Group Work

Econ 50 - Lecture 2

January 7, 2016

1. **Solving for Equilibrium**

   Suppose there are N_C identical consumers, each of whom spends $\frac{1}{4}$ of their income on a good:

   $$q^D(P) = \frac{\frac{1}{4}I}{P}$$

   There are N_F identical firms, each of which has an individual supply curve given by

   $$q^S(P) = \frac{P}{w}$$

   where w is the wage rate.

   (a) Write down the equations for the market supply $Q^S(P, w, N_F)$ and market demand $Q^D(P, I, N_C)$.

   $$Q^S(P, w, N_F) = N_F q^S(P, w) = N_F \frac{P}{w}$$
   $$Q^D(P, I, N_C) = N_C q^D(P, I) = N_C \frac{\frac{1}{4}I}{P}$$

   (b) Sketch the supply and demand curves for this market if $N_C = 100$, $N_F = 36$, $I = 64$, and $w = 9$. Indicate the equilibrium price and quantity on your diagram.

   Plugging these values into our equations for the supply and demand equations, and the equilibrium price and quantity, we obtain:

   $$Q^S(P, w, N_F) = N_F \frac{P}{w} = 36\frac{P}{9} = 4P$$
   $$Q^D(P, I, N_C) = N_C \frac{\frac{1}{4}I}{P} = 100\frac{\frac{1}{4}64}{P} = \frac{1600}{P}$$

   Setting these equal to each other, we can solve for the equilibrium price, P^E:

   $$4P^E = \frac{1600}{P^E}$$
   $$(P^E)^2 = \frac{1600}{4}$$
   $$P^E = 20$$

   Plugging the equilibrium price back into either equation, we obtain the equilibrium quantity:

   $$Q^S(P^E) = 4P^E = 4 \times 20 = 80$$
   $$Q^D(P^E) = \frac{1600}{P} = \frac{1600}{20} = 80$$

(c) Suppose the number of firms in the market quadrupled, to 144. Find the new equilibrium price, and calculate the change in consumer surplus.
The new supply curve is given by

$$Q^S(P, w, N_F) = N_F \frac{P}{w} = 36\frac{P}{9} = 16P$$

Following the same logic as above, this intersects the demand curve at a price of 10:

$$\begin{aligned} 16P^E &= \frac{1600}{P^E} \\ (P^E)^2 &= \frac{1600}{16} \\ P^E &= 10 \end{aligned}$$

The change in consumer surplus is the area bounded by the new price of 10, the old price of 20, and the demand curve. To find this, we take the integral of the quantity demanded from $P = 10$ to $P = 20$:

$$\begin{aligned} \Delta CS &= \int_{10}^{20} Q^D(P)dP \\ &= \int_{10}^{20} \frac{1600}{P}dP \\ &= 1600 \int_{10}^{20} \frac{1}{P}dP \\ &= 1600[\ln 20 - \ln 10] \\ &= 1600 \ln 2 \\ &\approx 1109 \end{aligned}$$

(d) Write P^E and Q^E as general functions of I, N_C, w, and N_F.
Here we follow the same procedure as above, setting $Q^S = Q^D$, only now we have more variables and fewer numbers:

$$Q^S(P^E, w, N_F) = Q^D(P^E, I, N_C) \tag{1}$$

$$N_F \frac{P^E}{w} = N_C \frac{\frac{1}{4}I}{P^E} \tag{2}$$

Solving equation (2) for P^E gives us:

$$(P^E)^2 = \frac{\frac{1}{4}IwN_C}{N_F} \tag{3}$$

$$P^E = \sqrt{\frac{\frac{1}{4}IwN_C}{N_F}} = \frac{1}{2}\sqrt{\frac{IwN_C}{N_F}} \tag{4}$$

This gives us our equilibrium price; again, to get the equilibrium quantity, we just need to plug this into either of the two equations. Either one gives us the same answer:

$$Q^E = Q^S(P^E, w, N_F) = N_F \frac{P^E}{w} = N_F \frac{\frac{1}{2}\sqrt{\frac{IwN_C}{N_F}}}{w} = \frac{1}{2}\sqrt{\frac{N_C N_F I}{w}} \tag{5}$$

$$Q^E = Q^D(P^E, I, N_C) = N_C \frac{\frac{1}{4}I}{P^E} = N_C \frac{\frac{1}{4}I}{\frac{1}{2}\sqrt{\frac{IwN_C}{N_F}}} = \frac{1}{2}\sqrt{\frac{N_C N_F I}{w}} \tag{6}$$

(e) In which of the exogenous variables is P^E increasing? What about Q^E?

We found before that the equilibrium price was given by

$$P^E = \frac{1}{2}\sqrt{\frac{IwN_C}{N_F}}$$

Because N_C, I, and w are in the numerator of the expression for P^E, the equilibrium price is increasing in the number of consumers, income, and wages (and decreasing in the number of firms). This makes sense: if there are more consumers demanding a good, or those consumers are richer, the demand curve will shift to the right, causing an increase in equilibrium price; and if wages increase, the supply curve will shift to the left, causing an increase in price. On the other hand, an increase in the number of firms will shift the supply curve to the right, decreasing the equilibrium price.

Similarly, we found before that the equilibrium quantity was given by

$$Q^E = \frac{1}{2}\sqrt{\frac{N_C N_F I}{w}}$$

Because N_C, N_F, and I are in the numerator of the expression for Q^E, the equilibrium quantity is increasing in the number of consumers or the number of firms, as well as income (and decreasing in wages). Again, this confirms our intuition: an increase (rightward shift) in either supply or demand will increase the equilibrium quantity, so consumers or firms entering the market will drive up quantity, as will an increase in consumer income. An increase in wages, on the other hand, causes a leftward shift in the supply curve, and therefore a decrease in equilibrium quantity.