Profit Maximization

Econ 50 | Lecture 15 | February 25, 2016

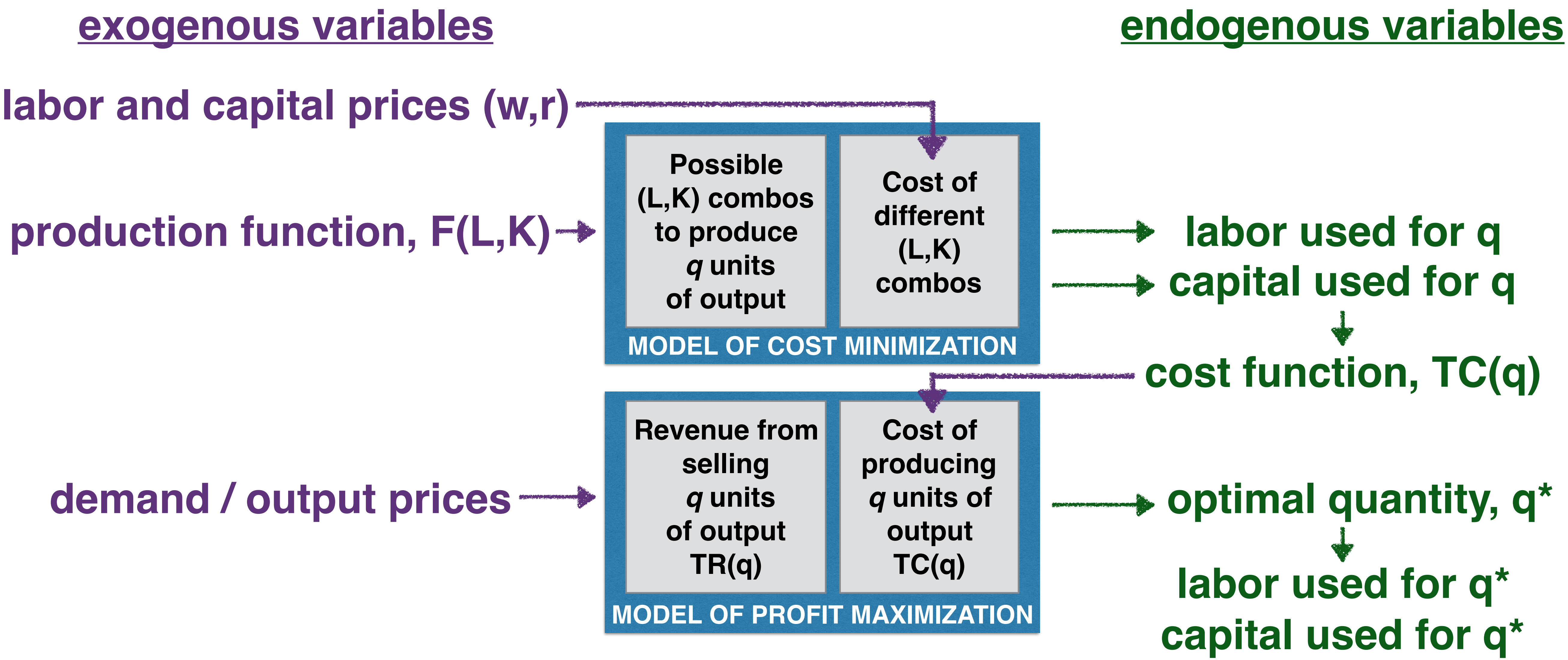
Lecture

- Profit-maximizing choice of inputs
- Profit-maximizing choice of output (and Hotelling's Lemma)
- The shutdown decision

