

БФ.1)

$$\partial_t n = D \partial_x^2 n - \alpha n \Rightarrow \partial_t \tilde{n} = -Dk^2 \tilde{n} - \alpha \tilde{n} \Rightarrow$$

$$\Rightarrow \tilde{n} = \tilde{n}(0) \exp[-(Dk^2 + \alpha)t]$$

$$n(0) = \delta(x-L) - \delta(x+L), \text{ м.к. смекка - возмущения } \Rightarrow$$

$$\Rightarrow \tilde{n}(0) = \int e^{ikx} [\delta(x-L) - \delta(x+L)] dx = e^{ikL} - e^{-ikL} \Rightarrow$$

$$\Rightarrow n = \frac{1}{2\pi} \int e^{-ikx} (e^{ikL} - e^{-ikL}) e^{-\alpha t - Dk^2 t} dk = \frac{e^{-\alpha t}}{2\pi} \left[ \int e^{-Dt(k^2 + ik \frac{(x-L)}{Dt})} dk - \right.$$

$$\left. - \int e^{-Dt(k^2 + ik \frac{(x+L)}{Dt})} dk \right] = \frac{e^{-\alpha t}}{2\pi} \left[ e^{-\frac{(x-L)^2}{4Dt}} \int e^{-Dt(k + i \frac{(x-L)}{2Dt})^2} dk - \int e^{-Dt(k + i \frac{(x+L)}{2Dt})^2} dk \right]$$

$$= \frac{e^{-\alpha t}}{2\pi} \sqrt{\frac{\pi}{Dt}} \left[ e^{-\frac{(x-L)^2}{4Dt}} - e^{-\frac{(x+L)^2}{4Dt}} \right] =$$

$$= \frac{e^{-\alpha t}}{\sqrt{4\pi Dt}} \left[ e^{-\frac{(x-L)^2}{4Dt}} - e^{-\frac{(x+L)^2}{4Dt}} \right] = n(x,t) \Rightarrow$$

$$P_{\text{surv}}(t) = \int_0^\infty dx n(x) = \frac{e^{-\alpha t}}{\sqrt{4\pi Dt}} \left[ \int_0^\infty e^{-\frac{(x-L)^2}{4Dt}} dx - \int_0^\infty e^{-\frac{(x+L)^2}{4Dt}} dx \right] =$$

$$= \frac{e^{-\alpha t}}{\sqrt{4\pi Dt}} \left[ \int_{-L}^\infty e^{-\frac{x^2}{4Dt}} dx - \int_L^\infty e^{-\frac{x^2}{4Dt}} dx \right] = \frac{e^{-\alpha t}}{\sqrt{4\pi Dt}} \left[ \int_0^{L/\sqrt{4Dt}} e^{-z^2} dz + \int_0^{L/\sqrt{4Dt}} e^{-z^2} dz \right] =$$

$$= \frac{e^{-\alpha t}}{\sqrt{\pi}} 2 \int_0^{L/\sqrt{4Dt}} e^{-z^2} dz = \boxed{e^{-\alpha t} \cdot \text{Erf} \left[ \frac{L}{\sqrt{4Dt}} \right] = P_{\text{surv}}(t)}$$

(ii) замечим, что  $P_{\text{surv}}(t) = P_{\text{survL}}(t) \cdot P_{\text{survR}}(t)$ ,

где  $P_{\text{survL}}(t)$  - вер-ть не падаетея к поверхности  $t$   
 $P_{\text{survR}}(t)$  - вер-ть не ~~попадаетея~~ ~~попадаетея~~ к поверхности  $t$

Ф-ция распределения "смерти" каемцы:

$$P_{\text{surv}}(t) = \int_t^\infty P_{\text{die}}(t) dt \Rightarrow P_{\text{die}}(t) = -\frac{d}{dt} P_{\text{surv}}(t) = \alpha e^{-\alpha t} \text{Erf} \left[ \frac{L}{\sqrt{4Dt}} \right] +$$

$$+ e^{-\alpha t} \frac{L}{\sqrt{4\pi D} t^{3/2}} e^{-\frac{L^2}{4Dt}} \Rightarrow$$

$$\Rightarrow \langle t \rangle = \int_0^{\infty} t \cdot P_{\text{die}}(t) dt = - \int_0^{\infty} t \frac{d}{dt} P_{\text{surv}} dt = - t \cdot P_{\text{surv}} \Big|_0^{\infty} + \int_0^{\infty} P_{\text{surv}} dt =$$

