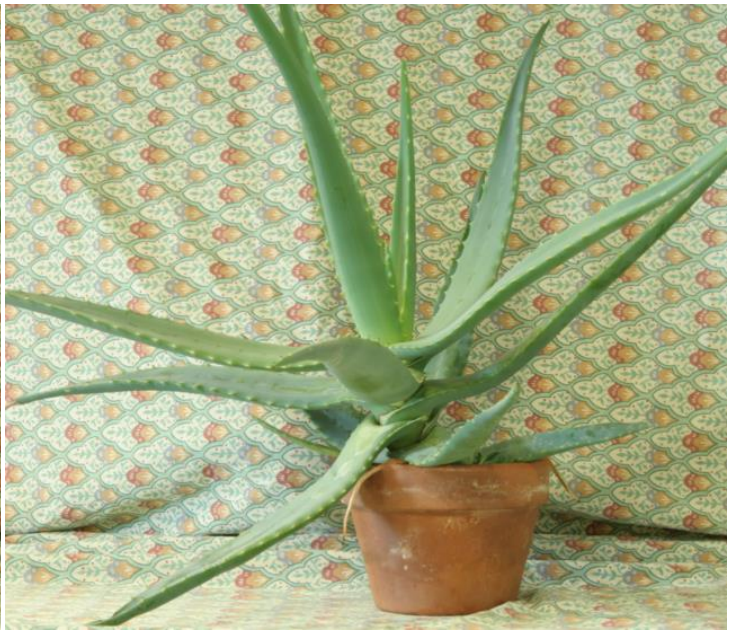
Cách tính cosine similarity thông thường

