Kinks and Corners, with Applications

Econ 50 | Lecture 7 | January 26, 2016

Lecture

- Kinks and corners
- Buying and selling
- Intertemporal consumption and interest rates

Group Work

- Trading game

Part I: Kinks and Corners

The Good Case

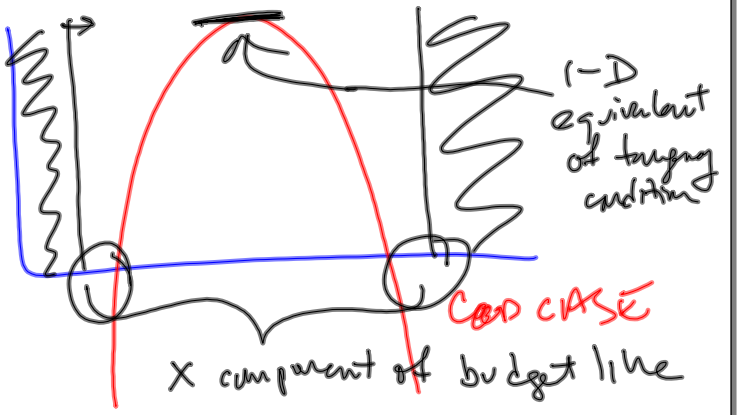
SMOOTH

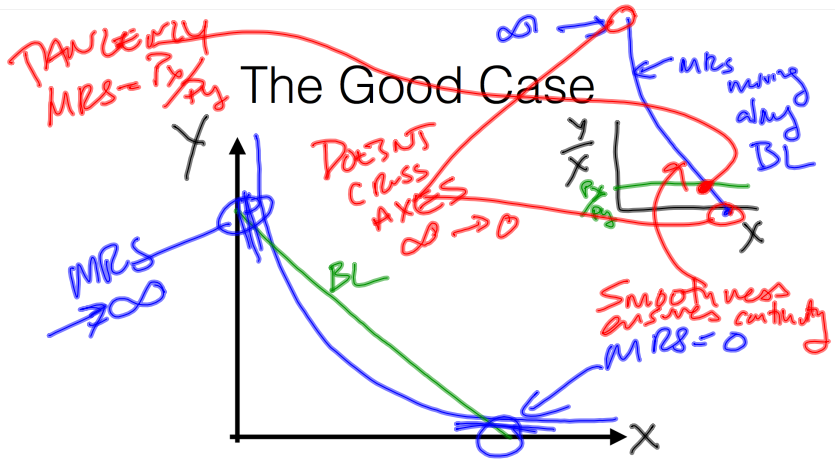
IF

- The consumer's preferences over goods X and Y are **continuously differentiable, strictly monotonic**, and **strictly convex**
- The indifferences curves **do not cross the axes**
- The budget constraint is a **simple straight line**

THEN THE SOLUTION…

- will be an **interior solution** (involve strictly positive quantities of X and Y)
- can be found using the **Lagrange method**
- is characterized by:
  $MRS_{x,y} = P_x/P_y$
  $P_x x + P_y y = I$





Corner Solution with Smooth Preferences

IF

- The consumer's preferences over goods X and Y are **continuously differentiable, strictly monotonic**, and **strictly convex**
- The indifferences curves ~~do not~~ **cross the axes**
- The budget constraint is a **simple straight line**

