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CSC 217
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Problem Set 1

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Problem 1 (25 points)

You flip a fair coin 4 times, determine the probability of the below events (Tosses are independent).

- (a) Four heads: HHHH

  The sample space contains 16 distinct elements and HHHH only occurs once. Therefore, the probability of four heads is 1/16.

- (b) The sequence head, tail, head, tail: HTHT

  The sample space contains 16 elements and HTHT only occurs once. Therefore, the probability of the sequence head, tail, head, tail occurring is 1/16.

- (c) Any sequence with 3 heads and 1 tail

  The sample space contains 16 elements and there are 4 sequences that contain 3 heads and 1 tail: HHHT, HHTH, HTHH, and THHH. Therefore, the probability of any sequence containing 3 heads and 1 tail occurring is 4/16 or ¼.

- (d) Any sequence where the number of heads is greater than or equal to the number of tails

  The sample space contains 16 elements and the number of sequences where the #heads >= #tails is 11: HHHH, HHHT, HHTH, HHTT, HTHH, HTHT, HTTH, THHH, THHT, THTH, and TTHH. Therefore, the probability of any sequence where the number of heads is greater than or equal to the number of tails is 11/16.

Problem 2 (25 points)

You have an exceptional pair of four-sided dice. When you roll the dice, the probability of any particular outcome is proportional to the sum of the results of each dice. Meaning that $P((x,y)) = (x + y)\alpha$ and α is a constant. Also, if x 6= y, (x,y) and (y,x) are considered as two different outcomes , but they are equally likely: $P((x,y)) = P((y,x))$ based on definition.

- What is the probability of the sum being odd?

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If P(x,y) = α(x+y) then the Σ P(x,y) is equal to the sum of the probabilities of all outcomes of rolling the pair of four-sided dices. Thus, Σ P(x,y) = 80α, where Σ P = 1 and α = 1/80. The probability of the sum being odd is the sum of the probability where the outcomes of the dice are 3,5,7 which is 40α. Therefore, the probability of the sum being odd is ½.

- What is the probability of rolling a 3 and a 4, in any order?

The probability of rolling a 3 and a 4 in any order is equal to sum of the probability of outcome (3,4) + (4,3) which is (7α + 7α)/80α. Therefore, the probability of rolling a 3 and a 4 is 7/40.

Problem 3 (25 points)

Alice and Bob each choose at random a number in the interval [0,2]. We assume a uniform probability law under which the probability of an event is proportional to its area. Consider the following events:

- A: The magnitude of the difference of the two numbers is greater than $\frac{1}{3}$.
- B: At least one of the numbers is greater than $\frac{1}{3}$.
- C: The two numbers are equal.
- D: Alice's number is greater than $\frac{1}{3}$.

Find the probabilities *P*(*B*), *P*(*C*), and *P* (*A* ∩ *D*).

Let the number that Alice chooses be x and the number that Bob chooses be y. Then the set of numbers (x, y) that Alice and Bob choose respectively will lie within a square measuring 2 by 2.

The probability that at least one of numbers is greater than 1/3 is the sum of the areas where x > 1/3 and y > 1/3. The sum of the areas would be $\left[\left(2-\frac{1}{3}\right)*\frac{1}{3}\right]+\left[\left(2-\frac{1}{3}\right)\left(2-\frac{1}{3}\right)\right]+\left[\left(2-\frac{1}{3}\right)*\frac{1}{3}\right]=\frac{35}{9}$. Therefore, the probability of P(B) is $\frac{\frac{35}{9}}{4}=\frac{35}{36}$.

The probability that the two numbers are equal is the area of the line x = y. Because x = y is a line, the area is equal to 0. Therefore, the probability of P(C) is 0.

The probability of *P* (*A* ∩ *D*) is the sum of the areas where Alice's number x > 1/3 and the magnitude of the difference of the two numbers is greater than 1/3. The probability of *P* (*A* ∩ *D*) is ½ $*\frac{25}{36}+\frac{\frac{1}{2}*\frac{4}{3}*\frac{4}{3}}{4}=\frac{25}{72}+\frac{16}{72}=\frac{41}{72}$.

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Problem 4 (25 points)

You have a fair five-sided dice. The sides of the dice are numbered from 1 to 5. Each dice roll is independent of all others, and all faces are equally likely to come out on top when the dice is rolled.
Suppose you roll the dice twice.

Part a

Let event A to be "the total of two rolls is 10", event B be "at least one roll resulted in 5", and event C be "at least one roll resulted in 1".

- Is event A independent of event B?

  P(A) = 1/25 and P(B) = 9/25 and the P (A ∩ B) = 1/25. This implies that A and B have non-zero probabilities of occurring. Thus, if any of the following statements P (A ∩ B) = P(A)*P(B), P(A|B) = P(A), P(B|A) = P(B) is true then A and B are independent. P(A|B) = P (A ∩ B)/ P(B) = 1/9. The probability of P(A|B) = 1/9 is not equal to P(A) = 1/25, therefore events A and B are not independent.

- Is event A independent of event C?

  P(A) = 1/25 and P(C) = 9/25 and P (A ∩ C) = 0. If P(A|C) = P(C) then A and B are independent, if it is false, then A and B are not independent. P(A|C) = 0 and 0 is not equal to 9/25. Therefore, events A and C are not independent.

Part b

Let event D be "the total of two rolls is 7", event E be "the difference between the two roll outcomes is exactly 1", and event F be "the second roll resulted in a higher number than the first roll".

- Are events E and F independent?

  The probability of P(E) occurring is 8/25 and the probability of P(F) occurring is 10/25. If P(F|E) is equal to P(F) then the events are independent. P(F|E) = P (F ∩ E)/P(E) = ½ . ½ is not equal to 10/25, therefore Event E and Event F are not independent.

- Are events E and F independent given event D?

  Events E and F are independent given event D if P (E ∩ F|D) = P (E|D) * P (F|D). The probability of P (E ∩ F|D) = ¼. The probability of P (E|D) is ½ and P (F|D) is ½. Therefore, events E and F are independent given D.