Cost Minimization

Econ 50 | Lecture 13 | February 18, 2016

Elasticity of Substitution

$$\sigma = \frac{\%\text{ change in capital-labor ratio}}{\%\text{ change in } MRTS_{L,K}}$$

$$= \frac{1}{\frac{\%\text{ change in } MRTS_{L,K}}{\%\text{ change in capital-labor ratio}}}$$

Calculating elasticity of MRTS with respect to K/L

Lecture

- Cost Minimization
- Expansion Paths
- Deriving Short-Run and Long-Run Total Cost Curves

Group Work

- Calculate short-run and long-run cost curves
- Understanding what the "lower envelope" means

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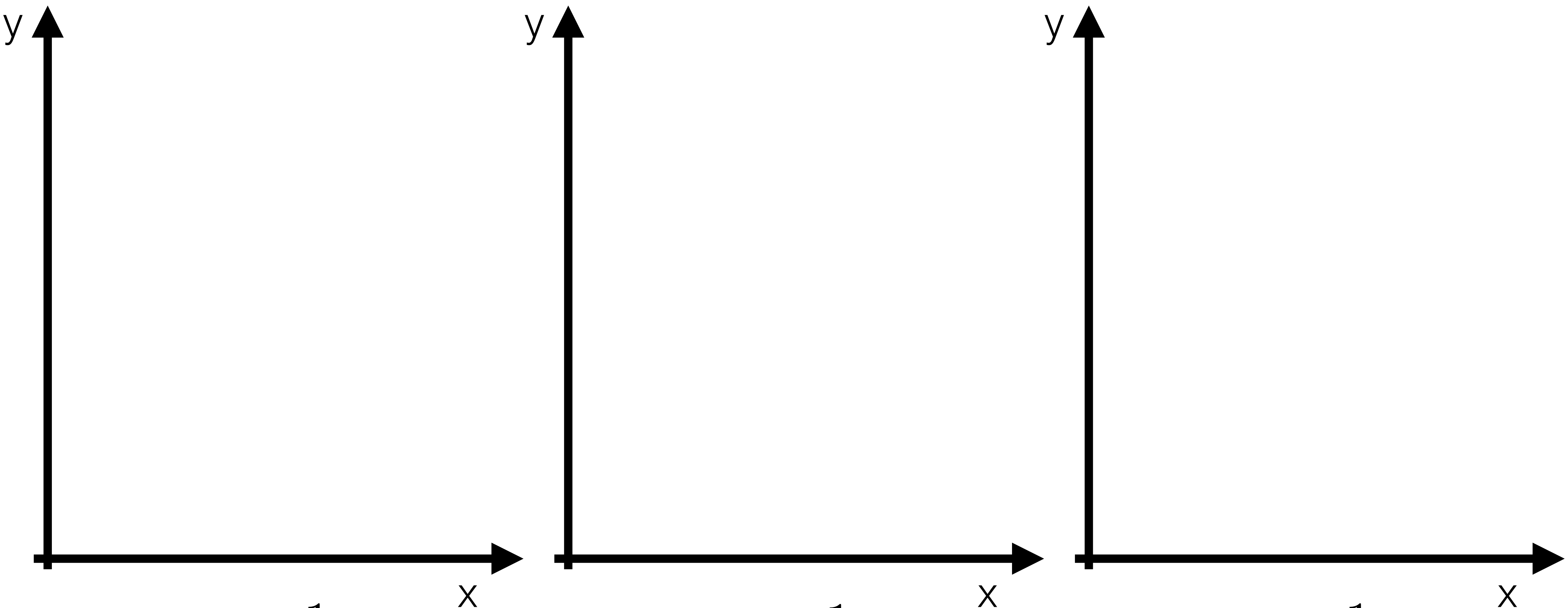
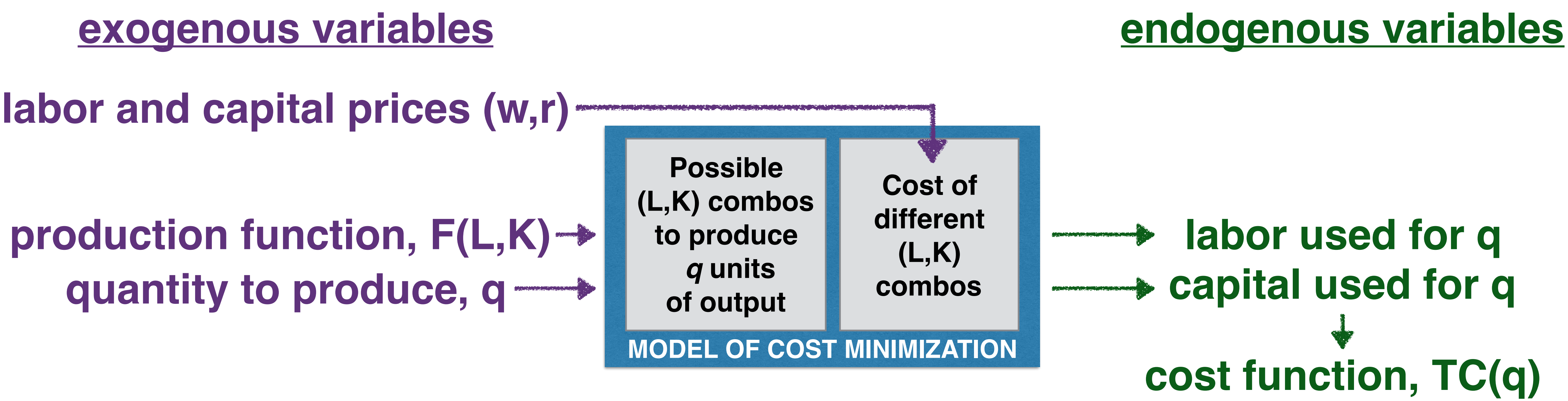
MOST IMPORTANT

