NORTH CAROLINA STATE UNIVERSITY

Homework 1 for Chapter 2

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PROBLEM 1

PROBLEM 2 PAGE 119 PART A Let $g(t)$ be the approximation to f,

$$\|f-g\|_\infty = \sup_{1\le i\le N} |f(t)-g(t)|$$
$$g(t) = c = \frac{y+1}{2}$$

Therefore, the L_∞ approximation for $f(x)$ is $g(t) = \frac{y+1}{2}$, with error $\|f-g\|_\infty = \frac{y-1}{2}$.

PROBLEM 2 PAGE 119 PART B Let $g(t)$ be the approximation to f,

$$\begin{aligned}\|f-g\|_2 &= (\sum_{i=1}^{N} |f(t_i)-g(t_i)|^2)^{\frac{1}{2}}\\ &= \sqrt{(N-1)(c-1)^2+(c-y)^2} = \sqrt{Nc^2-2(N-1+y)c+(N-1)+y^2}\end{aligned}$$

Solve this the function $h(c) = Nc^2-2(N-1+y)c+(N-1)+y^2$ for the minimum value and we get $c = \frac{N-1+y}{N}$.
Therefore, the L_2 approximation for $f(x)$ is $g(t) = \frac{N-1+y}{N}$, with error $\|f-g\|_2 = y^2(1-\frac{1}{N})-y(\frac{2N-2}{N})+\frac{N-1}{N}$

PROBLEM 2 PAGE 119 PART C As $N \to \infty$, the constant in the least square approximation goes to 1. It shows the least square approximation weights less on the outliers than the infinity approximation. Request more input.

PROBLEM 5 PAGE 119 PART A Define the following $\hat{f}(x) = 1 + cx$ and $f(x) = e^x$.

$$\begin{aligned}
\|\hat{f} - f\|_2^2 &= \int_0^1 |e^x - 1 - cx|^2 dx = \int_0^1 e^{2x} - 2e^x(1+cx) + (1-cx)^2 dx \\
&= \int_0^1 e^{2x} - (2e^x + 2ce^x x) + (1 - 2cx + c^2x^2) dx \\
&= \frac{1}{3}c^2 - c + \frac{e^2}{2} - 2e + \frac{5}{2}
\end{aligned}$$

Minimize the function $h(c) = \frac{1}{3}c^2 - c + \frac{e^2}{2} - 2e + \frac{5}{2}$ and the minimum reaches at $c = \frac{3}{2}$.

PROBLEM 5 PAGE 119 PART B Solve for the general case, $\max_{0\le x\le 1} |e^x - (1+cx)|$. Let $f_c(x) = e^x - (1+cx)$ and,

$$f_c'(x) = e^x - c$$

$c = 1$. $f_1(x)$ is monotonic increasing in the interval $[0,1]$ and the minimum is $f_1(0) = 0$. Then $|e_1(x)| = f_1(x)$. The maximum is at $x = 1$ and the max error is $e - 2$. For the case $c = \frac{3}{2}$, $f_{\frac{3}{2}}(x)$ is decreasing at $[0, ln\frac{3}{2}]$ and increasing at $[ln\frac{3}{2}, 1]$. The minimum is $f_{\frac{3}{2}}(ln\frac{3}{2}) < 0$. Therefore $\max e_2(x) = |f_{\frac{3}{2}}(x)|$ is either reached at the endpoint, $\{0,1\}$, or at $x = \ln\frac{3}{2}$. Plug in and the maximum of $e_2(x)$ at the interval $[0,1]$ is $e - 2.5$ at the endpoint $x = 1$.

PROBLEM 5 PAGE 119 PART C Define the following minimization problem,

$$\min_{0\le x\le 1} \|e^x - (1 + c_1x + c_2x^2)\|_2^2$$

$$\begin{aligned}
g(c_1, c_2) &= \int_0^1 (e^x - (1 + c_1x + c_2x^2))^2 dx \\
&= \frac{1}{3}c_1^2 + \frac{1}{2}(c_2 - 2)c_1 + \frac{1}{5}c_2^2 + \left(\frac{14}{3} - 2e\right)c_2 + \frac{1}{2}\left(5 - 4e + e^2\right)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial g}{\partial c_1} &= \frac{1}{2}(c_2 - 2) + \frac{2}{3}c_1 = 0 \\
\frac{\partial g}{\partial c_2} &= \frac{1}{2}c_1 + \frac{2}{5}c_2 + \frac{14}{3} - 2e = 0
\end{aligned}$$

Solve this system of equations and get $c_1 = 164 - 60e = 0.9031$ and $c_2 = 80e - \frac{650}{3} = 0.7959$.

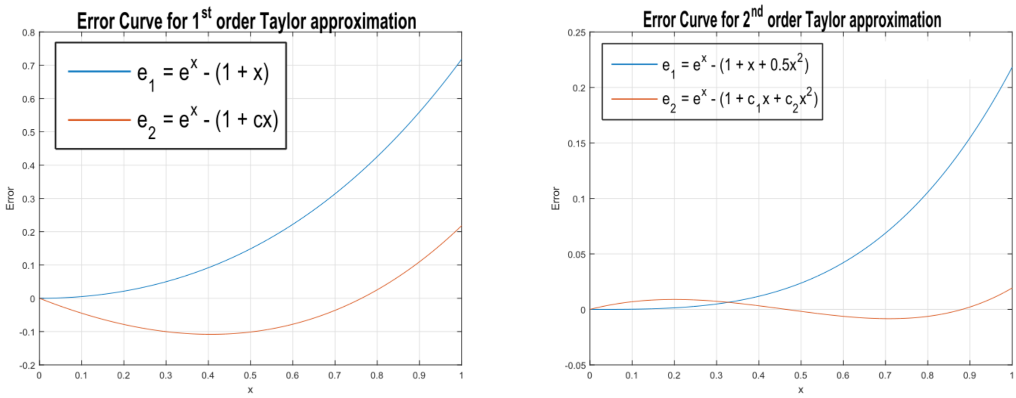

Figure 0.1: Error curves of $e_1(x)$ and $e_2(x)$

PROBLEM 33 PAGE 126 We use two different method to solve this problem.

