Fixed Income Derivatives: Models & Strategies in Practice

Project 1 (Option on ED Futures) P&L Attribution

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Agenda

- One-day P&L
- Break-even daily volatility
- Gamma/Theta P&L
- Delta-Hedge
- Stub rate
- Discussions

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EDZ0: Dec 2020 Eurodollar futures contract settling on 12/14/20

- On 10/08/20
  - Trading price: $99.65
  - Stub rate (from 10/08 to 12/14): 0.30% (Act/360 basis)
  - “95 Put” (strike = 99.50) price: 5 ticks ($125.00)

- On 10/09/20
  - Trading price: $99.60
  - Stub rate (from 10/09 to 12/14): 0.35%
  - “95 Put” (strike = 99.50) price: 6.25 ticks ($156.25)

One-day P&L: 25%

One-Day P&L Explanations

- Movement of contract price: 5 bp
- Moneyness of put:
  - 10/08: 15 bp out of the money
  - 10/09: 10 bp out of the money
- Forward rate: (100 - 99.65) / 100 = 0.35%
- Strike: (100 - 99.50) / 100 = 0.50%

One-Day P&L Explanations Cont.

$$\$125 = \frac{1}{1+0.30\% * \sqrt{67/365}} * 1mm * \sigma \sqrt{\frac{67}{365}} [N'(d) + dN(d)] * \frac{90}{360}$$

$$d = \frac{F-K}{\sigma\sqrt{t}} = \frac{0.35\% - 0.50\%}{\sigma\sqrt{\frac{67}{365}}}$$

- Implied volatility = 0.638% or 63.8bp
- Delta = 0.2977
- Contribution to price: 1.4883 ticks

Break-even Daily Volatility

- Given the price (5 cents), what volatility for the forward rate would need to produce the price of 5 cents
- By borrowing price (5 cents), how much you want to move to adjust for hedging
- How much option price change due to underlying & delta changing

$$\sigma_{IV_{daily-break-even}} = \frac{\sigma_{IV_{annum}}}{\sqrt{252}}$$

In [51]:
```python
def call_sigma(s):
    d=(F-K)/(s*np.sqrt(t))
    p=np.exp(-r*t) * 1000000 *s*np.sqrt(t)*(norm.pdf(d)+d*norm.cdf(d))*90/360
    return p-p0

F=(100-99.65)/100
K=(100-99.5)/100
t=67/365
r=0.003
p0=5*25
implied_oct8=fsolve(call_sigma,0.0068)[0]

even_break_oct8=(lambda x:x*np.sqrt(1/252))(implied_oct8)
print('Oct8 Implied Vol =',implied_oct8, 'and Oct 8 even-break Vol =',even_break_oct8)
```

```
Oct8 Implied Vol = 0.006377233300602508 and Oct 8 even-break Vol = 0.0004017279372865203
```

In [52]:
```python
F=(100-99.60)/100
K=(100-99.5)/100
t=66/365
r=0.0035
p0=6.25*25
implied_oct9=fsolve(call_sigma,0.0068)[0]


even_break_oct9=(lambda x:x*np.sqrt(1/252))(implied_oct9)
print('Oct9 Implied Vol =',implied_oct9, 'and Oct 9 even-break Vol =',even_break_oct9)
```

```
Oct9 Implied Vol = 0.006192681317977953 and Oct 9 even-break Vol = 0.0003901022551439373
```

Break-even Daily Volatility on Oct 8th

- Implied volatility = 0.638%
- Break-even daily volatility = implied_volatility * sqrt(1/252)

  = 0.638% * sqrt(1/252) = 0.0401%
- Realized volatility = price_change = forward_rate_difference

  = 0.4% - 0.35% = 0.05%
- Break-even daily volatility 4bp < Realized volatility 5bp

Greeks, P&L Attribution?

Calculation Code

By definition of Greeks & Taylor Expansion,

$$\triangle P = PnL = \delta(\triangle F) + \frac{1}{2}\gamma(\triangle F)^2 + \theta(\triangle T) + \nu(\triangle \sigma) + \ldots$$

	delta	gamma	theta	vega	otherPnL	Realized PnL
bumping in Forward rate	+1bp	-1bp,+1bp	-1/365	+1bp	N\A	N\A
Greeks(tick)	0.2977	0.0126	-0.0701	0.1473	N\A	N\A
P&L(tick)	1.4883	0.1569	-0.0701	-0.2718	-0.0533	1.2500
pct on realized PnL	1.1906	0.1255	-0.0561	-0.2174	-0.0427	1.0000

Naked Option: delta PnL predominant

How about a delta-hedged option?

