Comparative Statics II: Income and Substitution Effects

Econ 50 | Lecture 9 | February 2, 2016

Lecture

- Income and Substitution Effects: Intuitive Review
- Analyzing a Price Change: Slutsky Decomposition
- Finding the Decomposition Point: The “Dual” Problem

Group Work

- Finding the Decomposition Point: Cobb-Douglas

Part I

Income and Substitution Effects: An Intuitive Review

What happens when the price of X rises?

- Tangency condition: $\boldsymbol{MRS_{x,y} = P_x/P_y}$
    - X is now **relatively more expensive**
    - You will **substitute** Y for X
- Budget set: $\boldsymbol{P_x x + P_y y = I}$
    - You can **no longer afford** to be as happy as you were before.
    - You will **buy fewer of both goods**, relative to…some point

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Formal Definitions

- The **substitution effect** is the change in the quantity demanded resulting from a change **relative prices**, holding the **level of utility** constant

- The **income effect** is the change in the quantity demanded resulting from a change in **purchasing power**, holding **all prices** constant.

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Informal Definitions

Suppose the price of a good goes down.

You could now afford to be
just as happy as you were before

(move **along** your indifference curve)

by buying **more of that good**
and **less of other goods**
and save some money in the meantime.

SUBSTITUTION EFFECT

…but suppose you don't save the money.

You could spend that money to be
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Part II

Analyzing a Price Change: Slutsky Decomposition

Slutsky Decomposition Diagram

Point	Description	Utility	Price	Income
A	Initial Bundle	Initial Utility	Initial Price	Actual Income
B	“Decomposition” Bundle	Initial Utility	Final Price	Compensated Income
C	Final Bundle	Final Utility	Final Price	Actual Income

