Homework 2 Solutions

Econ 50 - Stanford University - Winter Quarter 2015/16

January 22, 2016

Exercise 1: *Based on Varian, Microeconomics, 8e, Problem 4.1*

A college football coach says that given any two linemen, A and B, he always prefers the one who is bigger and faster.

(a) Is this preference relation transitive? Why or why not?

(b) Is this preference relation complete? Why or why not?

(c) Is this preference relation monotonic? Why or why not?

Answer:

(a) Yes. A being bigger and faster than B and B being bigger and faster than C implies that A is bigger and faster than C. Therefore, $A \succ B, B \succ C \implies A \succ C$

(b) No. If A is bigger but B is faster, then the preference is not defined.

(c) Yes. More of either quality is better.

Exercise 2: Utility Functions (Lecture 5)

For each of the following utility functions, sketch the indifference curve $u(x, y) = 128$, and write the expressions for the partial derivatives $\frac{\partial u(x,y)}{\partial x}$ and $\frac{\partial u(x,y)}{\partial y}$ and the marginal rate of substitution.

(a) $u(x, y) = x^2 y^3$

(b) $u(x, y) = 2x + 4y$

(c) $u(x, y) = \min\{x, 2y\}$

(d) $u(x, y) = \sqrt{x} + \sqrt{y}$

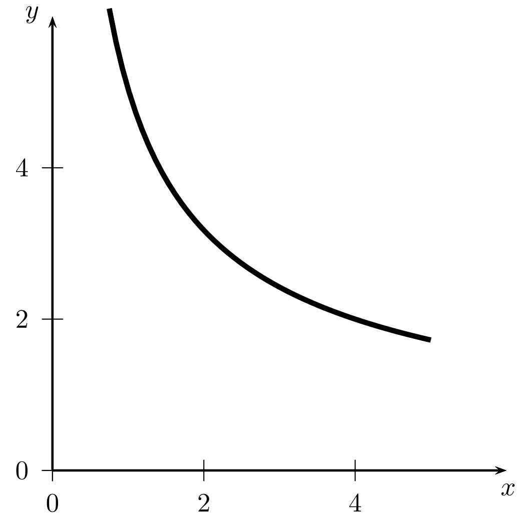
(e) $u(x, y) = x + \ln y$

(f) $u(x, y) = x^2 + y^2$

(g) $u(x, y) = x - y$

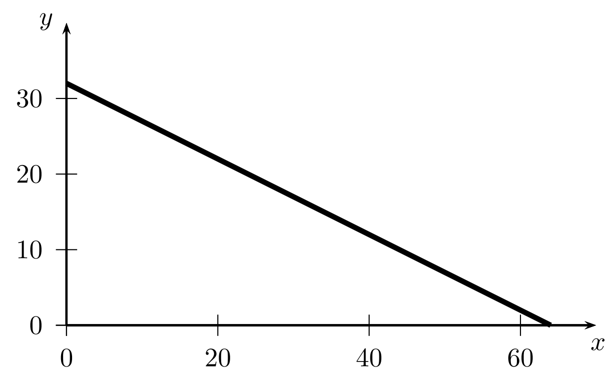
Answer:

(a)



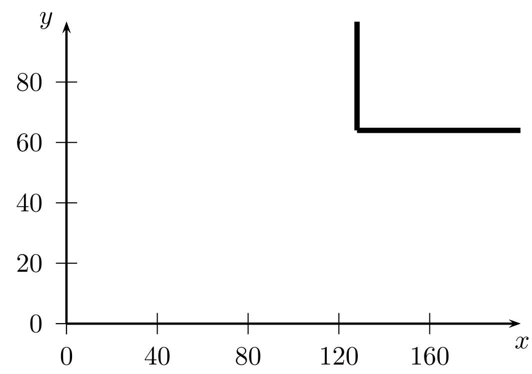
$\frac{\partial u}{\partial x} = 2xy^3, \frac{\partial u}{\partial y} = 3x^2y^2, \text{MRS} = \frac{2y}{3x}$.

(b)



$\frac{\partial u}{\partial x} = 2, \frac{\partial u}{\partial y} = 4, \text{MRS} = \frac{1}{2}$.

