Kinks and Corners, with Applications

Econ 50 | Lecture 7 | January 26, 2016

Lecture

Group Work

- Kinks and corners
- Buying and selling
- Intertemporal consumption and interest rates

Trading game

Part I: Kinks and Corners

The Good Case

IF

- The consumer's preferences over goods X and Y are continuously differentiable, strictly monotonic, and strictly convex
- The indifferences curves
 do not cross the axes
- The budget constraint is a simple straight line

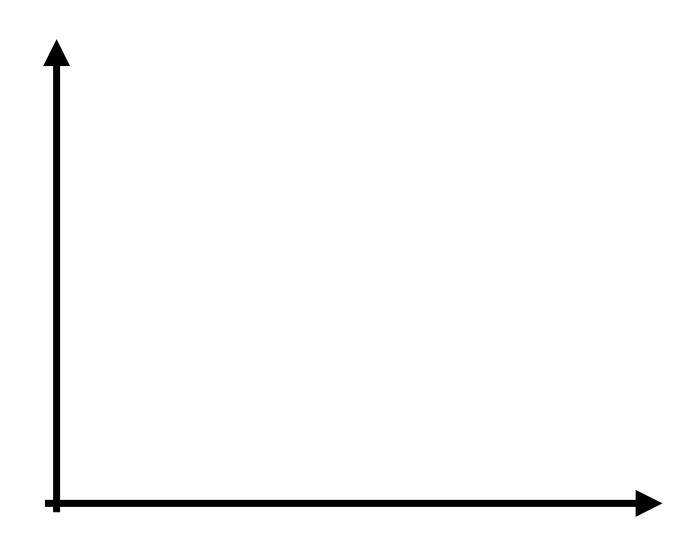
THEN THE SOLUTION...

- will be an interior solution (involve strictly positive quantities of X and Y)
- can be found using the Lagrange method
- is characterized by:

$$MRS_{x,y} = P_x/P_y$$

 $P_x x + P_y y = I$

The Good Case



IF

- The consumer's preferences over goods X and Y are continuously differentiable, strictly monotonic, and strictly convex
- The indifferences curves denot cross the axes
- The budget constraint is a simple straight line

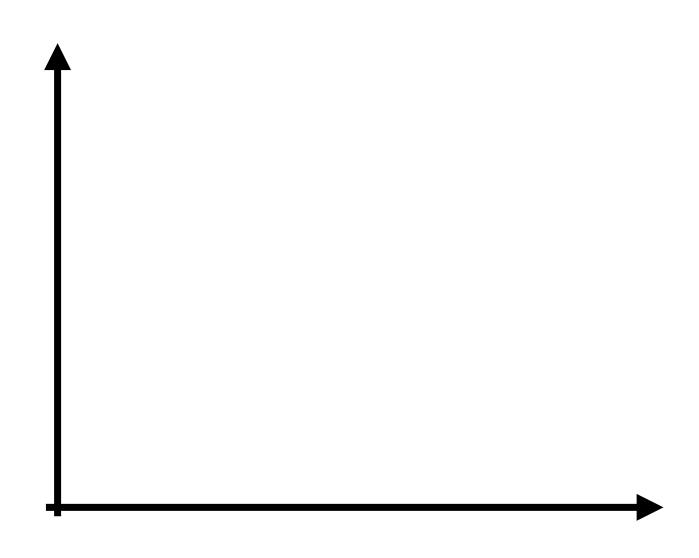
THEN THE SOLUTION...

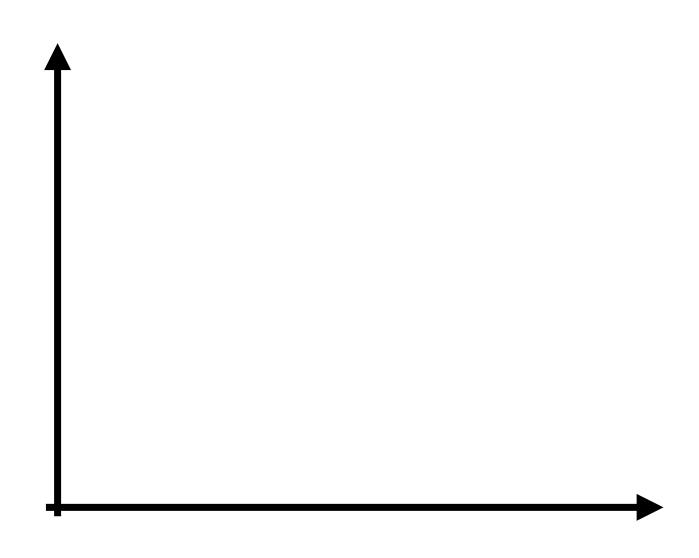
- Mightbe an interior solution (involve strictly positive quantities of X and Y)
- can be found using the Lagrange method

