Midterm - Part I - Questions 1-4 - 55 Points - Solutions

Econ 50 - Stanford University - Winter Quarter 2014/15

February 9, 2015

The exam is divided into two parts, Part I and Part II.
When you are done, please put each part in the appropriate box in the lobby outside Cubberley Auditorium.

Write your name and your TA's name (**Rui** Xu, **Michael** Zhang, or **Connor** Scherer), and sign the statements on the covers of Part I and Part II of the exam.

You will have a total of 110 minutes to complete this exam. You can work on Part I and Part II in any order. The exam is worth a total of 100 points, so you should allocate approximately one minute per point. Pace yourself carefully, and provide clear, concise answers – lengthy explanations are not necessary! If you finish early, there is an extra credit question worth 5 points (truly extra credit – it won't be used in normalizing scores!) if you want to show off. :)

Write all of your answers in the space provided. If you need extra room, please use the back of each sheet. Your numerical answers should be as precise as possible. If you're pressed for time, don't worry about simplifying your answers perfectly. Make sure you show your work.

If you must make any additional assumptions in order to answer a question, please state what those assumptions are. At least one member of the Econ 50 staff will be available in the lobby outside Cubberley Auditorium at all times. We usually cannot answer questions, but please notify us if you feel you've found a mistake in the exam or if you observe a classmate engaging in suspicious behavior.

Remember that the only aid you may use for this exam is a simple four-function calculator (not a graphing calculator, programmable calculator, etc). No notes, books, headphones, cell phones, etc. may be used to help you.

"The answers written on these pages are entirely my own. I attest that in taking this exam, I am fully complying with all provisions of Stanford's Fundamental Standard and Honor Code."

Signature:

Printed Name:

TA's Name:

Please do not open this exam until it is time to begin. Good luck!

Question 1: How elastic are those sweatpants? [15 points]

Suppose the market demand for sweatpants (good X) is given by $Q_x = 20 + I - P_x - \frac{1}{2}P_y$, where I is the average income of consumers, P_x is the price of sweatpants, and P_y is the price of T-shirts.

(a) Compute the **own-price** elasticity of demand for sweatpants (ϵ_{Q_x,P_x}), the **cross-price** elasticity of demand for sweatpants with respect to T-shirts (ϵ_{Q_x,P_y}), and the **income** elasticity of demand for sweatpants ($\epsilon_{Q_x,I}$). [6 points]

Answer:

$$\epsilon_{Q_x,P_x} = \frac{\partial Q_x}{\partial P_x}\frac{P_x}{Q_x} = -\frac{P_x}{Q_x} = -\frac{P_x}{20 + I - P_x - \frac{1}{2}P_y}$$

$$\epsilon_{Q_x,P_y} = \frac{\partial Q_x}{\partial P_y}\frac{P_y}{Q_x} = -\frac{1}{2}\frac{P_y}{Q_x} = -\frac{1}{2}\frac{P_y}{20 + I - P_x - \frac{1}{2}P_y}$$

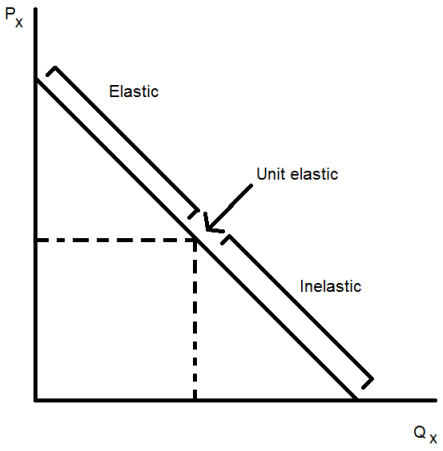
$$\epsilon_{Q_x,I} = \frac{\partial Q_x}{\partial I}\frac{I}{Q_x} = \frac{P_y}{Q_x} = \frac{I}{20 + I - P_x - \frac{1}{2}P_y}$$

(b) On a carefully drawn diagram of the demand for sweatpants, show where the demand for sweatpants is elastic, unit elastic, and inelastic. [6 points]

Answer: From part (a)

$$\epsilon_{Q_x,P_x} = -\frac{P_x}{Q_x}$$

Elastic: $\epsilon_{Q_x,P_x} < -1 \implies P_x > Q_x$
Unit elastic: $\epsilon_{Q_x,P_x} = -1 \implies P_x = Q_x$
Inelastic: $\epsilon_{Q_x,P_x} > -1 \implies P_x < Q_x$
Graphically, the demand function is a line with slope -1.



