

Б8.1

$$\partial_t n = D \partial_x^2 n - \alpha n \Rightarrow \partial_t \tilde{n} = -Dk^2 \tilde{n} - \alpha \tilde{n} \Rightarrow \\ \Rightarrow \tilde{n} = \tilde{n}(0) \exp[-(Dk^2 + \alpha)t]$$

$$n(0) = \delta(x-L) - \delta(x+L), \text{ m.k. симметрия - ненулевые } \Rightarrow$$

$$\Rightarrow \tilde{n}(0) = \int e^{ikx} [\delta(x-L) - \delta(x+L)] dx = e^{ikL} - e^{-ikL} \Rightarrow$$

$$\Rightarrow n = \frac{1}{2\pi} \int e^{-ikx} (e^{ikL+i\pi} - e^{-ikL}) e^{-\alpha t - Dk^2 t} dk = \frac{e^{-\alpha t}}{2\pi} \left[\int e^{-Dt(k^2 + ik\frac{(x-L)}{Dt})} dk - \int e^{-Dt(k^2 + ik\frac{(x+L)}{Dt})} dk \right] = \frac{e^{-\alpha t}}{2\pi} \left[e^{-\frac{(x-L)^2}{4Dt}} \int e^{-Dt(k + i\frac{(x-L)}{2Dt})^2} dk - e^{-\frac{(x+L)^2}{4Dt}} \int e^{-Dt(k + i\frac{(x+L)}{2Dt})^2} dk \right] =$$

$$\cdot \int e^{-Dt(k + i\frac{(x+L)}{2Dt})^2} dk] = \frac{e^{-\alpha t}}{2\pi} \sqrt{\frac{\pi}{Dt}} \left[e^{-\frac{(x-L)^2}{4Dt}} - e^{-\frac{(x+L)^2}{4Dt}} \right] =$$

$$\approx \frac{e^{-\alpha t}}{\sqrt{4\pi Dt}} \left[e^{-\frac{(x-L)^2}{4Dt}} - e^{-\frac{(x+L)^2}{4Dt}} \right] = n(x,t) \Rightarrow$$

$$P_{\text{surv}}(t) = \int_0^\infty dx n(x) = \frac{e^{-\alpha t}}{\sqrt{4\pi Dt}} \left[\int_0^\infty e^{-\frac{(x-L)^2}{4Dt}} dx - \int_0^\infty e^{-\frac{(x+L)^2}{4Dt}} dx \right] =$$

$$= \frac{e^{-\alpha t}}{\sqrt{4\pi Dt}} \left[\int_{-L}^\infty e^{-\frac{x^2}{4Dt}} dx - \int_L^\infty e^{-\frac{x^2}{4Dt}} dx \right] = \frac{e^{-\alpha t}}{\sqrt{4\pi Dt}} \left[\int_0^{L/\sqrt{4Dt}} e^{-z^2} dz + \int_0^{L/\sqrt{4Dt}} e^{-z^2} dz \right] =$$

$$= \frac{e^{-\alpha t}}{\sqrt{\pi}} 2 \int_0^{L/\sqrt{4Dt}} e^{-z^2} dz = \boxed{\underbrace{e^{-\alpha t} \cdot \operatorname{Erf} \left[\frac{L}{\sqrt{4Dt}} \right]}_{P_{\text{surv}}(t)}} = P_{\text{surv}}(t)$$

$$(ii) \text{ замечание, что } P_{\text{surv}}(t) = P_{\text{surv}}(t) \cdot P_{\text{surv}_{LD}}(t),$$

т.е. $P_{\text{surv}}(t)$ — бр-мб не зависит от времени t
 $P_{\text{surv}_{LD}}(t)$ — бр-мб не зависит от времени t

Ф-зак. пренебрежение "сверху" начального:

$$P_{\text{surv}}(t) = \int_t^\infty P_{\text{die}}(t) dt \Rightarrow P_{\text{die}}(t) = -\frac{d}{dt} P_{\text{surv}}(t) = \alpha e^{-\alpha t} \operatorname{Erf} \left[\frac{L}{\sqrt{4Dt}} \right] +$$

