Section A) Based on the info given by the problem, this option is a cash-or-nothing Call:

$$V(S,T) = \begin{cases} H & \text{if } S > X \\ 0 & \text{o/w} \end{cases}$$

Similar with the value of European Call under B-S Model, the value of the cash-or-nothing call is just the discounted expected payoff of the call, which should be

call : $\boxed{He^{-rT}N(d_2)}$ at the time of maturity T

where H is the constant payoff, r is the risk-free rate, $N(d_2)$ is the probability that $S>X$ at T.

$$N(d) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{d} e^{-\frac{x^2}{2}} dx$$

$$d_2 = \frac{\ln(S/X) + (r - \frac{\sigma^2}{2})T}{\sigma\sqrt{T}} \qquad d_1 = \frac{\ln(S/X) + (r - \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}$$

(Note: If the option is purchased at time $t \in [0,T)$, we replace T by $(T-t)$ in the formula.).

- Standard BS European Call Option:

$$C_X(S,t) = SN(d_1) - Xe^{-r(T-t)}N(d_2)$$

defined by

$$V(S,T) = \begin{cases} S-X & \text{if } S > X \\ 0 & \text{o/w} \end{cases}$$

Easily see that cash-or-nothing call is just the B-S call with $S=0$ $X=H$

Also, as we know there is another type of call, called Asset-or-nothing call, whose value is just BS call case with $X=0$.

Therefore, the standard BS Europ. Call can be considered as a portfolio:

buy one Asset-or-nothing Call AND sell one cash-or-nothing Call.

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Section B)

① For a portfolio of options, use an approxi. linear relationship :

$$\boxed{\Delta P \approx \sum_{i=1}^{n} S_i \delta_i \Delta X_i}$$

where $\delta_i = \dfrac{\Delta P}{\Delta S_i}$

$\Delta X_i = \dfrac{\Delta S_i}{S_i}$

P : Value of portfolio
n : # of assets
S_i : price of asset i
δ_i : delta of option i
ΔX_i : percentage change in asset i price in a day

In this problem,

- ΔP = delta of call $\times S_1 \times \Delta X_1$ + delta of put $\times S_2 \times \Delta X_2$

Under assumptions : B-S European Option & normal distribution of returns,

delta of call $= e^{-rt} N(d_1) = -0.589$

delta of put $= -e^{-rt} N(-d_1) = 0.284$

$\Rightarrow$

$$\Delta P = -29.45 \Delta X_1 + 5.68 \Delta X_2$$

$$Var(\Delta P) = 29.45^2 \underbrace{Var(\Delta X_1)}_{(\text{daily vola.})^2} + 5.68^2 \underbrace{Var(\Delta X_2)}_{0.0157^2} + 2 \times (-29.45)(5.68) \times \text{Correlation} \underbrace{\sqrt{Var(\Delta X_1) Var(\Delta X_2)}}$$

daily vola. = annum vol $/\sqrt{252} = 0.28/\sqrt{252} = 0.0176$

$= 0.239631$

$\Rightarrow$ Daily SD of the portfolio $= \sqrt{0.239631} = 0.48952$

10-day 99% : $\underbrace{N^{-1}(1-1\%)}_{\text{the 99th percentile is 2.33}} \times \sqrt{10} \times 0.48952 = 3.6068$ #

③ Need price histories for the two assets in the portfolio for an appropriate length of history.