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Cs325

Q1

a) **The shortest path from A to G is 19 (a – f – e – g = 10 + 2 +7)**

```
max dg
st
    da = 0
    df - da <= 10
    db - da <= 8
    dc - df <= 3
    db - df <= 7
    dc - db <= 4
    dd - dc <= 3
    df - dd <= 18
    de - df <= 2
    de - dd <= 25
    dg - de <= 7
    dd - dg <= 2
    dh - dg <= 3
    da - dh <= 4
    da - df <= 5
    de - db <= 10
    dd - de <= 9
    db - dh <= 9
end
```

```
LP OPTIMUM FOUND AT STEP      7

       OBJECTIVE FUNCTION VALUE

       1)      19.00000

 VARIABLE         VALUE          REDUCED COST
       DG        19.000000          0.000000
       DA         0.000000          0.000000
       DF        10.000000          0.000000
       DB         2.000000          0.000000
       DC         0.000000          0.000000
       DD         0.000000          0.000000
       DE        12.000000          0.000000
       DH         0.000000          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.000000          1.000000
        3)         0.000000          1.000000
        4)         6.000000          0.000000
        5)        13.000000          0.000000
        6)        15.000000          0.000000
        7)         6.000000          0.000000
        8)         3.000000          0.000000
        9)         8.000000          0.000000
       10)         0.000000          1.000000
       11)        13.000000          0.000000
       12)         0.000000          1.000000
       13)        21.000000          0.000000
       14)        22.000000          0.000000
       15)         4.000000          0.000000
       16)        15.000000          0.000000
       17)         0.000000          0.000000
       18)        21.000000          0.000000
       19)         7.000000          0.000000

NO. ITERATIONS=       7
```

b) **Shortest path a to all other vertices (since we found the shortest path to g above, so I didn't include here)**
(the sum of the shortest distances method**)**
From A

- **To a = 0 / To b = 8 / To c = 12 / To d = 15 / To e = 12 / To f = 10 / To h = 22**

```
max dg + db + dc + dd + de + df + dh
st
   da = 0
   df - da <= 10
   db - da <= 8
   dc - df <= 3
   db - df <= 7
   dc - db <= 4
   dd - dc <= 3
   df - dd <= 18
   de - df <= 2
   de - dd <= 25
   dg - de <= 7
   dd - dg <= 2
   dh - dg <= 3
   da - dh <= 4
   da - df <= 5
   de - db <= 10
   dd - de <= 9
   db - dh <= 9
end
```

```
LP OPTIMUM FOUND AT STEP     11

       OBJECTIVE FUNCTION VALUE

       1)       98.00000

 VARIABLE         VALUE          REDUCED COST
       DG        19.000000          0.000000
       DB         8.000000          0.000000
       DC        12.000000          0.000000
       DD        15.000000          0.000000
       DE        12.000000          0.000000
       DF        10.000000          0.000000
       DH        22.000000          0.000000
       DA         0.000000          0.000000


      ROW    SLACK OR SURPLUS      DUAL PRICES
        2)         0.000000          7.000000
        3)         0.000000          4.000000
        4)         0.000000          3.000000
        5)         1.000000          0.000000
        6)         9.000000          0.000000
        7)         0.000000          2.000000
        8)         0.000000          1.000000
        9)        23.000000          0.000000
       10)         0.000000          3.000000
       11)        28.000000          0.000000
       12)         0.000000          2.000000
       13)         6.000000          0.000000
       14)         0.000000          1.000000
       15)        26.000000          0.000000
       16)        15.000000          0.000000
       17)         6.000000          0.000000
       18)         6.000000          0.000000
       19)        23.000000          0.000000

NO. ITERATIONS=      11
```

(Individually computed way)

