Progress_Report_FeedingAmerica

July 28, 2025

1 Introduction: Milk allocation in grand-WI region for Feeding America

Food insecurity remains a widespread challenge across the United States, with millions of individuals relying on food banks for essential nutrition. This project, *Delivering Food to Communities in the US*, aims to formulate an optimization model to improve the efficiency and equity of food distribution. Our primary objective is to minimize transportation costs while ensuring that food reaches regions with the greatest need.

We abstract the real-world context into a **network flow model**, where each **state is simplified as a node**, and food logistics are represented as flows along arcs between them. The current formulation focuses on **Wisconsin (WI)** as the primary source, with potential redistribution through **Illinois (IL)**, and demand nodes located in neighboring states including **Michigan (MI), Minnesota (MN), and Iowa (IA)**. Due to current limitations in accessing detailed datasets, we approximate demand values using summary statistics from Feeding America's 2023 report, and supplement our model with publicly available information on transportation costs, supply chain parameters, and general nutritional guidelines. These sources inform the construction of model variables, constraints, and objective functions.

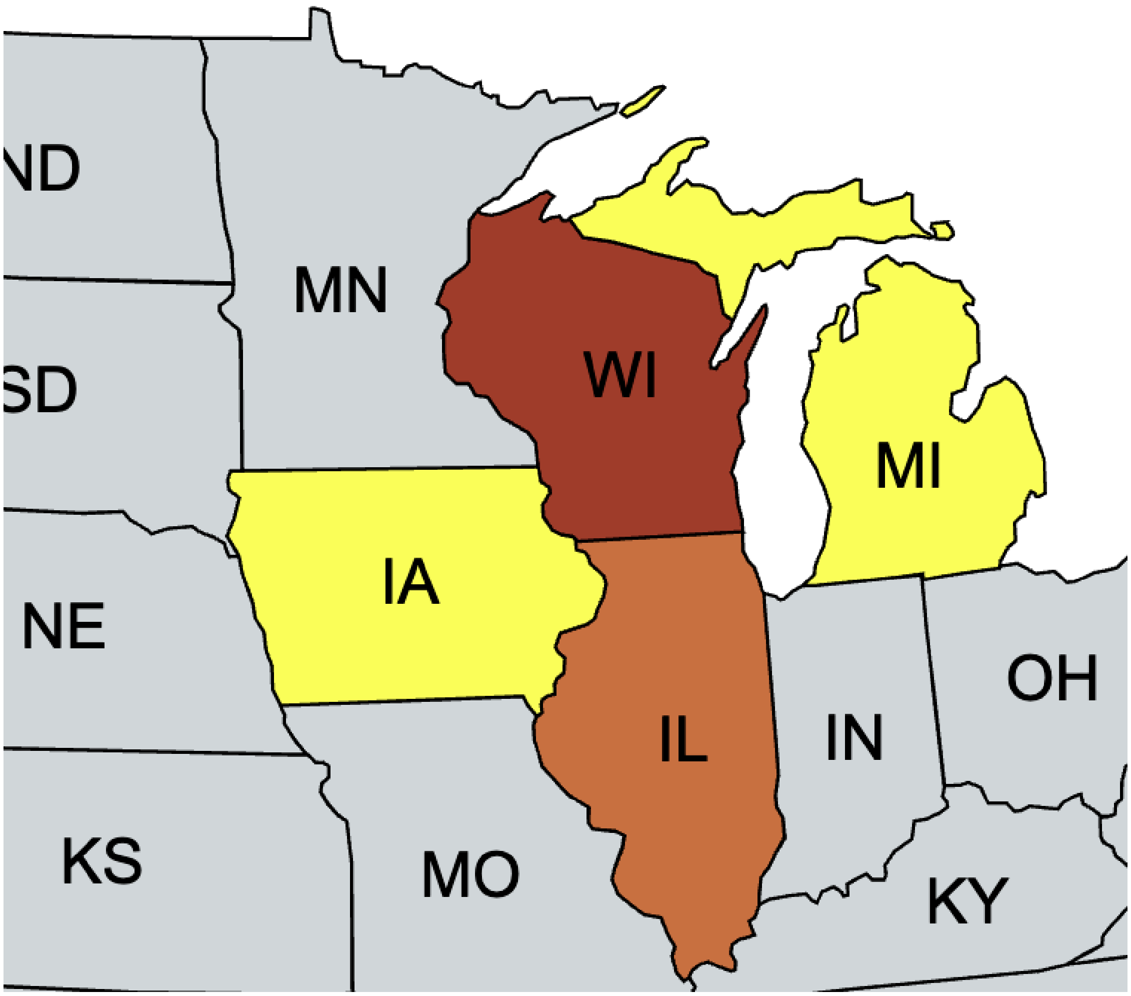
Our initial framework uses **Linear Programming (LP)** to solve a **minimum-cost flow problem** under supply, demand, and capacity constraints. Depending on time and feasibility, we may later extend the model to include **Mixed-Integer Programming (MIP)** features, such as routing restrictions or discrete truck assignments.

