Utility

Econ 50 | Lecture 5 | January 19, 2013

Lecture

- Quantifying Utility
- MRS, with Math
- Five New-ish Friends

Group Work

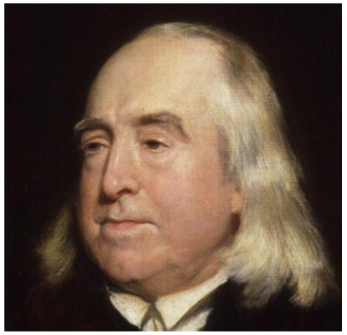
- Deep dive into three utility functions

Part I: Quantifying Utility

18th/19th Centuries: Utilitarianism

"...the **greatest happiness of the greatest number** is the foundation of morals and legislation…"

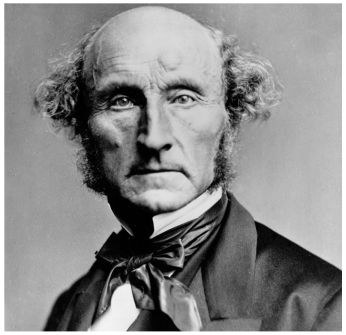
— Jeremy Bentham



18th/19th Centuries: Utilitarianism

"the utilitarian standard... is **not the agent's own greatest happiness**, but the **greatest amount of happiness, altogether**."

— John Stuart Mill



20th Century: Revealed Preference

"**Desires cannot be measured directly**, but only indirectly, by the outward phenomena to which they give rise...

the measure is found in the **price which a person is willing to pay** for the fulfilment or satisfaction of his desire."

— Paul Samuelson

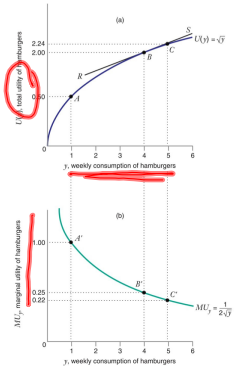
21st Century: Neuroeconomics

"When study participants sipped soda without knowing which it was, **[Coke and Pepsi] prompted equal reactions in the area of the brain associated with satisfaction**.

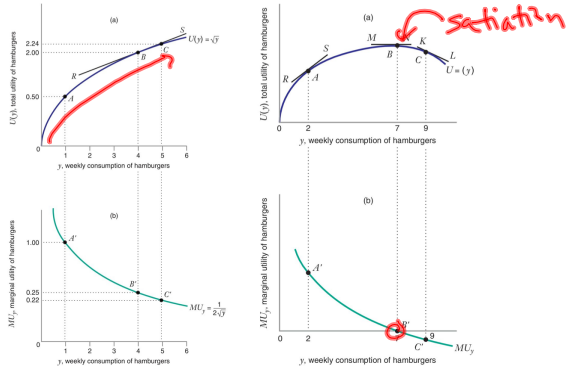
When participants knew which brand they were drinking, **Coke suddenly tasted better**."

— American Psychological Association Science Directorate, 2007

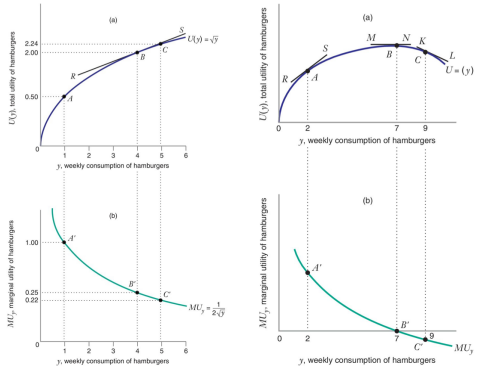
Utility Functions

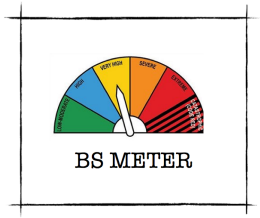


Utility Functions



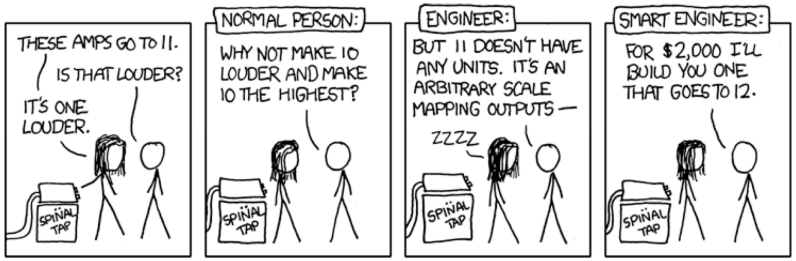
Utility Functions





Why?

