

## Comparative Statics I: Plotting Changes to Price and Income

Econ 50 | Lecture 8 | January 28, 2016

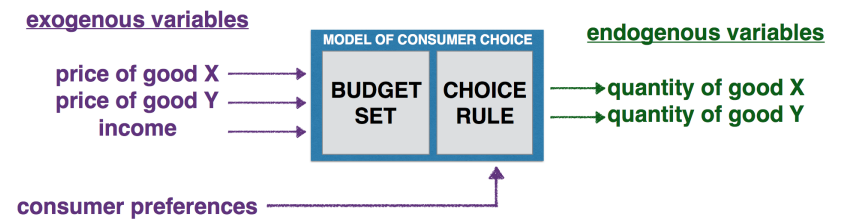
## Lecture

## Group Work

- Comparative statics: a conceptual introduction
- Four ways of looking at changes in prices and income
- Worked example: Cobb-Douglas
- Worked example: Quasilinear

## Part I Comparative Statics: A Conceptual Introduction

## Recall: Exogenous and Endogenous Variables



## Comparative Statics

- choose an **exogenous variable** of interest (price, income, etc.)
- hold all other **exogenous variables** constant
- examine how the **endogenous variables** change
- two potential plots:
  - exogenous variable vs. endogenous variable (e.g., demand curve)
  - multiple endogenous variables (e.g., price-consumption curve)

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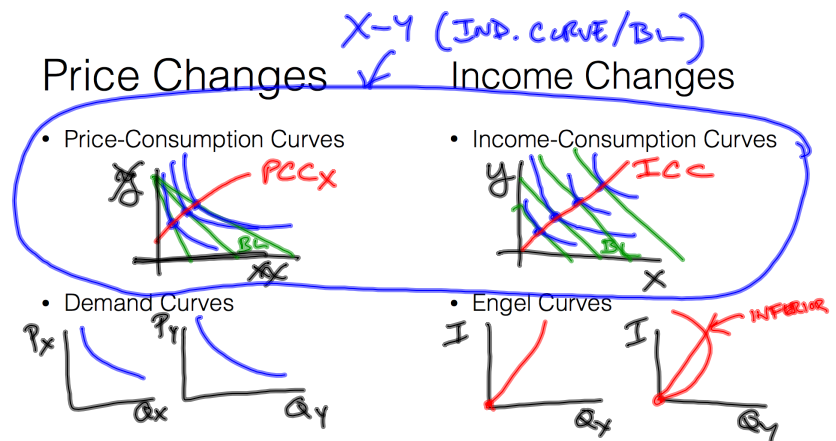
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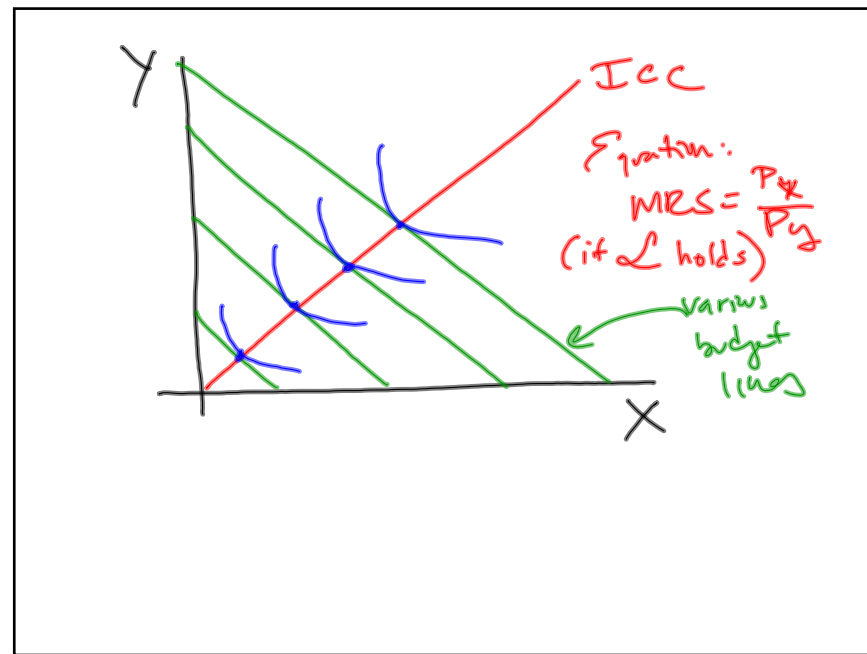
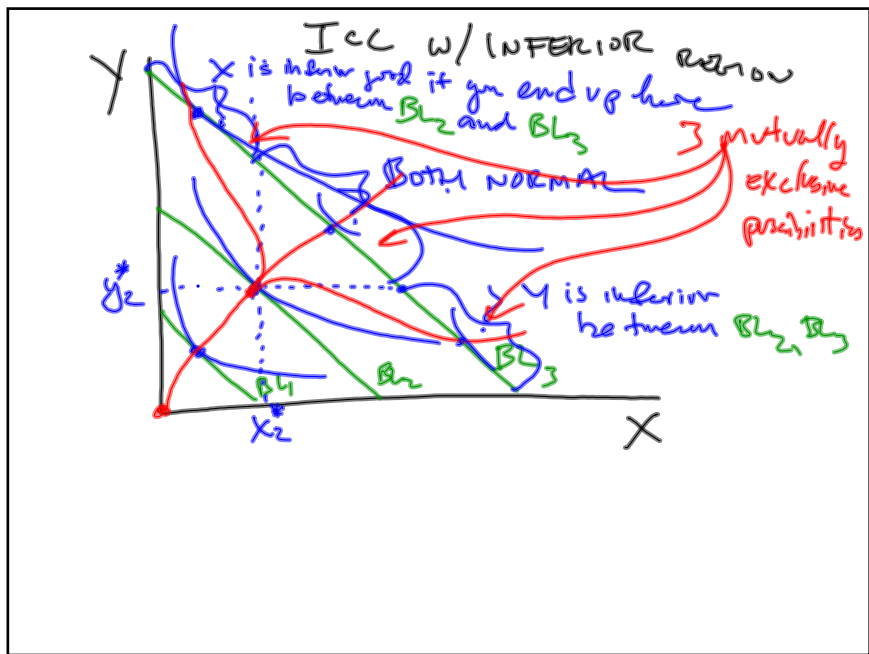
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## Part II

