Lecture Notes

Econ 50 - Lecture 2

January 7, 2016

These notes are meant to cover the material missing from the PowerPoint slides, since it didn't save my annotations. They're a bit more verbose than slides but I hope they help.

1. **Individual and Market Demand and Supply**

   Individual supply and demand represents the individual decisions of a consumer or firm. We write the quantity demanded or supplied with a lowercase q to indicate that this is an individual quantity, not a market quantity. If we're talking about different consumers, we'll subscript the index of the consumer. Therefore $q_i^D(P)$ represents the individual demand of consumer i given market price p.

   Market demand is the summation of all the individual demands. Thus if there are N_C consumers in a market, the market demand is given by

   $$Q^D(P) = \sum_{i=1}^{N_C} q_i^D(P)$$

   If it's possible to sum up the "average" demand behavior of consumers by a "representative agent" with demand $q^D(P)$, then you can just multiply the individual demand of the representative agent by the number of consumers:

   $$Q^D(P) = \sum_{i=1}^{N_C} q^D(P) = N_C \times q^D(P)$$

   In a slightly more complex model, you might have two "types" of consumers, type 1 and type 2, with N_{C_1} type-1 consumers, each of whom has a (representative) demand function $q_1^D(P)$, and N_{C_2} type-2 consumers, each of whom has a (representative) demand function $q_2^D(P)$. In this case the market demand is the sum of the two types' summed demand:

   $$Q^D(P) = \underbrace{\left[N_{C_1} \times q_1^D(P)\right]}_{\text{total type-1 demand}} + \underbrace{\left[N_{C_2} \times q_2^D(P)\right]}_{\text{total type-2 demand}}$$

   To make things concrete, let's think of an example of two types of consumers with linear demand curves. In particular, suppose there are 100 type-1 consumers who each have a (representative) demand curve given by

   $$q_1^D(P) = \begin{cases} 0 \text{ if } P > 10 \\ 10 - P \text{ if } P \leq 10 \end{cases}$$

   There are also 200 type-2 consumers who each have a (representative) demand curve given by

   $$q_2^D(P) = \begin{cases} 0 \text{ if } P > 5 \\ 5 - P \text{ if } P \leq 5 \end{cases}$$

What is the market demand? Well, if the price is above 10, neither type buys; if it's between 5 and 10, only type-1 consumers buy, and if it's below 5 both types buy. So we can write the market demand as follows:

$$
\begin{aligned}
Q^D(P) &= 100q_1^D(P) + 200q_2^D(P) \\
&= \begin{cases} 0 \text{ if } P > 10 \\ 100 \times (10 - P) \text{ if } 5 < P \leq 10 \\ 100 \times (10 - P) + 200 \times (5 - P) \text{ if } P \leq 5 \end{cases} \\
&= \begin{cases} 0 \text{ if } P > 10 \\ 1000 - 100P \text{ if } 5 < P \leq 10 \\ 2000 - 300P \text{ if } P \leq 5 \end{cases}
\end{aligned}
$$

The important thing is to remember that it is NOT the case that you just sum up the functions, without thinking about prices where demand is zero. For example, if you just wrote

$$Q^D(P) = 100q_1^D(P) + 200q_2^D(P) = 100 \times (10 - P) + 200 \times (5 - P) = 2000 - 300P$$

this would be incorrect for prices over 5, when type-2 consumers demand zero.

The logic for supply is exactly the same; the tricky thing to remember is that there might be prices where firms choose to produce zero.

2. **Consumer and Producer Surplus**

Consumer surplus is represented by the area below the demand curve and above the price consumers pay for a good. It represents the summation of the marginal welfare consumers get by paying less than the marginal value they place on a good.

Producer surplus is represented by the area above the supply curve and below the price firms receive for a good. It represents the summation of the marginal welfare firms get by selling units above their marginal cost of production. (Note that even when producer surplus is positive, a firm's overall profits may be negative since producer surplus only looks at marginal costs, not fixed costs.)

It's easy to calculate CS and PS when demand or supply curves are linear, because their shapes are regular geometric objects (usually triangles). However, for the more general case, it's necessary to take integrals to find their values.

It is often the case that CS is undefined: for example, consider the canonical demand function of $Q^D(P) = \frac{a}{P}$, which will occur over and over in this course. The integral of that function is

$$
\begin{aligned}
CS &= \int Q^D(P) dP \\
&= \int_{P^*}^{\infty} \frac{a}{P} dP \\
&= a \ln P
\end{aligned}
$$

If you take the definite integral of that function from some price P^* to infinity, you get

$$CS = a(\ln \infty - \ln P^*)$$

However, even with this function, the *change* in consumer surplus between two prices is well defined:

$$
\begin{aligned}
\Delta CS &= \int_{P_0}^{P_1} Q^D(P)dP \\
&= \int_{P_0}^{P_1} \frac{a}{P}dP \\
&= a\int_{P_0}^{P_1} \frac{1}{P}dP \\
&= a(\ln P_1 - \ln P_0) \\
&= a\ln\frac{P_1}{P_0}
\end{aligned}
$$

Note that this is negative if $P_1 < P_0$ and positive if $P_1 > P_0$. The sign isn't as important as the magnitude; the change in CS is the absolute value of this amount. (The magnitude is the same even if you reverse P_0 and P_1.) If the price increases, consumer surplus declines by this amount; if the price drops, consumer surplus increases by this amount.

Note that visually, what we're doing here is taking the integral by adding up small *horizontal* slivers. This is because quantity is a function of price, so the dependent variable is on the x-axis.

The logic for producer surplus is exactly the same, although the problem of the unbounded total producer surplus goes away because you're taking the integral of the supply function from $P = 0$ to $P = P^*$, not $P = P^*$ to $P = \infty$.