SOLUTION SKETCHES

Midterm

Econ 50 - Stanford University - Winter Quarter 2015/16

February 11, 2016

Write your name and your TA's name (**Joy** Chen, **Danny** Wright, or **Sindy** Li), and sign the statement below.

You will have a total of 110 minutes to complete this exam. The exam is worth a total of 100 points, so you should allocate approximately one minute per point. Pace yourself carefully, and provide clear, concise answers – lengthy explanations are not necessary!

Write all of your answers in the space provided. If you need extra room, please use the back of each sheet. Your numerical answers should be as precise as possible. If you're pressed for time, don't worry about simplifying your answers perfectly. Make sure you show your work.

If you must make any additional assumptions in order to answer a question, please state what those assumptions are. At least one member of the Econ 50 staff will be available outside the testing room at all times. We usually cannot answer questions, but please notify us if you feel you've found a mistake in the exam or if you observe a classmate engaging in suspicious behavior.

Remember that the only aid you may use for this exam is a simple calculator (not a graphing calculator). No notes, books, headphones, cell phones, etc. may be used to help you.

"The answers written on these pages are entirely my own. I attest that in taking this exam, I am fully complying with all provisions of Stanford's Fundamental Standard and Honor Code."

Signature:

Printed Name:

TA's Name:

Please do not open this exam until it is time to begin. Good luck!

Question 1: Fantastic Elastics [10 points]

(a) Answer the following questions for a consumer who spends all their income I on goods X and Y, at constant prices P_x and P_y, and whose demand for X is given by $q_x^D(P_x, P_y, I)$. [4 points]

- What can we say about the way this consumer views goods X and Y if $\frac{\partial q_x^D(P_x,P_y,I)}{\partial P_y} > 0$?

$\uparrow P_y \Rightarrow \uparrow q_x^D$

There-fore the goods are substitutes.

2 pts for "substitutes"
1 pt if partially correct
but don't mention
substitutes

- What can we say about the way this consumer views goods X and Y if $\frac{\partial q_x^D(P_x,P_y,I)}{\partial I} < 0$?

$\uparrow I \Rightarrow \downarrow q_x^D$

X must be inferior (1 pt.)

Y must be normal (1 pt.)

since both goods cannot be inferior.

(b) True or false: "Supply elasticity is the slope of the supply curve." Explain, and support your explanation with an example or counterexample. [6 points]

FALSE! (2 pts.)

$$\varepsilon_{Q^s,P} = \frac{dQ^s}{dP} \cdot \frac{P}{Q^s}$$

(inverse) slope

so in general not equal to slope (2 pts for good explanation)

e.g. $Q^s(P) = P^2$ has a slope of $\frac{1}{2P}$ (or an inverse slope of $2P$) and constant elasticity of 2.
(2 pts for counterexample)

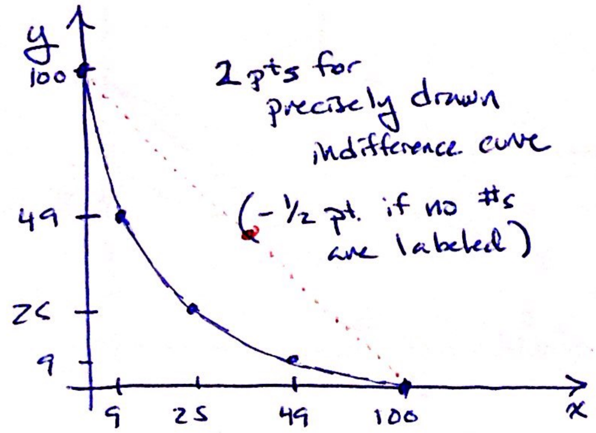
Question 2: A touching example [15 points]

(a) Clearly state the conditions under which the Lagrange method will always find the optimal consumption bundle. [5 points]

- Utility function is (strictly) monotonic (1 pt.)
  (strictly) convex (1 pt.)
  + "smooth" (cont. diff) (1 pt.)
- Indifference curves do not cross the axes (1 pt.)
- Budget constraint is a simple line (1 pt.)