$$+ e^{-\alpha t} \frac{L}{\sqrt{4\pi D t}} e^{-\frac{L^2}{4Dt}}$$

$$\begin{aligned}
\textcircled{D} \quad \langle t \rangle &= \int_0^\infty t \cdot P_{\text{die}}(t) dt = - \int_0^\infty t \cdot \frac{d}{dt} P_{\text{surv}} dt = - t \cdot P_{\text{surv}} \Big|_0^\infty + \int_0^\infty P_{\text{surv}} dt = \\
&= \int_0^\infty e^{-\alpha t} \cdot \operatorname{Erf}\left[\frac{L}{\sqrt{4Dt}}\right] dt = - \frac{e^{-\alpha t}}{\alpha} \cdot \operatorname{Erf}\left[\frac{L}{\sqrt{4Dt}}\right] \Big|_0^\infty - \int_0^\infty \frac{e^{-\alpha t}}{\alpha} \frac{L}{\sqrt{\pi D t^{3/2}}} e^{-\frac{L^2}{4Dt}} dt = \\
&= \frac{1}{\alpha} - \frac{L}{\alpha \sqrt{4\pi D}} \underbrace{\int_0^\infty \frac{dt}{t^{3/2}} e^{-\alpha t - \frac{L^2}{4Dt}}}_{I} \\
I &= \int_0^\infty \frac{dt}{t^{3/2}} e^{-\alpha t - \frac{L^2}{4Dt}} = -2 \int_0^\infty dt^{1/2} e^{-\alpha t - \frac{L^2}{4Dt}} = -2 \int_0^\infty du e^{-\frac{\alpha}{u^2} - \frac{L^2}{4D} u^2} = \\
&= 2 \int_0^\infty du \exp\left[-\frac{L^2}{4D} u^2 - \frac{\alpha}{u^2}\right] = \left[u^2 = \frac{\sqrt{4\alpha D}}{L} s^2 \atop du = \frac{\sqrt{4\alpha D}}{L} s ds \right] = 2 \int_0^\infty \left(\frac{\sqrt{4\alpha D}}{L}\right)^{1/2} ds \exp\left[-\frac{L}{2\sqrt{D}}\left(s^2 + \frac{1}{s^2}\right)\right] = \\
&= 2 \int_0^\infty \left(\frac{\sqrt{4\alpha D}}{L}\right)^{1/2} ds \exp\left[-\frac{L}{2\sqrt{D}}\left(s - \frac{1}{s}\right)^2 - L\sqrt{\frac{\alpha}{D}}\right] = 2 \left(\frac{\sqrt{4\alpha D}}{L}\right)^{1/2} e^{-L\sqrt{\frac{\alpha}{D}}} \underbrace{\int_0^\infty ds e^{-\frac{L}{2\sqrt{D}}\left(s - \frac{1}{s}\right)^2}}_J \\
J &= \int_0^\infty ds e^{-\frac{L}{2\sqrt{D}}\left(s - \frac{1}{s}\right)^2} = \int_0^\infty du \cdot \frac{1}{u^2} e^{-\frac{L}{2\sqrt{D}}\left(u - \frac{1}{u}\right)^2} \Rightarrow \\
\Rightarrow J &= \int_0^\infty \left(1 + \frac{1}{u^2}\right) du e^{-\frac{L}{2\sqrt{D}}\left(u - \frac{1}{u}\right)^2} = \left[z = u - \frac{1}{u} \atop dz = du \left(1 + \frac{1}{u^2}\right) \right] \cdot \int_{-\infty}^\infty dz e^{-\frac{L}{2\sqrt{D}}z^2} \Rightarrow \\
\Rightarrow J &= \frac{\sqrt{\pi}}{2} \left(\frac{D}{\alpha}\right)^{1/4} \sqrt{\frac{2}{L}} \Rightarrow \\
\Rightarrow \langle t \rangle &= \frac{1}{\alpha} - \frac{L}{\alpha \sqrt{4\pi D}} \cdot 2 \left(\frac{\sqrt{4\alpha D}}{L}\right)^{1/2} e^{-L\sqrt{\frac{\alpha}{D}}} \cdot \frac{\sqrt{\pi}}{2} \left(\frac{D}{\alpha}\right)^{1/4} \sqrt{\frac{2}{L}} = \frac{1}{\alpha} - \frac{e^{-L\sqrt{\frac{\alpha}{D}}}}{\alpha} = \\
&= \boxed{\frac{1}{\alpha} \left(1 - e^{-L\sqrt{\frac{\alpha}{D}}}\right) = \langle t \rangle}
\end{aligned}$$

(ii) plumb package - α

$$\text{plumb package} - \beta = \frac{1}{\langle t \rangle} = \frac{1}{\frac{1}{\alpha}(1 - e^{-L\sqrt{\frac{\alpha}{D}}})} = \frac{\alpha}{1 - e^{-L\sqrt{\frac{\alpha}{D}}}} \quad | \Rightarrow$$