https://youtu.be/KOO5S4vxi0o



http://xkcd.com/670/

Cardinality:
Numbers are important.

Cardinality:
Numbers are important.

Ordinality:
Ranking is important.

Recall: Completeness and Transitivity

- **Completeness**: any two bundles can be compared.
- **Transitivity**: if A is preferred to B, and B is preferred to C, then A is preferred to C.
- Group work: "is at least as tall as" is **complete** and **transitive**.
- Why? **Numerical comparison is complete and transitive.**

Recall: Completeness and Transitivity

- **Completeness**: any two bundles can be compared.
- **Transitivity**: if A is preferred to B, and B is preferred to C, then A is preferred to C.
- Group work: "is at least as tall as" is **complete** and **transitive**.
- Why? **Numerical comparison is complete and transitive.**

Recall: Completeness and Transitivity

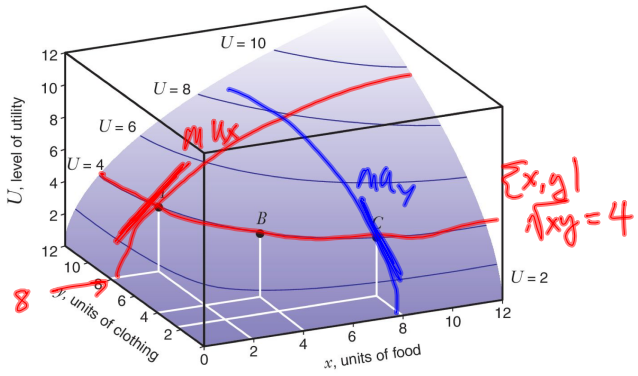
- **Completeness**: any two bundles can be compared.
- **Transitivity**: if A is preferred to B, and B is preferred to C, then A is preferred to C.
- Group work: "is at least as tall as" is **complete** and **transitive**.
- Why? **Numerical comparison is complete and transitive.**

Utility Functions That Aren't Silly

- A utility function **u()** assigns a real number to every possible bundle.

$A \succ B$ if and only if $u(A) > u(B)$.
$A \sim B$ if and only if $u(A) = u(B)$.
$A \prec B$ if and only if $u(A) < u(B)$.

Indifference Curves are Level Curves of u(x,y)



Marginal Utilities

are the partial derivatives of u().

$$MU_x = \frac{\partial u(x,y)}{\partial x} \qquad MU_y = \frac{\partial u(x,y)}{\partial y}$$

Marginal Utilities
are the partial derivatives of u().

Silly on their own

Meaningful in generating MRS

Marginal Utilities
are the partial derivatives of u().

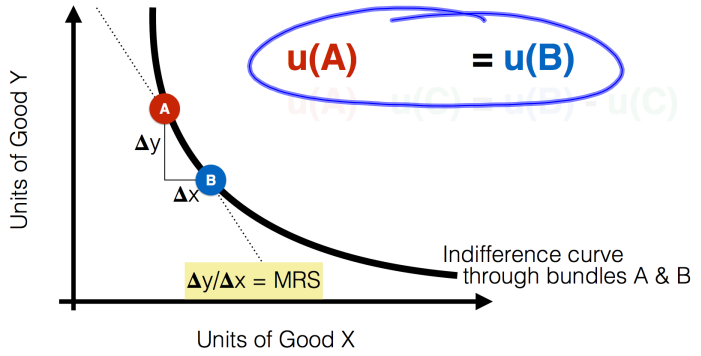
Silly on their own
Meaningful in generating **MRS**