(b) Suppose Jeremy's demand for cashmere sweaters (good X) and Bulgari watches (good Y) is given by $u(x, y) = \sqrt{x} + \sqrt{y}$. Draw Jeremy's indifference curve for $u(x, y) = 10$. (Hint: plot the points for $x = 0, 9, 25, 49, 100$). Does this utility function represent preferences that are convex and monotonic? Support your answer with visual or mathematical logic. [5 points]

1 pt. for drawing graph with correct axes



2 pts for precisely drawn indifference curve

(−½ pt. if no #s are labeled)

It is convex (1 pt.)
because a line connecting any two points on the ind. curve lies in the preferred region.
e.g. (50,50) is a convex combination of (0,100) and (100,0) and is preferred to (25,25)

It is monotonic (1 pt.)
because $MU_x > 0$, $MU_y > 0$

(c) It might seem like Jeremy's utility function might violate one of the conditions from part (a), but it actually doesn't. Calculate Jeremy's marginal rate of substitution ($MRS_{x,y}$) for this utility function, and use your expression for $MRS_{x,y}$ to explain why in this particular case, the Lagrange method will indeed work because the solution must be interior. [5 points]

The condition that might be violated is the "crossing the axes" condition, because these indifference curves touch the axes. However, $\boxed{MRS_{x,y} = \frac{MU_x}{MU_y} = \sqrt{\frac{y}{x}}}$ (2 pts).

(3 pts. for explanation)

So when $\boxed{x = 0,\ MRS = \infty}$; and when $\boxed{y = 0,\ MRS = 0}$.
Since the utility function is smooth, MRS is continuously decreasing as you move along a simple budget line; therefore $MRS = \frac{P_X}{P_Y}$ must occur at an interior point for any strictly positive and finite P_X/P_Y.

Question 3: Chips and salsa [20 points]

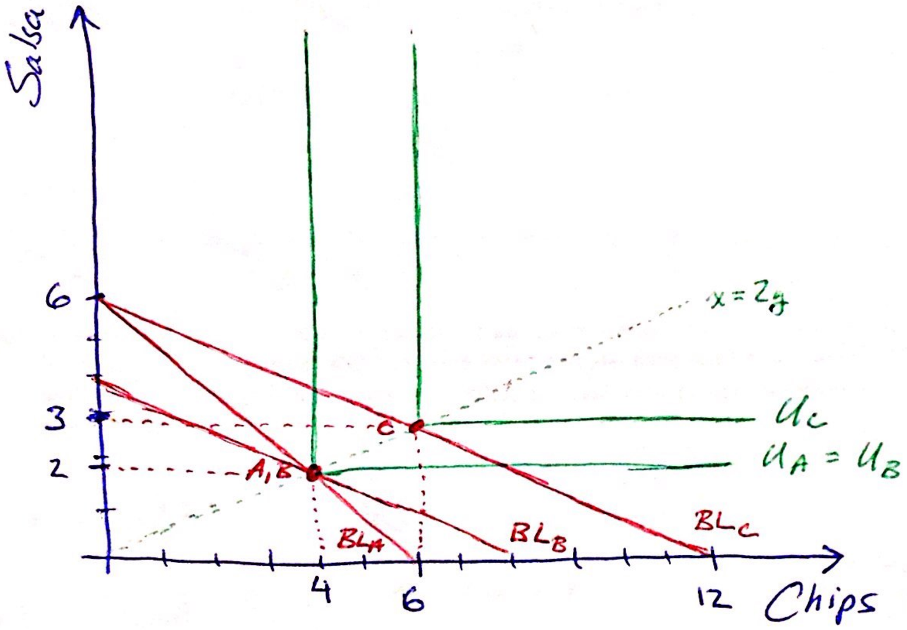
Morgan is a very finicky eater. He loves chips (good X) and salsa (good Y), but only eats them in a strict ratio of 2 bags of chips per 1 can of salsa. If he has more than twice as many chips as cans of salsa, he just doesn't eat the additional chips; and if he has more than one can of salsa for every two bags of chips, the extra salsa just goes to waste. He has a chips-and-salsa budget of $12.

(a) Write down a utility function that is consistent with this behavior. [2 points]

Always choose $x = 2y \Rightarrow u(x,y) = \min\{x, 2y\}$

(1 pt. for general form of utility fn; 1 pt for correct values)

(b) Suppose a bag of chips and a can of salsa each cost $2. On a diagram with chips on the X axis and salsa on the Y axis, plot his budget line (labeled BL_A), optimal consumption bundle (point A), and the indifference curve passing through point A (labeled U_A). [6 points] → 2 pts. each for A, BL_A, U_A



(c) Now suppose the price of chips drops to $1 per bag. Following the Slutsky procedure, indicate Morgan's decomposition point (B) and his final consumption bundle (C) to your diagram above, as well as the budget lines passing through these bundles at the new prices (labeled BL_B and BL_C) and indifference curves passing through those two bundles (labeled U_B and U_C). [8 points]

1 pts. each for B, BL_B, U_B; C, BL_C, U_C; +1 for B = A, +1 for $U_B = U_A$

(d) What are the magnitudes of the income and substitution effects of a change in the price of chips (good X) on Morgan's consumption of salsa (good Y)? What does this tell you about how Morgan views the substitutability or complementarity of chips and salsa? [4 points]

Substitution effect is zero since $y_A = y_B = 2$ (1 pts.)

