Zi Xuan Li

Professor Vahdani

CSC 21700

December 4th, 2022

CSc 217

Problem Set 3

Instructor: *Elahe Vahdani*

Problem 1 (20 points)

Let X be a continuous random variable with PDF given by

$$f_X(x) = \begin{cases} cx^2 & \text{if } |x| \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

- A) Find constant c.
- B) Calculate $E[3X+6]$
- C) Calculate $\text{Var}[3X+6]$.
- D) Calculate $P(X \geq \frac{1}{3})$, $P(X \leq \frac{7}{3})$ and $P(X \leq \frac{7}{3} | X \geq \frac{1}{3})$.

A) Find constant C

$$\int_{-3}^{3} cx^2 dx = c * \left[\frac{x^3}{3}\right]_{-3}^{3} = c\left[\frac{(3)^3}{3} - \frac{(-3)^3}{3}\right] = c[9 - -9] = c[18] = 1$$

$$c[18] = 1 \rightarrow c = \frac{1}{18}$$

B) Calculate E [3x + 6]

$$E[3x+6] = 3 * E[x] + 6$$

$$E[x] = \int_{-3}^{3} \frac{x^3}{18} dx = \frac{1}{18}\int_{-3}^{3} x^3 = \frac{1}{18} * \left[\frac{x^4}{4}\right]_{-3}^{3} = \frac{1}{18}\left[\frac{(3)^4}{4} - \frac{(-3)^4}{4}\right] = \frac{1}{18}[0] = 0$$

$$E[3x+6] = 3 * 0 + 6 = 6$$

C) Calculate Var [3x + 6]

Zi Xuan Li

Professor Vahdani

CSC 21700

December 4th, 2022

$$Var[3x+6] = 3^2 * Var[x] = 9 * Var[x]$$

$$Var(x) = E(x^2) - [E(x)]^2 = E(x^2) - [0]\hat{}2$$

$$E[x^2] = \int_{-3}^{3} \frac{x^4}{18} dx = \frac{1}{18}\int_{-3}^{3} x^4 = \frac{1}{18} * \left[\frac{x^5}{5}\right]_{-3}^{3} = \frac{1}{18}\left[\frac{(3)^5}{5} - \frac{(-3)^5}{5}\right] = \frac{1}{18}[97.2] = 5.4$$

$$Var(x) = E(x^2) - [E(x)]^2 = 5.4 - [0]^2 = 5.4$$

$$Var[3x+6] = 9 * Var[x] = 9 * 5.4 = 48.6$$

D) Calculate P (X => 1/3), P (X <= 7/3) and P (X <= 7/3 | X => 1/3)

$$P\left(x \geq \frac{1}{3}\right) = \int_{1/3}^{3} cx^2 dx = \int_{1/3}^{3} \frac{x^2}{18} dx = \frac{1}{18} * \left[\frac{x^3}{3}\right]_{\frac{1}{3}}^{3} = \frac{1}{18}\left[\frac{(3)^3}{3} - \frac{\left(\frac{1}{3}\right)^3}{3}\right] = \frac{1}{18} * 8.9877 = 0.4993$$

$$P\left(x \leq \frac{7}{3}\right) = \int_{-3}^{7/3} cx^2 dx = \int_{-3}^{7/3} \frac{x^2}{18} dx = \frac{1}{18} * \left[\frac{x^3}{3}\right]_{-3}^{\frac{7}{3}} = \frac{1}{18}\left[\frac{\left(\frac{7}{3}\right)^3}{3} - \frac{(-3)^3}{3}\right] = \frac{1}{18} * 13.235 = 0.7353$$

$$\mathrm{P}\left(\mathrm{x} \leq \frac{7}{3} \mid \mathrm{x} \geq \frac{1}{3}\right) = \frac{P(\frac{1}{3} \leq x \leq \frac{7}{3})}{P\left(x \geq \frac{1}{3}\right)}$$

$$P\left(\frac{1}{3} \leq x \leq \frac{7}{3}\right) = \int_{1/3}^{7/3} cx^2 dx = \int_{1/3}^{7/3} \frac{x^2}{18} dx = \frac{1}{18} * \left[\frac{x^3}{3}\right]_{\frac{1}{3}}^{\frac{7}{3}} = \frac{1}{18}\left[\frac{\left(\frac{7}{3}\right)^3}{3} - \frac{\left(\frac{1}{3}\right)^3}{3}\right] = \frac{1}{18} * 4.2\overline{2} = 0.2346$$

$$\mathrm{P}\left(\mathrm{x} \leq \frac{7}{3} \mid \mathrm{x} \geq \frac{1}{3}\right) = \frac{P(\frac{1}{3} \leq x \leq \frac{7}{3})}{P\left(x \geq \frac{1}{3}\right)} = \frac{0.2346}{0.4993} = 0.4699$$

