Homework 1

Econ 50 - Stanford University - Winter Quarter 2015/16

Due at the beginning of section on Friday, January 15

Exercise 1: Constrained Optimization with One Variable (Lecture 1)

Recall on the six functions from Group Work, Exercise 1:

i. $f(x) = 5 + 4x - x^2$

ii. $f(x) = 10 - |2 - x|$

iii. $f(x) = 9 - (x - 11)^2$

iv. $f(x) = 1 + \frac{1}{5}(x - 5)^2$

v. $f(x) = 10 - x$

vi. $f(x) = 3$

Thinking about these six functions, answer the following questions.

(a) For each function, write the "optimal" value(s) of x on the domain $0 \leq x \leq 10$; i.e., the value(s) that maximize(s) $f(x)$ on that domain. (This is what we did in group work in class.)

(b) For each of the cases above, explain why taking the derivative $f'(x)$ and setting it equal to zero works, or does not work.

(c) More generally, under what conditions will setting $f'(x) = 0$ get you to the solution to that problem?

(d) Write down a series of steps (i.e., an algorithm or flowchart) which, if followed, would allow you to find the optimal value(s) of x for each of the functions above. Here are some steps to get you started:

S1: Is $f(x)$ continuously differentiable on the domain $0 \leq x \leq 10$?
If so, go to S2.
If not, go to S3.

S2: Take the derivative $f'(x)$. How many values of x are there that set $f'(x) = 0$?
If none, go to S4.
If one, go to S5.
If a finite number more than one, go to S6.
If an infinite number, then $f(x)$ is a horizontal line and all values of x are optimal [END].

S3: (fill in the rest)

(e) Can you draw or describe a different kind of function that your algorithm would not work for?

Exercise 2: Constrained optimization with more than one variable (Lecture 1)

This is from Besanko and Braeutigam, 5e, Problem 1.4, with an extension.

A firm produces cellular telephone service using equipment and labor. When it uses E machine-hours of equipment and hires L person-hours of labor, it can provide up to Q units of telephone service. The relationship between Q,E, and L is as follows: $Q = \sqrt{EL}$. The firm must always pay P_E for each machine-hour of equipment is uses and P_L for each person-hour of labor it hires. Suppose the production manager is told to produce $Q = 200$ units of telephone service and that she wants to choose E and L to minimize costs while achieving that production target.

(a) What is the objective function for this problem?

(b) What is the constraint?

(c) Which of the variables $(Q, E, L, P_E$, and $P_L)$ are exogenous? Which are endogenous? Explain.

(d) Write a statement of the constrained optimization problem.

(e) Suppose $P_E = \$36$ and $P_L = \$9$. Use the method of Lagrange multipliers to find this firm's optimal choice of E and L. Show your work. Informally "check" your answer by showing two other combinations of E and L that would produce exactly 200 units of telephone service but that would be more costly than the one you identified in your solution.

Exercise 3: Market Supply and Producer Surplus (Lecture 2)

You might want to refer to the lecture notes posted after class to help you on this question...

Suppose there are two types of firms in the marketplace. The first type has a supply curve given by $q_1(P) = P$. The second type has a supply curve given by $q_2(P) = \sqrt{P}$. However, the second type has a "shutdown" price of $P = 9$; that is, it produces zero output if the price falls below \$9. There are 30 firms of each type.

(a) Sketch the individual supply curve for a representative firm of each type.

(b) Sketch the market supply curve.

(c) Suppose the price rises from \$4 to \$8. Illustrate the change in producer surplus and calculate its magnitude.

(d) Suppose the price rises from \$8 to \$16. Illustrate the change in producer surplus and calculate its magnitude.

Exercise 4: Parsing Demand Expressions (Lecture 2)

Consider the following expressions representing individual demand for good X:

i. $q_x^D(P_x, P_x, I) = \frac{I}{P_x + P_y}$

ii. $q_x^D(P_x, P_y, I) = \frac{I}{2P_x}$

iii. $q_x^D(P_x, P_y, I) = \frac{I}{P_x}$ if $P_x < P_y$, otherwise 0

iv. $q_x^D(P_x, P_y, I) = \left(\frac{P_y}{P_x + P_y}\right) \frac{I}{P_x}$

where I is income, P_x is the price of good X, and P_y is the price of another good.

(a) For each expression, identify whether two goods are complements, substitutes, or neither; explain how you arrived at your conclusion.

(b) To the best of your ability, give a verbal description of the kind of consumer behavior each expression describes. (For example: one describes a consumer who always spends half their income on good X...which one?)

Exercise 5: How elastic are those sweatpants? (Lecture 3)

This was a 15-point question on the midterm last year...

Suppose the market demand for sweatpants (good X) is given by $Q_x = 20 + I - P_x - \frac{1}{2}P_y$, where I is the average income of consumers, P_x is the price of sweatpants, and P_y is the price of T-shirts.

(a) Compute the **own-price** elasticity of demand for sweatpants (ϵ_{Q_x,P_x}), the **cross-price** elasticity of demand for sweatpants with respect to T-shirts (ϵ_{Q_x,P_y}), and the **income** elasticity of demand for sweatpants ($\epsilon_{Q_x,I}$).

(b) On a carefully drawn diagram of the demand for sweatpants, show where the demand for sweatpants is elastic, unit elastic, and inelastic.

(c) According to this demand function, are sweatpants and T-shirts complements, substitutes, or neither? How do you know?

Exercise 6: Elasticity and Logs (Lecture 3)

This problem, without the optional extension, should take you roughly four minutes...

Recall that from the group work problem in Lecture 2, we found the following for the market supply and demand curves, and equilibrium price and quantity:

$$Q^S(P, w, N_F) = N_F \frac{P}{w}$$
$$Q^D(P, I, N_C) = N_C \frac{\frac{1}{4}I}{P}$$

We then found the equilibrium price and quantity by setting $Q^S = Q^D$ and solving for P. This got us the following expressions for equilibrium price and quantity:

$$P^E(I, N_C, w, N_F) = \frac{1}{2}\sqrt{\frac{IwN_C}{N_F}}$$
$$Q^E(I, N_C, w, N_F) = \frac{1}{2}\sqrt{\frac{N_C N_F I}{w}}$$

(a) For each of these four functions, write them in log-log form. That is, write $\ln Q^S$ as a function of $\ln P$, $\ln w$, and $\ln N_F$; then do the same for the other three. (Once you do this, the following calculations should take approximately 10 seconds each!)

(b) Calculate the income elasticity of demand.

(c) Calculate the wage elasticity of supply.

(d) Calculate the elasticity of the equilibrium price with respect to the number of consumers.

(e) Calculate the elasticity of the equilibrium quantity with respect to consumer income.

Optional extension. Suppose a sales tax is levied on this market, so that consumers pay τP while firms still receive price P. (So for example, $\tau = 1.1$ would represent a 10% sales tax) In this case the supply and demand equations become:

$$Q^S(P, w, N_F) = N_F \frac{P}{w}$$
$$Q^D(P, I, N_C, \tau) = N_C \frac{\frac{1}{4}I}{\tau P}$$

Solve for the equilibrium price firms receive, and the equilibrium price consumers pay, as functions of τ. What is the elasticity of the equilibrium price that firms receive with respect to τ? What about the price that consumers pay? What does this say about whether consumers or producers bear the burden of the tax in this case?