Cost Curves

Econ 50 | Lecture 14 | February 23, 2016

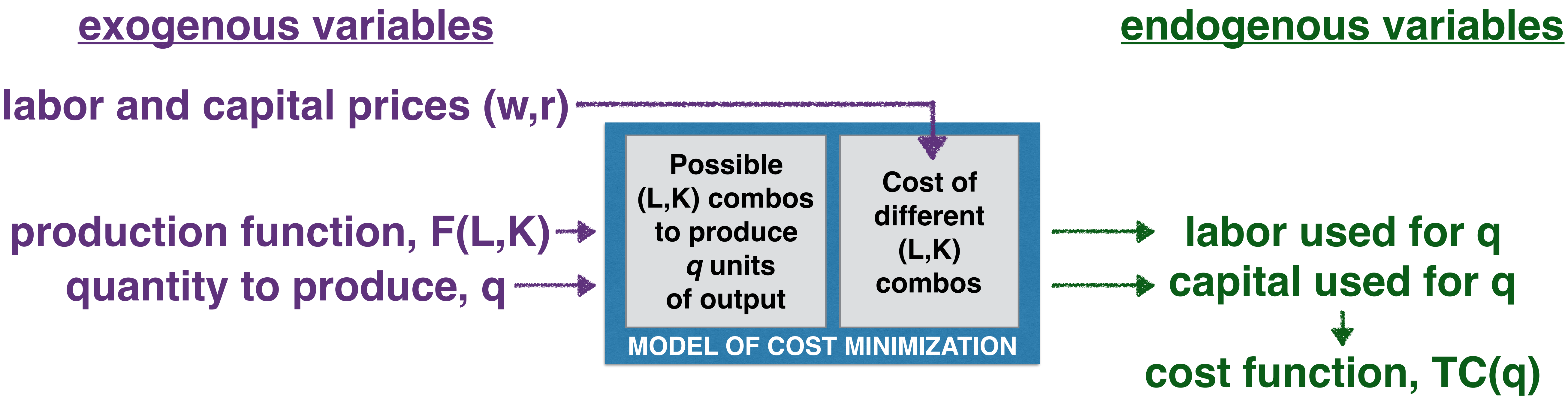
Lecture

- Production, Costs, and Returns to Scale [HW6 Q2]
- Drawing Cost Curves [HW6 Q3]
- “Backing out” the production function from the cost function [HW6 Q4]

Group Work

- HW6, Question 2, part (a)
- HW6, Question

Producer Theory, Part I: Cost Minimization



Producer Theory, Part II: Profit Maximization

exogenous variables

endogenous variables

cost function, TC(q)

demand / output prices

Revenue from selling *q* units of output TR(q)

Cost of producing *q* units of output TC(q)

