Section 1 Problems

Econ 50 - Stanford University - Winter Quarter 2015/16

Friday, January 15, 2016

Problem 1: Constrained Optimization

Maximize the function $f(x, y) = x^2 + y^2$ subject to the constraint $x + y \leq 4$. (And also that $x \geq 0$ and $y \geq 0$.)

Problem 2: Constrained Optimization

Maximize the function $f(x, y) = a \ln x + b \ln y$ subject to the constraint $p_x x + p_y y \leq I$.

Problem 3: Marginal Cost; Interpretation of Lagrangian Multiplier λ

Recall that in Homework 1, Exercise 2, we encountered a firm that produces cellular telephone service using equipment and labor. When it uses E machine-hours of equipment and hires L person-hours of labor, it can provide up to Q units of telephone service. The relationship between Q, E, and L is as follows: $Q = \sqrt{EL}$. The firm must always pay P_E for each machine-hour of equipment it uses and P_L for each person-hour of labor it hires.

We solved for the optimal choices of E and L given Q, P_E, and P_L, which we can denote by $E^*(Q, P_E, P_L)$ and $L^*(Q, P_E, P_L)$.

Now I would like to ask you to find the marginal cost of the firm, that is the cost of producing an additional unit, when the firm is producing at output level Q and facing prices P_E and P_L. You should be able to express your answer in terms of these 3 exogenous variables. See if you can find the connection between the marginal cost and the Lagrangian multiplier λ.