Assignment 07

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```
library(tidyverse)
```

```
## ── Attaching packages ─────────────────────────────────────── tidyverse 1.3.2 ──
## ✔ ggplot2 3.3.6      ✔ purrr   0.3.4
## ✔ tibble  3.1.8      ✔ dplyr   1.0.10
## ✔ tidyr   1.2.1      ✔ stringr 1.4.1
## ✔ readr   2.1.3      ✔ forcats 0.5.2
## ── Conflicts ────────────────────────────────────────── tidyverse_conflicts() ──
## ✖ dplyr::filter() masks stats::filter()
## ✖ dplyr::lag()    masks stats::lag()
```

Maximum likelihood estimates

Maximum likelihood estimates for Red tailed hawks

```
library(Stat2Data)
data("Hawks")
```

Q1

```
RedTailedDf<-Hawks%>%
  filter(Species == 'RT')%>%
  select(Weight,Tail,Wing)


head(RedTailedDf,5)
```

```
##   Weight Tail Wing
## 1    920  219  385
## 2    930  221  376
## 3    990  235  381
## 4   1090  230  412
## 5    960  212  370
```

```
dim(RedTailedDf)
```

```
## [1] 577   3
```

Q2

```
# estimate of mu and sigma

mu<-mean(RedTailedDf$Tail)
sigma<-sd(RedTailedDf$Tail)*sqrt(576/577)


mu
```

```
## [1] 222.149
```

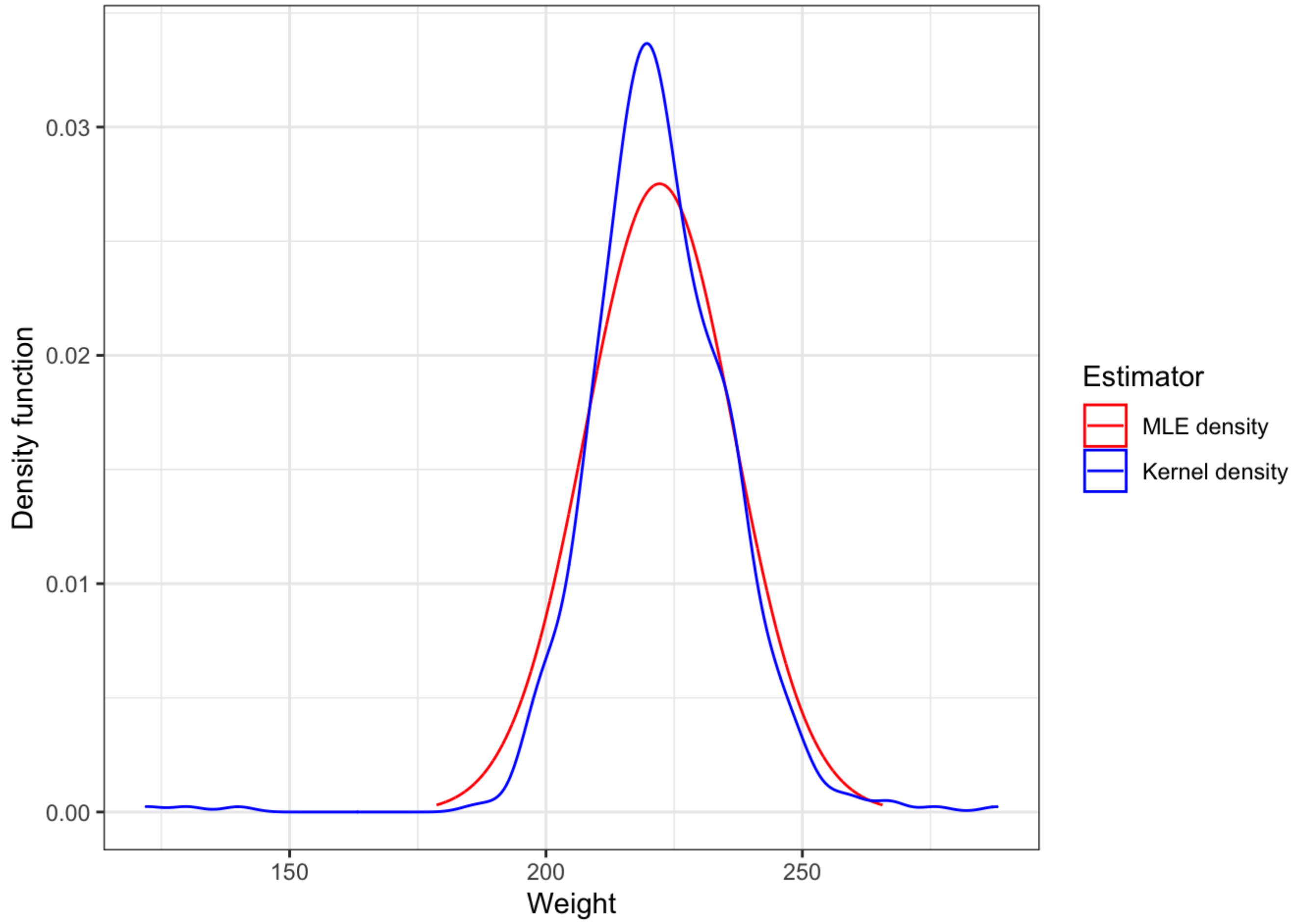
```
sigma
```

```
## [1] 14.49838
```

Q3

```
weights<- seq(mu-3*sigma,mu+3*sigma,sigma*0.001)

colors<- c("MLE density" = "red","Kernel density"="blue")

estimated_density = data.frame(
  Weight = weights,
  Density = dnorm(weights, mean = mu,sd = sigma)
)


plot_obj<- ggplot() + geom_line(data =estimated_density,
                                aes(x = Weight, y= Density, color = "MLE density"))


plot_obj + geom_density(data = RedTailedDf,
                        aes(x =RedTailedDf$Tail, color = "Kernel density")) + labs(y=
"Density function",color = "Estimator") + theme_bw() + scale_color_manual(values = co
lors)
```

```
## Warning: Use of `RedTailedDf$Tail` is discouraged. Use `Tail` instead.
```



Unbiased estimation of the population variance

Q1

```
num_trials<-1000 # set the number of trials
set.seed(0) # set the random seed
sampling_simulation<-data.frame(trial=seq(num_trials)) %>%
  mutate(sample_size = map(.x=trial, ~sample(c(5,100),5, replace = TRUE)))