$$y = x^{\alpha}$$



$\alpha < 1$ $\alpha = 1$ $\alpha > 1$

Producer Theory, Part I: Cost Minimization



Part I

Cost Minimization

“New” Graphical Element: Isocost Line

- Given input prices, the set of all combinations of L and K that cost the same amount

- E.g., the isocost line for 10 is the set of all combinations of L and K such that $\mathbf{wL + rK = 10}$.

- Like a budget line, but you’re not “given” an income

Consumer Theory

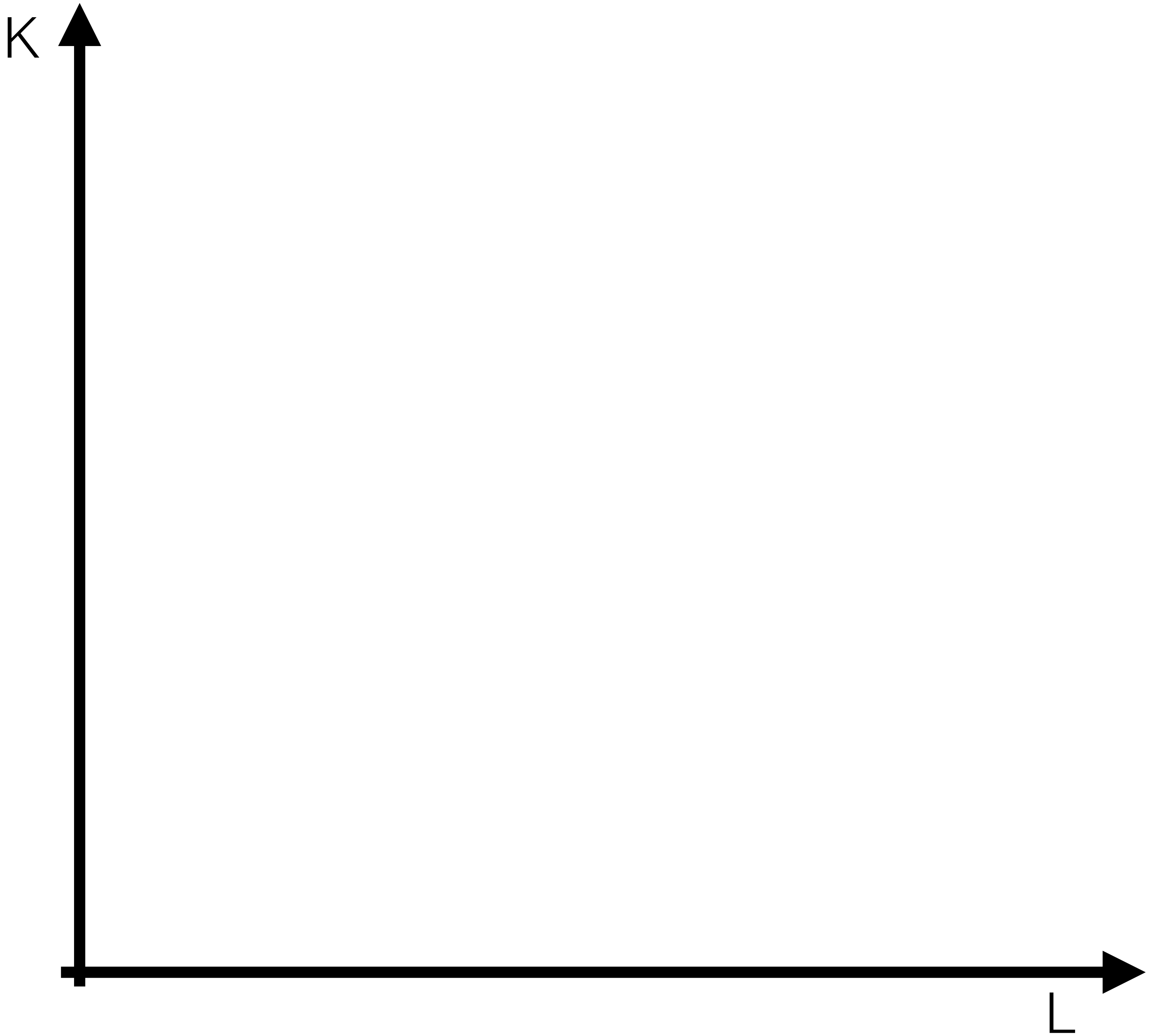
Hicksian Demand

“Given prices $\mathbf{P_x}$ and $\mathbf{P_y}$,
what combination of **X** and **Y**
gives me utility **U**
at the lowest cost?”

Producer Theory

Conditional Demands

“Given prices **w** and **r**,
what combination of **L** and **K**
can produce quantity **q**
at the lowest cost?”



Cost Minimization,
Visually

Conditional Demands and Total Cost

