Inputs and Production Functions

Econ 50 | Lecture 12 | February 16, 2016

Lecture

- Overview of Producer Theory
- Production Functions
  with Two Inputs, In General:

  Isoquants & MRTS (6.3)
  Returns to Scale (6.5)
  Elasticity of Substitution (6.4)
  Technological Change (6.6)

- Deep Dive: Cobb-Douglas

Group Work

- Your group is a team;
  analyze its production function

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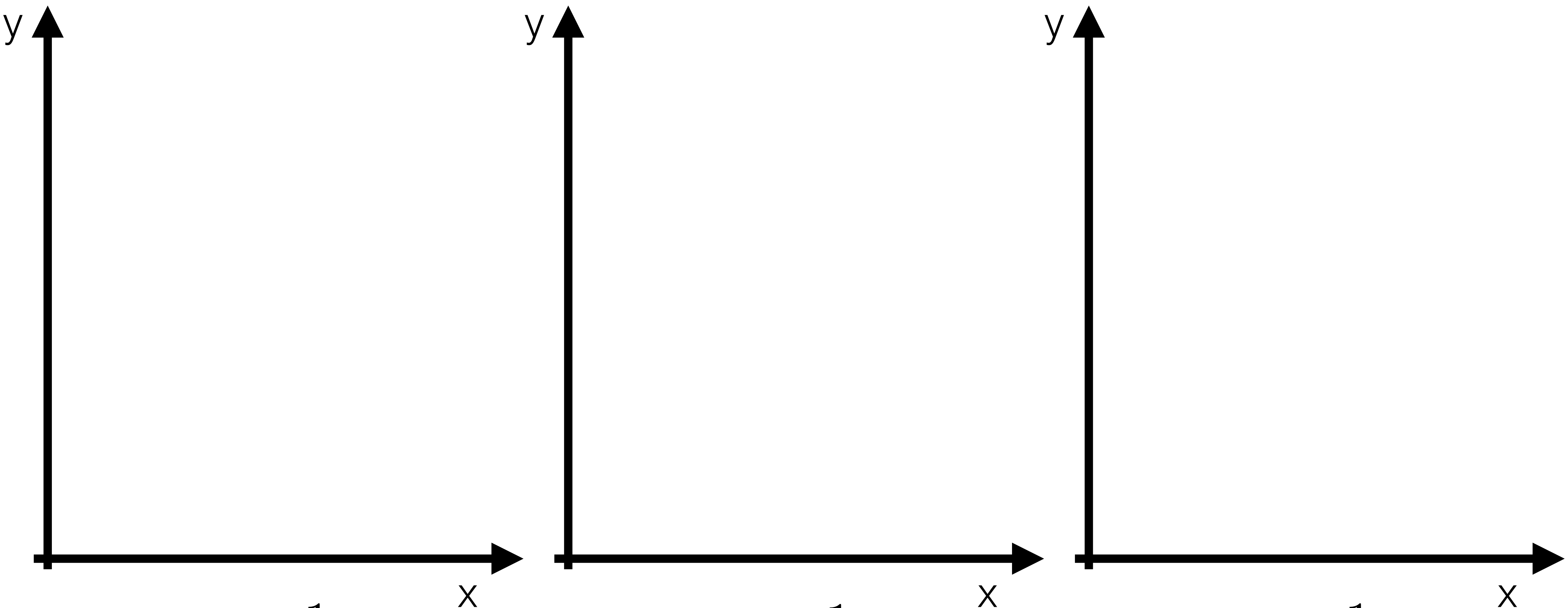
MOST IMPORTANT

Group Work

- Your group is a team;
  analyze its production function

$$y = x^{\alpha}$$

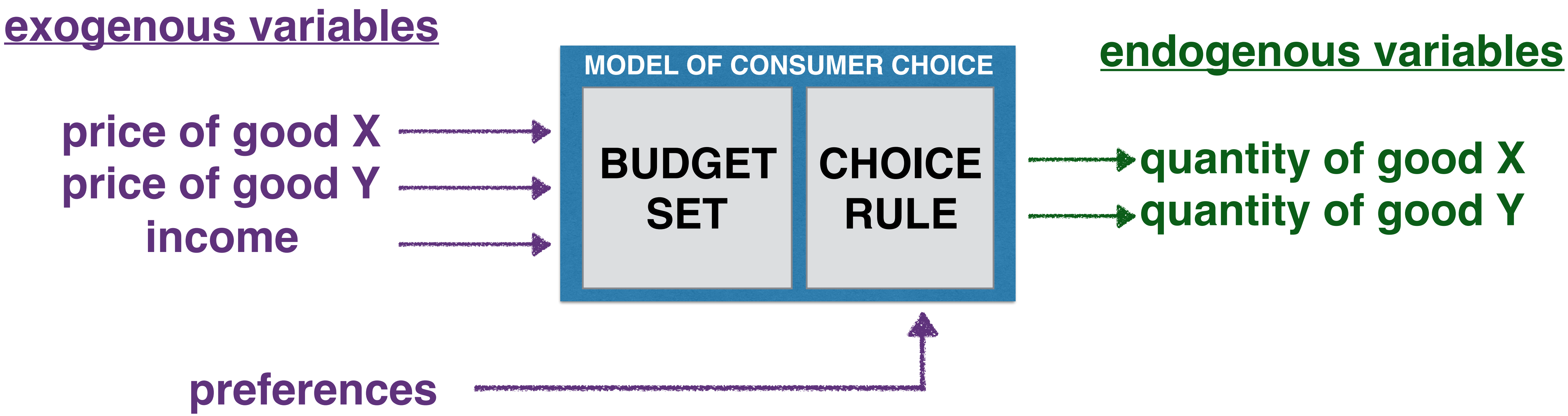


$\alpha < 1$ $\alpha = 1$ $\alpha > 1$

Part I

Overview of Producer Theory

Recall: Consumer Theory



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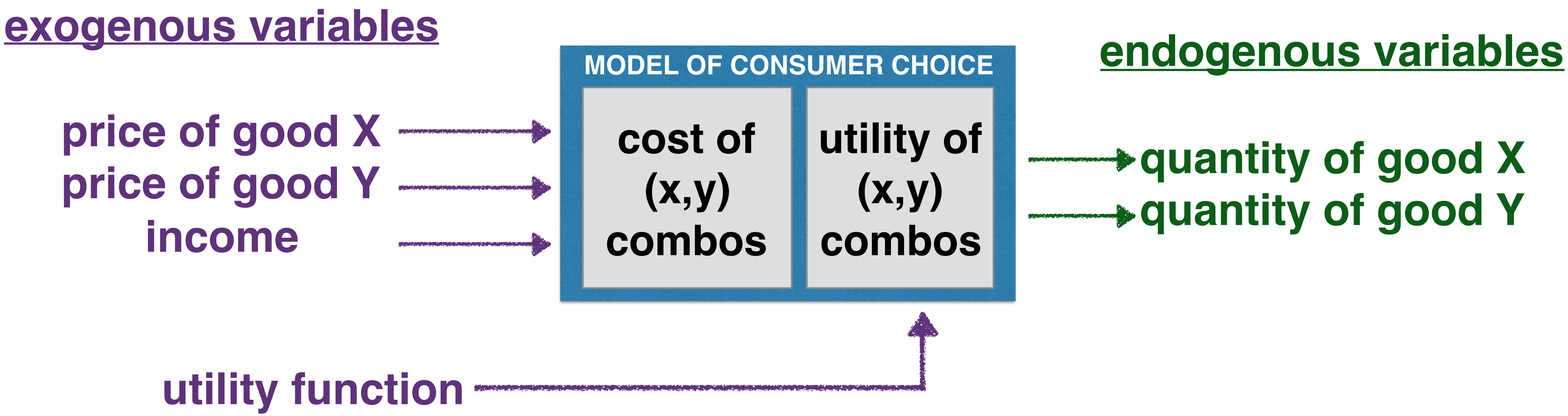