#mle_x<-sum((sampling_simulation-mean(sampling_simulation))^2) / (sample_size)
#v<-x<-sum((sampling_simulation-mean(sampling_simulation))^2) / (sample_size-1)
```

Q2

Maximum likelihood estimation with the Possion distribution

Q1

The natural log likelihood function:

$$\log l(\lambda) = -n\lambda + \ln(\lambda) \sum_{i=1}^{n} X_i - \sum_{i=1}^{n} \ln(X_i!)$$

derivative natural log likelihood function

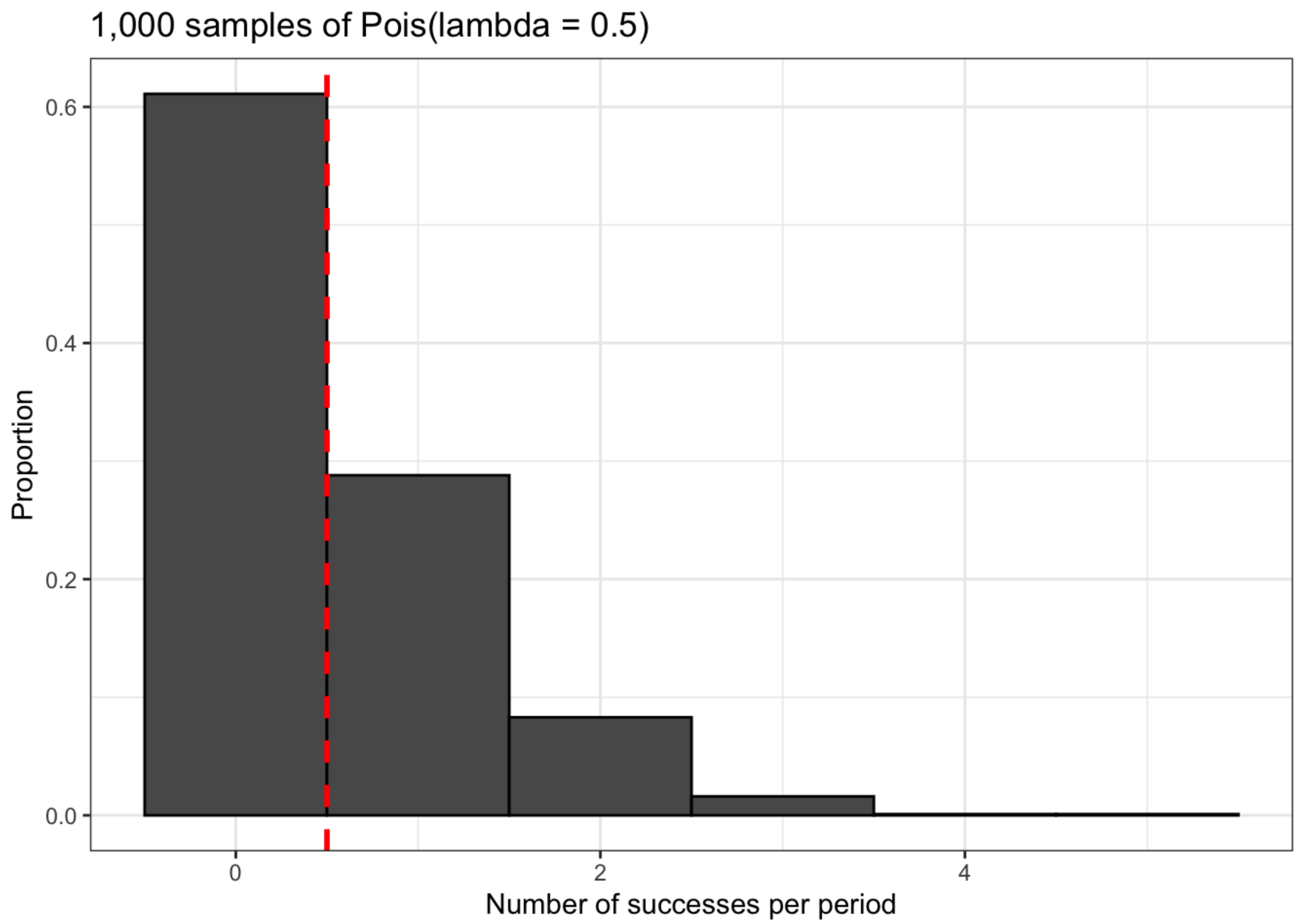
$$\frac{\partial}{\partial\lambda}\log l(\lambda) = -n + \frac{1}{\lambda}\sum_{i=1}^{n} X_i = -n + \frac{n}{\lambda}\overline{X}$$

Q2

$$-n + \frac{n}{\lambda}\overline{X} = 0$$
$$\lambda = \overline{X}$$

Q3

```
set.seed(0)

poisson_data <- data.frame('data' = rpois(1000, 0.5))

poisson_data %>% ggplot() +
  geom_histogram(aes(x = data,
                     y = stat(count / sum(count))),
                     color = 'black',
                 binwidth = 1) +
  geom_vline(xintercept = 0.5,
             size = 1,
             linetype = 'dashed',
             color = 'red') +
  theme_bw() +
  labs(x = 'Number of successes per period',
       y = 'Proportion',
       title = '1,000 samples of Pois(lambda = 0.5)')
```



Q4

```
Von_df<-read.csv("VonBortkiewicz.csv",header = TRUE,sep =",")

head(Von_df)
```

```
##   fatalities year corps fisher
## 1          0 1875     G     no
## 2          0 1875     I     no
## 3          0 1875    II    yes
## 4          0 1875   III    yes
## 5          0 1875    IV    yes
## 6          0 1875     V    yes
```

