Improved Dropout for Shallow and Deep Learning

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Joint work with Prof. Gong and Prof. Yang

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Outline

The success of deep learning

- Image Classification

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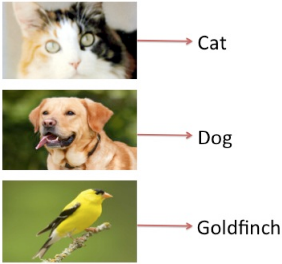

The success of deep learning

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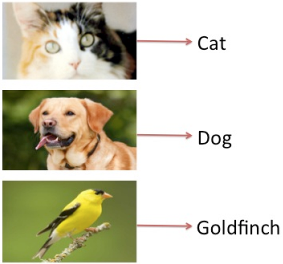
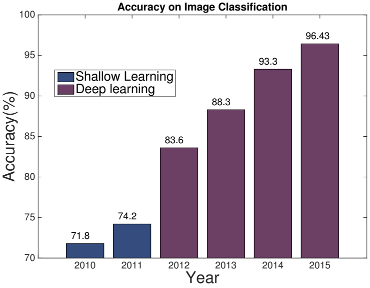
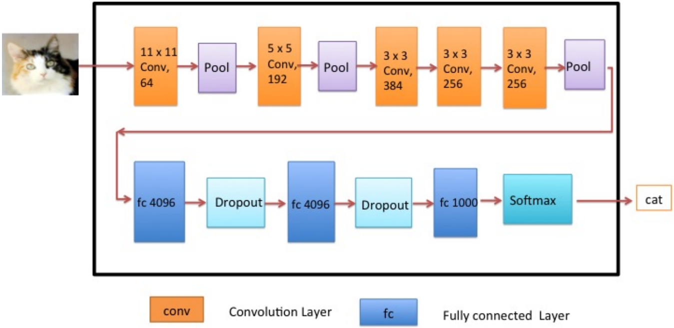




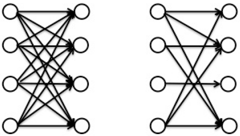
Deep Neural Network

The classical example: AlexNet [A Krizhevsky, et .al, 2012]



Dropout Layer

- Dropout Layer: Uniformly at randomly drop out features.



Before applying dropout

After applying dropout

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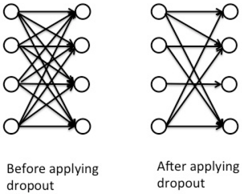

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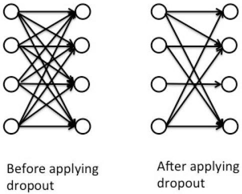

- Is uniformly dropout optimal?
  - Answered the above question in this work.

Improved Dropout

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- Dropping out the output of the neuron based on multinomial distribution computed from the training data.

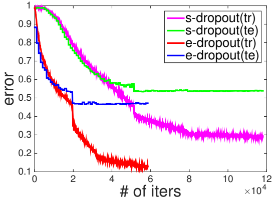

Figure: Evolutional dropout vs standard dropout on CIFAR100 datasets for deep learning

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- Denote by $\mathcal{P}$ the joint distribution of $(\mathbf{x}, y)$ and by $\mathcal{D}$ the marginal distribution of $\mathbf{x}$.
- The goal is to learn a linear prediction function ($f(x) = \mathbf{w}^\top \mathbf{x}$) that minimizes the expected risk (considering loss function $\ell(\cdot, y)$):

$$\min_{\mathbf{w} \in \mathbb{R}^d} \mathcal{L}(\mathbf{w}) \triangleq \mathrm{E}_{\mathcal{P}}[\ell(\mathbf{w}^\top \mathbf{x}, y)] \tag{1}$$

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- Denote by $\widehat{\mathcal{P}}$ the joint distribution of the new data $(\widehat{\mathbf{x}}, y)$ and by $\widehat{\mathcal{D}}$ the marginal distribution of $\widehat{\mathbf{x}}$.
- With the corrupted data, the risk minimization becomes

$$\min_{\mathbf{w} \in \mathbb{R}^d} \widehat{\mathcal{L}}(\mathbf{w}) \triangleq \mathrm{E}_{\widehat{\mathcal{P}}}[\ell(\mathbf{w}^\top(\mathbf{x} \circ \epsilon), y)] \tag{2}$$

Multinomial Dropout

Definition 1

A **multinomial dropout** is defined as $\widehat{\mathbf{x}} = \mathbf{x} \circ \epsilon$, where $\epsilon_i = \frac{m_i}{kp_i}, i \in [d]$ and $\{m_1, \ldots, m_d\}$ follow a multinomial distribution $Mult(p_1, \ldots, p_d; k)$ with $\sum_{i=1}^{d} p_i = 1$ and $p_i \geq 0$.