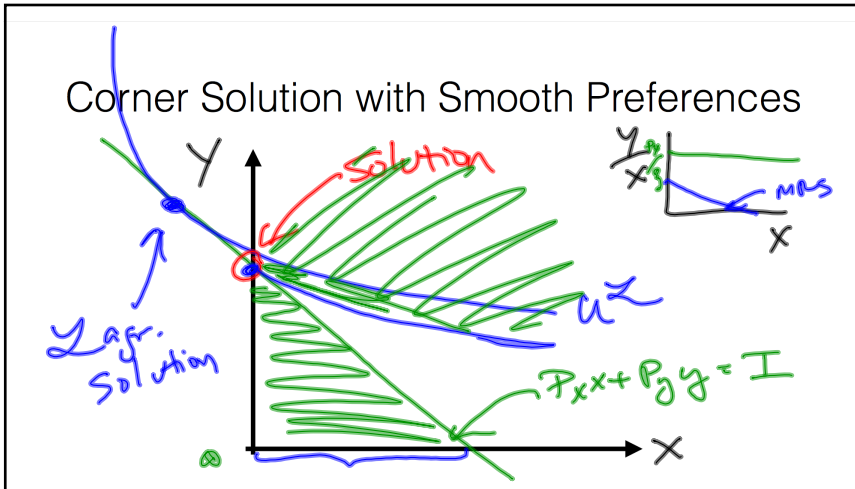
THEN THE SOLUTION…

Might

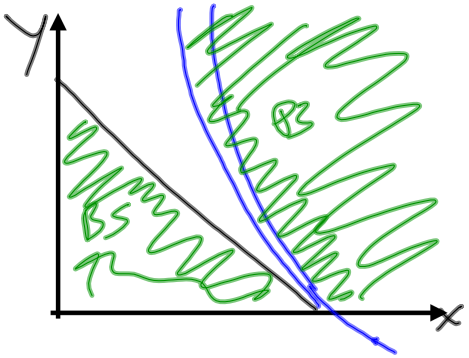
- ~~will~~ be an **interior solution** (involve strictly positive quantities of X and Y)
- can be found using the **Lagrange method**

Might be

- ~~is~~ characterized by:
  ***MRS*$_{x,y}$ = P_x/P_y**
  $P_x x + P_y y = I$



Corner Solution with Smooth Preferences



Corner Solution with Smooth Preferences

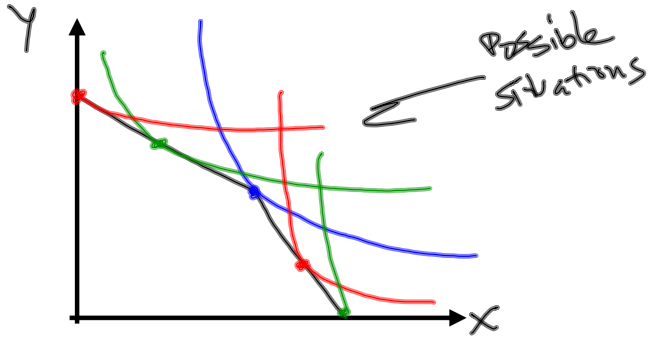
Approach for Smooth Preferences with Potential Corners

IF	THEN…
• The consumer's preferences over goods X and Y are **continuously differentiable, strictly monotonic**, and **strictly convex** • The indifferences curves ~~**do not**~~ **cross the axes** • The budget constraint is a **simple straight line**	• Use the **Lagrange method** to find the point of tangency • (the point characterized by:) $MRS_{x,y} = P_x/P_y$ $P_x x + P_y y = I$ • ***If that point lies outside the domain of the budget constraint, go to the "closest corner"***

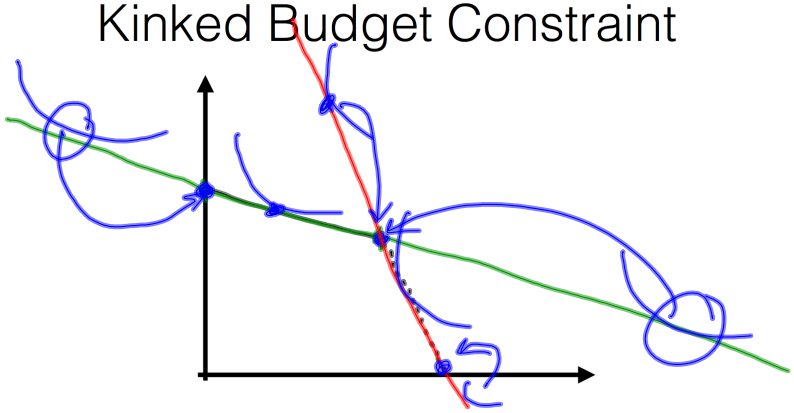
Kinked Budget Constraints

IF	THEN THE SOLUTION…
• The consumer's preferences over goods X and Y are **continuously differentiable, strictly monotonic**, and **strictly convex** • The indifferences curves ~~**do not**~~ **cross the axes** may or may not • The budget constraint is NOT a **simple straight line**	?

