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Fixed Income Derivatives: Models & Strategies in Practice

Project 1 (Option on ED Futures) P&L Attribution

Presenters: Zibin Zhen, Linglan Wang, Zhilin Liu, Yuhan Zhao, Wei-Han Huang, Chenhao Su, Shufan Zhang

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Agenda

- One-day P&L
- Break-even daily volatility
- Gamma/Theta P&L
- Delta-Hedge
- Stub rate
- Discussions

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EDZ0: Dec 2020 Eurodollar futures contract settling on 12/14/20

Options on Euro-dollar futures trade at Chicago Mercantile Exchange (CME) and are quoted in Euro-dollar ticks ($25 per contract). These options are treated as caplets/floorlets, using Black's formula to evaluate them.

- On Oct. 8
    - EDZ0 trading price: $99.65
    - Stub rate (from Oct. 8 to 12/14/20): 0.30% (Act/360 basis)
    - "95 Put" (strike = $99.50) price: 5 ticks ($125.00)

- On 10/09/20
    - EDZ0 trading price: $99.60
    - Stub rate (from 10/09 to 12/14/20): 0.35%
    - "95 Put" (strike = $99.50) price: 6.25 ticks ($156.25)

Note: A put on the future price is a call/cap on the implied underlying forward rate

In other words, when we analyze the ED put, we should use BS formula for call firstly, and then price the ED put. (Details in Jupyter)

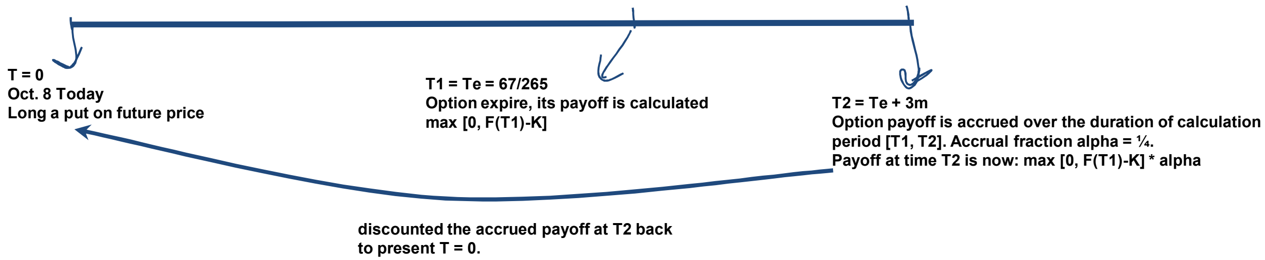
One-Day Movements

- Strike rate:
  - (100 - 99.50) / 100 = 0.50%
- Implied underlying forward rate:
  - Oct 8: (100 - 99.65) / 100 = 0.35%
  - Oct 9: (100 - 99.6) / 100 = 0.4%
- Movement of contract price: 99.60 - 99.65 = - 5 bp
- Movement of underlying rate: +5 bp
- Moneyness of put:
  - Oct 8: 99.5 - 99.65 = -15 bp, out of the money
  - Oct 9: 99.5 - 99.6 = -10 bp, out of the money

One-day P&L: (6.25-5) / 5 = 25%

Where does this come from? Before Greeks, volatility is still unknown.

Pricing options on ED Futures



Market practice is to use Black's formula to price options on futures.
In the case of a **put option on future price**:

For a given calculation period $[T_1, T_2]$, the risk-neutral formula for a **caplet on a rate** F is calculated as:

$$P(0) = D(r, T_2) \times \text{Notional Amount } \$1\text{mm} \times E[\max(0, F(T_1) - K)] \times \alpha(T_1, T_2)$$

and it is further assumed that forward rate is a tradable asset, hence **Black's call formula** is used to evaluate the E[.] term.

Implied Volatility

With this pricing formula, given option market price P(0), forward rate F(T1), strike K, time to expire Te, discounting rate "stub rate" r, use an optimizer in Python, we can obtain the Implied Volatility of the underlying forward rate.