-from A to A = 0

```
max da
st
        da = 0
        df-da <= 10
        db-da <=8
        dc-df <=3
        db-df <=7
        dc-db <=4
        dd-dc <=3
        df-dd <=18
        de-df <=2
        de-dd <=25
        dg-de <=7
        dd-dg <=2
        dh-dg <=3
        da-dh <=4
        da-df <=5
        de-db <=10
        dd-de <=9
        db-dh <=9
end
```

```
LP OPTIMUM FOUND AT STEP      1

       OBJECTIVE FUNCTION VALUE

       1)     0.0000000E+00

 VARIABLE         VALUE          REDUCED COST
       DA         0.000000          0.000000
       DF        10.000000          0.000000
       DB         8.000000          0.000000
       DC         0.000000          0.000000
       DD         0.000000          0.000000
       DE        12.000000          0.000000
       DG        19.000000          0.000000
       DH        22.000000          0.000000


      ROW    SLACK OR SURPLUS      DUAL PRICES
        2)         0.000000          1.000000
        3)         0.000000          0.000000
        4)         0.000000          0.000000
        5)        13.000000          0.000000
        6)         9.000000          0.000000
        7)        12.000000          0.000000
        8)         3.000000          0.000000
        9)         8.000000          0.000000
       10)         0.000000          0.000000
       11)        13.000000          0.000000
       12)         0.000000          0.000000
       13)        21.000000          0.000000
       14)         0.000000          0.000000
       15)        26.000000          0.000000
       16)        15.000000          0.000000
       17)         6.000000          0.000000
       18)        21.000000          0.000000
       19)        23.000000          0.000000

NO. ITERATIONS=       1
```

-from A to B = 8

```
max db
st
        da = 0
        df-da <= 10
        db-da <=8
        dc-df <=3
        db-df <=7
        dc-db <=4
        dd-dc <=3
        df-dd <=18
        de-df <=2
        de-dd <=25
        dg-de <=7
        dd-dg <=2
        dh-dg <=3
        da-dh <=4
        da-df <=5
        de-db <=10
        dd-de <=9
        db-dh <=9
end
```

```
LP OPTIMUM FOUND AT STEP      3

       OBJECTIVE FUNCTION VALUE

       1)      8.000000

 VARIABLE         VALUE          REDUCED COST
       DB         8.000000          0.000000
       DA         0.000000          0.000000
       DF        10.000000          0.000000
       DC        12.000000          0.000000
       DD         0.000000          0.000000
       DE        12.000000          0.000000
       DG        19.000000          0.000000
       DH         0.000000          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.000000          1.000000
        3)         0.000000          0.000000
        4)         0.000000          1.000000
        5)         1.000000          0.000000
        6)         9.000000          0.000000
        7)         0.000000          0.000000
        8)        15.000000          0.000000
        9)         8.000000          0.000000
       10)         0.000000          0.000000
       11)        13.000000          0.000000
       12)         0.000000          0.000000
       13)        21.000000          0.000000
       14)        22.000000          0.000000
       15)         4.000000          0.000000
       16)        15.000000          0.000000
```

-From A to C = 12

```
max dc
st
        da = 0
        df-da <= 10
        db-da <=8
        dc-df <=3
        db-df <=7
        dc-db <=4
        dd-dc <=3
        df-dd <=18
        de-df <=2
        de-dd <=25
        dg-de <=7
        dd-dg <=2
        dh-dg <=3
        da-dh <=4
        da-df <=5
        de-db <=10
        dd-de <=9
        db-dh <=9
end
```

```
LP OPTIMUM FOUND AT STEP      0

       OBJECTIVE FUNCTION VALUE

       1)      12.00000

 VARIABLE         VALUE          REDUCED COST
       DC        12.000000          0.000000
       DA         0.000000          0.000000
       DF        10.000000          0.000000
       DB         8.000000          0.000000
       DD         0.000000          0.000000
       DE        12.000000          0.000000
       DG        19.000000          0.000000
       DH         0.000000          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.000000          1.000000
        3)         0.000000          0.000000
        4)         0.000000          1.000000
        5)         1.000000          0.000000
        6)         9.000000          0.000000
        7)         0.000000          1.000000
        8)        15.000000          0.000000
        9)         8.000000          0.000000
       10)         0.000000          0.000000
       11)        13.000000          0.000000
       12)         0.000000          0.000000
       13)        21.000000          0.000000
       14)        22.000000          0.000000
       15)         4.000000          0.000000
       16)        15.000000          0.000000
       17)         6.000000          0.000000
       18)        21.000000          0.000000
       19)         1.000000          0.000000

NO. ITERATIONS=       0
```

