Inputs and Production Functions

Econ 50 | Lecture 12 | February 16, 2016

Lecture

- Overview of Producer Theory
- Production Functions
  with Two Inputs, In General:

  Isoquants & MRTS (6.3)
  Returns to Scale (6.5)
  Elasticity of Substitution (6.4)
  Technological Change (6.6)

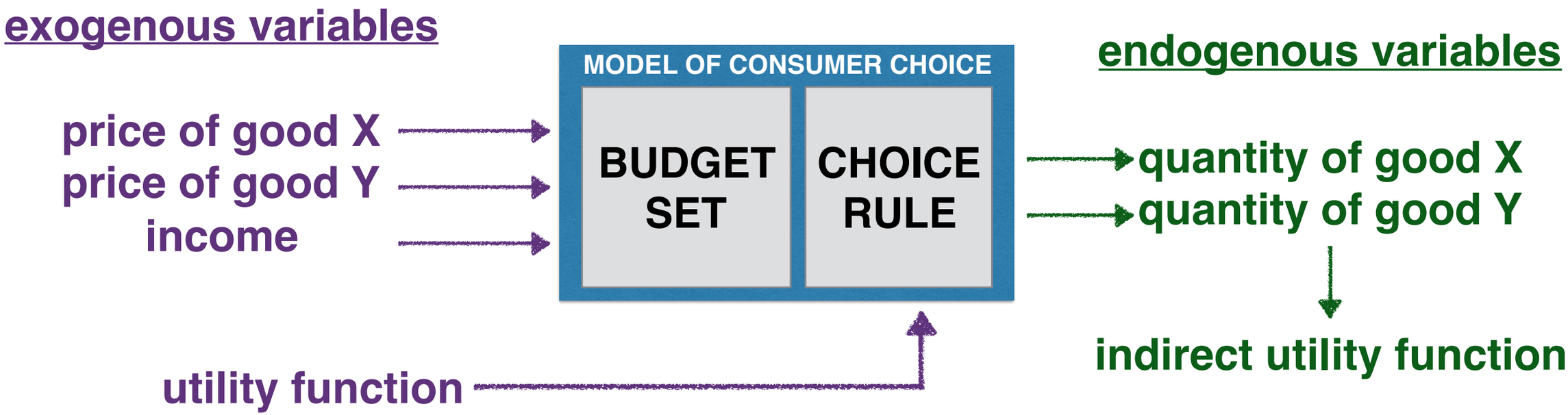
- Deep Dive: Cobb-Douglas

Group Work

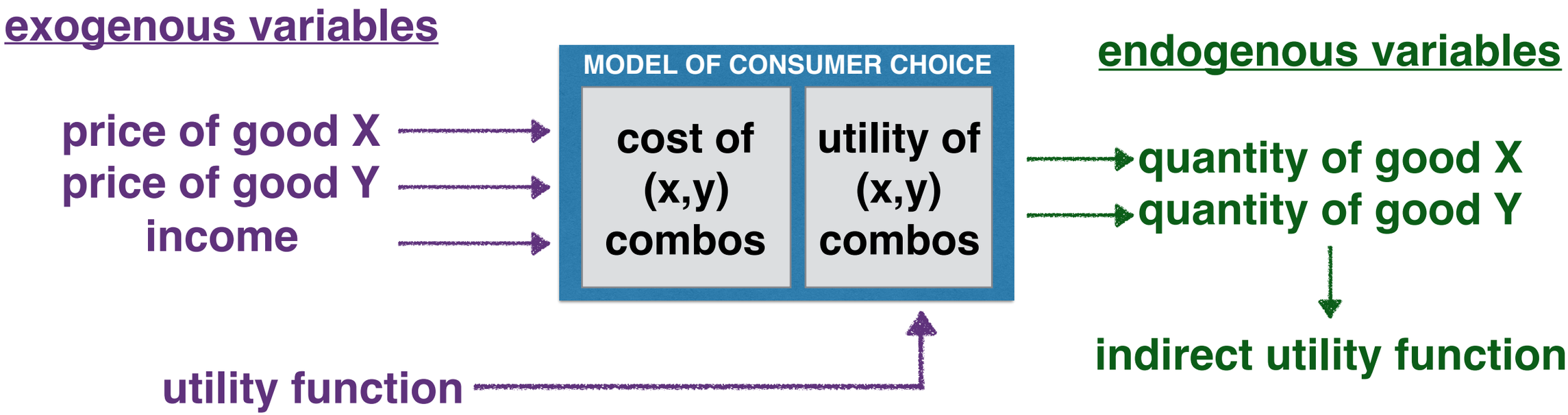
- Your group is a team;
  analyze its production function