Income effect is +1 since $y_B = 2$, $y_C = 3$ (1 pt.)

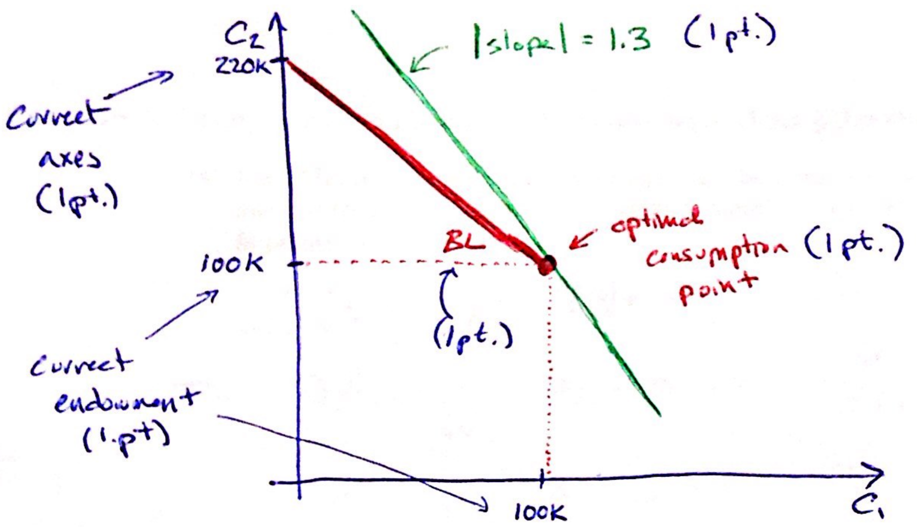
Therefore Morgan views these goods as perfect complements

(2 pts)

Question 4: No time like the present [10 points]

Beth has bizarre views about consumption today (c_1) versus consumption next year (c_2). In particular, her utility function is $u(c_1, c_2) = 1.3c_1 + c_2$. She has a steady income of $100,000 per year. (Note: she lives with her parents, who feed her, so spending zero of her own money in either period is OK with her.)

(a) Suppose Beth can earn an interest rate of 20% on any money she saves, but due to a bad credit history she cannot borrow. Draw her budget constraint, indicate her optimal consumption point, and carefully draw the indifference curve passing through that point. [5 points]



Saves 100K @ 20% interest => grows to 120K so max cons. in period 2 is I_2 + 120K = 220K.

(b) Now suppose she can save or borrow at an interest rate of r. Describe her optimal behavior as a function of r. That is, at what interest rates will she save, and how much? At what interest rates will she borrow, and how much? At what interest rate might she neither borrow nor save, and why? [5 points]

(1 pt.) $MRS_{1,2} = \frac{MU_1}{MU_2} = \frac{1.3}{1} = 1.3$

(1 pt.) Will borrow if $MRS > 1+r \iff 1.3 > 1+r \iff r < 0.3$
i.e., if interest rates are less than 30%

(1 pt.) Will save if $MRS < 1+r \iff r > 0.3$

(1 pt.) Will be indifferent if $r = 0.3$

(1 pt.) Because c_1 and c_2 are perfect substitutes, if she borrows or saves, it will be 100% (i.e., she will set $c_1 = 0$ if she saves, and set $c_2 = 0$ if she borrows)

For Questions 5 and 6, suppose Mindy's preferences over cronuts (X) and other goods (good Y, measured in dollars) are summarized by the utility function

$$u(x, y) = 12\sqrt{x} + y$$

Today Mindy has a total of $\$I$ available to spend on cronuts (delicious combinations of doughnuts and croissants) and other things. The price of a cronut is P. (Since each unit of "dollars spent on other things" is \$1, this is like saying $P_x = P$ and $P_y = 1$; we'll just fix $P_y = 1$ to make the math a little easier.)

