

# **AuditPCH: Auditable Payment Channel Hub with Privacy Protection**

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# Outline

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1. Background
2. Preliminaries
3. Linkable Randomizable Puzzle Scheme
4. Auditable Anonymous PCH
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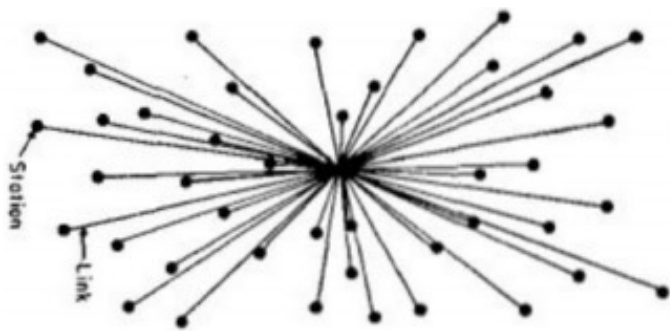
# 1 Background

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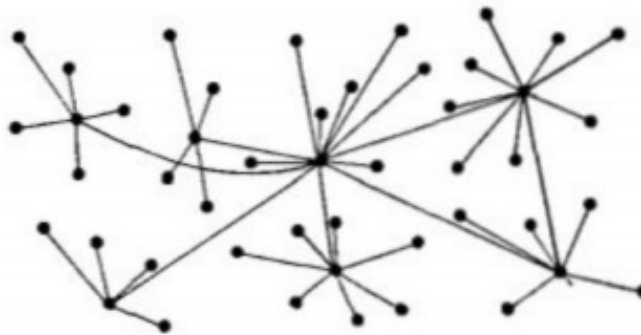
# 1.1 Payment Channel

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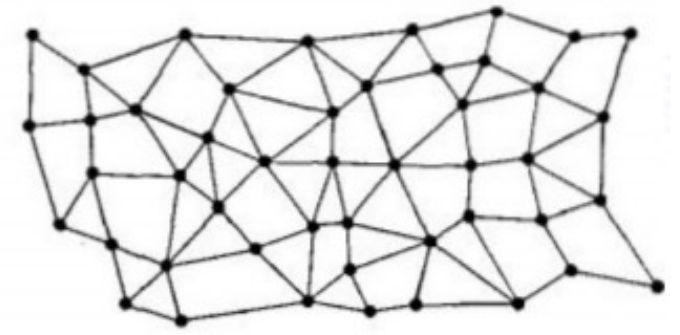
- **Two users** open a payment channel and perform **off-chain** payments by updating the channel state enjoying **high payment throughput** and **low confirmation delay**.
- If there are more than two users, each pair of users needs to establish their own payment channel to facilitate the payment, which is a **non-scalable approach**.
- Payment Channel Networks (PCN) enable two users with no direct payment channel to pay each other through the channels of **some intermediaries**.
- PCN payments may require multi-channel paths and intermediaries to actively participate in relaying the payments, which can lead to their failure.



Centralized



Decentralized



Distributed

## 1.2 Challenges in Auditable Anonymous PCH

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- **Atomicity:** For any payment of  $m$  coins from  $S$  to  $R$ , the PCH should ensure that either  $R$  receives  $m$  coins from  $T$  and  $T$  receives  $m$  coins from  $S$ , or both parties receive none.
- **Value Privacy:**  $T$  should not know the payment amount between  $S$  and  $R$ .
- **Relationship Anonymity:**  $T$  should not be able to find out if there is any relation between  $S$  and  $R$  of a specific payment.
- **Griefing Resistance:** The PCH should only initiate a payment procedure if  $R$  can prove that the payment request are previously backed by some coins locked by a  $S$  during the payment procedure.
- **Illegal Financial Activity Auditability:**  $A$  should know the relationship of  $S$  and  $R$  and verify the integrity of payments.

# 1.3 Abstract

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- **PCH Definition:**

Anonymous Payment Channel Hub (PCH), one of the most promising layer-two solutions, settles the **scalability** issue in blockchain while guaranteeing the **unlinkability** of transacting parties.

- **Problem:**

Developments bring conflicting requirements, i.e., **hiding the sender-to-receiver relationships from any third party but opening the relationship to the auditor**. Existing works do not support these requirements simultaneously since off-chain transactions are not recorded in the blockchain.

- **This work:**

The first anonymous PCH solution that provides **privacy & auditability**.

- ① Linkable randomizable puzzle for conditional transactions.
- ② A novel auditable solution for PCH.
- ③ Formal security proof & extensive experiments and evaluation.

