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CSC 217
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CSc 217

Problem Set 2

Instructor: *Elahe Vahdani*

Problem 1 (20 points)

4 buses carrying 148 job-seeking students arrive at a job convention. The buses carry 40, 33, 25, and 50 students, respectively. One of the students is randomly selected. Let X denote the number of students that were on the bus carrying this randomly selected student. One of the 4 bus drivers is also randomly selected. Let Y denote the number of students on his bus.

- (a) Which of E[X] or E [Y] do you think is larger? Give your reasoning in words.

  E[X] is greater than E[Y] because in E[X], there is a greater chance of choosing a student from a bus with a larger number of students, whereas for E[Y], the chance of picking each bus is equal when choosing one of the drivers.

- (b) Compute E[X] and E [Y].

  E[X] = 40 * P (40) + 33 * P (33) + 25 * P (25) + 50 * P (50)

  E[X] = 40*(40/148) + 33*(33/148) + 25*(25 /148) + 50*(50/148)

  E[X] = 39.284

  E[Y] = ¼ (40 + 33 + 25 + 50)

  E[Y] = ¼ (148)

  E[Y] = 37

Problem 2 (25 points)

Consider an experiment in which a fair four-sided die (with faces labeled 0, 1, 2, 3) is thrown once to determine how many times a fair coin is to be flipped. In the sample space of this experiment, random variables N and K are defined by

- N = the result of the die roll

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- K = the total number of heads resulting from the coin flips

- (a) Determine and sketch $p_N(n)$

  $p_N(n)$ = ¼ because it is a fair-four-sided die therefore, N is equally likely to be any of the four numbers: 0, 1, 2, 3.

- (b) Determine and tabulate $p_{N,K}(n,k)$

N	P(N=n)	K	P(K=k|N=n)	P(n,k) = P(K=k|N=n)P(N=n)
0	0.25	0	1	0.25
0	0.25	1	0	0
0	0.25	2	0	0
0	0.25	3	0	0
1	0.25	0	0.5	0.125
1	0.25	1	0.5	0.125
1	0.25	2	0	0
1	0.25	3	0	0
2	0.25	0	0.25	0.0625
2	0.25	1	0.5	0.125
2	0.25	2	0.25	0.0625
2	0.25	3	0	0
3	0.25	0	0.125	0.03125

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3	0.25	1	0.375	0.09375
3	0.25	2	0.375	0.09375
3	0.25	3	0.125	0.03125
0	0.25	0	1	0.25

- (c) Determine and sketch $p_{K|N}(k \mid 2)$

If k = 0, then $p_{K|N}(k \mid 2)$ = ¼

If k = 1, then $p_{K|N}(k \mid 2)$ = ½

If k = 2, then $p_{K|N}(k \mid 2)$ = ¼

If k = 3, then $p_{K|N}(k \mid 2)$ = 0

- (d) Determine and sketch $p_{N|K}(n \mid 2)$

If n = 0, then $p_{N|K}(n \mid 2) = 0$

If n = 1, then $p_{N|K}(n \mid 2) = 0$

If n = 2, then $p_{N|K}(n \mid 2) = 2/5$

If n = 3, then $p_{N|K}(n \mid 2) = 3/5$

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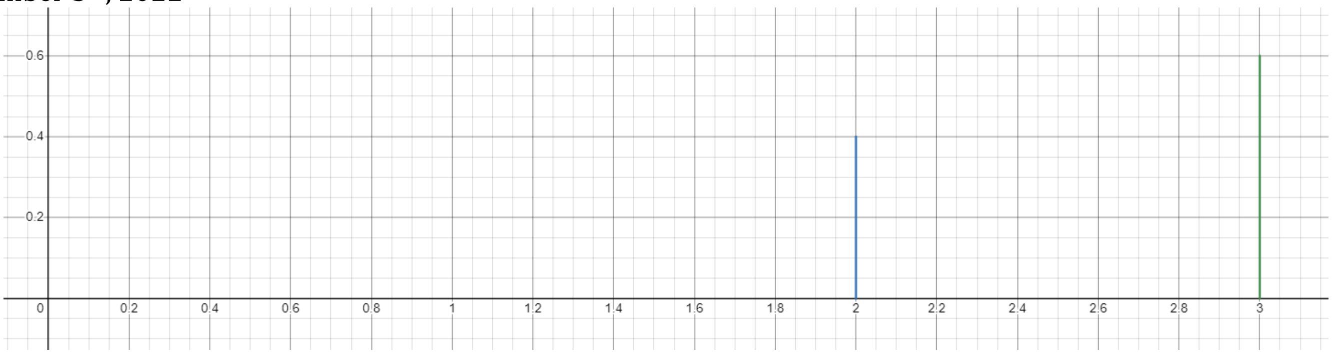
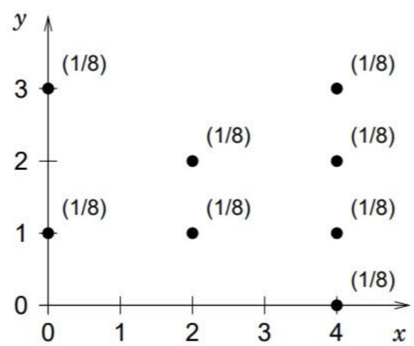


Problem 3 (25 points)

Consider an outcome space comprising eight equally likely event points, as shown below:



- (a) Which value(s) of x maximize(s) $E[Y \mid X = x]$?