MODEL OF PROFIT MAXIMIZATION

optimal quantity, q*

Part I

Production, Costs, and Returns to Scale

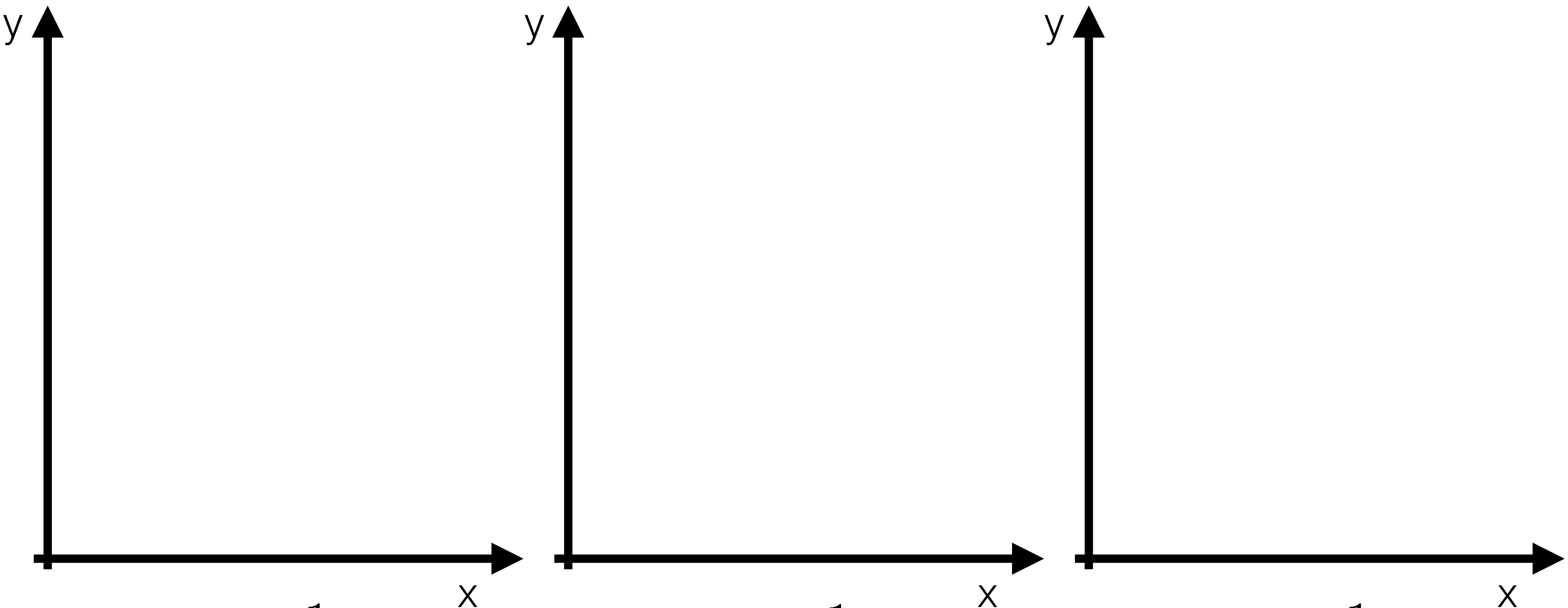
Long-Run Total Cost

$$TC(w,r,q) = wL^*(w,r,q) + rK^*(w,r,q)$$

Short-Run Total Cost

$$TC(w,r,q,\overline{K}) = wL(q|\overline{K}) + r\overline{K}$$

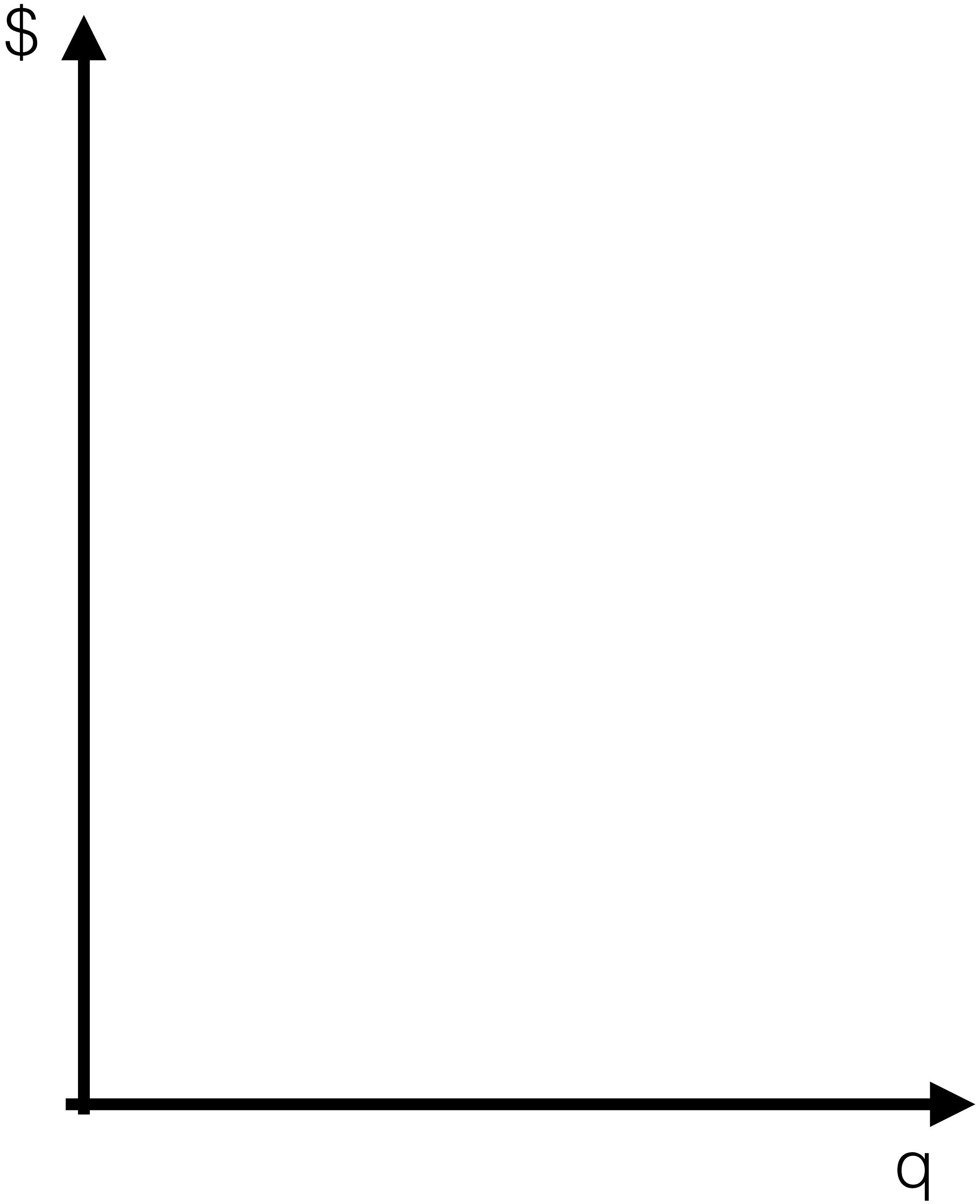
$$y = x^{\alpha}$$



$\alpha < 1$ $\alpha = 1$ $\alpha > 1$

SR and **LR** total cost: $f(L,K) = AL^{\alpha}K^{\beta}$

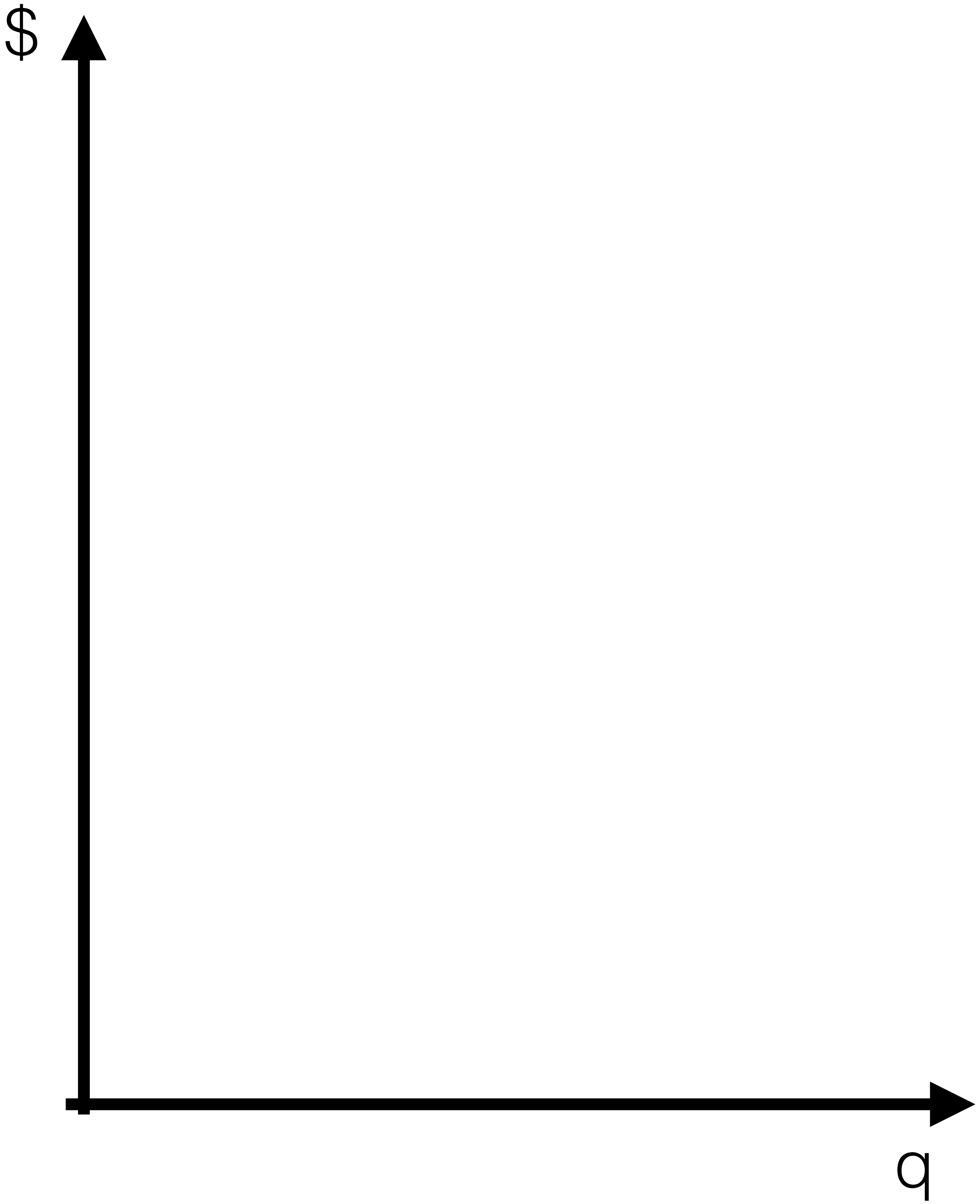
	$\alpha < 1$	$\alpha = 1$	$\alpha > 1$
$\alpha + \beta < 1$	●		
$\alpha + \beta = 1$			
$\alpha + \beta > 1$		●	



