Application of Discrete Models

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1 Representation of Integers

1.1 Euclidean division

If $a, b \in \mathbb{Z}$ with $b \neq 0$ then $\exists! q, r \in \mathbb{Z}$ such that $a = qb + r$ where $0 \leq r < |b|$. This is the *Euclidean division* or *long division* of the *dividend* a with the *divisor* b. The results of the division are the *quotient* q and the *remainder* r. The standard notation for the remainder is $a \bmod b$.

1.2 Number systems

Let $1 < b \in \mathbb{Z}$ be the *base* of the *number system.* For each $0 \leq n \in \mathbb{Z}$ there exists a unique $1 \leq d \in \mathbb{Z}$ and a unique set of *digits* $0 \leq n_1, n_2, \ldots, n_{d-1} < b$ all integers, such that

$$n = \sum_{k=0}^{d-1} n_k b^k.$$

If $n = 0$, then $d = 1$ and $n_0 = 0$. Otherwise $d = \lfloor \log_b n \rfloor + 1$ and we can extract the digits of n with long division, since

$$\begin{aligned} n &= n_{d-1} b^{d-1} + \cdots + n_2 b^2 + n_1 b + n_0 \\ &= \left(n_{d-1} b^{d-2} + \cdots + n_2 b + n_1 \right) b + n_0 \end{aligned}$$

where the quotient $n_{d-1} b^{d-2} + \cdots + n_2 b + n_1$ is a $d - 1$ digit number and n_0 is the extracted digit.

We call n_0 the *least significant digit* and n_{d-1} the *most significant digit.* The storage order of digits is called *little endian* if we start at the least significant digits and move towards the most significant one. Otherwise it is called *big endian.*