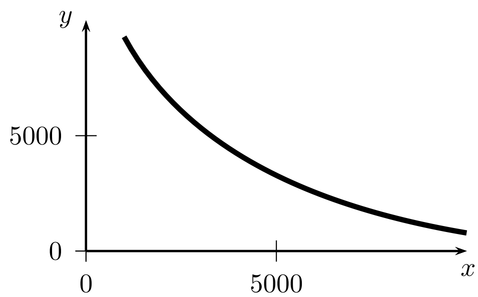
(c)



If $x < 2y$, $\frac{\partial u(x,y)}{\partial x} = 1$, $\frac{\partial u(x,y)}{\partial y} = 0$, and MRS $= \infty$. If $x > 2y$, $\frac{\partial u(x,y)}{\partial x} = 0$, $\frac{\partial u(x,y)}{\partial y} = 2$, and MRS

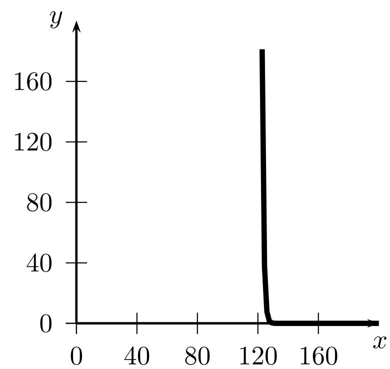
$= 0$. If $x = 2y$, the derivatives are undefined.

(d)



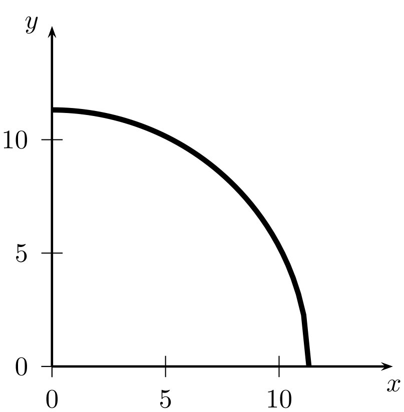
$\frac{\partial u}{\partial x} = \frac{1}{2\sqrt{x}}, \frac{\partial u}{\partial y} = \frac{1}{2\sqrt{y}}, \text{MRS} = \sqrt{\frac{y}{x}}$.

(e)



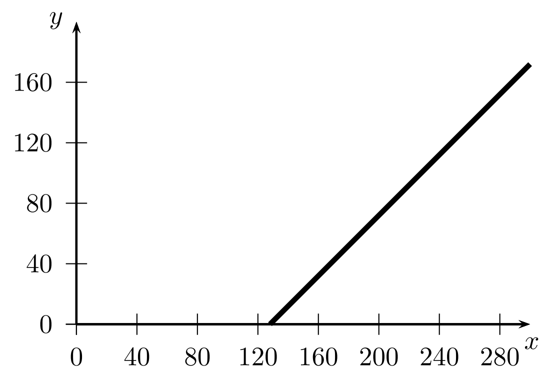
$\frac{\partial u}{\partial x} = 1, \frac{\partial u}{\partial y} = \frac{1}{y}, \text{MRS} = y$.

(f)



$\frac{\partial u}{\partial x} = 2x, \frac{\partial u}{\partial y} = 2y, \text{MRS} = \frac{x}{y}$.

(g)



$\frac{\partial u}{\partial x} = 1, \frac{\partial u}{\partial y} = -1, \text{MRS} = -1.$

Exercise 3: Preferences and Utility (Lectures 4 and 5)

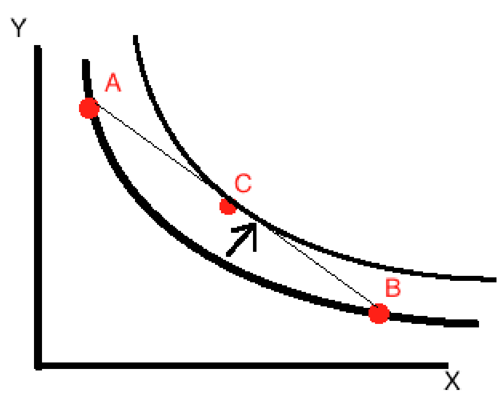
In Lecture 4, we said that a preference relation was **convex** if it satisfied the following condition: if A and B are on the same indifference curve, and C is a convex combination of A and B (i.e., lies along a line connecting A and B), then C is preferred to both A and B.

For each of the utility functions from Exercise 2, determine whether the preferences represented by that utility function is convex or not. In particular, for each function, choose points A, B, and C that fit the above definition, draw the relevant diagram showing those points, and show whether the utility at point C is higher than, equal to, or less than the utility at points A and B.

Answer:

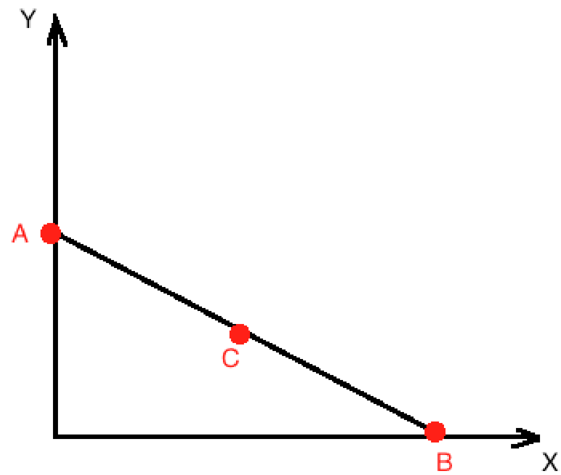
(a) $u(x, y) = x^2y^3$

As shown in the indifference curves below, the utility at point C is higher than the utility at A or B, as C is on an indifference curve with higher utility. The preference relation is convex.



