№2

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}$$

1) $P(A) = A(A^TA)^{-1}A^T$; $A^TA = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow (A^TA)^{-1} = \begin{pmatrix} 1/2 & 0 \\ 0 & 1 \end{pmatrix}$

$P(A) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1/2 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \end{pmatrix}$

$(B^TB) = \begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 2 \\ 2 & 4 \end{pmatrix} \Rightarrow (B^TB)^{-1}: \begin{pmatrix} 2 & 2 & | & 1 & 0 \\ 2 & 4 & | & 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 2 & 2 & | & 1 & 0 \\ 0 & 2 & | & -1 & 1 \end{pmatrix} \rightsquigarrow \begin{pmatrix} 2 & 0 & | & 2 & -1 \\ 0 & 2 & | & -1 & 1 \end{pmatrix} \Rightarrow$

$\Rightarrow (B^TB)^{-1} = \begin{pmatrix} 1 & -1/2 \\ -1/2 & 1/2 \end{pmatrix}$

$P(B) = \begin{pmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & -1/2 \\ -1/2 & 1/2 \end{pmatrix}\begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -1/2 & 1 \\ 1/2 & 1/2 & -1/2 \end{pmatrix} = \begin{pmatrix} 1 & 1/2 & 0 \\ 1/2 & 1/2 & -1/2 \\ 0 & -1/2 & 1 \end{pmatrix}$

2) $a_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$, $a_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \Rightarrow$ с помощью ортонализации Грама - Шмидта получаем:

$\vec{e}_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$, $\vec{e}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} - (\hat{a}_2 \cdot e_1)\cdot \vec{a}_1 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \Rightarrow Q = \begin{pmatrix} 1/\sqrt{2} & 0 \\ 0 & 1 \\ 1/\sqrt{2} & 0 \end{pmatrix}$, откуда $R = Q^TA = \begin{pmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} \sqrt{2} & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow$

$\Rightarrow A = \begin{pmatrix} 1/\sqrt{2} & 0 \\ 0 & 1 \\ 1/\sqrt{2} & 0 \end{pmatrix}\cdot\begin{pmatrix} \sqrt{2} & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}$

$b_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$, $b_2 = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \Rightarrow \vec{e}_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$, $\vec{e}_2 = \frac{1}{\ell}\left(\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} - (\hat{e}_1 \cdot \vec{e}_1)\cdot \vec{b}_2\right) = \frac{1}{\ell}\left(\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} - \frac{\sqrt{2}}{2}\cdot\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}\right) =$

$\vec{e}_2 = \frac{1}{\ell}\left(\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} - \vec{e}_1(\hat{b}_2\cdot\hat{e}_1)\right) = \ell$

$\vec{e}_2 = \frac{1}{\ell}\left(\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}\cdot\frac{(\hat{b}_2\cdot\sqrt{2}\hat{e}_1)}{2}\right) = \frac{1}{\ell}\left(\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}\right) = \frac{1}{\ell}\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ -1/\sqrt{3} \end{pmatrix} \Rightarrow$

$\Rightarrow Q = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{3} \\ 0 & 1/\sqrt{3} \\ 1/\sqrt{2} & -1/\sqrt{3} \end{pmatrix} \Rightarrow R = Q^TB = \begin{pmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 1/\sqrt{3} & 1/\sqrt{3} & -1/\sqrt{3} \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} \sqrt{2} & \sqrt{2} \\ 0 & \sqrt{3} \end{pmatrix} \Rightarrow$

$\Rightarrow B = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{3} \\ 0 & 1/\sqrt{3} \\ 1/\sqrt{2} & -1/\sqrt{3} \end{pmatrix}\begin{pmatrix} \sqrt{2} & \sqrt{2} \\ 0 & \sqrt{3} \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}$