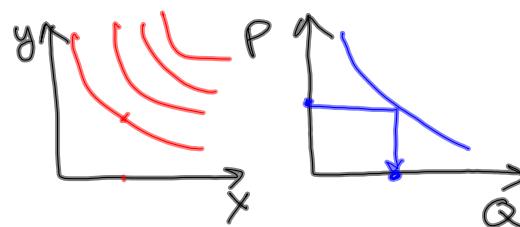


Budget Constraints and Optimal Choice

Econ 50 | Lecture 6 | January 21, 2016

Text



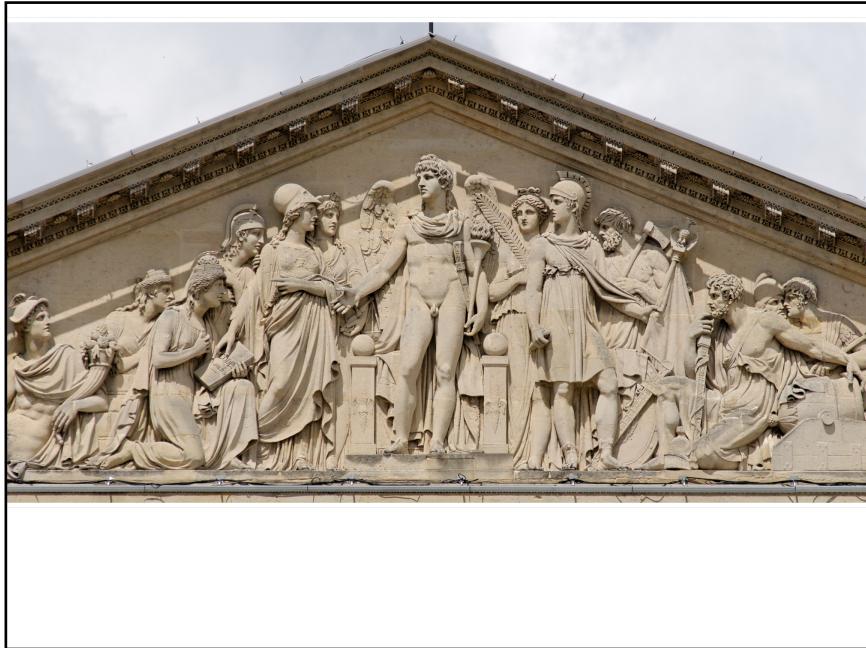
Lecture

- Review: The Story So Far
- Budget constraints
- Constrained optimization
- Optimization for each of our utility functions (graphical)

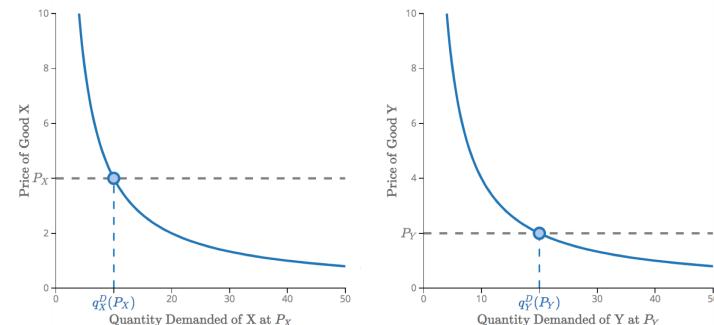
Group Work

- Playing with budget constraints
- The relationship between MRS and the price ratio
- Solving Cobb-Douglas

Part I: The Story So Far



Demand Curves for X and Y



Unit Goal

$$q_x^D(P_x, P_y, I)$$

$$q_y^D(P_x, P_y, I)$$

exogenous variables

price of good X
price of good Y
income

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Unit Goal

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endogenous variables

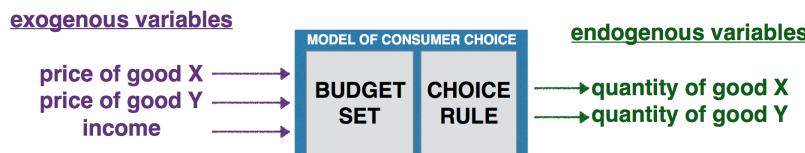
quantity of good X
quantity of good Y

$$q_y^D(P_x, P_y, I)$$

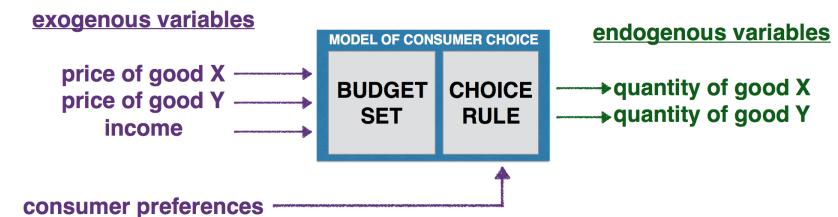
Unit Approach



Unit Approach



Unit Approach



Unit Overview

- Preferences (Lectures 4 & 5, Homework 2)**

How do consumers rank alternatives?

