№21

а) $A=\begin{pmatrix}3&0\\0&-2\end{pmatrix}=U\Sigma V^+$

$A^T\cdot A^*=\begin{pmatrix}9&0\\0&4\end{pmatrix}\Rightarrow \begin{matrix}\sigma_1=3\\\sigma_2=2\end{matrix}\Rightarrow \Sigma=\begin{pmatrix}3&0\\0&2\end{pmatrix}\Rightarrow \begin{matrix}v_1=\begin{pmatrix}1\\0\end{pmatrix}\\ v_2=\begin{pmatrix}0\\1\end{pmatrix}\end{matrix}\Rightarrow V^+=\begin{pmatrix}1&0\\0&1\end{pmatrix}$

$A\cdot\begin{pmatrix}1\\0\end{pmatrix}=3\cdot u_1\Rightarrow u_1=\begin{pmatrix}1\\0\end{pmatrix}$

$A\cdot\begin{pmatrix}0\\1\end{pmatrix}=2\cdot u_2\Rightarrow u_2=\begin{pmatrix}0\\-1\end{pmatrix}$ $\Rightarrow U=\begin{pmatrix}1&0\\0&-1\end{pmatrix}\Rightarrow\begin{pmatrix}1&0\\0&-1\end{pmatrix}\begin{pmatrix}3&0\\0&2\end{pmatrix}\begin{pmatrix}1&0\\0&1\end{pmatrix}=\begin{pmatrix}3&0\\0&-2\end{pmatrix}=\underline{\underline{A}}$

б) $B=\begin{pmatrix}0&2\\0&0\\0&0\end{pmatrix}$; $B^TB=\begin{pmatrix}0&0&0\\2&0&0\end{pmatrix}\begin{pmatrix}0&2\\0&0\\0&0\end{pmatrix}=\begin{pmatrix}0&0\\0&4\end{pmatrix}\Rightarrow\begin{cases}\sigma_1=2\\\sigma_2=0\end{cases}\Rightarrow\begin{cases}v_1=\begin{pmatrix}0\\1\end{pmatrix}\\v_2=\begin{pmatrix}1\\0\end{pmatrix}\end{cases}\Rightarrow V^+=\begin{pmatrix}0&1\\1&0\end{pmatrix}$ – т.к. $v_1\perp v_2$

$B\cdot\begin{pmatrix}0\\1\end{pmatrix}=2\cdot u_1\Rightarrow u_1=\begin{pmatrix}1\\0\\0\end{pmatrix}$, $B\cdot\begin{pmatrix}1\\0\end{pmatrix}=0$ и $u_1\perp u_2,u_3\Rightarrow u_2=\begin{pmatrix}0\\1\\0\end{pmatrix}$ $u_3=\begin{pmatrix}0\\0\\1\end{pmatrix}\Rightarrow$

$\Rightarrow\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}\cdot\begin{pmatrix}2&0\\0&0\\0&0\end{pmatrix}\begin{pmatrix}0&1\\1&0\end{pmatrix}=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}\cdot\begin{pmatrix}0&2\\0&0\\0&0\end{pmatrix}=\begin{pmatrix}0&2\\0&0\\0&0\end{pmatrix}=\underline{\underline{B}}$

в) $C=\begin{pmatrix}1&1\\1&1\end{pmatrix}$; $C^TC=\begin{pmatrix}2&2\\2&2\end{pmatrix}\Rightarrow\begin{cases}\sigma_1=2\\\sigma_2=0\end{cases}\Rightarrow\begin{cases}v_1=\begin{pmatrix}1\\1\end{pmatrix}\cdot\frac{1}{\sqrt2}\\v_2=\begin{pmatrix}1\\-1\end{pmatrix}\cdot\frac{1}{\sqrt2}\end{cases}\Rightarrow V^+=\frac{1}{\sqrt2}\begin{pmatrix}1&1\\1&-1\end{pmatrix}$

$C\cdot\frac{1}{\sqrt2}\begin{pmatrix}1\\1\end{pmatrix}=2\cdot u_1\Rightarrow u_1=\begin{pmatrix}1\\1\end{pmatrix}\frac{1}{\sqrt2}$; $C\cdot\frac{1}{\sqrt2}\begin{pmatrix}1\\-1\end{pmatrix}=0$ и $u_2\perp u_1\Rightarrow u_2=\frac{1}{\sqrt2}\begin{pmatrix}-1\\1\end{pmatrix}\Rightarrow$

$\Rightarrow\frac{1}{2}\begin{pmatrix}1&-1\\1&1\end{pmatrix}\begin{pmatrix}2&0\\0&0\end{pmatrix}\begin{pmatrix}1&1\\1&-1\end{pmatrix}=\begin{pmatrix}1&-1\\1&1\end{pmatrix}\begin{pmatrix}1&1\\0&0\end{pmatrix}=\begin{pmatrix}1&1\\1&1\end{pmatrix}=\underline{\underline{C}}$