Comparative Statics II: Income and Substitution Effects

Econ 50 | Lecture 9 | February 2, 2016

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Lecture

- Income and Substitution Effects: Intuitive Review
- Analyzing a Price Change: Slutsky Decomposition
- Finding the Decomposition Point: The "Dual" Problem

Group Work

- Finding the Decomposition Point: Cobb-Douglas

Part I
Income and Substitution Effects: An Intuitive Review

What happens when the price of X rises?

- Tangency condition: $MRS_{x,y} = P_x/P_y$
  - X is now **relatively more expensive**
  - You will **substitute** Y for X
- Budget set: $P_x x + P_y y = I$
  - You can **no longer afford** to be as happy as you were before.
  - You will **buy fewer of both goods**, relative to…some point

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INC

Formal Definitions

- The **substitution effect** is the change in the quantity demanded resulting from a change **relative prices**, holding the **level of utility** constant
- The **income effect** is the change in the quantity demanded resulting from a change in **purchasing power**, holding **all prices** constant.

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Informal Definitions

Suppose the price of a good goes down. You could now afford to be **just as happy as you were before** (move **along** your indifference curve) by buying **more of that good** and **less of other goods** and save some money in the meantime. **SUBSTITUTION EFFECT**	*…but suppose you don't save the money.* You could spend that money to be **happier than you were before** (move to a **higher** indifference curve) by buying **more of one or both goods** (depending on whether they're normal or inferior goods) **INCOME EFFECT**

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Part II
Analyzing a Price Change: Slutsky Decomposition

Slutsky Decomposition Diagram

Point	Description	Utility	Price	Income
A	Initial Bundle	Initial Utility	Initial Price	Actual Income
B	"Decomposition" Bundle	Initial Utility	Final Price	Compensated Income
C	Final Bundle	Final Utility	Final Price	Actual Income

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$ needed to afford u_1 at new price

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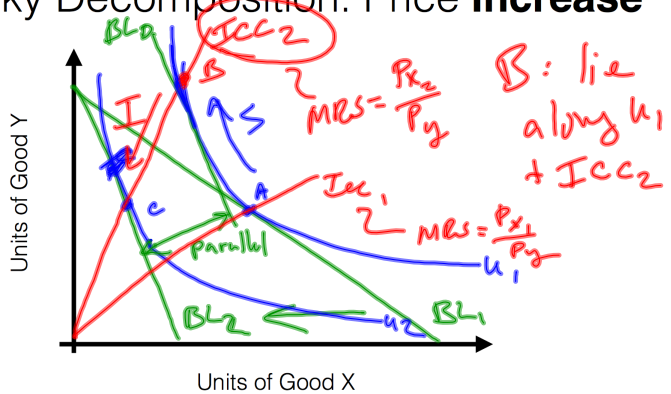
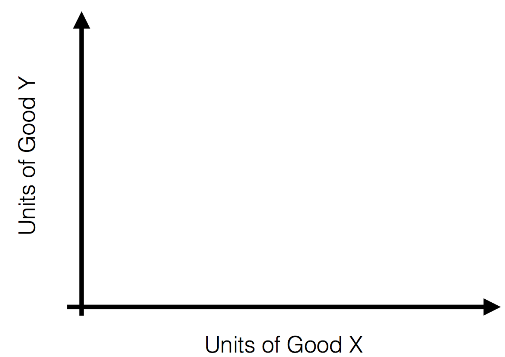
Slutsky Decomposition Diagram

	Initial Utility	Final Utility
Initial Price	Bundle **A** (Actual Income)	NEXT LECTURE
Final Price	Bundle **B** (Compensated Income)	Bundle **C** (Actual Income)

Slutsky Decomposition: Price **Increase**



Slutsky Decomposition: Price **Decrease**



Substitutes and Complements

- Suppose the price of good X changes.
- If the **substitution effect on Y dominates the income effect on Y**, then X and Y are substitutes and the **PCC is downward sloping**.
- If the **income effect on Y dominates the substitution effect**, then X and Y are complements and the **PCC is upward sloping**.
- If the **income effect on Y exactly offsets the substitution effect**, then X and Y are independent and the **PCC is horizontal**.

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Normal, Inferior, and Giffen Goods

- Consider just the income effect of an increase in the price of X.
- If both goods are **normal**, the **final point** will have **more of both** than the **decomposition point**.
- If one good is **inferior** in the relevant income range, the **final point** will have **less of the inferior good** than the **decomposition point**.
- If good X is a **Giffen** good in the relevant income range, the **final point** will have **less of good X** than the **initial point**.