Group Work

- Derive supply curve for Cobb-Douglas

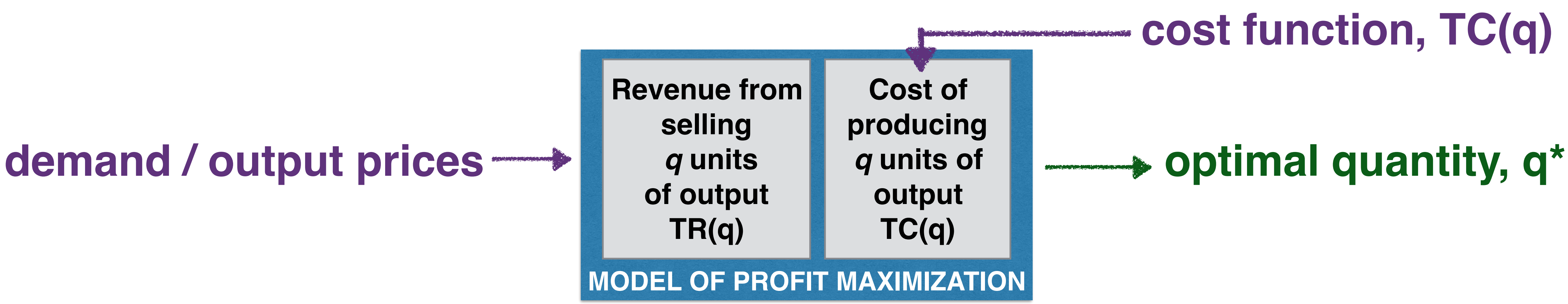
Unified Producer Theory



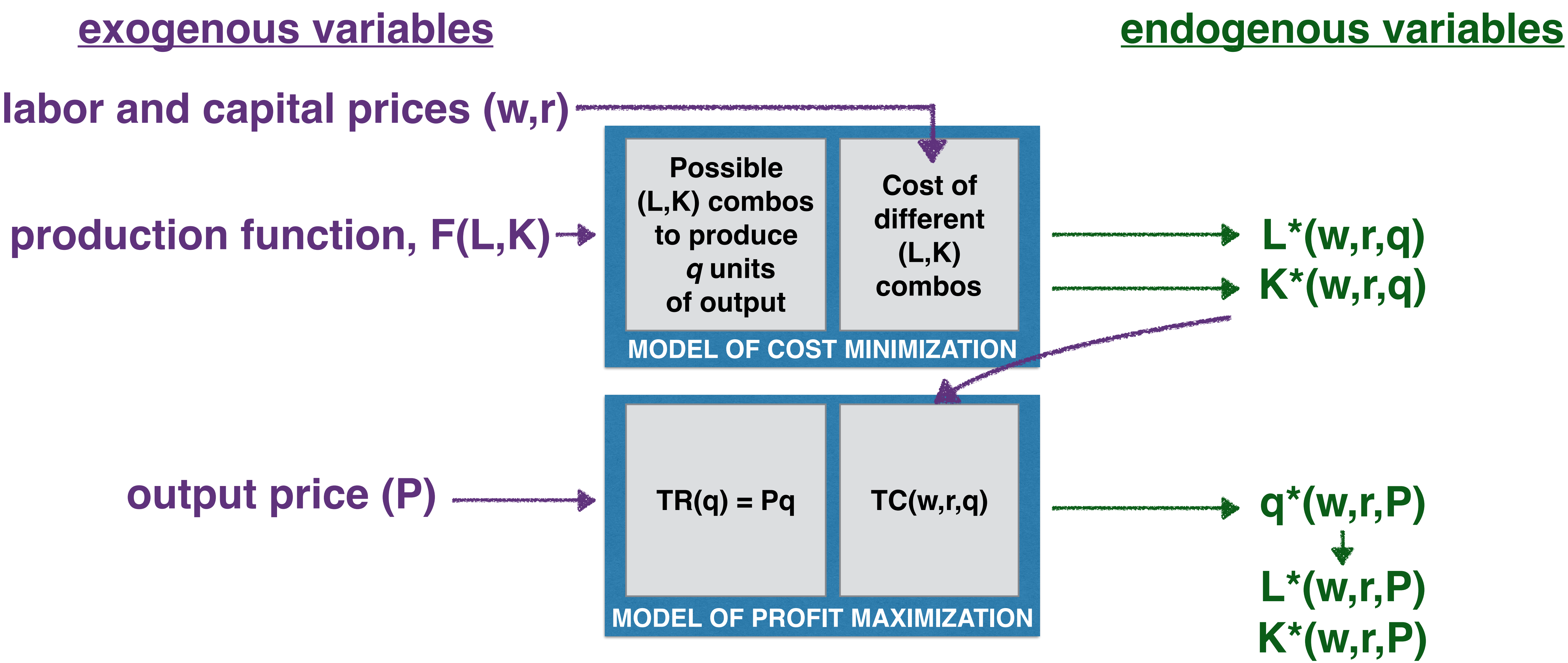
Producer Theory, Part II: Profit Maximization