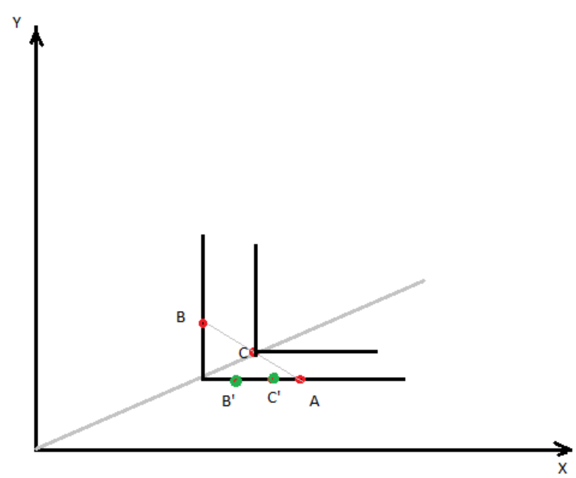
(b) $u(x, y) = 2x + 4y$

As shown in the indifference curve below, the utility at point C is the same as the utility at A or B, as the three points are on the same indifference curve. The utility function is not strictly convex (though it is weakly convex). In fact, it's a straight line and goods x and y are perfect substitutes.



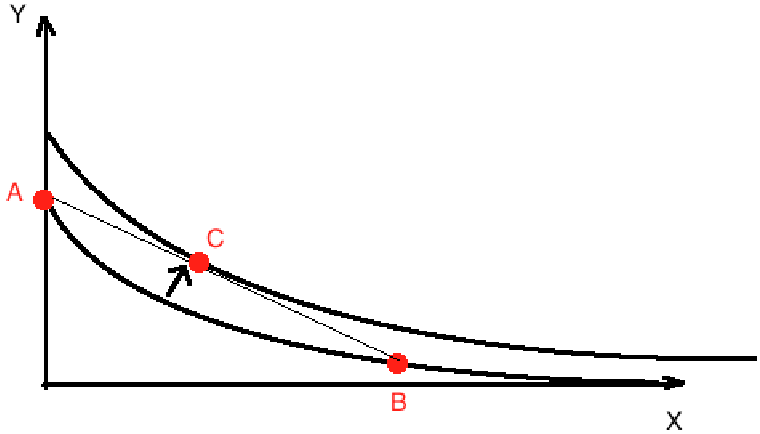
(c) $u(x, y) = \min(x, 2y)$

If we choose two points A and B on different sides of the grey line, the linear combination C has a higher utility. However, if you choose two points A and B' on the same sides of the grey line, the linear combination C has the same utility as A and B'. Therefore, the preference as a whole is not strictly convex. (It is weakly convex.)



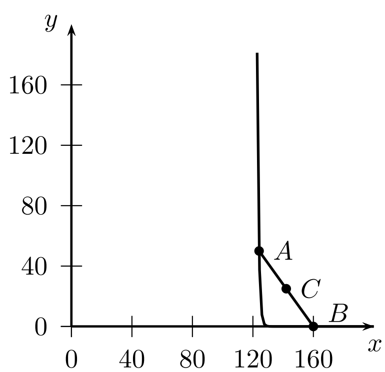
(d) $u(x, y) = \sqrt{x} + \sqrt{y}$

As shown in the indifference curves below, the utility at point C is higher than the utility at A or B, as C is on an indifference curve with higher utility. The utility function is convex.



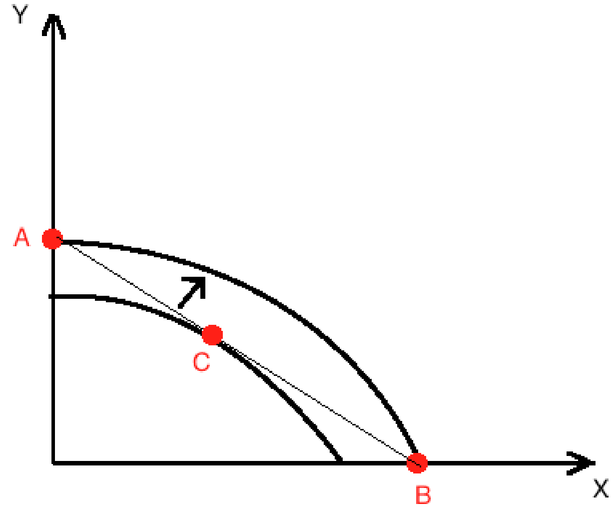
(e) $u(x, y) = x + \ln y$

As shown in the indifference curves below, the utility at point C is higher than the utility at A or B, as C is on an indifference curve with higher utility. The utility function is convex.