Break-even Daily Volatility on Oct 8th

$$\sigma_{IV_{daily-break-even}} = \frac{\sigma_{IV_{annum}}}{\sqrt{252}}$$

- Implied volatility = 0.638%
- Break-even daily volatility = implied_volatility * sqrt(1/252)

  = 0.638% * sqrt(1/252) = 0.0401%

Realized Daily Volatility

- Realized one-day volatility = change in underlying forward rate

  = 0.4% - 0.35% = 0.05%

- Break-even daily volatility 4bp **<** Realized volatility 5bp

One-day PnL Explanation:

Betting on movements in Forward rate

As we buy a put for 5 ticks, the volatility for the forward rate that is required to produce this price is implied as 4bp. But we ended up with a 5bp realized volatility, which is more than required.

→ We are outperforming → Positive profit

Greeks, P&L Attribution?

By definition of Greeks & Taylor Expansion,

$$\triangle P = PnL = \delta(\triangle F) + \frac{1}{2}\gamma(\triangle F)^2 + \theta(\triangle T) + \nu(\triangle \sigma) + ...$$

	delta	gamma	theta	vega	otherPnL	Realized PnL
bumping in Forward rate	+1bp	-1bp,+1bp	-1/365	+1bp	N\A	N\A
Greeks(tick)	0.2977	0.0126	-0.0701	0.1473	N\A	N\A
P&L(tick)	1.4883	0.1569	-0.0701	-0.2718	-0.0533	1.2500
pct on realized PnL	1.1906	0.1255	-0.0561	-0.2174	-0.0427	1.0000

Naked Option: delta PnL predominant

How about a delta-hedged option?

If delta-hedged, we should expect a positive PnL in our portfolio. Because Gamma PnL > Theta PnL in absolute values.

https://quant.stackexchange.com/questions/39619/gamma-pnl-vs-vega-pnl

Consider the delta neutral portfolio $\Pi = C - \frac{\partial C}{\partial S} S$. Assuming that the interest rate and volatility are not change during the small time period Δt. The P&L of the portfolio is given by

$$P\&L_{\Delta t}^{\Pi} = \frac{1}{2}\gamma(\Delta S)^2 + \theta \Delta t,$$

where θ is the theta hedge ratio. For small interest rate, which we assume to be zero, $\theta \approx -\frac{1}{2}\gamma S^2 \sigma^2$ and $\gamma = \frac{\nu}{S^2 \sigma T}$; see, for example, Black–Scholes model. Then

$$\begin{aligned} P\&L_{\Delta t}^{\Pi} &\approx \frac{1}{2}\gamma S^2 \frac{1}{\Delta t}\left(\frac{\Delta S}{S}\right)^2 \Delta t - \frac{1}{2}\gamma S^2 \sigma^2 \Delta t \\ &\approx \frac{1}{2}\gamma S^2 \sigma_{Re}^2 \Delta t - \frac{1}{2}\gamma S^2 \sigma_{IV}^2 \Delta t \end{aligned}$$

implied vol Oct 8. = 4bp

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If delta-hedged, what would have been the hedge? What would have been the delta-neutral P&L?

delta-hedge: short delta unit of underlying and long a put

- Hedge the option with underlying EDZ0
- Long 1 put option and long 0.297741 EDZ0 (short Forward rate)
- P&L = Gamma P&L + Theta P&L + Vega P&L + Rho P%L + ...

Underlying	Position	Oct 8	Oct 9	P&L
Forward rate	-0.297741 ticks	35 bp	40 bp	-0.2387 ticks
Put Option on EDZ0	1	5 ticks	6.25 ticks	

Vega PnL:

The option we buy at 5 ticks goes from 15 OTM to 10 OTM, more close to have intrinsic value. → I should make money.

Exceptional case:

- I long a put for 5 ticks on first day
- market sold off on second day which is good to the put
- However, the put goes down in its value, now traded at 4 ticks only.