```
dim(Von_df)
```

```
## [1] 280   4
```

```
mean(Von_df$fatalities)
```

```
## [1] 0.7
```

```
data<-Von_df$fatalities
dpois(data[1],lambda = 1)
```

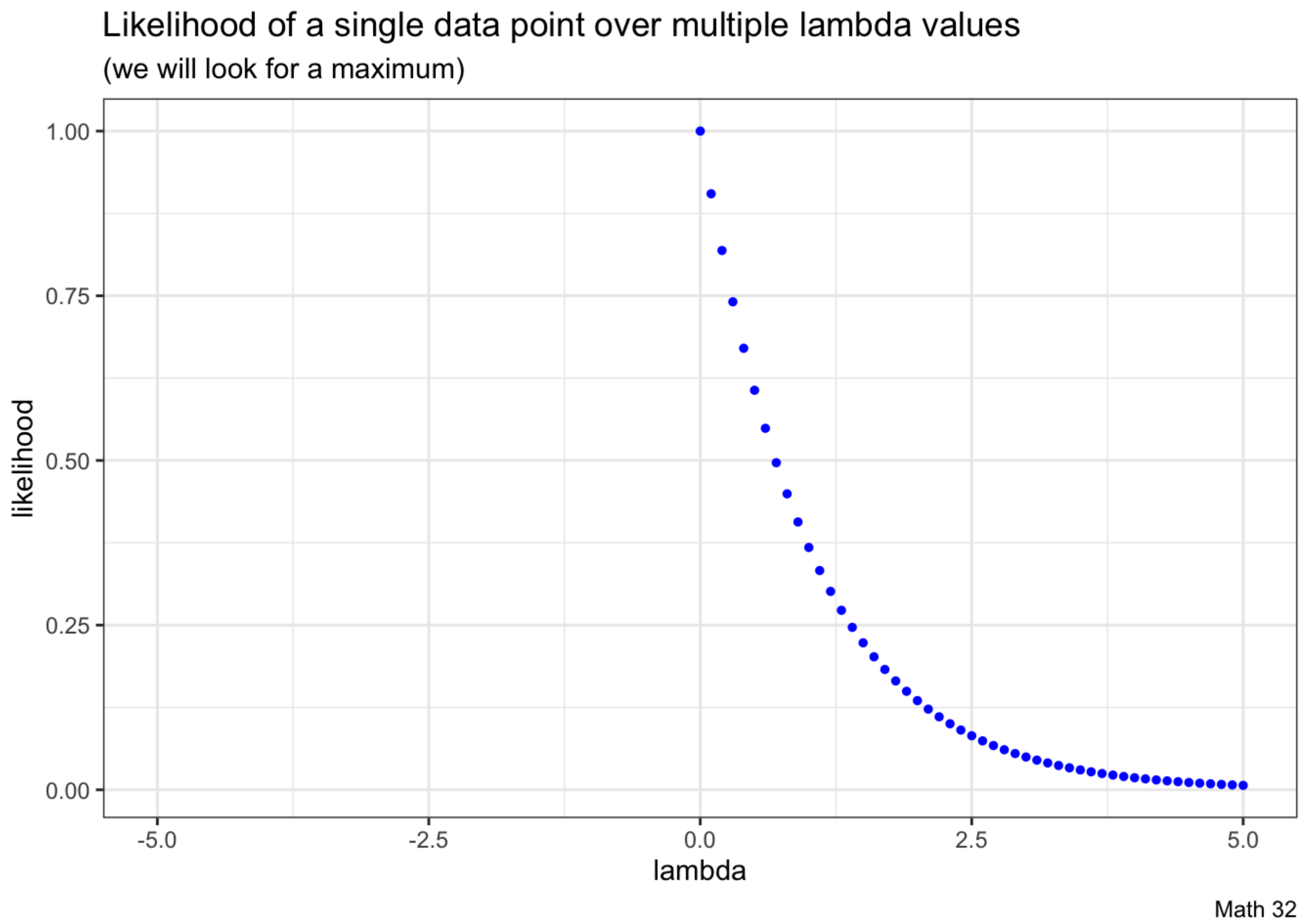
```
## [1] 0.3678794
```

```
lambda_values <- seq(-5, 5,by = 0.1)
likelihood <- dpois(data[1], lambda = lambda_values)
```

```
## Warning in dpois(data[1], lambda = lambda_values): NaNs produced
```

```
# arranged into a data frame
lh_single <- data.frame(lambda_values = lambda_values, likelihood = likelihood )

lh_single %>%
  ggplot(aes(x = lambda_values,y = likelihood))+
  geom_point(size = 1, color = "blue")+
  labs(title = "Likelihood of a single data point over multiple lambda values",
       subtitle = "(we will look for a maximum)",
       caption = "Math 32",
       x = "lambda",
       y = "likelihood") +
  theme_bw()
```

```
## Warning: Removed 50 rows containing missing values (geom_point).
```



Maximum likehood estimation for the exponential distribution

Q1

$$\lambda_0 = \frac{1}{\overline{X}}$$

Q2

```
CP_df<-read.csv("CustomerPurchase.csv",header = TRUE,sep =",")

dim(CP_df)
```

```
## [1] 640   2
```

```
time_d<-data.frame(CP_df$Time)
```

```
for (i in 1:nrow(time_d)){
  time_d[i,] <- time_d[i+1,]-time_d[i,]
}
```

```
time_d = as.numeric(as.character(time_d$CP_df.Time))
```

```
CP_df%>%
  mutate(time_diffs = lead(time_d))
```

```
##      Time Purchase time_diffs
## 1     564     3.25          7
## 2     571   504.85         22
## 3     578     7.60        145
## 4     600    43.45         61
## 5     745     9.30         27
## 6     806   352.80         48
## 7     833   182.05          7
## 8     881     8.55         70
## 9     888    65.35         38
## 10    958   211.00         62
## 11    996   471.30        221
## 12   1058    76.30         53
## 13   1279     0.05         52
## 14   1332     0.00         94
## 15   1384   406.50         33
## 16   1478    51.55         17
## 17   1511   740.20         29
## 18   1528    24.55        118
## 19   1557    13.35         32
## 20   1675    60.25         15
## 21   1707   168.20         28
## 22   1722    76.35          5
## 23   1750    41.10          3
## 24   1755    82.40         29
## 25   1758    94.65        198
## 26   1787   123.90         59
## 27   1985     0.00         50
## 28   2044    84.55         72
## 29   2094    20.50          2
## 30   2166     0.10         16
## 31   2168     2.45         66
## 32   2184     0.05         10
## 33   2250    16.30         51
## 34   2260    21.30         15
## 35   2311   145.15         36
## 36   2326   177.55         38
## 37   2362     1.85         12
## 38   2400    21.45         54
## 39   2412    83.40         51
## 40   2466   258.90         65
## 41   2517    40.05         63
## 42   2582    21.95         28
## 43   2645    68.25         15
## 44   2673    34.90         65
## 45   2688   345.90         50
## 46   2753    81.20         26
## 47   2803     6.35        100
## 48   2829    14.10         21
## 49   2929    11.00        109
## 50   2950    63.30        161
## 51   3059    62.25         28
## 52   3220   165.15         30
## 53   3248    78.40         49
## 54   3278    18.00         10
```

