This project contains five sections.

Section A)

Assume that random variables X and$Y$ are normally distributed.

$$X \sim N(\mu_X, \sigma_X^2)$$

$$Y \sim N(\mu_Y, \sigma_Y^2)$$

The correlation between $X$and$Y$ is ρ. How can you choose constants a and b such that you minimize the variance of the random variable sum $S=aX+bY$ under the constraints that a$+b=1$, $0 \leq a \leq 1$ and $0 \leq b \leq 1$?

Section B)

You are given a set of normally distributed random variables X_i $(i = 1,2,\cdots,n)$ with covariance matrix$C = c_{i,j}$, assuming all the X_i $(i = 1,2,\cdots,n)$ have 0 mean. Please outline an algorithm to transform $X_i, i = 1,2,\cdots,n$ into a new set of random variables Y_i $(i = 1,2,\cdots,n)$ using linear transformation, i.e, please find coefficients $b_{i,j}$ such that

$$Y_j = \sum_{i=1}^{n} b_{i,j} X_i \, , j = 1,2,\cdots n$$

Where Y_i and Y_j have 0 correlation for every $i,j = 1,2,\cdots n, i \neq j$.

Section C)

Write a Matlab or R function to find the square root of 300 using Newton-Raphson method. Also write a Matlab or R function to find the value of “x” in following equation (Using Newton-Raphson method).

$$x^3 e^{-x^2} = 0$$

Section D)

Write a Matlab or R function to compute the value of an American styled option written on non-dividend paying stock. Use the Monte Carlo methodology for pricing the option. Assume required data.

Section E)

Let S(t) be the price of dollar at time t, i.e. the number of euros per dollar. The behavior of S(t) through time is modeled by $\frac{dS(t)}{S(t)} = \mu dt + \sigma dW(t)$ for a standard Brownian motion and real value μ and $\sigma > 0$. Now, let $U(t) = \frac{1}{S(t)}$ be the exchange rate of euro against the dollar. Show that U(t) satisfies the following stochastic differential equation.

$$\frac{dU(t)}{U(t)} = (\sigma^2 - \mu)dt - \sigma dW(t)$$