Section 3 Solutions

Econ 50 - Stanford University - Winter Quarter 2015/16

Friday, January 29, 2016

1. **Part (a) solution**

   Using chain rule

$$MU_x = [-(x^{-1} + y^{-1})]^{-2}(-x^{-2})$$
$$MU_y = [-(x^{-1} + y^{-1})]^{-2}(-y^{-2})$$

   Therefore

$$MRS_{x,y} = \frac{MU_x}{MU_y} = \frac{x^{-2}}{y^{-2}} = \frac{y^2}{x^2}$$

2. **Part (b) solution**

   The indifference curve for any given u can be written as

$$y = \frac{ux}{x-u}$$

   We don't consider $x < u$ because then $y < 0$. The indifference curves never cross the axes. Drawing the indifference curves also show that utility is convex and monotonic. The optimal is given by the tangent condition

$$MRS_{x,y} = \frac{y^2}{x^2} = \frac{P_x}{P_y} \implies y = \sqrt{\frac{P_x}{P_y}}x$$

   Substitute this equation into the budget constraint

$$I = P_x x + P_y y = P_x x + P_y \sqrt{\frac{P_x}{P_y}}x \implies x^* = \frac{I}{P_x + \sqrt{P_x P_y}}$$

   Substitute this back into either the MRS or budget equation (or by symmetry) gives

$$y^* = \frac{I}{P_y + \sqrt{P_x P_y}}$$

3. **Part (c) solution**

   Using the demand functions in Problem 1, for $P_x = 9$

$$x^* = \frac{288}{9 + \sqrt{9}} = 24$$

$$y^* = \frac{288}{1 + \sqrt{9}} = 72$$

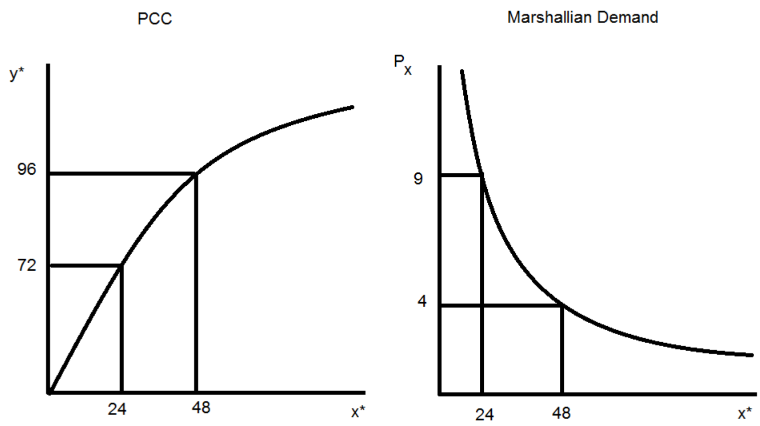
   For $P_x = 4$

$$x^* = \frac{288}{4 + \sqrt{4}} = 48$$

$$y^* = \frac{288}{1 + \sqrt{4}} = 96$$

4. **Part (d) solution**

The functions x^* and y^* both increase when P_x decreases, implying that the PCC is upward sloping. This calculation shows that when x^* doubles, y^* less than doubles, meaning the curve is concave. For the demand curve, the function x^* shows that the curve asymptotes towards both axes. When $P_x \to 0$, $x^* \to \infty$. When $P_x \to \infty$, $x^* \to 0$.



5. **Part (e) solution**

From part (a),

$$y^* = \frac{I}{P_y + \sqrt{P_x P_y}}$$

$$\frac{\partial y^*}{\partial P_x} < 0$$

They are complements. A decrease in price of X leads to an increase in the demand for Y.