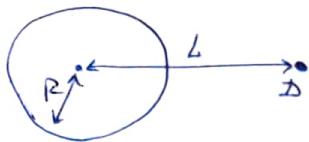
$$\Rightarrow \text{left-most nonremovable go packages: } 1 - \frac{\alpha}{\beta} = 1 - \frac{\alpha}{\alpha/(1 - e^{-L\sqrt{\frac{\alpha}{D}}})} = \\ = 1 - \alpha + e^{-L\sqrt{\frac{\alpha}{D}}} = \boxed{e^{-L\sqrt{\frac{\alpha}{D}}} = P}$$

(where some more useful property w/ $\langle t \rangle = \frac{1}{\alpha}(1 - e^{-L\sqrt{\frac{\alpha}{D}}}) = \frac{1}{\alpha} \frac{N_{\text{pack}}}{N} \Rightarrow$

$$\Rightarrow P = 1 - \frac{N_{\text{pack}}}{N} = 1 - 1 + e^{-L\sqrt{\frac{\alpha}{D}}} = e^{-L\sqrt{\frac{\alpha}{D}}} \quad)$$

Б8.2]

1) $\partial_t n = D \Delta n \Rightarrow$



\Rightarrow при $n = n(r)$ (рассматриваем модельно-однородную):

$$\partial_t n = \frac{D}{r} \partial_r^2 (nr) \Rightarrow \partial_t (nr) = D \partial_r^2 (nr)$$

2) ближайшее $\tilde{n} = nr \Rightarrow \partial_t \tilde{n} = D \partial_r^2 \tilde{n}$ (предположение о том что $\int_R^L r^2 n(r,0) = 1$)

$$n(r,0) = \cancel{\frac{1}{L^2}} \frac{1}{L^2} (\delta(r-L) - \delta(r-(2R-L))) \Rightarrow$$

$$\Rightarrow \tilde{n} = \frac{1}{L^2} [\delta(r-L) - \delta(r-(2R-L))] \text{ симметрическое распределение}$$

3) $G(x,t) = \frac{1}{\sqrt{4\pi Dt}} e^{-\frac{x^2}{4Dt}}$ где (*) \Rightarrow

$$\Rightarrow \tilde{n}(r,t) = \frac{1}{\sqrt{4\pi Dt} L^2} \int_{-\infty}^{\infty} [\delta(r'-L) - \delta(r'-(2R-L))] \cdot r' e^{-\frac{(r-r')^2}{4Dt}} dr' =$$

$$= \frac{1}{\sqrt{4\pi Dt} L} \left[e^{-\frac{(r-L)^2}{4Dt}} - e^{-\frac{(r-(2R-L))^2}{4Dt}} \right]$$

4) $P_{\text{surv}}(t) = \int_R^{\infty} r^2 n dr = \int_R^{\infty} r^2 \frac{\tilde{n}}{r} dr = \int_R^{\infty} r \tilde{n} dr =$
 $= \frac{1}{\sqrt{4\pi Dt} L} \left[\int_R^{\infty} r e^{-\frac{(r-L)^2}{4Dt}} dr - \int_R^{\infty} r e^{-\frac{(r-(2R-L))^2}{4Dt}} dr \right] = \frac{1}{\sqrt{4\pi Dt} L} \left[\int_0^{\infty} (r+R) e^{-\frac{(r+R-L)^2}{4Dt}} dr - \right.$
 $- \left. \int_0^{\infty} (r+R) e^{-\frac{(r+R+L)^2}{4Dt}} dr \right] = \frac{1}{\sqrt{4\pi Dt} L} \left[\int_0^{\infty} r e^{-\frac{(r+R-L)^2}{4Dt}} dr + \int_0^{\infty} r e^{-\frac{(r+R+L)^2}{4Dt}} dr + \right]$
 $+ R \int_0^{\infty} e^{-\frac{(r+R-L)^2}{4Dt}} dr - R \int_0^{\infty} e^{-\frac{(r+R+L)^2}{4Dt}} dr = \frac{1}{\sqrt{4\pi Dt} L} \left[\int_{-\infty}^{\infty} (r-R+L) e^{-\frac{r^2}{4Dt}} dr + R \left(+ \int_0^{\infty} e^{-\frac{r^2}{4Dt}} dr + \right. \right.$
 $+ R \left(\int_{-R+L}^{\infty} e^{-\frac{r^2}{4Dt}} dr - \int_{-R+L}^{\infty} e^{-\frac{r^2}{4Dt}} dr \right) \left. \right] = \frac{1}{\sqrt{4\pi Dt} L} \left[\int_{-\infty}^{\infty} (r-R+L) e^{-\frac{r^2}{4Dt}} dr + R \left(+ \int_0^{\infty} e^{-\frac{r^2}{4Dt}} dr + \right. \right.$
 $+ \left. \int_0^{L-R} e^{-\frac{r^2}{4Dt}} dr \right) \left. \right] = \frac{1}{\sqrt{4\pi Dt} L} \left[(L-R) \int_{-\infty}^{\infty} e^{-\frac{r^2}{4Dt}} dr + \int_{-\infty}^{\infty} r e^{-\frac{r^2}{4Dt}} dr + 2R \sqrt{\frac{4\pi D t}{2}} \operatorname{erf} \left[\frac{L-R}{\sqrt{4Dt}} \right] \right]$