$$TC(w, r, q) = wL^*(w, r, q) + rK^*(w, r, q)$$

Lagrange Method

$$\min_{L,K} wL + rK$$
$$\text{s.t. } f(L,K) = q$$

$$\mathcal{L}(L,K,\lambda) = wL + rK + \lambda(q - f(K,L))$$

Example: Cobb-Douglas $F(L, K) = L^{\frac{1}{4}} K^{\frac{1}{4}}$

1. Sketch an isoquant for **q = 10**.

Example: Cobb-Douglas $F(L, K) = L^{\frac{1}{4}} K^{\frac{1}{4}}$

2. Calculate the **MRTS**

Example: Cobb-Douglas $F(L,K) = L^{\frac{1}{4}} K^{\frac{1}{4}}$

3. Derive the **conditional demands** for labor and capital:
L*(w,r,q), **K*(w,r,q)**

Example: Cobb-Douglas $F(L, K) = L^{\frac{1}{4}} K^{\frac{1}{4}}$

4. Find the **total cost** of producing **q** units, for general **w** and **r**

Example: Cobb-Douglas $F(L,K) = L^{\frac{1}{4}} K^{\frac{1}{4}}$

5. Confirm that when **w = 9** and **r = 16**, we obtain
L* = 133, **K* = 75**, **TC = $2,400** if we want to produce **q = 10**