exogenous variables

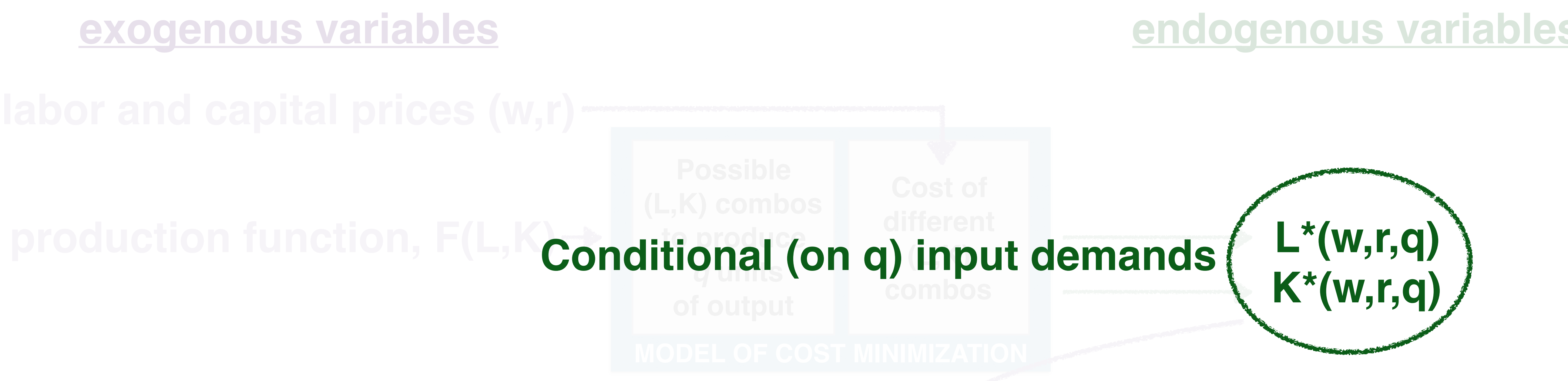
endogenous variables



Unified Producer Theory: **Perfect Competition**



Unified Producer Theory: **Perfect Competition**



Conditional (on q) input demands L*(w,r,q) K*(w,r,q)

Profit-maximizing quantity q*(w,r,P)

Profit-maximizing input demands L*(w,r,P) K*(w,r,P)

Unified Producer Theory: **Perfect Competition**

exogenous variables

endogenous variables

labor and capital prices (w,r)

production function, F(L,K)

output price (P)

Conditional (on q) input demands L*(w,r,q) K*(w,r,q)

MOST IMPORTANT TAKEAWAY:
BE ABLE TO DERIVE THESE
AND UNDERSTAND HOW THEY ARE
DIFFERENT FROM CONDITIONAL

Profit-maximizing quantity q*(w,r,P)

Profit-maximizing input demands L*(w,r,P) K*(w,r,P)

Given a production function $q = f(L, K)$

input prices (w, r),

and the market price of output (P),

there are two ways of thinking about the firm's problem:

	Profit =	Total Revenue	– Total Costs
Choosing **inputs**:	$\pi(L, K) =$	$P \times f(L, K)$	$-\ (wL + rK)$
Choosing **output**:	$\pi(q) =$	$P \times q$	$-\ TC(q)$
		(Value of output)	(Cost of inputs)