If delta-hedged, what would have been the hedge? What would have been the P&L?

- Hedge the option with underlying EDZ0
- Long 1 put option and long 0.297741 EDZ0
- P&L = ∑ price changes * position

  = Gamma P&L + Theta P&L + Vega P&L

Asset	Position	Price on Oct 8	Price on Oct 9	P&L ($)
EDZ0	0.297741	99.65%	99.60%	-5.96765
Put Option on EDZ0	1	5 ticks	6.25 ticks	

If delta-hedged, we should expect a positive PnL in our portfolio. Because Gamma PnL > Theta PnL in absolute values.

https://quant.stackexchange.com/questions/39619/gamma-pnl-vs-vega-pnl

Consider the delta neutral portfolio $\Pi = C - \frac{\partial C}{\partial S} S$. Assuming that the interest rate and volatility are not change during the small time period Δt. The P&L of the portfolio is given by

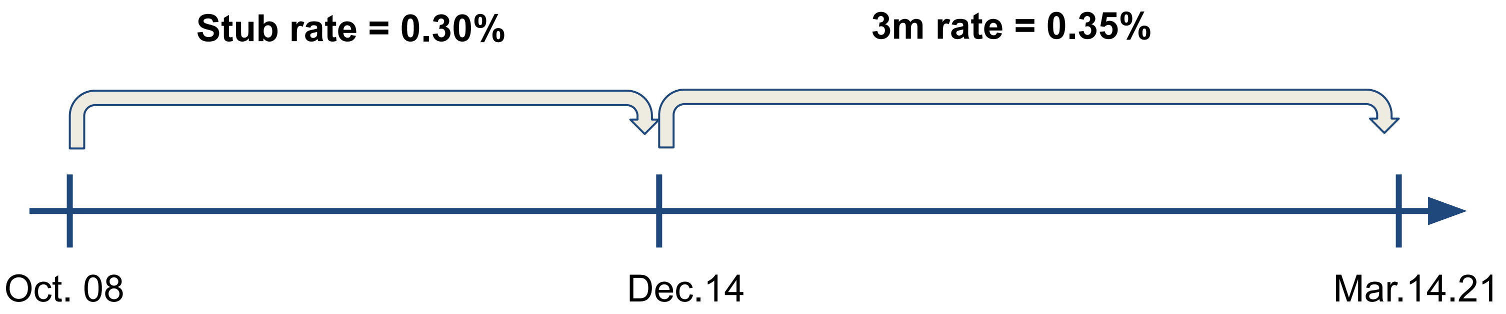
$$P\&L^{\Pi}_{\Delta t} = \frac{1}{2}\gamma(\Delta S)^2 + \theta \Delta t,$$

where θ is the theta hedge ratio. For small interest rate, which we assume to be zero, $\theta \approx -\frac{1}{2}\gamma S^2\sigma^2$ and $\gamma = \frac{\nu}{S^2\sigma T}$; see, for example, Black–Scholes model. Then

$$\begin{aligned} P\&L^{\Pi}_{\Delta t} &\approx \frac{1}{2}\gamma S^2 \frac{1}{\Delta t}\left(\frac{\Delta S}{S}\right)^2 \Delta t - \frac{1}{2}\gamma S^2\sigma^2\Delta t \\ &\approx \frac{1}{2}\gamma S^2 \sigma_{Re}^2 \Delta t - \frac{1}{2}\gamma S^2 \sigma^2 \Delta t \end{aligned}$$

Stub rate:

- On 10/08/20
  - Trading price: \$99.65
  - Stub rate (from 10/08 to 12/14): 0.30% (Act/360 basis)
  - “95 Put” (strike = 99.50) price: \$125.00
- On 10/09/20
  - Trading price: \$99.60
  - Stub rate (from 10/09 to 12/14): 0.35%
  - “95 Put” price: \$156.25



Put(0) = D(Dec.14) E [max(K-S , 0)]

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Discussion

Assume that we like our position and think it has more to give, but want to take some profits. What would we do?

