1.46 Prove that the following languages are not regular. You may use the pumping lemma and the closure of the class of regular languages under union, intersection, and complement.

(a) $L = \{0^n 1^m 0^n \mid m, n \geq 0\}$

Solution.

Proof: Assume that L is regular. Then by the Pumping Lemma for regular languages, there exists a pumping length, p, for L such that for any string $s \in L$ where $|s| \geq p, s = xyz$ subject to the following conditions:

$$1)\ |y| > 0$$
$$2)\ |xy| \leq p$$
$$3)\ xy^i z \in L,\ \forall i \geq 0$$

Let $s = 0^p 1 0^p$. $|s| \geq p$ and $s \in L$. By condition 2, it follows that x and y are composed of only zeros. By condition 1, it follows that $y = 0^k$ for some $k > 0$. By condition 3, we can take $i = 0$ and the resulting string will still be in L. Thus, $xy^0 z$ should be in L. $xy^0 z = xz = 0^{p-k} 1 0^p$. But this is not in L because $p - k < k$, which is a contradiction with the pumping lemma. Therefore our assumption that L is regular is false, and L is not a regular language. ■