Part II: MRS, with Math

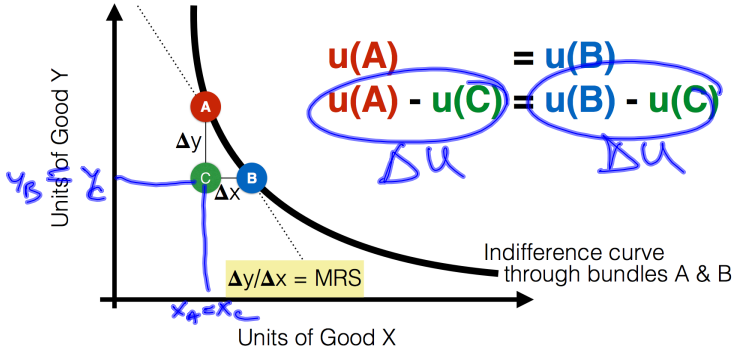
Marginal Rate of Substitution

- **Intuitively**: rate at which a consumer is willing to give up good Y to get an additional unit of good X.
- **Visually**: absolute value of the slope of an indifference curve
- **Mathematically**: $\frac{MU_x}{MU_y}$

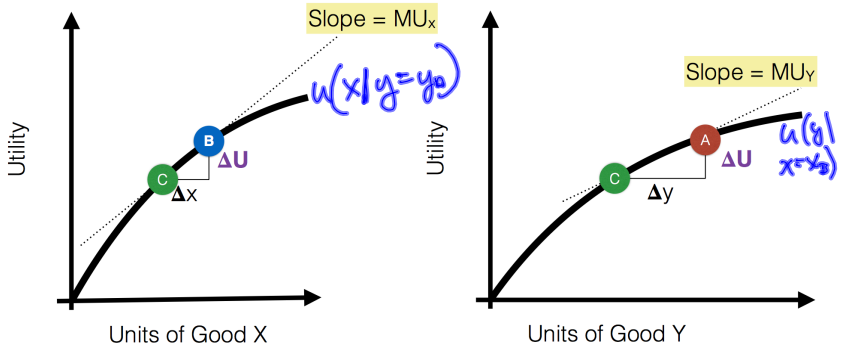
Marginal Rate of Substitution



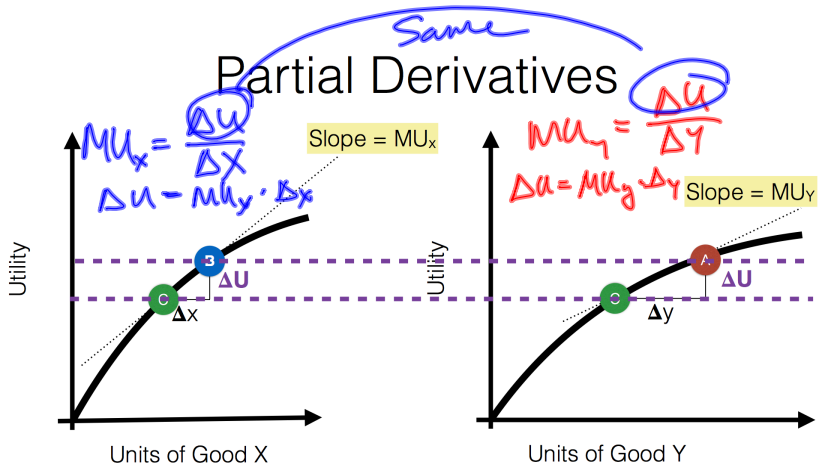
Marginal Rate of Substitution



Partial Derivatives



Partial Derivatives

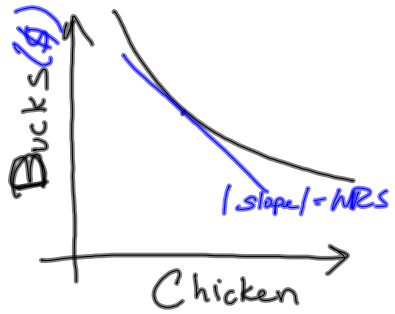


Solving for MRS

$$\Delta U = MU_x \Delta x - MU_y \Delta y$$

$$MRS_{x,y} = \frac{MU_x}{MU_y} = \frac{\Delta y}{\Delta x}$$

A Special Case: "Composite Good"



$MRS = \frac{MU_{ch}}{MU_{\$}}$

"How many $ would you pay for a chicken?"

