There are two sections in this project.

Section A)

Consider the European digital option that pays a constant H if the stock price is above strike price X at maturity and zero otherwise. Assuming stock price S follows the following SDE under physical measure

$$\frac{dS}{S} = \mu dt + \sigma dB_t$$

Assuming the risk-free rate is constant r. Please write down the price of this option and explain how it is related to the price of the standard Black-Scholes European call option.

Section B)

A bank has written a call option on one stock and a put option on another stock. For the first option the stock price is 50, the strike price is 51, the volatility is 28% per annum, and the time to maturity is 9 months. For the second option the stock price is 20, the strike price is 19, and the volatility is 25% per annum, and the time to maturity is 1 year. Neither stock pays a dividend. The risk-free rate is 6% per annum, and the correlation between stock price returns is 0.4.

1) Please derive an approximate linear relationship between the change in the portfolio value and the change in the underlying stocks, and then estimate the 10-day 99% VaR based on this relation.

2) Using C/C++ or Java or Matlab to calculate the 10-day 99% Monte Carlo Simulation based VaR for the portfolio. Set the number of simulation to 5000.

3) What else data is required to calculate the 10-day 99% Historical based VaR for the portfolio?