Section 4 Solutions

Econ 50 - Stanford University - Winter Quarter 2015/16

Friday, February 5, 2016

Problem 1: Utility function deep dive: demand derivations and comparative statics

(From Midterm, Winter 2015)

Suppose Wilson's preferences over X and Y are summarized by the utility function

$$u(x, y) = (x^{-1} + y^{-1})^{-1}$$

As usual, he has a total of $\$I$ available to spend on X and Y at prices P_x and P_y per unit, respectively.

Last week, we found that Wilson's Marshallian demand functions are given by:

$$x^* = \frac{I}{P_x + \sqrt{P_x P_y}}$$

$$y^* = \frac{I}{P_y + \sqrt{P_x P_y}}$$

(a) Write down expressions for Wilson's **indirect utility function** $V(P_x, P_y, I)$ and his **expenditure function** $E(P_x, P_y, U)$.

Answer: Substitute x^* and y^* into the utility function:

$$V = \left(\left(\frac{I}{P_x + \sqrt{P_x P_y}}\right)^{-1} + \left(\frac{I}{P_y + \sqrt{P_x P_y}}\right)^{-1}\right)^{-1}$$

$$= \left(\frac{P_x + \sqrt{P_x P_y}}{I} + \frac{P_y + \sqrt{P_x P_y}}{I}\right)^{-1}$$

$$= \frac{I}{P_x + P_y + 2\sqrt{P_x P_y}}$$

The expenditure function is obtained by inverting V

$$I = V(P_x + P_y + 2\sqrt{P_x P_y}) \implies E = U(P_x + P_y + 2\sqrt{P_x P_y})$$

(b) Write down expressions for Wilson's **Hicksian demand functions**, $x^H(P_x, P_y, U)$ and $y^H(P_x, P_y, U)$.

Answer: Take the Marshallian demands x^* and y^* and substitute out the I for U using the expenditure function in part (b)

$$x^H = \frac{I}{P_x + \sqrt{P_x P_y}} = \frac{U(P_x + P_y + 2\sqrt{P_x P_y})}{P_x + \sqrt{P_x P_y}} = \frac{U(\sqrt{P_x} + \sqrt{P_y})^2}{\sqrt{P_x}(\sqrt{P_x} + \sqrt{P_y})} = \frac{U(\sqrt{P_x} + \sqrt{P_y})}{\sqrt{P_x}}$$

$$y^H = \frac{I}{P_y + \sqrt{P_x P_y}} = \frac{U(P_x + P_y + 2\sqrt{P_x P_y})}{P_y + \sqrt{P_x P_y}} = \frac{U(\sqrt{P_x} + \sqrt{P_y})^2}{\sqrt{P_y}(\sqrt{P_x} + \sqrt{P_y})} = \frac{U(\sqrt{P_x} + \sqrt{P_y})}{\sqrt{P_y}}$$

(c) Now assume Wilson's income is $I = \$288$ and the price of good Y is $P_y = \$1$ per unit. On a carefully drawn Slutsky diagram, show the effect of a price change from $P_x = 9$ to $P_x = 4$. Label your initial point A, the final point C, and the Slutsky decomposition point B. Clearly show the coordinates for those points, as well as the coordinates of the intercepts of all relevant budget lines. Recall that last week we found:
When $P_x = 9, (x^*, y^*) = (24, 72)$ and when $P_x = 4, (x^*, y^*) = (48, 96)$.

Answer: From last week: $A(24, 72)$ and $C(48, 96)$. The utility at A is

$$u(A) = \frac{24 \cdot 72}{24 + 72} = 18$$

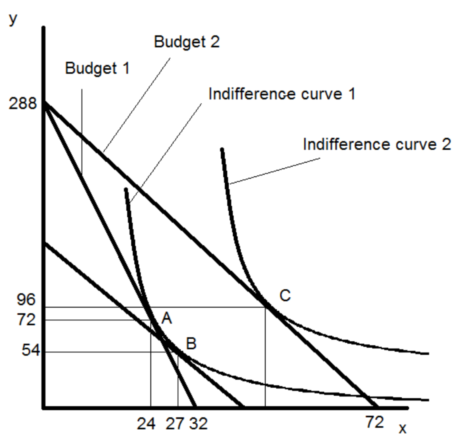
This is also $u(B)$. Using the Hicksians in Question 5, with new prices $P_x = 4$ and $P_y = 1$,

$$x^H = \frac{18(\sqrt{4} + \sqrt{1})}{\sqrt{4}} = 27, y^H = \frac{18(\sqrt{4} + \sqrt{1})}{\sqrt{1}} = 54$$

which are the coordinates for $B(27, 54)$. Note that even if you didn't compute the Hicksians in Question 5, you could get this directly using the tangency condition: we're looking for the point along the indifference curve $u(x, y) = 18$ where $MRS = 4$. Since $MRS = 4$ when $y = 2x$, you can plug $y = 2x$ into the utility function to obtain

$$u(x, 2x) = \frac{x \times 2x}{x + 2x} = \frac{2x^2}{3x} = \frac{2}{3}x.$$

Setting this equal to 18 gives us $x = 27$, so $y = 2x = 54$.



(d) Compute the **compensating variation** and **equivalent variation** for this price change.

Answer: The utility at C is

$$u(C) = \frac{48 \cdot 96}{48 + 96} = 32$$

Using the expenditure function from Question 5, CV is the difference in expenditure evaluated at the old utility of 18 while EV is the difference in expenditure evaluated at the new utility of 32.

$$CV = E(9, 1, 18) - E(4, 1, 18) = 18(9 + 1 + 2\sqrt{9 \cdot 1}) - 18(4 + 1 + 2\sqrt{4 \cdot 1}) = 126$$

$$EV = E(9, 1, 32) - E(4, 1, 32) = 32(9 + 1 + 2\sqrt{9 \cdot 1}) - 32(4 + 1 + 2\sqrt{4 \cdot 1}) = 224$$

As above, if you hadn't been able to find a good expenditure function, you could also compute the coordinates for the decomposition points directly from the tangency condition, and evaluate the expenditure required to purchase them at the appropriate prices.

(e) Illustrate the compensating variation in a diagram showing the relevant Marshallian and Hicksian demand curves.

Answer: Graph the Hicksian for the old utility of 18. The CV is the shaded area.