  At X = 0, E (Y | X = x) is at it's maximum.

  E [Y | X = 0] = ½ + 3/2 = 2

  E [Y | X = 2] = ½ + 1 = 3/2

  E [Y | X = 4] = ¼ + ½ + 3/4 = 3/2

- (b) Which value(s) of y maximize(s) var $(X \mid Y = y)$?

  At Y = 0, V (X | Y = y) is at it's maximum.

  Var (Y | X = 0) = 10/2 – 4 = 1

  Var (Y | X = 2) = 5/2 – 9/4 = ¼

  Var {Y | X = 4) = 11/4 – 9/4 = ½

- (c) Let $R = min(X, Y)$. Sketch $p_R(r)$

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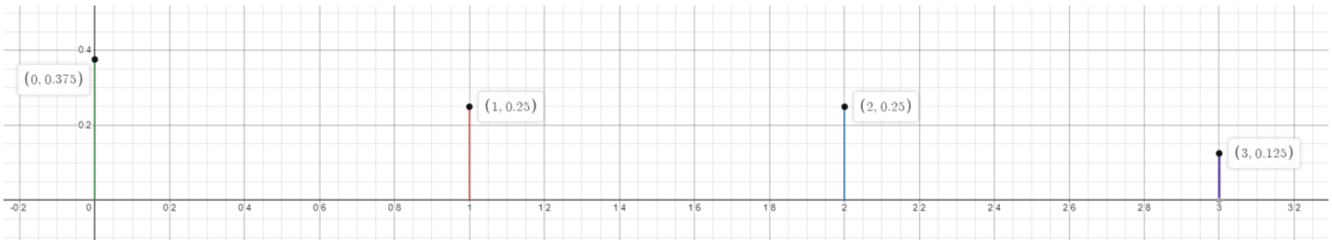
$p_R(0)$ = P (X = 0, Y = 0) + P (X = 0, Y = 3) + P (X = 4, Y = 0) = 3/8

$p_R(1)$ = P (X = 2, Y = 1) + P (X = 4, Y = 1) = ¼

$p_R(2)$ = P (X = 2, Y = 2) + P (X = 4, Y = 2) = ¼

$p_R(3)$ = P (X = 4, Y = 3) = 1/8



- (d) Let A denote the event $X^2 \geq Y$. Determine numerical values for the quantities $E[XY]$ and $E[XY|A]$.

  E [XY] = 2 (1/8) + 4(1/8) + 4 (1/8) + 8(1/8) + 12(1/8) = 30/8

  E [XY | A] = E [XY], therefore E [XY | A] = 30/8.

Problem 4 (30 points)

The joint PMF of the random variables X and Y is given by the following table:

$y = 3$	c	c	$2c$
$y = 2$	$2c$	0	$4c$
$y = 1$	$3c$	c	$6c$
	$x = 1$	$x = 2$	$x = 3$

- (a) Find the value of the constant c.

  The sum of the probabilities must be 1 and c + 2c + 3c + c + 0 + c + 2c + 4c + 6c = 20c, therefore the constant c must be equal to 1/20.

- (b) Find $p_Y(2)$.

  $p_Y(2)$ = P (x = 1, y = 2) + P (x = 2, y = 2) + P (x = 3, y = 2) = 6c = 6 * 1/20 = 3/10.

- (c) Consider the random variable $Z = YX^2$. Find $E[Z | Y = 2]$.

  $E[Z | Y = 2]$ = E [2x^2] = 2(1 * 6c + 4 * 2c + 9 * 12c) = 244c = 244 * 1/20 = 12.2

- (d) Conditioned on the event that $X \neq 2$, are X and Y independent? Give a one-line justification.

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Conditioned on the event that $X \neq 2$, X and Y are not independent because $P[X = x, Y = y] \neq P(X = x) * P(Y = y)$ for all x, y.

$P[X = 1, Y = 3] = c = 1/20$

$P[X = 1] * P[Y = 3] = 6c * 4c = 24c = 6/100$

$P[X = 1, Y = 3] \neq P[X = 1] * P[Y = 3]$

- (e) Find the conditional variance of Y given that X = 2.

$V(Y|X = 2) = E[Z^2| T = 2] – E[Z | Y = 2] = 4(6c + 32c + 972c)/20 – (12.2)^2 = 202 – 148.84 = 53.16.$