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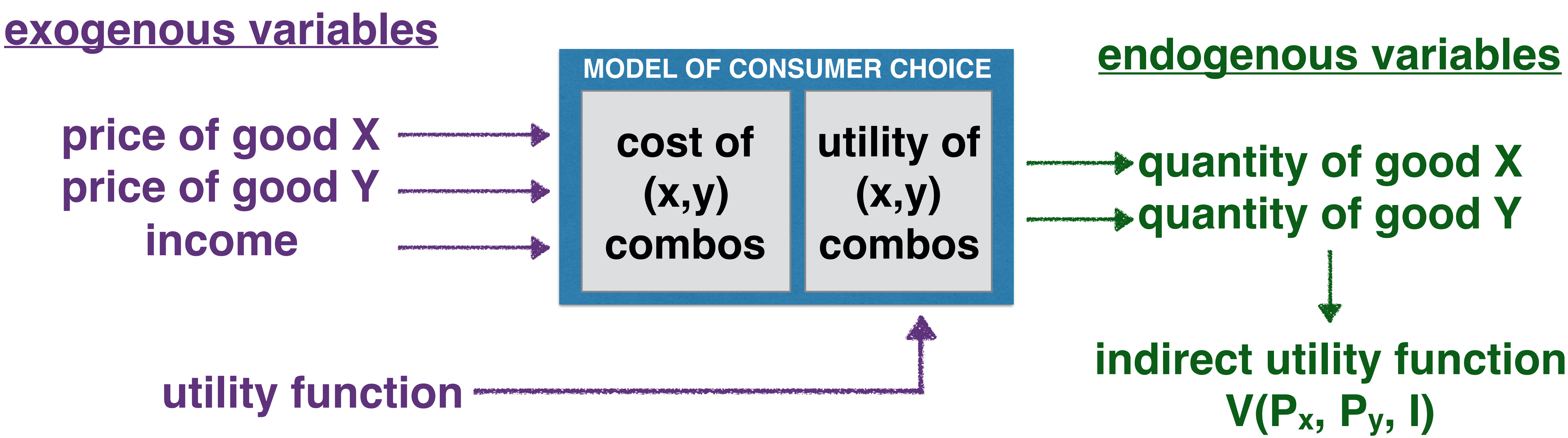

Consumer Theory

- Buy two goods,
  X and **Y**,
  at prices $\mathbf{P_x}$ and $\mathbf{P_y}$
- **Utility function**:
  transform those goods into
  "**utility**"

Producer Theory

- Buy two inputs,
  labor (L) and **capital (K)**,
  at prices **w** and **r**.
- **Production function**:
  transform those inputs into
  output
- Sell that output at price **P**

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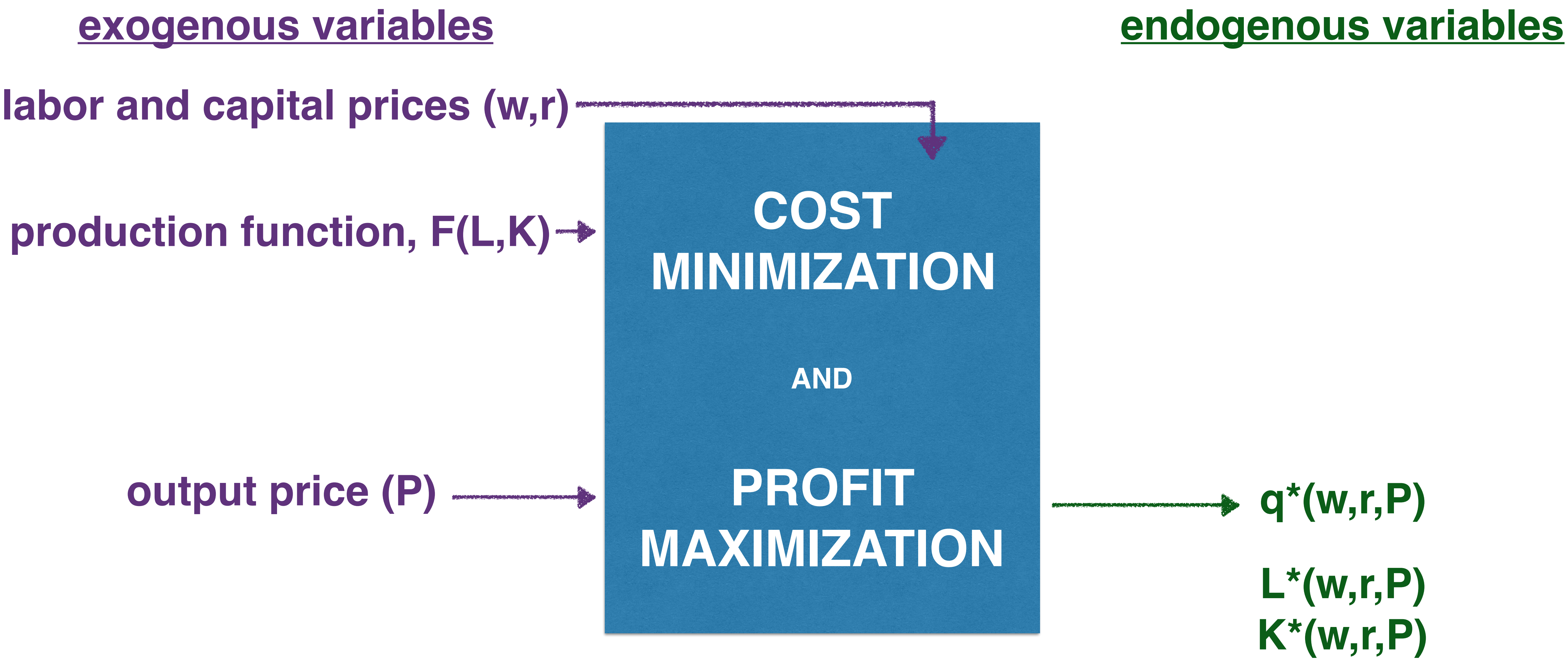
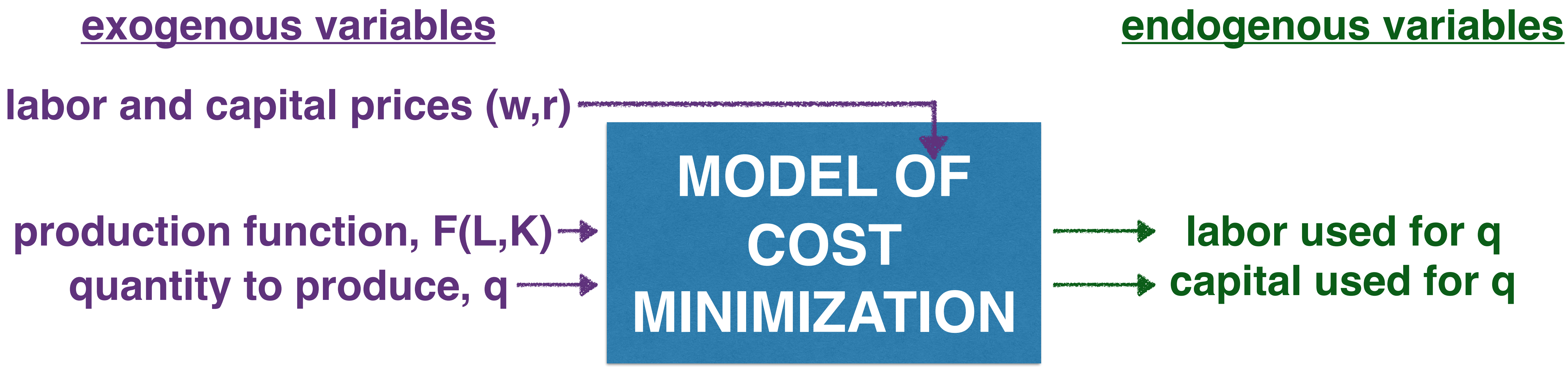
Unified Producer Theory: Perfect Competition