Time = ~36 seconds

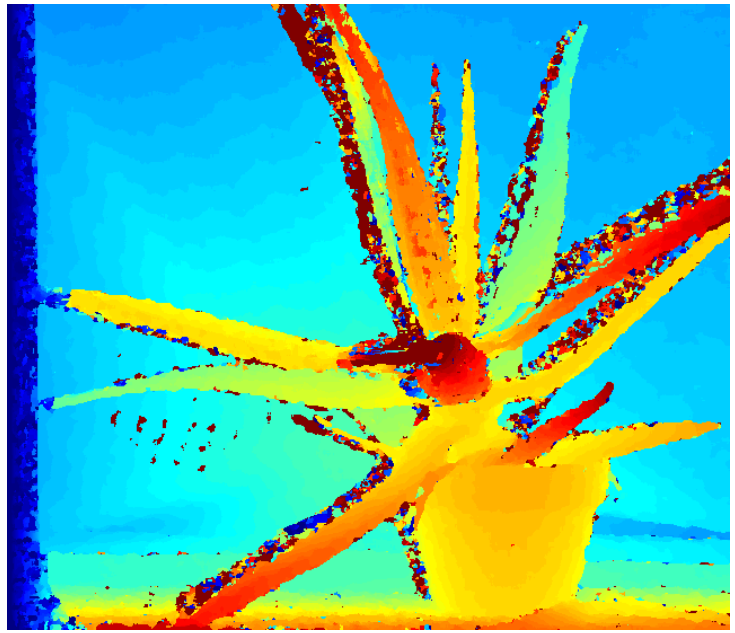
Dùng integral image

Time < 1 second

Ảnh trái



Ảnh phải



Ảnh chiều sâu cho ảnh trái dùng cosine similarity

Discussion