Readings: B&B Ch. 3; Varian Ch. 3 & 4

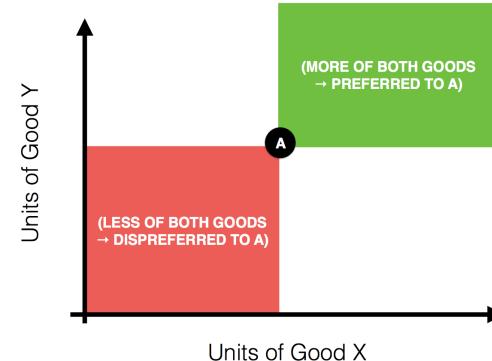
- Choice (Lectures 6 & 7, Homework 3)**

How do consumers choose to allocate their income among goods?

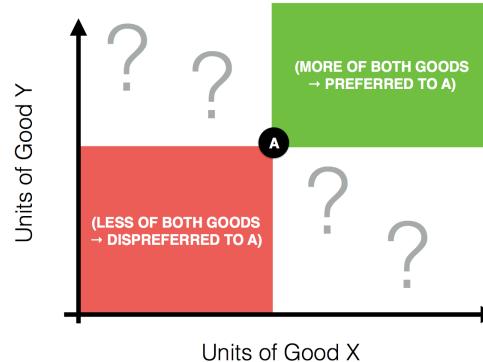
~~What do their observed choices "reveal" about their preferences?~~

