# Section 4 Problems

Econ 50 - Stanford University - Winter Quarter 2015/16

Friday, February 5, 2016

**Problem 1: Utility function deep dive: demand derivations and comparative statics**

(From Midterm, Winter 2015)

Suppose Wilson's preferences over $X$ and $Y$ are summarized by the utility function

$$u(x,y) = (x^{-1} + y^{-1})^{-1}$$

As usual, he has a total of $\$I$ available to spend on $X$ and $Y$ at prices $P_x$ and $P_y$ per unit, respectively.

Last week, we found that Wilson's Marshallian demand functions are given by:

$$x^* = \frac{I}{P_x + \sqrt{P_x P_y}}$$

$$y^* = \frac{I}{P_y + \sqrt{P_x P_y}}$$

(a) Write down expressions for Wilson's **indirect utility function** $V(P_x, P_y, I)$ and his **expenditure function** $E(P_x, P_y, U)$. Use the fact that $u(x,y) = \frac{xy}{x+y}$.

(b) Write down expressions for Wilson's **Hicksian demand functions**, $x^H(P_x, P_y, U)$ and $y^H(P_x, P_y, U)$.

(c) Now assume Wilson's income is $I = \$288$ and the price of good $Y$ is $P_y = \$1$ per unit. On a carefully drawn Slutsky diagram, show the effect of a price change from $P_x = 9$ to $P_x = 4$. Label your initial point $A$, the final point $C$, and the Slutsky decomposition point $B$. Clearly show the coordinates for those points, as well as the coordinates of the intercepts of all relevant budget lines. Recall that last week we found:
When $P_x = 9, (x^*, y^*) = (24, 72)$ and when $P_x = 4, (x^*, y^*) = (48, 96)$.

(d) Compute the **compensating variation** and **equivalent variation** for this price change.

(e) Illustrate the compensating variation in a diagram showing the relevant Marshallian and Hicksian demand curves.