Ứng dụng Cosine Similarity để tính mức độ giống nhau giữa hai hình

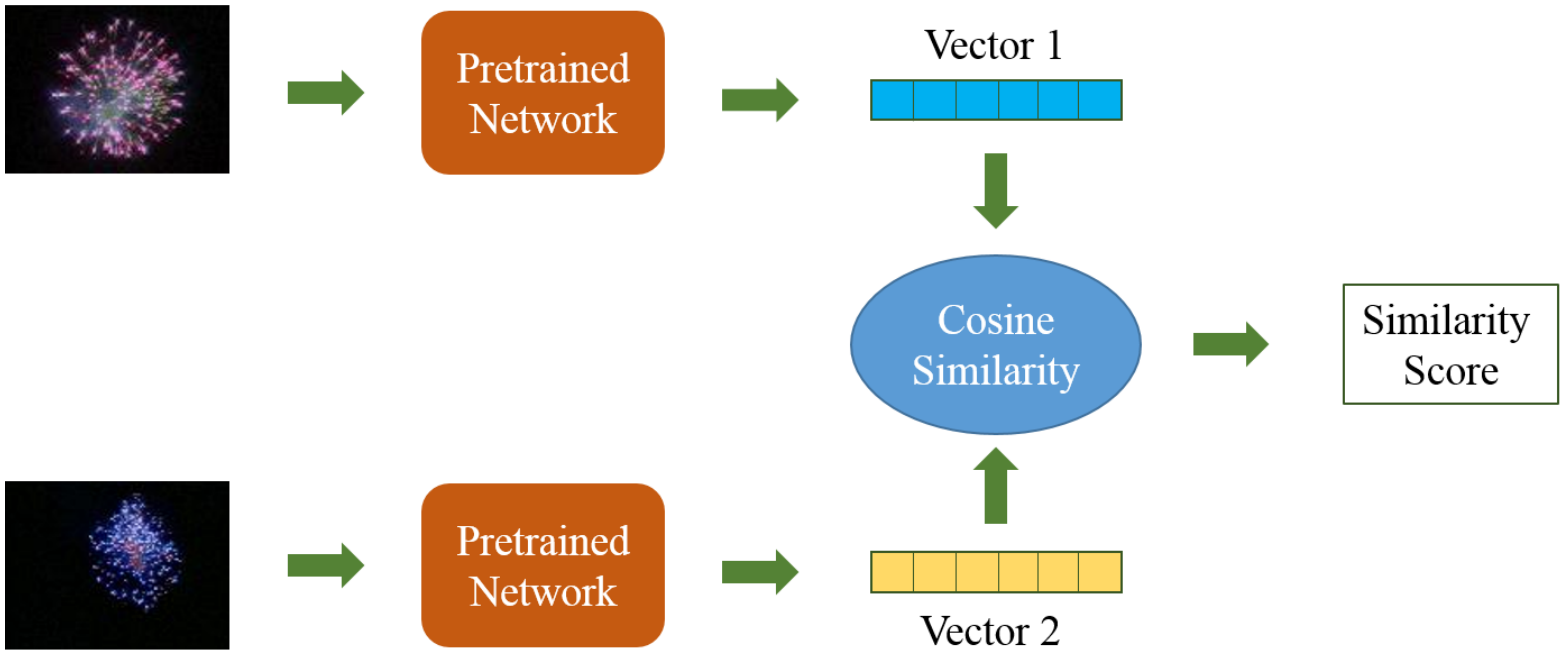
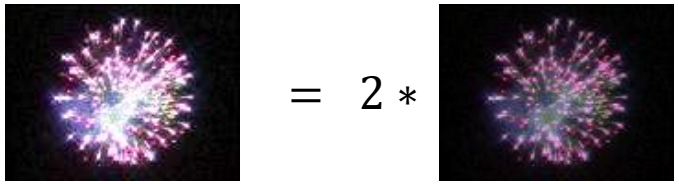
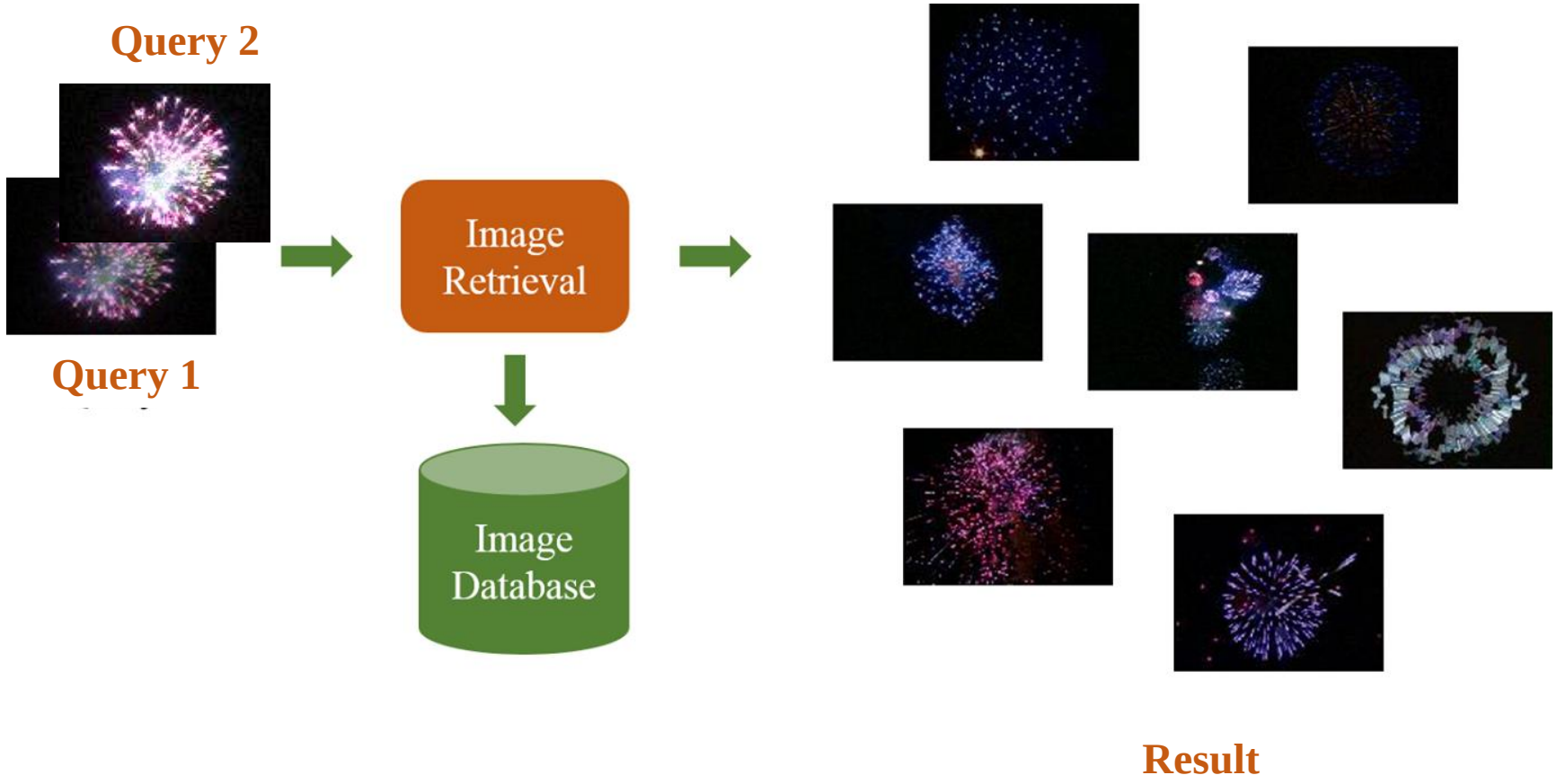


