N1)

$$I_n(\alpha) = \int_0^1 \frac{x^n}{x+\alpha}\,dx$$

i)

$$I_{n+1}(\alpha) = \int_0^1 \frac{x^{n+1}}{x+\alpha}\,dx = \int_0^1 x^{n+1} x^{-1}\left(1+\frac{\alpha}{x}\right)^{-1} = \int_0^1 x^n\left(1-\frac{\alpha}{x}+\frac{\alpha^2}{x^2}-\dots\right)dx = \int_0^1 x^n\,dx -$$

$$- \int_0^1 x^n\left(\frac{\alpha}{x}-\frac{\alpha^2}{x^2}+\dots\right) = \frac{x^{n+1}}{n+1}\Big|_0^1 - \alpha\int_0^1 x^n\cdot x^{-1}\left(1-\frac{\alpha}{x}+\frac{\alpha^2}{x^2}-\dots\right)dx = \frac{1}{n+1} - \alpha\int_0^1 \frac{x^n}{x+\alpha}\,dx \Rightarrow$$

$$\Rightarrow \boxed{I_{n+1}(\alpha) = \frac{1}{n+1} - \alpha I_n(\alpha)}$$

ii)

$$I_0(\alpha) = \int_0^1 \frac{dx}{x+\alpha} = \ln|x+\alpha|\Big|_0^1 = \boxed{\ln\left|\frac{1+\alpha}{\alpha}\right|}$$