Budget Constraints and Optimal Choice

Econ 50 | Lecture 6 | January 21, 2016

Lecture

- Review: The Story So Far
- Budget constraints
- Constrained optimization
- Optimization for each of our utility functions (graphical)

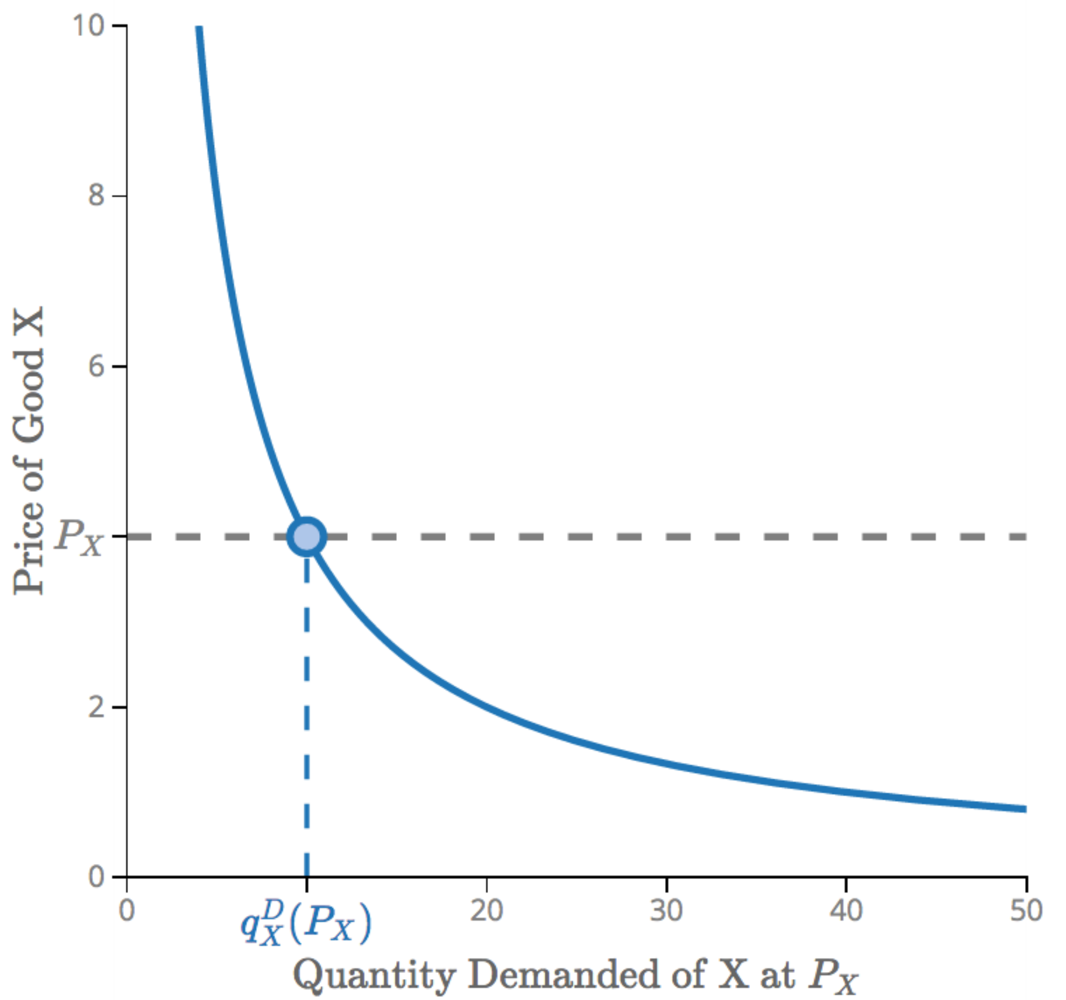
Group Work

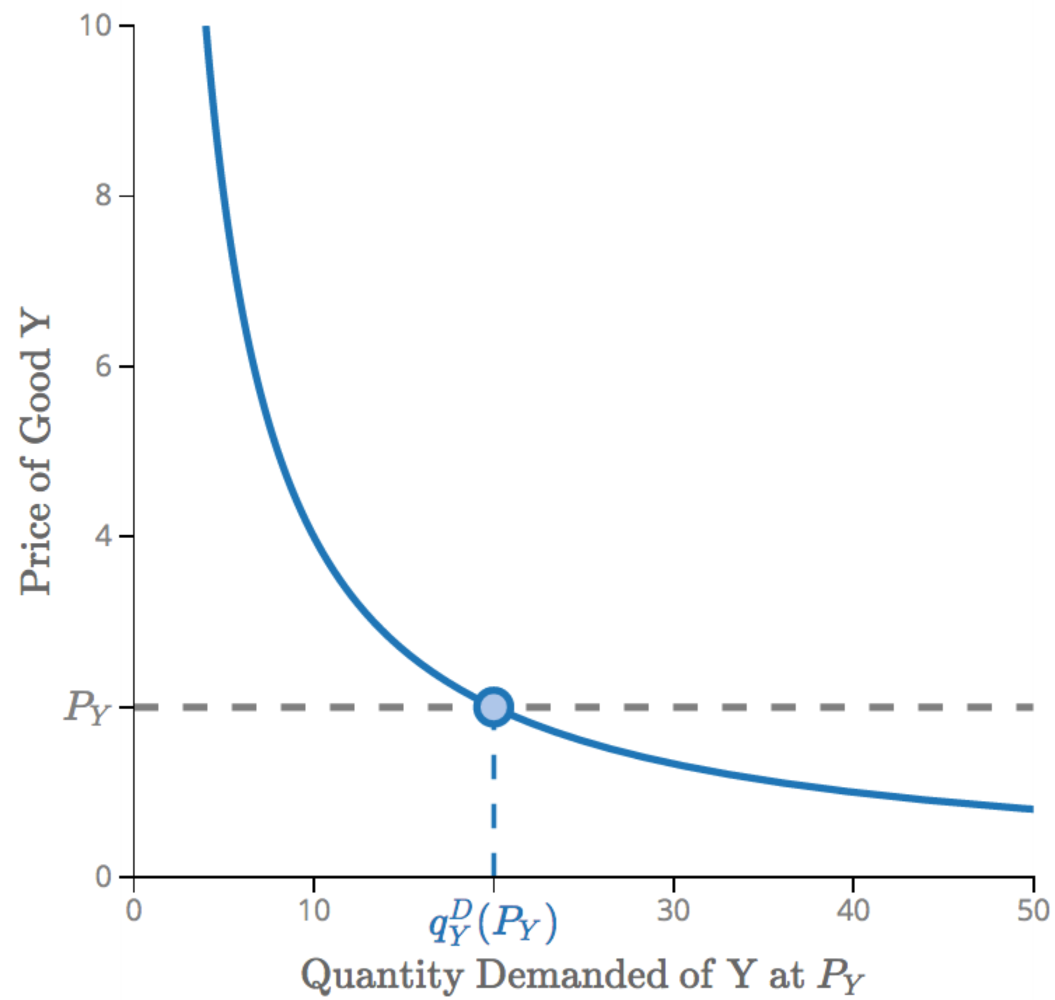
- Playing with budget constraints
- The relationship between MRS and the price ratio
- Solving Cobb-Douglas

