Group Work Sample Solutions

Econ 50 - Stanford University - Winter Quarter 2015/16

February 4, 2016

Suppose your team has a utility function $u(x, y) = x^{\alpha}y^{1-\alpha}$. For this entire question, assume $P_y = 3$.

1. **Pick a random number for your team's value of** α.

   I'll choose $\alpha = 0.4$.

2. **Draw your Hicksian and Marshallian Demand curves for X**.

   If we look at the graph for $\alpha = 0.25$, we see the following bundles (as indicated by the coordinates of points A, B, C, and E):

   - Bundle A is the initial bundle; here, $P_{x_1} = 2$ and $U = U_1$. Since bundle A has an X coordinate of 24, the point $(Q = 24, P = 2)$ is the point in a demand diagram that corresponds to point A.
   - Bundle B is the decomposition bundle; here $P_{x_2} = 4$ and $U = U_1$. Since bundle B has an X coordinate of 16, the point $(Q = 16, P = 4)$ is the point in a demand diagram that corresponds to point B.
   - Bundle C is the final bundle; here, $P_{x_2} = 4$ and $U = U_2$. Since bundle C has an X coordinate of 12, the point $(Q = 12, P = 4)$ is the point in a demand diagram that corresponds to point C.
   - Bundle E is the "equivalent" bundle; here $P_{x_1} = 2$ and $U = U_2$. Since bundle C has an X coordinate of 18, the point $(Q = 18, P = 2)$ is the point in a demand diagram that corresponds to point E.

   Once we've added these four points to the demand diagram, what remains is to connect the dots in the right way. The Marshallian demand curve passes through points A and C in the demand diagram, since they represent the choices holding income constant. There are two Hicksian demand curves: one passes through points A and B in the demand diagram, since they represent the choices holding utility constant at the initial utility level; the other passes through points C and E in the demand diagram, since they represent the choices holding utility constant at the final utility level.

3. **Calculate CV and EV.**

   (a) Calculate the cost of the four bundles (A, B, C, and E) at the **relevant** prices.

   - Bundle A is the initial bundle; here, $P_{x_1} = 2$ and $U = U_1$. Since bundle A has the coordinates $(24, 24)$, the cost of bundle A is

     $$E(P_{x_1}, P_y, U_1) = P_{x_1}x_A + P_y y_A = 2 \times 24 + 3 \times 24 = 120,$$

     so the original income is \$120.
   - Bundle B is the decomposition bundle; here $P_{x_2} = 4$ and $U = U_1$. Since bundle B has the coordinates $(16, 32)$, the cost of bundle B is

     $$E(P_{x_2}, P_y, U_1) = P_{x_2}x_B + P_y y_B = 4 \times 16 + 3 \times 32 = 160,$$

     so you would need \$160 to afford U_1 at the new prices.

- Bundle C is the final bundle; here, $P_{x_2} = 4$ and $U = U_2$. Since bundle C has the coordinates $(12, 24)$, the cost of bundle C is

$$E(P_{x_2}, P_y, U_2) = P_{x_2} x_C + P_y y_C = 4 \times 24 + 3 \times 24 = 120,$$

which is the same as our original income is \$120. (Note that because of rounding, you might not have found exactly the same income here; that's OK.)

- Bundle E is the "equivalent" bundle; here $P_{x_1} = 2$ and $U = U_2$. Since bundle E has the coordiantes $(18, 18)$, the cost of bundle E is

$$E(P_{x_1}, P_y, U_2) = P_{x_2} x_E + P_y y_E = 2 \times 18 + 3 \times 18 = 90,$$

so you would need \$90 to afford U_2 at the old prices.

(b) Calculate your **Compensating Variation**, which is the difference in cost of bundles B and C.

We found before that B would cost \$160, and C costs your actual income of \$120; so your compensating variation is \$40.

(c) Calculate your **Equivalent Variation**, which is the difference in cost of bundles A and E.

We found before that E would cost \$90, and A costs your actual income of \$120; so your equivalent variation is \$30.

4. **How much are you hurt?**

(a) The diagram shows a doubling of the price of X, from \$2 to \$4. How much would your team be willing to pay to avoid this price increase?

After the price increase, your utility is lower. In particular, your utility after the price change is the same as at point E. Point E represents the bundle you would buy if your income were \$90 and prices remained unchanged; so the price change is "equivalent" to a loss of \$30 of income. Therefore you would be willing to pay up to \$30 to avoid the price change.

(b) Suppose I want you to be as happy as you were before the price increase. How much would I have to pay you?

Before the price increase, your utility is higher. In particular, your utility before the price change is the same as at point B. Point B represents the bundle you would buy if prices changed, but your income was \$160; so if I "compensated" you \$40 for the price change, you would be no worse off than you were initially.

Note that the change in consumer surplus lies in between these two values. In particular, since this is Cobb-Douglas with $\alpha = 0.4$ and $I = 120$, the change in consumer surplus is given by

$$\begin{aligned} \Delta CS &= \int_2^4 \frac{0.4 \times 120}{P_x} \\ &= 48 \times (\ln 4 - \ln 2) \\ &\approx 33.27 \end{aligned}$$

which is between the equivalent variation of 30 and the compensating variation of 40.