```
## 55   3327     0.15          15
## 56   3337   120.25          55
## 57   3352    39.55          39
## 58   3407    62.95           4
## 59   3446   199.10          55
## 60   3450   340.80          12
## 61   3505     0.05          79
## 62   3517   314.60         242
## 63   3596     0.75          22
## 64   3838    72.15         137
## 65   3860    67.20          57
## 66   3997    23.50          41
## 67   4054   257.95          42
## 68   4095     5.20          89
## 69   4137   124.05         116
## 70   4226    43.80         145
## 71   4342     0.20          14
## 72   4487   220.70          19
## 73   4501    53.30           3
## 74   4520    10.10          18
## 75   4523    13.15          78
## 76   4541    64.15          41
## 77   4619    71.90         138
## 78   4660     0.60          19
## 79   4798   122.65          50
## 80   4817     0.00          41
## 81   4867   123.85           3
## 82   4908    68.90         114
## 83   4911   132.35          40
## 84   5025     0.15          79
## 85   5065    53.95          62
## 86   5144    12.95          67
## 87   5206     9.85         105
## 88   5273    59.75          52
## 89   5378   346.20          22
## 90   5430     5.55          52
## 91   5452    28.80          13
## 92   5504   128.40          34
## 93   5517    48.30          13
## 94   5551   389.85          22
## 95   5564   135.80          11
## 96   5586   496.40           7
## 97   5597    81.85          17
## 98   5604     0.80          95
## 99   5621    90.90          26
## 100  5716     0.25          22
## 101  5742     0.00          19
## 102  5764    29.55          62
## 103  5783   168.30         135
## 104  5845    81.25          10
## 105  5980   148.05         115
## 106  5990   165.05          28
## 107  6105    30.85          38
## 108  6133    88.45          79
## 109  6171    30.60          92
## 110  6250   451.90           2
```

```
## 111  6342    50.40           6
## 112  6344     5.65          71
## 113  6350    17.95          36
## 114  6421     0.15          22
## 115  6457    41.30           3
## 116  6479   126.40          11
## 117  6482   156.75          15
## 118  6493    68.65          12
## 119  6508     0.15          37
## 120  6520   147.95          85
## 121  6557   248.15          49
## 122  6642   475.85           1
## 123  6691    11.50          11
## 124  6692    97.75         163
## 125  6703   216.55          68
## 126  6866    35.10           5
## 127  6934   104.60          60
## 128  6939    66.35          13
## 129  6999    33.70          63
## 130  7012    61.40          19
## 131  7075   173.70          19
## 132  7094    89.30           5
## 133  7113    12.80          45
## 134  7118    69.15           8
## 135  7163     1.85           5
## 136  7171    43.20         104
## 137  7176   103.20          39
## 138  7280   463.95          26
## 139  7319    10.70          51
## 140  7345     1.15         113
## 141  7396   271.80          59
## 142  7509    63.70          11
## 143  7568   136.05          40
## 144  7579    19.35          29
## 145  7619    95.35          20
## 146  7648   158.70          85
## 147  7668     0.05         106
## 148  7753     3.85          25
## 149  7859   179.45          12
## 150  7884     0.05          87
## 151  7896    85.70         142
## 152  7983   101.90          96
## 153  8125   412.30          36
## 154  8221    51.00          92
## 155  8257    96.40          36
## 156  8349    51.95           3
## 157  8385    11.85           2
## 158  8388   137.60          75
## 159  8390    96.45          49
## 160  8465    57.40          81
## 161  8514     1.50          72
## 162  8595    41.45          74
## 163  8667   142.80          11
## 164  8741   126.45         133
## 165  8752    12.00         106
## 166  8885     4.90          17
```

```
## 167  8991   466.25          130
## 168  9008   188.90           48
## 169  9138    56.55           90
## 170  9186    21.20           31
## 171  9276     0.60          109
## 172  9307    15.15           65
## 173  9416    10.45           76
## 174  9481     4.10           97
## 175  9557   205.10           34
## 176  9654   200.70           22
## 177  9688     0.75           38
## 178  9710    65.95           38
## 179  9748    51.45           12
## 180  9786    19.75           16
## 181  9798   147.55           47
## 182  9814    37.45           66
## 183  9861     2.70           27
## 184  9927   179.65           38
## 185  9954     9.45           26
## 186  9992    27.60           10
## 187 10018    17.40          116
## 188 10028     9.15           57
## 189 10144    82.10           40
## 190 10201    30.65            6
## 191 10241     7.50           62
## 192 10247     5.00           18
## 193 10309     0.40           39
## 194 10327   414.80           41
## 195 10366    20.40           23
## 196 10407   142.80           37
## 197 10430   143.35            3
## 198 10467   139.90            4
## 199 10470    75.40           51
## 200 10474     2.85           34
## 201 10525     0.00           40
## 202 10559   138.00            9
## 203 10599     4.25           34
## 204 10608   105.00          111
## 205 10642     9.75           20
## 206 10753    16.25           20
## 207 10773     1.95           50
## 208 10793    26.65            8
## 209 10843   158.55           69
## 210 10851    95.10           51
## 211 10920    26.50            6
## 212 10971   157.35           54
## 213 10977   317.40            8
## 214 11031    44.40           33
## 215 11039    97.30          109
## 216 11072    73.50           77
## 217 11181     0.00            5
## 218 11258     0.20           54
## 219 11263   383.60           16
## 220 11317   159.25           47
## 221 11333   136.55           55
## 222 11380    59.25           78
```

```
## 223 11435    99.25          16
## 224 11513    59.70          93
## 225 11529    48.85          31
## 226 11622    67.35          22
## 227 11653     0.00          59
## 228 11675   520.55          28
## 229 11734   146.10         229
## 230 11762    25.20          12
## 231 11991   220.35          25
## 232 12003     6.25          29
## 233 12028   330.75          21
## 234 12057    16.50          45
## 235 12078    78.55          20
## 236 12123   251.50          32
## 237 12143   204.55          52
## 238 12175     2.25          77
## 239 12227     7.55          54
## 240 12304    31.30          66
## 241 12358    24.00         122
## 242 12424    31.25          29
## 243 12546     1.50           3
## 244 12575    60.90           4
## 245 12578   154.90         138
## 246 12582     1.45          30
## 247 12720    34.55          24
## 248 12750    70.85         172
## 249 12774   332.65          36
## 250 12946     0.05          43
## 251 12982    16.40           0
## 252 13025    32.15          11
## 253 13025   108.90          38
## 254 13036    39.75          69
## 255 13074   141.30          58
## 256 13143    33.20           2
## 257 13201    87.30          13
## 258 13203    67.40          76
## 259 13216    47.10         159
## 260 13292     6.35           2
## 261 13451   230.25          35
## 262 13453    15.85          37
## 263 13488   125.00           6
## 264 13525    61.00          51
## 265 13531   348.15          14
## 266 13582   131.75         112
## 267 13596    19.70          52
## 268 13708    44.15          26
## 269 13760    34.40          81
## 270 13786    53.50          20
## 271 13867   327.40           0
## 272 13887   191.65          95
## 273 13887     2.45         317
## 274 13982    85.90         196
## 275 14299    69.40           6
## 276 14495     0.75          64
## 277 14501   278.95          50
## 278 14565    25.40          35
```