$$f(L, K) = AL^{\alpha}K^{\beta}$$

$$\alpha < 1$$

$$\alpha + \beta < 1$$



$$f(L,K) = AL^{\alpha}K^{\beta}$$

$$\alpha = 1$$

$$\alpha + \beta > 1$$

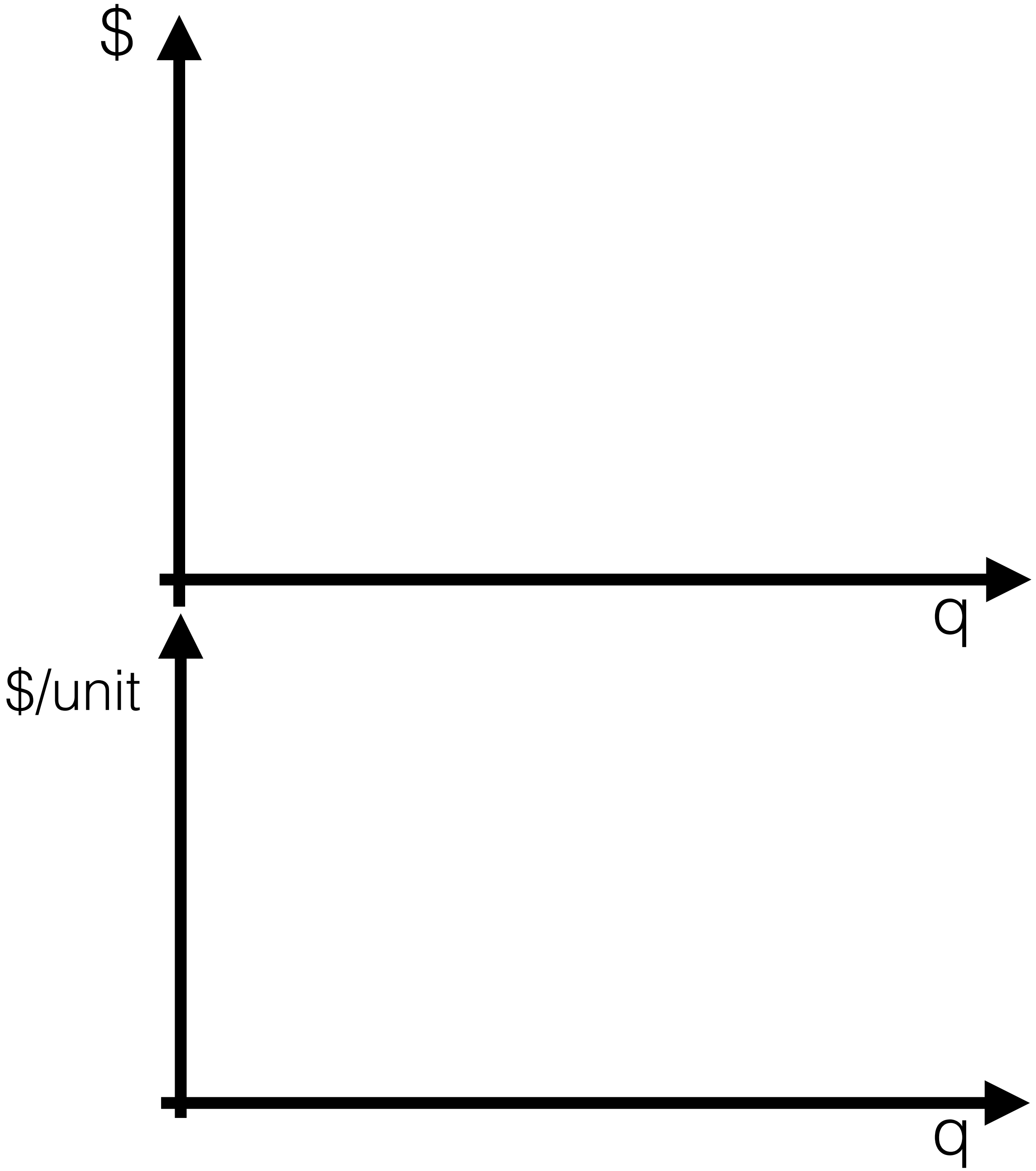
Output Elasticity and Economies of Scale

- Output elasticity of total cost: $\dfrac{\text{\% change in total cost}}{\text{\% increase in quantity}}$

Value of $\epsilon_{TC,Q}$	*MC* Versus *AC*	How *AC* Varies as *Q* Increases	Economies/ Diseconomies of Scale
$\epsilon_{TC,Q} < 1$	$MC < AC$	Decreases	Economies of scale
$\epsilon_{TC,Q} > 1$	$MC > AC$	Increases	Diseconomies of scale
$\epsilon_{TC,Q} = 1$	$MC = AC$	Constant	Neither

Economies of Scale and Returns to Scale

- Economies of scale refer to ***costs*** in the short-run or long-run
- Returns to scale refers to the ***production function*** in the long run
- Returns to scale implies economies of scale exist in the long run (decreasing long-run average cost)



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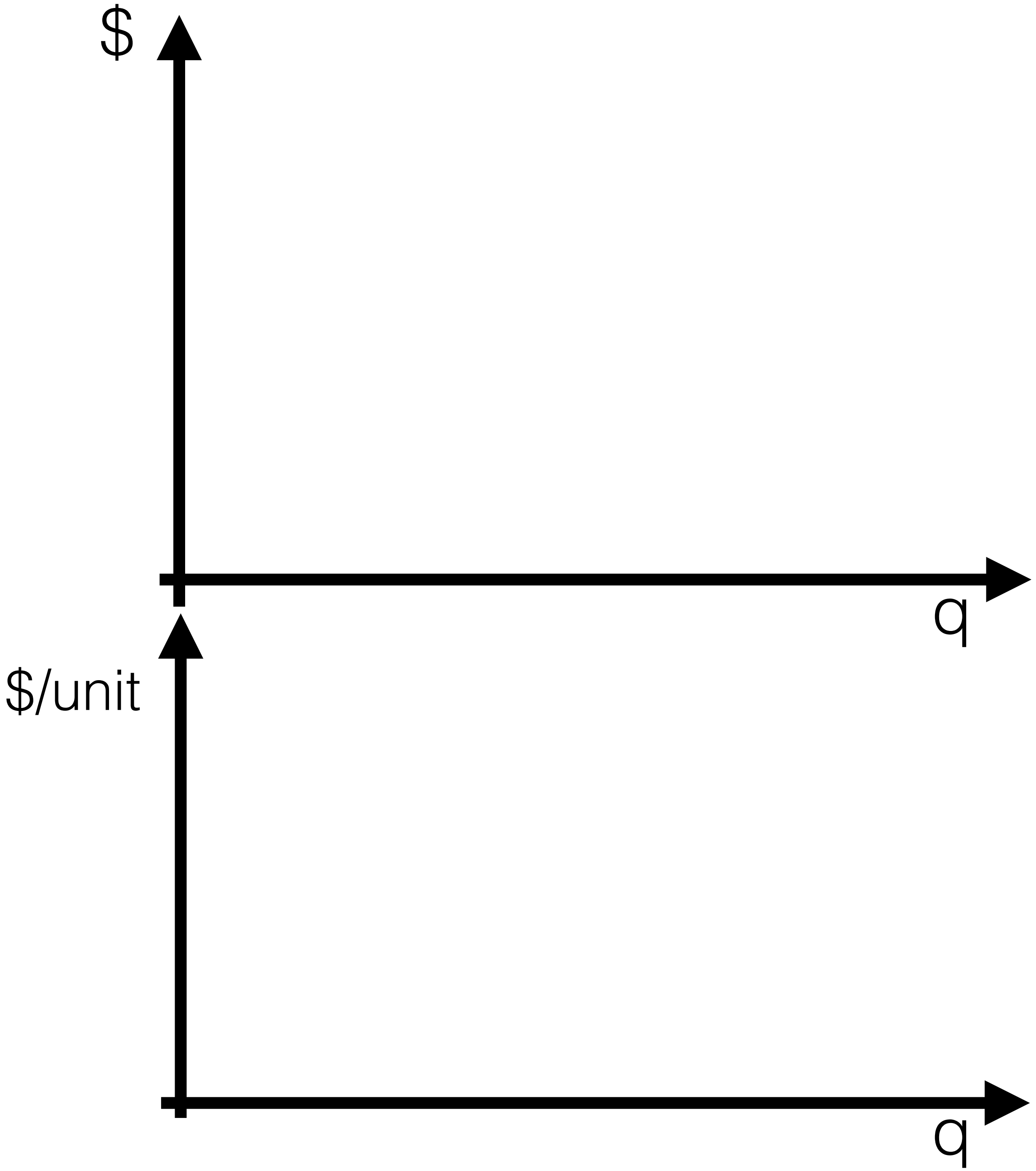
$$\alpha < 1$$

$$\alpha + \beta < 1$$



$$f(L, K) = AL^{\alpha}K^{\beta}$$

$$\alpha = 1$$

$$\alpha + \beta > 1$$

Homework Problem 2:
Plot TC, STC, AC, and SAC
for each cell of the table

Key Takeaways from Part I

- There is an **inverse relationship** between the shape of a production function and the shape of the associated total cost function

- **Decreasing** returns to scale => **increasing** long-run average cost
  Constant returns to scale => **constant** long-run average cost
  Increasing returns to scale => **decreasing** long-run average cost

- In the short run, **fixed costs** make things a little more complicated!