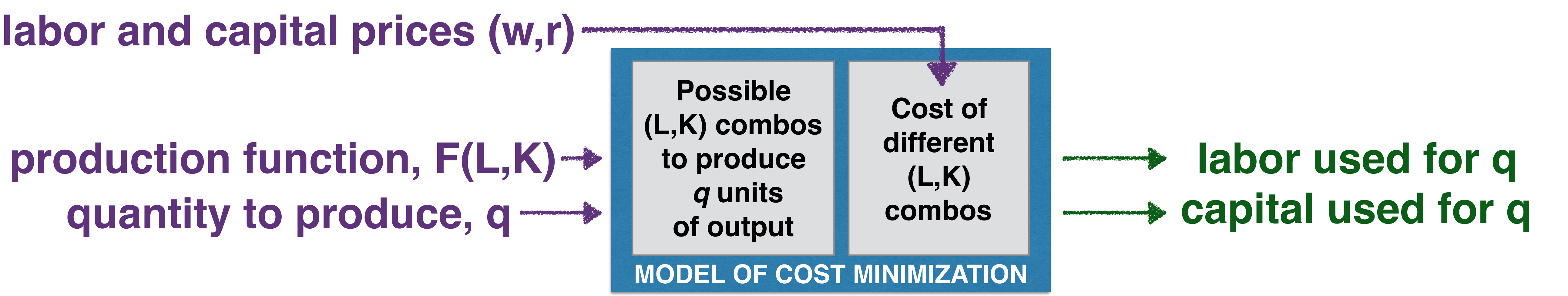


Producer Theory, Part I: Cost Minimization



Producer Theory, Part I: Cost Minimization

exogenous variables | **endogenous variables**



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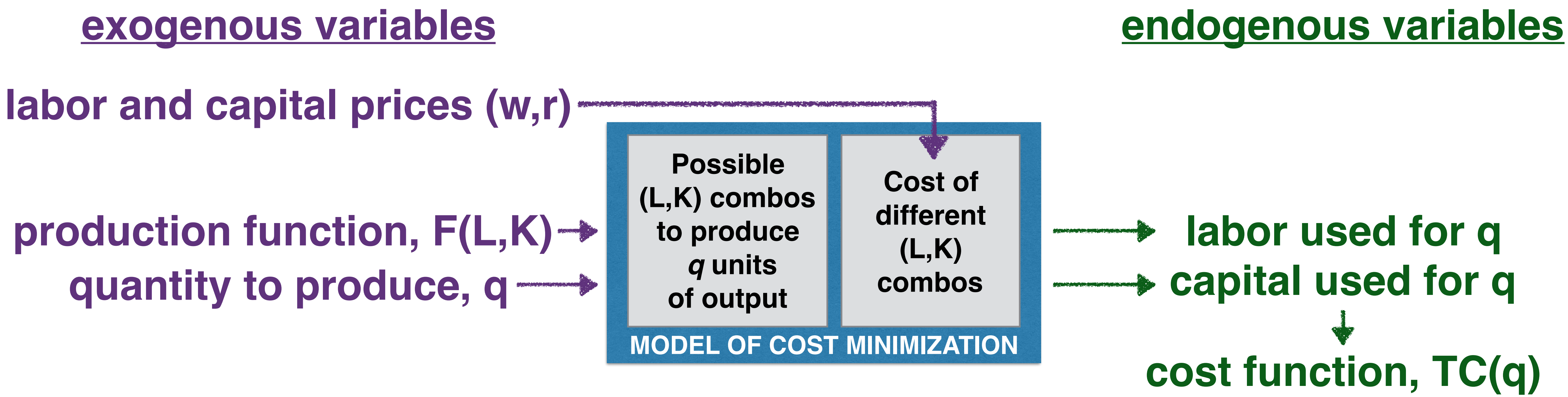
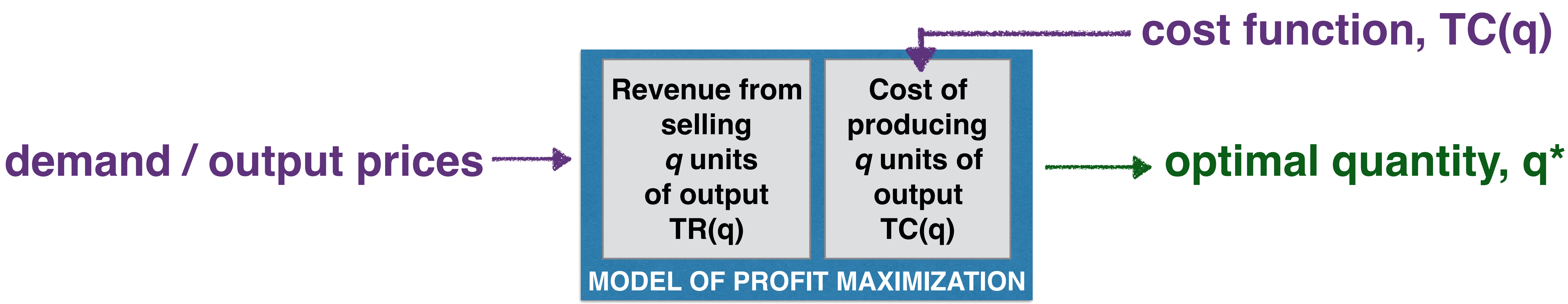