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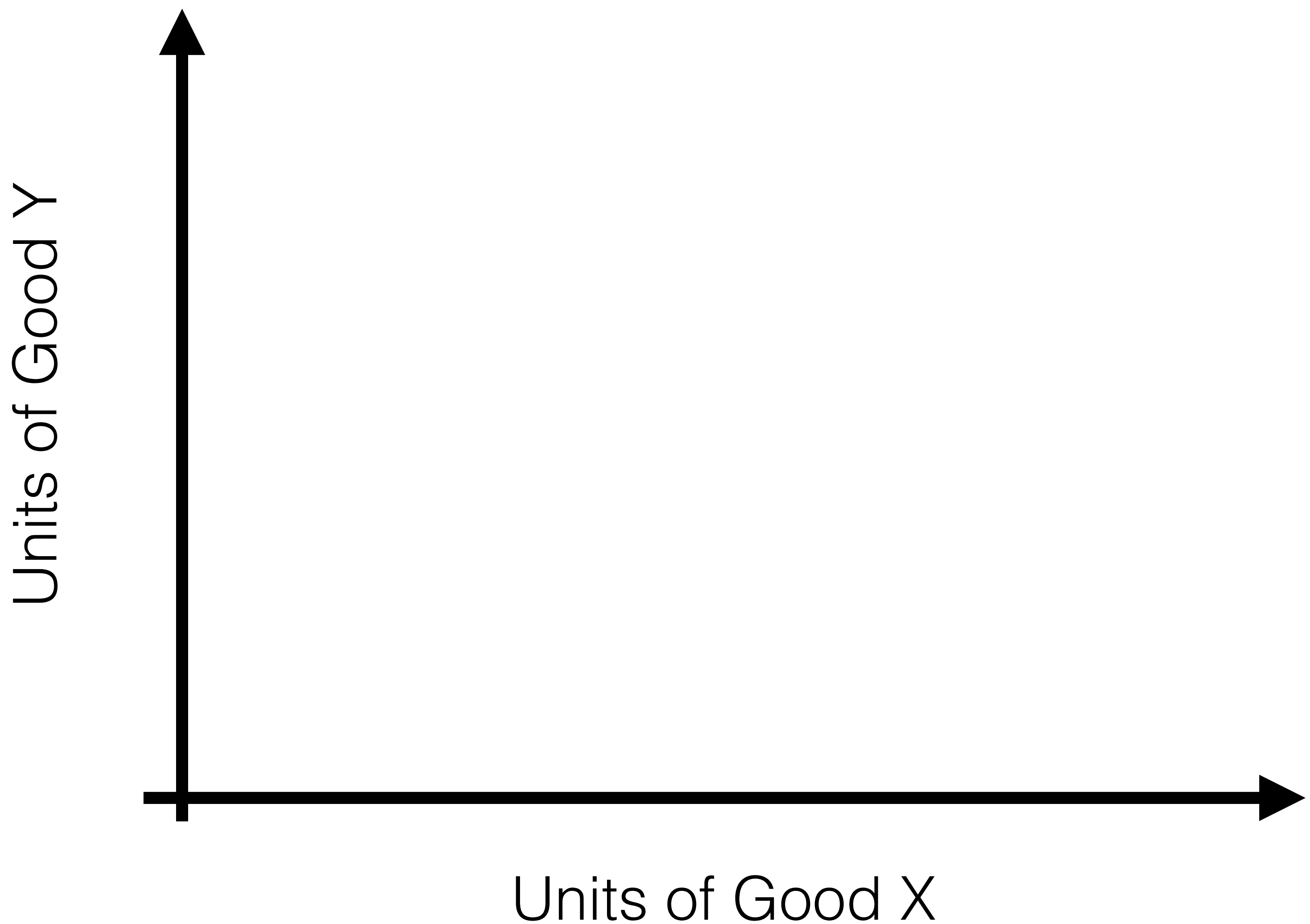