Relationship with One Input: Simpler!

	Production Function		
	$Q = L^2$	$Q = \sqrt{L}$	$Q = L$
Labor requirements function	$L = \sqrt{Q}$	$L = Q^2$	$L = Q$
Long-run total cost	$TC = w\sqrt{Q}$	$TC = wQ^2$	$TC = wQ$
Long-run average cost	$AC = w\sqrt{Q}$	$AC = wQ$	$AC = w$
How does long-run average cost vary with Q?	Decreasing	Increasing	Constant
Economies/diseconomies of scale?	Economies of scale	Diseconomies of scale	Neither
Returns to scale	Increasing	Decreasing	Constant

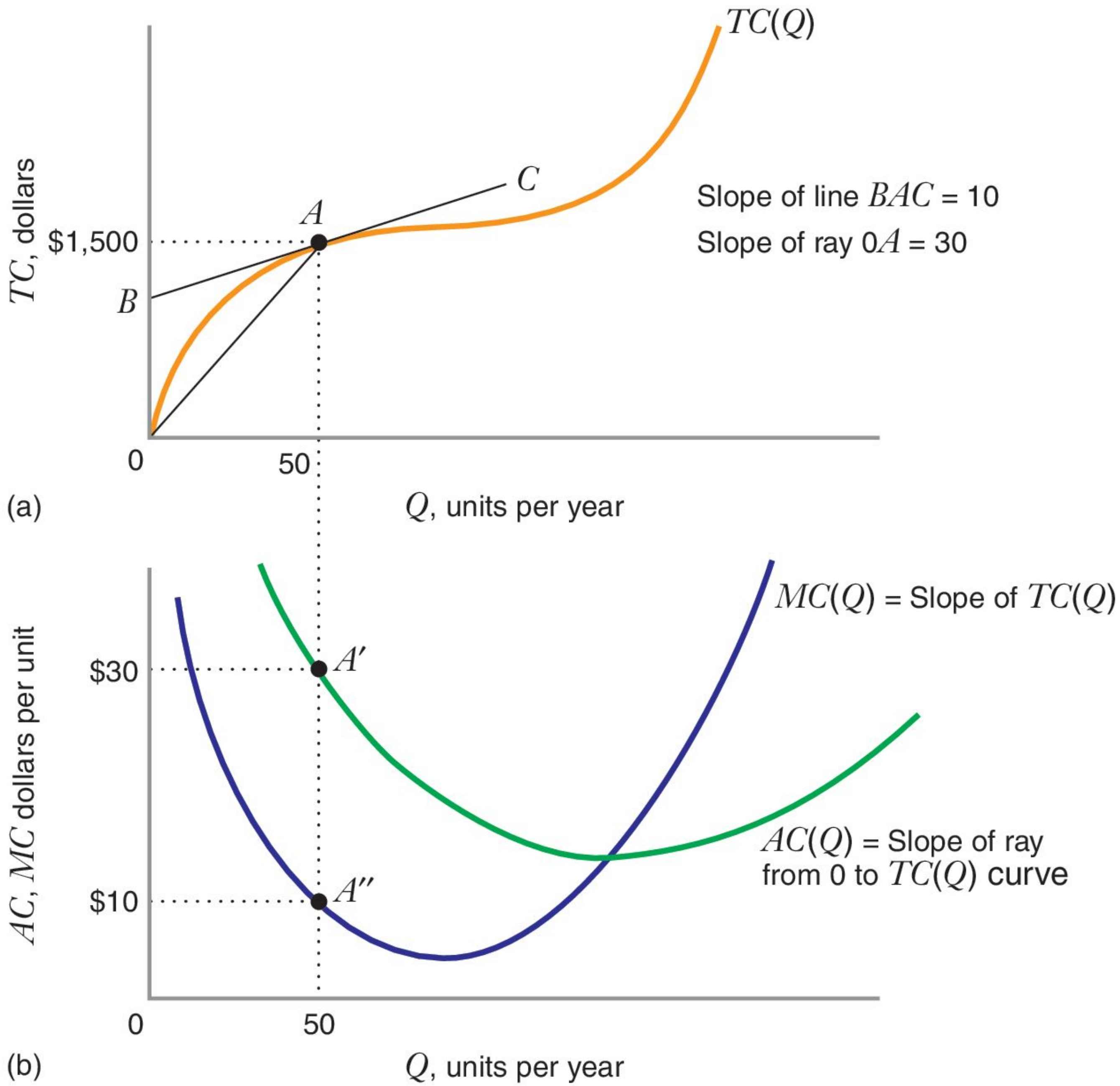
Part II
Cost Curves

Cost Curves

- We now leave behind **w** and **r**, and focus on **variable** and **fixed** costs.
- We will express costs in terms of a single variable, **q**.
- **Total costs**: **TC(q) = FC + VC(q)**
- **Unit costs**:
  - Marginal cost: **MC(q) = TC'(q) = 0 + VC'(q)**
  - Average fixed cost: **AFC(q) = FC/q**
    Average variable cost: **AVC(q) = VC(q)/q**
    Average total cost: **ATC(q) = AFC(q) + AVC(q)**

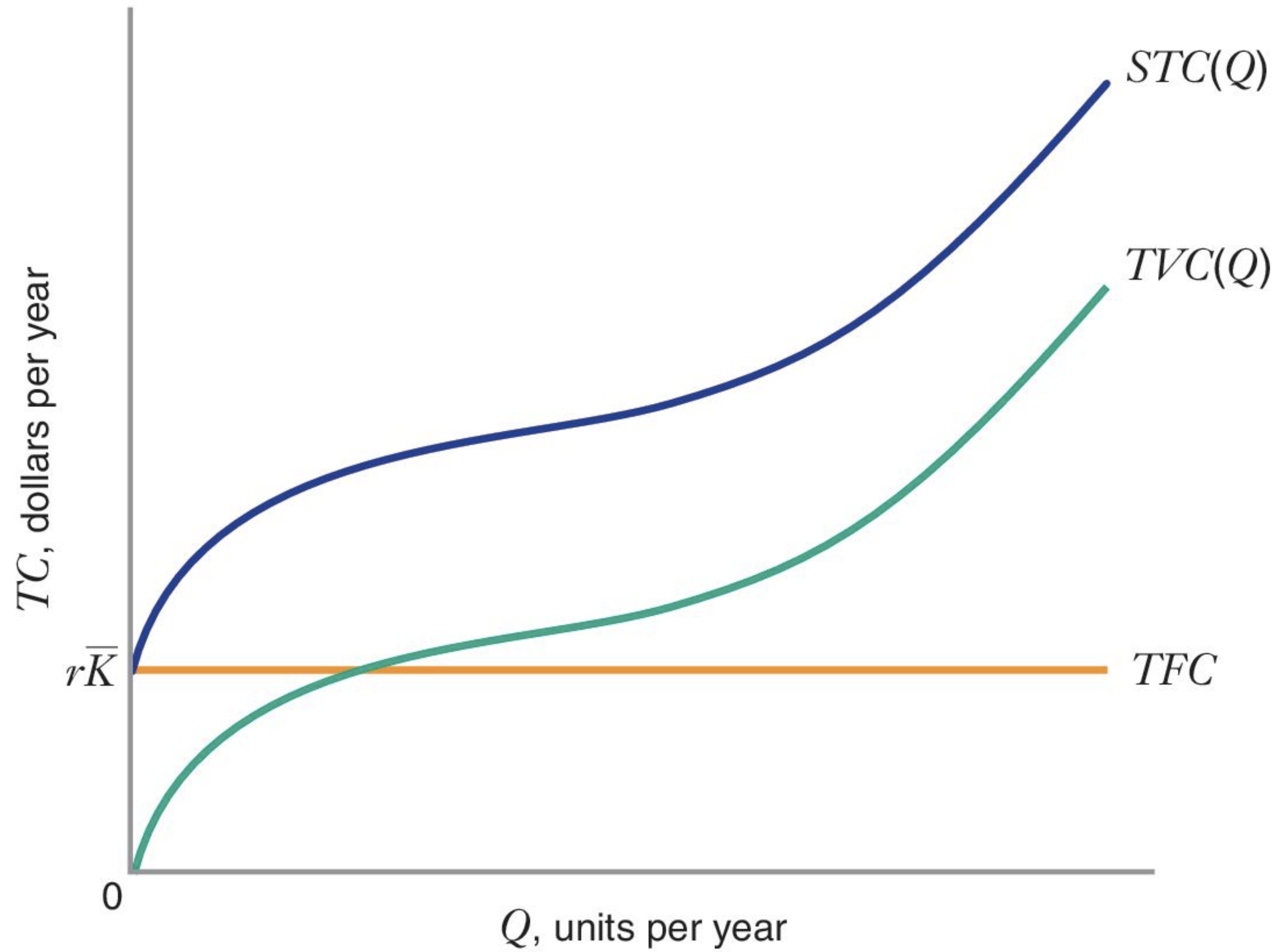
Relationship between MC, AC, and Fixed Costs