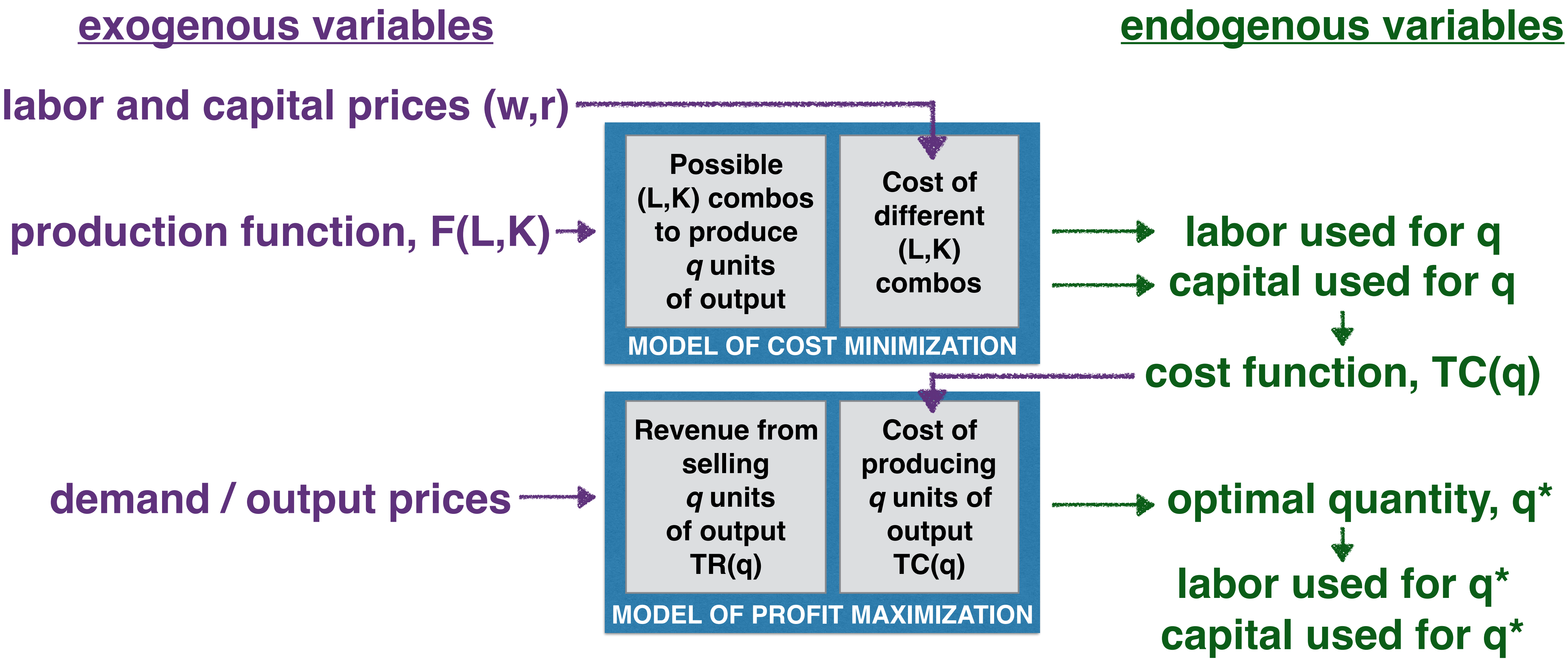
Producer Theory, Part II: Profit Maximization

exogenous variables

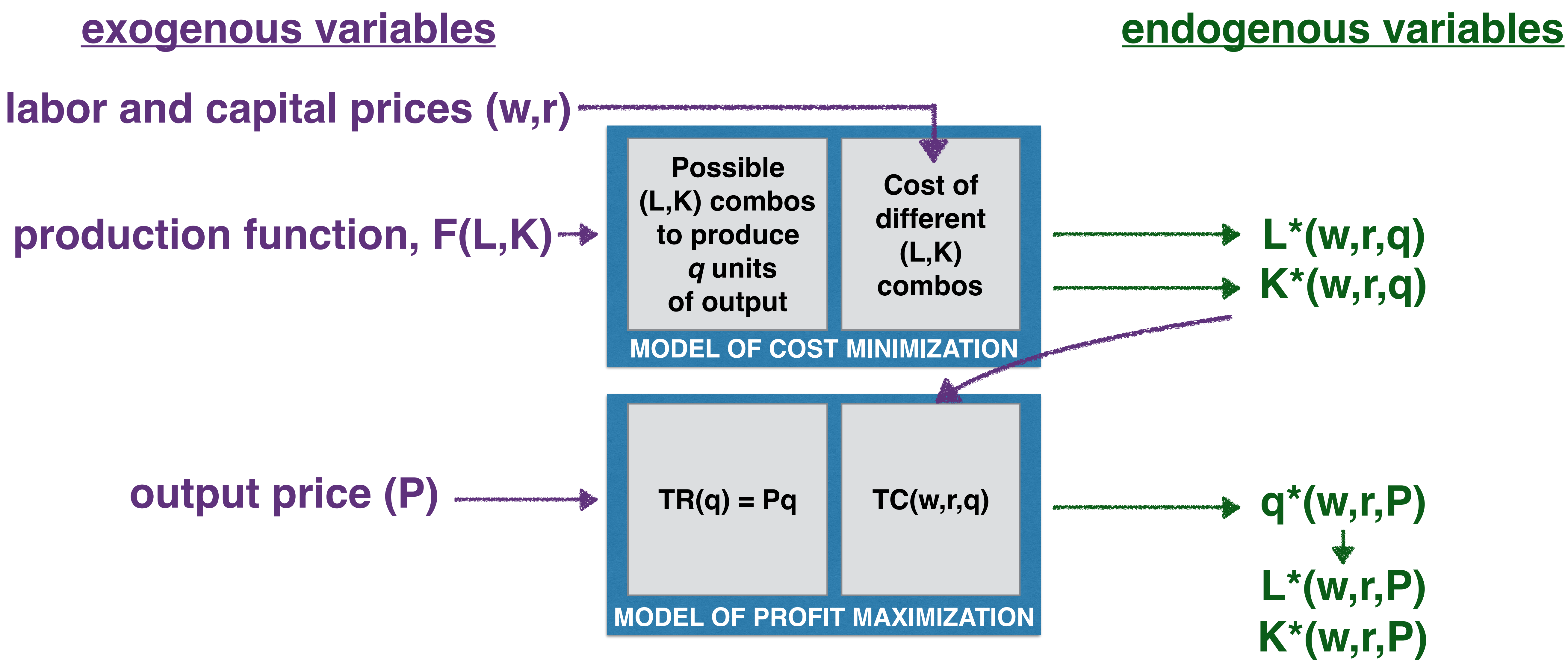
endogenous variables



Unified Producer Theory



Unified Producer Theory: **Perfect Competition**



Unified Producer Theory: **Perfect Competition**

exogenous variables

endogenous variables

labor and capital prices (w,r)

production function, F(L,K)

Possible (L,K) combos to produce q units of output

Cost of different (L,K) combos

MODEL OF COST MINIMIZATION

Conditional (on q) input demands

$L^*(w,r,q)$
$K^*(w,r,q)$

output price (P)