This progress report outlines the model design and assumptions, summarizes work completed to date, and lays out the next steps, including potential extensions such as subgroup prioritization and sensitivity analyses.

[1]:
```
# dependencies
# modeling
using JuMP, HiGHS
# visualization
using Plots, FileIO, Images
# data structure
using DataFrames, CSV, NamedArrays
```

2 Theoretic model construction

[2]:
```
# show neighbor states
# https://www.mapchart.net/usa.html
neighbors = load("./img/wi_neighbors.png")
plot(neighbors, size=(100, 50), axis=nothing, border=:none)
display(neighbors)
```



Abstracted from the broader **Feeding America (FA)** food distribution context, we focus on the optimization of food allocation among states. For our current project, the problem is refined to center around **Wisconsin (WI)** and its neighboring states, including **Michigan (MI), Minnesota (MN), Iowa (IA), and Illinois (IL)**. We utilize the annual food insecurity population data from FA and plan to use the latest data from 2023 as the starting point for building our models. Considering Wisconsin's reputation as America's Dairyland, we further instantiate the context as a dairy product allocation and transportation problem. As the hub, IL has some advantage on the general transportation with cold-chain storage (e.g. Food banks in IL collaborating with

FA), which reflects on its capability to re-distribute the product. But there is some upper limit of available storage in IL, which will be reflected into the models. The models are constructed within a **Min-Cost Network Flow (MCNF)** problem framework. And the details will be delineated with mode details in later steps.

We differentiate the granularity of the problem in two aspects:
1. Number and scale of the receiving nodes
2. Whether to include a benefit trade-off term in the objective function

These aspects jointly influence the model construction. Consequently, we develop the model in a graded manner:
1. **Basic model**: Each state is treated as a holistic node with aggregated demand, focusing solely on minimizing transportation cost.
2. **Subgroup trade-off model**: Each state node is further split into subgroup nodes (**Children, Adults, Seniors**) with differentiated demands and priorities. The prioritization of specific subgroups introduces a trade-off term that balances subgroup benefit with transportation cost in the objective function.

