NORTH CAROLINA STATE UNIVERSITY

Homework 1 for Chapter 2

Jinwen Zhao, Chang Liu, Ling Zhang

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PROBLEM 1

PROBLEM 2 PAGE 119 PART A Let $g(t)$ be the approximation to f,

$$\|f-g\|_\infty = \sup_{1\le i\le N} |f(t)-g(t)|$$
$$g(t) = c = \frac{y+1}{2}$$

Therefore, the L_∞ approximation for $f(x)$ is $g(t) = \frac{y+1}{2}$, with error $\|f-g\|_\infty = \frac{y-1}{2}$.

PROBLEM 2 PAGE 119 PART B Let $g(t)$ be the approximation to f,

$$\begin{aligned}\|f-g\|_2 &= (\sum_{i=1}^{N} |f(t_i)-g(t_i)|^2)^{\frac{1}{2}} \\ &= \sqrt{(N-1)(c-1)^2+(c-y)^2} = \sqrt{Nc^2-2(N-1+y)c+(N-1)+y^2}\end{aligned}$$

Solve this the function $h(c) = Nc^2-2(N-1+y)c+(N-1)+y^2$ for the minimum value and we get $c = \frac{N-1+y}{N}$.

Therefore, the L_2 approximation for $f(x)$ is $g(t) = \frac{N-1+y}{N}$, with error $\|f-g\|_2 = y^2(1-\frac{1}{N}) - y(\frac{2N-2}{N}) + \frac{N-1}{N}$

PROBLEM 2 PAGE 119 PART C As $N \to \infty$, the constant in the least square approximation goes to 1. It shows the least square approximation weights less on the outliers than the infinity approximation. Request more input.

PROBLEM 5 PAGE 119 PART A Define the following $\hat{f}(x) = 1 + cx$ and $f(x) = e^x$.

$$
\begin{aligned}
\|\hat{f} - f\|_2^2 &= \int_0^1 |e^x - 1 - cx|^2 dx = \int_0^1 e^{2x} - 2e^x(1+cx) + (1-cx)^2 dx \\
&= \int_0^1 e^{2x} - (2e^x + 2ce^x x) + (1 - 2cx + c^2x^2) dx \\
&= \frac{1}{3}c^2 - c + \frac{e^2}{2} - 2e + \frac{5}{2}
\end{aligned}
$$

Minimize the function $h(c) = \frac{1}{3}c^2 - c + \frac{e^2}{2} - 2e + \frac{5}{2}$ and the minimum reaches at $c = \frac{3}{2}$.

PROBLEM 5 PAGE 119 PART B Solve for the general case, $\max_{0\leq x\leq 1} |e^x - (1+cx)|$. Let $f_c(x) = e^x - (1+cx)$ and,

$$
f_c'(x) = e^x - c
$$

$c = 1$. $f_1(x)$ is monotonic increasing in the interval $[0,1]$ and the minimum is $f_1(0) = 0$. Then $|e_1(x)| = f_1(x)$. The maximum is at $x = 1$ and the max error is $e - 2$. For the case $c = \frac{3}{2}$, $f_{\frac{3}{2}}(x)$ is decreasing at $[0, ln\frac{3}{2}]$ and increasing at $[ln\frac{3}{2}, 1]$. The minimum is $f_{\frac{3}{2}}(ln\frac{3}{2}) < 0$. Therefore $\max e_2(x) = |f_{\frac{3}{2}}(x)|$ is either reached at the endpoint, $\{0,1\}$, or at $x = \ln\frac{3}{2}$. Plug in and the maximum of $e_2(x)$ at the interval $[0,1]$ is $e - 2.5$ at the endpoint $x = 1$.

PROBLEM 5 PAGE 119 PART C Define the following minimization problem,

$$
\min_{0\leq x\leq 1} \|e^x - (1 + c_1x + c_2x^2)\|_2^2
$$

$$
\begin{aligned}
g(c_1, c_2) &= \int_0^1 (e^x - (1 + c_1x + c_2x^2))^2 dx \\
&= \frac{1}{3}c_1^2 + \frac{1}{2}(c_2 - 2)c_1 + \frac{1}{5}c_2^2 + \left(\frac{14}{3} - 2e\right)c_2 + \frac{1}{2}\left(5 - 4e + e^2\right)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial g}{\partial c_1} &= \frac{1}{2}(c_2 - 2) + \frac{2}{3}c_1 = 0 \\
\frac{\partial g}{\partial c_2} &= \frac{1}{2}c_1 + \frac{2}{5}c_2 + \frac{14}{3} - 2e = 0
\end{aligned}
$$

Solve this system of equations and get $c_1 = 164 - 60e = 0.9031$ and $c_2 = 80e - \frac{650}{3} = 0.7959$.

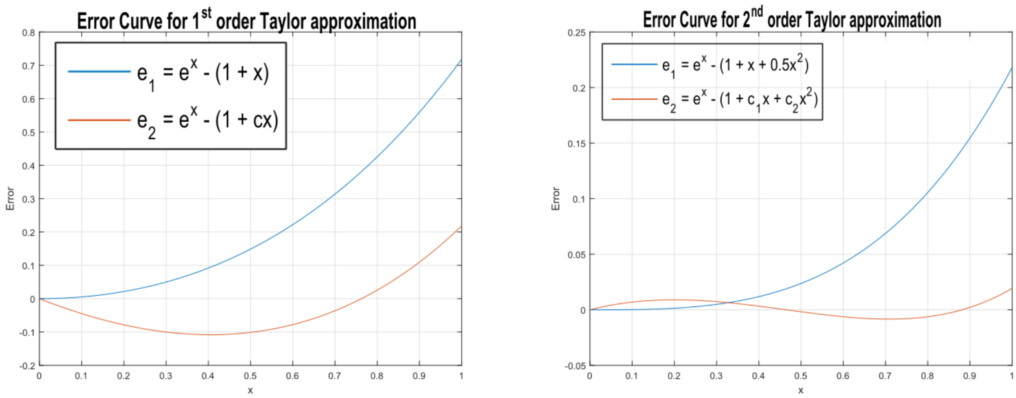

Figure 0.1: Error curves of $e_1(x)$ and $e_2(x)$

PROBLEM 33 PAGE 126 We use two different method to solve this problem.