Part I: The Story So Far


CODE

Demand Curves for X and Y





Unit Goal

$$q_x^D(P_x, P_y, I)$$

$$q_y^D(P_x, P_y, I)$$

Unit Goal

<u>**exogenous variables**</u>

price of good X
price of good Y
income

$$q_x^D(P_x, P_y, I)$$

$$q_y^D(P_x, P_y, I)$$

Unit Goal

exogenous variables

price of good X
price of good Y
income

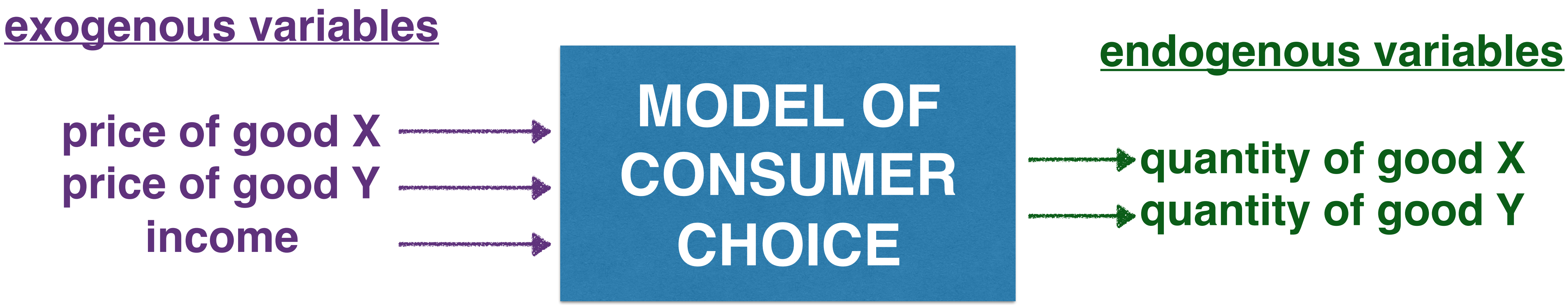