Part III: Five New-Ish Friends

Utility Function 1: Cobb-Douglas

$u(x,y) = x^a y^b$

$MRS = \frac{MU_x = a x^{a-1} y^b}{MU_y = b x^a y^{b-1}} = \frac{ay}{bx}$ ← MRS

Utility Function 2: Perfect Substitutes

$u(x,y) = ax + by$

$MRS = \frac{MU_x = a}{MU_y = b}$

Utility Function 3: Perfect Complements

$u(x,y) = \min\left\{\frac{x}{a}, \frac{y}{b}\right\}$

$a=2$, $b=1$: $u(10,10) = \min\{5, 10\} = 5$

$u(12,10) = \min\{6, 10\} = 6$

$u(20,10) = \min\{10, 10\} = 10$

$u(40,10) = \min\{20, 10\} = 10$

$u(x,y) = \min\left\{\frac{x}{a}, \frac{y}{b}\right\} = \begin{cases} x/a & \text{if } x/a < y/b \\ y/b & \text{if } x/a > y/b \end{cases}$

$MU_x = \begin{cases} 1/a & \text{if } \frac{x}{a} < \frac{y}{b} \\ 0 & \text{if } \frac{x}{a} > \frac{y}{b} \end{cases}$

$MU_y = \begin{cases} 0 & \text{if } \frac{x}{a} < \frac{y}{b} \\ 1/b & \text{if } \frac{x}{a} > \frac{y}{b} \end{cases}$

$MRS = \infty$ if $\frac{x}{a} < \frac{y}{b}$, 0 if $\frac{x}{a} > \frac{y}{b}$

$u(x,y) = x^a y^b$ $\quad \alpha = \frac{a}{a+b}$

$MRS = \frac{a}{b}\frac{y}{x}$ $\quad 1-\alpha = \frac{b}{a+b}$

$u(x,y) = x^\alpha y^{1-\alpha}$

$MRS = \frac{\alpha}{1-\alpha}\frac{y}{x}$ $\quad \frac{\alpha}{1-\alpha} = \frac{\frac{a}{a+b}}{\frac{b}{a+b}} = \frac{a}{b}$

Utility Function 4: CES

$u(x,y) = (ax^r + by^r)^{1/r}$

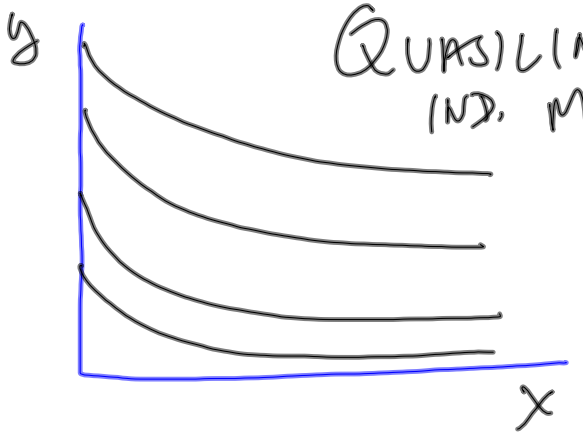
$\frac{MU_x}{MU_y} = \frac{\frac{1}{r}(ax^r+by^r)^{1/r-1} \cdot r a x^{r-1}}{\frac{1}{r}(ax^r+by^r)^{1/r-1} \cdot r b y^{r-1}} = \frac{a y^{1-r}}{b x^{1-r}}$

$r=0 \Rightarrow MRS = \frac{ay}{bx}$ (CD) | $r=1 \Rightarrow \frac{a}{b}$ (PS) | $r \to -\infty \Rightarrow$ { : (PC)

Utility Function 5: Quasilinear

$u(x,y) = a\ln x + by$

$\frac{MU_x}{MU_y} = \frac{\frac{a}{x}}{b} \Rightarrow MRS = \frac{a}{bx}$



QUASILINEAR IND. MAP