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Part I

Profit-Maximizing Choice of **Inputs**

Choosing Inputs

	Profit =	Total Revenue	−	Total Costs
		(Value of output)		(Cost of inputs)
Total Profit:	$\pi(L, K) =$	$P \times f(L, K)$	−	$(wL + rK)$
Marginal Profit (L):	$\frac{\partial \pi(L,K)}{\partial L} =$	$P \times MP_L$	−	w
Marginal Profit (K):	$\frac{\partial \pi(L,K)}{\partial K} =$	$P \times MP_K$	−	r

Marginal profit = 0 when $P \times MP_L = w$ and $P \times MP_K = r$

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Marginal Profit (K):	$\frac{\partial \pi(L,K)}{\partial K}$	=	$P \times MP_K$	$-\ r$

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MARGINAL REVENUE PRODUCT OF LABOR

$$MRP_L = P \times MP_L$$

"Additional revenue from an additional unit of labor"

MARGINAL REVENUE PRODUCT OF CAPITAL

$$MRP_K = P \times MP_K$$

"Additional revenue from an additional unit of capital"

Note

Short-run conditional demand for labor was **perfectly inelastic**.

Short-run profit-maximizing demand for labor is not! ...but why not?

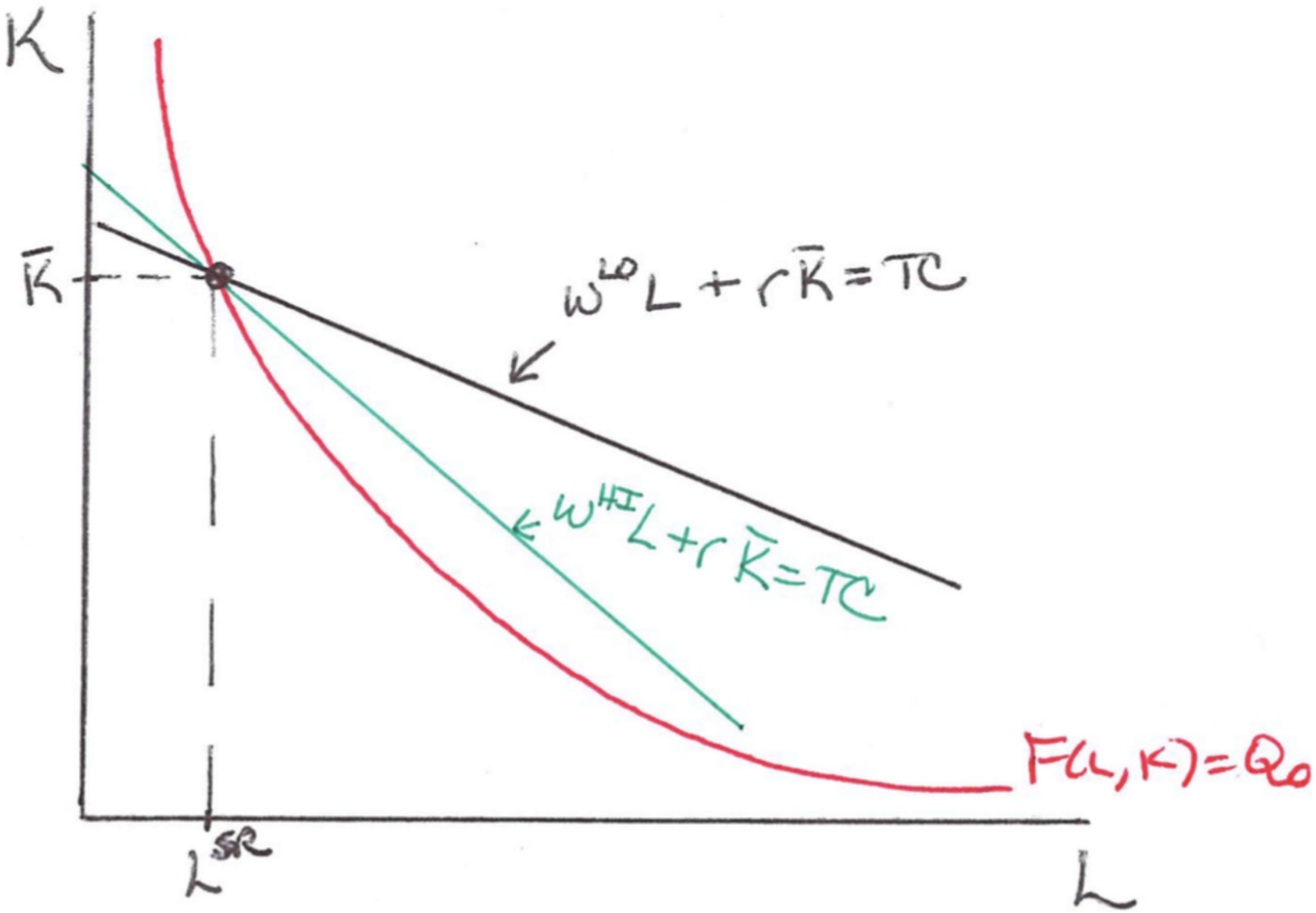
Note

- Short-run **conditional** demand for labor was **perfectly inelastic**.
- Short-run **profit-maximizing** demand for labor is not.
- Why the difference?

Why is the short-run profit-maximizing demand for labor not perfectly inelastic?

A	The firm is adjusting its capital	- (%)
B	The firm has some pricing power	- (%)
C	The firm is adjusting quantity in response to prices	- (%)
D	The firm has some wage-setting power	- (%)