Question 5: Utility function deep dive: Demand derivations [20 points]

(a) Find Mindy's marginal rate of substitution between cronuts and other goods. Use the Lagrange method to find her optimal consumption bundle. Under what conditions will that method work? [8 points]

$$\mathcal{L}(x,y,\lambda) = 12\sqrt{x} + y + \lambda(\cancel{}I - Px - y)$$

2 pts. for approach

FOC: $\frac{\partial \mathcal{L}}{\partial x} = \frac{6}{\sqrt{x}} - P\lambda = 0 \Rightarrow \lambda = \frac{6}{P\sqrt{x}}$

$\frac{\partial \mathcal{L}}{\partial y} = 1 - \lambda = 0 \Rightarrow \lambda = 1$

$\frac{6}{P\sqrt{x}} = 1$ (2 pts)

$\Rightarrow \boxed{x = \frac{36}{P^2}}$

Fine to get this by setting MRS = P

$\frac{\partial \mathcal{L}}{\partial \lambda} = I - Px - y = 0 \Rightarrow Px + y = I$

$\frac{36}{P} + y = I$ (2 pts.)

$\boxed{y = I - \frac{36}{P}}$

Finds solution as long as $y > 0 \Leftrightarrow \boxed{I > \frac{36}{P}}$ (2 pts.)

(b) Carefully write down Mindy's **Marshallian demand functions**, $x^*(P, I)$ and $y^*(P, I)$. [3 points]

$$x^*(P,I) = \begin{cases} \frac{36}{P^2} & \text{if } I \geq \frac{36}{P} \quad (1 \text{ pt.}) \\ \frac{I}{P} & \text{if } I \leq \frac{36}{P} \quad (\tfrac{1}{2} \text{ pt.}) \end{cases}$$

$$y^*(P,I) = \begin{cases} I - \frac{36}{P} & \text{if } I \geq \frac{36}{P} \quad (1 \text{ pt.}) \\ 0 & \text{if } I \leq \frac{36}{P} \quad (\tfrac{1}{2} \text{ pt.}) \end{cases}$$

(c) Derive Mindy's **indirect utility function** $V(P, I)$. [3 points]

If $I \geq \frac{36}{P} \Rightarrow x^* = \frac{36}{P^2},\ y^* = I - \frac{36}{P}$

$V(P, I) = u(x^*(P,I), y^*(P,I)) \Leftarrow$ 1pt. for following this procedure

$= 12\sqrt{x^*} + y^*$

$= 12 \cdot \frac{6}{P} + I - \frac{36}{P} = \frac{72}{P} + I - \frac{36}{P} = \boxed{I + \frac{36}{P}}$ (1pt.)

If $I \leq \frac{36}{P} \Rightarrow 12\sqrt{x^*} + y^* = 12 \cdot \sqrt{\frac{I}{P}} + 0 = \boxed{12\sqrt{\frac{I}{P}}}$ (1 pt.)

(d) Derive Mindy's **expenditure function** $E(P, U)$. [3 points]

First we need the cutoff utility (not income) for the corner solution.

When $I = \frac{36}{P}$, $V(P, I) = \frac{36}{P} + \frac{36}{P} = \frac{72}{P}$ so that is the cutoff. (1 pt.)

When $U \geq \frac{72}{P} \Rightarrow U = I + \frac{36}{P} \Leftrightarrow \boxed{E(P, I) = U - \frac{36}{P}}$ (1pt.)

$U \leq \frac{72}{P} \Rightarrow U = 12\sqrt{\frac{I}{P}} \Leftrightarrow \boxed{E(P, I) = P \cdot \frac{U^2}{144}}$ (1pt.)

− 1 pt. for using I cutoff

(e) Carefully write down Mindy's **Hicksian demand functions**, $x^H(P, U)$ and $y^H(P, U)$. [3 points]

$$x^H(P, u) = \begin{cases} \boxed{\frac{36}{P^2} \quad \text{if } U \geq \frac{72}{P}} & (1\text{ pt.}) \\ \frac{E(P,u)}{P} = \frac{P \cdot \frac{U^2}{144}}{P} = \boxed{\frac{U^2}{144} \quad \text{if } U \leq \frac{72}{P}} & (1/2\text{ pt.}) \end{cases}$$

$$y^H(P, u) = \begin{cases} E(P,u) - \frac{36}{P} = (U - \frac{36}{P}) - \frac{36}{P} = \boxed{U - \frac{72}{P} \quad \text{if } U \geq \frac{72}{P}} & (1\text{ pt.}) \\ \boxed{0 \quad \text{if } U \leq \frac{72}{P}} & (1/2\text{ pt.}) \end{cases}$$

− 1 pt. for using I cutoff

Question 6: Utility function deep dive: Comparative statics analysis (25 points)

Now assume Mindy's income is $I = \$36$. This question will ask you to carefully analyze the effect of a price increase in P from \$2 to \$3 per cronut.