$$= \int_0^{\infty} e^{-\alpha t} \cdot \text{Erf}\left[\frac{L}{\sqrt{4Dt}}\right] dt = - \frac{e^{-\alpha t}}{\alpha} \cdot \text{Erf}\left[\frac{L}{\sqrt{4Dt}}\right] \Big|_0^{\infty} - \int_0^{\infty} \frac{e^{-\alpha t}}{\alpha} \frac{L}{\sqrt{4\pi D}} t^{-3/2} e^{-\frac{L^2}{4Dt}} dt =$$

$$= \frac{1}{\alpha} - \frac{L}{\alpha \sqrt{4\pi D}} \underbrace{\int_0^{\infty} \frac{dt}{t^{3/2}} e^{-\alpha t - \frac{L^2}{4Dt}}}_I$$

$$I = \int_0^{\infty} \frac{dt}{t^{3/2}} e^{-\alpha t - \frac{L^2}{4Dt}} = -2 \int_0^{\infty} dt^{-1/2} e^{-\alpha t - \frac{L^2}{4Dt}} = -2 \int_0^{\infty} du e^{-\frac{\alpha}{u^2} - \frac{L^2}{4D} u^2} =$$

$$= 2 \int_0^{\infty} du \exp\left[-\frac{L^2}{4D} u^2 - \frac{\alpha}{u^2}\right] = \left[ u^2 = \frac{\sqrt{4D\alpha}}{L} \xi^2 \right] = 2 \int_0^{\infty} \left(\frac{\sqrt{4D\alpha}}{L}\right)^{1/2} d\xi \exp\left[-\frac{L}{2\sqrt{D}} \left(\xi^2 + \frac{1}{\xi^2}\right)\right] =$$

$$= 2 \int_0^{\infty} \left(\frac{\sqrt{4\alpha D}}{L}\right)^{1/2} d\xi \exp\left[-\frac{L}{2\sqrt{D}} \left(\xi - \frac{1}{\xi}\right)^2 - L\sqrt{\frac{\alpha}{D}}\right] = 2 \left(\frac{\sqrt{4\alpha D}}{L}\right)^{1/2} e^{-L\sqrt{\frac{\alpha}{D}}} \underbrace{\int_0^{\infty} d\xi e^{-\frac{L}{2\sqrt{D}} \left(\xi - \frac{1}{\xi}\right)^2}}_J$$

$$J = \int_0^{\infty} d\xi e^{-\frac{L}{2\sqrt{D}} \left(\xi - \frac{1}{\xi}\right)^2} = \int_0^{\infty} du \cdot \frac{1}{u^2} e^{-\frac{L}{2\sqrt{D}} \left(u - \frac{1}{u}\right)^2} \Rightarrow$$

$$\Rightarrow 2J = \int_0^{\infty} \left(1 + \frac{1}{u^2}\right) du e^{-\frac{L}{2\sqrt{D}} \left(u - \frac{1}{u}\right)^2} = \left[ z = u - \frac{1}{u} \right] = \int_{-\infty}^{\infty} dz e^{-\frac{L}{2\sqrt{D}} z^2} \Rightarrow$$

$$\Rightarrow J = \frac{\sqrt{\pi}}{2} \left(\frac{D}{\alpha}\right)^{1/4} \sqrt{\frac{2}{L}} \Rightarrow$$

$$\Rightarrow \langle t \rangle = \frac{1}{\alpha} - \frac{L}{\alpha \sqrt{4\pi D}} \cdot \left(\frac{\sqrt{4\alpha D}}{L}\right)^{1/2} e^{-L\sqrt{\frac{\alpha}{D}}} \cdot \frac{\sqrt{\pi}}{2} \left(\frac{D}{\alpha}\right)^{1/4} \sqrt{\frac{2}{L}} = \frac{1}{\alpha} - \frac{e^{-L\sqrt{\frac{\alpha}{D}}}}{\alpha} =$$

$$= \boxed{\frac{1}{\alpha} (1 - e^{-L\sqrt{\frac{\alpha}{D}}}) = \langle t \rangle}$$

(iii) пусть распада —  $\alpha$

$$\text{пусть ~~всех~~ среднее} - \beta = \frac{1}{\langle t \rangle} = \frac{1}{\frac{1}{\alpha} (1 - e^{-L\sqrt{\frac{\alpha}{D}}})} = \frac{\alpha}{1 - e^{-L\sqrt{\frac{\alpha}{D}}}} \quad \Big| \Rightarrow$$