Kinked Budget Constraint



Kinked Budget Constraint



Kinked Budget Constraint

Approach for Kinked Budget Constraints

IF	THEN…
• The consumer's preferences over goods X and Y are **continuously differentiable, strictly monotonic**, and **strictly convex** • The indifferences curves ~~do not~~ may or may not **cross the axes** • The budget constraint is NOT a **simple straight line**	• Analyze each segment of the budget constraint • In each case, the solution may be interior to the segment, or at the end (corner) of the segment • Compare the sub-solutions and take the one with the highest utility.

```
optimalBundle(budget:BudgetConstraint) {
    var u = this;
    var candidateBundles: TwoGoodBundle[] = budget.budgetSegments.map(
        function(segment) { return u.optimalBundleAlongSegment(segment)}
    );
    var maxUtilityBundle = candidateBundles[0];
    candidateBundles.forEach(function(bundle) {
        if(u.utility(bundle) > u.utility(maxUtilityBundle)) {
            maxUtilityBundle = bundle;
        }
    });
    return maxUtilityBundle;
}

optimalBundleAlongSegment(budgetSegment:BudgetSegment) {
    var u = this;
    var constrainedX, unconstrainedX;
    unconstrainedX = u._unconstrainedOptimalX(budgetSegment);
    constrainedX = budgetSegment.xDomain.closestValueTo(unconstrainedX);
    return {x: constrainedX, y: budgetSegment.linear.yValue(constrainedX)};
}
```

When all else fails: Compare **MRS** to the **price ratio**

$$MRS_{x,y} > \frac{P_x}{P_y}$$

YOU LIKE X RELATIVE TO Y
MORE THAN THE MARKET
AT YOUR CURRENT BUNDLE

MOVE RIGHT ALONG
THE BUDGET CONSTRAINT

$$MRS_{x,y} < \frac{P_x}{P_y}$$

YOU LIKE X RELATIVE TO Y
LESS THAN THE MARKET
AT YOUR CURRENT BUNDLE

MOVE LEFT ALONG
THE BUDGET CONSTRAINT

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MOVE RIGHT ALONG THE BUDGET CONSTRAINT

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YOU LIKE X RELATIVE TO Y **MORE** THAN THE MARKET AT YOUR CURRENT BUNDLE

MOVE RIGHT ALONG THE BUDGET CONSTRAINT

$$MRS_{x,y} < \frac{P_x}{P_y}$$

YOU LIKE X RELATIVE TO Y **LESS** THAN THE MARKET AT YOUR CURRENT BUNDLE

MOVE LEFT ALONG THE BUDGET CONSTRAINT

Everything goes wrong, Example 1: Kinked Budget Constraint and Perfect Complements

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Everything goes wrong, Example 2: Kinked Budget Constraint and Perfect Substitutes

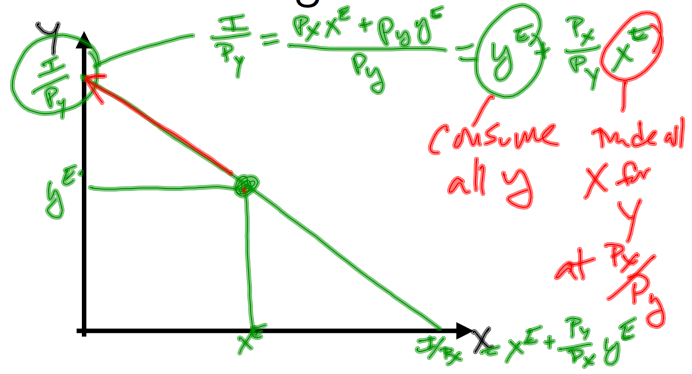
Everything goes wrong, Example 2: Kinked Budget Constraint and Perfect Substitutes