Part I
Overview of Producer Theory

Model of Consumer Theory



Model of Consumer Theory



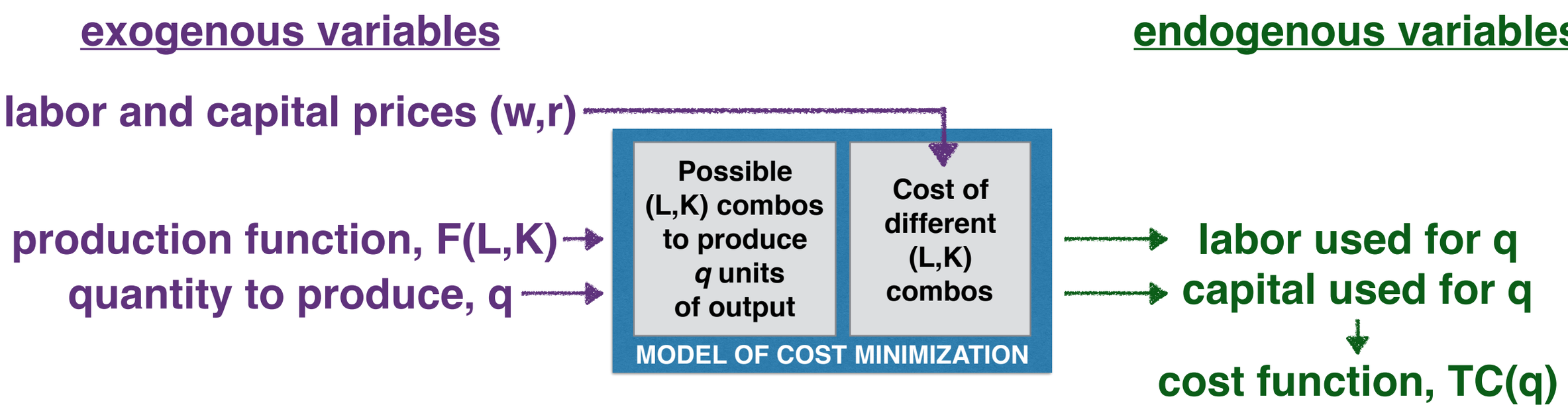
Consumer Theory

- Buy two goods,
  X and **Y**,
  at prices $\mathbf{P_x}$ and $\mathbf{P_y}$
- **Utility function**:
  transform those goods into
  "**utility**"

Producer Theory

- Buy two inputs,
  labor (L) and **capital (K)**,
  at prices **w** and **r**.
- **Production function**:
  transform those inputs into
  output
- Sell that output at price **P**

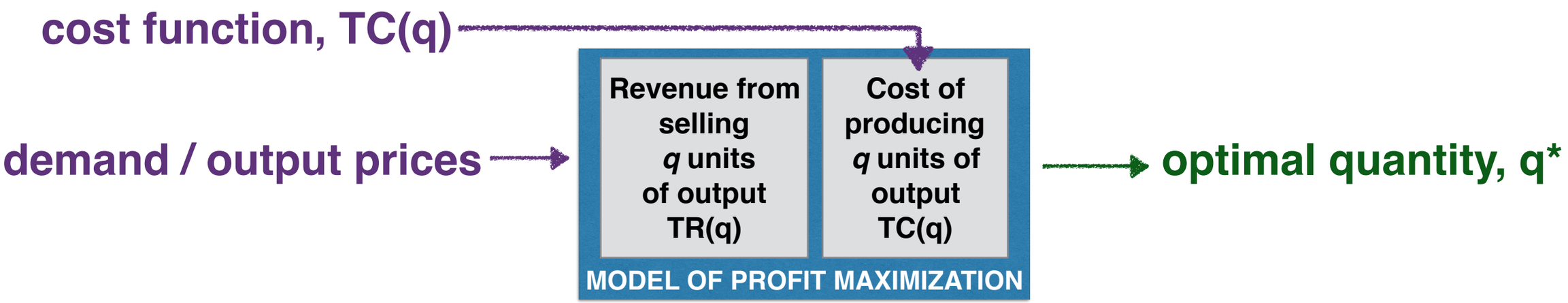
Producer Theory, Part I: Cost Minimization



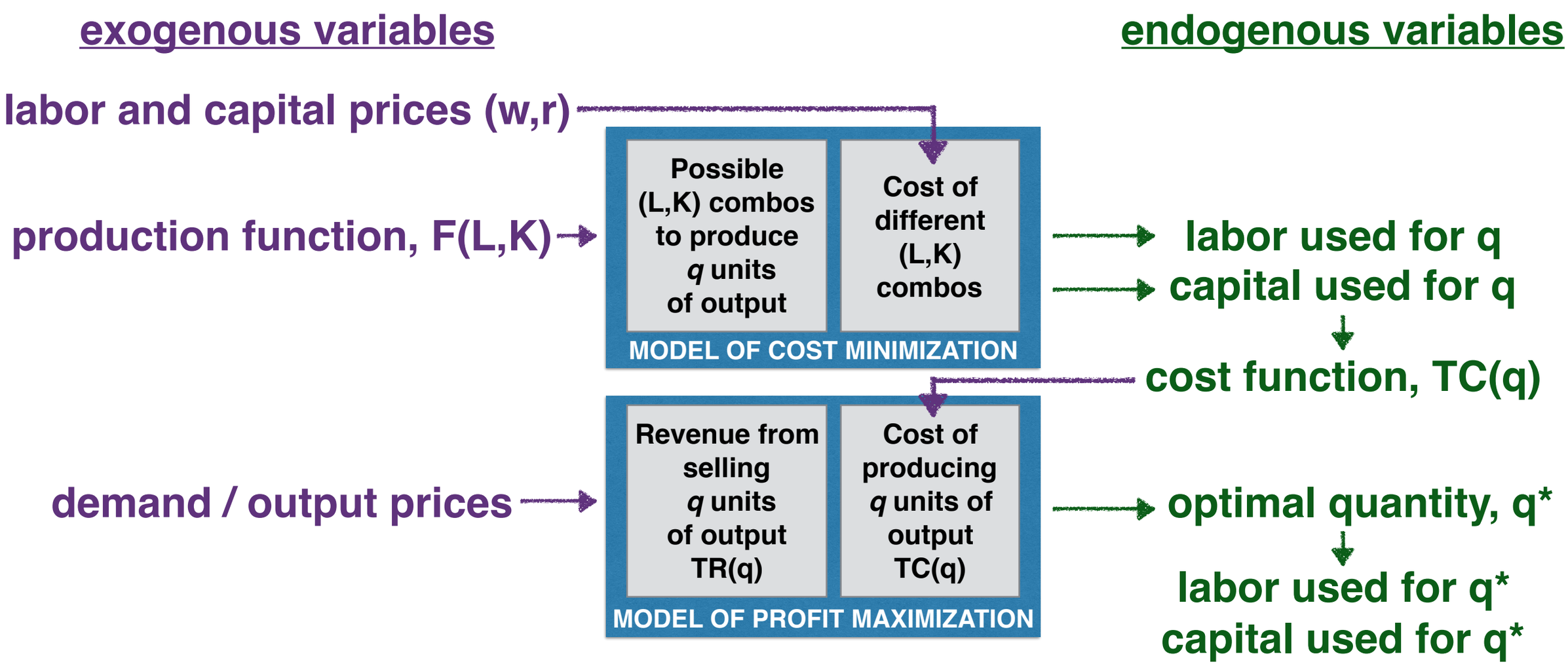
Producer Theory, Part II: Profit Maximization