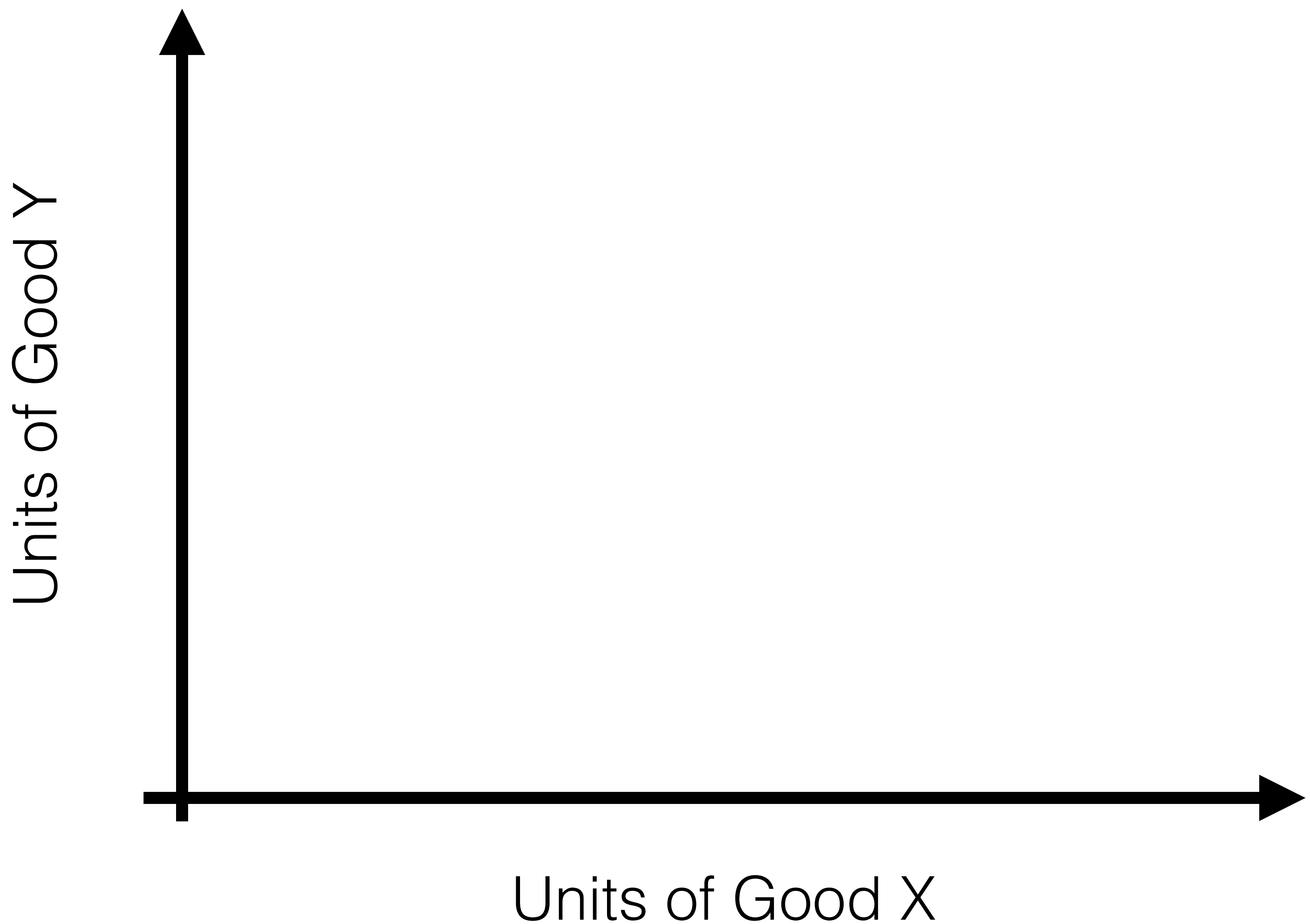
Slutsky Decomposition Diagram