2.1 Basic model

2.1.1 Model structure

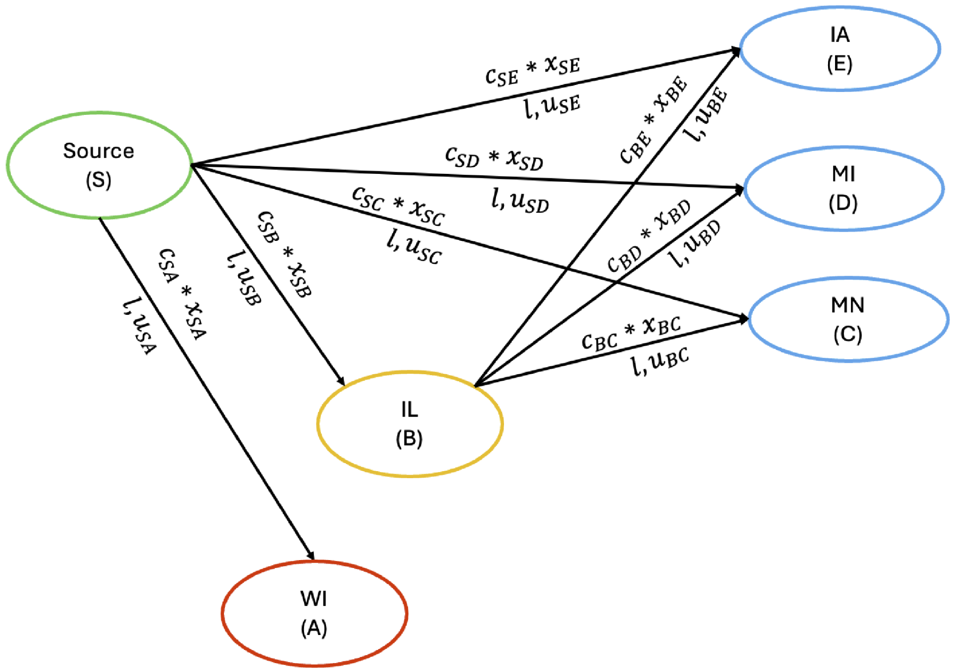
For the basic model, we consider the demand of each state generally, which means the demand is approximated by the overall population of all subgroups in each state.

The involved states will be structured as a network, and in the basic framework, we define:

- There is a **source** geographically in WI, but WI is treated as an independent node, and has its own local demand from food-insecure population.
- IL functions as both a **demand node** (to cover its own food-insecure population) and a **redistribution hub**, capable of forwarding surplus food to MI, MN, and IA.
- MI, MN, and IA are purely **demand nodes** with requirements derived from food insecurity statistics.

[3]:
```
# show the network
basic_model_img = load("./img/basic_model.png")
plot(basic_model_img, axis=nothing, border=:none)
```

[3]:



2.1.2 Decision Variables

In a MCNF model, we define the total of food allocation (more specifically, milk in gallons) flow x_{ij} on each edge (i, j) (i.e., e_{ij}) as the decision variables. Here, $i, j \in N = \{S, A, B, C, D, E\}$.

Note here we define the edge set as $Edge$ to distinguish it from the node E. And $Edge = \{SA, SB, SC, SD, SE, BC, BD, BE\}$.

2.1.3 Objective function of basic model

The objective is described by the transportation and cold-chain storage cost in the basic model. We're aiming to:

- **Minimize total transportation and cold-chain storage cost** (given as c_{ij} per unit of x_{ij} flow on the corresponding edge) across the network, given estimated per-unit costs based on transportation and cold-chain logistics of dairy product.

$$\min_x \sum_{(i,j)\in L} c_{ij}x_{ij}$$

2.1.4 Constraints

Capacity constraint on each edge

$$l_{ij} \leq x_{ij} \leq u_{ij}, \forall (i, j) \in Edge$$

The limitations on the edge capacity are given by: 1 cold-chain transportation budget; (2) (IL as hub) max storage and transfer capability, (3) nonnegativity. More details are given after data collection and estimation.

Flow balance constraint

$$\sum_{j \in N} x_{kj} - \sum_{i \in N} x_{ik} = b_k, \forall k \in N$$

2.1.5 Data collection and estimation (Based on 2023 data)

- Demand of dairy product (estimated as milk) of each state node ($-b_i, i \neq S$):
  - Population of WI, IL, IA, MI and MN in food insecurity:
    - We requested data from: https://www.feedingamerica.org/research/senior-hunger-research
    - Overall Food-insecure population: WI (694,710), IL (1,647,630), IA (385,130), MI (1,544,250) and MN (596,190)
  - Suggested consumption of dairy product:
    - Since the reported average consumption of milk (~15 gallons per person per year) is far higher than the satisfied among the population supported by FA (<1 gallon per person peryear), we'd like to make a reasonbly developmental suggestion that the expected milk consumption of each person on average is 1 gallon per year here. Which means the total demand of a state will be computed numerically as the population number in unit of gallon.
    - Average: https://agnet.mdac.ms.gov/agManage/uploads/1274.pdf
    - FA: https://www.feedingamerica.org/hunger-blog/three-items-hungry-families-need
  - So, the demand of milk in gallon of each state:
    - WI (694,710), IL (1,647,630), IA (385,130), MI (1,544,250) and MN (596,190)
- Milk production in 2023 in WI (b_S):
  - 2,760,000,000 lb = 321,088,290.67434 gal (~321,088,291)
  - https://data.nass.usda.gov/Statistics_by_State/Wisconsin/Publications/Dairy/2023/WI-Milk-Production-09-23.pdf
- Cold-chain transportation cost for each inter-state route:
  - Route distance: https://www.travelmath.com/distance