```
## 279 14615    63.00          158
## 280 14650    88.80           16
## 281 14808     0.00           16
## 282 14824     0.30           49
## 283 14840    40.70           18
## 284 14889   111.75            0
## 285 14907    51.85           23
## 286 14907   130.95           22
## 287 14930    80.15           94
## 288 14952   107.45           15
## 289 15046     7.30            4
## 290 15061   126.55           52
## 291 15065     1.95           14
## 292 15117    12.80           33
## 293 15131    36.05           21
## 294 15164    18.85           54
## 295 15185    36.45           71
## 296 15239    14.10           62
## 297 15310     3.55           26
## 298 15372     8.90           21
## 299 15398   226.85          183
## 300 15419     3.00          105
## 301 15602    27.70           52
## 302 15707    97.50           89
## 303 15759     4.75           12
## 304 15848    11.15           64
## 305 15860   290.85           11
## 306 15924     0.05           98
## 307 15935     0.70          117
## 308 16033     7.10            6
## 309 16150    62.60           23
## 310 16156   208.80            0
## 311 16179     1.75           13
## 312 16179    14.80          129
## 313 16192   251.95          168
## 314 16321     4.45           32
## 315 16489   127.80           33
## 316 16521    86.85           15
## 317 16554     4.15           72
## 318 16569   221.15           53
## 319 16641   145.00           23
## 320 16694    11.75            8
## 321 16717     0.40           14
## 322 16725    16.70           41
## 323 16739   148.05            5
## 324 16780     8.50           35
## 325 16785    34.35          129
## 326 16820    33.05           33
## 327 16949   141.25            3
## 328 16982     1.10           17
## 329 16985    26.10           66
## 330 17002     2.20           34
## 331 17068    56.10           24
## 332 17102    33.85           54
## 333 17126    42.00            1
## 334 17180     6.40           10
```

```
## 335 17181    3.80         68
## 336 17191  157.60        115
## 337 17259   54.95          3
## 338 17374   29.70         19
## 339 17377  220.40         92
## 340 17396    3.40         15
## 341 17488  139.85         48
## 342 17503   15.80         63
## 343 17551    1.95         78
## 344 17614   14.10         25
## 345 17692   17.50         12
## 346 17717   11.15          4
## 347 17729  100.75         54
## 348 17733  285.95          3
## 349 17787  177.85          4
## 350 17790  227.00         21
## 351 17794   20.20         66
## 352 17815  153.05         80
## 353 17881    0.00         53
## 354 17961   33.30         13
## 355 18014  456.60        146
## 356 18027   43.85         27
## 357 18173   21.70         74
## 358 18200    7.75         50
## 359 18274  141.15         12
## 360 18324  434.15         36
## 361 18336   89.70         13
## 362 18372   24.55         48
## 363 18385  425.15         47
## 364 18433    2.20         33
## 365 18480  118.90         38
## 366 18513   15.05        106
## 367 18551    0.00         19
## 368 18657  221.40         17
## 369 18676   28.75         10
## 370 18693   21.50        100
## 371 18703   46.40         29
## 372 18803   13.95         70
## 373 18832   18.30         19
## 374 18902    5.75         23
## 375 18921   13.95         57
## 376 18944  384.50         63
## 377 19001   33.00          7
## 378 19064   55.05        130
## 379 19071  129.65         97
## 380 19201  163.70          9
## 381 19298   29.05         25
## 382 19307   28.30         13
## 383 19332  133.90        151
## 384 19345  258.90          9
## 385 19496   15.65          7
## 386 19505  238.95         17
## 387 19512   64.35         27
## 388 19529   19.60          2
## 389 19556  230.85         33
## 390 19558  169.30          3
```

```
## 391 19591   233.50           0
## 392 19594    22.60          69
## 393 19594   141.95          57
## 394 19663   169.10         114
## 395 19720    10.45          28
## 396 19834   342.10          77
## 397 19862    60.70           2
## 398 19939    40.20          10
## 399 19941   105.15           1
## 400 19951    86.90          62
## 401 19952     3.85          35
## 402 20014    40.30         118
## 403 20049   585.25           1
## 404 20167    38.70          32
## 405 20168    22.25          10
## 406 20200   135.35          54
## 407 20210    88.35          86
## 408 20264     9.55          16
## 409 20350     0.30          25
## 410 20366   153.60          18
## 411 20391    56.30          15
## 412 20409    98.85         113
## 413 20424    22.55          53
## 414 20537    19.10          21
## 415 20590    51.40          22
## 416 20611   517.30          39
## 417 20633     9.55          36
## 418 20672     0.00          78
## 419 20708     5.05          69
## 420 20786    18.10          27
## 421 20855    97.65         254
## 422 20882    32.25          10
## 423 21136    27.30           9
## 424 21146     3.70          12
## 425 21155    51.05          36
## 426 21167   186.40           5
## 427 21203   478.30          25
## 428 21208   241.10         170
## 429 21233     0.75          21
## 430 21403    11.10          44
## 431 21424   116.25          41
## 432 21468    44.10          33
## 433 21509    73.15         123
## 434 21542     4.35         157
## 435 21665    14.75          54
## 436 21822     0.20           4
## 437 21876     0.20          11
## 438 21880     3.55          39
## 439 21891    15.40         101
## 440 21930    44.45          61
## 441 22031   120.05          39
## 442 22092    10.80          18
## 443 22131     0.90          39
## 444 22149   307.95          41
## 445 22188   178.85          85
## 446 22229   129.55          28
```