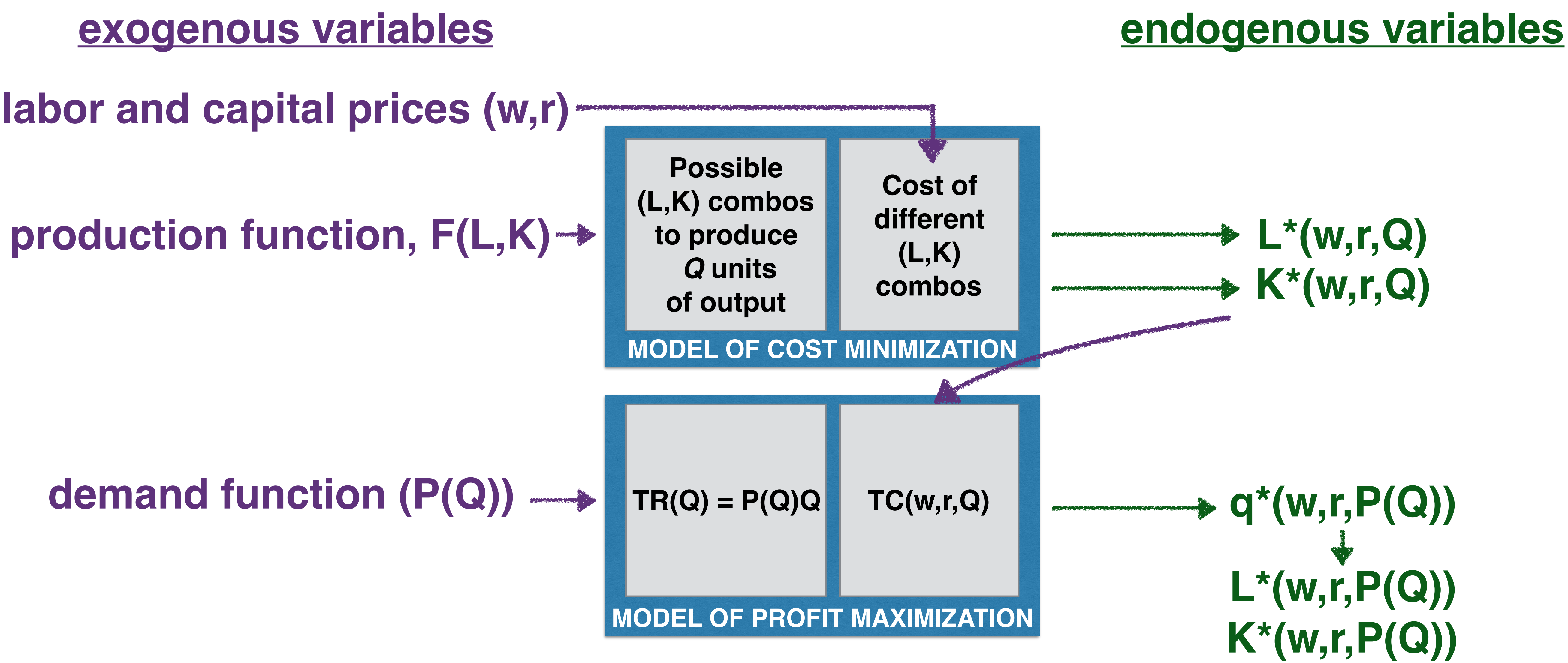
Profit-maximizing quantity

$q^*(w,r,P)$

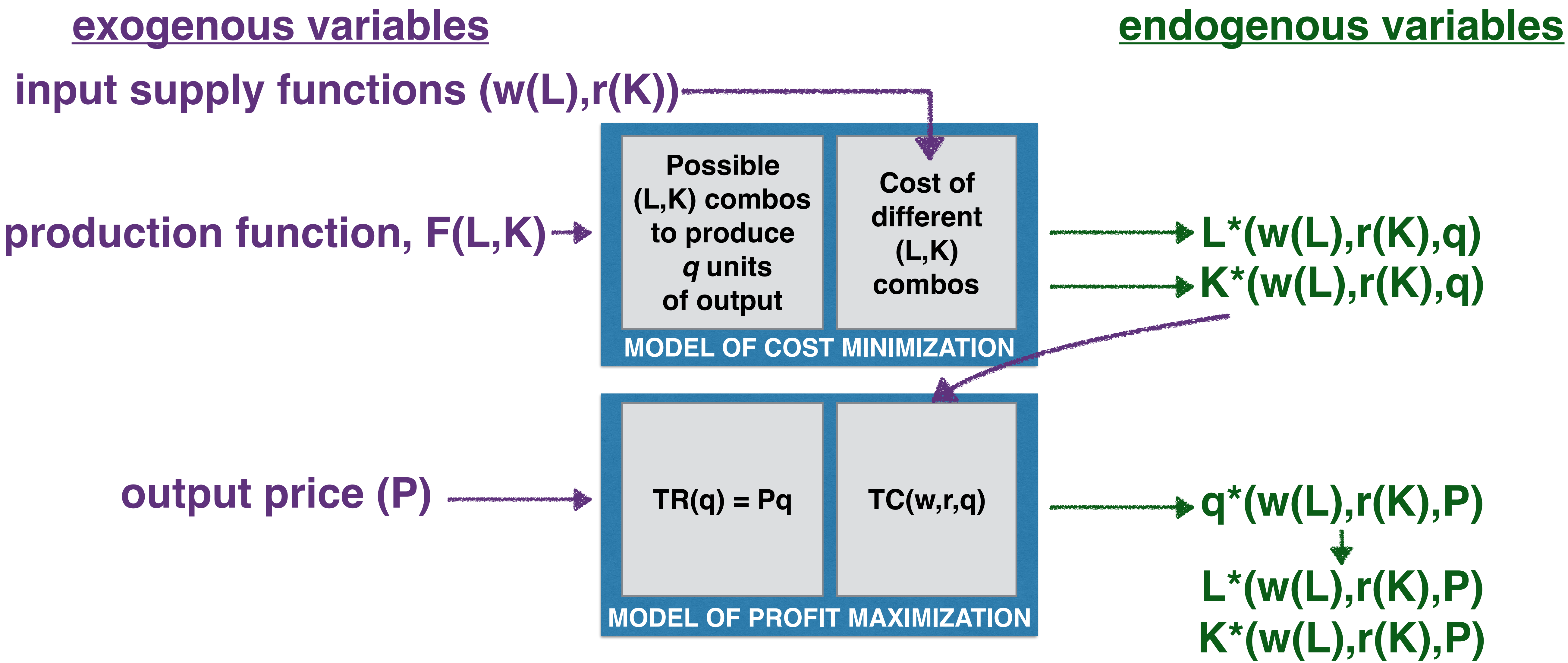
Profit-maximizing input demands

$L^*(w,r,P)$
$K^*(w,r,P)$

Unified Producer Theory: **Monopoly**



Unified Producer Theory: **Monopsony**



Part IV: Producer Choice (Cost Minimization)
Given input prices and an output target, how does a firm choose inputs to minimize cost?

- Lecture 12: Inputs and Production Functions (B&B chapter 6)
- Lecture 13: Costs and Cost Minimization (B&B chapter 7)

Part V: Deriving the Supply Curve (Profit Maximization for a Price Taker)
Given output prices and a cost function, how does a firm choose output quantity to maximize profit?

- Lecture 14: Cost Curves (B&B chapter 8)
- Lecture 15: Profit Maximization for the Competitive Firm (B&B, chapter 9, sections 1-2)
- Lecture 16: Supply Curves; Short-Run and Long-Run Equilibrium (B&B, chapter 9, sections 3-5)
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Part VI: Market Power (Profit Maximization for a Price Setter)
If a firm can set prices in either the output or input markets, what input-output combination maximizes its profit?

- Lecture 18: Monopoly and Monopsony (B&B, chapter 11)

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Standing Assignment

Please read the chapter **before** class
and answer the Review Questions
at the end of the chapter.

I will assume you have done this!

<u>Part II</u>

Production Functions with Two Inputs

Production Functions: 4 Key Concepts

1. **Isoquants and Marginal Rate of Technical Substitution (MRTS)**
   u(x,y) : indifference curves : MRS :: F(L,K) : isoquants : MRTS

2. **Returns to Scale**
   How does quantity respond to doubling all inputs?

3. **Elasticity of Substitution**
   How does the MRTS change if F(L,K) stays constant and K/L changes?

4. **Technological Progress**
   How does the MRTS change if F(L,K) changes and K/L stays constant?

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Consumer Theory

- Buy goods, **X** and **Y**, at prices $\mathbf{P_x}$ and $\mathbf{P_y}$
- **Utility function**: transform those goods into "**utility**"
- **Indifference curves:** Combinations of (x,y) that give same utility

Producer Theory

- Buy inputs, **labor (L)** and **capital (K)**,at prices **w** and **r**.
- **Production function**: transform those inputs into **output**
- **Isoquants:** Combinations of (L,K) that produce the same quantity

Major difference:
output means something!