## **2 Preliminaries**

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## 2.1 Public Key Encryption

**Public Key Encryption.** The encryption scheme includes the algorithms (Gen, Enc, Dec) [18].  $(pk, sk) \leftarrow \text{Gen}(\lambda)$  is the key generation algorithm to produce a key pair  $(sk, pk)$ , where  $sk$  is a selected value.  $c \leftarrow \text{Enc}(pk, m)$  is the encryption algorithm to encrypt a message  $m$  with the public key  $pk$  as a ciphertext  $c$ .  $m' \leftarrow \text{Dec}(c, sk)$  decrypts the ciphertext  $c$  as the plaintext  $m'$  via the private key  $sk$ . We utilize the ElGamal encryption scheme  $\Pi_{\text{El}}$  [18] and the Castagnos-Laguillaumie (CL) encryption scheme  $\Pi_{\text{CL}}$  which satisfy the Indistinguishability under Chosen Plaintext Attacks (IND-CPA).

**SysGen:** The system parameter generation algorithm takes as input a security parameter  $\lambda$ . It chooses a cyclic group  $(\mathbb{G}, p, g)$  and returns the system parameters  $SP = (\mathbb{G}, p, g)$ .

**KeyGen:** The key generation algorithm takes as input the system parameters  $SP$ . It randomly chooses  $\alpha \in \mathbb{Z}_p$ , computes  $g_1 = g^\alpha$ , and returns a public/secret key pair  $(pk, sk)$  as follows:

$$pk = g_1, \quad sk = \alpha.$$

**Encrypt:** The encryption algorithm takes as input a message  $m \in \mathbb{G}$ , the public key  $pk$ , and the system parameters  $SP$ . It chooses a random number  $r \in \mathbb{Z}_p$  and returns the ciphertext  $CT$  as  $CT = (C_1, C_2) = (g^r, g_1^r \cdot m)$ .

**Decrypt:** The decryption algorithm takes as input a ciphertext  $CT$ , the secret key  $sk$ , and the system parameters  $SP$ . Let  $CT = (C_1, C_2)$ . It decrypts the message by computing

$$C_2 \cdot C_1^{-\alpha} = g_1^r m \cdot (g^r)^{-\alpha} = m.$$

### Algorithm KeyGen( $1^\lambda$ )

1.  $(B, n, p, s, g, f, G, F) \xleftarrow{\$} \text{Gen}(1^\lambda, 1^\mu)$
2. Pick<sup>a</sup>  $x \xleftarrow{\$} \{0, \dots, Bp - 1\}$  and set  $h \leftarrow g^x$
3. Set  $pk \leftarrow (B, p, g, h, f)$  and  $sk \leftarrow x$ .
4. Return  $(pk, sk)$

### Algorithm Encrypt( $1^\lambda, pk, m$ )

1. Pick  $r \xleftarrow{\$} \{0, \dots, Bp - 1\}$
2. Compute  $c_1 \leftarrow g^r$
3. Compute  $c_2 \leftarrow f^m h^r$
4. Return  $(c_1, c_2)$

### Algorithm Decrypt( $1^\lambda, pk, sk, (c_1, c_2)$ )

1. Compute  $M \leftarrow c_2 / c_1^x$
2.  $m \leftarrow \text{Solve}(p, g, f, G, F, M)$
3. Return  $m$

### Algorithm EvalSum( $1^\lambda, pk, (c_1, c_2), (c'_1, c'_2)$ )

1. Compute  $c''_1 \leftarrow c_1 c'_1$  and  $c''_2 \leftarrow c_2 c'_2$
2. Pick  $r \xleftarrow{\$} \{0, \dots, Bp - 1\}$
3. Return  $(c''_1 g^r, c''_2 h^r)$

### Algorithm EvalScal( $1^\lambda, pk, (c_1, c_2), \alpha$ )

1. Compute  $c'_1 \leftarrow c_1^\alpha$  and  $c'_2 \leftarrow c_2^\alpha$
2. Pick  $r \xleftarrow{\$} \{0, \dots, Bp - 1\}$
3. Return  $(c'_1 g^r, c'_2 h^r)$



## 2.2 Commitment Scheme

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**Commitment scheme.** Our construction requires a commitment scheme that allows users to commit a message (e.g., token identification) and verify its correctness. A commitment scheme  $\Pi_{\text{com}}$  is composed by three algorithms (CMSetup, Com, CMVerify).  $pp \leftarrow \text{CMSetup}(\lambda)$  inputs  $\lambda$  and generates the public parameter  $pp$ .  $(com, r) \leftarrow \text{Com}(pp, m, r)$  commits the message  $m$  in the commitment  $com$  with the randomness coin  $r$ .  $\{0, 1\} \leftarrow \text{CMVerify}(com, r, m)$  verifies if the message  $m$  is committed in  $com$ . Our solution introduces the Pedersen commitment scheme [14], satisfying the information-theoretically hiding and computationally binding properties.