### Four Ways of Looking at Changes in Prices and Income





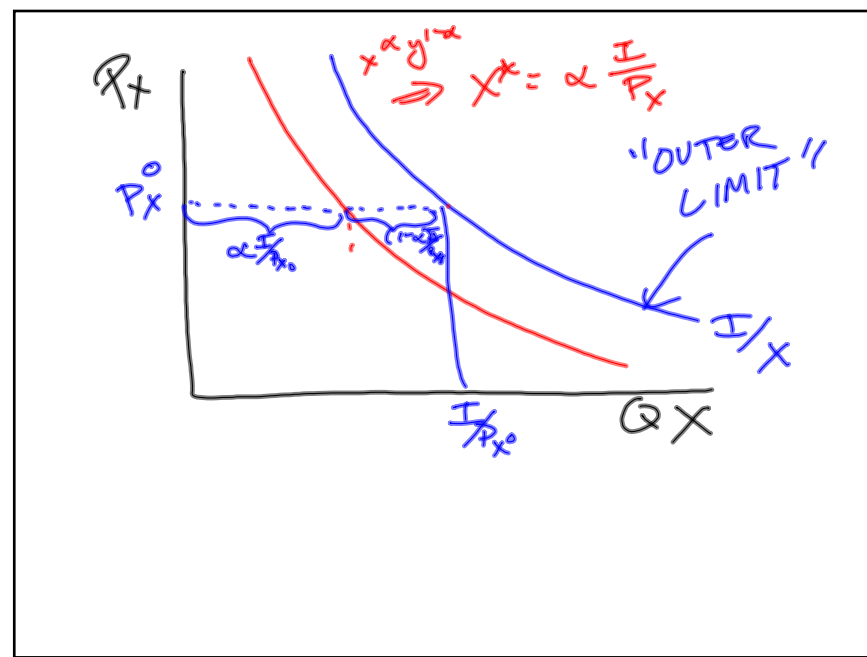
C-D MRS

$a \ln x + b \ln y$

Price ratio

$\frac{a/x}{b/y} = \frac{P_x}{P_y}$

$y = \frac{b}{a} \frac{P_x}{P_y} x$



## Part III

### Worked Example: Cobb-Douglas

$$x^* = \frac{1}{3} \frac{I}{P_x}, y^* = \frac{2}{3} \frac{I}{P_y}$$

### Worked Example $u(x,y) = xy^2$

#### Initial optimization:

Solve for  $(x^*, y^*)$  as a function of  $P_x$ ,  $P_y$ , and  $I$ .  
Find optimal consumption if  $P_x = 4$ ,  $P_y = 4$ ,  $I = 120$

#### Price-consumption curves:

Hold  $P_y = 4$  and  $I = 120$ ; vary  $P_x$ ; plot optimal consumption bundles in X-Y space.  
Hold  $P_x = 4$  and  $I = 120$ ; vary  $P_y$ ; plot optimal consumption bundles in X-Y space.

#### Income-consumption curves:

Hold  $P_x = 4$  and  $P_y = 4$ ; vary  $I$ ; plot optimal consumption bundles in X-Y space.