-From A to D = 15

```
max dd
st
        da = 0
        df-da <= 10
        db-da <=8
        dc-df <=3
        db-df <=7
        dc-db <=4
        dd-dc <=3
        df-dd <=18
        de-df <=2
        de-dd <=25
        dg-de <=7
        dd-dg <=2
        dh-dg <=3
        da-dh <=4
        da-df <=5
        de-db <=10
        dd-de <=9
        db-dh <=9
end
```

```
LP OPTIMUM FOUND AT STEP      2

       OBJECTIVE FUNCTION VALUE

       1)      15.00000

 VARIABLE         VALUE          REDUCED COST
       DD        15.000000          0.000000
       DA         0.000000          0.000000
       DF        10.000000          0.000000
       DB         8.000000          0.000000
       DC        12.000000          0.000000
       DE        12.000000          0.000000
       DG        19.000000          0.000000
       DH         0.000000          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.000000          1.000000
        3)         0.000000          0.000000
        4)         0.000000          1.000000
        5)         1.000000          0.000000
        6)         9.000000          0.000000
        7)         0.000000          1.000000
        8)         0.000000          1.000000
        9)        23.000000          0.000000
       10)         0.000000          0.000000
       11)        28.000000          0.000000
       12)         0.000000          0.000000
       13)         6.000000          0.000000
       14)        22.000000          0.000000
       15)         4.000000          0.000000
       16)        15.000000          0.000000
       17)         6.000000          0.000000
       18)         6.000000          0.000000
       19)         1.000000          0.000000

NO. ITERATIONS=       2
```

-From A to E = 12

```
max de
st
        da = 0
        df-da <= 10
        db-da <=8
        dc-df <=3
        db-df <=7
        dc-db <=4
        dd-dc <=3
        df-dd <=18
        de-df <=2
        de-dd <=25
        dg-de <=7
        dd-dg <=2
        dh-dg <=3
        da-dh <=4
        da-df <=5
        de-db <=10
        dd-de <=9
        db-dh <=9
end
```

```
LP OPTIMUM FOUND AT STEP      0

       OBJECTIVE FUNCTION VALUE

       1)      12.00000

 VARIABLE        VALUE          REDUCED COST
       DE        12.000000          0.000000
       DA         0.000000          0.000000
       DF        10.000000          0.000000
       DB         8.000000          0.000000
       DC        12.000000          0.000000
       DD        15.000000          0.000000
       DG        19.000000          0.000000
       DH         0.000000          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
        2)         0.000000          1.000000
        3)         0.000000          1.000000
        4)         0.000000          0.000000
        5)         1.000000          0.000000
        6)         9.000000          0.000000
        7)         0.000000          0.000000
        8)         0.000000          0.000000
        9)        23.000000          0.000000
       10)         0.000000          1.000000
       11)        28.000000          0.000000
       12)         0.000000          0.000000
       13)         6.000000          0.000000
       14)        22.000000          0.000000
       15)         4.000000          0.000000
       16)        15.000000          0.000000
       17)         6.000000          0.000000
       18)         6.000000          0.000000
       19)         1.000000          0.000000

NO. ITERATIONS=       0
```