Conditional on q, a change in w will not affect the amount of labor used



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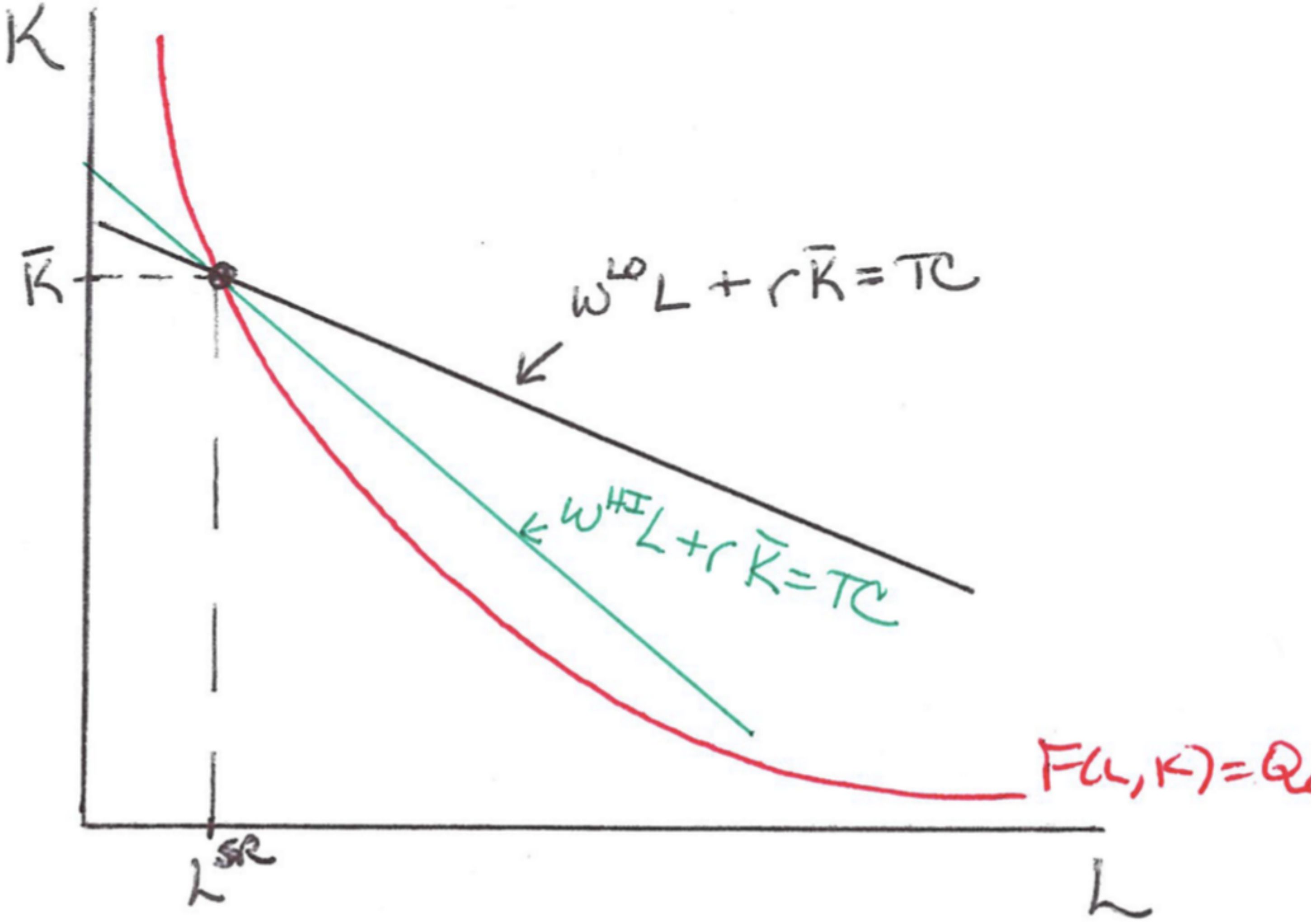