(a) Find the cronuts that Mindy will choose to buy if $P = 2$ and if $P = 3$. Use these two points to sketch a reasonable price-consumption curve for cronuts (PCC_X) and (Marshallian) demand curve for cronuts (D_X) in two carefully-drawn diagrams. *Be sure to label your axes!* [8 points]

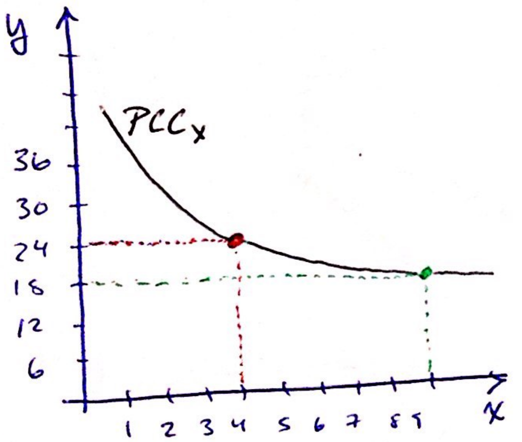
Note: for this question we always have $I > \frac{36}{P}$

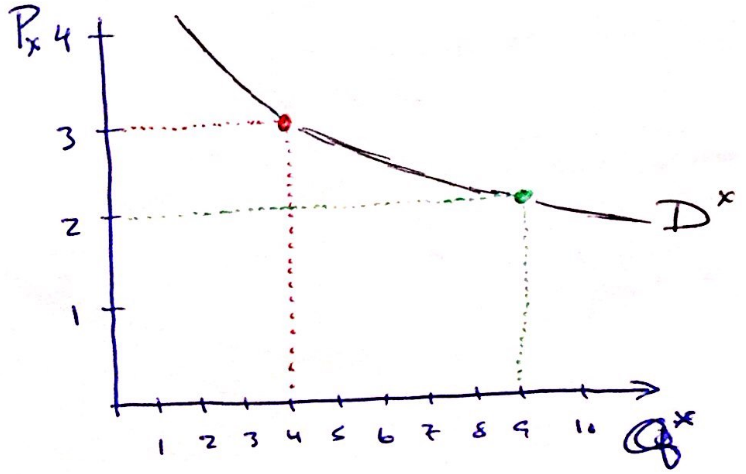
$\therefore\ x^*(P=2,\ I=36) = \frac{36}{2^2} = 9$ (1 pt.)

$y^*(P=2,\ I=36) = 36 - \frac{36}{2} = 18$ (1 pt.)

$x^*(P=3,\ I=36) = \frac{36}{3^2} = 4$ (1 pt.)

$y^*(P=3,\ I=36) = \cancel{36}\ \frac{36}{3} = 24$ (1 pt.)