1. Lagrange Interpolation

$$
\begin{aligned}
i = 0,\ l_0(x) &= \frac{x - x_1}{x_0 - x_1}\frac{x - x_2}{x_0 - x_2} \\
i = 1,\ l_0(x) &= \frac{x - x_0}{x_1 - x_0}\frac{x - x_2}{x_1 - x_2} \\
i = 2,\ l_0(x) &= \frac{x - x_0}{x_2 - x_0}\frac{x - x_1}{x_2 - x_1} \\
p(x) = \ln(10) * l_0(x) &+ \ln(11) * l_1(x) + \ln(12) * l_2(x)
\end{aligned}
$$

   The relative error is 0.10282%

2. Newton's Interpolation
   Chang Chang

PROBLEM 36 PAGE 126 PART A We know that,

$$
\begin{aligned}
|E(x)| = &|e^x - p_n(f;x)| = |\frac{e^{\xi(x)}}{(n+1)!}\prod_{i=0}^{n}(x-x_i)| \\
&= \frac{e^{\xi(x)}}{(n+1)!}\prod_{i=0}^{n}|(x-\frac{i}{n})| \\
&= \frac{e^{\xi(x)}}{(n+1)!}\prod_{i=0}^{n}\sqrt{|(x-\frac{i}{n})(x-\frac{n-i}{n})|}
\end{aligned}
$$

We then show the hint is true by a simple maximization problem, $\forall i = 0\cdots n$

$$
\max_{0\le x\le 1}|(x-\frac{i}{n})(x-\frac{n-i}{n})|
$$

Define $f(x) = (x-\frac{i}{n})(x-\frac{n-i}{n})$ and the minimum point is achieved at $x=\frac{1}{2}$. That is, $|f(x)|$ achieves maximum of $max = |(\frac{1}{2}-\frac{i}{n})(\frac{1}{2}-\frac{n-i}{n})| \le |(\frac{1}{2}-0)(\frac{1}{2}-1)| = \frac{1}{4}$ at $x=\frac{1}{2}$.

$$
\max_{0\le x\le 1}|E(x)| \le \frac{e^1}{(n+1)!}\frac{1}{2^n}
$$

Solve the following inequality,

$$
\frac{e^1}{(n+1)!}\frac{1}{2^n} \le 10^{-6}
$$

The smallest n is 9.

PROBLEM 36 PAGE 126 PART B What is connection between the Talyor polynomial and the interpolation polynomial???

PROBLEM 46 PAGE 128 We know that,

$$T_n(\cos\theta) = \cos(n\theta)$$

Take derivative with respect to θ and set it to $\frac{\pi}{2}$,

$$T'_n(\cos\theta) = n\sin(n\theta)\sin(\theta)$$
$$T'_n(0) = n\sin(\frac{n\pi}{2})$$

PROBLEM 9 PAGE 137

1 PROBLEM TITLE

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$$\begin{aligned}(x+y)^3 &= (x+y)^2(x+y) \\ &= (x^2+2xy+y^2)(x+y) \\ &= (x^3+2x^2y+xy^2)+(x^2y+2xy^2+y^3) \\ &= x^3+3x^2y+3xy^2+y^3\end{aligned} \tag{1.1}$$

Phasellus viverra nulla ut metus varius laoreet. Quisque rutrum. Aenean imperdiet. Etiam ultricies nisi vel augue. Curabitur ullamcorper ultricies

1.1 HEADING ON LEVEL 2 (SUBSECTION)

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$$A = \begin{bmatrix} A_{11} & A_{21} \\ A_{21} & A_{22} \end{bmatrix} \tag{1.2}$$

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1.1.1 HEADING ON LEVEL 3 (SUBSUBSECTION)

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HEADING ON LEVEL 4 (PARAGRAPH) Suspendisse vel felis. Ut lorem lorem, interdum eu, tincidunt sit amet, laoreet vitae, arcu. Aenean faucibus pede eu ante. Praesent enim elit, rutrum at, molestie non, nonummy vel, nisl. Ut lectus eros, malesuada sit amet, fermentum eu, sodales cursus, magna. Donec eu purus. Quisque vehicula, urna sed ultricies auctor, pede lorem egestas dui, et convallis elit erat sed nulla. Donec luctus. Curabitur et nunc. Aliquam dolor odio, commodo pretium, ultricies non, pharetra in, velit. Integer arcu est, nonummy in, fermentum faucibus, egestas vel, odio.

2 LISTS

2.1 EXAMPLE OF LIST (3*ITEMIZE)

- First item in a list
    - First item in a list
        - First item in a list
        - Second item in a list
    - Second item in a list
- Second item in a list

2.2 EXAMPLE OF LIST (ENUMERATE)

1. First item in a list
2. Second item in a list
3. Third item in a list