Homework 1

Econ 50 - Stanford University - Winter Quarter 2015/16

Due at the beginning of section on Friday, January 15

Exercise 1: Constrained Optimization with One Variable (Lecture 1)

Recall on the six functions from Group Work, Exercise 1:

i. $f(x) = 5 + 4x - x^2$

ii. $f(x) = 10 - |2 - x|$

iii. $f(x) = 9 - (x - 11)^2$

iv. $f(x) = 1 + \frac{1}{5}(x - 5)^2$

v. $f(x) = 10 - x$

vi. $f(x) = 3$

Thinking about these six functions, answer the following questions.

(a) For each function, write the "optimal" value(s) of x on the domain $0 \leq x \leq 10$; i.e., the value(s) that maximize(s) $f(x)$ on that domain. (This is what we did in group work in class.)

(b) For each of the cases above, explain why taking the derivative $f'(x)$ and setting it equal to zero works, or does not work.

(c) More generally, under what conditions will setting $f'(x) = 0$ get you to the solution to that problem?

(d) Write down a series of steps (i.e., an algorithm or flowchart) which, if followed, would allow you to find the optimal value(s) of x for each of the functions above. Here are some steps to get you started:

S1: Is $f(x)$ continuously differentiable on the domain $0 \leq x \leq 10$?
If so, go to S2.
If not, go to S3.

S2: Take the derivative $f'(x)$. How many values of x are there that set $f'(x) = 0$?
If none, go to S4.
If one, go to S5.
If a finite number more than one, go to S6.
If an infinite number, then $f(x)$ is a horizontal line and all values of x are optimal [END].

S3: (fill in the rest)

(e) Can you draw or describe a different kind of function that your algorithm would not work for?

Exercise 2: Constrained optimization with more than one variable (Lecture 1)

Answer Besanko and Braeutigam, 5e, Problem 1.4. [I'll type this up soon in case you don't own the textbook.] Also answer the following:

(e) Suppose $P_E = \$36$ and $P_L = \$9$. Use the method of Lagrange multipliers to find this firm's optimal choice of E and L. Show your work. informally "check" your answer by showing two other combinations of E and L that would produce exactly 200 units of telephone service but that would be more costly than the one you identified in your solution.

Exercise 3: Market Supply and Producer Surplus (Lecture 2)

You might want to refer to the lecture notes posted after class to help you on this question...

Suppose there are two types of firms in the marketplace. The first type has a supply curve given by $q_1(P) = P$. The second type has a supply curve given by $q_2(P) = \sqrt{P}$. However, the second type has a "shutdown" price of $P = 9$; that is, it produces zero output if the price falls below \$9. There are 30 firms of each type.

(a) Sketch the individual supply curve for a representative firm of each type.

(b) Sketch the market supply curve.

(c) Suppose the price rises from \$4 to \$8. Illustrate the change in producer surplus and calculate its magnitude.

(d) Suppose the price rises from \$8 to \$16. Illustrate the change in producer surplus and calculate its magnitude.

Exercise 4: Parsing Demand Expressions (Lecture 2)

Consider the following expressions representing individual demand for good X:

i. $q_x^D(P_x, P_x, I) = \frac{I}{P_x+P_y}$

ii. $q_x^D(P_x, P_y, I) = \frac{I}{2P_x}$

iii. $q_x^D(P_x, P_y, I) = \frac{I}{P_x}$ if $P_x < P_y$, otherwise 0

iv. $q_x^D(P_x, P_y, I) = \left(\frac{P_y}{P_x+P_y}\right)\frac{I}{P_x}$

where I is income, P_x is the price of good X, and P_y is the price of another good.

(a) For each expression, identify whether two goods are complements, substitutes, or neither; explain how you arrived at your conclusion.

(b) To the best of your ability, give a verbal description of the kind of consumer behavior each expression describes. (For example: one describes a consumer who always spends half their income on good X...which one?)

Exercise 5: How elastic are those sweatpants? (Lecture 3)

This was a 15-point question on the midterm last year...

Suppose the market demand for sweatpants (good X) is given by $Q_x = 20 + I - P_x - \frac{1}{2}P_y$, where I is the average income of consumers, P_x is the price of sweatpants, and P_y is the price of T-shirts.

(a) Compute the **own-price** elasticity of demand for sweatpants (ϵ_{Q_x,P_x}), the **cross-price** elasticity of demand for sweatpants with respect to T-shirts (ϵ_{Q_x,P_y}), and the **income** elasticity of demand for sweatpants ($\epsilon_{Q_x,I}$).

(b) On a carefully drawn diagram of the demand for sweatpants, show where the demand for sweatpants is elastic, unit elastic, and inelastic.

(c) According to this demand function, are sweatpants and T-shirts complements, substitutes, or neither? How do you know?

Exercise 6: TBD (Lecture 3)

We may add a sixth problem after Lecture 3.