$$q_x^D(P_x, P_y, I)$$

$$q_y^D(P_x, P_y, I)$$

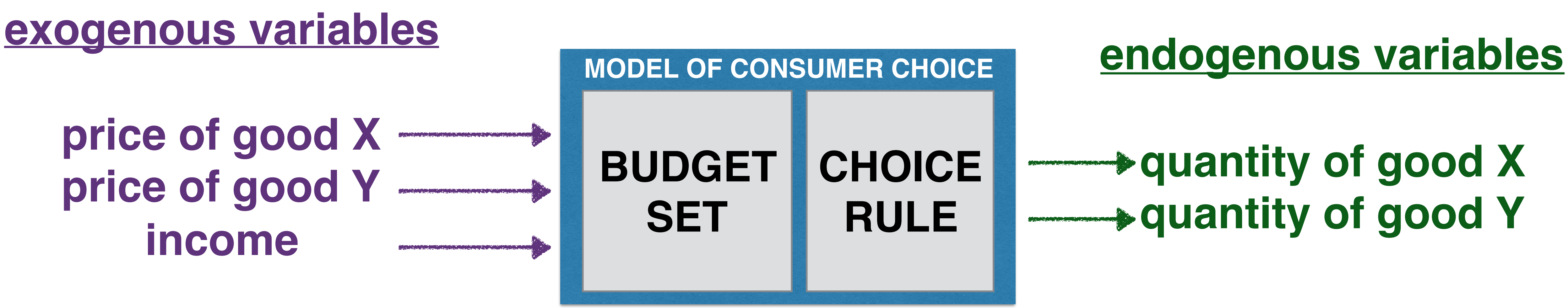
endogenous variables

quantity of good X
quantity of good Y

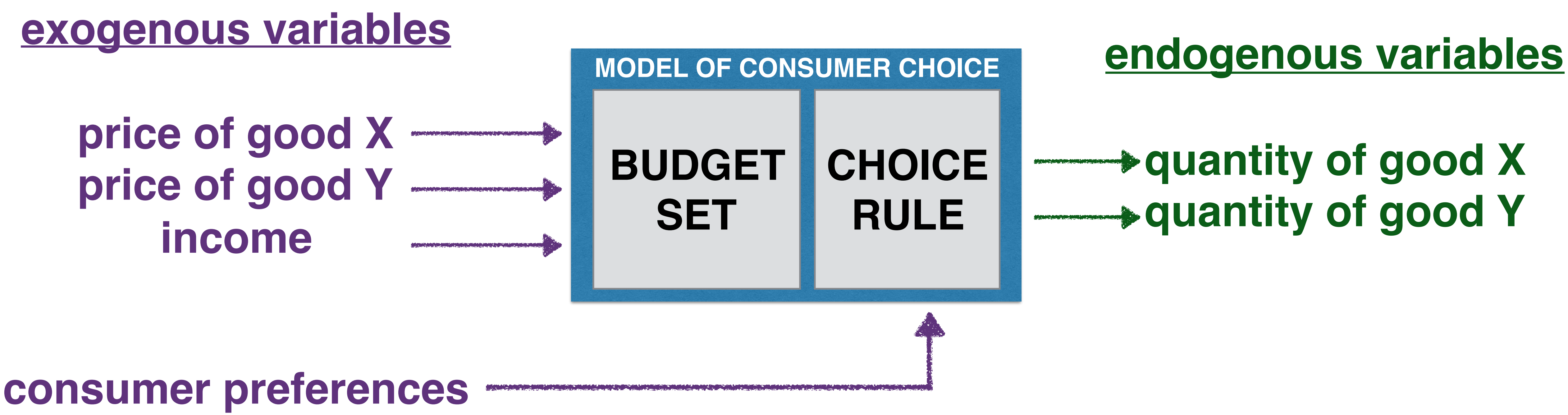
Unit Approach



Unit Approach



Unit Approach

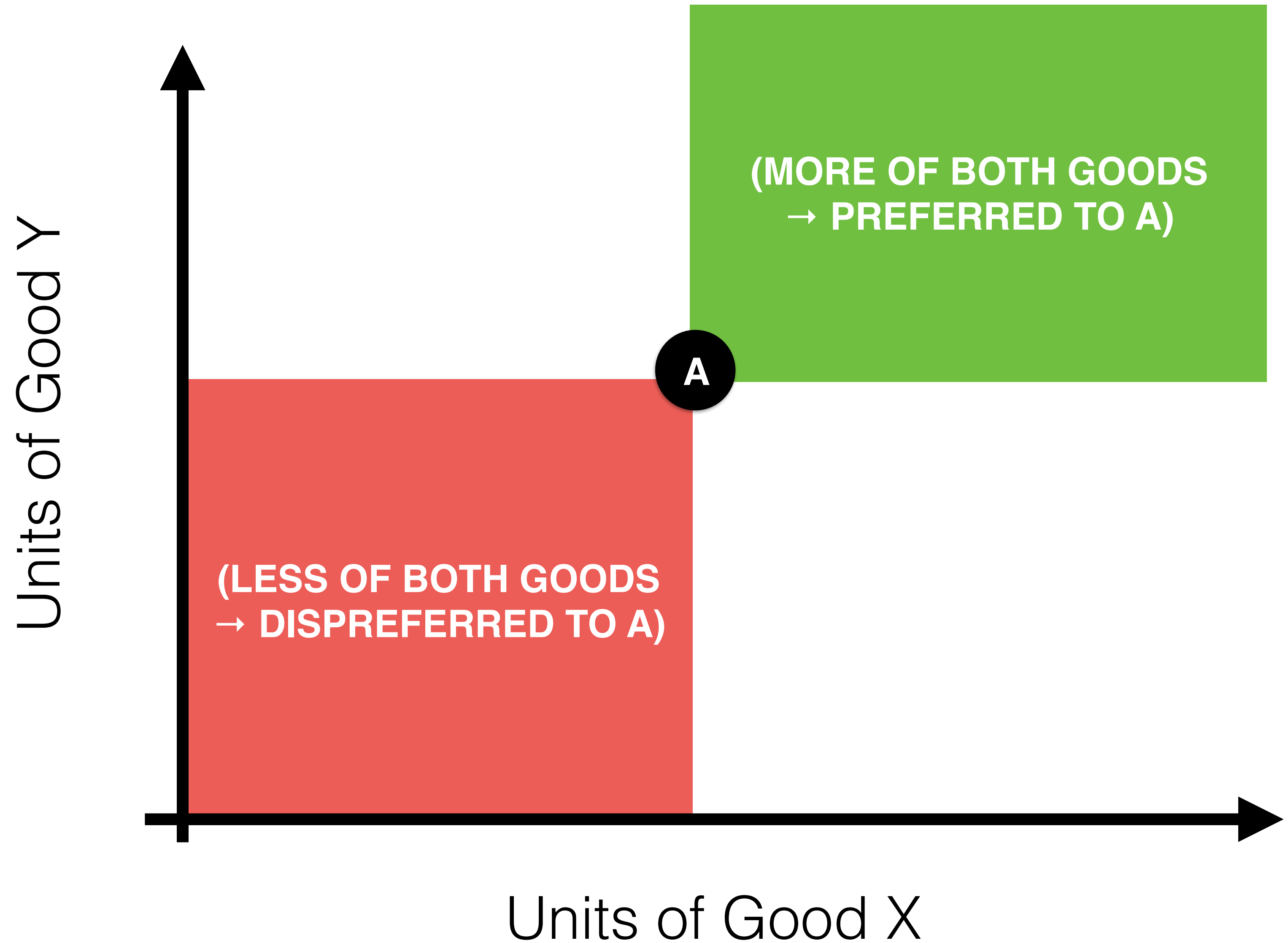


Unit Overview

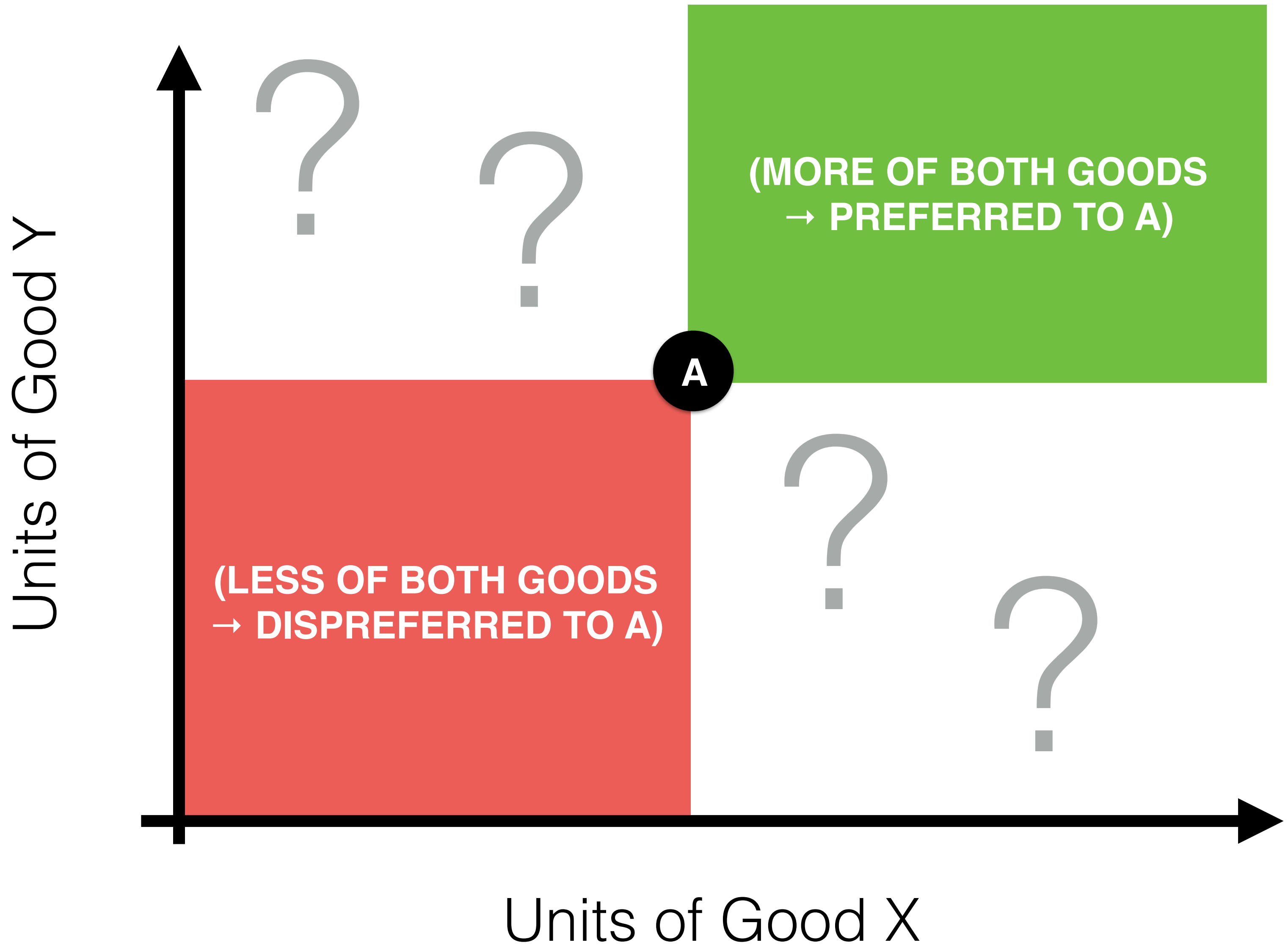
- **Preferences (Lectures 4 & 5, Homework 2)**
  How do consumers rank alternatives?
  Readings: B&B Ch. 3; Varian Ch. 3 & 4

- **Choice (Lectures 6 & 7, Homework 3)**
  How do consumers choose to allocate their income among goods?
  What do their observed choices "reveal" about their preferences?
  Readings: B&B Chapter 4; Varian Ch. 2, 5, 7