…so monotonic transformation of production functions also mean something…

Example: Cobb-Douglas

$$F(L, K) = L^{\frac{1}{4}} K^{\frac{1}{4}}$$

$$F(L, K) = L^{\frac{1}{2}} K^{\frac{1}{2}}$$

$$F(L, K) = LK$$

Consumer Theory

- Utility function: **u(x,y)**
- Marginal utility of X:
- Marginal utility of Y:
- Marginal rate of substitution:

Producer Theory

- Production function: **F(L,K)**
- Marginal product of labor:
- Marginal product of capital:
- Marginal rate of technical substitution:

Consumer Theory

Marginal Rate of Substitution

"If I reduced my consumption of
good X by one unit,
how much more of **good Y**
would I need in order to
achieve the same utility
as we were before?"

Producer Theory

Marginal Rate of Technical Substitution

"If we reduced our use of
labor by one unit,
how much more **capital**
would we need in order to
produce the same output
as we were before?"

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Returns to Scale (B&B 6.5)

- Response to a change in **all inputs** (not just one at a time). Easiest to think about **doubling all inputs** from **(L, K)** to **(2L, 2K)**:

- **Decreasing returns to scale:** doubling all inputs less than doubles output: $F(2L, 2K) < 2F(L, K)$

- **Constant returns to scale:** doubling all inputs exactly doubles output: $F(2L, 2K) = 2F(L, K)$

- **Increasing returns to scale:** doubling all inputs more than doubles output: $F(2L, 2K) > 2F(L, K)$

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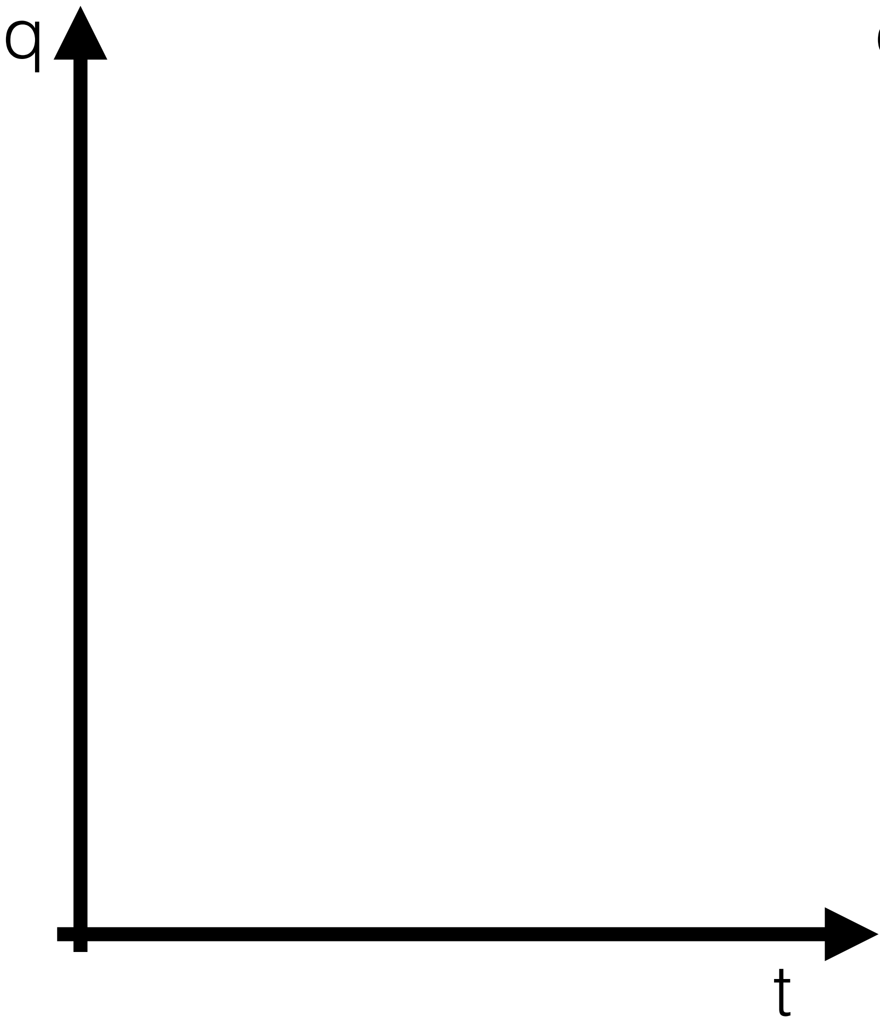
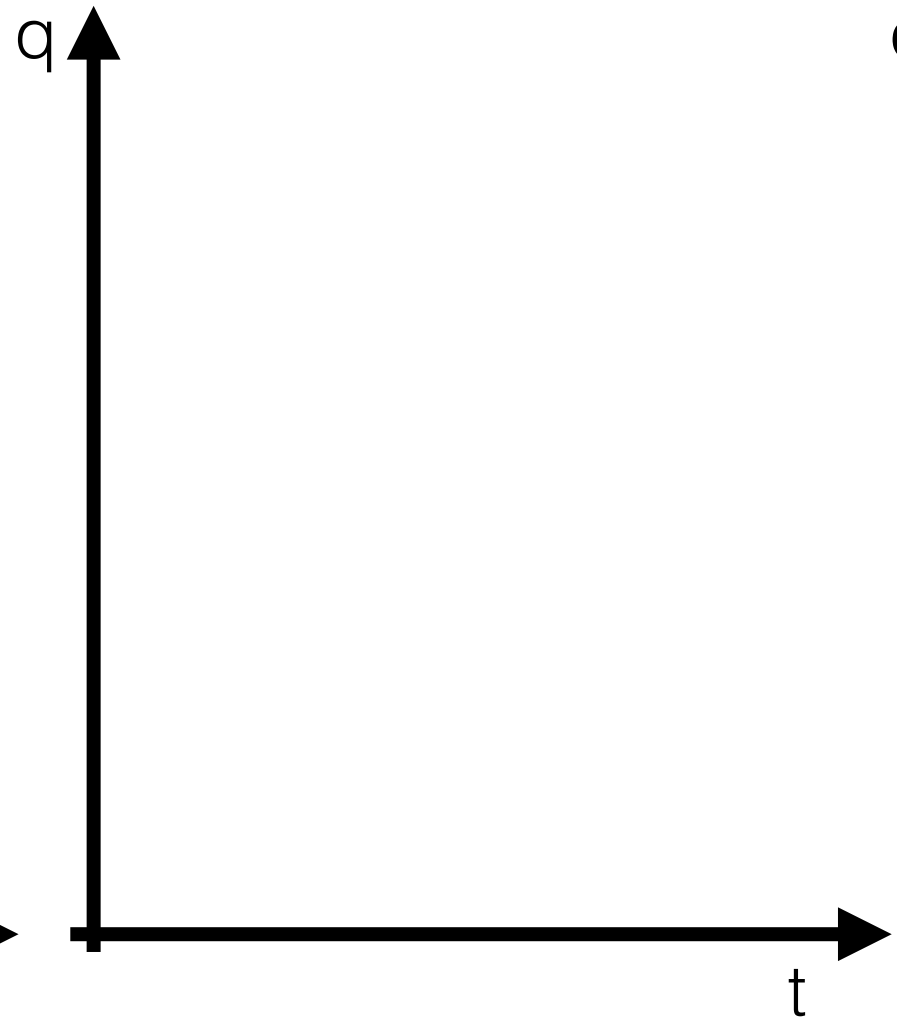
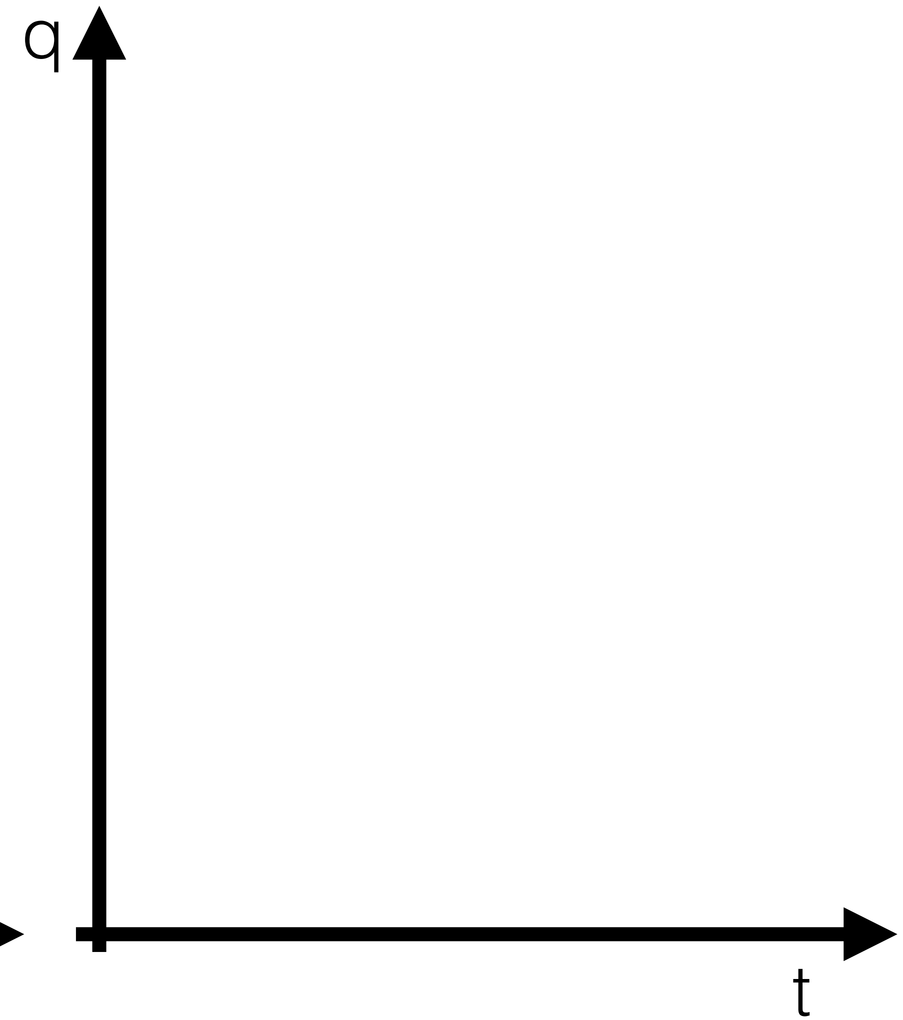
Returns to Scale, Graphically: $q = F(tL, tK)$