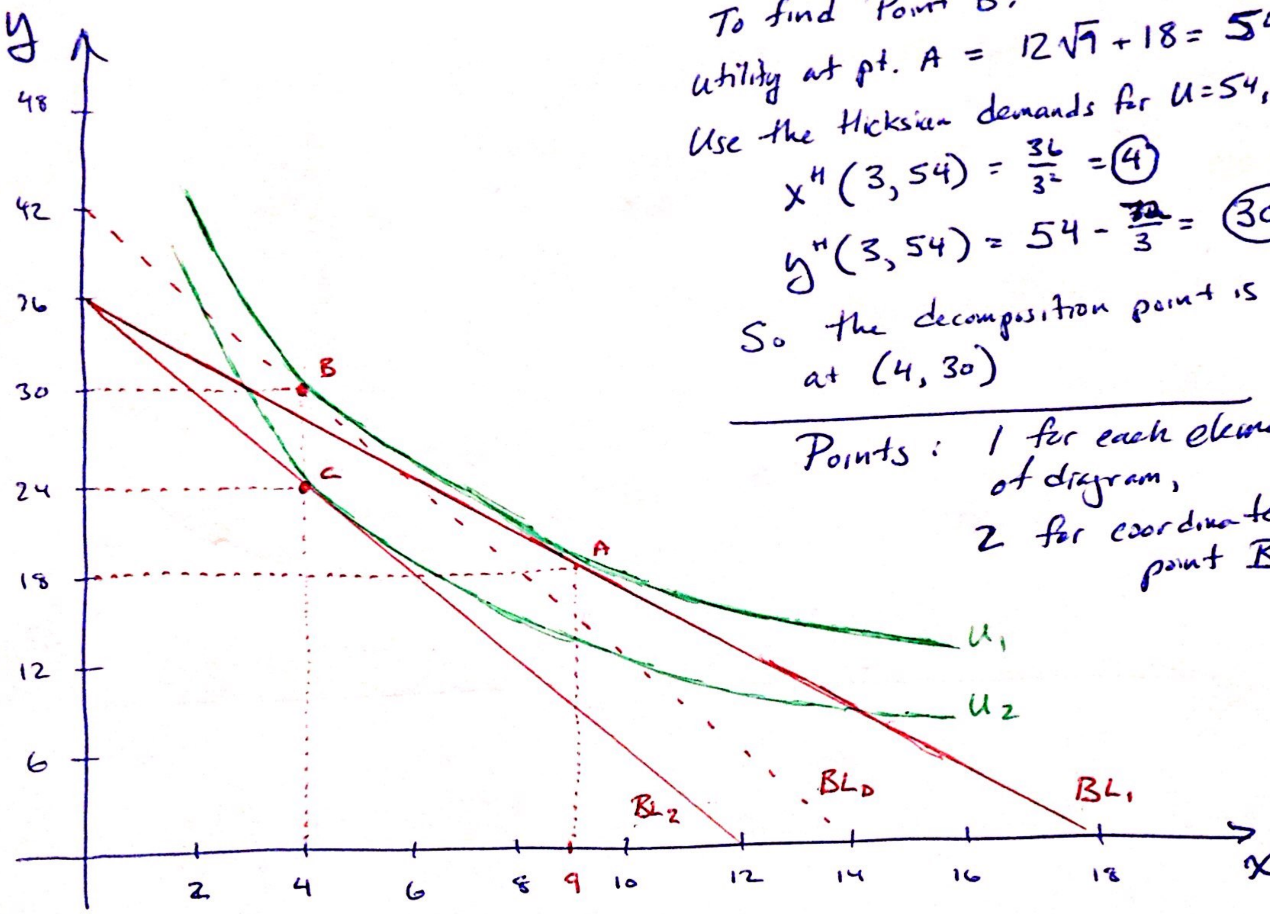


Each graph: 1 pt. for axes, 1 pt. for drawing curve properly

(b) Does Mindy view cronuts and "other things" as complements or substitutes? How do you know? [2 points]

$\underbrace{PCC_X \text{ is downward sloping}}_{1\text{ pt.}} \Rightarrow \underbrace{\text{substitutes}}_{1\text{ pt.}}$

(c) On a carefully drawn Slutsky diagram, show the effect of an increase in the price of cronuts from $P = 2$ to $P = 3$. Label your initial point A, the final point C, and the Slutsky decomposition point B. Clearly show the coordinates for those points, the coordinates of the intercepts of the three budget lines that correspond to the three points, and the utility levels at the indifference curves passing through those three points. [10 points]



To find Point B:

utility at pt. A $= 12\sqrt{9} + 18 = 54$

Use the Hicksian demands for $U = 54$, $P = 3$

$x^H(3, 54) = \frac{36}{3^2} = 4$

$y^H(3, 54) = 54 - \frac{\cancel{72}}{3} = 30$

So the decomposition point is at $(4, 30)$

Points: 1 for each element of diagram,
2 for coordinates of point B.

(d) What is the compensating variation, equivalent variation, and change in consumer surplus for this price change? [5 points]

For Quasilinear, $CV = EV = \Delta CS$ so this is easier than it could be!

At point B: $E(3, 54) = 54 - \frac{36}{3} = 42$

~~Thus~~ Since initial income is 36, this means

the magnitude of $CV = EV = \Delta CS = |42 - 36| = 6$

This is a sufficient answer, but you could also go through the exercise of calculating EV and ΔCS directly.

3 pts. for right approach, 2 pts for correct magnitude; no points deducted for showing as +/-.