- MC < AC => AC is decreasing
- MC = AC => AC is constant
- MC > AC => AC is increasing
- Fixed costs => MC < AC when q = 0
- Fixed costs + increasing MC => AC is U-shaped
- Fixed costs + constant MC => AC is always decreasing

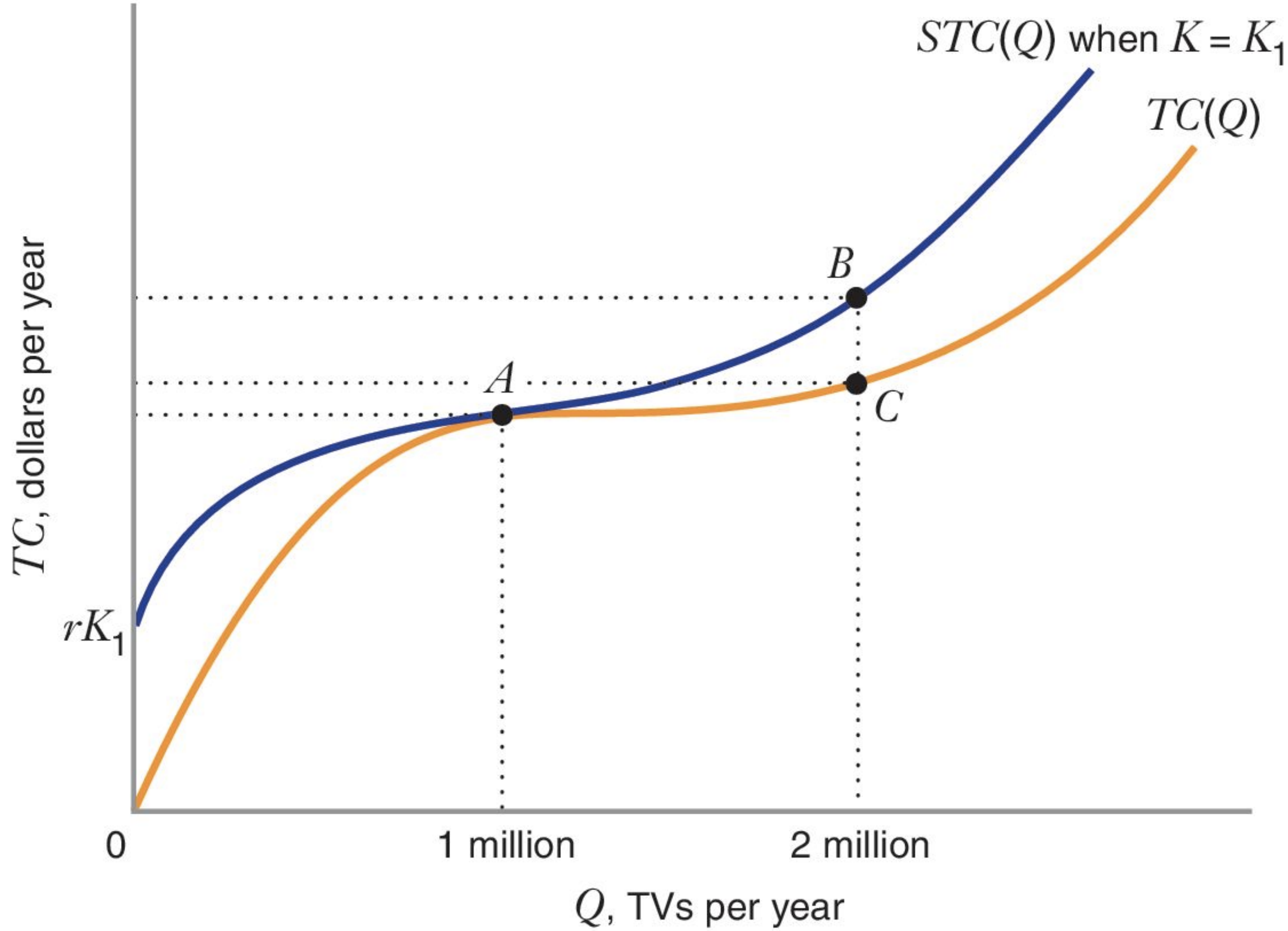


TC(Q)
C
Slope of line BAC = 10
Slope of ray 0A = 30
A
$1,500
TC, dollars
B
0
50
(a)
Q, units per year
MC(Q) = Slope of TC(Q)
$30
A′
AC, MC dollars per unit
AC(Q) = Slope of ray from 0 to TC(Q) curve
$10
A″
0
50
(b)
Q, units per year

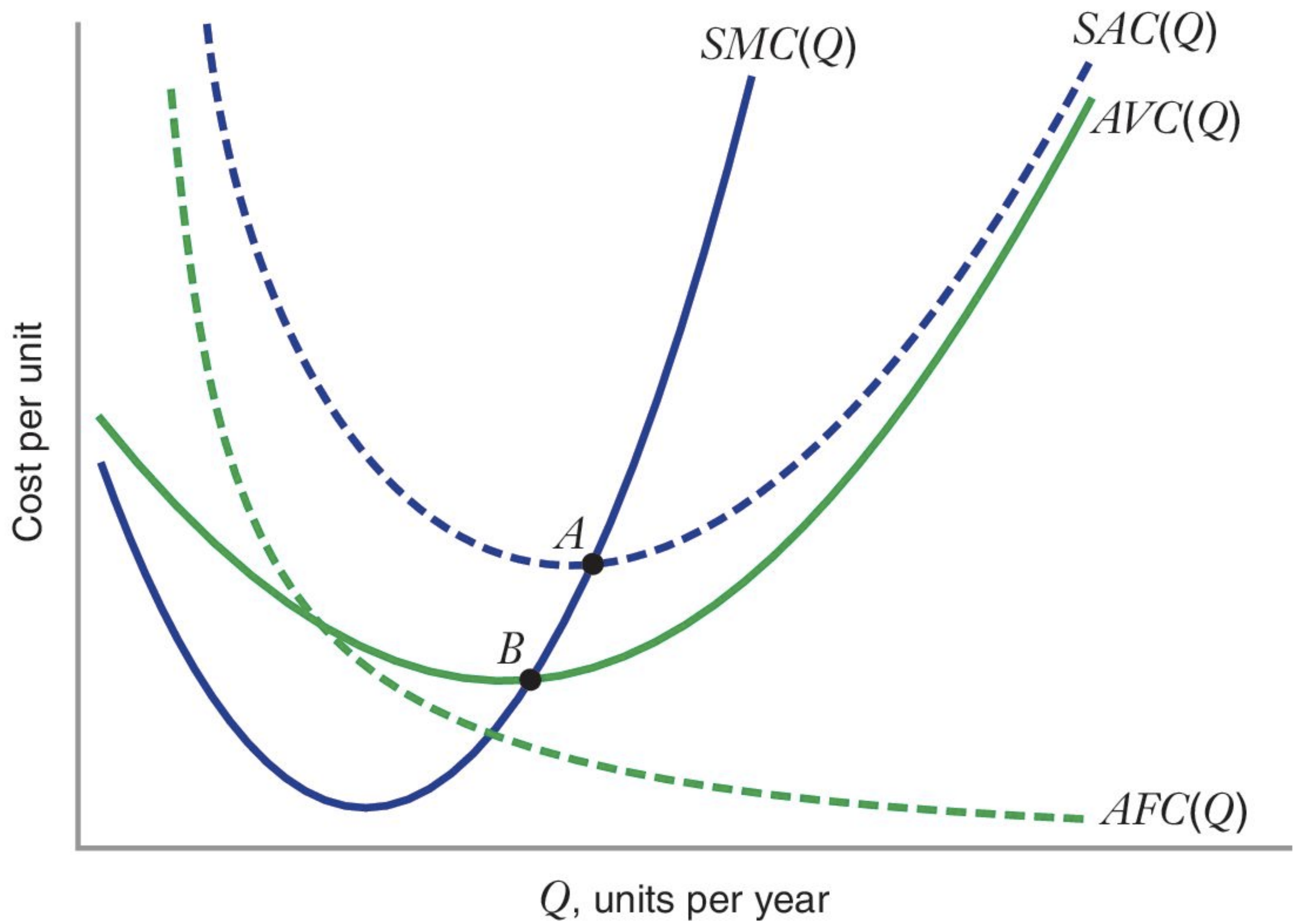
B&B Figure 8.13: Short-Run Fixed, Total, and Variable Costs