	Initial Utility	**Final Utility**
Initial Price	Bundle **A** (Actual Income)	
Final Price	Bundle **B** (Compensated Income)	Bundle **C** (Actual Income)

Slutsky Decomposition: Price **Increase**



Slutsky Decomposition: Price **Decrease**



Substitutes and Complements

- Suppose the price of good X changes.
- If the **substitution effect on Y dominates the income effect on Y**, then X and Y are substitutes and the **PCC is downward sloping**.
- If the **income effect on Y dominates the substitution effect**, then X and Y are complements and the **PCC is upward sloping**.
- If the **income effect on Y exactly offsets the substitution effect**, then X and Y are independent and the **PCC is horizontal**.

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Normal, Inferior, and Giffen Goods

- Consider just the income effect of an increase in the price of X.
- If both goods are **normal**, the **final point** will have **more of both** than the **decomposition point**.
- If one good is **inferior** in the relevant income range, the **final point** will have **less of the inferior good** than the **decomposition point**.
- If good X is a **Giffen** good in the relevant income range, the **final point** will have **less of good X** than the **initial point**.

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Slutsky Diagram: Inferior and Giffen Goods

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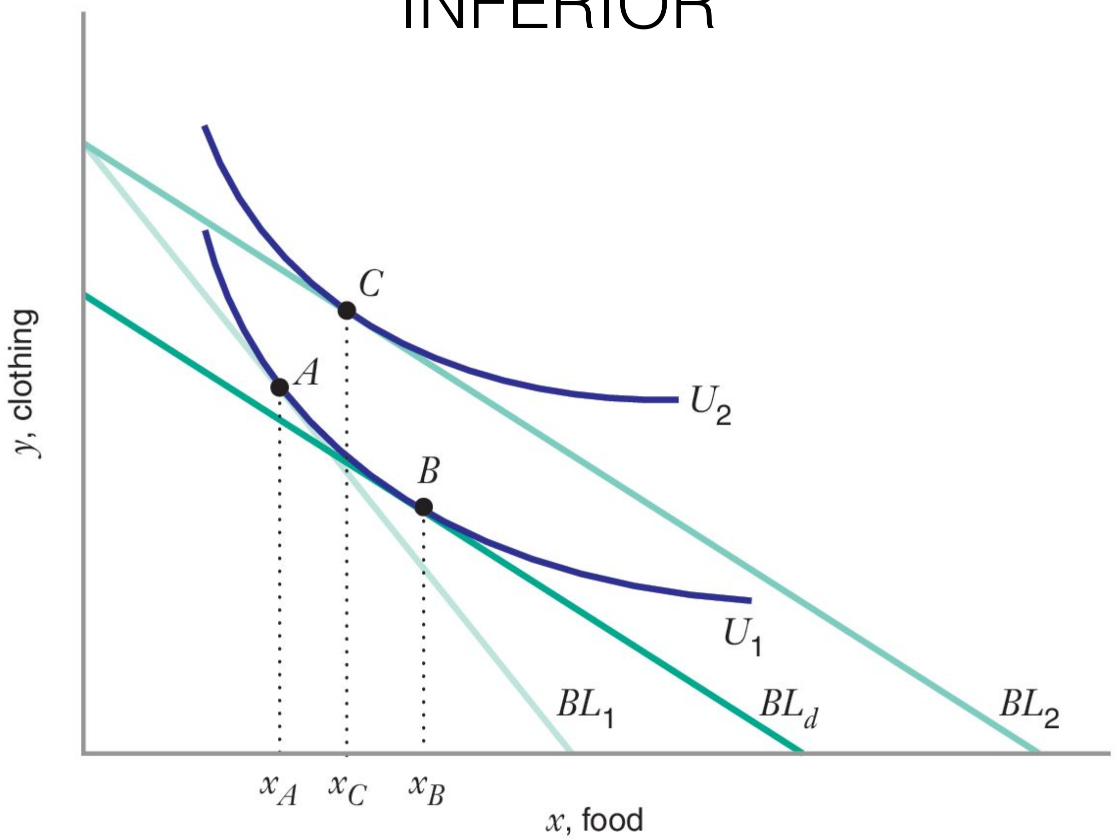

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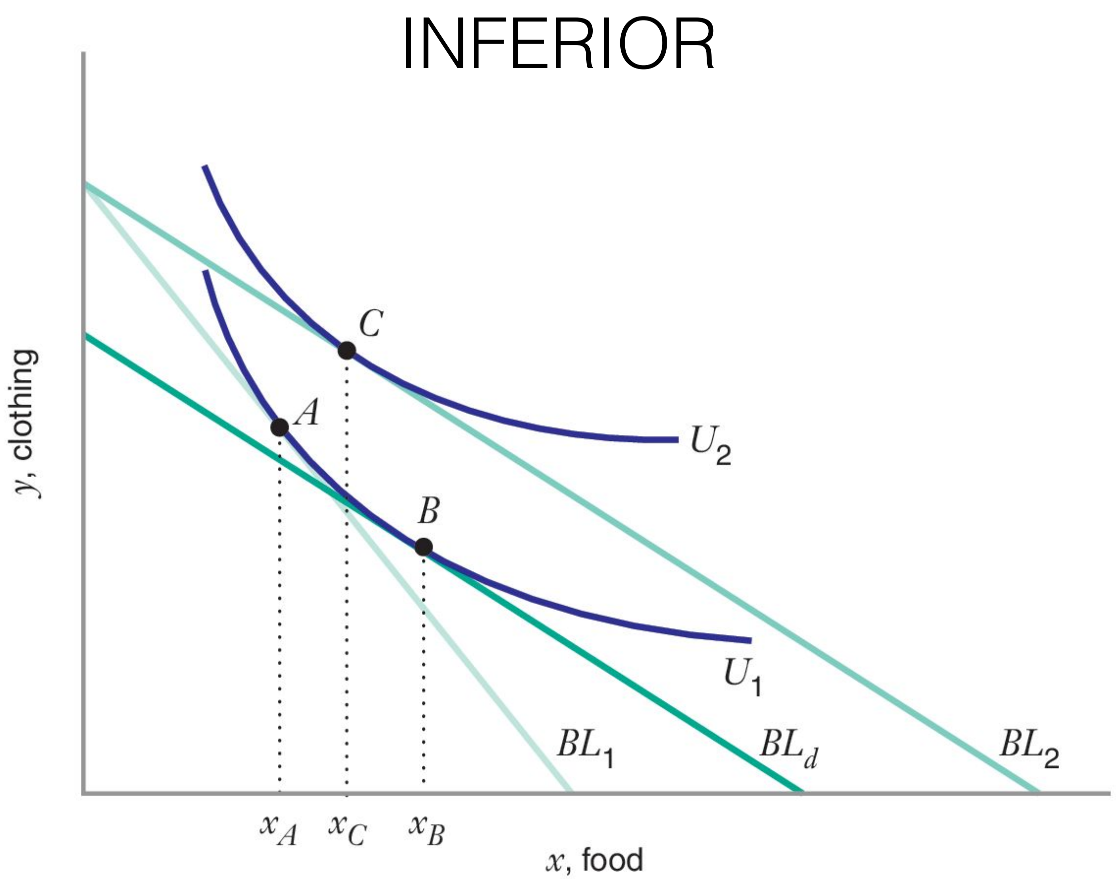