exogenous variables

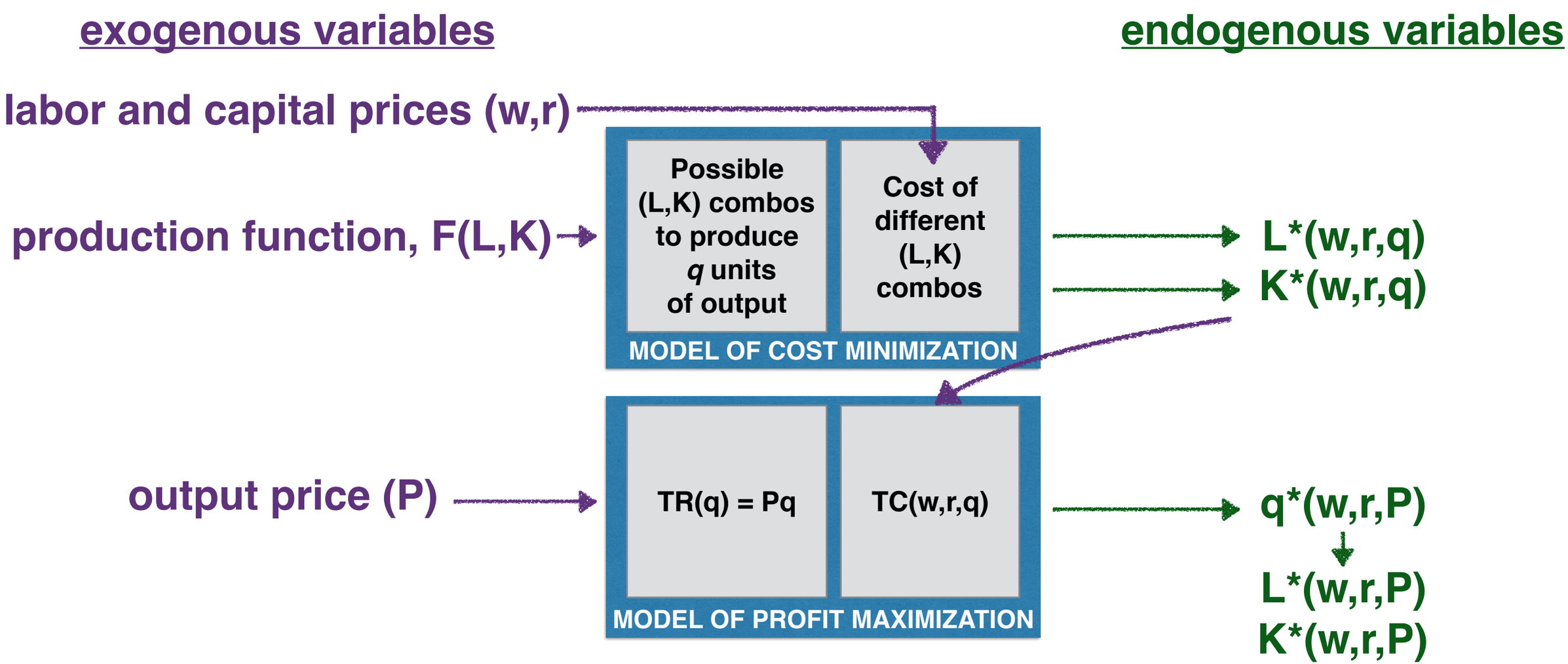
endogenous variables



Unified Producer Theory



