CS 325 Summer 2020

Homework 5

Problem 1 (7 points)

Shortest Paths using LP: Shortest paths can be cast as an LP using distances dv from the source s to a particular vertex v as variables.

- We can compute the shortest path from s to t in a weighted directed graph by solving.

  max dt
  subject to
  ds = 0
  dv – du ≤ w(u,v) for all (u,v)∈ E

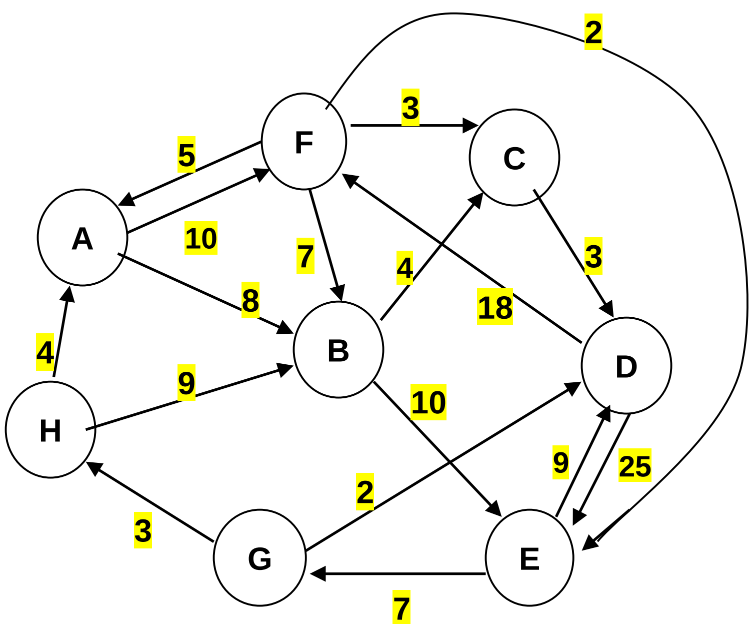
- We can compute the single-source by changing the objective function to

  max $\sum_{v \in V} dv$

Use linear programming to answer the questions below. State the objective function and constraints for each problem and include a copy of the LP code and output.

(a) Find the distance of the shortest path from A to G in the graph below.

(b) Find the distances of the shortest paths from A to all other vertices.



Problem 2 (7 points)

Product Mix: Acme Industries produces four types of men's ties using three types of material. Your job is to determine how many of each type of tie to make each month. The goal is to maximize profit, profit per tie = selling price - labor cost – material cost. Labor cost is \$0.75 per tie for all four types of ties. The material requirements and costs are given below.

Material	**Cost per yard**	**Yards available per month**
Silk	\$20	1,000
Polyester	\$6	2,050
Cotton	\$12	1,250

Product Information	**Type of Tie**			
	Silk = s	**Poly = p**	**Blend1 = b**	**Blend2 = c**
Selling Price per tie	\$6.75	\$3.50	\$4.31	\$4.81
Monthly Minimum units	6,000	10,000	14,000	6,000
Monthly Maximum units	7,000	14,000	16,000	8,500

Material Information in yards	**Type of Tie**			
	Silk	**Polyester**	**Blend 1 (50/50)**	**Blend 2 (30/70)**
Silk	0.125	0	0	0
Polyester	0	0.08	0.05	0.03
Cotton	0	0	0.05	0.07

Formulate the problem as a linear program with an objective function and all constraints. Determine the optimal solution for the linear program using any software you want. Include a copy of the code and output. What are the optimal numbers of ties of each type to maximize profit?

Problem 3 (6 points)

Making Change: Given coins of denominations (value) $1 = v1 < v2 < \ldots < vn$, we wish to make change for an amount A using as few coins as possible. Assume that vi's and A are integers. Since v1= 1 there will always be a solution. Solve the coin change using integer programming. For each of the following denomination sets and amounts, formulate the problem as an integer program with an objective function and constraints. Determine the optimal solution. What is the minimum number of coins used in each case and how many of each coin is used? Include a copy of your code.

(a) V = [1, 5, 10, 25] and A = 202
(b) V = [1, 3, 7, 12, 27] and A = 293

Problem 4 (5 points)

Consider the following linear program.

(a) Write the following linear program in slack form.

(b) State what are the basic and non-basic variables in your slack form.

Maximize $2x_1 - 6x_3$

Subject to

$x_1 + x_2 - x_3 \leq 14$

$6x_1 - x_2 \geq 8$

$-x_1 + 2x_2 + 2x_3 \geq 0$

$x_1 \geq 0$

$x_2 \geq 0$

$x_3 \geq 0$

You may solve Problems 1 to 3 using your choice of software, state which software package/language(s) you used and provide the code or spreadsheet.

Note: There is no submission to TEACH this week.