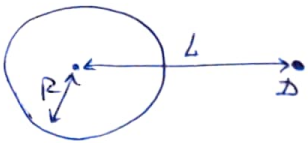
$$\Rightarrow \text{вер-ть пошитоюся до распада: } 1 - \frac{\alpha}{\beta} = 1 - \frac{\alpha}{\alpha / (1 - e^{-L\sqrt{\frac{\alpha}{D}}})} =$$
$$= 1 - 1 + e^{-L\sqrt{\frac{\alpha}{D}}} = \boxed{e^{-L\sqrt{\frac{\alpha}{D}}} = P}$$

$$\left( \text{или это можно увидеть сразу из } \langle t \rangle = \frac{1}{\alpha} (1 - e^{-L\sqrt{\frac{\alpha}{D}}}) = \frac{1}{\alpha} \frac{N_{\text{расп}}}{N} \Rightarrow$$
$$\Rightarrow \text{или } P = 1 - \frac{N_{\text{расп}}}{N} = 1 - 1 + e^{-L\sqrt{\frac{\alpha}{D}}} = \underline{\underline{e^{-L\sqrt{\frac{\alpha}{D}}}}} \right)$$

БФ.2]

$$1) \partial_t n = D \Delta n \Rightarrow$$

$\Rightarrow$  при  $n = n(r)$  (равномерное движение по окружности):



$$\partial_t n = \frac{D}{r} \partial_r^2 (nr) \Rightarrow \partial_t (nr) = D \partial_r^2 (nr)$$

$$2) \text{ введём } \tilde{n} = nr \Rightarrow \partial_t \tilde{n} = D \partial_r^2 \tilde{n} \quad \text{нормировка для } \int_R^\infty r^2 n(r,0) = 1$$

$$n(r,0) = \frac{1}{L^2} (\delta(r-L) - \delta(r-(2R-L))) \Rightarrow$$

$$\Rightarrow \tilde{n} = \frac{r}{L^2} (\delta(r-L) - \delta(r-(2R-L))) \quad \text{м.н. центра симметрии}$$

$$3) G(x,t) = \frac{1}{\sqrt{4\pi Dt}} e^{-\frac{x^2}{4Dt}} \quad \text{функция } \Rightarrow$$

$$\Rightarrow \tilde{n}(r,t) = \frac{1}{\sqrt{4\pi Dt} L^2} \int_{-\infty}^{\infty} [\delta(r'-L) - \delta(r'-(2R-L))] \cdot r' e^{-\frac{(r-r')^2}{4Dt}} dr' =$$

$$= \frac{1}{\sqrt{4\pi Dt} L} \left[ e^{-\frac{(r-L)^2}{4Dt}} - e^{-\frac{(r-(2R-L))^2}{4Dt}} \right]$$

$$4) P_{\text{surv}}(t) = \int_R^\infty r^2 n dr = \int_R^\infty r^2 \frac{\tilde{n}}{r} dr = \int_R^\infty r \tilde{n} dr =$$

$$= \frac{1}{\sqrt{4\pi Dt} L} \left[ \int_R^\infty r e^{-\frac{(r-L)^2}{4Dt}} dr - \int_R^\infty r e^{-\frac{(r-(2R-L))^2}{4Dt}} dr \right] = \frac{1}{\sqrt{4\pi Dt} L} \left[ \int_0^\infty (r+R) e^{-\frac{(r+R-L)^2}{4Dt}} dr - \right.$$

$$\left. - \int_0^\infty r e^{-\frac{(r-R+L)^2}{4Dt}} dr \right] = \frac{1}{\sqrt{4\pi Dt} L} \left[ \int_0^\infty r e^{-\frac{(r+R-L)^2}{4Dt}} dr + \int_0^\infty r e^{-\frac{(r+R-L)^2}{4Dt}} dr + \right.$$

$$\left. + R \int_0^\infty e^{-\frac{(r+R-L)^2}{4Dt}} dr - R \int_0^\infty e^{-\frac{(r-R+L)^2}{4Dt}} dr \right] = \frac{1}{\sqrt{4\pi Dt} L} \left[ \int_{-\infty}^\infty r e^{-\frac{(r+R-L)^2}{4Dt}} dr + \right.$$

$$\left. + R \left( \int_{-R+L}^\infty e^{-\frac{v^2}{4Dt}} dv - \int_{-R+L}^\infty e^{-\frac{r^2}{4Dt}} dr \right) \right] = \frac{1}{\sqrt{4\pi Dt} L} \left[ \int_{-\infty}^\infty (r-R+L) e^{-\frac{r^2}{4Dt}} dr + R \left( \int_0^{L-R} e^{-\frac{r^2}{4Dt}} dr + \right. \right.$$

