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This is for the homework of week6.

Game description:

This is a Tetris game which is block matching. You will have differently shaped block in random order. In order to win this game, you should complete lines with given blocks.

Earn points as much as you can, if you get fail to complete line with blocks and if your last block hit the top, then you lost. Game is over.

Complexity of the Game:

In order to check the line complete, I used loop. Since it I must check the entire board to see how many lines there are, and then also check the line itself to see where the line is completed, and another loop for checking number of removable lines. So, entirely, I have four inner loops which is take n^4 time.

So, the complexity of the Tetris is O(n^4).

Proof of NP_completeness

NP-Complete Proof Template:

Prove that Q is in NP-Complete

1) Show that Q ∈NP.
   Give a polynomial time algorithm that verifies an instance of Qin polynomial time. In other words if I give you an "answer" can you verify it polynomial time.

   ➔ Basically, Tetris is checking row-width, if the row is filled and will be removed.
   ➔ Set B is number of row width, if the width is 10, this mean the Board is consist of 10 of 1 x 1 size of block.
   ➔ Each Tetris block is consist of 1 x 1 size of block.
   ➔ Tetris is the game you can play on board size of N and M. N and M represents the height and width. If the board size if too large, it takes exponential time, however, with reasonable size of N and M, it takes (pseudo) poly time as O(N*M) with dynamic programing. So, this is in NP
   .
2) Show that R ≤P Q for some R∈NP-Complete.
   a. Pick an instance, R, of your favorite NP-Complete problem. Usually the R you select has a structure similar to Q.

   ➔ I will use the 3 partition problem which is known as NP –Complete. Sum of subsets sum is equal to sum of SET.

➔ Initial board is B, and partially filled block is B'. The sum of B' is equal to B. And when the sum of B' equal to B, you can win and continue the game Tetris

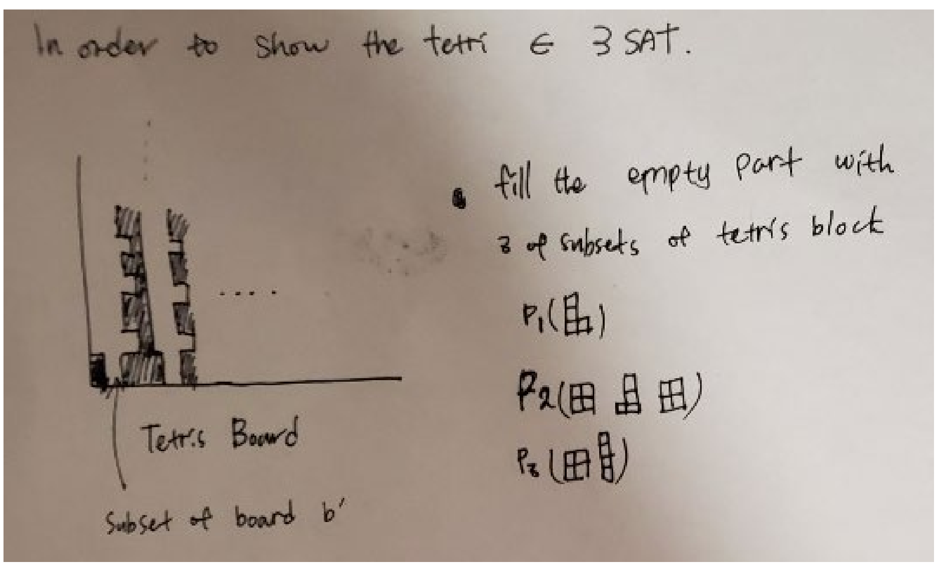
b. Show a polynomial algorithm to transform an arbitrary instance x of R into an instance of x' of Q.
As I simply mentioned above, there is initial Board B, and we intestinally created Tetris board that partially filled blocks in specific way (and I will call it as bucket). And it called B' and, I will fill the B' with Tetris block with p1 to pn. (In order to reduce the problem, remove randomness of block creation) If I can fill the buckets with Tetris blocks without any empty part on the block, I can show the 3 partition problem can be reduced to Tetris. Creating buckets which b' is taking poly time by adding 1 on each designated position of complexity of O(MN)

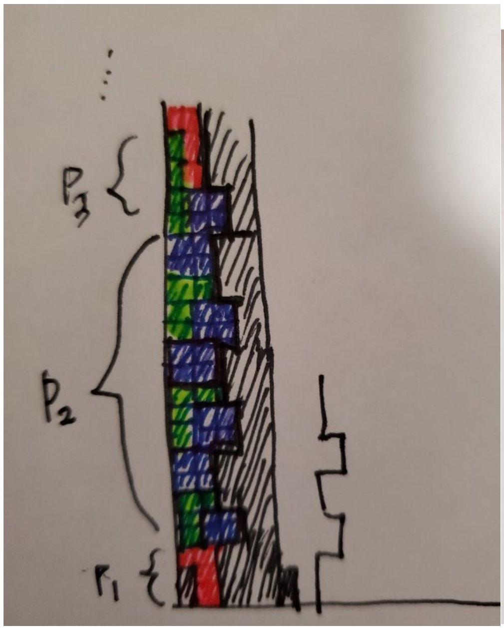
c. Prove that R(x) = yes if and only if Q(x') = yes. That is.
   i. If x is a "yes" solution of R then x' is a "yes" solution of Q.

➔ Now the Tetris Board B has < a1,…,a3, T> and B' has < p1, …. , p3, T'>
➔ Here 'a' represent the buckets and 'p' represent the subset of Tetris blocks.

   ii. If x' is a "yes" solution of Q then x is a "yes" solution of R.

➔





Successfully filled the blanks

So, on this image.

board partitioned into 3 sets, and filled with blocks ~~and block~~ which is multiset of $\langle P_1 \cdots P_n \rangle$ and it fill the bucket without any empty block. it shows the Sum of subset is equal to target sum.

So it shows that the Tetris problem solved the 3 Sat problem.

If both 1) and 2) are true then Q is in NP-Complete

➔ Since I proved the i and ii of c, Tetris game is in NP-Complete

Reference for NP Tetris game

Erik D. Demaine, Susan Hohenberger, and David Liben-Nowell / Tetris is Hard, Even to Approximate / MIT Report / 2003 / https://erikdemaine.org/papers/Tetris_COCOON2003/paper.pdf