Example: Cobb-Douglas

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CD: Decreasing Returns to Scale

$$F(L,K) = L^{\frac{1}{4}} K^{\frac{1}{4}}$$

$$F(L,K) = L^{\frac{1}{2}} K^{\frac{1}{2}}$$

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CD: Constant Returns to Scale

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CD: Increasing Returns to Scale

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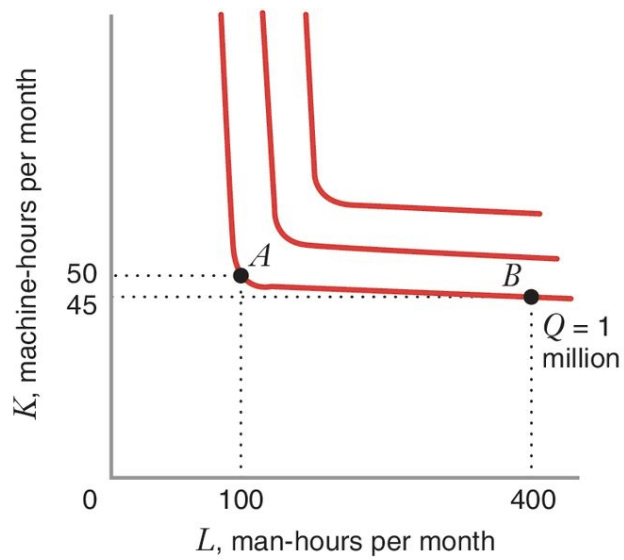
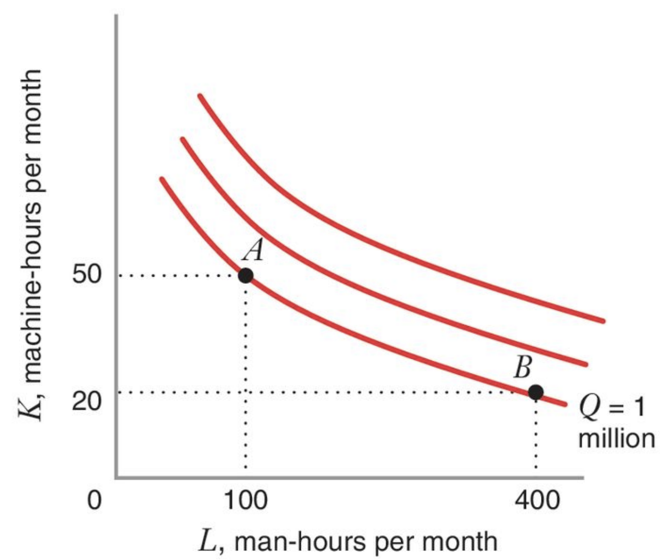
Elasticity of Substitution

- **How substitutable** are capital and labor for each other?
- Specifically: how does the MRTS <u>change</u>
  as we increase the capital-labor ratio K/L?



(a) Production Function
with Limited Input
Substitution Opportunities



(b) Production Function
with Abundant Input
Substitution Opportunities

Math: Elasticity of Substitution

$$\sigma = \frac{\text{\% change in capital-labor ratio}}{\text{\% change in } MRTS_{L,K}}$$

$$= \frac{1}{\frac{\text{\% change in } MRTS_{L,K}}{\text{\% change in capital-labor ratio}}}$$