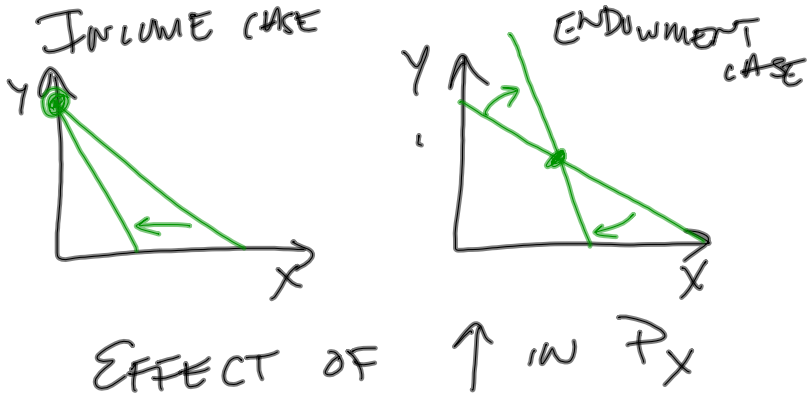
Part II: Endowment Economies

Endowment Budget Constraint

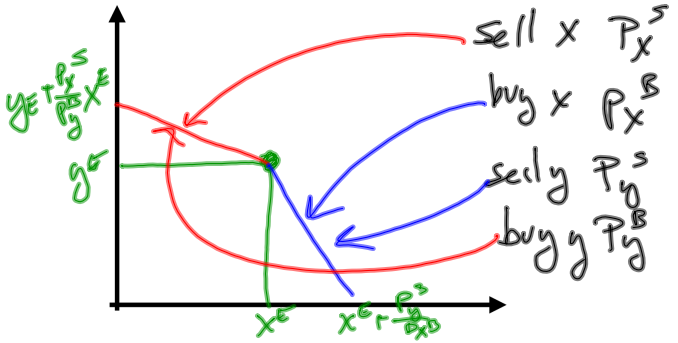
- Instead of starting with money income (I), you start with an **endowment of X and Y** (x^e, y^e)
- You can buy and sell **X** and **Y** for prices P_x and P_y
- If you sold your endowment for money, you would get: $P_x x^e + P_y y^e =$ "income"
- Therefore your budget constraint may be written: $P_x x + P_y y = P_x x^e + P_y y^e$ (spend)

Endowment Budget Constraint





Endowment Budget Constraint : Different Prices for Buying and Selling



Part III: Intertemporal Consumption and Interest Rates

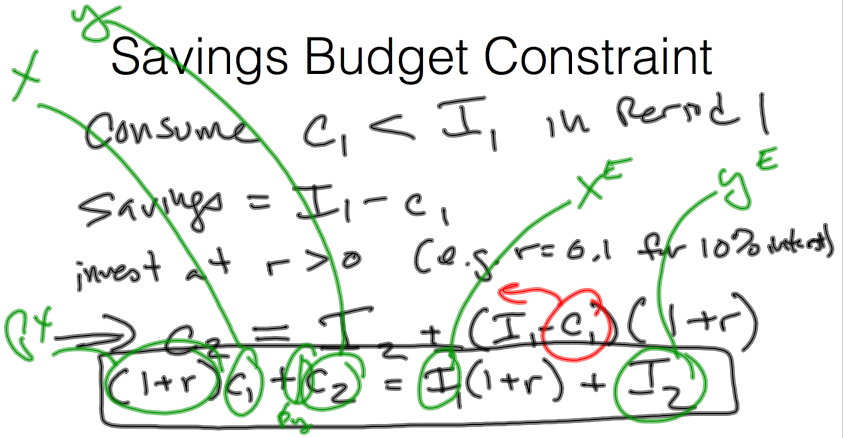
Intertemporal Choice

- Instead of goods X and Y, our goods are "**Consumption this year**" and "**Consumption next year**"
- You have an endowment of "**Income this year**" and "**Income next year**"
- You can spend more than this year's income (C1 > I1) by **borrowing** against next year's income

  OR

  You can spend more than next year's income (C2 > I2) by **saving** some of this year's income

Savings Budget Constraint



Borrowing Budget Constraint

INTERTEMPORAL BUDGET CONSTRAINTS

For Savings:	For Borrowing:
$(1+r)c_1 + c_2 = (1+r)I_1 + I_2$	$c_1 + \frac{c_2}{1+r} = I_1 + \frac{I_2}{1+r}$
"Future Value"	"Present Value"
If save 100%:	If borrow 100%:
$c_2 = (1+r)I_1 + I_2$	$c_1 = I_1 + \frac{I_2}{1+r}$

Intertemporal Budget Constraint

Intertemporal Budget Constraint : Different Interest Rates for Saving and Borrowing