Unified Producer Theory: Perfect Competition



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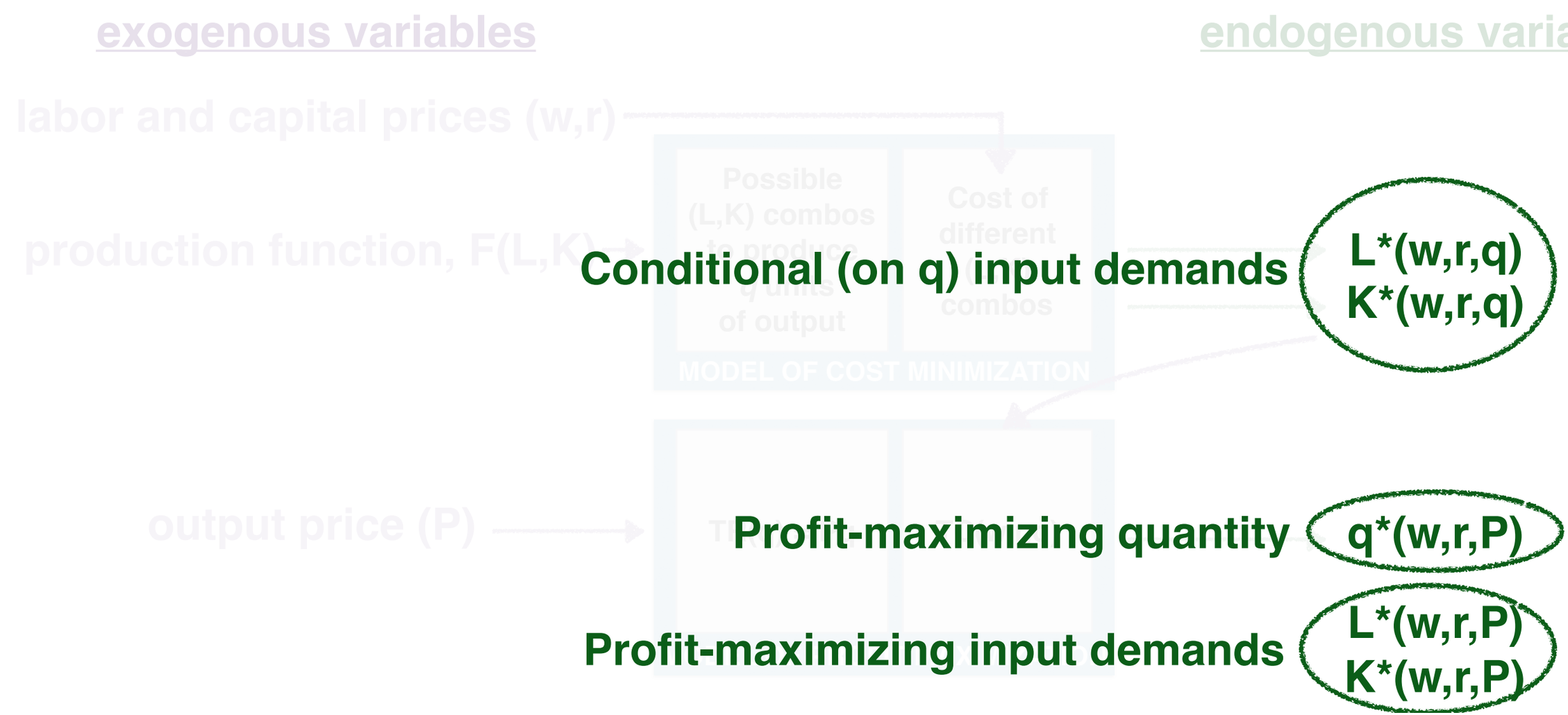
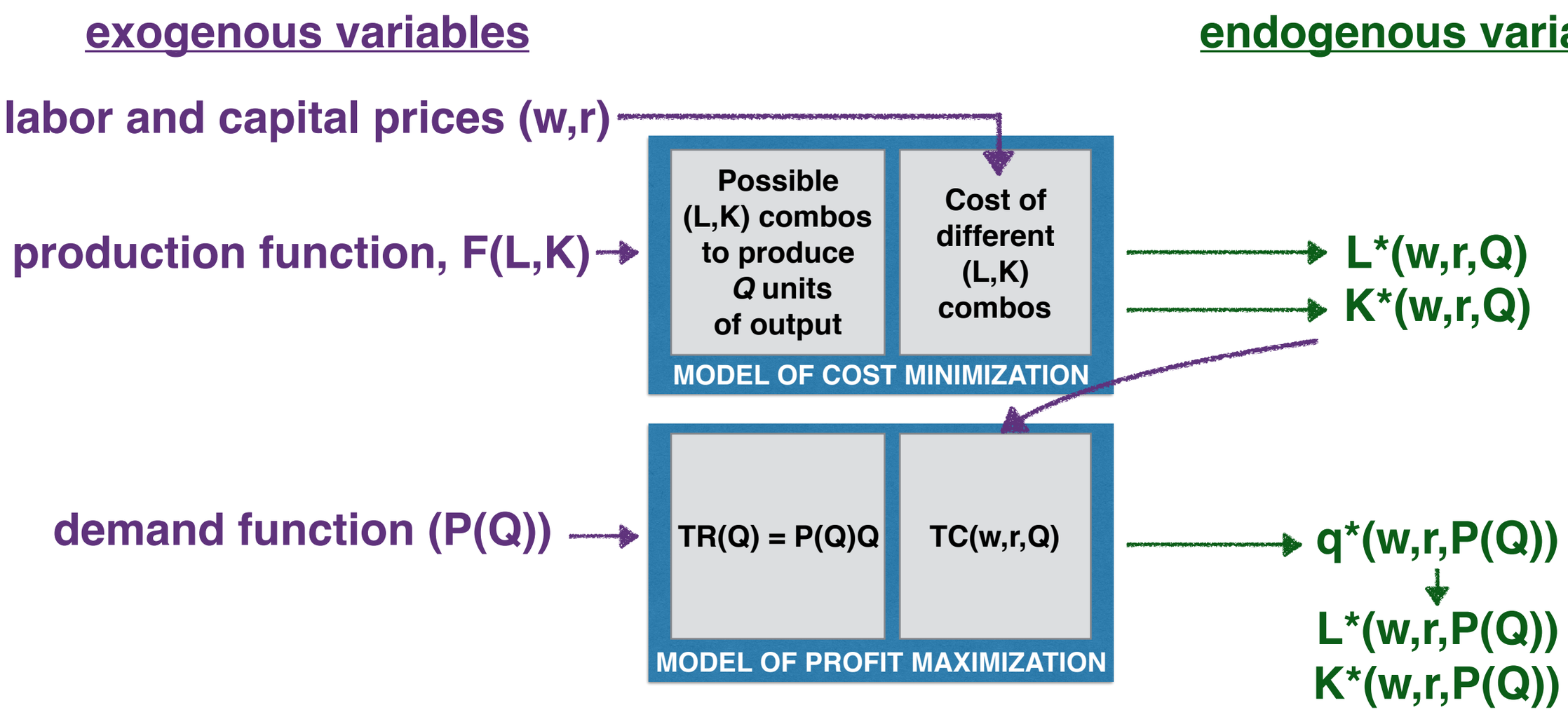
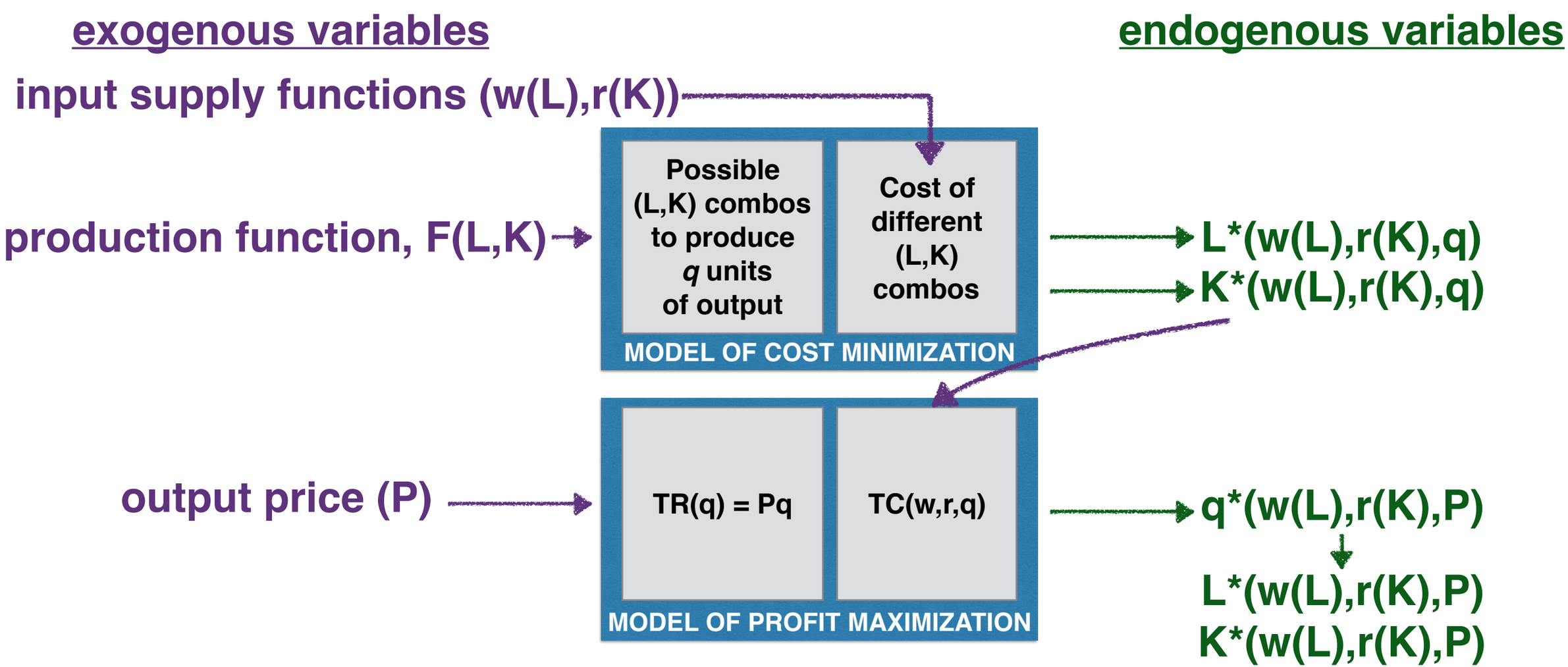