```
## 447 22314   328.00          83
## 448 22342     0.00           6
## 449 22425   418.25          85
## 450 22431   223.05           7
## 451 22516    76.85          23
## 452 22523   118.70          17
## 453 22546    50.65          32
## 454 22563    35.55         176
## 455 22595    14.90           1
## 456 22771   136.60         103
## 457 22772   444.30          16
## 458 22875   429.60          85
## 459 22891    61.55          12
## 460 22976    35.80          43
## 461 22988    62.85          36
## 462 23031   207.20         151
## 463 23067   276.15           2
## 464 23218    89.20          13
## 465 23220   201.15          87
## 466 23233    42.55          54
## 467 23320   100.90           1
## 468 23374   214.95          25
## 469 23375   968.65          39
## 470 23400    40.25          18
## 471 23439     4.95          54
## 472 23457    94.05           1
## 473 23511    15.85          55
## 474 23512   120.30          62
## 475 23567    80.95          56
## 476 23629   241.40          13
## 477 23685     0.05           8
## 478 23698     1.90          22
## 479 23706   296.65          54
## 480 23728    67.55           5
## 481 23782     0.25          24
## 482 23787     0.20          32
## 483 23811     9.00           1
## 484 23843   113.80          58
## 485 23844    10.05          46
## 486 23902   246.75          23
## 487 23948    69.60          20
## 488 23971    64.35          27
## 489 23991    43.60          65
## 490 24018    22.20           8
## 491 24083    31.15         115
## 492 24091   209.25          18
## 493 24206     5.15           5
## 494 24224   195.95          36
## 495 24229    18.95          14
## 496 24265   135.40          22
## 497 24279    39.85          55
## 498 24301     5.35           2
## 499 24356    81.60           1
## 500 24358     5.85          22
## 501 24359   203.05          70
## 502 24381     1.30          19
```

```
## 503 24451    27.90          144
## 504 24470     0.95          104
## 505 24614     8.20           64
## 506 24718     1.35           37
## 507 24782    26.15           69
## 508 24819     7.35            8
## 509 24888    15.60          231
## 510 24896    79.35           36
## 511 25127     1.20           60
## 512 25163   139.40           51
## 513 25223    44.75           31
## 514 25274    41.60           58
## 515 25305    48.30            8
## 516 25363    21.95           51
## 517 25371     8.85          329
## 518 25422   153.55          107
## 519 25751   257.20          123
## 520 25858   224.00          242
## 521 25981     0.55           52
## 522 26223     1.85          105
## 523 26275    37.30          219
## 524 26380     0.15           84
## 525 26599    16.40           33
## 526 26683   144.40           27
## 527 26716    51.30           39
## 528 26743   117.25           95
## 529 26782     1.85           71
## 530 26877    19.55           86
## 531 26948     1.30          112
## 532 27034     1.30          157
## 533 27146    67.95            2
## 534 27303     4.40           29
## 535 27305     0.00           19
## 536 27334     4.20          107
## 537 27353     8.45          181
## 538 27460     0.80           40
## 539 27641   328.95           20
## 540 27681     5.30           11
## 541 27701     0.55            1
## 542 27712     7.35           22
## 543 27713    32.45           42
## 544 27735    16.50            0
## 545 27777    70.50           56
## 546 27777    23.15           79
## 547 27833   156.95           25
## 548 27912   133.10           16
## 549 27937   190.20           47
## 550 27953    69.55          103
## 551 28000   201.15            2
## 552 28103    38.35           49
## 553 28105   168.10           26
## 554 28154    31.90            3
## 555 28180     4.55           90
## 556 28183     2.15           14
## 557 28273     3.80            9
## 558 28287     2.80            7
```

```
## 559 28296   111.75         221
## 560 28303    40.40          57
## 561 28524     6.10          39
## 562 28581     0.45          45
## 563 28620     8.00          82
## 564 28665    26.20          19
## 565 28747     2.75          40
## 566 28766    14.60         107
## 567 28806    53.75         201
## 568 28913   151.20          11
## 569 29114    37.65          20
## 570 29125    55.60          30
## 571 29145     0.05          74
## 572 29175   145.20          83
## 573 29249    54.40           7
## 574 29332   135.75           4
## 575 29339   291.85          50
## 576 29343    62.50           5
## 577 29393    23.15          28
## 578 29398     2.40          14
## 579 29426     8.05          41
## 580 29440     0.30          96
## 581 29481    84.40          38
## 582 29577     9.80           3
## 583 29615    25.00          18
## 584 29618     4.95          61
## 585 29636   195.45           1
## 586 29697    56.70           0
## 587 29698    11.50           8
## 588 29698   125.40          34
## 589 29706    18.90          15
## 590 29740    35.20           3
## 591 29755    51.80          10
## 592 29758   196.30          72
## 593 29768   236.50          38
## 594 29840   234.40         266
## 595 29878   176.20          32
## 596 30144    79.50          89
## 597 30176   373.15          34
## 598 30265   416.65          86
## 599 30299   244.00          26
## 600 30385    27.40           2
## 601 30411     0.05          39
## 602 30413    64.00          53
## 603 30452     8.90          21
## 604 30505    73.15           9
## 605 30526     9.05          46
## 606 30535   166.10          15
## 607 30581   457.75          98
## 608 30596   173.55          56
## 609 30694   136.50           1
## 610 30750    48.40           5
## 611 30751   455.25          85
## 612 30756    38.10          11
## 613 30841   254.10           0
## 614 30852   298.45          95
```

```
## 615 30852    60.55          66
## 616 30947   228.45           8
## 617 31013    20.20          46
## 618 31021    95.50          36
## 619 31067     4.15           3
## 620 31103     3.80          65
## 621 31106    78.10          85
## 622 31171   145.10          25
## 623 31256    68.45          65
## 624 31281    24.00           6
## 625 31346     5.20         118
## 626 31352    50.85         134
## 627 31470    50.00          85
## 628 31604    69.95           3
## 629 31689    56.85           6
## 630 31692     9.70           8
## 631 31698    71.25         279
## 632 31706   256.55          12
## 633 31985   125.05          65
## 634 31997   280.55          18
## 635 32062    23.65         186
## 636 32080   112.75          52
## 637 32266   101.70          58
## 638 32318     2.95          14
## 639 32376     1.50          NA
## 640 32390   370.70          NA
```

Q3

```
x<-CP_df$Purchase

nloglik<- function(x,theta) sum(-dexp(x=x,rate=theta,log=T))

optimize(f=nloglik,x=x,interval = c(0,1000))$minimum
```

```
## [1] 0.01074833
```

```
1/mean(x)
```

```
## [1] 0.01072986
```

Q4

Confidence intervals

Student’s t-confidence intervals

Q1

does not change wider narrower

Q2