Part II
Expansion Path

Consumer Theory

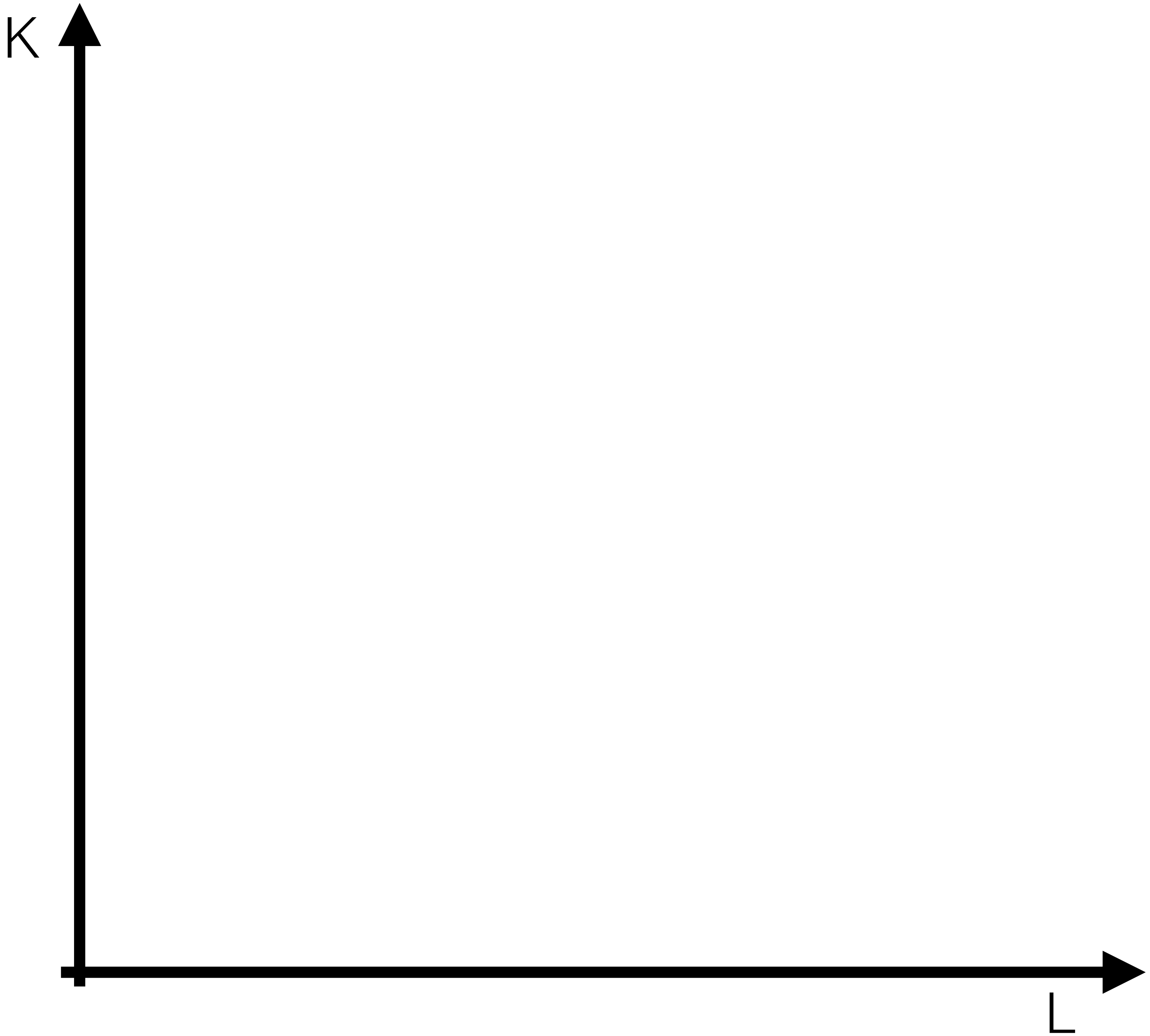
Income-Consumption Curve

"Given prices $\mathbf{P_x}$ and $\mathbf{P_y}$,
what combinations of **X** and **Y**
would I buy if I had
various different incomes?"

Producer Theory

Expansion Path

"Given prices **w** and **r**,
what combination of **L** and **K**
would we employ to produce
various different quantities?"



Expansion Path, Visually

Sidebar: The Long Run and the Short Run

Expansion Path in the Long Run and Short Run

- “Long Run” from an input selection perspective: can vary K and L
- “Short Run” from an input selection perspective: K is fixed

Part III

Long Run and Short Run Total Cost

Long-Run Total Cost

$$TC(w, r, q) = wL^*(w, r, q) + rK^*(w, r, q)$$

Short-Run Total Cost

$$TC(w, r, q, \overline{K}) = wL(q|\overline{K}) + r\overline{K}$$

Example: Cobb-Douglas $F(L,K) = L^{\frac{1}{4}}K^{\frac{1}{4}}$

6. Find the **short run total cost** of producing **q** units, for general **w** and **r**