Unified Producer Theory: Monopoly



Unified Producer Theory: Monopsony



Part IV: Producer Choice (Cost Minimization)
Given input prices and an output target, how does a firm choose inputs to minimize cost?

- Lecture 12: Inputs and Production Functions (B&B chapter 6)
- Lecture 13: Costs and Cost Minimization (B&B chapter 7)

Part V: Deriving the Supply Curve (Profit Maximization for a Price Taker)
Given output prices and a cost function, how does a firm choose output quantity to maximize profit?

- Lecture 14: Cost Curves (B&B chapter 8)
- Lecture 15: Profit Maximization for the Competitive Firm (B&B, chapter 9, sections 1-2)
- Lecture 16: Supply Curves; Short-Run and Long-Run Equilibrium (B&B, chapter 9, sections 3-5)
- Lecture 17: Competitive Markets; Applications and Extensions (B&B, chapter 10)

Part VI: Market Power (Profit Maximization for a Price Setter)
If a firm can set prices in either the output or input markets, what input-output combination maximizes its profit?

- Lecture 18: Monopoly and Monopsony (B&B, chapter 11)

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Standing Assignment

Please read the chapter **before** class
and answer the Review Questions
at the end of the chapter.

I will assume you have done this!

Part II
Production Functions with Two Inputs

Production Functions: 4 Key Concepts

1. **Isoquants and Marginal Rate of Technical Substitution (MRTS)**
   u(x,y) : indifference curves : MRS :: F(L,K) : isoquants : MRTS

2. **Returns to Scale**
   How does quantity respond to doubling all inputs?

3. **Elasticity of Substitution**
   How does the MRTS change if F(L,K) stays constant and K/L changes?

4. **Technological Progress**
   How does the MRTS change if F(L,K) changes and K/L stays constant?

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Consumer Theory

- Buy goods, **X** and **Y**, at prices $\mathbf{P_x}$ and $\mathbf{P_y}$
- **Utility function**: transform those goods into "**utility**"
- **Indifference curves:** Combinations of (x,y) that give same utility

Producer Theory

- Buy inputs, **labor (L)** and **capital (K)**,at prices **w** and **r**.
- **Production function**: transform those inputs into **output**
- **Isoquants:** Combinations of (L,K) that produce the same quantity

Major difference:
output means something!

…so monotonic transformation of production functions also mean something…

Example: Cobb-Douglas

$$F(L,K) = L^{\frac{1}{4}} K^{\frac{1}{4}}$$

$$F(L,K) = L^{\frac{1}{2}} K^{\frac{1}{2}}$$

$$F(L,K) = LK$$

Consumer Theory

- Utility function: **u(x,y)**
- Marginal utility of X:
- Marginal utility of Y:
- Marginal rate of substitution:

Producer Theory

- Production function: **F(L,K)**
- Marginal product of labor:
- Marginal product of capital:
- Marginal rate of technical substitution:

Consumer Theory

Marginal Rate of Substitution

"If I reduced my consumption of **good X** by one unit,
how much more of **good Y**
would I need in order to
achieve the same utility
as we were before?"

Producer Theory

Marginal Rate of Technical Substitution

"If we reduced our use of **labor** by one unit,
how much more **capital**
would we need in order to
produce the same output
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Returns to Scale (B&B 6.5)

- Response to a change in **all inputs** (not just one at a time).
  Easiest to think about **doubling all inputs** from **(L, K)** to **(2L, 2K)**:

- **Decreasing returns to scale:**
  doubling all inputs less than doubles output: **F(2L, 2K) < 2F(L, K)**

- **Constant returns to scale:**
  doubling all inputs exactly doubles output: **F(2L, 2K) = 2F(L, K)**

- **Increasing returns to scale:**
  doubling all inputs more than doubles output: **F(2L, 2K) > 2F(L, K)**