Route	Driving distance (miles)
WI → IL	148
WI → IA	293
WI → MI	372
WI → MN	273
IL → IA	332
IL → MI	217
IL → MN	409

  - Reefer Rental in midwest region:
    - cost (per mile): according to a review on national reefer rate from DAT, the spot reefer rate was 2.44 dollars/mile ($cost_{reefer}$ = 2.44 dollars/mile)
      - https://www.dat.com/company/news-events/news-releases/dat-truckload-volume-index-july-freight-volumes-and-rates-chilled-by-seasonality

* capacity of milk per reefer:
  * According to some public report, we could estimate the average volume V_{reefer} of a reefer as 7500 gallons.
  * https://dairy-cattle.extension.org/how-many-gallons-does-the-typical-milk-transport-carry-what-is-their-empty-and-full-weights-i-am-using-this-to-determine-bridge-specifications-in-a-rural-area/?utm_source=chatgpt.com

- Cost on each route (per gallon of milk):
  * Using the formula: $c_{\text{route}} = \frac{2.44\times\text{miles}}{7{,}500}$ \$/gal

Route	Distance (miles)	Cost per gallon per route($/gal)
WI → IL	148	0.0481
WI → IA	293	0.0953
WI → MI	372	0.1210
WI → MN	273	0.0887
IL → IA	332	0.1080
IL → MI	217	0.0706
IL → MN	409	0.1331

  * Those are the c_{ij}
- Limitations on the capacity of edge/routes (u_{route})
  * These are given indirectly by the budget on transportation of the food banks collaborating with FA in each state (WI and IL). Besides, since the information on the budge estimation in WI is not adequate, it will be similarly estimated referring the public report of food bank financial condition for IL. Meanwhile, since not all the transportation is used for milk transferring, we estimate that only 10% of the total upper limitation will be the upper limit in current problem.
  * According to Feeding America's 2023 annual report, the Northern Illinois Food Bank spent approximately $38,107,504 on food operations. Assuming this represents about half of Illinois's total spending, we estimate the statewide amount to be twice that figure. Based on standard supply-chain budgeting assumptions—where 70% of the food operations budget is allocated to storage and transportation, and 10% of that is dedicated to milk—we estimate the transportation budget for milk in Illinois as:

$$\text{Budget}_{\text{IL milk transportation}} = 2 \times 38{,}107{,}504 \times 0.70 \times 0.10 \approx \$5{,}335{,}051$$

  * For Wisconsin, we assume the corresponding capacity is about 60% of Illinois's budget:

$$\text{Budget}_{\text{WI milk transportation}} = 0.60 \times 5{,}335{,}051 \approx \$3{,}201{,}030$$

  * So, the capacity of each route from WI and IL will be given as the $u_{route} = \frac{TransBudget}{costMilkPerGallon}$

Route	Cost ($/gal)	Budget ($)	Capacity (u_{route}) (gal)
WI → IL	0.0481	3,201,030	66,549,480
WI → IA	0.0953	3,201,030	33,588,982

Route	Cost ($/gal)	Budget ($)	Capacity (u_{route}) (gal)
WI → MI	0.1210	3,201,030	26,454,793
WI → MN	0.0887	3,201,030	36,088,275
IL → IA	0.1080	5,335,051	49,398,620
IL → MI	0.0706	5,335,051	75,567,295
IL → MN	0.1331	5,335,051	40,083,028