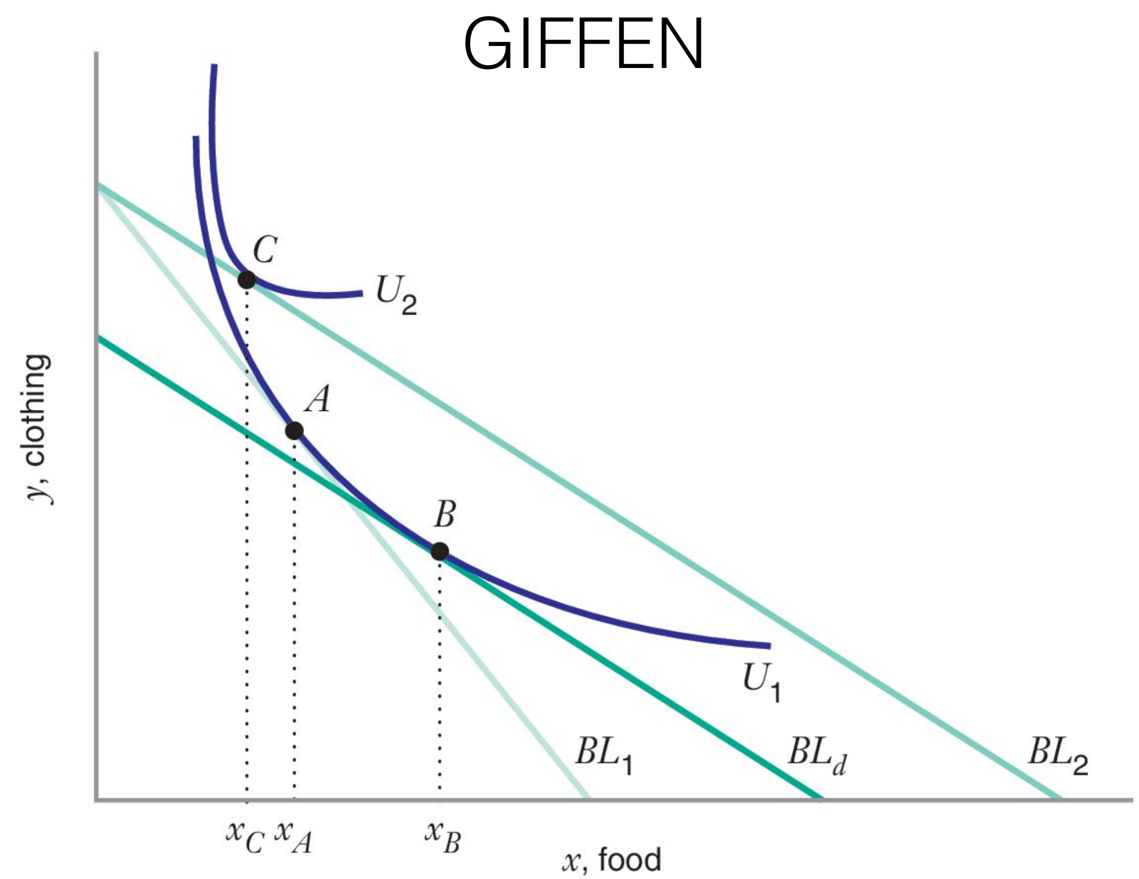

Part III

Finding the Decomposition Point

The “Dual” Problem

The **Dual Problem**: Two Equivalent Ways to Optimize

Utility Maximization	**Cost Minimization**
$\max_{x,y}\ u(x,y)$ s.t. $P_x x + P_y y = I$	$\min_{x,y}\ P_x x + P_y y$ s.t. $u(x,y) = U$

Solve for x^* and y^*; the solutions are:

Marshallian Demand Functions: $x^*(P_x, P_y, I), y^*(P_x, P_y, I)$	**Hicksian Demand Functions:** $x^*(U, P_x, P_y), y^*(U, P_x, P_y)$

Plug x^* and y^* back into the objective function:

Indirect Utility Function: $V(P_x, P_y, I) = u[x^*(P_x, P_y, I), y^*(P_x, P_y, I)]$ (Utility from utility-maximizing choice)	**Expenditure Function:** $E(U, P_x, P_y) = P_x x^*(U, P_x, P_y) + P_y y^*(U, P_x, P_y)$ (Cost of cost-minimizing choice)

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Group Work

How to Derive Hicksian Demand from Marshallian Demand

Example: Cobb-Douglas $u(x, y) = xy$

Start with Marshallian demand:	$x^* = \dfrac{I}{2P_x}, y^* = \dfrac{I}{2P_y}$
Plug (x^*, y^*) back into $u(x, y) = xy$ to find the indirect utility function:	$u(x^*, y^*) = \left(\dfrac{I}{2P_x}\right)\left(\dfrac{I}{2P_y}\right) \Rightarrow V(P_x, P_y, I) = \dfrac{I^2}{4P_xP_y}$
Set the indirect utility function equal to U and solve for I to find the expenditure function:	$\dfrac{I^2}{4P_xP_y} = U \Rightarrow I^2 = 4P_xP_yU$ $\Rightarrow E(P_x, P_y, U) = 2P_x^{\frac{1}{2}}P_y^{\frac{1}{2}}U^{\frac{1}{2}}$
Plug this "lowest cost" back into the Marshallian demand (as the income) to get the Hicksian demand:	$x^* = \dfrac{E(P_x, P_y, U)}{2P_x} = \dfrac{2P_x^{\frac{1}{2}}P_y^{\frac{1}{2}}U^{\frac{1}{2}}}{2P_x}$ $\Rightarrow x^H(P_x, P_y, U) = \left(\dfrac{P_y}{P_x}\right)^{\frac{1}{2}}U^{\frac{1}{2}}$

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$$\frac{I^2}{4P_xP_y} = U \Rightarrow I^2 = 4P_xP_yU$$

$$\Rightarrow E(P_x, P_y, U) = 2P_x^{\frac{1}{2}}P_y^{\frac{1}{2}}U^{\frac{1}{2}}$$

Plug this "lowest cost" back into the Marshallian demand (as the income) to get the Hicksian demand:

$$x^* = \frac{E(P_x, P_y, U)}{2P_x} = \frac{2P_x^{\frac{1}{2}}P_y^{\frac{1}{2}}U^{\frac{1}{2}}}{2P_x}$$

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How to Derive Hicksian Demand from Marshallian Demand

Example: Cobb-Douglas $u(x, y) = xy$

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