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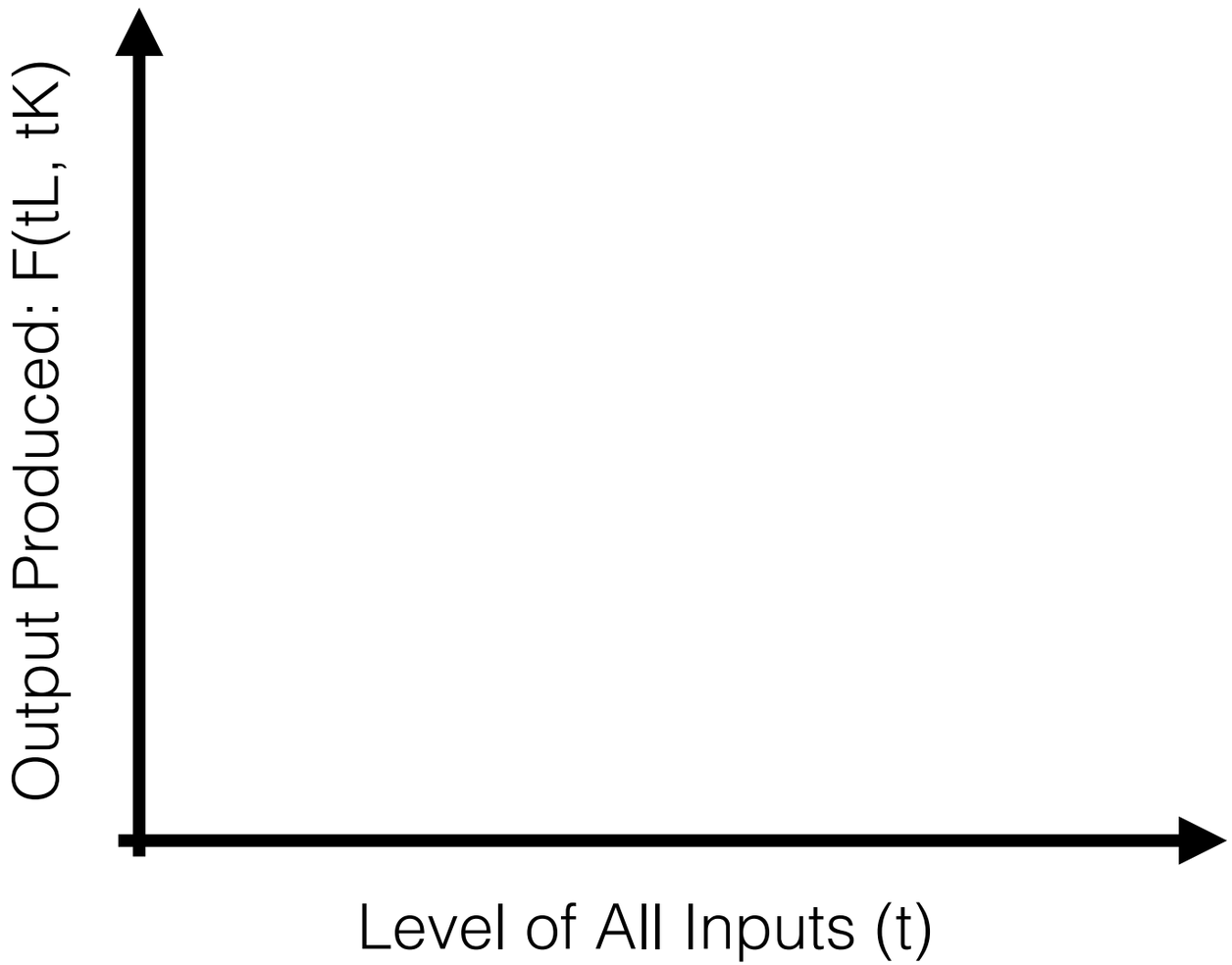
Returns to Scale, Graphically: F(tL, tK)



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CD: Decreasing Returns to Scale

$$F(L,K) = L^{\frac{1}{4}} K^{\frac{1}{4}}$$

$$F(L,K) = L^{\frac{1}{2}} K^{\frac{1}{2}}$$

$$F(L,K) = LK$$

CD: Constant Returns to Scale

$$F(L,K) = L^{\frac{1}{4}} K^{\frac{1}{4}}$$

$$F(L,K) = L^{\frac{1}{2}} K^{\frac{1}{2}}$$

$$F(L,K) = LK$$

CD: Increasing Returns to Scale

$$F(L,K) = L^{\frac{1}{4}} K^{\frac{1}{4}}$$

$$F(L,K) = L^{\frac{1}{2}} K^{\frac{1}{2}}$$

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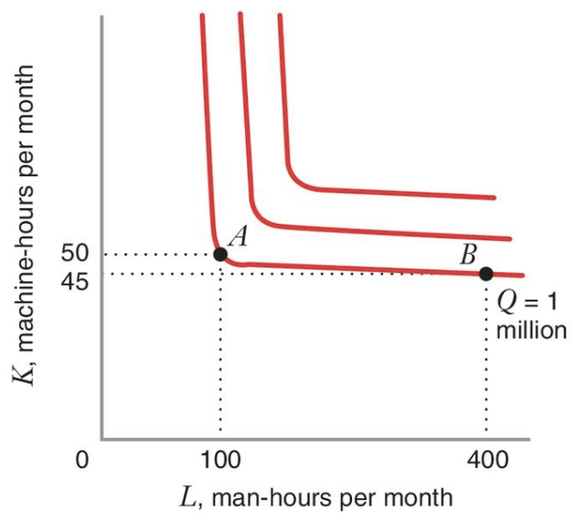
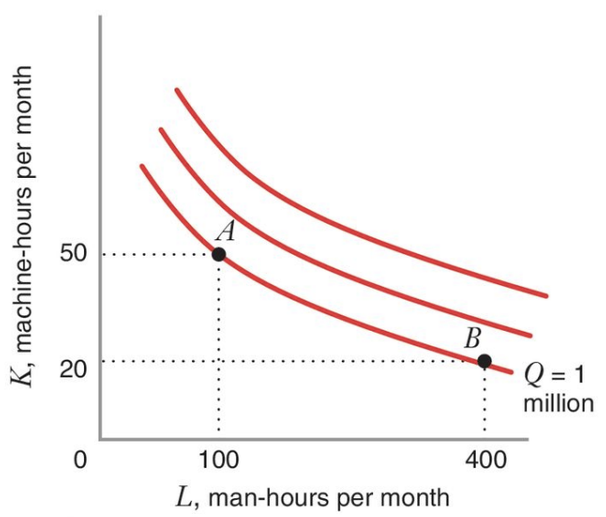
Elasticity of Substitution

- **How substitutable** are capital and labor for each other?
- Specifically: how does the MRTS change as we increase the capital-labor ratio K/L?



