What do we mean Gamma PnL? That is the 2^{nd} contribution to the change of value of the derivative, the put. One half gamma times the change of underlying squared. This term, for gamma, always positive if we are saying I am the owner of the option. And we also know what theta is, the one-day worth time-delay, so the theta PnL is always negative. And what was suggesting here is that the gamma contribution should be a little bit higher than the positive part of theta. And we suggest here, because the contract moved 5bp from 99.65 (0.35%) to 99.60 (0.4%), and the implied break even daily vol is about 4bp, then the gamma pnl should be a little bit greater than the positive part of theta pnl.

Let's consider an example:

What you are paying now, is not 5ticks=5cent%=0.05%=5bps for the option but the vol you paid is 15bp for the option. Now what is your daily move required? – break-even daily vol 7bp. Which is 7 bp per day movement. If you but the option at 15, and say one day goes by, the contract moves only 5, then I underperformed, didn't move enough for the time decay to be offset by the gamma. But Could you pay more for the option? You brought in to an argument about more movement is supposed to happen. For the first day, the amount of movement didn't happen, then for that day, if you were delta hedged, then what you should find is you will loss money because your first order term will be canceled, but you would not make it enough from long gamma to offset the decay of the option value due to be one-day shorter.

Implied vol: how much does the underlying need to move day by day to justify what you paid for the option.

In our problem, the option, when you buy it, it out of the money (underlying contract price 99.65 > strike price 99.50~~, or put on price is call on rate, trading time forward rate 0.35% < strike rate 0.5%)~~, so the underlying doesn't move at all, you will loss all your premium cuz you could never finish below the strike price. But the point is you will be delta-hedging this, it be better you are gonna sit there and do nothing, and hope after 67 days, the contract is below the strike 99.50. so you really want to is somehow control the daily violability. You kind of like sit there and pay 5 cents buy the option and nothing in 67days, the contract finish below 99.50. you are trying to control the daily movement. That is what delta-hedging is doing, you are controlling the daily violability. You are pay for a certain amount of variability. How much did you pay for it? You paid 5 cents, the price, which is the amount to 4bp a daily movement. So the principal is, if the daily movement average is more than 4bp over the life of the option, you should make money; if the average is less than 4, you should loss money. This is not exactly true for some reason, but it is the general spear of it. Now for one day period, how much movement did the implied vol says what we are suppose to get? And that is simply 4bps. So you get the option price, you get the implied vol, express it in daily basis, and ask yourself: did the contract price move for that one day more or less than the daily implied vol. in our example, it did. The contract moves 5 with the implied vol on which we buy the option being only 4. I mean the contract move is intrinsic, it happens, while the implied daily movement we are pricing in depends on what we paid for the option. The more we paid for the option, the more we are betting on more daily movement in the underlying.

The option we buy at 5 cents goes from 15 OTM to 10 OTM. I should make money. Go to the exchange, saying hey the put I brought yesterday at 5 cents. What is its trading price today? They said, sorry, the best bid is 4 cents. That is possible, but how it could be 4 cents? I brought it at 5 cents with 15 OTM, now it is only one day shorter. Yesterday the contract is sold off by 5 from 99.65 to 99.6, why the put is only worth 4 cent now? what is the answer, mathematically? It means implied volatility from the second day is less than the implied vol I paid for on the first day. Which can be true! It is possible that You can be long a put, and the market sold off which is good to the put, but the put goes down in its value. Why? Because the market price of the put now has a lower implied vol than the one when you brought it. And what measure that sensitivity to implied vol? it is Vega. The 1st derivative of option price over implied vol. so now when we go to the problem, we should take a look of that. We are sot of saying, let s assume that the same implied vol, better to say that, we know what is the option price is.

For example, let's say, the option price goes crazily from 5 ticks to 11 ticks, and I know what delta was, what gamma was, what time-decay was, how could be when the put went from 5 to 11 over 1 day when the contract went down only by 5? The mathematical answer is, the implied vol that is trading is at a crazy amount. Now if you want to explain that PnL, you have po2 the Vega. The contract is expressed in price, but what we are modeling is the rate, one hundred minus the price. When we say the vol of the option when it is trading at 5 cents, the implied vol is 63 bp a year, or 4bp today, I don't mean the vol of the price, I mean the vol of the yield(rate), is 100 – price. That is what we have the Vega yield. It is the yield model, the random variable is the yield. But the contract is quoted in price. So when I say over one year, the standard deviation of the contract is 15bp, in normal black model, if the price has 15 standard deviation, so does the 100-price, the yield/rate also have std 15 bp.

1) Delta term times -5bp (from 99.65/0.35% to 99.6/0.4%) = move of the contract times the delta of the option. Theta term: short the time by 1 day, how much the option price goes down. Then the gamma term simply the 2nd derivative. These 3 terms together should explain the 1.25 cents increase in the option price. Most of it is explain by the delta, since we have see the delta is about minus one quarter 【(6.25-5)ticks/(-5bp) = -0.25ticks/per bp??】 And minus one quarter times 5 is exactly 1.25 ticks. So the most of the pnl is explained by the delta. Which is not surprising, usually it is the case. If there is a huge huge move in the underlying, then the gamma will come to be important. But for a small move, when you squaring the change, the gamma pnl will be really small.
2) Break even

3) implied vols, and mentioned about the relationship between break even daily vol and our realized vol. That is sort of an important for us to see if we can really make money through the option or say delta-hedge option we brought on Oct8.

Before we step into the PnL of delta-hedge portfolio, we want to firstly study on the PnL sources under a naked option scenario. And Here comes our Greeks!
On this slide, it is quite straightforward. This table contained Greeks and their corresponding pnL we calculated for the option we brought today, Oct8.
So, from this equation, we can easily see for a naked option, our profit mainly comes from four terms, which are delta, gamma, theta, and Vega.
In Greek calculation, for example, the delta, we bump the forward rate on Oct8 by +1bp and revaluate the option price, take the difference and this gives us the delta value.

Also we know what is the forward rate on the second day oct 9, so we can then calculated the realized PnL for each of the Greeks on Oct 8.
For example, the Gamma pl, we computed the gamma firstly, then one half gamma times the change of the underlying squared gonna be our gamma PnL. And Similarly with others.

And looks at the bottom line here in the table, obviously, the deltaPnL predominant the total PnL, this is for the naked option scenario as mentioned before.

But how about in a more reality setting, saying we are more intended to explore the PnL of a delta-hedged option instead of a naked option. Next, we will show how we would do the delta hedging and what PnL we'll obtain from that strategy.

In this example, we would expect gamma is a little bit more positive than theta is in negative. Because the contract movement in reality is 5 versus the impl vol is only 4. So we expect marginally that the gamma PnL as a positive number is little bit greater than the theta pnl term.

Long put on contract price = long call on rate

Gamma why always positive

Theta why always negative