(c) According to this demand function, are sweatpants and T-shirts complements, substitutes, or neither? How do you know? [3 points]

Answer: The change in quantity demanded for x in response to a change in price of y is $\frac{\partial Q_x}{\partial P_y} = -\frac{1}{2} < 0$. They are compliments.

Question 2: Thinking on the margin [15 points]

(a) What does it mean if $MRS_{x,y} < \frac{P_x}{P_y}$ at a point along a consumer's budget constraint? [5 points]

Answer: The relative cost of an additional unit of x (given by the price ratio $\frac{P_x}{P_y}$) is greater than the relative benefit (given by $MRS_{x,y}$). That means the consumer can increase utility by selling X (to buy more Y).

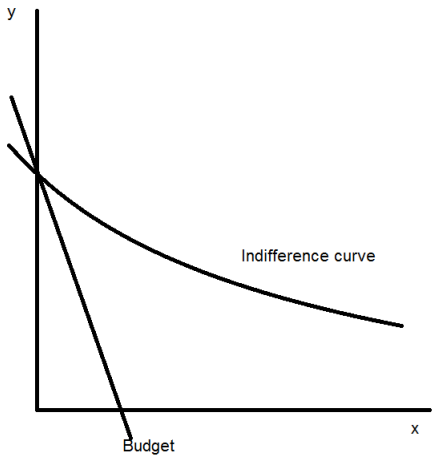
Another way to see this is rearranging the equation into

$$MRS_{x,y} = \frac{MU_x}{MU_y} < \frac{P_x}{P_y} \implies \frac{MU_x}{P_x} < \frac{MU_y}{P_y}$$

The additional utility of every dollar spent on x is less than that spent on y. Therefore if the consumer sells a dollar worth of X to buy a dollar worth of Y, the utility gained in Y exceeds the utility lost in X.

(b) If a consumer is in a position where that is true, can they always improve their utility by changing their consumption bundle? Why or why not? Illustrate your answer with one or two carefully drawn budget-line/indifference-curve diagrams. [5 points]

Answer: If the consumer is already consuming 0 units of X, then it's not possible to sell X to buy more Y. In that case the consumer is at an optimal corner solution where equality of the above equation doesn't hold. In the graph below, the magnitude of the slope of the budget line $\frac{P_x}{P_y}$ is greater than the slope of the tangent to the indifference curve $MRS_{x,y}$. Seeking a higher indifference curve while staying on the budget line requires the consumer to consume negative X which is not possible.



(c) If they could improve their utility by changing their consumption bundle, would it involve consuming more X and less Y, more Y and less X, or would it depend upon the exact form of the utility function in question? Carefully state the assumptions underlying your answer. [5 points]

Answer: The argument in part (a) assumes that both MU_x and MU_y are positive. This means that the utility function is monotonic. Under this assumption, consuming more Y and less X increases utility.

Question 3: It's all the same to me [15 points]

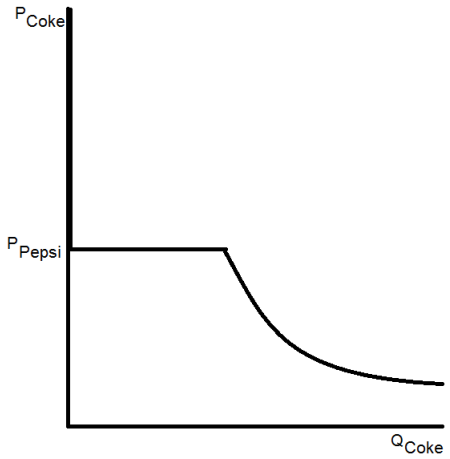
Nick cannot for the life of him tell the difference between Coke and Pepsi; he views them as perfect substitutes for one another.