Normal, Inferior, and Giffen Goods

- Consider just the income effect of ~~an in~~ a decrease in the price of X.
- If both goods are **normal**, the **final point** will have **more of both** than the **decomposition point**.
- If one good is **inferior** in the relevant income range, the **final point** will have **less of the inferior good** than the **decomposition point**.
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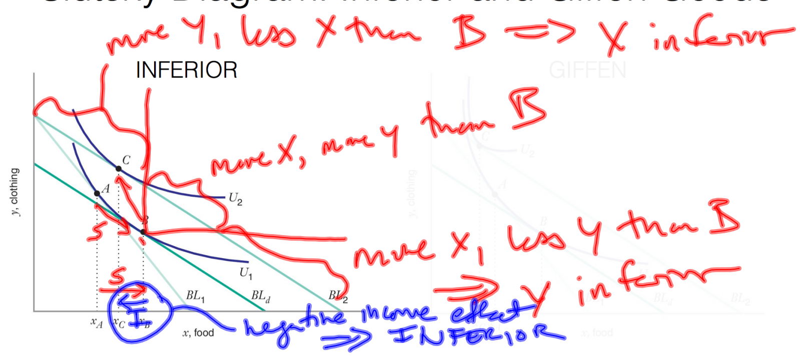
Slutsky Diagram: Inferior and Giffen Goods



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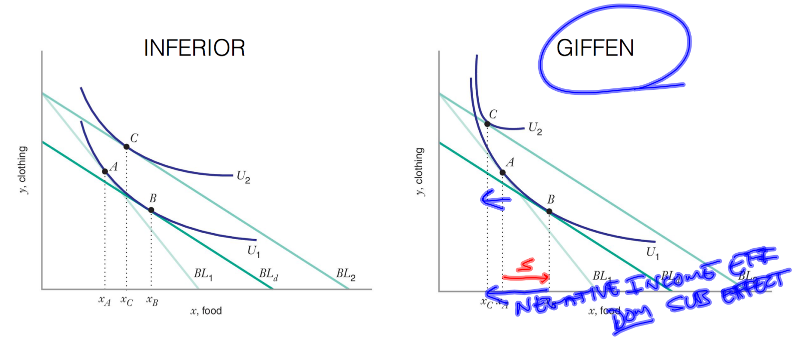

Part III
Finding the Decomposition Point
The "Dual" Problem

The **Dual Problem**: Two Equivalent Ways to Optimize

Utility Maximization	**Cost Minimization**
$\max_{x,y}\ u(x,y)$ s.t. $P_x x + P_y y = I$	$\min_{x,y}\ P_x x + P_y y$ s.t. $u(x,y) = U$

Solve for x^* and y^*; the solutions are:

Marshallian Demand Functions:	**Hicksian Demand Functions:**
$x^*(P_x, P_y, I), y^*(P_x, P_y, I)$	$x^*(U, P_x, P_y), y^*(U, P_x, P_y)$

Plug x^* and y^* back into the objective function:

Indirect Utility Function:	**Expenditure Function:**
$V(P_x, P_y, I) = u[x^*(P_x, P_y, I), y^*(P_x, P_y, I)]$	$E(U, P_x, P_y) = P_x x^*(U, P_x, P_y) + P_y y^*(U, P_x, P_y)$
(Utility from utility-maximizing choice)	(Cost of cost-minimizing choice)

The **Dual Problem**: Two Equivalent Ways to Optimize

Utility Maximization | **Cost Minimization**

$\max_{x,y}\ u(x,y)$ — OBJECTIVE FUNCTION (MAX)

s.t. $P_x x + P_y y = I$ — CONSTRAINT

$\min_{x,y}\ P_x x + P_y y$

s.t. $u(x,y) = U$

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Utility Maximization | **Cost Minimization**

EX VARS
P_x, P_y, I

$\max_{x,y}\ u(x,y)$

s.t. $P_x x + P_y y = I$

$\min_{x,y}\ P_x x + P_y y$

s.t. $u(x,y) = U$

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Two Equations Relating Utility and Income

Indirect Utility function:
$V(P_x,P_y,I)$

Expenditure function:
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Indirect Utility function:
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"How much utility
can I buy with income I?"