(a) Production Function with Limited Input Substitution Opportunities



(b) Production Function with Abundant Input Substitution Opportunities

Math: Elasticity of Substitution

$$\sigma = \frac{\%\text{ change in capital-labor ratio}}{\%\text{ change in } MRTS_{L,K}}$$

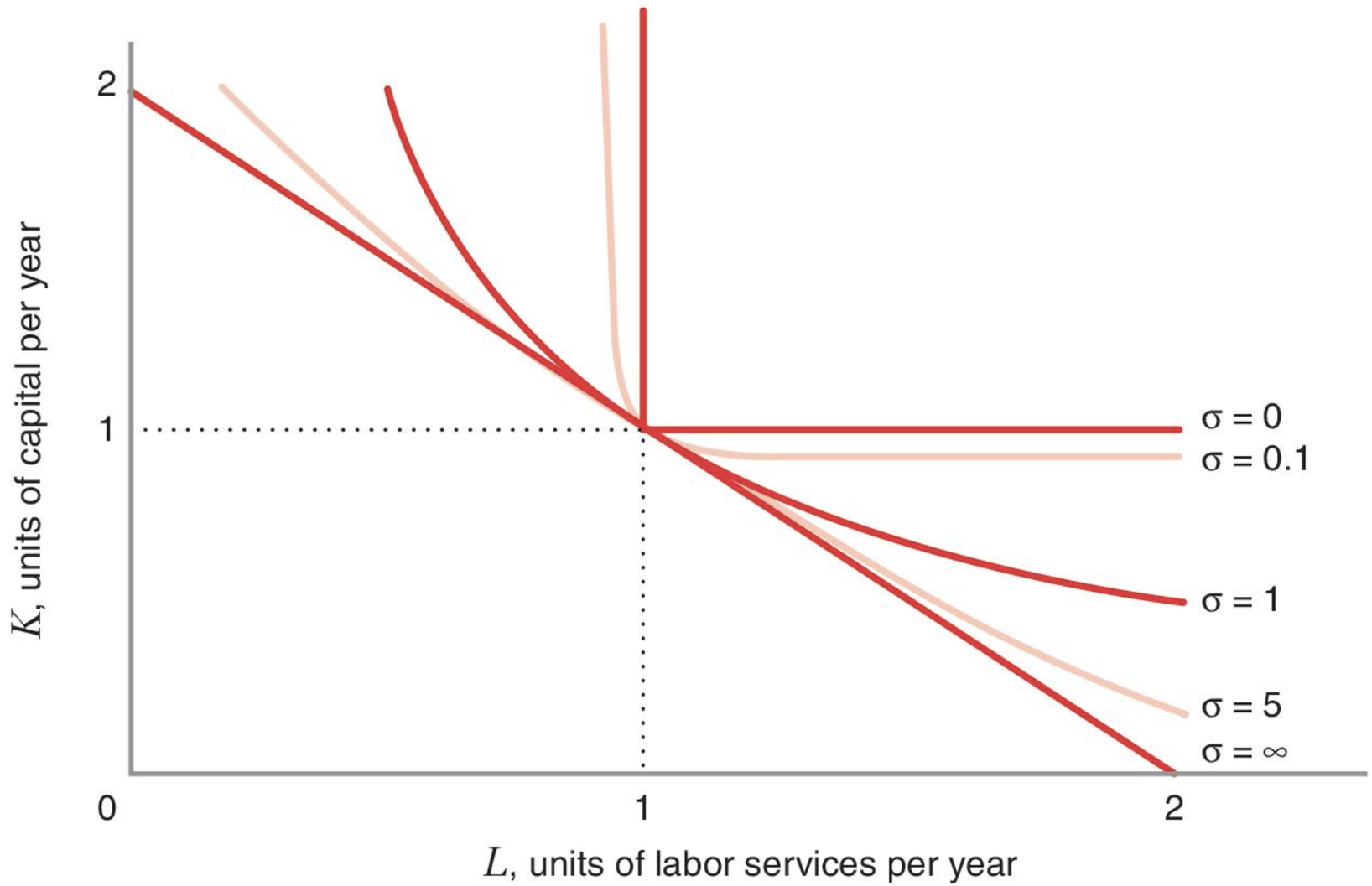
$$= \frac{1}{\frac{\%\text{ change in } MRTS_{L,K}}{\%\text{ change in capital-labor ratio}}}$$

Elasticity of Substitution

$$\sigma = \frac{\%\text{ change in capital-labor ratio}}{\%\text{ change in } MRTS_{L,K}}$$

$$= \frac{1}{\frac{\%\text{ change in } MRTS_{L,K}}{\%\text{ change in capital-labor ratio}}}$$

Elasticity of Substitution, Graphically



Interpretation of Production Functions

Production Function	Elasticity of Substitution (σ)	Other Characteristics
Linear production function	$\sigma = \infty$	Inputs are perfect substitutes Isoquants are straight lines
Fixed-proportions production function	$\sigma = 0$	Inputs are perfect complements Isoquants are L-shaped
Cobb—Douglas production function	$\sigma = 1$	Isoquants are curves
CES production function	$0 \leq \sigma \leq \infty$	Includes other three production functions as special cases Shape of isoquants varie

Part III
Mathematical Deep Dive: Cobb-Douglas

Example: General Cobb-Douglas

$$f(L, K) = AL^{\alpha}K^{\beta}$$

Calculate:
MRTS
Elasticity of Substitution
Returns to Scale

See “Derivations” PDF for detailed notes! :)