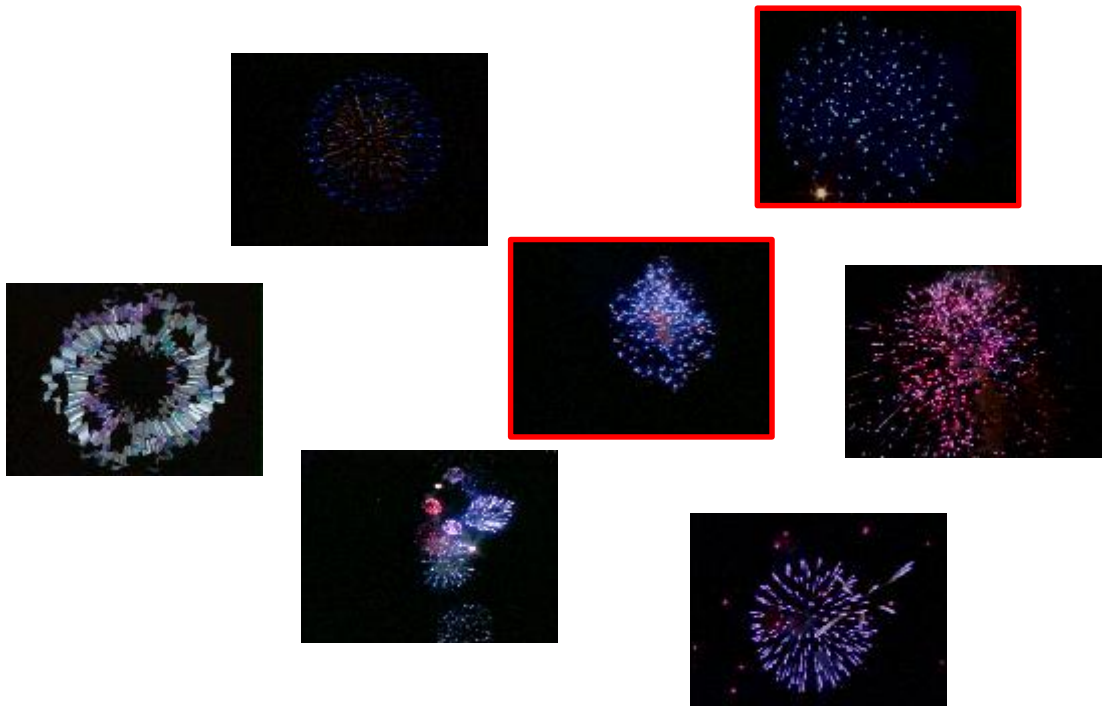
Image Retrieval





Hai hình query cho cùng kết quả như nhau

Query 1

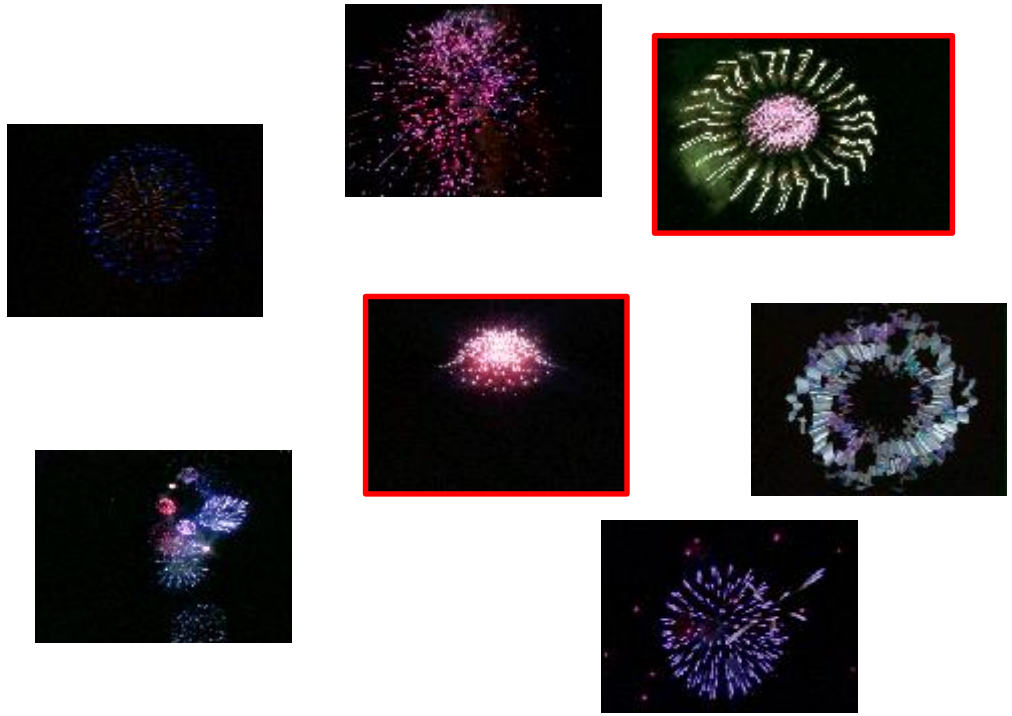


Result 1

Query 2

= 50 +

Query 1



Result 2

Result 1 ≠ Result 2

Summary

- Derivative/Gradient
- Interpolation
- Vector and Matrix
- Cosine Similarity
- Integral


Thank you!