-From A to F = 10

```
max df
st
        da = 0
        df-da <= 10
        db-da <=8
        dc-df <=3
        db-df <=7
        dc-db <=4
        dd-dc <=3
        df-dd <=18
        de-df <=2
        de-dd <=25
        dg-de <=7
        dd-dg <=2
        dh-dg <=3
        da-dh <=4
        da-df <=5
        de-db <=10
        dd-de <=9
        db-dh <=9
end
```

```
LP OPTIMUM FOUND AT STEP      0

       OBJECTIVE FUNCTION VALUE

       1)      10.00000

 VARIABLE        VALUE          REDUCED COST
       DF       10.000000          0.000000
       DA        0.000000          0.000000
       DB        8.000000          0.000000
       DC       12.000000          0.000000
       DD       15.000000          0.000000
       DE       12.000000          0.000000
       DG       19.000000          0.000000
       DH        0.000000          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
        2)        0.000000          1.000000
        3)        0.000000          1.000000
        4)        0.000000          0.000000
        5)        1.000000          0.000000
        6)        9.000000          0.000000
        7)        0.000000          0.000000
        8)        0.000000          0.000000
        9)       23.000000          0.000000
       10)        0.000000          0.000000
       11)       28.000000          0.000000
       12)        0.000000          0.000000
       13)        6.000000          0.000000
       14)       22.000000          0.000000
       15)        4.000000          0.000000
       16)       15.000000          0.000000
       17)        6.000000          0.000000
       18)        6.000000          0.000000
       19)        1.000000          0.000000

NO. ITERATIONS=       0
```

From A to H = 22

```
max dh
st
        da = 0
        df-da <= 10
        db-da <=8
        dc-df <=3
        db-df <=7
        dc-db <=4
        dd-dc <=3
        df-dd <=18
        de-df <=2
        de-dd <=25
        dg-de <=7
        dd-dg <=2
        dh-dg <=3
        da-dh <=4
        da-df <=5
        de-db <=10
        dd-de <=9
        db-dh <=9
end
```

```
LP OPTIMUM FOUND AT STEP      6

       OBJECTIVE FUNCTION VALUE

       1)      22.00000

 VARIABLE        VALUE          REDUCED COST
       DH       22.000000          0.000000
       DA        0.000000          0.000000
       DF       10.000000          0.000000
       DB        8.000000          0.000000
       DC        0.000000          0.000000
       DD        3.000000          0.000000
       DE       12.000000          0.000000
       DG       19.000000          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
        2)        0.000000          1.000000
        3)        0.000000          1.000000
        4)        0.000000          0.000000
        5)       13.000000          0.000000
        6)        9.000000          0.000000
        7)       12.000000          0.000000
        8)        0.000000          0.000000
        9)       11.000000          0.000000
       10)        0.000000          1.000000
       11)       16.000000          0.000000
       12)        0.000000          1.000000
       13)       18.000000          0.000000
       14)        0.000000          1.000000
       15)       26.000000          0.000000
       16)       15.000000          0.000000
       17)        6.000000          0.000000
       18)       18.000000          0.000000
       19)       23.000000          0.000000

NO. ITERATIONS=       6
```

Q2

-Linear program code

```
MAX 3.5w + 2.27x + 2.66y + 3.04z
ST
    0.125w <= 1000
    0.08x + 0.05y + 0.03z <= 2050
    0.05y + 0.07z <= 1250
    w >= 6000
    w <= 7000
    x >= 10000
    x <= 14000
    y >= 14000
    y <= 16000
    z >= 6000
    z <= 8500
    w >= 0
    x >= 0
    y >= 0
    z >= 0
END
GIN w
GIN x
GIN y
GIN z
```