```
x<-c(RedTailedDf$Weight)

m<-mean(x,na.rm = TRUE)
s<-sd(x,na.rm = TRUE)
l<-length(x)
```

```
error<- qt(0.99,df= l-1)*s/sqrt(l)

left<-m-error
right<- m+error
left
```

```
## [1] 1076.054
```

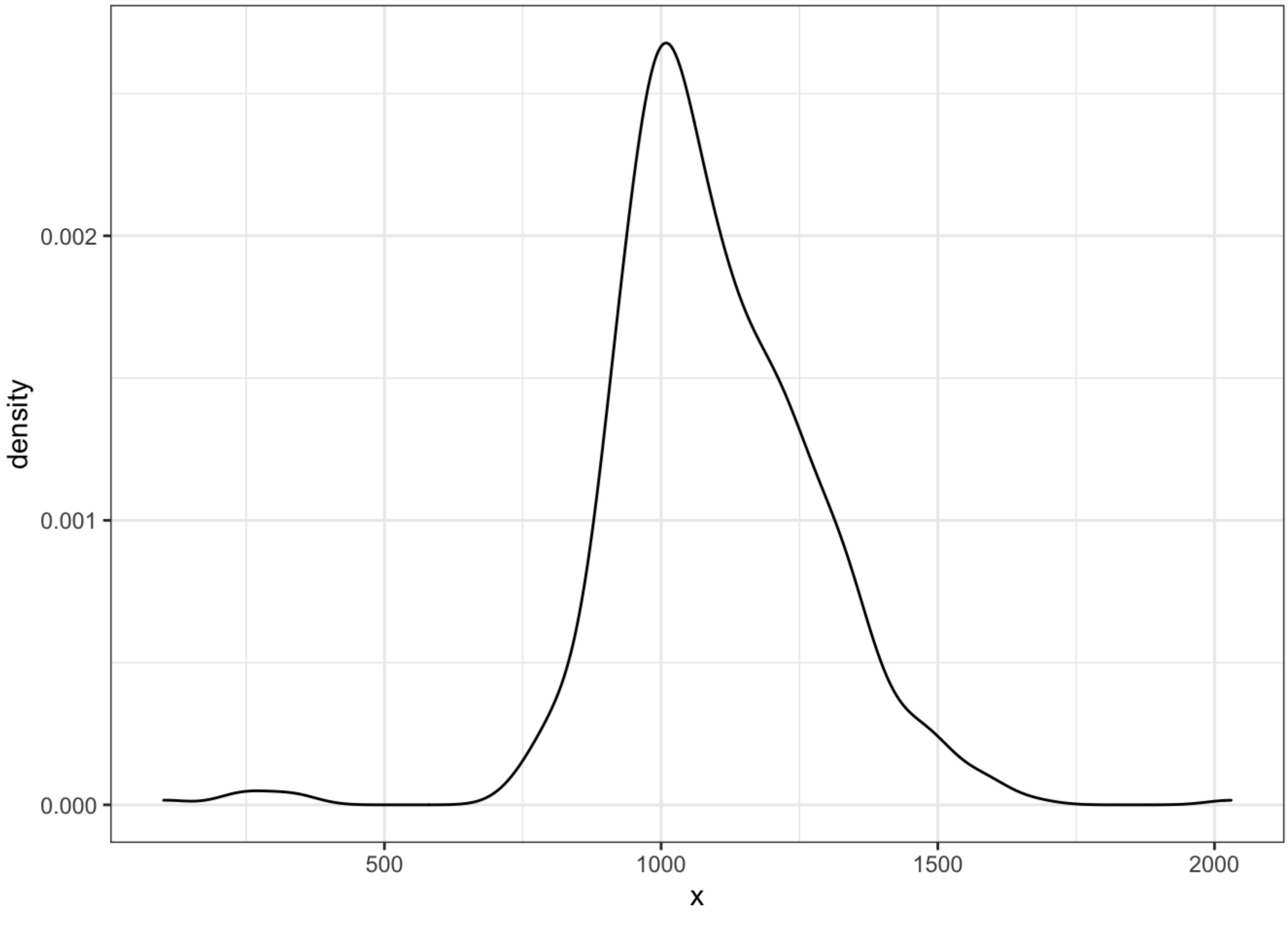
```
right
```

```
## [1] 1112.806
```

Q3

```
ggplot(data = RedTailedDf,aes(x=x)) + geom_density() + theme_bw()
```

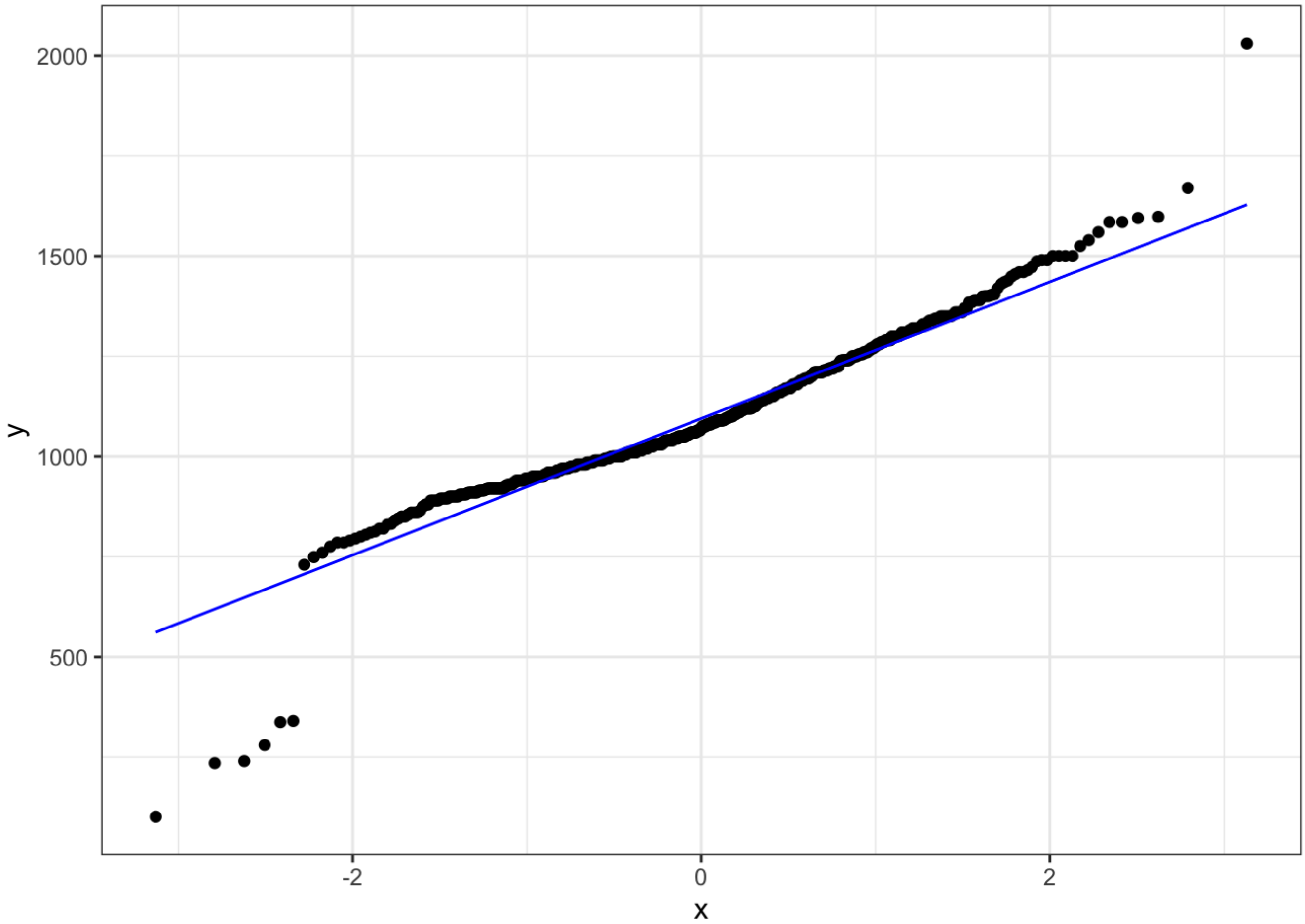
```
## Warning: Removed 5 rows containing non-finite values (stat_density).
```



