Section E) Known: ① $\frac{\Delta S(t)}{S(t)} = \mu \Delta t + \sigma \Delta X$ where $\Delta X \sim N(0, \Delta t)$ ie. $dW(t)$

② $U(t) = \frac{1}{S(t)}$ exchange rate given

③ $\Delta f(S,t) = \frac{\partial f}{\partial S}\Delta S + \frac{\partial f}{\partial t}\Delta t + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2}\Delta t$ Ito's lemma

Let $f(S,t) = \frac{1}{S}$,

by ③ $\Delta(\frac{1}{S}) = -S^{-2}\Delta S + 0 + \frac{1}{2}\sigma^2 S^2 (2S^{-3})\Delta t$

$= -\frac{\Delta S}{S^2} + \sigma^2 \frac{1}{S}\Delta t$ (*)

$\frac{dU(t)}{U(t)} = \frac{d\frac{1}{S(t)}}{\frac{1}{S(t)}} = S(t) \cdot \Delta(\frac{1}{S(t)})$ by ②

$= S(t) \cdot (-\frac{\Delta S}{S^2} + \sigma^2 \frac{1}{S}\Delta t)$ (*)

$= -\frac{\Delta S}{S(t)} + \sigma^2 \Delta t$

$= -\mu \Delta t - \sigma \Delta X + \sigma^2 \Delta t$ by ①

$= (\sigma^2 - \mu)\Delta t - \sigma \Delta X$

ie. $\frac{dU(t)}{U(t)} = (\sigma^2 - \mu)dt - \sigma dW(t)$