-Output

```
LP OPTIMUM FOUND AT STEP      7
OBJECTIVE VALUE =   117243.570

SET        Z TO <=  7856 AT    1, BND=  0.1172E+06 TWIN=-0.1000E+31     28
SET        Y TO <= 14001 AT    2, BND=  0.1172E+06 TWIN= 0.1172E+06     34
SET        X TO <= 13928 AT    3, BND=  0.1172E+06 TWIN= 0.1172E+06     38

NEW INTEGER SOLUTION OF    117241.461     AT BRANCH      3 PIVOT      38
BOUND ON OPTIMUM:  117242.9
DELETE         X AT LEVEL     3
FLIP        Y TO >=     14002 AT    2 WITH BND=    117242.93
SET        Z TO <=  7855 AT    3, BND=  0.1172E+06 TWIN=-0.1000E+31     41
SET        X TO >= 13928 AT    4, BND=  0.1172E+06 TWIN= 0.1172E+06     45
SET        X TO <= 13928 AT    5, BND=  0.1172E+06 TWIN=-0.1000E+31     45
SET        Z TO >=  7855 AT    6, BND=  0.1172E+06 TWIN=-0.1000E+31     45
SET        Y TO <= 14002 AT    7, BND=  0.1172E+06 TWIN=-0.1000E+31     46
DELETE         Y AT LEVEL     7
DELETE         Z AT LEVEL     6
DELETE         X AT LEVEL     5
FLIP        X TO <=     13927 AT    4 WITH BND=    117242.16
SET        X TO >= 13927 AT    5, BND=  0.1172E+06 TWIN=-0.1000E+31     46
SET        Y TO <= 14004 AT    6, BND=  0.1172E+06 TWIN= 0.1172E+06     50
SET        Y TO >= 14004 AT    7, BND=  0.1172E+06 TWIN=-0.1000E+31     50
SET        Z TO <=  7854 AT    8, BND=  0.1172E+06 TWIN=-0.1000E+31     52
DELETE         Z AT LEVEL     8
DELETE         Y AT LEVEL     7
DELETE         Y AT LEVEL     6
DELETE         X AT LEVEL     5
DELETE         X AT LEVEL     4
DELETE         Z AT LEVEL     3
DELETE         Y AT LEVEL     2
DELETE         Z AT LEVEL     1
ENUMERATION COMPLETE. BRANCHES=     8 PIVOTS=      52
```

```
LAST INTEGER SOLUTION IS THE BEST FOUND
RE-INSTALLING BEST SOLUTION...

       OBJECTIVE FUNCTION VALUE

       1)      117241.5

 VARIABLE        VALUE          REDUCED COST
        W      7000.000000         -3.500000
        X     13928.000000         -2.270000
        Y     14001.000000         -2.660000
        Z      7856.000000         -3.040000


      ROW    SLACK OR SURPLUS      DUAL PRICES
        2)       125.000000          0.000000
        3)         0.030020          0.000000
        4)         0.029987          0.000000
        5)      1000.000000          0.000000
        6)         0.000000          0.000000
        7)      3928.000000          0.000000
        8)        72.000000          0.000000
        9)         1.000000          0.000000
       10)      1999.000000          0.000000
       11)      1856.000000          0.000000
       12)       644.000000          0.000000
       13)      7000.000000          0.000000
       14)     13928.000000          0.000000
       15)     14001.000000          0.000000
       16)      7856.000000          0.000000

NO. ITERATIONS=      53
BRANCHES=     8 DETERM.=  1.000E     0
```

-Answer: in order to maximize the profit, *this company should produce 7000 of silk ties, 13928 of polyester ties, 14001 of blend 1 ties, and 7856 blend2 ties.*

