0.1 Examine the following formal descriptions of sets so that you understand which members they contain. Write a short informal English description of each set.

(a) $\{1, 3, 5, 7, ...\}$

This set contains all positive odd natural numbers.

(b) $\{..., -4, -2, 0, 2, 4, ...\}$

This set contains all even integers.

(c) $\{n \mid n = 2m \text{ for some } m \text{ in } \mathbb{N}\}$

This set contains all even natural numbers.

(d) $\{n \mid n = 2m \text{ for some } m \text{ in } \mathbb{N}, \text{ and } n = 3k \text{ for some } k \text{ in } \mathbb{N}\}$

This set contains all even natural numbers that are multiples of 3. In other words, it includes all natural numbers that can be expressed as 6 times a natural number.

(e) $\{w \mid w \text{ is a string of 0s and 1s and } w \text{ equals the reverse of } w\}$

This set contains strings that are palindromes. In other words, each string in the set read the same backwards and forwards.

(f) $\{n \mid n \text{ is an integer and } n = n + 1\}$

This is an empty set because there are no integers that are equal to their own value plus 1.

0.2 Write formal descriptions of the following sets.

(a) The set containing the numbers 1, 10, and 100

$\{1, 10, 100\}$

(b) The set containing all integers that are greater than 5

$\{n \mid n \text{ is greater than 5 in } \mathbb{Z}\}$

(c) The set containing all natural numbers that are less than 5

$\{n \mid n \text{ is less than 5 in } \mathbb{N}\}$

(d) The set containing the string *aba*

$\{aba\}$

(e) The set containing the empty string

$\{\varepsilon\}$

(f) The set containing nothing at all

$\{\}$

0.3 Let A be the set $\{x, y, z\}$ and B be the set $\{x, y\}$.

(a) Is A a subset of B?

No, A is not a subset of B because every member of A is not a member of B.

(b) Is B a subset of A?

Yes, B is a subset of A because every member of B is a member of A.

(c) What is $A \cup B$?

$\{x, y, z\}$

(d) What is $A \cap B$?

$\{x, y\}$

(e) What is $A \times B$?

$\{(x,x),(x,y),(y,x),(y,y),(z,x),(z,y)\}$

(f) What is the power set of B?

$\{\{\ \},\{x\},\{y\},\{x,y\}\}$

0.4 If A has a elements and B has b elements, how many elements are in $A \times B$? Explain your answer.

Solution.

The Cartesian product would have $A * B$ elements because for each element in A, there would be each element in B that could be paired up with it. Resulting in $A * B$ ordered pairs in the Cartesian product. ■

0.5 If C is a set with c elements, how many elements are in the power set of C? Explain your answer.

Solution.

The power set of C would contain 2^c elements because there are two options for every element in C, to include it or to exclude it in a subset. This results in 2 choices for the first element, second element, and so on. Resulting in 2^c combinations. ■

0.6 Let X be the set $\{1,2,3,4,5\}$ and Y be the set $\{6,7,8,9,10\}$. The unary function $f : X \to Y$ and the binary function $g : X \times Y \to Y$ are described in the following tables.

(a) What is the value of $f(2)$?

$f(2) = 7$

(b) What are the range and domain of f?

The range of f is $\{6,7\}$ and domain of f is $\{1,2,3,4,5\}$

(c) What is the value of $g(2,10)$?

The value of $g(2,10)$ is 6.

(d) What are the range and domain of g?

The range of g is $\{6,7,8,9,10\}$ and the domain of g is the Cartesian product of set X and set Y.

(e) What is the value of $g(4, f(4))$?

The value of $f(4)$ is 7, so the value of $g(4,7)$ is 8.

0.7 For each part, give a relation that satisfies the condition.

(a) Reflexive and symmetric but not transitive

A relation that satisfies the condition is ”friends with”. This is reflexive because everyone is friends with themselves, and is symmetric because if X is friends with Y, then Y is friends with X. This is not transitive because if X is friends with Y, and Y is friends with Z, it does not guarantee that X is friends with Z.

(b) Reflexive and transitive but not symmetric

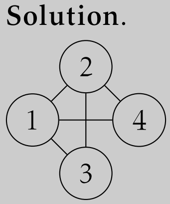
A relation that satisfies the condition is ”older than or equal to”. This is reflexive because a person can be older than or equal to themselves in age. This is transitive because if X is older than or equal to Y, and Y is older than or equal to Z, then X is older than or equal to Z. This is not symmetric because if X is older than or equal to Y, Y cannot be older than or equal to X.

(c) Symmetric and transitive but not reflexive

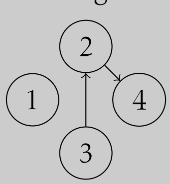
A relation that satisfies the condition is ”sibling of”. This is symmetric because if X is a sibling of Y, then Y is a sibling of X. This is also transitive because if X is a sibling of Y, and Y is a sibling of Z, then X is a sibling of Z. This is not reflexive because one cannot be a sibling of themselves.

0.8 Consider the undirected graph $G = (V, E)$ where V, the set of nodes, is $\{1,2,3,4\}$ and E, the set of edges, is $\{\{1,2\},\{2,3\},\{1,3\},\{2,4\},\{1,4\}\}$. Draw the graph G. What are the degrees of each node? Indicate a path from node 3 to node 4 on your drawing of G.

Solution.

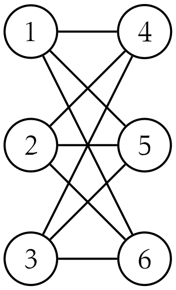


The degree of node 1 is 3, node 2 is 3, node 3 is 2, and node 4 is 2.



A path from node 3 to 4 is indicated above. ■

0.9 Write a formal description of the following graph.



Solution.

This is a Graph $G = (V, E)$ where V, the set of nodes, is $\{1, 2, 3, 4, 5, 6\}$ and E, the set of edges, is $\{(1, 4), (1, 5), (1, 6)(2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6)\}$ ■