Expenditure function:
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Two Equations Relating Utility and Income

Indirect Utility function:
V(P_x,P_y,I)

measured in "utils"

"How much utility
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Expenditure function:
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Expenditure function:
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"How much money does it cost
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Two Equations Relating Utility and Income

Indirect Utility function:
Utils = **V(P_x,P_y,I)**
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Group Work

$$u(x, y) = xy$$

$$x^*(P_x, P_y, I) = \frac{I}{2P_x}$$

$$y^*(P_x, P_y, I) = \frac{I}{2P_y}$$

$$V(P_x, P_y, I) = u\left(\frac{I}{2P_x}, \frac{I}{2P_y}\right) = \frac{I^2}{4P_xP_y}$$

$$\boxed{U = \frac{I^2}{4P_xP_y}}$$ IND UTIL FN

$$I^2 = 4UP_xP_y$$

$$\boxed{I = 2\sqrt{UP_xP_y}}$$ EXP FN

How to Derive Hicksian Demand from Marshallian Demand

Example: Cobb-Douglas $u(x, y) = xy$

Start with Marshallian demand:	$x^* = \frac{I}{2P_x}, y^* = \frac{I}{2P_y}$
Plug (x^*, y^*) back into $u(x, y) = xy$ to find the indirect utility function:	$u(x^*, y^*) = \left(\frac{I}{2P_x}\right)\left(\frac{I}{2P_y}\right) \Rightarrow V(P_x, P_y, I) = \frac{I^2}{4P_xP_y}$
Set the indirect utility function equal to U and solve for I to find the expenditure function:	$\frac{I^2}{4P_xP_y} = U \Rightarrow I^2 = 4P_xP_yU$ $\Rightarrow E(P_x, P_y, U) = 2P_x^{\frac{1}{2}}P_y^{\frac{1}{2}}U^{\frac{1}{2}}$
Plug this "lowest cost" back into the Marshallian demand (as the income) to get the Hicksian demand:	$x^* = \frac{E(P_x, P_y, U)}{2P_x} = \frac{2P_x^{\frac{1}{2}}P_y^{\frac{1}{2}}U^{\frac{1}{2}}}{2P_x}$ $\Rightarrow x^H(P_x, P_y, U) = \left(\frac{P_y}{P_x}\right)^{\frac{1}{2}}U^{\frac{1}{2}}$

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Set the indirect utility function equal to U and solve for I to find the expenditure function:

$$\frac{I^2}{4P_xP_y} = U \Rightarrow I^2 = 4P_xP_yU$$
$$\Rightarrow E(P_x, P_y, U) = 2P_x^{\frac{1}{2}}P_y^{\frac{1}{2}}U^{\frac{1}{2}}$$

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Plug (x^*, y^*) back into $u(x,y) = xy$ to find the indirect utility function: $u(x^*, y^*) = \left(\frac{I}{2P_x}\right)\left(\frac{I}{2P_y}\right) \Rightarrow V(P_x, P_y, I) = \frac{I^2}{4P_xP_y}$

Set the indirect utility function equal to U and solve for I to find the expenditure function:

$$\frac{I^2}{4P_xP_y} = U \Rightarrow I^2 = 4P_xP_yU$$
$$\Rightarrow E(P_x, P_y, U) = 2P_x^{\frac{1}{2}}P_y^{\frac{1}{2}}U^{\frac{1}{2}}$$

Plug this "lowest cost" back into the Marshallian demand (as the income) to get the Hicksian demand:

$$x^* = \frac{E(P_x, P_y, U)}{2P_x} = \frac{2P_x^{\frac{1}{2}}P_y^{\frac{1}{2}}U^{\frac{1}{2}}}{2P_x}$$

How to Derive Hicksian Demand from Marshallian Demand

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$$\Rightarrow x^H(P_x, P_y, U) = \left(\frac{P_y}{P_x}\right)^{\frac{1}{2}}U^{\frac{1}{2}}$$

$$E(P_x, P_y, u) = 2\sqrt{P_x P_y u}$$

$$x^*(P_x, P_y, E(P_x, P_y, u)) = \frac{E(P_x, P_y, u)}{2P_x} = \frac{\cancel{2}\sqrt{P_x P_y u}}{\cancel{2}P_x} = \boxed{\sqrt{\frac{P_y}{P_x} u}} \equiv x^H(P_x, P_y, u)$$

ICC $\Rightarrow MRS_{xy} = \frac{P_x}{P_y}$

$\Downarrow$

$\frac{y}{x} = \frac{4}{1}$

$\boxed{y = 4x}$

$u_A = u(4;4) = 16$

$xy = 16$

$x \cdot 4x = 16$

$4x^2 = 16$

$x^2 = 4$

$\boxed{x = 2 \quad y = 4 \cdot 2 = 8}$