Q3

a) Linear program code & output

```
min a+b+c+d
st
    1a + 5b + 10c + 25d = 202
    a >= 0
    b >= 0
    c >= 0
    d >= 0
end
GIN a
GIN b
GIN c
GIN d
```

```
FIX ALL VARS.(    2)  WITH RC >  0.000000E+00
SET        A TO >=     1 AT    1, BND=  -9.040     TWIN=-0.1000E+31      5
SET        D TO <=     8 AT    2, BND=  -10.00     TWIN=-0.1000E+31      6

NEW INTEGER SOLUTION OF    10.0000000     AT BRANCH      2 PIVOT       6
BOUND ON OPTIMUM:  9.000000
DELETE         D AT LEVEL     2
DELETE         A AT LEVEL     1
RELEASE FIXED VARIABLES
FIX ALL VARS.(    1)  WITH RC >  0.000000E+00
SET        C TO <=     1 AT    1, BND=  -9.000     TWIN= -9.280         15
SET        A TO >=     1 AT    2, BND=  -9.640     TWIN=-0.1000E+31     18
DELETE         A AT LEVEL     2
DELETE         C AT LEVEL     1
RELEASE FIXED VARIABLES
FIX ALL VARS.(    1)  WITH RC >  0.000000E+00
SET        C TO <=     0 AT    1, BND=  -9.000     TWIN= -9.480         25
SET        B TO >=     2 AT    2, BND=  -9.680     TWIN=-0.1000E+31     27
DELETE         B AT LEVEL     2
DELETE         C AT LEVEL     1
RELEASE FIXED VARIABLES
ENUMERATION COMPLETE. BRANCHES=     6 PIVOTS=      32

LAST INTEGER SOLUTION IS THE BEST FOUND
RE-INSTALLING BEST SOLUTION...

       OBJECTIVE FUNCTION VALUE

       1)      10.00000

 VARIABLE        VALUE          REDUCED COST
        A         2.000000          1.000000
        B         0.000000          1.000000
        C         0.000000          1.000000
        D         8.000000          1.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.000000          0.000000
       3)         2.000000          0.000000
       4)         0.000000          0.000000
       5)         0.000000          0.000000
       6)         8.000000          0.000000

NO. ITERATIONS=      32
BRANCHES=     6 DETERM.=  1.000E    0
```

In order to minimize the number of coin used for the amount of 202: *you can include 8 of D-coin (25) and 2 of A-coin (1).*

b) Linear program code & output

```
min a+b+c+d+e
st
    1a + 3b + 7c + 12d + 27e= 293
    a >= 0
    b >= 0
    c >= 0
    d >= 0
    e >= 0
end
GIN a
GIN b
GIN c
GIN d
GIN e
```