```
ggplot(data = RedTailedDf,aes(sample=x)) +theme_bw() +stat_qq()+ stat_qq_line(color =
"blue")
```

```
## Warning: Removed 5 rows containing non-finite values (stat_qq).
```

```
## Warning: Removed 5 rows containing non-finite values (stat_qq_line).
```



Investigating coverage for Student's intervals

Q1

```
student_t_confidence_interval<-function(sample,confidence_level){
  sample<-sample[!is.na(sample)] #removeanymissingvalues
  n<-length(sample) #computesamplesize
  mu_est<-mean(sample) #computesamplemean
  sig_est<-sd(sample) #computesamplesd
  alpha=1-confidence_level #alphafromgamma
  t<-qt(1-alpha/2,df=n-1) #getstudenttquantile
  l=mu_est-(t/sqrt(n))*sig_est #lower
  u=mu_est+(t/sqrt(n))*sig_est #upper
  return(c(l,u)) }
```

```
num_trials<-100000
sample_size<-30
mu_0<-1
sigma_0<-3
alpha<-0.05
set.seed(0) #setrandomseedforreproducibility


single_alpha_coverage_simulation_df<-data.frame(trial=seq(num_trials))%>%
  #generaterandomGaussiansamples:
  mutate(sample=map(.x=trial,.f=~rnorm(n=sample_size,mean=mu_0,sd=sigma_0)))%>%
  #generateconfidenceintervals:
  mutate(ci_interval=map(.x=sample,.f=~student_t_confidence_interval(.x,1-alpha)))%>%
  #checkifintervalcoversmu_0:
  mutate(cover=map_lgl(.x=ci_interval,.f=~((min(.x)<=mu_0)&(max(.x)>=mu_0))))%>%
  #computeintervallength:
  mutate(ci_length=map_dbl(.x=ci_interval,.f=~(max(.x)-min(.x))))



#estimateofcoverageprobability:
single_alpha_coverage_simulation_df%>%
  pull(cover)%>%
  mean()
```

```
## [1] 0.95003
```

One sample hypothesis testing

One sample t-test on penguins data

Q1

```
library(palmerpenguins)

data(package = 'palmerpenguins')

head(penguins)
```

```
## # A tibble: 6 × 8
##   species island    bill_length_mm bill_depth_mm flipper_l…¹ body_…² sex    year
##   <fct>   <fct>              <dbl>         <dbl>       <int>   <int> <fct> <int>
## 1 Adelie  Torgersen           39.1          18.7         181    3750 male   2007
## 2 Adelie  Torgersen           39.5          17.4         186    3800 fema…  2007
## 3 Adelie  Torgersen           40.3          18           195    3250 fema…  2007
## 4 Adelie  Torgersen           NA            NA            NA      NA <NA>   2007
## 5 Adelie  Torgersen           36.7          19.3         193    3450 fema…  2007
## 6 Adelie  Torgersen           39.3          20.6         190    3650 male   2007
## # … with abbreviated variable names ¹flipper_length_mm, ²body_mass_g
```

```
bill_adelie<-penguins%>%
  filter(species == 'Adelie')%>%
  select(bill_length_mm)
bill_adelie<-bill_adelie$bill_length_mm
```

```
sample_size<-length(bill_adelie)

sample_mean<-mean(bill_adelie,na.rm = TRUE)

sample_sd<-sd(bill_adelie,na.rm = TRUE)

test_statistic<-(sample_mean-40)/(sample_sd/sqrt(sample_size))


test_statistic
```

```
## [1] -5.594619
```

```
# compute the p-value
2*(1-pt(abs(test_statistic),df = sample_size-1))
```

```
## [1] 1.011578e-07
```

```
t.test(x=bill_adelie,mu = 40)
```

```
##
##  One Sample t-test
##
## data:  bill_adelie
## t = -5.5762, df = 150, p-value = 1.114e-07
## alternative hypothesis: true mean is not equal to 40
## 95 percent confidence interval:
##  38.36312 39.21966
## sample estimates:
## mean of x
##  38.79139
```

```
alpha<-0.01
t<-qt(1-alpha/2,df = sample_size-1)
confidence_interval_l<-sample_mean-t*sample_sd/sqrt(sample_size)
confidence_interval_u<-sample_mean+t*sample_sd/sqrt(sample_size)
confidence_interval<-c(confidence_interval_l,confidence_interval_u)


confidence_interval
```

```
## [1] 38.22781 39.35497
```

Implementing a one-sample t-test

Q1

```
t.test(x=bill_adelie,mu =40,alternative = "two.sided")
```

```
##
##  One Sample t-test
##
## data:  bill_adelie
## t = -5.5762, df = 150, p-value = 1.114e-07
## alternative hypothesis: true mean is not equal to 40
## 95 percent confidence interval:
##  38.36312 39.21966
## sample estimates:
## mean of x
##  38.79139
```