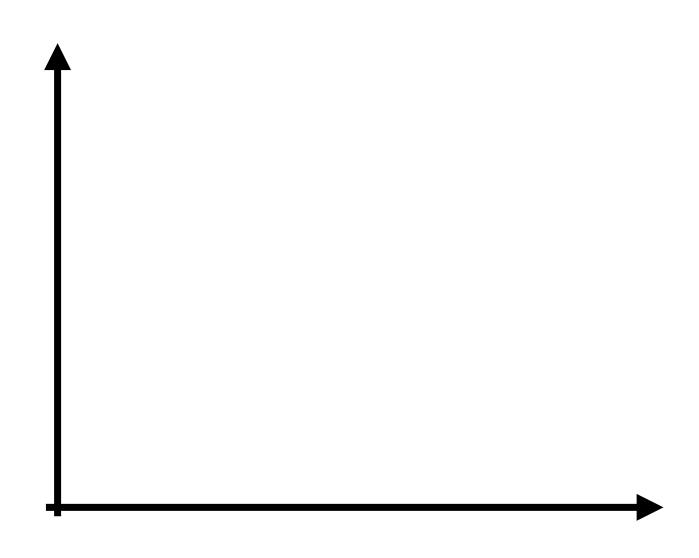
Might be characterized by:

$$MRS_{x,y} = P_x/P_y$$

 $P_x x + P_y y = I$







Approach for Smooth Preferences with Potential Corners

IF

- The consumer's preferences over goods X and Y are continuously differentiable, strictly monotonic, and strictly convex
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 denot cross the axes
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THEN...

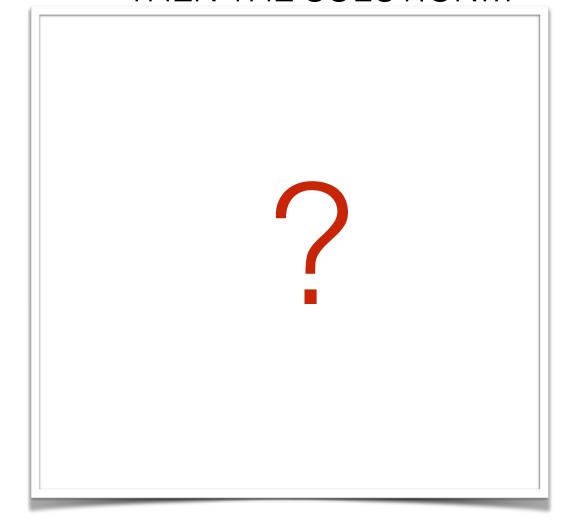
- Use the Lagrange method to find the point of tangency
- (the point characterized by:) $MRS_{x,y} = P_x/P_y$ $P_x x + P_y y = I$
- · If that point lies outside the domain of the budget constraint, go to the "closest corner"

Kinked Budget Constraints

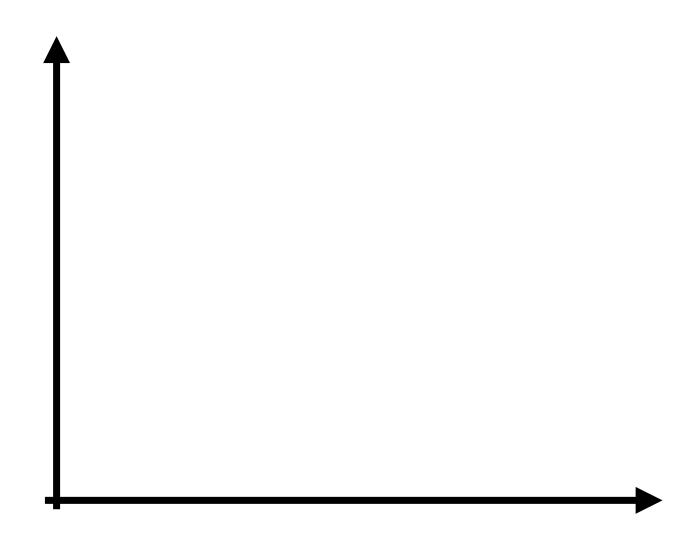
IF

- The consumer's preferences over goods X and Y are continuously differentiable, strictly monotonic, and strictly convex
- The indifferences curves
 denot cross the axes
 may or may not
 - The budget constraint is NOT a simple straight line