1. Lagrange Interpolation

$$
\begin{aligned}
i = 0,\ l_0(x) &= \frac{x - x_1}{x_0 - x_1}\frac{x - x_2}{x_0 - x_2} = \frac{(x-11)(x-12)}{2} \\
i = 1,\ l_0(x) &= \frac{x - x_0}{x_1 - x_0}\frac{x - x_2}{x_1 - x_2} = -(x-10)(x-12) \\
i = 2,\ l_0(x) &= \frac{x - x_0}{x_2 - x_0}\frac{x - x_1}{x_2 - x_1} = \frac{(x-10)(x-11)}{2} \\
p(x) &= \ln(10) * l_0(x) + \ln(11) * l_1(x) + \ln(12) * l_2(x) \\
p(11.1) &= 2.406969856623995
\end{aligned}
$$

   The relative error is 1.028×10^{-5}

2. Newton's Interpolation
   To solve the problem in Newton's form, first let's creating the divided difference table using the given points.

x_i	$f(x)$	$f[x_i, x_{i-1}]$	$f[x_i, x_{i-1}, x_{i-2}]$
10	$\ln(10)$		
11	$\ln(11)$	$\ln(11/10)$	
12	$\ln(12)$	$\ln(12/11)$	$0.5\ln(121/120)$

   Then, the interpolation polynomial can be written as

$$p(x) = \ln(10) + (x-10)\ln(11/10) + 0.5(x-10)(x-11)\ln(121/120).$$

   therefore, $p(11.1) = 2.407882724933612$, the relative error for the interpolation is

$$\epsilon_{rel} = \frac{|p(11) - \ln(11.1)|}{\ln(11.1)} = 3.895 \times 10^{-4}$$

PROBLEM 36 PAGE 126 PART A We know that,

$$\begin{aligned}|E(x)| =&|e^x - p_n(f;x)| = |\frac{e^{\xi(x)}}{(n+1)!}\prod_{i=0}^{n}(x-x_i)| \\ &= \frac{e^{\xi(x)}}{(n+1)!}\prod_{i=0}^{n}|(x-\frac{i}{n})| \\ &= \frac{e^{\xi(x)}}{(n+1)!}\prod_{i=0}^{n}\sqrt{|(x-\frac{i}{n})(x-\frac{n-i}{n})|}\end{aligned}$$

We then show the hint is true by a simple maximization problem, $\forall i = 0\cdots n$

$$\max_{0\le x\le 1}|(x-\frac{i}{n})(x-\frac{n-i}{n})|$$

Define $f(x) = (x-\frac{i}{n})(x-\frac{n-i}{n})$ and the minimum point is achieved at $x=\frac{1}{2}$. That is, $|f(x)|$ achieves maximum of $max = |(\frac{1}{2}-\frac{i}{n})(\frac{1}{2}-\frac{n-i}{n})| \le |(\frac{1}{2}-0)(\frac{1}{2}-1)| = \frac{1}{4}$ at $x=\frac{1}{2}$.

$$\max_{0\le x\le 1}|E(x)| \le \frac{e^1}{(n+1)!}\frac{1}{2^n}$$

Solve the following inequality,

$$\frac{e^1}{(n+1)!}\frac{1}{2^n} \le 10^{-6}$$

The smallest n is 9.

PROBLEM 36 PAGE 126 PART B What is connection between the Talyor polynomial and the interpolation polynomial???

PROBLEM 46 PAGE 128 We know that,

$$T_n(\cos\theta) = \cos(n\theta)$$

Take derivative with respect to θ and set it to $\frac{\pi}{2}$,

$$T_n'(\cos\theta) = n\sin(n\theta)\sin(\theta)$$
$$T_n'(0) = n\sin(\frac{n\pi}{2})$$

PROBLEM 9 PAGE 137

1 PROBLEM TITLE

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$$
\begin{aligned}
(x+y)^3 &= (x+y)^2(x+y) \\
&= (x^2+2xy+y^2)(x+y) \\
&= (x^3+2x^2y+xy^2)+(x^2y+2xy^2+y^3) \\
&= x^3+3x^2y+3xy^2+y^3
\end{aligned} \tag{1.1}
$$

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1.1 HEADING ON LEVEL 2 (SUBSECTION)

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$$
A = \begin{bmatrix} A_{11} & A_{21} \\ A_{21} & A_{22} \end{bmatrix} \tag{1.2}
$$

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1.1.1 HEADING ON LEVEL 3 (SUBSUBSECTION)

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2 LISTS

2.1 EXAMPLE OF LIST (3*ITEMIZE)

- First item in a list
  - First item in a list
    - First item in a list
    - Second item in a list
  - Second item in a list
- Second item in a list

2.2 EXAMPLE OF LIST (ENUMERATE)

1. First item in a list
2. Second item in a list
3. Third item in a list