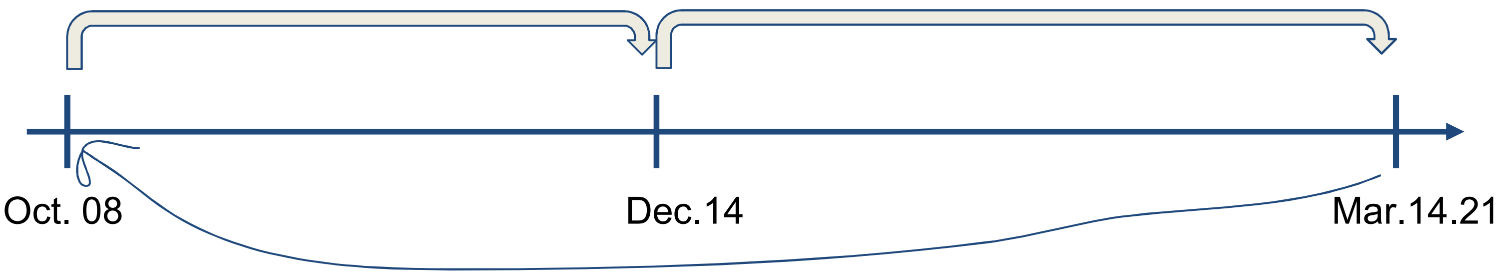
How?

- Implied vol of underlying rate on second day < implied vol on first day
- **Vega**: measure of sensitivity to implied vol

Rho PnL: Stub rate

- On 10/08/20
  - Trading price: $99.65
  - Stub rate (from 10/08 to 12/14): 0.30%
  - “95 Put” (strike = 99.50) price: $125.00
- On 10/09/20
  - Trading price: $99.60
  - Stub rate (from 10/09 to 12/14): 0.35%
  - “95 Put” price: $156.25

Stub rate = 0.30%



Put(0) = D(Dec.14) E [max(0 , F-K)]

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Discussion

Assume that we like our position and think it has more to give, but want to take some profits. What would we do?

Solution 1: Partly delta hedge

- If things keep going better, then we will reduce the delta-hedge position. If delta brings benefits, then embrace it
- In detail, if forward rates go up everyday, then reduce the position for delta hedging or don’t even do any hedge to keep the PnL come from delta exposure

Example

- Assume forward contract will go down for 2 bp every day
- Compare partly delta hedged portfolio with fully hedged portfolio

Underlying asset changes over 5 days

	Oct.12th	Oct.13th	Oct.14th	Oct.15th	Oct.16th
option price	239.57	261.71	285.34	310.51	337.24
forward rate	0.0046	0.0048	0.005	0.0052	0.0054
delta	0.3772	0.4073	0.438	0.4689	0.4999

Partly hedged vs Fully hedged:

position of future(fully hedged)	0.3772	0.4073	0.4380	0.4689	0.4999
position of future(half hedged)	0.1886	0.2037	0.219	0.2344	0.2499

Conclusion

P&L of fully hedged portfolio	NA	1.77	1.74	1.73	1.73
P&L of partly hedged portfolio	NA	11.95	12.69	13.45	14.23

- If the market acts just as we expect, we can take more benefits by exposing more to delta

What if there is undesired change (forward rate decreases) in the market?

Solution 2: Construct a straddle to hedge

- On Oct 8th, the **call (on rate)** gives an implied volatility is **63.8bp** and its price is for **$125**. Using Black Normal Formula for put price, **the put (on rate)** is for **$498.88** on Oct 8th

- On Oct 9th, the **call (on rate)** value reaches to **$156.25**, and we can calculate its implied volatility is **62.0bp** now. Therefore, **the put (on rate)** is for **$405.38** on Oct 9th

- If buying **call (on rate)** on Oct 8th and **the put (on rate)** on Oct 9th, total cost is $125 + $405.38 = **$530.38**. Then, we calculate the expected excess return of this straddle

Date /Price	Oct 8th	Oct 9th
call	$125	$156.25
put	$498.88	$405.38
total	$623.88	$561.63

Cost of our straddle (buy a call on Oct 8th and a put on Oct 9th): $530.38

Total price of a straddle constructed on Oct 8th and 9th are $623.88 and $561.63, respectively, which are also the market view on expected return of the straddle on Oct 8th and Oct 9th

If the market turns back to Oct 8th (the rate falls), then the excess return of our straddle is:

$$\$(623.88 - 530.38) * (1 + 0.30\%) = \$93.78$$

If the market stays with the situation on Oct 9th, then the excess return is:

$$\$(561.63 - 530.38) * (1 + 0.35\%) = \$31.36$$

Q & A