Zi Xuan Li

Professor Vahdani

CSC 21700

December 4th, 2022

Problem 2 (10 points)

Let X be a continuous random variable with PDF given by

$$f_X(x) = e^{-2|x|} \quad \text{for all } x \in R$$

Find the CDF function of Y (for all the points), where

- $Y = X^2$

A) Find the CDF function of Y (for all the points), where Y = X^2

$$P(Y \leq y) = P(x^2 \leq y) = P\left(-\sqrt{y} \leq x \leq \sqrt{y}\right) = \int_{-\sqrt{y}}^{\sqrt{y}} e^{-2|x|}\,dx = 2\int_{0}^{\sqrt{y}} e^{-2x}\,dx = [-e^{-2x}]_0^{\sqrt{y}}$$

$$0 \; if \; y \leq 0$$

$$1 - e^{-2\sqrt{y}} \; if \; y > 0$$

Problem 3 (10 points)

Let X be a continuous random variable with PDF given by

$$f_X(x) = \begin{cases} c(x^3 + x^2) & \text{if } 0 < x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Find the value of c and Calculate Var$[\frac{1}{X}]$.

A) Find the value of c and Calculate Var [1/x]

$$\int_0^1 c(x^3 + x^2)dx = c * \left[\frac{x^4}{4} + \frac{x^3}{3}\right]_0^1 = c\left[\frac{1}{4} + \frac{1}{3}\right] = c\left[\frac{7}{12}\right] = 1$$

$$c\left[\frac{7}{12}\right] = 1 \rightarrow c = \frac{12}{7}$$

$$E\left[\frac{1}{x}\right] = \int_0^1 c * \frac{1}{x}(x^3 + x^2)dx = \frac{12}{7} * \left[\frac{x^3}{3} + \frac{x^2}{2}\right]_0^1 = \left(\frac{10}{7}\right)$$

$$E\left[\frac{1}{x^2}\right] = \int_0^1 c * \frac{1}{x^2}(x^3 + x^2)dx = \frac{12}{7} * \left[\frac{x^2}{2} + x\right]_0^1 = \left(\frac{18}{7}\right)$$

$$Var\left[\frac{1}{x}\right] = E\left(\frac{1}{x^2}\right) - [(E(x)]^2 = \frac{18}{7} - \left(\frac{10}{7}\right)^2 = 26/49$$

Zi Xuan Li

Professor Vahdani

CSC 21700

December 4th, 2022

Problem 4 (10 points)

Distribution of random variable X is given as $X \sim N(2, 16)$. Calculate

- $\mathrm{P}(X \leq 5)$
- $\mathrm{P}(X \geq 2)$
- $\mathrm{P}(2 \leq X \leq 5)$

$$\mathrm{X} \sim \mathrm{N}(2, 16) \rightarrow \mu = 2, \sigma = 4, \mathrm{z} = \frac{\mathrm{x} - \mu}{\sigma}$$

A) P (X <= 5)

$$P(X \leq 5) = P\left(Z \leq \frac{5-2}{4}\right)$$

$$P(X \leq 5) = P\left(Z \leq \frac{3}{4}\right)$$

$$P(X \leq 5) = 0.7734$$

B) P (X => 2)

$$P(X \geq 2) = P\left(Z \geq \frac{2-2}{4}\right)$$

$$P(X \geq 2) = P(Z \geq 0)$$

$$P(X \geq 2) = 0.5$$

C) P (2 <= X <= 5)

$$P(2 \leq X \leq 2) = P\left(\frac{2-2}{4} \leq Z \leq \frac{5-2}{4}\right)$$

$$P(2 \leq X \leq 2) = P\left(0 \leq Z \leq \frac{3}{4}\right)$$

$$P(2 \leq X \leq 2) = P\left(Z \leq \frac{3}{4}\right) - P(Z \leq 0)$$

$$P(2 \leq X \leq 2) = 0.7734 - 0.5$$

$$P(2 \leq X \leq 2) = 0.2734$$