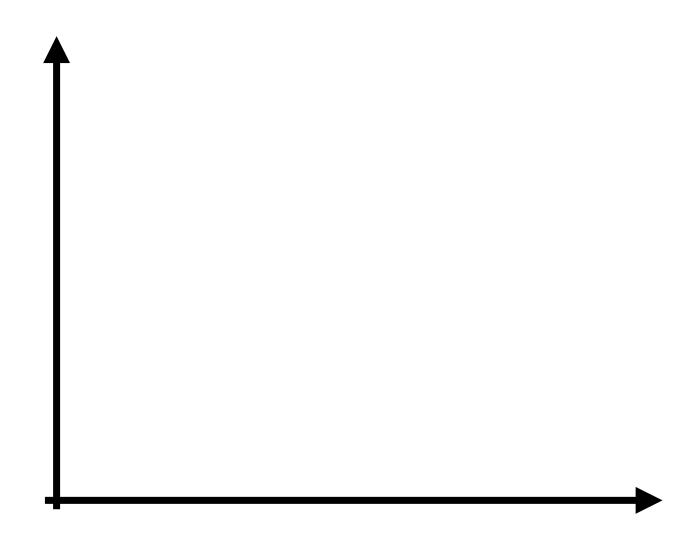
THEN THE SOLUTION...



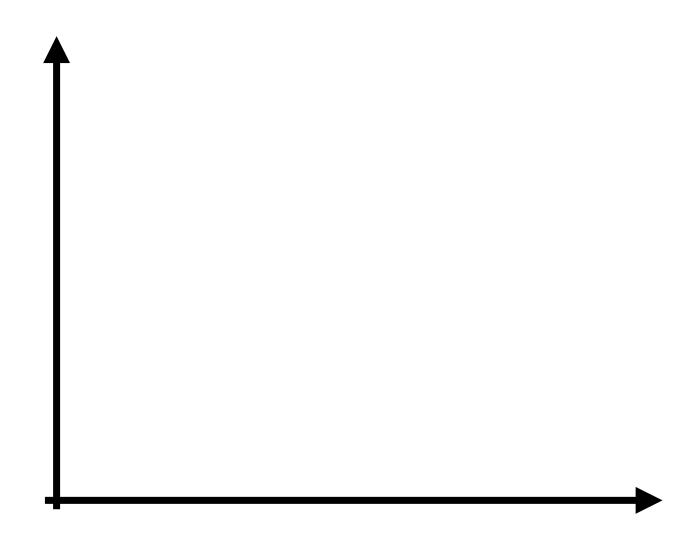
Kinked Budget Constraint



Kinked Budget Constraint



Kinked Budget Constraint



Approach for Kinked Budget Constraints

IF

- The consumer's preferences over goods X and Y are continuously differentiable, strictly monotonic, and strictly convex
- The indifferences curves
 denot cross the axes
 may or may not
 - The budget constraint is NOT a simple straight line

THEN...

- Analyze each segment of the budget constraint
- In each case, the solution may be interior to the segment, or at the end (corner) of the segment
- Compare the sub-solutions and take the one with the highest utility.

```
optimalBundle(budget:BudgetConstraint) {
    var u = this;
    var candidateBundles: TwoGoodBundle[] = budget.budgetSegments.map(
        function(segment) { return u.optimalBundleAlongSegment(segment)}
    );
    var maxUtilityBundle = candidateBundles[0];
    candidateBundles.forEach(function(bundle) {
        if(u.utility(bundle) > u.utility(maxUtilityBundle)) {
            maxUtilityBundle = bundle;
    });
    return maxUtilityBundle;
optimalBundleAlongSegment(budgetSegment:BudgetSegment) {
    var u = this;
    var constrainedX, unconstrainedX;
    unconstrainedX = u._unconstrainedOptimalX(budgetSegment);
    constrainedX = budgetSegment.xDomain.closestValueTo(unconstrainedX);
    return {x: constrainedX, y: budgetSegment.linear.yValue(constrainedX)};
```

 $MRS_{x,y} > \frac{P_x}{P_y}$

YOU LIKE X RELATIVE TO Y MORE THAN THE MARKET AT YOUR CURRENT BUNDLE

MOVE RIGHT ALONG
THE BUDGET CONSTRAINT

 $MRS_{x,y} < \frac{P_x}{P_y}$

YOU LIKE X RELATIVE TO Y
LESS THAN THE MARKET
AT YOUR CURRENT BUNDLE

 $MRS_{x,y} > \frac{P_x}{P_y}$

YOU LIKE X RELATIVE TO Y
MORE THAN THE MARKET
AT YOUR CURRENT BUNDLE

MOVE RIGHT ALONG
THE BUDGET CONSTRAINT

$$MRS_{x,y} < \frac{P_x}{P_y}$$

YOU LIKE X RELATIVE TO Y
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MOVE RIGHT ALONG
THE BUDGET CONSTRAINT

$$MRS_{x,y} < \frac{P_x}{P_y}$$

YOU LIKE X RELATIVE TO Y **LESS** THAN THE MARKET

AT YOUR CURRENT BUNDLE

$$MRS_{x,y} > \frac{P_x}{P_y}$$

YOU LIKE X RELATIVE TO Y
MORE THAN THE MARKET
AT YOUR CURRENT BUNDLE

MOVE RIGHT ALONG
THE BUDGET CONSTRAINT

$$MRS_{x,y} < \frac{P_x}{P_y}$$

YOU LIKE X RELATIVE TO Y
LESS THAN THE MARKET
AT YOUR CURRENT BUNDLE

Everything goes wrong, Example 1: Kinked Budget Constraint and Perfect Complements