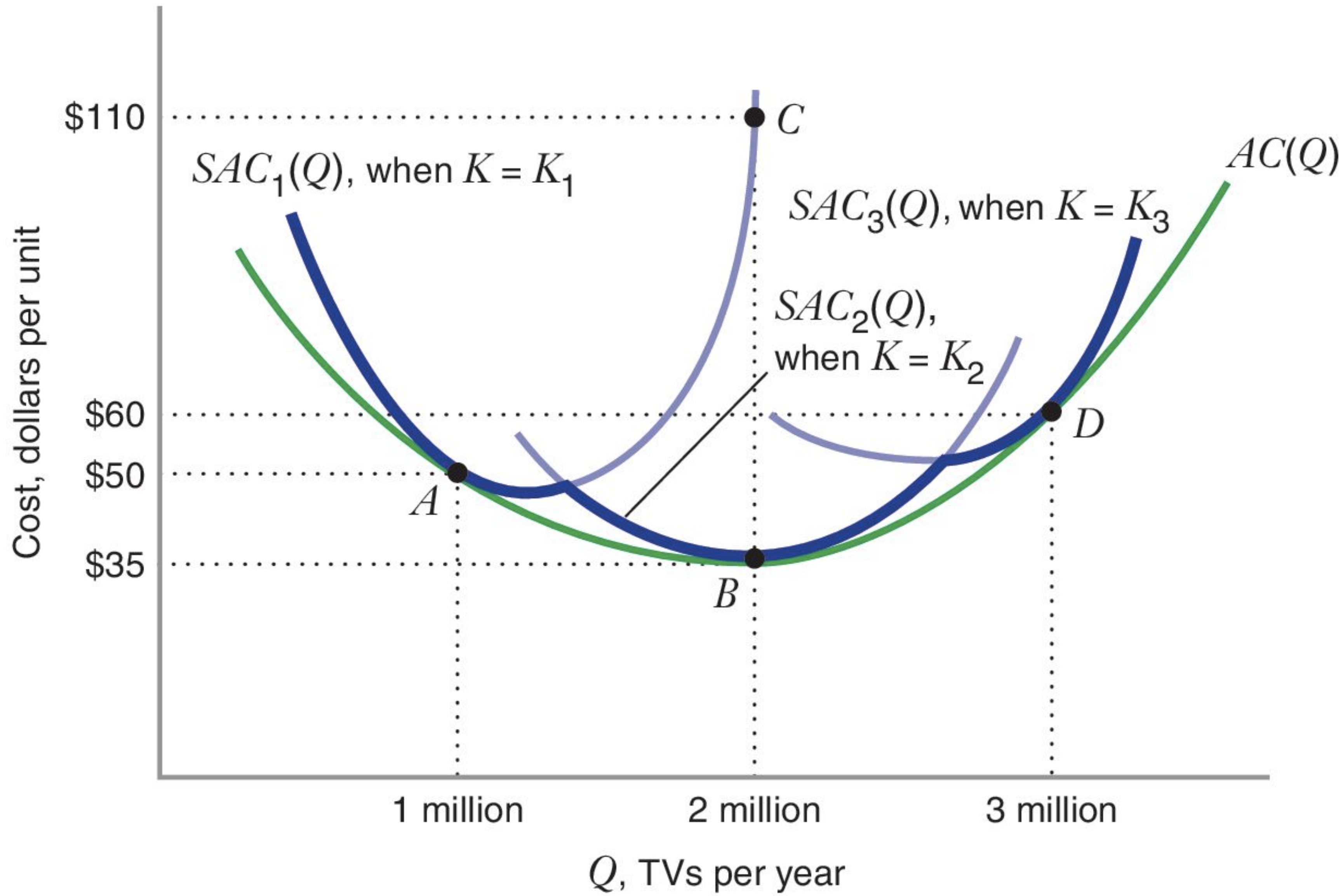
B&B Figure 8.15: Short-Run and Long-Run Total Costs