Readings: B&B Chapter 4; Varian Ch. 2, 5, 7

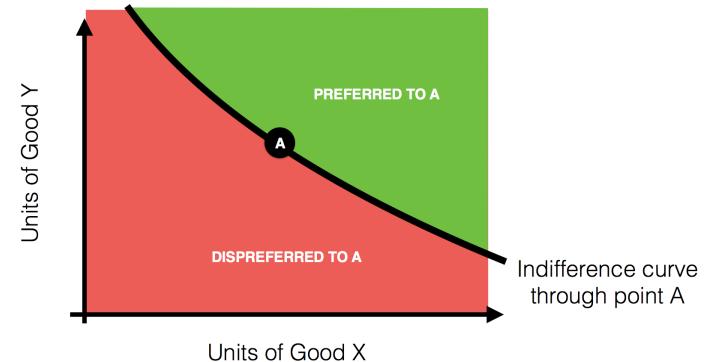
Monotonicity Assumption



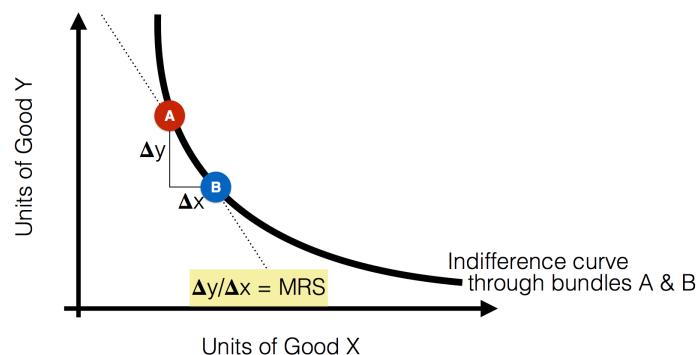
Monotonicity Assumption



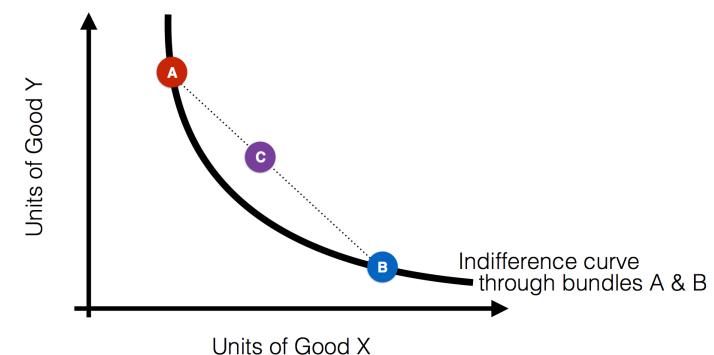
Indifference curves



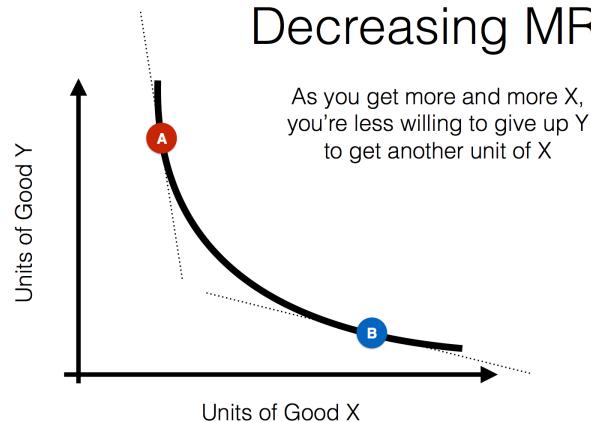
Marginal Rate of Substitution



Convexity Assumption



Decreasing MRS



Marginal Rate of Substitution

- **Intuitively:** rate at which a consumer is willing to give up good Y to get an additional unit of good X.
- **Visually:** absolute value of the slope of an indifference curve
- **Mathematically:**
$$\frac{MU_x}{MU_y}$$

Utility Functions

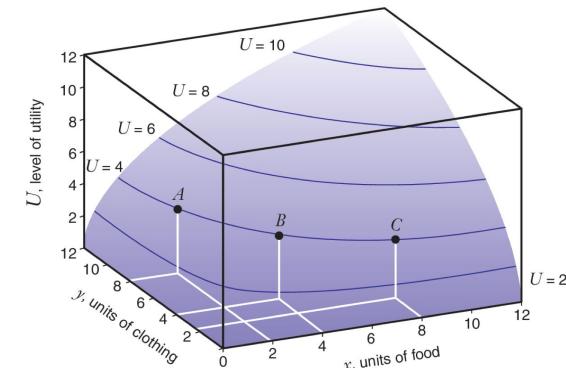
- A utility function $u()$ assigns a real number to every possible bundle.

$A > B$ if and only if $u(A) > u(B)$.

$A \sim B$ if and only if $u(A) = u(B)$.

$A < B$ if and only if $u(A) < u(B)$.

Indifference Curves are Level Curves of $u(x,y)$



Marginal Rate of Substitution
= Ratio of the Marginal Utilities

$$MRS_{x,y} = \frac{\frac{\partial u(x,y)}{\partial x}}{\frac{\partial u(x,y)}{\partial y}} = \frac{MU_x}{MU_y}$$

What We've Established

- (1) A way of **ranking** bundles (utility function)
- (2) A way of analyzing **tradeoffs** (MRS)

What We've Established

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Next Question:

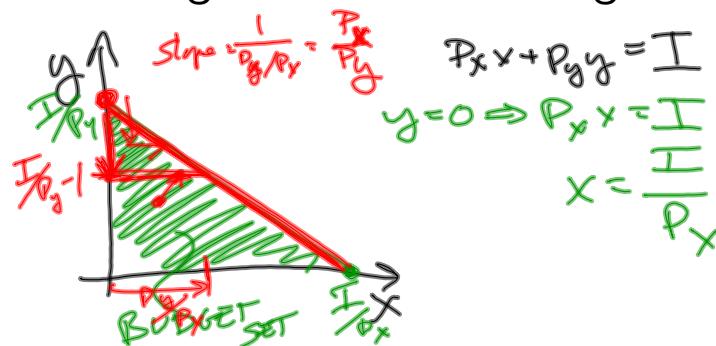
Given a **limited choice set**,
which bundle
will the consumer choose?

Part II: Budget Constraints

Simple Budget Constraint

- You have a given income I
- You're choosing quantities of two goods: x units of X, y units of Y
- Each good has a constant price (P_x and P_y , respectively)
- Total expenditure on good X: $P_x x$
Total expenditure on good Y: $P_y y$
- Budget Line: $P_x x + P_y y = I$

Budget Constraint Diagram



Slope of the Budget Line

- Budget Line: $P_x x + P_y y = I$
- Solve for y: $\frac{P_y y}{P_y} = \frac{I - P_x x}{P_y}$
 $y = \frac{I}{P_y} - \frac{P_x}{P_y} x$
- Magnitude (absolute value) of the slope is the **price ratio**

Interpretation of the Price Ratio

- The price ratio represents the rate at which the market allows you to substitute good X for good Y
- Moving along the budget line (down and to the right): "How many units of good Y do I need to sell for price P_y each in order to buy one more unit of X at a price of P_x ?"