Let us consider  $\mathbb{G} = \langle g \rangle$ , a cyclic group of prime order  $q$ , and two random generators  $g, h \in \mathbb{G}$ . The Pedersen commitment scheme allows to commit to scalar elements from  $\mathbb{Z}_q$ :

**Commitment:** to commit to a scalar  $m \in \mathbb{Z}_q$ , one chooses a random  $r \xleftarrow{\$} \mathbb{Z}_q$ , and sets  $c \leftarrow g^m h^r$ , while the opening value is set to  $r$ ;

**Opening:** to open a commitment  $c \in \mathbb{G}$ , one reveals the pair  $(m, r)$ . If  $c = g^m h^r$ , the receiver accepts the opening to  $m$ , otherwise it refuses.

## 2.3 Malleable Proof Scheme

**Malleable proof scheme.** The malleable proof schemes [16, 19] support users to transform their receiving proofs into new proofs against the converted witness (e.g., a randomized solution of a randomized puzzle) and statement. Let  $R(x, w)$  be a relation related to the language  $L := \{x \mid \exists w \text{ such that } (x, w) \in R\}$ , where  $x$  is a statement and  $w$  is a witness of the statement. Two transformation functions ( $w' = \mathcal{T}_{wit}(w)$ ,  $x' = \mathcal{T}_{stmt}(x)$ ) are defined to restrict the allowed transformation of users. The malleable proof scheme is formulated as:  $\Pi_{MP} = (\text{CRSSetup}, \text{Vry}, \text{Prove}, \text{ZKEval})$ .  $crs \leftarrow \text{CRSSetup}(\lambda)$  generates the Common Reference Strings (CRS)  $crs$ .  $\pi \leftarrow \text{Prove}(w, crs, x)$  is the prover algorithm with the witness  $w$ , the CRS  $crs$ , and the statement  $x$  as inputs to produce a proof  $\pi$  stating  $(x, w) \in R$ .  $\{0, 1\} \leftarrow \text{Vry}(\pi, crs, x)$  is the verifier algorithm to check whether the existence of  $w$  and  $x$  satisfies the relation  $R$ .  $\pi' \leftarrow \text{ZKEval}(crs, x, \{\mathcal{T}_{wit}, \mathcal{T}_{stmt}\}, \pi)$  produces a transformed proof  $\pi'$  for stating  $(x', w') \in R$ , where  $x'$  and  $w'$  come from the transformation functions  $\{\mathcal{T}_{wit}, \mathcal{T}_{stmt}\}$ . Note that the transformed proof  $\pi'$  still can be verified by the algorithm  $\text{Vry}$ . Here, a malleable proof scheme [16] is initialized by the Groth-Sahai proof scheme [19] and satisfies the Witness Indistinguishability (WI) property.

### Malleable Proof Systems and Applications

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### Efficient Non-interactive Proof Systems for Bilinear Groups \*

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## 2.4 Adaptor Signature

**Adaptor signature scheme.** Different from traditional signature schemes, the adaptor signature scheme  $\Pi_{\text{ad}}$ , which consists of five algorithms (SKeyGen, PSign, Adapt, PVrfy, Ext), enables signers to give a *pre-signature* concerning the revelation of a secret value. We define a statement  $Y = g^y$ , where  $g$  is the generator of the group.  $(pk, sk) \leftarrow \text{SKeyGen}(\lambda)$  initializes a key pair  $(pk, sk)$  for signing.  $\tilde{\sigma} \leftarrow \text{PSign}(m, Y, sk)$  inputs the secret key  $sk$ , a message  $m$ , and a statement  $Y$  to generate a pre-signature  $\tilde{\sigma}$ .  $\{0,1\} \leftarrow \text{PVrfy}(pk, m, \tilde{\sigma}, Y)$  checks the validity of the pre-signature  $\tilde{\sigma}$ .  $\sigma \leftarrow \text{Adapt}(y, \tilde{\sigma})$  takes the witness  $y$  and the pre-signature  $\tilde{\sigma}$  as inputs to produce a valid signature  $\sigma$ .  $y \leftarrow \text{Ext}(\tilde{\sigma}, \sigma, Y)$  computes the witness  $y$  via the inputs  $\tilde{\sigma}$  and  $\sigma$ . The adaptor signature scheme satisfies pre-signature adaptability, which guarantees parties collect a valid signature from a valid pre-signature, and witness extractability, which ensures parties extract a valid witness via a valid signature and its pre-signature. Here, the scheme is formalized in [20, 21] and matches the security property against Existential Unforgeability under Chosen Message Attack (EUF-CMA) [18].

