## CSCI046 Notes: Runtime Analysis

## 1 Counting

Our goal is to count how many times something will happen in our code. This is often used as a proxy for how long it takes a program to run.

## Example 1.

```
print('x')
print('x')
print('x')

for i in range(10):
    print('y')

for i in range(10,20):
    print('z')
print('z')
print('z')
print('z')
```

1. What is the exact number of times that the letter x will be printed?

2. What is the exact number of times that the letter y will be printed?

**Example 2.** Answer the questions below based on the following python code:

```
for i in range(10):
2
       print('x')
3
   for i in range(20):
       print('x')
4
5
   for i in range(10):
6
       for j in range(20):
7
            for k in range(30):
8
9
                print('y')
10
   print('z')
11
   for i in range(10):
12
       print('z')
13
       for j in range(10):
14
            print('z')
15
            print('z')
16
       for j in range(10):
17
            print('z')
18
   for i in range(10):
19
20
       print('z')
```

1. What is the exact number of times that the letter x will be printed?

2. What is the exact number of times that the letter y will be printed?

**Example 3.** Answer the questions below based on the following python code:

```
for i in range(n):
2
       print('x')
3
   for i in range(n*2):
       print('x')
4
5
   for i in range(n):
6
       for j in range(n*2):
7
            for k in range(n*3):
8
9
                print('y')
10
   print('z')
11
   for i in range(n):
12
       print('z')
13
       for j in range(n):
14
            print('z')
15
            print('z')
16
       for j in range(n):
17
            print('z')
18
   for i in range(n):
19
20
       print('z')
```

1. What is the exact number of times that the letter x will be printed?

2. What is the exact number of times that the letter y will be printed?

Example 4. Answer the questions below based on the following python code:

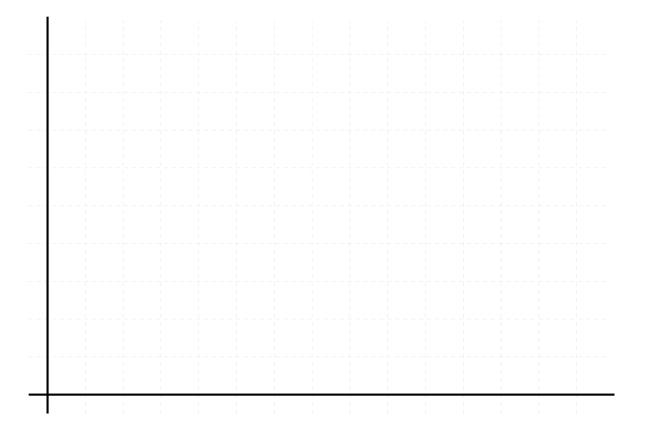
```
1
   for i in range(n):
2
       for j in range(0,i):
            for k in range(0,j):
3
                print('x')
4
5
   for i in range(n):
6
       for j in range(i,n):
7
            for k in range(j,n):
8
                print('y')
9
10
   for i in range(n):
11
       for j in range(i,n):
12
            for k in range(i,j):
13
                print('z')
14
```

1. What is the exact number of times that the letter x will be printed?

2. What is the exact number of times that the letter y will be printed?

2 Math: Big-O/ $\Theta/\Omega$  Notation

Key Ideas:



**Definition 1.** Let f, g be functions from  $\mathbb{R}^+ \to \mathbb{R}^+$ . Then,

- 1. If  $\lim_{x\to\infty} \frac{f(x)}{g(x)} < \infty$ , then we say f = O(g).
- 2. If  $\lim_{x\to\infty} \frac{f(x)}{g(x)} > 0$ , then we say  $f = \Omega(g)$ .
- 3. We say that  $f = \Theta(g)$  if both f = O(g) and  $f = \Omega(g)$ .

Intuitively, you should think of O as  $\leq$ ,  $\Omega$  as  $\geq$ , and  $\Theta$  as =.

Example 5.

1. 
$$f(x) = x$$
$$g(x) = x^2$$

$$2. \ f(x) = x^2$$
$$g(x) = x$$

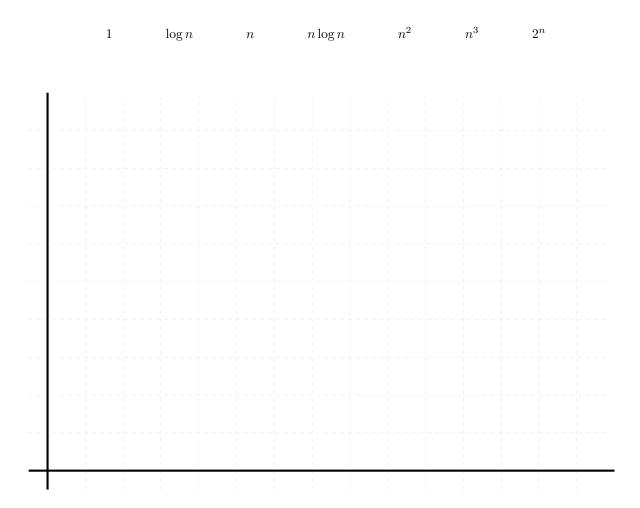
3. 
$$f(x) = x^2 + 2x + 5$$
  
 $g(x) = x$ 

4. 
$$f(x) = x^2 + 2x + 5$$
  
 $g(x) = x^2$ 

5. 
$$f(x) = x^2 + 2x + 5$$
  
 $g(x) = x^3$ 

**Example 6.** What happens when we double the size of the inputs? f(x) = x

**Example 7.** You should memorize the relationship between the following functions:



**Example 8.** Complete each equation below by adding the symbol O if f = O(g),  $\Omega$  if  $f = \Omega(g)$ , or  $\Theta$  if  $f = \Theta(g)$ . The first row is completed for you as an example.

f(n)		g(n)
1	=	O(n)
$3n\log n$	=	$n^2$
1	=	1/n
$\log_2 n$	=	$\log_3 n$
$\log n$	=	$\frac{1}{\log n}$
$5 \cdot 10^{30}$	=	$\log n$
$\log n$	=	$\log(n^2)$
$2^n$	=	$3^n$
	_	$\sqrt{rac{1}{n}}$
$\frac{1}{n}$	=	<u> </u>
$\log n$	=	$(\log n)^2$

**Example 9.** Simplify the following expressions:

$$1. \ O\left(n^3 + n^2\right)$$

$$2. \ O\left(n^3 + 5n^2 \log n + \log n\right)$$

3. 
$$O(100000000000)$$

4. 
$$O\left(\log n + 1000000000000\right)$$

$$5. \ O\left(\frac{1}{n} + \frac{1}{n^2}\right)$$