Group Work

Econ 50 - Lecture 3

January 12, 2016

1. **Calculating elasticity of a linear demand curve**

   Use the point method to find the price elasticity of demand if $Q^D(P) = 12 - 2P$.

   The general formula for the "X elasticity of Y" is

   $$\epsilon_{Y,X} = \frac{dY/Y}{dX/X} = \frac{dY}{dX} \times \frac{X}{Y}$$

   where Y is the dependent (endogenous) variable and X is the independent (exogenous) variable. In this case we're looking for the price elasticity of demand, so the endogenous variable Y is Q^D and the exogenous variable X is P. **Note that this is true even though we plot P on the vertical ("y") axis and Q^D on the horizontal ("x") axis!!!**. Thus the formula in this case is:

   $$\epsilon_{Q^D,P} = \frac{dQ^D}{dP} \times \frac{P}{Q^D}$$

   We want to express elasticity only in terms of the exogenous variable (in this case P). So we evaluate $\frac{dQ^D}{dP}$ and substitute that in, and also substitute the expression for Q^D in the denominator:

   $$\frac{dY}{dX} = -2$$
   $$P = P$$
   $$Q^D = 12 - 2P$$

   Plugging these into the equation gives us

   $$\epsilon_{Q^D,P} = -2 \times \frac{P}{12 - 2P} = -\frac{P}{6 - P}$$

   (a) When is ϵ perfectly inelastic?
   $|\epsilon| = 0$ when $P = 0$. This is the horizontal (quantity) intercept of the linear demand curve.

   (b) When is ϵ perfectly elastic?
   $|\epsilon| = \infty$ when $6 - P = 0$; i.e., when $P = 6$. This is the vertical (price) intercept of the linear demand curve.

   (c) When is ϵ unit elastic?
   $|\epsilon| = 1$ when $P = 6 - P$; i.e., when $P = 3$. This is the midpoint of the linear demand curve.

2. **Calculating elasticity of a general demand curve**

   Repeat question 1 for a general linear demand function $Q^D(P) = a - bP$.

   This example is worked in B&B.

3. **Calculating Elasticity for a Constant Elasticity Supply Curve**

   Consider the supply function $Q^S(P) = P^2Q$. ?

   (a) Use the midpoint method to calculate the price elasticity of supply between $P = 99$ and $P = 101$.

   For price, $\Delta P = 101 - 99 = 2$ and $P_M = \frac{1}{2}(99 + 101) = 100$.

   At $P = 99$, $Q(P) = P^2 = 99^2 = 9{,}801$. At $P = 101$, $Q(P) = 101^2 = 10{,}201$. Therefore $\Delta Q = 10{,}201 - 9{,}801 = 400$ and $Q_M = \frac{1}{2}(9{,}801 + 10{,}201) = 10{,}001$.

   Thus the midpoint method yields

$$\epsilon_{Q^S,P} = \frac{\Delta Q^S / Q^S_M}{\Delta P / P_M} = \frac{400/10{,}001}{2/100} \approx 2$$

   (b) Use the point method to calculate the price elasticity of supply at $P = 100$.

$$\epsilon_{Q^S,P} = \frac{dQ^S}{dP} \times \frac{P}{Q^S} = 2P \times \frac{P}{P^2} = 2$$

   (c) Use the log-log method to calculate the price elasticity of supply at $P = 100$.

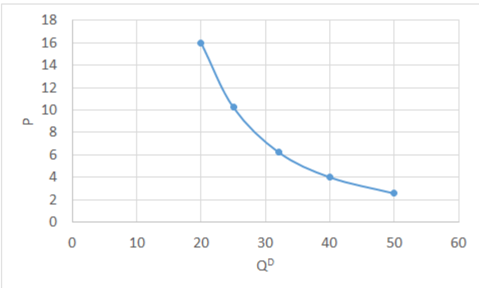
$$\epsilon_{Q^S,P} = \frac{d \ln Q^S}{d \ln P}$$

   Since $\ln Q^S = 2 \ln P$, this is also equal to 2.

4. **Deriving the linear relationship between log data**

   Yule for You, Inc. has a popular brand of "Natural Yule Logs" which burn with 50% less smoke than real wood. Ella Sticetti, the owner of Yule for You, decided to experiment a bit with pricing on her web site, randomly choosing different prices for different customers to see how responsive demand was to changes in price. The data she found is shown in the table.

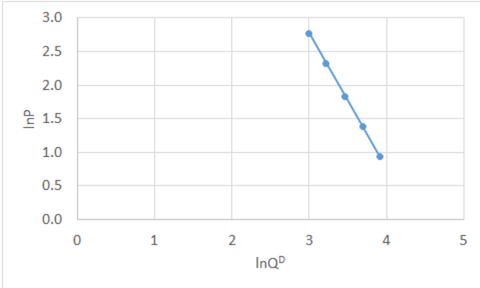
   (a) Using this data, sketch this demand curve in a diagram with Q^D on the horizontal axis and P on the vertical axis.



   (b) Now fill out the rest of the table by calculating $\ln P$ and $\ln Q^D$ for each line of the table, using a calculator or spreadsheet. Plot these points on a new graph with $\ln Q^D$ on the horizontal axis and $\ln P$ on the vertical axis.

   Completed chart and graph:

Price (P)	**Quantity Demanded (Q^D)**	$\ln P$	$\ln Q^D$
\$2.56	50	0.94	3.91
\$4.00	40	1.39	3.69
\$6.25	32	1.83	3.47
\$10.24	25	2.33	3.22
\$16.00	20	2.77	3.00



(c) All of the dots should appear in a line; find the slope, and use the point-slope formula to calculate the equation of that line and use the result to write $\ln Q^D$ as a function of $\ln P$. (Be sure to remember that the dependent variable is on the horizontal axis!)

Visually the slope $= -2$ and the intercept $= 8.76$. However, remember that we're plotting $\ln P$ on the vertical axis and $\ln Q^D$ on the horizontal axis. That means that the equation of the line is given by

$$\ln P = -2 \ln Q^D + 8.76$$

We're interested in expressing quantity as a function of price, though, so we want to rearrange the equation to read

$$\ln Q^D = \frac{1}{2}(8.76 - \ln P) = 4.38 - \frac{1}{2} \ln P$$

(d) Write down the demand curve that Yule for You faces for their Natural Yule Logs, $Q^D(P)$. (Hint: what's the inverse function of a natural log...?)

The inverse of the natural log is e. Therefore we raise e to the power of each side of the equation:

$$e^{\ln Q^D} = e^{4.38 - \frac{1}{2} \ln P} = 80P^{-\frac{1}{2}}$$

since $e^{4.38} = 80$.

(e) What is Yule for You's price elasticity of demand for Natural Yule Logs at $P = \$4.00$? What about at $P = \$6.25$?

$$\epsilon = \frac{d \ln Q^D}{d \ln P} = -\frac{1}{2}$$

The elasticity is independent of the value of P.