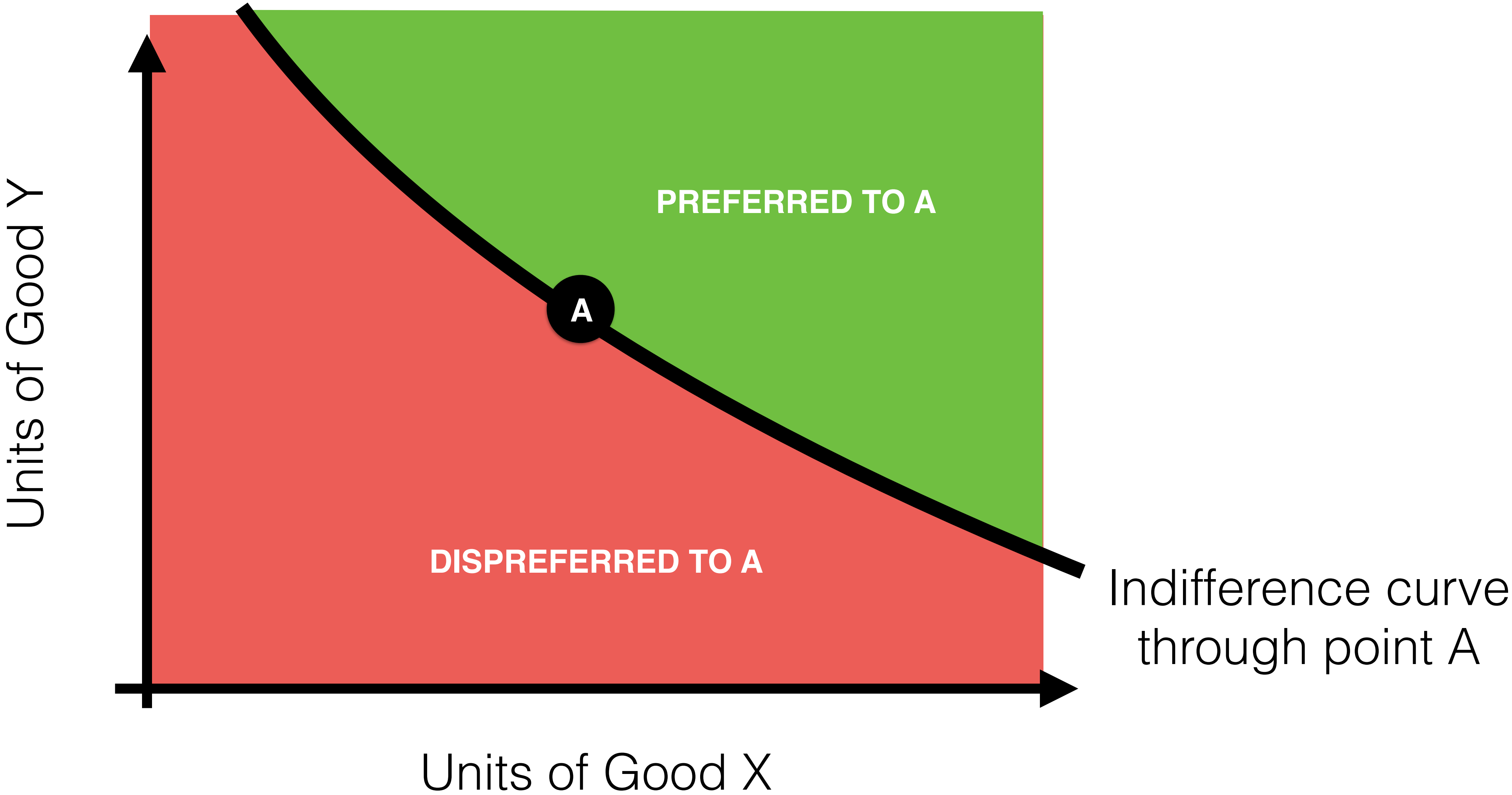
Monotonicity Assumption



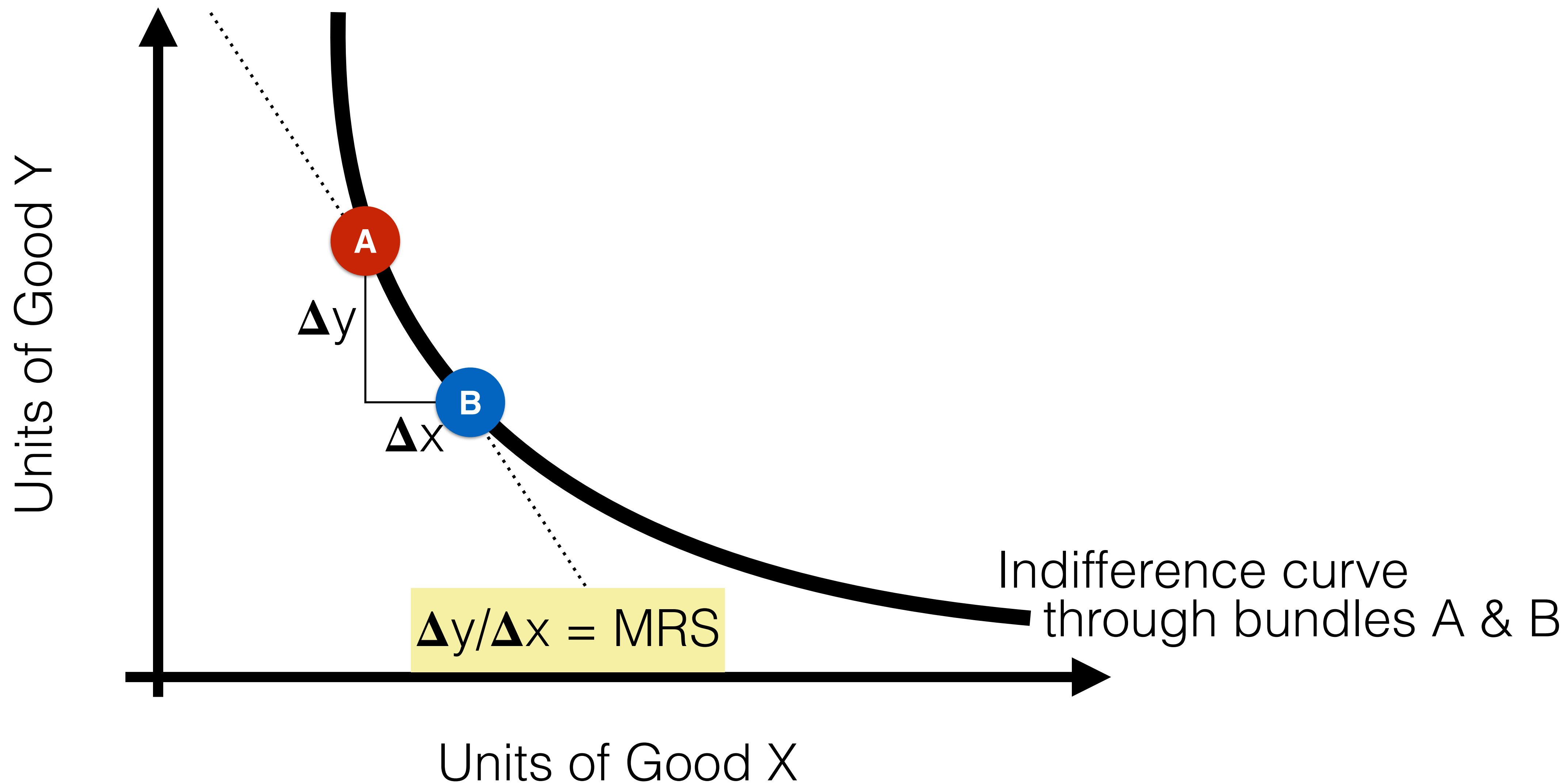
Monotonicity Assumption



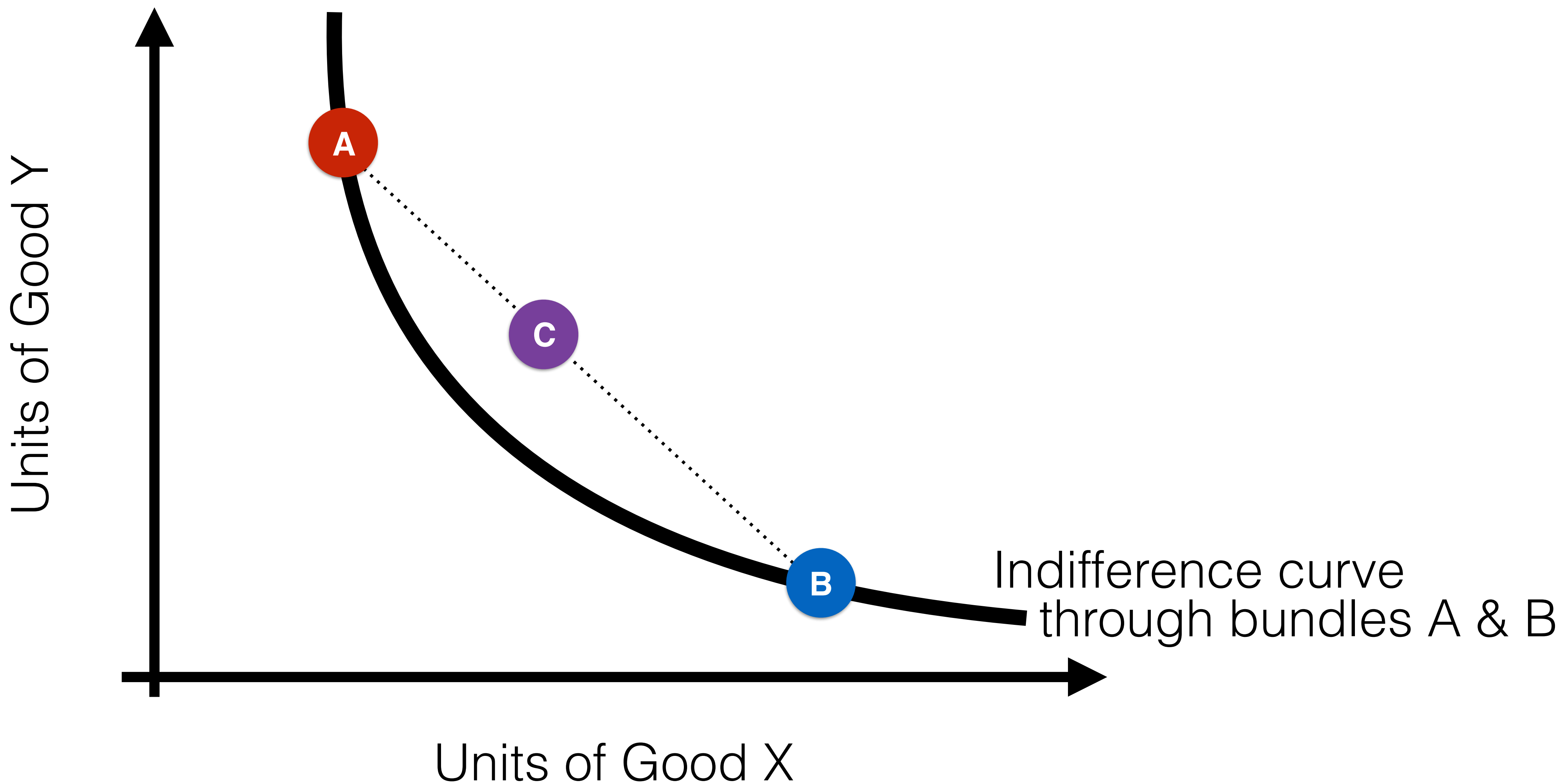
Indifference curves



Marginal Rate of Substitution

