

Homework 4

Econ 50 - Stanford University - Winter Quarter 2015/16

Due at the beginning of section on Tuesday, February 9

Exercise 1: Math warmup

In Lecture 9 we will show how to derive the Hicksian demand from the Marshallian demand. However, the reverse is also possible.

For the canonical utility function $u(x, y) = \alpha \ln x + (1 - \alpha) \ln y$:

- Solve the cost-minimization problem $\min_{x,y} P_x x + P_y y$ subject to the utility constraint $u(x, y) = U$ to find the Hicksian demand functions $x^H(P_x, P_y, U)$ and $y^H(P_x, P_y, U)$.
- Plug the optimal values $x^H(P_x, P_y, U)$ and $y^H(P_x, P_y, U)$ back into the objective function $P_x x + P_y y$ to get the expenditure function $E(P_x, P_y, U) = P_x x^H(P_x, P_y, U) + P_y y^H(P_x, P_y, U)$.
- Set income I equal to the expenditure function $E(P_x, P_y, U)$ and solve for U to get the indirect utility function $V(P_x, P_y, I)$.
- Substitute this value of U into the original Hicksian demand functions to obtain the Marshallian demand functions $x^*(P_x, P_y, I)$ and $y^*(P_x, P_y, I)$.

Exercise 2: It's all the same to me (Lecture 8)

This was a midterm question last year.

Nick cannot for the life of him tell the difference between Coke and Pepsi; he views them as perfect substitutes for one another.

- Clearly draw his demand curve for Coke on a carefully labeled diagram.
- Pick any point on the demand curve above, and label it point A. Use a budget-line/indifference curve diagram to explain what's going on at that point.
- Explain what would happen if the price of Coke dropped a little bit (say, a penny) from the point you chose in part (b).

Exercise 3: Define and Quantify (Lectures 9 & 10)

Suppose Buster's preferences over bats (x) and mitts (y) are summarized by the utility function $u(x, y) = x + 2y$.

- For review, write down Buster's demand functions, $x^*(P_x, P_y, I)$ and $y^*(P_x, P_y, I)$. Use these demand functions to write down Buster's indirect utility function $V(P_x, P_y, I)$, his expenditure function $E(P_x, P_y, U)$, and his "Hicksian" demands, $x^H(P_x, P_y, U)$ and $y^H(P_x, P_y, U)$.
- Calculate and describe the substitution and income effects (e.g. "from Bundle A to Bundle B; from Bundle B to Bundle C") when the price of bats increases from $P_{x_1} = 1$ to $P_{x_2} = 2$, given that the price of mitts remains at $P_y = 3$ and Buster has a total income of $I = 60$ available for spending on bats and mitts. Illustrate your answer with a Slutsky diagram, drawn by hand.
- Numerically, calculate Buster's compensating variation (CV) and equivalent variation (EV) for the price change described in part (b). Show your work.

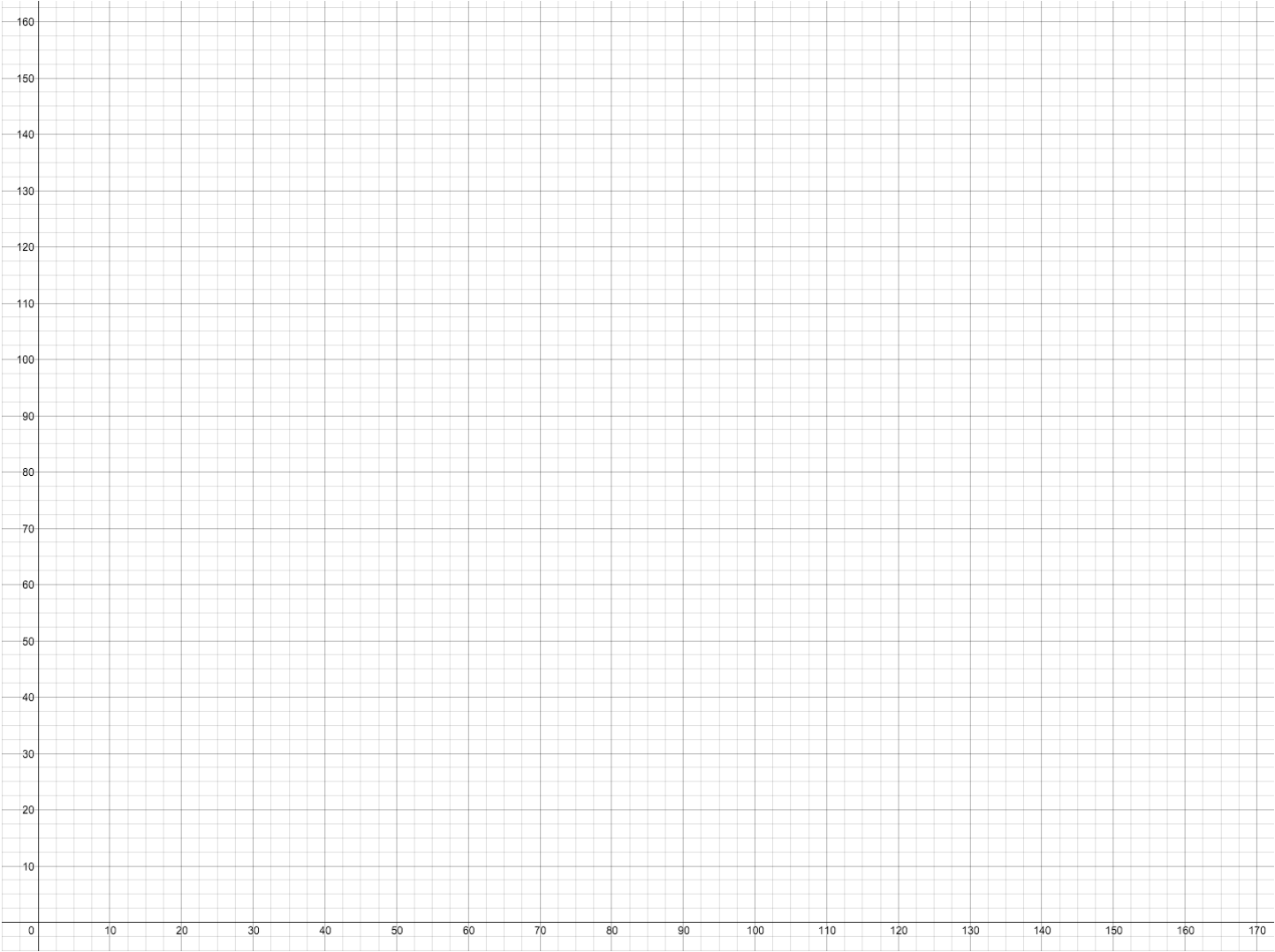
Exercise 4: A calculation and drawing exercise (Lectures 8, 9 & 10)