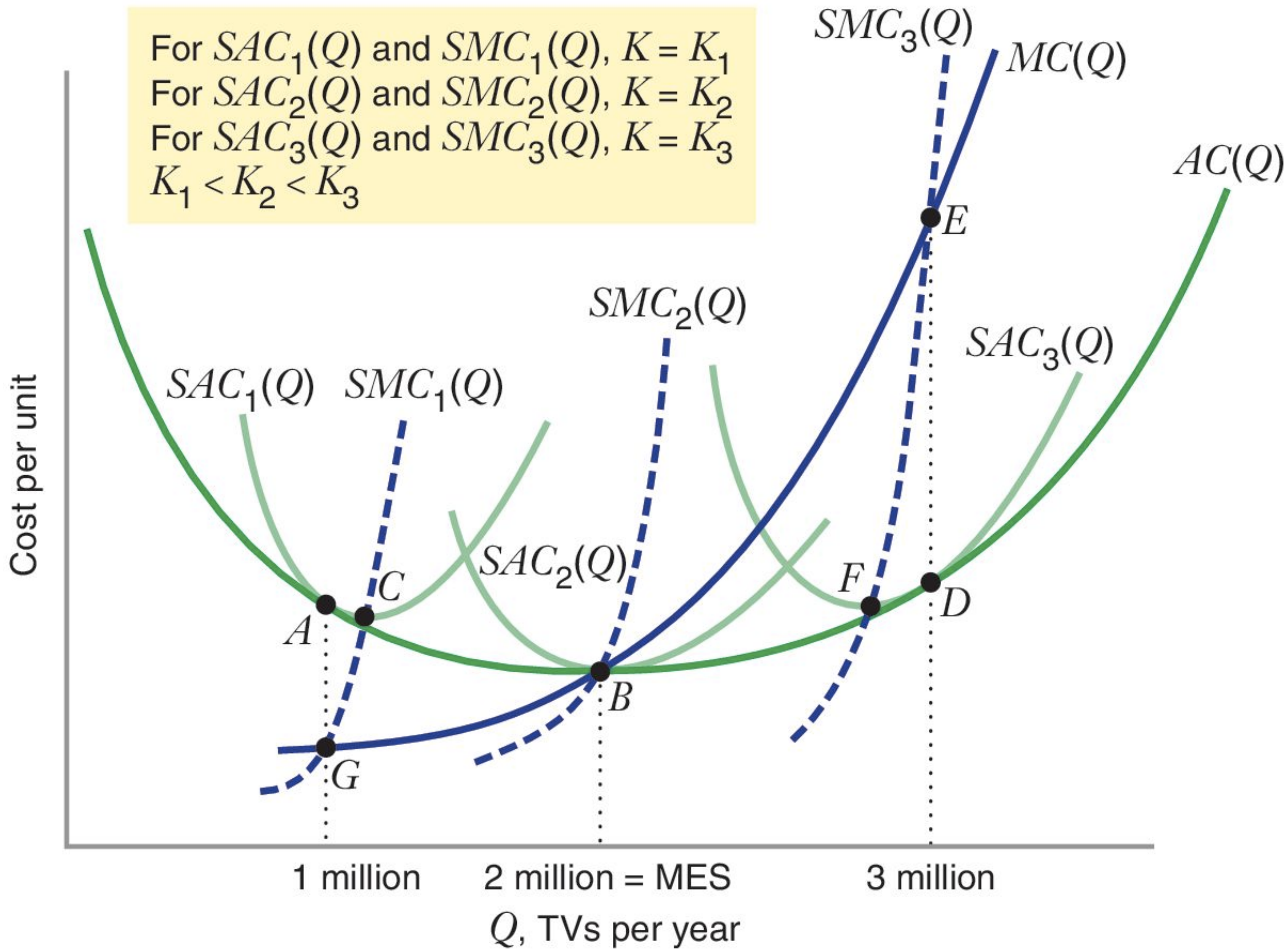
B&B Figure 8.16: Short-Run Unit Costs



B&B Figure 8.17: Short-Run and Long-Run Average Cost Curves



B&B Figure 8.18: Short-Run and Long-Run Marginal and Average Cost Curves



Let's derive some curves!

$$F(L, K) = L^{\frac{1}{4}} K^{\frac{1}{4}}$$

With w = r = 1, our cost functions from last time become:

$$TC(q) = 2q^2$$

$$STC(q) = 4 + \frac{1}{4}q^4$$

For the total cost curves:

$$TC(q) = 2q^2$$

$$STC(q) = 4 + \frac{1}{4}q^4$$

Calculate:

$\boxed{\begin{array}{c}\text{MC(q)}\\ \text{AC(q)}\end{array}}$

$\boxed{\begin{array}{c}\text{FC}\\ \text{AFC(q)}\\ \text{VC(q)}\\ \text{AVC(q)}\\ \text{SAC(q)}\\ \text{SMC(q)}\end{array}}$

Homework Problem 3:
Re-create these graphs for the cost functions associated with $Q = L^{1/2}K$

Key Takeaways from Part II

- Given a total cost function, you should know how to calculate marginal and average costs.

- The long-run total cost and average cost curves are the **lower envelope** of their short-run counterparts

- The long-run marginal cost curve is **not** the lower envelope of the short-run marginal cost curves

- Each MC curve intersects its AC curve at the minimum AC.
  At the **minimum efficient scale**, AC = SAC = MC = SMC!

Part III

"Backing out" the Production Function from the Cost Function

Review: Deriving Long-Run Total Cost

- Last time: given a **production function F(L,K)**,
  we derived the **conditional labor demands L*(w,r,q) and K*(w,r,q)**
  and therefore the **cost function TC(w,r,q) = wL*(w,r,q) + rK*(w,r,q)**.

- We can go the other way!

  - From cost function to conditional demand: **Shephard's Lemma**

  - From conditional demands to production function: **"dual method"**

Last time: Production to Total Cost

PRODUCTION FUNCTION

[COST MINIMIZATION]

CONDITIONAL INPUT DEMANDS

[PLUG INTO EXPENDITURE]

TOTAL COST

Today: Total Cost to Production Function

PRODUCTION FUNCTION

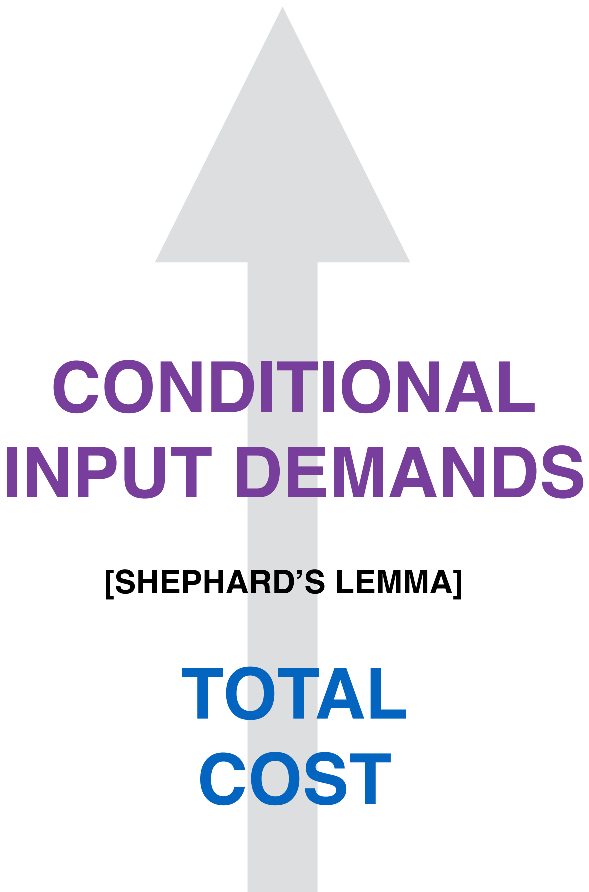
["DUAL METHOD"]

CONDITIONAL INPUT DEMANDS

[SHEPHARD'S LEMMA]

TOTAL COST

Shephard’s Lemma

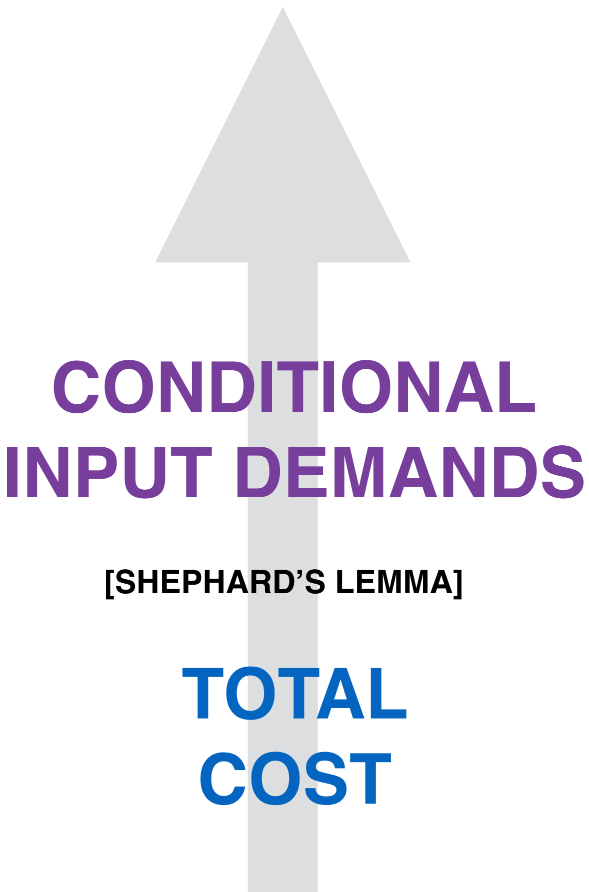


Given **TC(w,r,q)**,
the conditional demands **L*(w,r,q)** and **K*(w,r,q)**
are the partial derivatives of total cost
with respect to **w** and **r** respectively.