Solution 1: Partly delta hedge

- If things keep going better, then we will reduce the delta-hedge position. If delta brings benefits, then embrace it
- In detail, if forward rates go up everyday, then reduce the position for delta hedging or don't even do any hedge

Example

- Assume forward contract will go down for 2 bp every day
- Compare partly delta hedged portfolio with fully hedged portfolio

Underlying asset changes over 5 days

	Oct.12th	Oct.13th	Oct.14th	Oct.15th	Oct.16th
option price	239.57	261.71	285.34	310.51	337.24
forward rate	0.0046	0.0048	0.005	0.0052	0.0054
delta	0.3772	0.4073	0.438	0.4689	0.4999

Partly hedged vs Fully hedged:

position of future(fully hedged)	0.3772	0.4073	0.4380	0.4689	0.4999
position of future(half hedged)	0.1886	0.2037	0.219	0.2344	0.2499

Conclusion

P&L of fully hedged portfolio	NA	1.77	1.74	1.73	1.73
P&L of partly hedged portfolio	NA	11.95	12.69	13.45	14.23

- If the market acts just as we expect, we can take more benefits by exposing more to delta

What if there is undesired change in the market?

Solution 2: Construct a straddle to hedge

- On Oct 8^{th}, implied volatility of **the call** is **63.8bp** and its price is for **$125**. Using Black Normal Formulae for put price, **the put** is for **$498.88** on Oct 8^{th}

- On Oct 9^{th}, the price for **the call** reaches to **$156.25**, and we can calculate its implied volatility is **62.0bp** now. Therefore, **the put** is for **$405.38** on Oct 9^{th}

- If buying **a call** on Oct 8^{th} and **a put** on Oct 9^{th}, total cost is $125 + $405.38 = **$530.38**. Then, we calculate the expected excess return of this straddle

Date /Price	Oct 8th	Oct 9th
call	\$125	\$156.25
put	\$498.88	\$405.38
total	\$623.88	\$561.63

Cost of our straddle (buy a call on Oct 8th and a put on Oct 9th): \$530.38

Total price of a straddle constructed on Oct 8th and 9th are \$623.88 and \$561.63, respectively, which are also the market view on expected return of the straddle on Oct 8th and Oct 9th

If the market turns back to Oct 8th (the rate falls), then the excess return of our straddle is:

$$\$(623.88 - 530.38) * (1 + 0.30\%) = \$93.78$$

If the market stays with the situation on Oct 9th, then the excess return is:

$$\$(561.63 - 530.38) * (1 + 0.35\%) = \$31.36$$

Q & A

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Project 1 – P&L Attribution

On Thu, 8-Oct-2020: EDZ0, the Dec 2020 Eurodollar futures contract settling on Mon, 14-Dec-2020, is the "1st White, ED1" contract and is trading at 99.65. The "stub rate" (from 8-Oct-2019 to 14-Dec-2020) is 0.30% (Act/360 basis). You buy a "95 Put" (strike = 99.50) for 5 ticks. Expiration date of the put is the same as ED1's settlement date (14-Dec-2020).

The next day, Fri, 9-Oct-2020, the market has sold off by 5 bp and EDZ0 is now trading at 99.60. The stub rate (from 9-Oct-2020 to 14-Dec-2020) is now 0.35%. Your "95 Put" is now trading at 6.25 ticks.

Nice job: 1-day return of 25%! Who needs Tesla?

1) *Can you explain your one-day P&L?*

2) *On Thu, what was your break-even daily volatility? Was it realized?*

3) *Over the one-day horizon what was your Gamma P&L vs. Theta P&L?*

4) *If you had delta-hedged, what would have been the hedge? What would have been your P&L?*

Hint: A put on a Eurodollar contract is a call on its implied rate. Calculate the implied Normal volatility on Oct 8th (= 63.8 bp, second hint!). Calculate the option Greeks (Delta, Gamma, Vega, and Theta) using "bump and reval" on the same day. Solve for the implied Normal vol on Oct 9th, and relate the realized 1-day P&L to the expected P&L.

Discuss: Let's assume that you like your position and think it has more to give, but want to take some profit. What would you do?