Elasticity of Substitution

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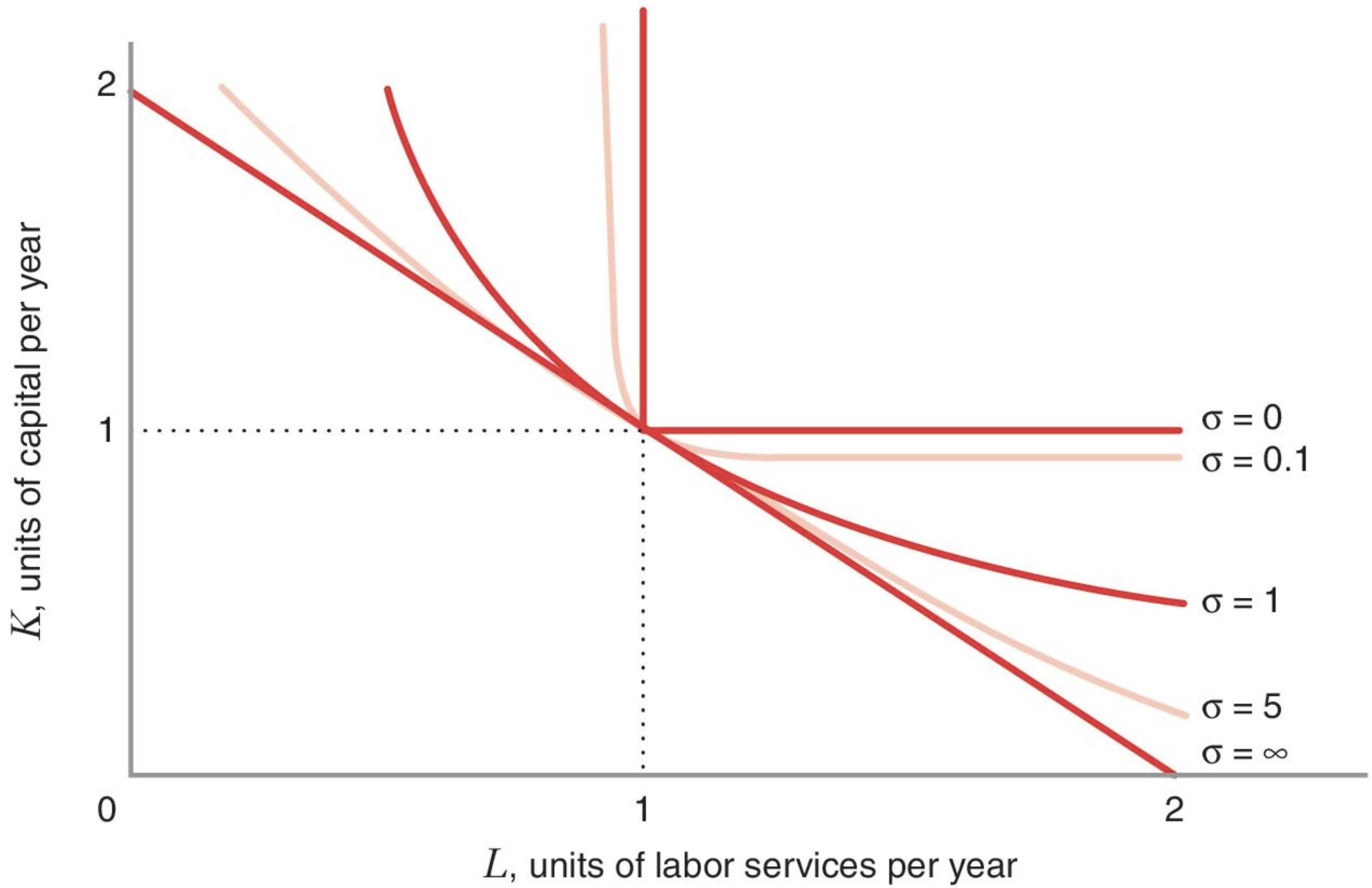
Elasticity of Substitution, Graphically



Interpretation of Production Functions

Production Function	Elasticity of Substitution (σ)	Other Characteristics
Linear production function	$\sigma = \infty$	Inputs are perfect substitutes Isoquants are straight lines
Fixed-proportions production function	$\sigma = 0$	Inputs are perfect complements Isoquants are L-shaped
Cobb—Douglas production function	$\sigma = 1$	Isoquants are curves
CES production function	$0 \leq \sigma \leq \infty$	Includes other three production functions as special cases Shape of isoquants varie

Part III

Mathematical Deep Dive: Cobb-Douglas

Example: General Cobb-Douglas

$$F(L, K) = AL^{\alpha}K^{\beta}$$

Calculate:

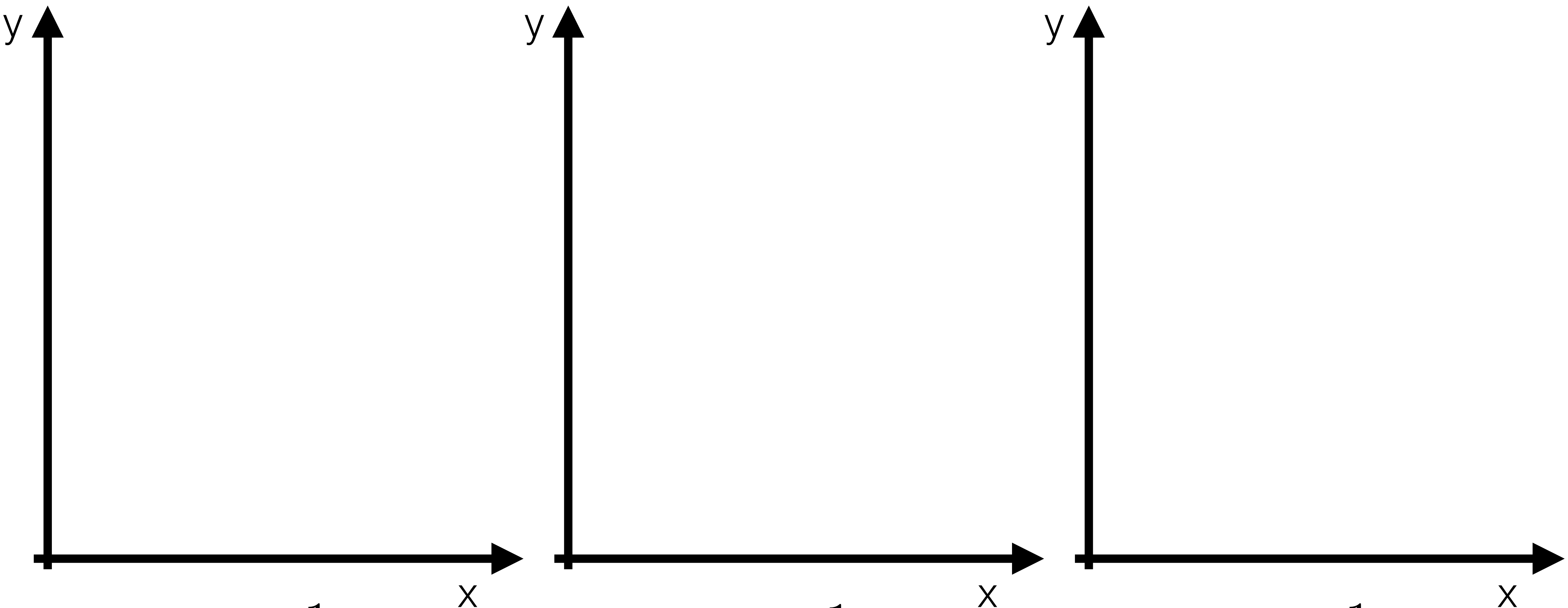
MP_L, MP_K, MRTS

Elasticity of Substitution

Returns to Scale

See “Derivations” PDF for detailed notes! :)

$$y = x^{\alpha}$$



$\alpha < 1$ $\alpha = 1$ $\alpha > 1$

Marginal Productivity and Returns to Scale

$$F(L, K) = AL^{\alpha}K^{\beta}$$

$$\alpha < 1 \Rightarrow \text{Diminishing } MP_L$$

$$\beta < 1 \Rightarrow \text{Diminishing } MP_K$$

$$\alpha + \beta < 1 \Rightarrow \text{Decreasing returns to scale}$$

$$\alpha + \beta = 1 \Rightarrow \text{Constant returns to scale}$$

$$\alpha + \beta > 1 \Rightarrow \text{Increasing returns to scale}$$

Cobb-Douglas: Marginal Productivity

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Marginal Productivity and Returns to Scale

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Cobb-Douglas: Calculating Returns to Scale

$$F(L, K) = AL^{\alpha}K^{\beta}$$

$$F(tL, tK) = A(tL)^{\alpha}(tK)^{\beta}$$

Marginal Productivity and Returns to Scale

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