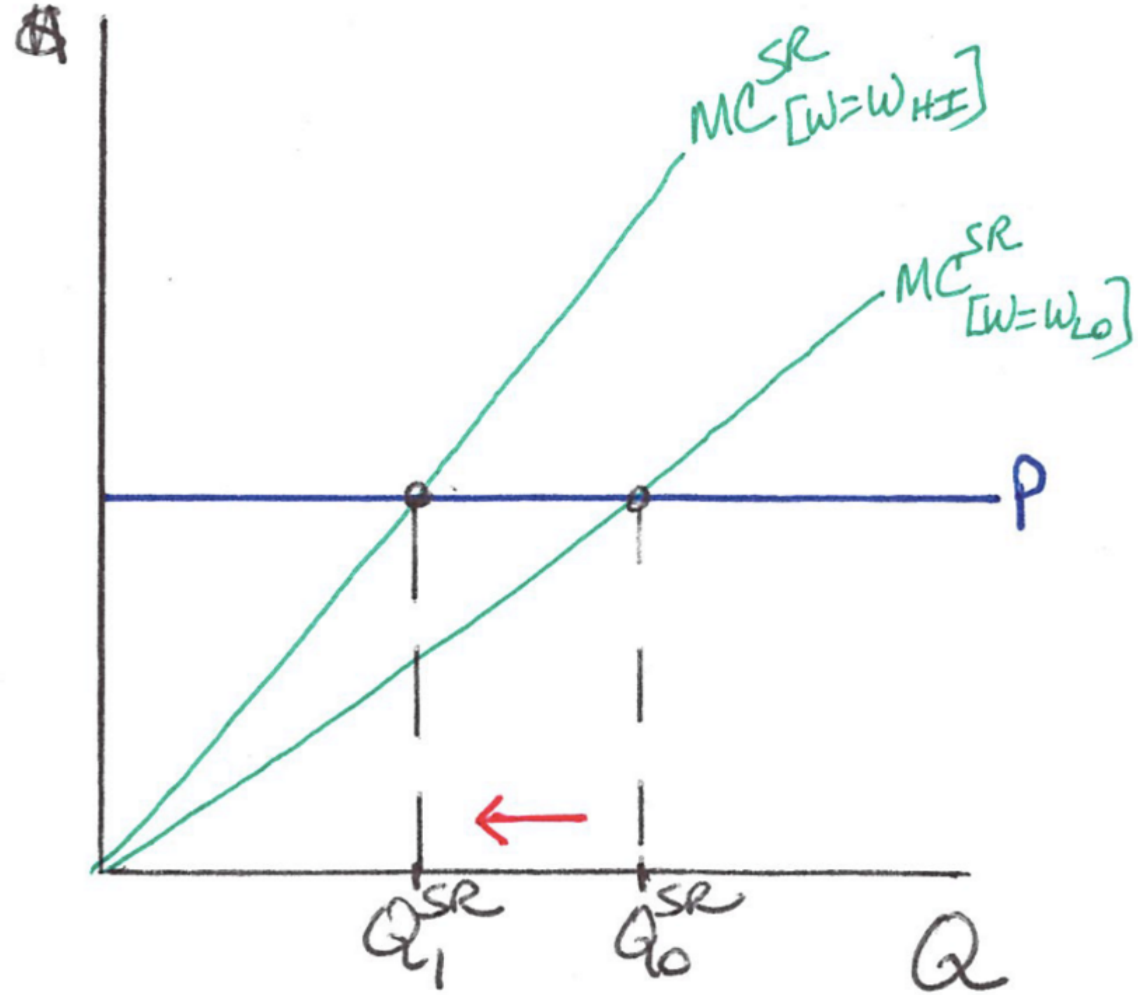
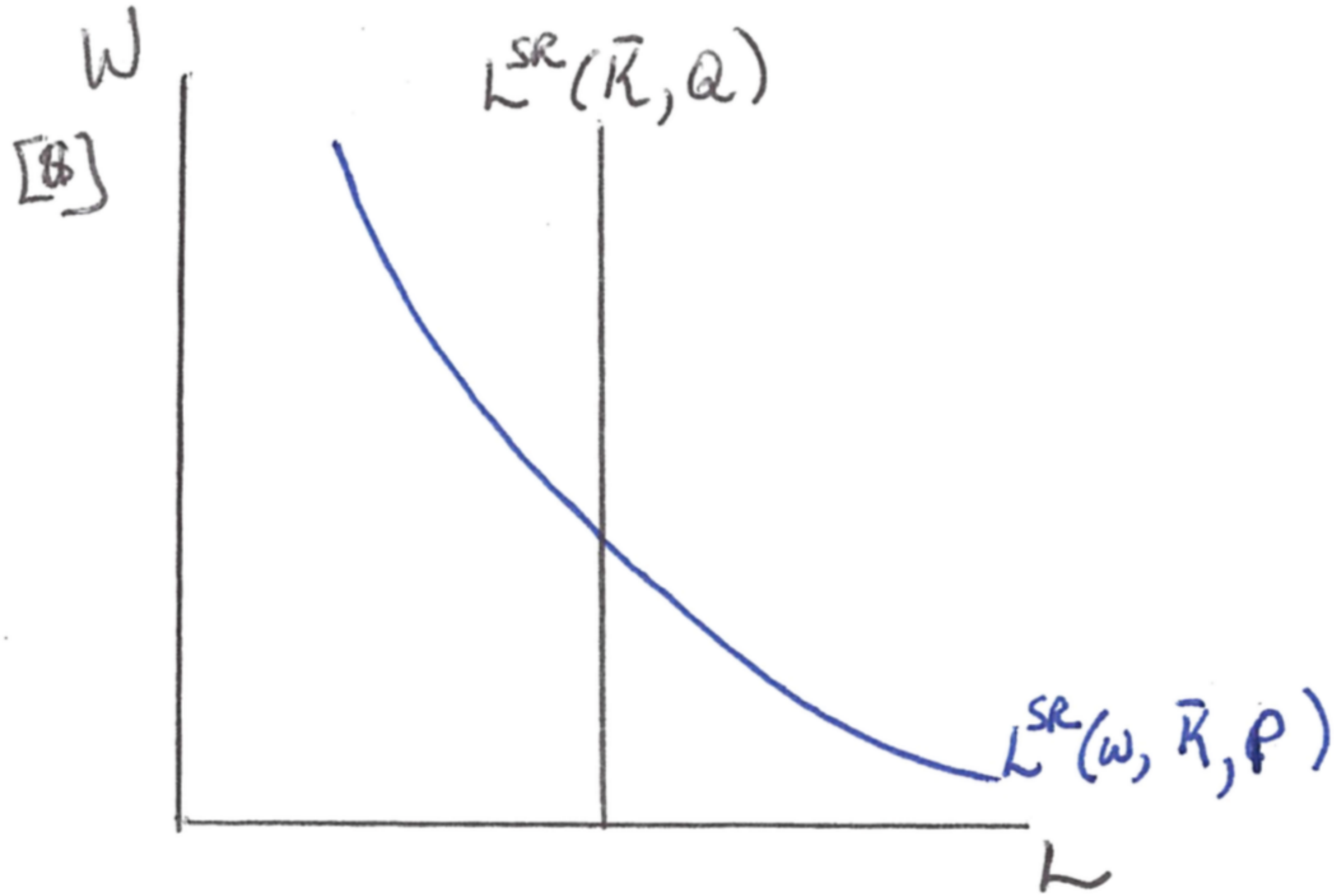