$$= \frac{1}{\sqrt{4\pi Dt} L} \left[(L-R) \sqrt{\pi D t} + R \sqrt{4\pi D t} \operatorname{Erf} \left[\frac{L-R}{\sqrt{4Dt}} \right] \right] = \boxed{1 - \frac{R}{L} + \frac{R}{L} \operatorname{Erf} \left[\frac{L-R}{\sqrt{4Dt}} \right] \xrightarrow{t \rightarrow \infty} 1 - \frac{R}{L}}$$

5.10.1

$$\begin{cases} P(y=0|x=0) = 1 \\ P(y=1|x=0) = 0 \end{cases} \quad \begin{cases} P(y=0|x=1) = f = 0,1 \\ P(y=1|x=1) = 1-f = 0,9 \end{cases}$$

$$P(x=0) = 0,8$$

$$P(x=1) = 0,2$$

$$(i) \quad \begin{cases} P(y=0) = P(x=0) \cdot P(y=0|x=0) + P(x=1) \cdot P(y=0|x=1) = \\ = 0,8 \cdot 1 + 0,2 \cdot 0,1 = \boxed{0,82 = P(y=0)} \\ P(y=1) = P(x=0) \cdot P(y=1|x=0) + P(x=1) \cdot P(y=1|x=1) = \\ = 0,8 \cdot 0 + 0,2 \cdot 0,9 = \boxed{0,18 = P(y=1)} \end{cases}$$

$$(ii) \quad P(x=1|y=0) = \frac{P(y=0|x=1) \cdot P(x=1)}{P(y=0)} = \frac{0,2 \cdot 0,1}{0,82} = \boxed{\frac{1}{41} = P(x=1|y=0)}$$

↗
~~популярное~~
 единство

$$\begin{cases} P(x=0|y=0) = \frac{P(y=0|x=0) \cdot P(x=0)}{P(y=0)} = \frac{1 \cdot 0,8}{0,82} = \frac{40}{41} \\ P(x=0|y=1) = 0 \\ P(x=1|y=1) = 1 \end{cases} \quad - \text{здесь симметрическое условие}$$

$$(iii) \quad I(X,Y) = S(X) - S(X|Y)$$

$$S(X) = - \sum p(x) \log_2 p(x) = -0,8 \log_2 0,8 - 0,2 \log_2 0,2$$

$$S(X|Y) = - \sum \sum p(x,y) \log_2 \frac{p(x,y)}{p(y)} = - \sum \sum p(x,y) \log_2 p(x|y) =$$

$$\begin{cases} p(x=0,y=0) = 0,8 \\ p(x=0,y=1) = 0 \\ p(x=1,y=0) = \frac{1}{41} \cdot \frac{82}{100} = 0,02 \\ p(x=1,y=1) = 0,18 \end{cases} \quad \begin{aligned} &= -0,8 \log_2 \frac{40}{41} - 0,1 \cancel{\log_2 0} - \\ &\quad - 0,02 \log_2 \frac{1}{41} - 0,18 \cancel{\log_2 1} = \\ &= -0,8 \log_2 0,8 + 0,8 \log_2 0,82 - 0,02 \log_2 0,02 + \\ &\quad + 0,02 \log_2 0,02 \Rightarrow \end{aligned}$$

$$\Rightarrow I(X,Y) = -0,8 \cancel{\log_2 0,8} - 0,2 \log_2 0,2 + 0,8 \cancel{\log_2 0,8} - 0,8 \log_2 0,82 + 0,02 \log_2 0,02 - 0,02 \cancel{\log_2 0,02} = -0,82 \cancel{\log_2 0,82} - 0,18 \log_2 0,2 - 0,02 \log_2 10 \approx 0,5863 \approx 0,6$$