#### Demand curves:

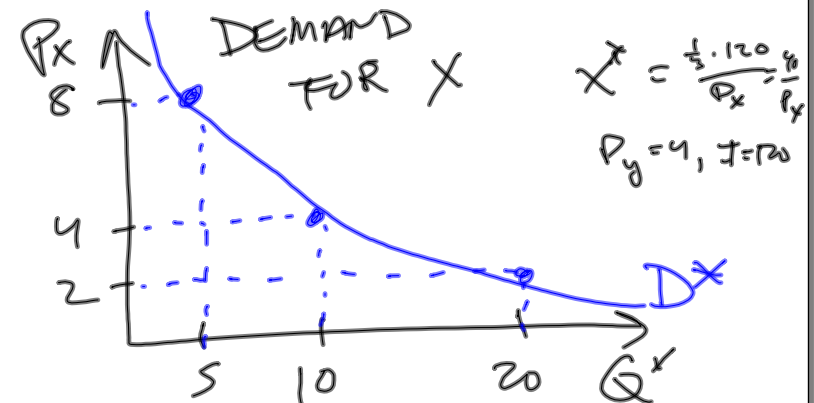
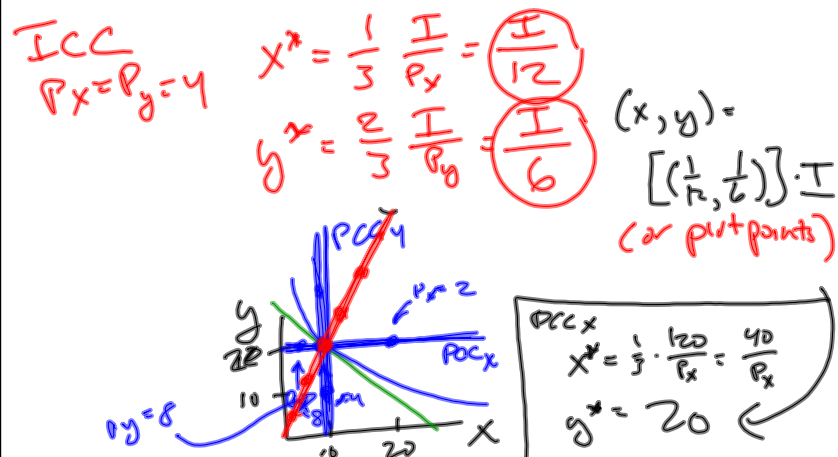
Hold  $P_y = 4$  and  $I = 120$ ; plot demand curve for X (quantity of X as a function of  $P_x$ ) X in P-Q space.  
Hold  $P_x = 4$  and  $I = 120$ ; plot demand curve for Y (quantity of Y as a function of  $P_y$ ) in P-Q space.

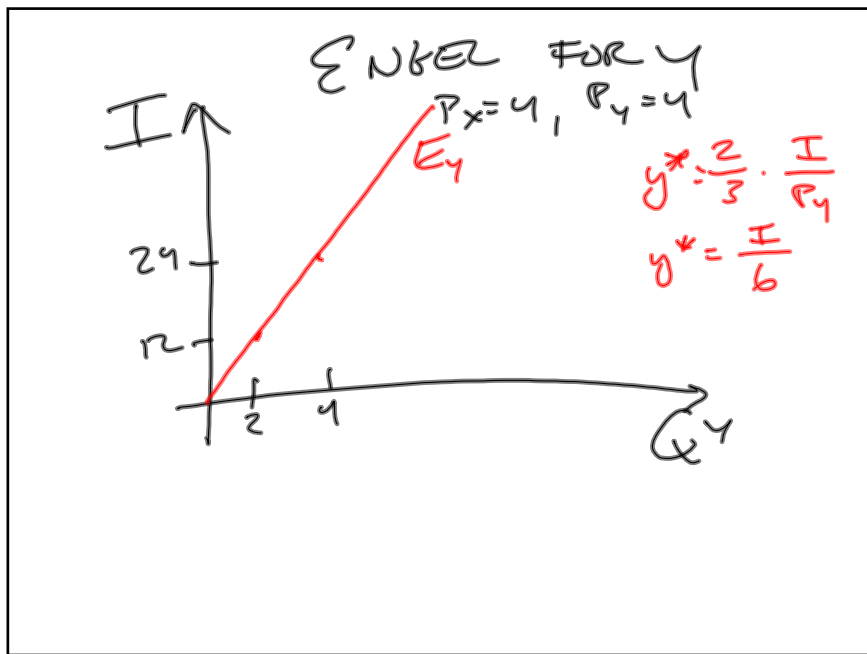
#### Engel curves:

Hold  $P_x = 4$  and  $P_y = 4$ ; plot consumption of X and Y as a function of income in I-Q space.

$$x^\alpha y^{1-\alpha}$$

$$\alpha = 1/3$$





$$u(x, y) = 12 \ln x + y$$

$$MU_x = \frac{12}{x}, MU_y = 1 \Rightarrow MRS = \frac{12}{x}$$

$$\text{Lagrange} \Rightarrow MRS = \frac{P_x}{P_y} \Rightarrow P_x x + P_y y = I$$

$$\frac{12}{x} = \frac{P_x}{P_y} \Rightarrow P_x x = 12 P_y \Rightarrow x = \frac{12 P_y}{P_x}$$

$$12 P_y + P_y y = I$$

$$P_y y = I - 12 P_y$$

$$y = \frac{I}{P_y} - 12$$

$$\text{Lagrange} \Rightarrow x = \frac{12 P_y}{P_x} \quad y = \frac{I}{P_y} - 12$$

$$y > 0 \Leftrightarrow \frac{I}{P_y} > 12 \text{ (or } I > 12 P_y)$$

$$\text{If } I < 12 P_y \Rightarrow \begin{cases} x = I/P_x \\ y = 0 \end{cases}$$