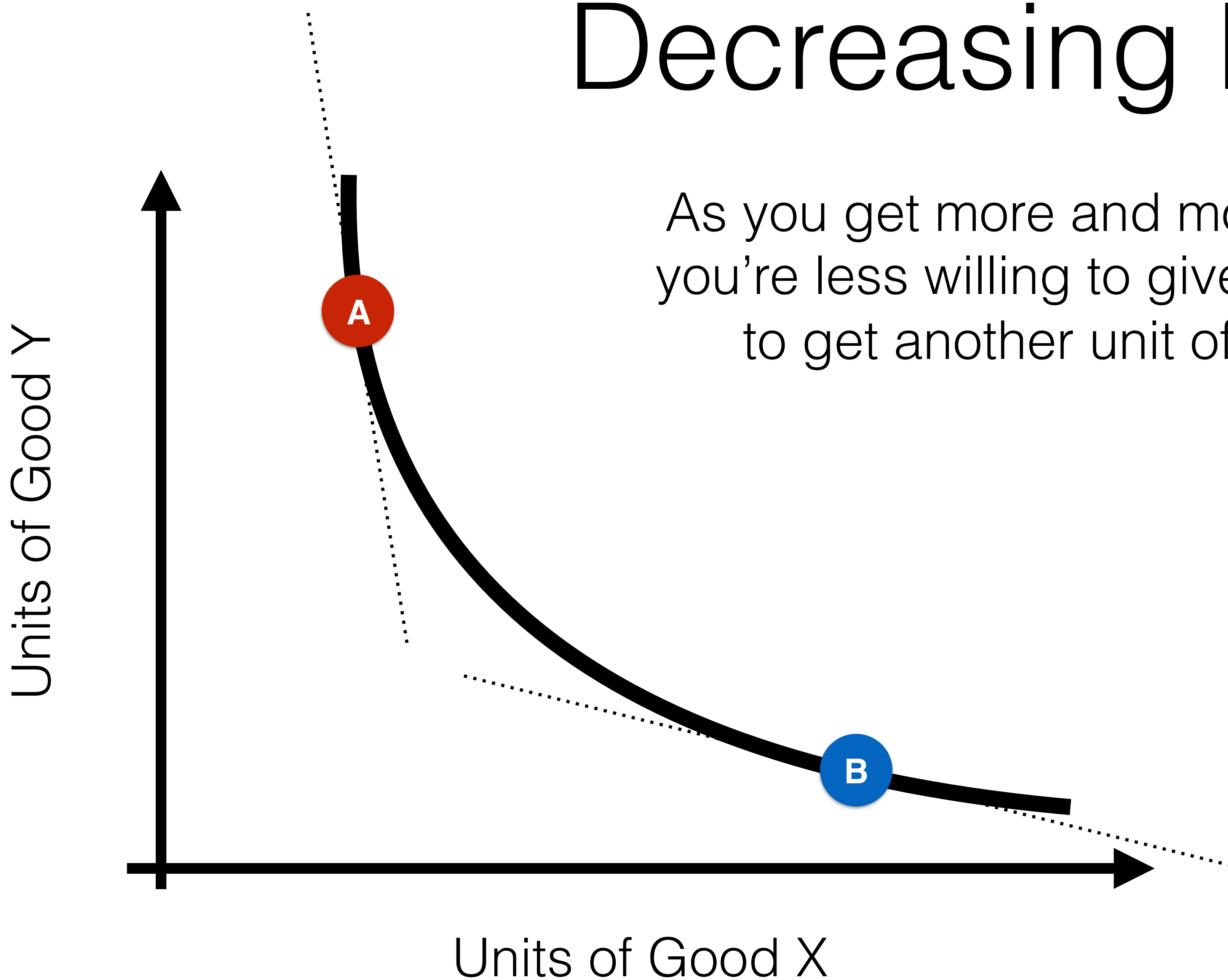


Convexity Assumption



Decreasing MRS

As you get more and more X, you're less willing to give up Y to get another unit of X



Marginal Rate of Substitution

- **Intuitively**: <u>rate</u> at which a consumer is willing to give up good Y to get an additional unit of good X.
- **Visually**: absolute value of the slope of an indifference curve
- **Mathematically**: $\frac{MU_x}{MU_y}$