$$\left. + \int_0^{L-R} e^{-\frac{r^2}{4Dt}} dr \right] = \frac{1}{\sqrt{4\pi Dt} L} \left[ (L-R) \int_{-\infty}^\infty e^{-\frac{r^2}{4Dt}} dr + \int_{-\infty}^\infty r e^{-\frac{r^2}{4Dt}} dr + 2R \sqrt{4Dt\pi} \operatorname{Erf}\left[\frac{L-R}{\sqrt{4Dt}}\right] \right]$$

$$= \frac{1}{\sqrt{4\pi Dt} L} \left[ (L-R) \sqrt{4\pi Dt} + R \sqrt{4\pi Dt} \operatorname{Erf}\left[\frac{L-R}{\sqrt{4Dt}}\right] \right] = \left[ 1 - \frac{R}{L} + \frac{R}{L} \operatorname{Erf}\left[\frac{L-R}{\sqrt{4Dt}}\right] \right] \xrightarrow{t \rightarrow \infty} 1 - \frac{R}{L}$$

510.1/

$$\begin{cases} p(y=0|x=0) = 1 \\ p(y=1|x=0) = 0 \end{cases} \quad \begin{cases} p(y=0|x=1) = f = 0,1 \\ p(y=1|x=1) = 1-f = 0,9 \end{cases}$$

$$p(x=0) = 0,8$$

$$p(x=1) = 0,2$$

$$\begin{aligned} (i) \quad p(y=0) &= p(x=0) \cdot p(y=0|x=0) + p(x=1) \cdot p(y=0|x=1) = \\ &= 0,8 \cdot 1 + 0,2 \cdot 0,1 = \boxed{0,82 = p(y=0)} \\ p(y=1) &= p(x=0) \cdot p(y=1|x=0) + p(x=1) \cdot p(y=1|x=1) = \\ &= 0,8 \cdot 0 + 0,2 \cdot 0,9 = \boxed{0,18 = p(y=1)} \end{aligned}$$

$$(ii) \quad p(x=1|y=0) = \frac{p(y=0|x=1) \cdot p(x=1)}{p(y=0)} = \frac{0,2 \cdot 0,1}{0,82} = \boxed{\frac{1}{41} = p(x=1|y=0)}$$

формула Байеса

$$\begin{cases} p(x=0|y=0) = \frac{p(y=0|x=0) \cdot p(x=0)}{p(y=0)} = \frac{1 \cdot 0,8}{0,82} = \frac{40}{41} \\ p(x=0|y=1) = 0 \\ p(x=1|y=1) = 1 \end{cases} \quad \text{— где через формулу Байеса}$$

$$(iii) \quad I(x,y) = S(x) - S(x|y)$$

$$S(x) = - \sum p(x) \log_2 p(x) = -0,8 \log_2 0,8 - 0,2 \log_2 0,2$$

$$S(x|y) = - \sum \sum p(x,y) \log_2 \frac{p(x,y)}{p(y)} = - \sum \sum p(x,y) \log_2 p(x|y) =$$

$$\begin{aligned} \begin{cases} p(x=0,y=0) = 0,8 \\ p(x=0,y=1) = 0 \\ p(x=1,y=0) = \frac{1}{41} \cdot \frac{82}{100} = 0,02 \\ p(x=1,y=1) = 0,18 \end{cases} &= -0,8 \log_2 \frac{40}{41} - 0 \log_2 0 - \\ &\quad - 0,02 \log_2 \frac{1}{41} - 0,18 \log_2 1 = \\ &= -0,8 \log_2 0,8 + 0,8 \log_2 0,82 - 0,02 \log_2 0,02 + \\ &\quad + 0,02 \log_2 0,82 \Rightarrow \end{aligned}$$

$$\begin{aligned} \Rightarrow I(x,y) &= -0,8 \log_2 0,8 - 0,2 \log_2 0,2 + 0,8 \log_2 0,82 - 0,8 \log_2 0,82 + 0,02 \log_2 0,02 - \\ &\quad - 0,02 \log_2 0,82 = -0,82 \log_2 0,82 - 0,18 \log_2 0,2 - 0,02 \log_2 10 \approx 0,5863 \approx \underline{\underline{0,6}} \end{aligned}$$