$$\frac{\partial TC(w,r,q)}{\partial w} = L^*(w,r,q)$$

$$\frac{\partial TC(w,r,q)}{\partial r} = K^*(w,r,q)$$

Shephard's Lemma



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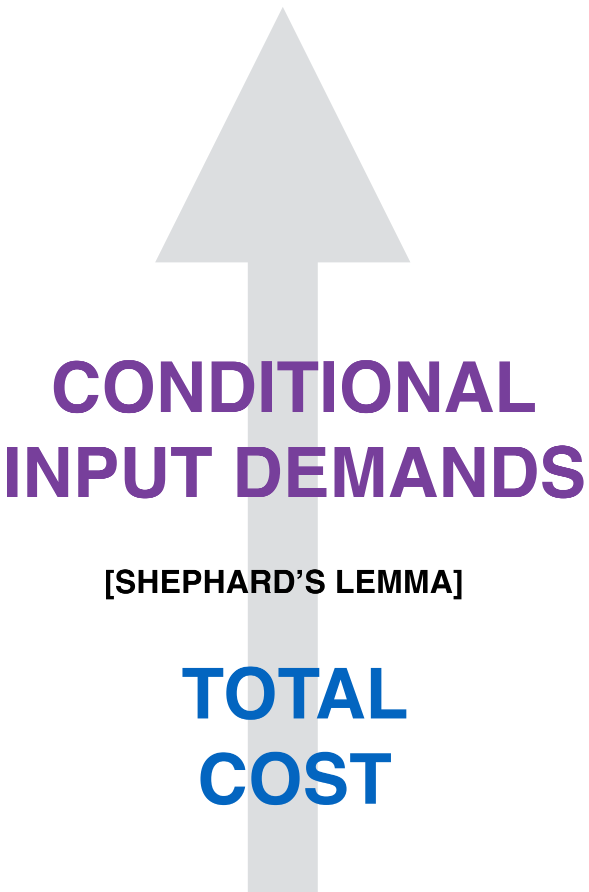

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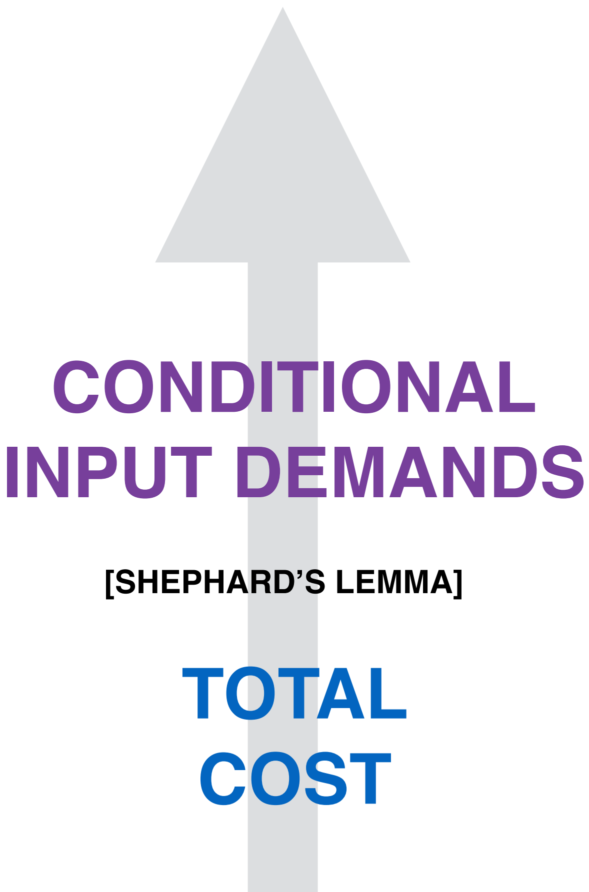

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$$\frac{\partial TC(w,r,q)}{\partial r} = K^*(w,r,q)$$

Why does Shephard's Lemma work? Start with total cost:

$$TC(w,r,q) = wL^*(w,r,q) + rK^*(w,r,q)$$

By the chain rule:

$$\frac{\partial TC(w,r,q)}{\partial w} = L^*(w,r,q) + \left[w\frac{\partial L^*(w,r,q)}{\partial w} + r\frac{\partial K^*(w,r,q)}{\partial w}\right]$$

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this must equal zero when MRTS = w/r

Today: Total Cost to Production Function

PRODUCTION FUNCTION

[“DUAL METHOD”]

CONDITIONAL INPUT DEMANDS

[SHEPHARD’S LEMMA]

TOTAL COST

"Dual Method"

PRODUCTION FUNCTION

["DUAL METHOD"]

CONDITIONAL INPUT DEMANDS

The conditional demands **L*(w,r,q)** and **K*(w,r,q)** are both functions of **w/r** (the slope of the isoquant), <u>not</u> w and r individually. So we can do this:

Step 1a: Set **L*(w,r,q) = L** and solve for **w/r**.
Step 1b: Set **K*(w,r,q) = K** and solve for **w/r**.

Step 2: Set the values of **w/r** from (1a) and (1b) equal to each other to get an equation that just has **L**, **K**, and **q**.

Step 3: Solve that equation for **q**; this is our production function!

"Dual Method"

PRODUCTION FUNCTION

["DUAL METHOD"]

CONDITIONAL INPUT DEMANDS

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“Dual Method”

PRODUCTION FUNCTION

[“DUAL METHOD”]

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Step 3: Solve that equation for **q**; this is our production function!

Cobb-Douglas Example: Shephard's Lemma

In your group: Find the **conditional input demands** by taking the partial derivatives of **TC(w,r,q)** with respect to **w** and **r**.

CONDITIONAL INPUT DEMANDS

$$L^*(w, r, q) =$$

$$K^*(w, r, q) =$$

[SHEPHARD'S LEMMA]

TOTAL COST

$$TC(w, r, q) = 2\sqrt{wr}q^2$$

Cobb-Douglas Example: Shephard's Lemma

CONDITIONAL INPUT DEMANDS

$$L^*(w, r, q) = \sqrt{\frac{r}{w}q^2}$$

$$K^*(w, r, q) = \sqrt{\frac{w}{r}q^2}$$

[SHEPHARD'S LEMMA]

TOTAL COST

$$TC(w, r, q) = 2\sqrt{wrq^2}$$

“Dual Method” : Cobb-Douglas Example

PRODUCTION FUNCTION

[“DUAL METHOD”]

CONDITIONAL INPUT DEMANDS

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$$K^*(w,r,q) = \sqrt{\frac{w}{r}q^2}$$

Step 1a: Set **L*(w,r,q)** equal to **L** and solve for w/r as a function of **L** and **q**.
Step 1b: Set **K*(w,r,q)** equal to **K** and solve for w/r as a function of **K** and **q**.
Step 2: Set (1a) and (1b) equal to each other to get an equation in **q**, **L**, and **K**.
Step 3: Solve the equation from (2) for **q** as a function of **L** and **K**.

$$L^*(w, r, q) = \sqrt{\frac{r}{w}} q^2$$

Step 1a: Set **L*(w,r,q)** equal to **L** and **solve for w/r** as a function of **L** and **q**.

$$K^*(w, r, q) = \sqrt{\frac{w}{r}} q^2$$

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Step 3: Solve the equation from (2) for **q** as a function of **L** and **K**.

Homework Problem 4:
Do this for a (slightly) more general Cobb-Douglas production function

Production and Costs: Summary 1

- Production and (total) costs are two views into a single process

- Given a production function **F(L,K)**, use cost minimization to derive conditional input demands **L*(w,r,q)** and **K*(w,r,q)**; plug into expenditure to get **TC(w,r,q) = wL*(w,r,q) + rK*(w,r,q)**.

- Given a cost function **TC(w,r,q)**, use Shephard's lemma to derive conditional input demands **L*(w,r,q)** and **K*(w,r,q)** and the dual method to get **F(L,K)**.