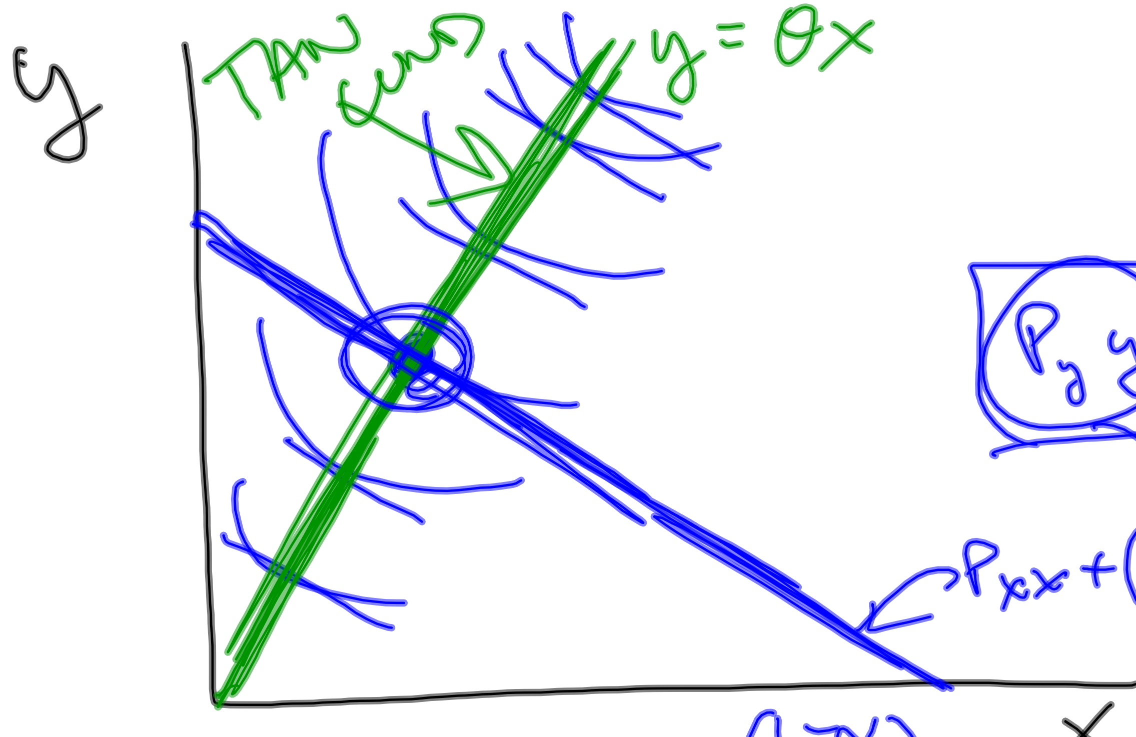
$$\mathcal{L}(x,y,\lambda) = \underbrace{\alpha \ln x + (1-\alpha)\ln y}_{u(x,y)} + \lambda(I - P_x x - P_y y)$$

$$\frac{\partial \mathcal{L}}{\partial x} = \underbrace{\frac{\alpha}{x}}_{MU_x} - \lambda P_x = 0 \Rightarrow \lambda = \frac{\alpha}{x} \cdot \frac{1}{P_x}$$

$$\frac{\partial \mathcal{L}}{\partial y} = \frac{1-\alpha}{y} - \lambda P_y = 0 \Rightarrow \lambda = \frac{1-\alpha}{y} \cdot \frac{1}{P_y}$$

MRS

$$\frac{\alpha}{1-\alpha}\frac{y}{x} = \frac{P_x}{P_y} \Rightarrow \boxed{\alpha P_y y = (1-\alpha) P_x x}$$



$$y = \underbrace{\frac{1-\alpha}{\alpha}\frac{P_x}{P_y}}_{\theta} x$$

$$P_y y = \frac{1-\alpha}{\alpha} P_x x$$

$$P_x x + P_y y = I$$

$$P_x x + \left(\frac{1-\alpha}{\alpha}\right) P_x x = I$$

$$\left(1 + \left(\frac{1-\alpha}{\alpha}\right)\right) P_x x = I$$

$$P_x x = \frac{I}{1 + \frac{1-\alpha}{\alpha}} \cdot \frac{\alpha}{\alpha} = \frac{\alpha I}{1}$$

\$ spent on X

$$P_x X = \alpha I$$

fract of I income

$$X = \alpha \frac{I}{P_x}$$

$$g^D(P_x, P_y, I) = \frac{\alpha I}{P_x}$$