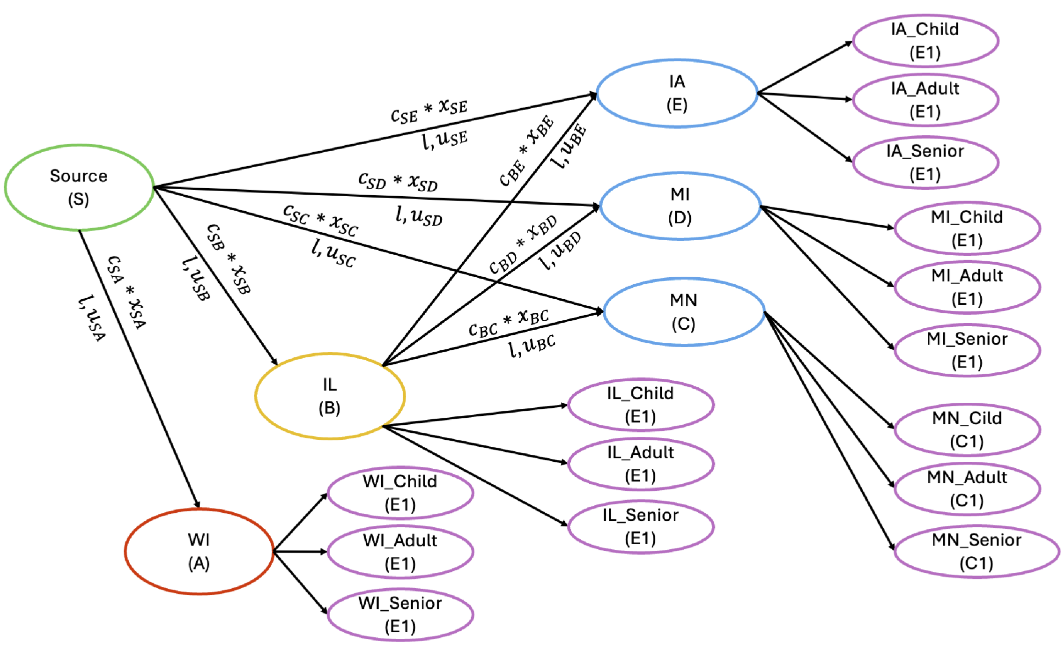
  – Special cost and limitation on Source(WI)-WI. To avoid a series of repeated solution with different amount of product allocated to WI local, we will manually set a small cost on the transportation from source to WI, which will affect the objective function with the optimal allocation to local.

After we have those data, we could then construct the model with numeric values in scripts.

2.2 Upgraded model

[4]:
```
# show the network
upgraded_model_img = load("./img/upgraded_model.png")
plot(upgraded_model_img, axis=nothing, border=:none)
```

[4]:



This model is similarly structured as the basic model, but we will then consider the priority of subgroups and consequent tradeoff in objective function.

3 Confusion and uncertain items

Considering trucks and drivers as decision variables: We're unsure if this can be efficiently solved within a Mixed-Integer Programming (MIP) framework, given the additional complexity it introduces.

Handling source and demand mismatches: If the total supply from sources cannot satisfy the total demand, should we introduce a norm-based relaxation (e.g., a least-squares penalty) to make the problem feasible, especially since this is a network flow problem?

Introducing a dummy extra source: Another possible approach for infeasibility is to add a dummy source with unlimited supply but a significantly higher cost, representing procurement from distant regions. Would this be a reasonable and standard way to handle infeasible demand?

Considering local production as part of demand/supply: Should each state/location be treated as both a source and a sink, incorporating local production as a supply node? Would this abstraction improve the generality and flexibility of the model?

4 Some discussion on the progress

We have collected the major relevant data from online and decided on some assumption or relaxation on the basis of those data. The subgroup information is accessible in the same report providing the total population data. So the next steps will mainly focus on complete the construction of upgraded model, and the solution of both models using Julia. Especially, we should elaborate more on the definition and analytic implementation of the subgroup-tradeoff condition.

Depending on the current progress, we estimate that we could complete the work within about 20 work hours. If we further extend the problem with finer granularity, which introduces the consideration of accurate number of hired drivers and rented reefers, the time could be even longer. But our best prospect is to finalize a complete analysis based on LP-MCNF models.