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- Ability of using non-uniformly sampling probabilities for different features.
- Easy to control the level of dropout by varying the value of k.

Multinomial Dropout

- Dropout is a data-dependent regularizer.

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Proposition 1

If $\ell(z, y) = \log(1 + \exp(-yz))$, then

$$\mathrm{E}_{\widehat{\mathcal{P}}}[\ell(\mathbf{w}^\top \widehat{\mathbf{x}}, y)] = \mathrm{E}_{\mathcal{P}}[\ell(\mathbf{w}^\top \mathbf{x}, y)] + R_{\mathcal{D},\mathcal{M}}(\mathbf{w})$$

where $\mathcal{M}$ denotes the distribution of ϵ and

$$R_{\mathcal{D},\mathcal{M}}(\mathbf{w}) = \mathrm{E}_{\mathcal{D},\mathcal{M}}\left[\log \frac{\exp(\mathbf{w}^\top \frac{\mathbf{x}\circ\epsilon}{2}) + \exp(-\mathbf{w}^\top \frac{\mathbf{x}\circ\epsilon}{2})}{\exp(\mathbf{w}^\top \mathbf{x}/2) + \exp(-\mathbf{w}^\top \mathbf{x}/2)}\right].$$

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- Give the initial solution $\mathbf{w}_1$.

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- Update the model at t^{th} iteration:

$$\mathbf{w}_{t+1} = \mathbf{w}_t - \eta_t \nabla \ell(\mathbf{w}_t^\top (\mathbf{x}_t \circ \boldsymbol{\epsilon}_t), y_t) \tag{3}$$

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- Output the final solution:

$$\widehat{\mathbf{w}}_n = \frac{1}{n}\sum_{t=1}^{n} \mathbf{w}_t$$

Outline

Improved dropout for Shallow Learning

Theorem 1:

Let $\mathcal{L}(\mathbf{w})$ be the expected risk of $\mathbf{w}$ defined in (1). Assume $\mathrm{E}_{\widehat{\mathcal{D}}}[\|\mathbf{x} \circ \boldsymbol{\epsilon}\|_2^2] \leq B^2$ and $\ell(z, y)$ is convex and G-Lipschitz continuous. For any $\|\mathbf{w}_*\|_2 \leq r$, by appropriately choosing η, we can have

$$\mathrm{E}[\mathcal{L}(\widehat{\mathbf{w}}_n) + R_{\mathcal{D},\mathcal{M}}(\widehat{\mathbf{w}}_n)] \leq \mathcal{L}(\mathbf{w}_*) + R_{\mathcal{D},\mathcal{M}}(\mathbf{w}_*) + \frac{GBr}{\sqrt{n}}$$

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- Standard SGD analysis.

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How to prove the above theorem?

- Standard SGD analysis.
- Dropout is a data-dependent regularizer.

Improved dropout for Shallow Learning

- Minimizing the term $\mathrm{E}_{\widehat{\mathcal{D}}}[\|\mathbf{x} \circ \boldsymbol{\epsilon}\|_2^2]$ and the relaxed upper bound of term $R_{\mathcal{D},\mathcal{M}}(\mathbf{w}_*)$ yields the optimal sampling probabilities:

$$p_i^* = \frac{\sqrt{\mathrm{E}_{\mathcal{D}}[x_i^2]}}{\sum_{j=1}^{d} \sqrt{\mathrm{E}_{\mathcal{D}}[x_j^2]}}, i = 1, \ldots, d \quad (4)$$

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- Practically, we use the empirical second-order statistics to compute the probabilities:

$$p_i = \frac{\sqrt{\frac{1}{n}\sum_{j=1}^{n}[[\mathbf{x}_j]_i^2]}}{\sum_{i'=1}^{d}\sqrt{\frac{1}{n}\sum_{j=1}^{n}[[\mathbf{x}_j]_{i'}^2]}}, i = 1, \ldots, d \tag{5}$$

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- Why not?
  - Too expensive to compute dropout probablity from all examples.
- How to address this issue?
  - Use a mini-batch of examples to calculate the dropout probablity.

