$$\begin{pmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{pmatrix} = \begin{pmatrix} b_{11} & b_{21} & \cdots & b_{n1} \\ b_{12} & & & \\ \vdots & & & \\ b_{1n} & & & b_{nn} \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{pmatrix}$$

Section B) For each $j \neq k$

$$Y_j = \sum_{i=1}^{n} b_{i,j} \cdot X_i = b_{1,j} \cdot X_1 + b_{2,j} \cdot X_2 + \cdots + b_{n,j} \cdot X_n$$

$$Y_k = \sum_{i=1}^{n} b_{i,k} \cdot X_i = b_{1,k} \cdot X_1 + b_{2,k} \cdot X_2 + \cdots + b_{n,k} \cdot X_n$$

Because we need $cor(Y_j, Y_k) = 0$, $\rho = \dfrac{cov(Y_j, Y_k)}{\sigma_j \sigma_k}$

We need $Cov(Y_j, Y_k) = 0$

$$\Rightarrow E(Y_j \cdot Y_k) - \cancel{E(Y_j)E(Y_k)}^{0} = 0$$

linear combination of X_i's with mean $= 0$

$$\Rightarrow E(Y_j \cdot Y_k) = 0$$

Y_k

$$\begin{aligned} E\Big(& b_{1,j} \cdot X_1 \cdot b_{1,k} \cdot X_1 + b_{1,j} \cdot X_1 \cdot b_{2,k} \cdot X_2 + \cdots + b_{1,j} X_1 \cdot b_{n,k} \cdot X_n \\ + \ & b_{2,j} \cdot X_2 \cdot b_{1k} \cdot X_1 + b_{2,j} \cdot X_2 \cdot b_{2,k} \cdot X_2 + \quad + \\ + \ & \vdots \\ + \ & b_{n,j} \cdot X_n \cdot b_{1k} \cdot X_1 + \quad + b_{n,j} X_n \cdot b_{n,k} \cdot X_n \Big) = 0 \end{aligned}$$

$$\begin{aligned} = \ & b_{1,j} b_{1,k} \cdot \underbrace{\cancel{E(X_1 \cdot X_1)}}_{C_{11}} + b_{1,j} b_{2,k} \underbrace{\cancel{E(X_1 \cdot X_2)}}_{C_{12}} + \cdots + b_{1,j} b_{n,k} \underbrace{\cancel{E(X_1 \cdot X_n)}}_{C_{1n}} \\ + \ & b_{2,j} b_{1,k} \cdot \underbrace{\cancel{E(X_2 \cdot X_1)}}_{C_{21}} + b_{2,j} b_{2,k} \underbrace{\cancel{E(X_2 X_2)}}_{C_{22}} + \cdots + b_{2,j} b_{n,k} \underbrace{\cancel{E(X_2 X_n)}}_{C_{2n}} \\ + \ & \vdots \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \vdots \quad C_{nn} \\ + \ & b_{n,j} b_{1,k} \underbrace{\cancel{E(X_n X_1)}}_{C_{n1}} + \quad \cdots \quad + b_{n,j} b_{n,k} \cancel{E(X_n X_n)} = 0 \end{aligned}$$

Since for each pair X_i, X_j $i \neq j$ $Cov(X_i, X_j) = E(X_i X_j) - \cancel{E(X_i)E(X_j)}^{0} = C_{ij}$

$$= (b_{1,j} \quad b_{2,j} \cdots b_{nj}) \left[C \cdot \begin{pmatrix} b_{1,k} \\ b_{2,k} \\ \vdots \\ b_{n,k} \end{pmatrix} \right] = 0 \quad \text{where } C = \begin{pmatrix} C_{11} & C_{12} & \cdots & C_{1n} \\ C_{21} & C_{22} & & \vdots \\ \vdots & & & \\ C_{n1} & \cdots & & C_{nn} \end{pmatrix}$$

(OR : For each pair Y_J, Y_K : $\sum_{\beta=1}^{n} \sum_{\alpha=1}^{n} b_{\alpha,J} b_{\beta,K} C_{\alpha,\beta} = 0.$)