OK, this is a super long question, but it has you do absolutely everything you need to do to understand the relationship between the Slutsky diagram, income and substitution effects, Hicksian and Marshallian demand...so take your time, do it carefully, and really take the time to understand the steps it guides you through. You should be able to do parts (a)-(e) after Lecture 8, parts (f)-(j) after Lecture 9, and parts (k) and (l) after Lecture 10. The use of colored pencils is highly recommended!

Fred's utility function is $u(x, y) = xy^2$. He has income $I = 120$ and faces prices $P_x = 1$ and $P_y = 1$. In this exercise you will draw and explain two large diagrams illustrating an increase in the price of X: one in $X - Y$ space, and one in $P_x - Q_x$ space. Please use the template in this homework (which you can print out as many times as you need!). Note: it's easiest to draw this graph if you do the steps in order...

- (a) Draw Fred's initial budget line if $P_x = 1$, $P_y = 1$, and $I = 120$; label it BL_A . Calculate and plot Fred's initial optimal consumption bundle; label it A . Plot the corresponding point along Fred's Marshallian demand curve in the lower diagram; also label that point A .
- (b) Draw Fred's new budget line if $P_x = 8$, $P_y = 1$, and $I = 120$; label it BL_C . Calculate and plot Fred's new optimal consumption bundle at the new price of X; label it C . Plot the corresponding point along Fred's Marshallian demand curve in the lower diagram; also label that point C .
- (c) Derive the equation for Fred's price-consumption curve for good X, holding $P_y = 1$ and $I = 120$ constant. Plot this in the upper diagram and label it PCC_x .
- (d) Derive the equations for Fred's income-consumption curves when $P_x = 1$ and when $P_x = 8$, holding $P_y = 1$ and $I = 120$ constant. Plot these in the upper graph and label them ICC_1 and ICC_8 respectively.
- (e) Derive the equation for Fred's Marshallian demand curve for good X, holding $P_y = 1$ and $I = 120$ constant. Plot this curve in the lower graph and label it D^M .
- (f) Derive Fred's indirect utility function. Determine Fred's utility at point A and at point C . Write down the equations for the indifference curves passing through those two points.
- (g) Calculate the point at which the indifference curve passing through point A intersects the ICC passing through point C . Add this point to your top diagram, and label it point B . In the bottom diagram, add a new point B with the same quantity of X as in this bundle B , at a price of $P_x = 8$.
- (h) Calculate the point at which the indifference curve passing through point C intersects the ICC passing through point A . Add this point to your diagram, and label it point E . In the bottom diagram, add a new point E with the same quantity of X as in this bundle E , at a price of $P_x = 1$.
- (i) Derive Fred's expenditure function. Using this expenditure function, determine the income $I_{A,8}$ required to afford the utility at point A if $P_x = 8$, and plot the budget line $8x + y = I_{A,8}$ in the top graph. Confirm that it passes through point B , and label it BL_B . Similarly, find the income $I_{C,1}$ required to afford the utility at point C if $P_x = 1$, and plot the budget line $x + y = I_{C,1}$ in the top graph; confirm that this passes through point E , and label it BL_E .
- (j) Carefully draw Fred's indifference curves passing through the points A , B , C , and E in the upper diagram. (Hint: How many are there? Which lines are they tangent to?)
- (k) Derive Fred's Hicksian demand function. Carefully draw Fred's Hicksian demand curves passing through the points A , B , C , and E in the lower diagram. (Hint: How many are there? What is their relationship with the Marshallian demand function?)
- (l) Compute the change in Fred's consumer surplus, his compensating variation, and his equivalent variation associated with this price change. Illustrate these in the bottom diagram.

Indifference Curves and Budget Constraints; ICC; Slutsky Decomposition



Marshallian and Hicksian Demands; Compensating and Equivalent Variation