Everything goes wrong, Example 1: Kinked Budget Constraint and Perfect Complements



Everything goes wrong, Example 2: Kinked Budget Constraint and Perfect Substitutes



Everything goes wrong, Example 2: Kinked Budget Constraint and Perfect Substitutes



Part II: Endowment Economies

Endowment Budget Constraint

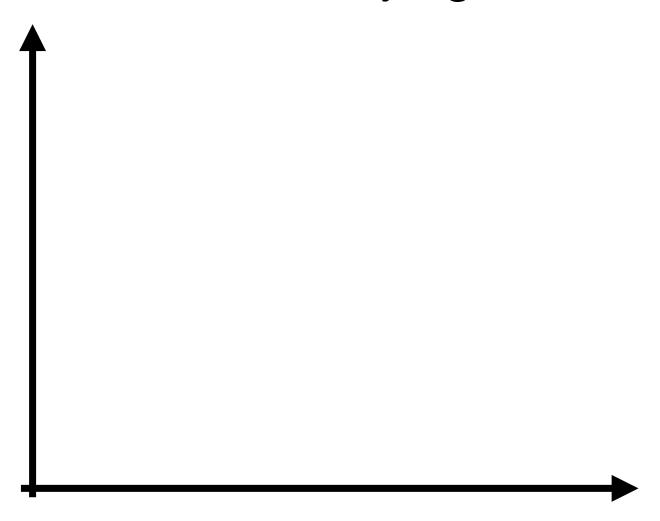
- Instead of starting with money income (I), you start with an endowment of X and Y
- You can buy and sell **X** and **Y** for prices P_x and P_y
- If you sold your endowment for money, you would get:

Therefore your budget constraint may be written:

Endowment Budget Constraint



Endowment Budget Constraint: Different Prices for Buying and Selling



Part III: Intertemporal Consumption and Interest Rates

Intertemporal Choice

- Instead of goods X and Y, our goods are
 "Consumption this year" and "Consumption next year"
- You have an endowment of "Income this year" and "Income next year"
- You can spend more than this year's income (C1 > I1)
 by borrowing against next year's income
 OR

You can spend more than next year's income (C2 > I2) by **saving** some of this year's income

Savings Budget Constraint

Borrowing Budget Constraint

INTERTEMPORAL BUDGET CONSTRAINTS

For Savings:

For Borrowing:

$$(1+r)c_1+c_2=(1+r)I_1+I_2$$

$$c_1 + \frac{c_2}{1+r} = I_1 + \frac{I_2}{1+r}$$

"Future Value"

"Present Value"

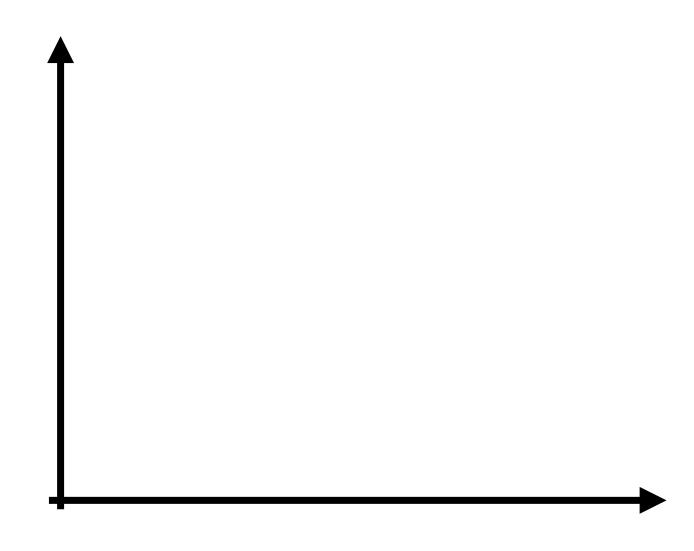
If save 100%:

If borrow 100%:

$$c_2 = (1+r)I_1 + I_2$$

$$c_1 = I_1 + \frac{I_2}{1+r}$$

Intertemporal Budget Constraint



Intertemporal Budget Constraint: Different Interest Rates for Saving and Borrowing