Utility Functions

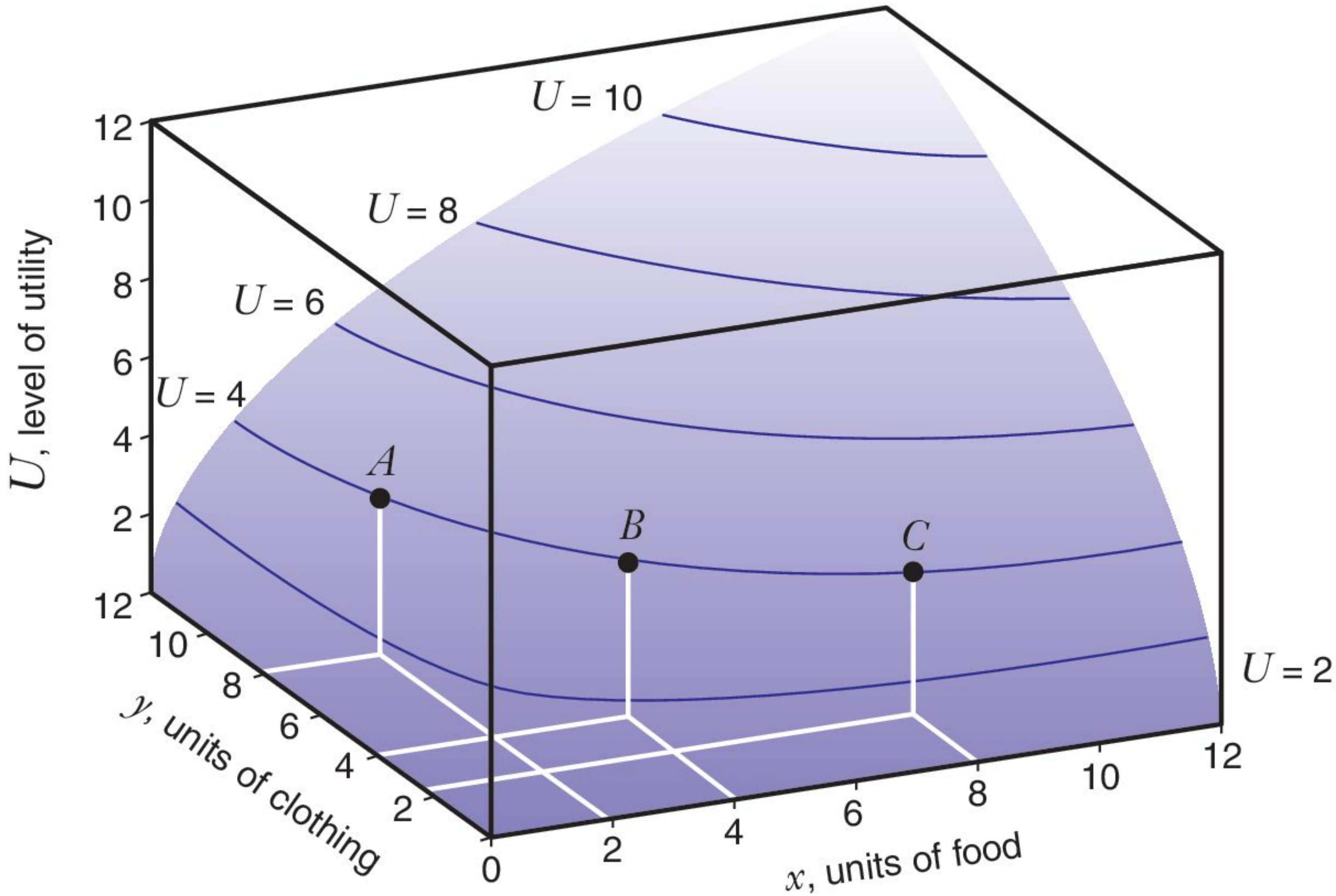
- A utility function **u()** assigns a real number to every possible bundle.

$$A \succ B \text{ if and only if } u(A) > u(B).$$

$$A \sim B \text{ if and only if } u(A) = u(B).$$

$$A \prec B \text{ if and only if } u(A) < u(B).$$

Indifference Curves are Level Curves of u(x,y)



Marginal Rate of Substitution = Ratio of the Marginal Utilities

$$MRS_{x,y} = \frac{\frac{\partial u(x,y)}{\partial x}}{\frac{\partial u(x,y)}{\partial y}} = \frac{MU_x}{MU_y}$$

What We've Established

(1) A way of **ranking** bundles
(utility function)
(2) A way of analyzing **tradeoffs**
(MRS)

What We've Established

(1) A way of **ranking** bundles
(utility function)
(2) A way of analyzing **tradeoffs**
(MRS)

Next Question:

Given a **limited choice set**,
which bundle
will the consumer choose?

Part II: Budget Constraints

Simple Budget Constraint

- You have a given income $\boldsymbol{I}$
- You're choosing quantities of two goods: $\boldsymbol{x}$ units of X, $\boldsymbol{y}$ units of Y
- Each good has a constant price ($\boldsymbol{P_x}$ and $\boldsymbol{P_y}$, respectively)
- Total expenditure on good X: $\boldsymbol{P_x x}$
  Total expenditure on good Y: $\boldsymbol{P_y y}$
- **Budget Line:** $\boldsymbol{P_x x + P_y y = I}$

Budget Constraint Diagram

Slope of the Budget Line

- **Budget Line: $P_x x + P_y y = I$**
- Solve for y:
- *Magnitude (absolute value) of the slope is the **price ratio***

Interpretation of the Price Ratio

- The price ratio represents the rate at which
  the market allows you to substitute good X for good Y

- Moving along the budget line (down and to the right):
  "**How many units of good Y** do I need to sell for price $\boldsymbol{P_y}$ each
  in order to **buy one more unit of of X** at a price of $\boldsymbol{P_x}$?"

Part III:
Constrained Optimization

The Good Case

IF

- The consumer's preferences over goods X and Y are **continuously differentiable, strictly monotonic**, and **strictly convex**
- The indifferences curves **do not cross the axes**
- The budget constraint is a **simple straight line**

THEN THE SOLUTION…

- will be an **interior solution** (involve strictly positive quantities of X and Y)
- can be found using the **Lagrange method**
- is characterized by:
  $MRS_{x,y} = P_x/P_y$
  $P_x x + P_y y = I$

The Canonical Econ 50 Diagram

Why does this work?

- ***Verbally***:
  Consumer compares their own willingness to trade Y for X (**MRS**)
  with the market's willingness to trade Y for X (**price ratio**)

- ***Graphically***:
  At the optimal point, there is **no overlap** between
  the **set of points preferred to that bundle** and the **budget set**.

- ***Mathematically***:
  FOC's of the Lagrange method find the tangency condition

Three ways of thinking about comparing **MRS** to the **price ratio**

$MRS_{x,y} > \frac{P_x}{P_y}$	$\frac{MU_x}{MU_y} > \frac{P_x}{P_y}$	$\frac{MU_x}{P_x} > \frac{MU_y}{P_y}$
The consumer is more willing to give up Y to get X than the market requires.	The consumer places relatively more value on X than the market in general.	The consumer gets more "bang for her buck" from good X than good Y..

Part IV: Optimization for our Five Utility Functions

Utility Function 1: Cobb-Douglas

Utility Function 2: Perfect Substitutes

Utility Function 3: Perfect Complements

Utility Function 4: CES

Utility Function 5: Quasilinear