...but a change in w will cause a change in the profit-maximizing q



…so the labor needed to produce the profit-maximizing q will depend on w



Part II
Profit-Maximizing Choice of **Outputs**

Choosing Output

	Profit =	Total Revenue	−	Total Costs
		(Value of output)		(Cost of inputs)
Total Profit:	$\pi(q)$ =	$P \times q$	−	$TC(q)$
Marginal Profit:	$\frac{d\pi(q)}{dq}$ =	P	−	$MC(q)$

Marginal profit = 0 when $P = MC(q)$

Choosing Output

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Visualizing Profit per Unit

	Profit =	Total Revenue	−	Total Costs
		(Value of output)		(Cost of inputs)
Total Profit:	$\pi(q) =$	$P \times q$	−	$TC(q)$
	$=$	$P \times q$	−	$ATC(q) \times q$
	$=$	$[P - ATC(q)] \times q$		

Visualizing Profit per Unit

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Total Profit:	$\pi(q)$ =	$P \times q$	–	$TC(q)$
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Visualizing Profit per Unit

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Hotelling's Lemma

$$\frac{\partial \pi(w, r, P)}{\partial P} = q^*(w, r, P)$$

...in the short run or the long run

Why? ...a little like Shephard's Lemma

$$\pi(P) = P \times q^*(P) - TC(q^*(P))$$

$$\frac{\partial \pi(P)}{\partial P} = q^*(P) + P\frac{\partial q^*}{\partial P} - \frac{\partial TC}{\partial q}\frac{\partial q^*}{\partial P}$$

$$= q^*(P) + [P - MC(q)]\frac{\partial q^*}{\partial P}$$

...and since $P = MC(q)$, the second term is zero.

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Part III

The shutdown decision

The Shutdown Decision

	Profit =	Total Revenue	−	Total Costs
		(Value of output)		(Cost of inputs)
Profit if Operate:	$\pi(q)$ =	$P \times q$	−	$[FC + TVC(q)]$
Profit if Shut Down:	$\pi(0)$ =	$P \times 0$	−	FC

The Shutdown Decision

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The Shutdown Decision

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A firm will shut down if:

Profit if Operate	$<$	Profit if Shut Down
$\pi(q)$	$<$	$\pi(0)$
$P \times q - [FC + TVC(q)]$	$<$	$-FC$
$P \times q$	$<$	$TVC(q)$
P	$<$	$AVC(q)$

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P	$<$	$AVC(q)$

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Profit if Operate	<	Profit if Shut Down
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$P \times q - [FC + TVC(q)]$	<	$-FC$
$P \times q$	<	$TVC(q)$
P	<	$AVC(q)$

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Profit if Operate	<	Profit if Shut Down
$\pi(q)$	<	$\pi(0)$
$P \times q - [FC + TVC(q)]$	<	$-FC$
$P \times q$	<	$TVC(q)$
P	<	$AVC(q)$

SUPPLY DECISIONS: SUMMARY

Short run supply: adjust variable inputs (e.g. L)

to set q so that $P = MC(q)$,

as long as $P >$ min $AVC(q)$

Long run supply: adjust all inputs (e.g., K and L)

to set q so that $P = MC(q)$,

as long as $P >$ min $AC(q)$,

including opportunity costs of leaving industry.