(a) Clearly draw his demand curve for Coke on a carefully labeled diagram. [5 points]

Answer: If they are perfect substitutes, then Nick buys only Coke if it's cheaper than Pepsi. Otherwise he buys no Coke. Mathematically this is

$$Q_{Coke} = \frac{I}{P_{Coke}} \; if \; P_{Coke} < P_{Pepsi}$$

$$Q_{Coke} = 0 \; if \; P_{Coke} > P_{Pepsi}$$

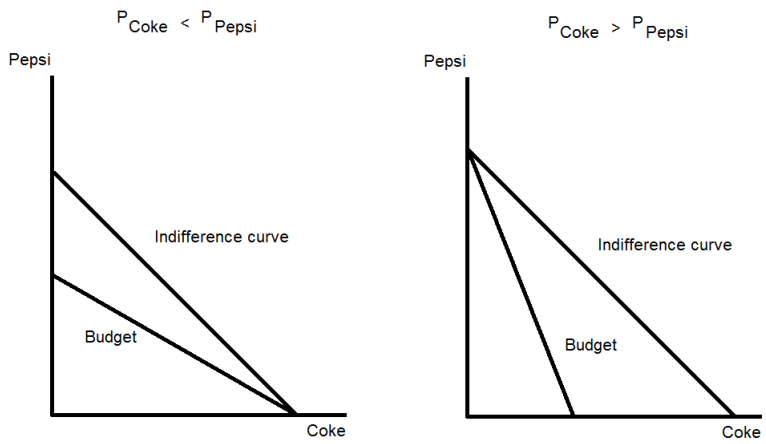


(b) Pick any point on the demand curve above, and label it point A. Use a budget-line/indifference curve diagram to explain what's going on at that point. [8 points]

Answer: The indifference curve is linear with a slope of -1 because consuming one less can of Coke and one more can of Pepsi leaves Nick indifference. The slope of the budget line is determined by the relative prices.

Two scenarios are given by the equations in part (a). In the first scenario, the slope of the budget line is shallower than the slope of the indifference curve, causing the optimal bundle to be at a corner solution on the Coke axis. Nick consumes all Coke and no Pepsi and this corresponds to the "curved" section in part (a).

In the second scenario, the slope of the budget line is steeper than the indifference curve, causing the optimal bundle to be at a corner solution on the Pepsi axis. Nick consumes no Coke, which corresponds to the vertical section in part (a).



There is a third scenario when the prices are equal. This corresponds to the horizontal segment in part (a). Nick is indifferent between Coke and Pepsi and can consume any amount of Coke up to the point that his budget is exhausted. The rest of the budget will be spent on Pepsi.

(c) Explain what would happen if the price of Coke dropped a little bit (say, a penny) from the point you chose in part (b). [2 points]

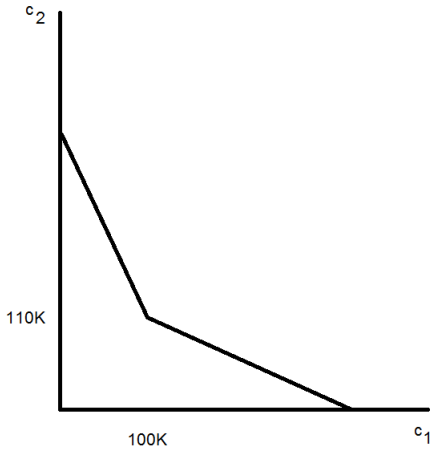
Answer: In the first scenario, Nick still consumes all Coke because it's still cheaper. He now consumes a bit more. In the second scenario, Pepsi is still cheaper so Nick still consumes 0 Coke. In the third scenario, he could have started with any amount of Coke. After the price change, he consumes all Coke.

Question 4: An unusually sweet deal [10 points]

Schmidt expects to earn \$100,000 this year and \$110,000 next year. He has a very unusual bank: it offers a *higher* interest rate on savings than it charges on loans. Specifically, it charges 10% annual interest on loans, but offers 15% interest on savings.

(a) Draw a precise diagram of Schmidt's intertemporal budget constraint for his decision of how much to consume this year and next. [5 points]

Answer: If Schmidt borrows, then $P_2 = \frac{1}{1.1}$. Since $P_1 = 1$, the magnitude of the slope of the budget line for borrowing is $\frac{P_1}{P_2} = 1.1$. Similarly, the magnitude of the slope of the budget line for saving is 1.15 which is steeper. This leads to the budget constraint



(b) If Schmidt has smooth (i.e., continuously differentiable) indifference curves, is there any way that he will choose to neither borrow nor save? Explain your answer, drawing some additional curves if necessary. [5 points]

Answer: With the indifference curve in part (a), suppose that the corner $A(100K, 110K)$ where Schmidt neither borrows nor saves is the optimal. A smooth indifference curve through that point has to be below one of the two segments of the budget. Then any point B on the budget line above the indifferent curve would be better than A. Therefore A cannot be optimal.