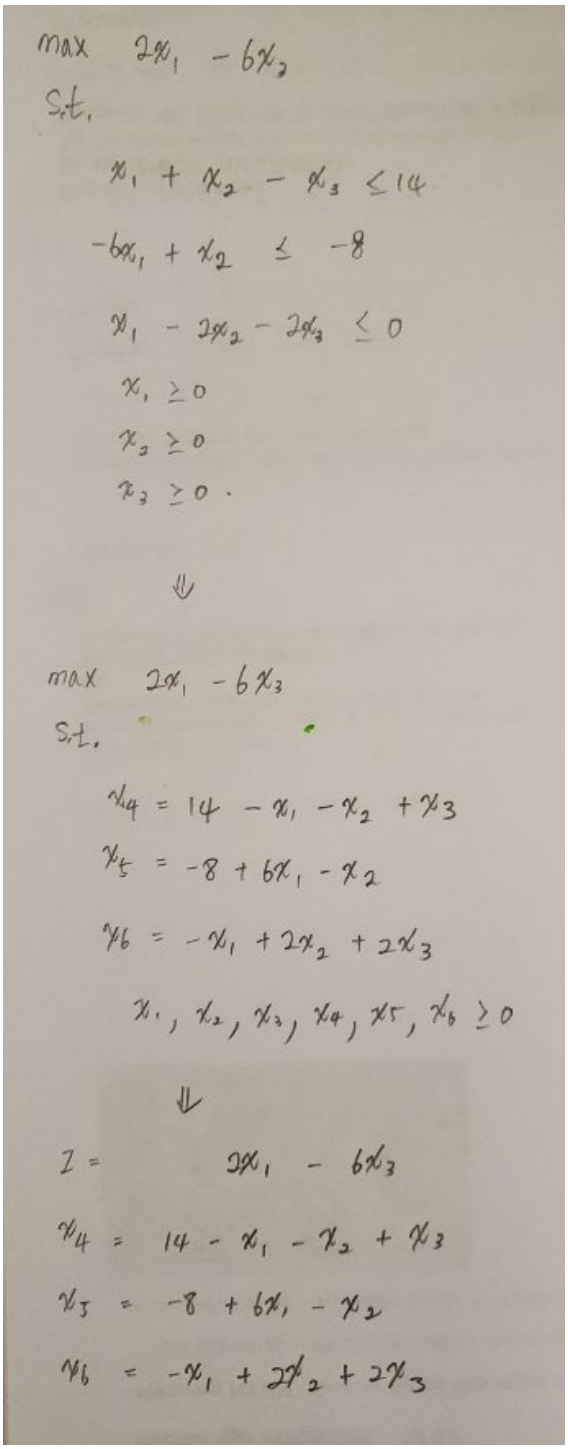
```
SET        D TO <=      3 AT    4, BND=  -14.00      TWIN=-0.1000E+31      75

NEW INTEGER SOLUTION OF    14.0000000     AT BRANCH     31 PIVOT      75
BOUND ON OPTIMUM:  12.33333
DELETE         D AT LEVEL     4
DELETE         E AT LEVEL     3
DELETE         E AT LEVEL     2
DELETE         D AT LEVEL     1
RELEASE FIXED VARIABLES
FIX ALL VARS.(    2)  WITH RC >  0.000000E+00
SET        D TO >=      2 AT    1, BND=  -12.85      TWIN= -14.67          84
SET        D TO <=      2 AT    2, BND=  -12.85      TWIN=-0.1000E+31      84
SET        E TO <=      9 AT    3, BND=  -19.67      TWIN=-0.1000E+31      86
DELETE         E AT LEVEL     3
DELETE         D AT LEVEL     2
DELETE         D AT LEVEL     1
RELEASE FIXED VARIABLES
SET        C TO <=      0 AT    1, BND=  -13.58      TWIN= -13.08          98
DELETE         C AT LEVEL     1
ENUMERATION COMPLETE. BRANCHES=    34 PIVOTS=      98

LAST INTEGER SOLUTION IS THE BEST FOUND
RE-INSTALLING BEST SOLUTION...

       OBJECTIVE FUNCTION VALUE

       1)      14.00000

 VARIABLE        VALUE          REDUCED COST
        A         0.000000          1.000000
        B         0.000000          1.000000
        C         2.000000          1.000000
        D         3.000000          1.000000
        E         9.000000          1.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.000000          0.000000
       3)         0.000000          0.000000
       4)         0.000000          0.000000
       5)         2.000000          0.000000
       6)         3.000000          0.000000
       7)         9.000000          0.000000

NO. ITERATIONS=      98
BRANCHES=   34 DETERM.=  1.000E    0
```

In order to minimize the number of coin used for the amount of 293: you can *include 9 of E-coin (27), 3 of D-coin (12), and 2 of C-coin (7).*

Q4

a)

first, make the formula to standard form, and then make it to slack form



max $2x_1 - 6x_2$

s.t.

$x_1 + x_2 - x_3 \leq 14$

$-6x_1 + x_2 \leq -8$

$x_1 - 2x_2 - 2x_3 \leq 0$

$x_1 \geq 0$

$x_2 \geq 0$

$x_3 \geq 0$.

⇓

max $2x_1 - 6x_3$

s.t.

$x_4 = 14 - x_1 - x_2 + x_3$

$x_5 = -8 + 6x_1 - x_2$

$x_6 = -x_1 + 2x_2 + 2x_3$

$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$

⇓

$Z = 2x_1 - 6x_3$

$x_4 = 14 - x_1 - x_2 + x_3$

$x_5 = -8 + 6x_1 - x_2$

$x_6 = -x_1 + 2x_2 + 2x_3$

b) Since the left-hand side of equality is basic variable, x4, x5, x6 are basic variables. And right-hand side of equality is nonbasic variable, x1, x2, x3 are nonbasic variables