Part III: Constrained Optimization

The Good Case

IF

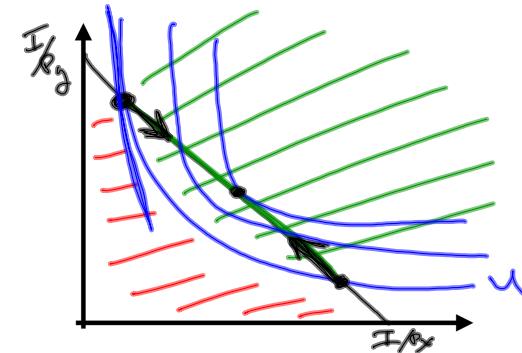
smooth

- The consumer's preferences over goods X and Y are **continuously differentiable**, **strictly monotonic**, and **strictly convex**
- The indifference curves **do not cross the axes**
- The budget constraint is a **simple straight line**

THEN THE SOLUTION...

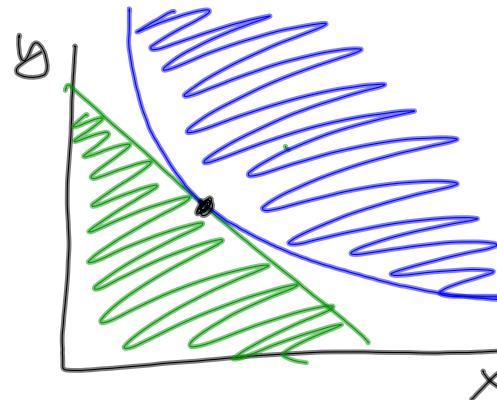
- will be an **interior solution** (involve strictly positive quantities of X and Y)
- can be found using the **Lagrange method**
- is characterized by:
 $MRS_{x,y} = P_x/P_y$
 $P_x x + P_y y = I$

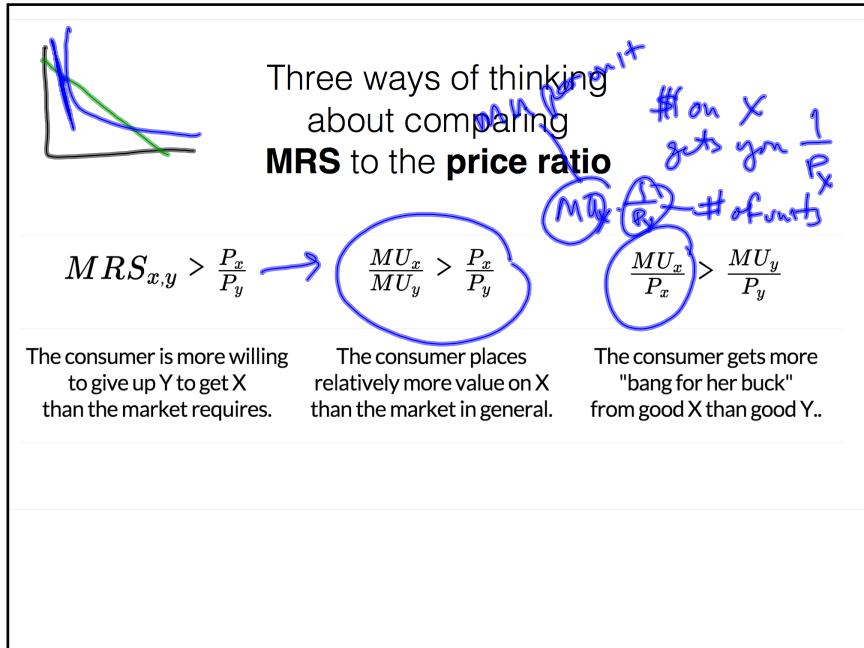
The Canonical Econ 50 Diagram



Why does this work?

- Verbally:**
Consumer compares their own willingness to trade Y for X (**MRS**) with the market's willingness to trade Y for X (**price ratio**)
- Graphically:**
At the optimal point, there is **no overlap** between the **set of points preferred to that bundle** and the **budget set**.
- Mathematically:**
FOC's of the Lagrange method find the tangency condition





Part IV: Optimization for our Five Utility Functions

Utility Function 1: Cobb-Douglas

Utility Function 2: Perfect Substitutes

Utility Function 3: Perfect Complements

Utility Function 4: CES

Utility Function 5: Quasilinear