Improved dropout for deep learning

- Let $X^l = (\mathbf{x}_1^l, \ldots, \mathbf{x}_m^l)$ denote the outputs of the l^{th} layer for a mini-batch of m examples, calculate the probabilities for dropout by

$$p_i^l = \frac{\sqrt{\frac{1}{m}\sum_{j=1}^{m}[[\mathbf{x}_j^l]_i^2]}}{\sum_{i'=1}^{d}\sqrt{\frac{1}{m}\sum_{j=1}^{m}[[\mathbf{x}_j^l]_{i'}^2]}}, i = 1, \ldots, d \tag{6}$$

Improved dropout for deep learning

Evolutional Dropout for Deep Learning

Input: a batch of outputs of a layer: $X^l = (\mathbf{x}_1^l, \ldots, \mathbf{x}_m^l)$ and dropout level parameter $k \in [0, d]$

Output: $\widehat{X}^l = X^l \circ \Sigma^l$

Compute sampling probabilities by (6)

For $j = 1, \ldots, m$

Sample $\mathbf{m}_j^l \sim Mult(p_1^l, \ldots, p_d^l; k)$

Construct $\epsilon_j^l = \frac{\mathbf{m}_j^l}{k\mathbf{p}^l} \in \mathbb{R}^d$, where $\mathbf{p}^l = (p_1^l, \ldots, p_d^l)^\top$

Let $\Sigma^l = (\epsilon_1^l, \ldots, \epsilon_m^l)$ and compute $\widehat{X}^l = X^l \circ \Sigma^l$

Figure: Evolutional Dropout applied to a layer over a mini-batch

Outline

Experimental Results for Shallow Learning

Training/test error between standard and improved dropout

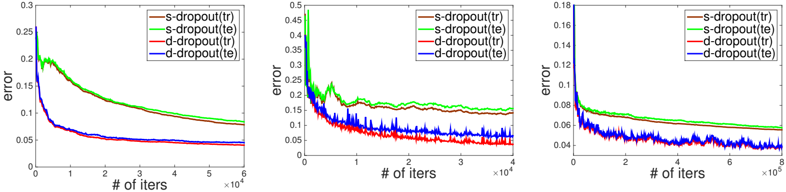

Figure: data-dependent dropout vs. standard dropout on three datasets (real-sim, news20 and RCV1) for logistic regression

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Experimental Results for Deep Learning

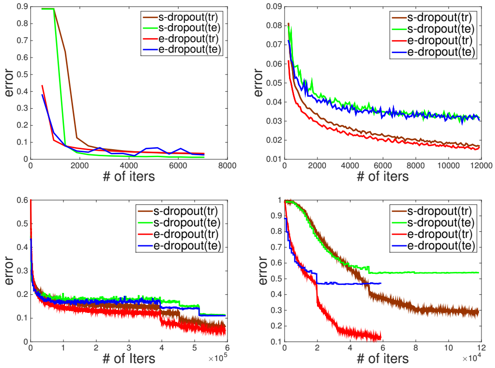

Figure: Evolutional dropout vs. standard dropout on four benchmark datasets (MNIST, SVHN, CIFAR-10 and CIFAR-100) for deep learning

Experimental Results for Deep Learning

Compared to Batch Normalization

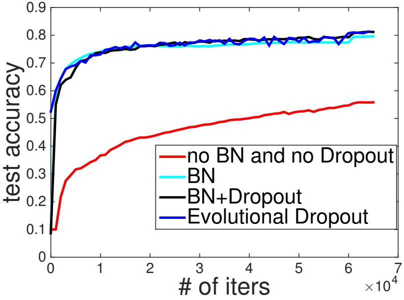

Figure: Evolutional dropout vs BN on CIFAR-10.

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- Proposed a multinomial dropout for shallow learning.
- Demonstrated that this proposed distribution-dependent dropout leads to a faster convergence and a smaller generalization error through the risk bound analysis.
- Proposed an efficient evolutional dropout for deep learning.
- Justified the proposed dropouts for both shallow and deep learning empirically.

Question?