Homework 3

Econ 50 - Stanford University - Winter Quarter 2015/16

Due at the beginning of section on Friday, January 29

Exercise 1: Math Warmup: The Canonical Optimization Problems (Lecture 6)

For each of the following five "canonical" utility functions, find the point (x^*, y^*) that maximizes utility subject to the standard budget constraint $P_x x + P_y y = I$. In each case, indicate whether the solution is sometimes, always, or never found using the Lagrange method, and provide a brief, intuitive reason why. *Note: we already did some of these in lecture and section...*

(a) Cobb-Douglas: $u(x, y) = \alpha \ln x + (1 - \alpha) \ln y$

(b) Perfect Substitutes: $u(x, y) = \alpha x + (1 - \alpha)y$

(c) Perfect Complements: $u(x, y) = \min\{\frac{x}{\alpha}, \frac{y}{1-\alpha}\}$

(d) Quasilinear: $u(x, y) = \alpha \ln x + (1 - \alpha)y$

(e) CES: $u(x, y) = [\alpha x^r + (1 - \alpha)y^r]^{\frac{1}{r}}$ (optional; the math on this one can get hairy!)

Exercise 2: Thinking on the Margin (Lecture 6)

This was a midterm question from last year.

(a) What does it mean if $MRS_{x,y} < \frac{P_x}{P_y}$ at a point along a consumer's budget constraint?

(b) If a consumer is in a position where that is true, can they always improve their utility by changing their consumption bundle? Why or why not? Illustrate your answer with one or two carefully drawn budget-line/indifference-curve diagrams.

(c) If they could improve their utility by changing their consumption bundle, would it involve consuming more X and less Y, more Y and less X, or would it depend upon the exact form of the utility function in question? Carefully state the assumptions underlying your answer.

Exercise 3: Choosing a Budget Constraint (Lecture 6)

Suppose I have a budget of \$240 per year to spend on puppy food (good X) and puppy toys (good Y). Puppy toys cost $p_y = \$10$ each, and I always buy them from the local supermarket. At that store, I can buy puppy food for $p_x^S = \$20$ per bag; at Costco, it's only $p_x^C = \$10$ per bag for puppy food, but a Costco membership costs \$60 per year. Assume for simplicity that I'd only go to Costco to buy puppy food, and that my preferences are given by $u(x, y) = xy$.

(a) Draw my possible budget constraints with and without a Costco membership. (Make this a pretty big and very precise graph!)

(b) Solve my constrained optimization problem if I choose **not** to buy a Costco membership. How many bags of puppy food, and how many puppy toys, would I choose to buy? Carefully add this point, and the indifference curve passing through it, to the graph you drew in part (a).

(c) Solve my constrained optimization problem if I **do** choose to buy a Costco membership. How many bags of puppy food, and how many puppy toys, would I choose to buy? Again, add this point, and the indifference curve passing through it, to the graph you drew in part (a).

(d) Should I buy a Costco membership in order to take advantage of the discount on puppy food? Why or why not?

(e) My friend has faces the same choice, but she has a different sized dog, so her preferences are given by $u(x, y) = xy^2$. Solve her optimization problem with and without a Costco membership. Should she buy a Costco membership? Why or why not? Show your work!

(f) Based on these two utility functions (and your answers to the last two questions): who do you think has a bigger dog? Why?

Exercise 4: Predatory Lending and Borrowing (Lecture 7)

Sam and Gianna are two shark tour operators on Fisherman's Wharf, and they're each trying to make their consumption and savings decisions. Sam is very successful, and will earn \$100K this year. He plans on taking next year off to travel the world, living off savings from this year. Gianna will earn \$45K this year, but next year expects to take over over some of Sam's business, and so expects her earnings to increase to \$55K. They each have a utility function $u(c_1, c_2) = c_1 c_2$ over consumption this year (c_1) and consumption next year (c_2). Assume that each of them only plans two years in advance (so we can ignore decisions beyond next year, including savings decisions next year).

(a) Suppose both Sam and Gianna can borrow money interest-free, and also get no interest on savings. Write down their budget constraints and solve their optimization problem for c_1^* and c_2^*. How much will Sam save for next year? How much will Gianna borrow against her future earnings? Illustrate their consumption/savings decisions on a budget line-indifference curve diagram.

(b) Repeat part (a) if both Sam and Gianna can either borrow or save at 10% interest. Who is made better off, relative to the situation in part (a)? Who is made worse off?

(c) Suppose Gianna can borrow at 10% interest, but receives no interest on savings. Draw her new budget constraint, and again solve for her optimal consumption/savings decision. Will she still borrow against her future income?

(d) Suppose Gianna's credit isn't great, and the only person who will lend her money is a "loan shark" who charges 30% interest. Will she borrow any money at that rate? Why or why not? Illustrate this decision in a budget line-indifference curve diagram.

(e) What is the (approximate) highest interest rate Gianna will be willing to pay to borrow against her future earnings, if she receives no interest on savings? Give an intuitive explanation of your answer.

(f) Suppose Gianna's credit is so bad that she can't borrow money at all. What is the lowest interest rate that would make her save some of her current year's income?

Exercise 5: Endowment Budget Constraint (Lecture 7)

Suppose that instead of having a fixed income I, you have an endowment of $y^E = 12$ units of good Y. You can sell each of these units of good Y at price P_y and use the proceeds to buy good X, which has a price of P_x.

(a) Draw your budget line if $P_x = 3$ and $P_y = 4$. Draw what happens to your budget line if P_x increases from 3 to 4, or if P_y decreases from 4 to 3. How is this different from the effect of price changes if you had a fixed dollar income?

(b) Suppose you have the simple Cobb-Douglas utility function $u(x, y) = xy$. Solve for your optimal consumption (x^*, y^*) as a function of the exogenous variables y^E, P_x, and P_y.

(c) In the canonical Cobb-Douglas case, X and Y are neither complements nor substitutes, because the quantity demanded of each depends only on its own price. Is that still the case with an endowment budget line? Why or why not? (What is particularly weird about your demand for good Y in this scenario...?)