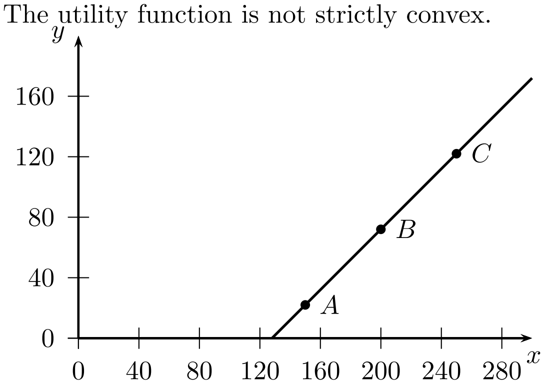
(f) $u(x, y) = x^2 + y^2$

As shown in the indifference curves below, the utility at point C is lower than the utility at A or B, as C is on an indifference curve with lower utility. The utility function is not convex, but concave.



(g) $u(x, y) = x - y$

As shown in the indifference curve below, the utility at point C is the same as the utility at A or B, as the three points are on the same indifference curve because the indifference curves are lines. The utility function is not strictly convex.



Exercise 4: Utility Functions (Lecture 5)

Which sets of the following utility functions represent the same preferences? How do you know?

(a) $u(x, y) = 3x + 4y$

(b) $u(x, y) = x^3 + y^4$

(c) $u(x, y) = 3 \ln x + 4 \ln y$

(d) $u(x, y) = x^3 y^4$

(e) $u(x, y) = x^{\frac{1}{3}} y^{\frac{1}{4}}$

(f) $u(x, y) = x^{\frac{3}{4}} y$

(g) $u(x, y) = \frac{3}{4}x + y$

(h) $u(x, y) = \frac{1}{3} \ln x + \frac{1}{4} \ln y$

Answer: Compute the MRS for each of the utility functions. Utility functions with the same MRS represent the same preferences.

(a) MRS $= \frac{3}{4}$

(b) MRS $= \frac{3x^2}{4y^3}$

(c) MRS $= \frac{3y}{4x}$

(d) MRS $= \frac{3y}{4x}$

(e) MRS $= \frac{4y}{3x}$

(f) MRS $= \frac{3y}{4x}$

(g) MRS $= \frac{3}{4}$

(h) MRS $= \frac{4y}{3x}$

So (a) and (g) represent the same preferences. Similarly, (c), (d), and (f) represent the same preferences. Finally, (e) and (h) represent the same preferences. No other utility function represents the same preferences as (b).

This problem could also be solved using monotonic transformations. If one utility function is a monotonic transformation of another, then the two utility functions represents the same preferences.

Exercise 5: Wow. I feel completely satisfied! (Lectures 4 and 5)

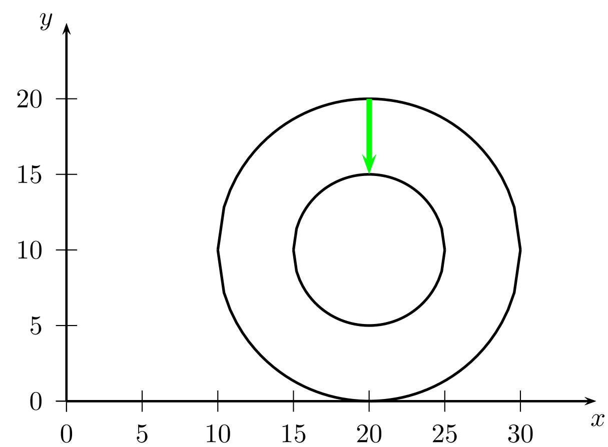
Suppose you only consume two goods: chocolate kisses (x) and peanuts (y). Your marginal utility of chocolate kisses is positive for the first **twenty** kisses you consume each day; beyond the twentieth kiss, though, any additional kisses will make you less and less happy. Similarly, you enjoy positive marginal utility for the first **ten ounces** of peanuts you consume each day, but negative marginal utility for any peanuts beyond that.

(a) Write down a utility function $U(x, y)$ that is consistent with these preferences; also sketch an indifference curve diagram, indicating with arrows the direction of "higher utility."

(b) Are your preferences convex? Explain intuitively, making reference to the indifference curves you drew in part (a).

Answer:

(a) $U(x, y) = -(x-20)^2 - (y-10)^2$. Other functions are possible but this is the easiest. Any function that correctly follows the instructions will be accepted.



(b) Yes, these preferences are convex. Note that the indifference curves are circles and that the direction of increasing utility is inwards. The convex combination of any two points on an indifference curve lie inside the circle, which is preferred.