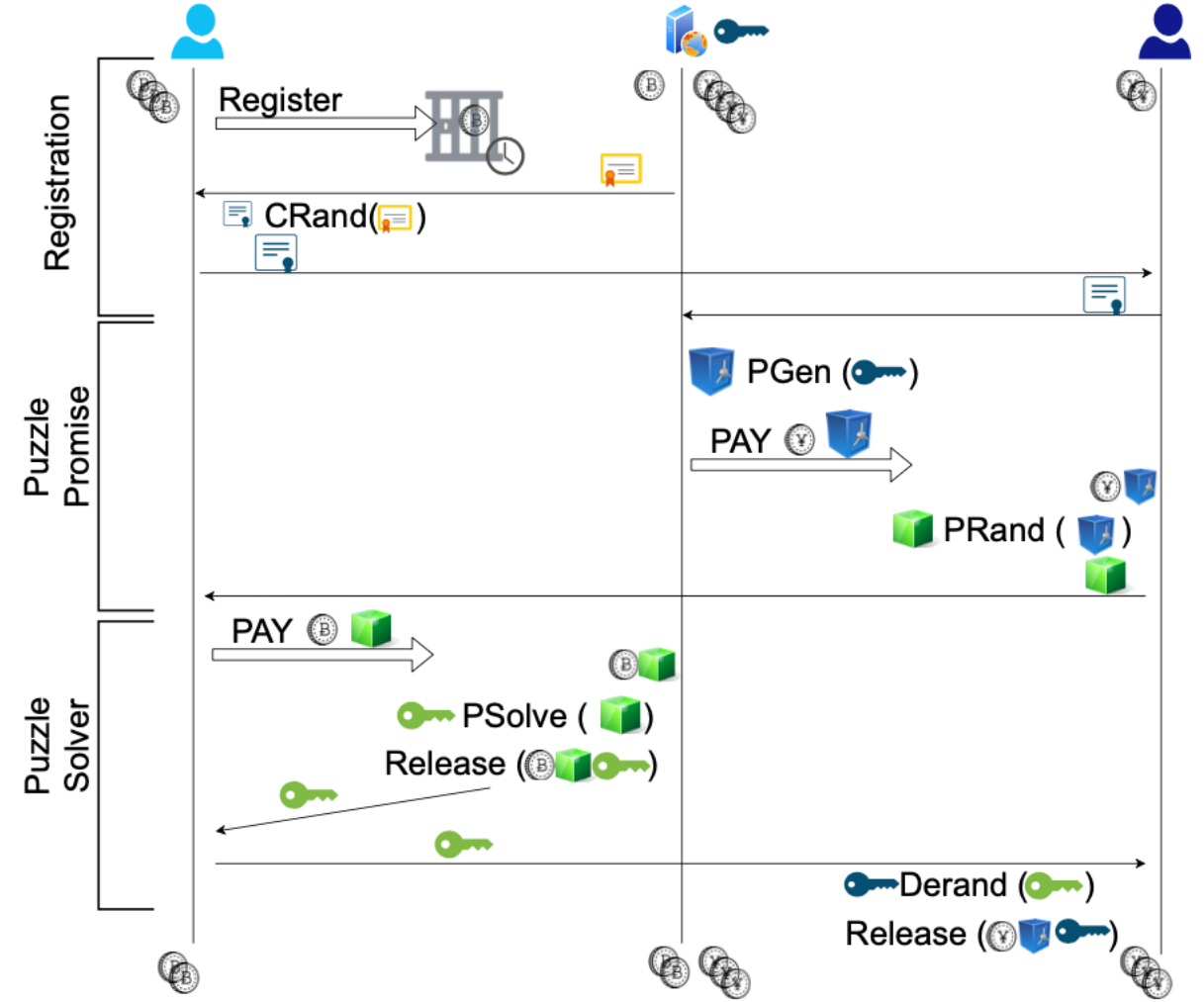
$\text{pSign}_{sk}(m, I_Y)$	$\text{pVrfy}_{pk}(m, I_Y; \tilde{\sigma})$	$\text{Adapt}(\tilde{\sigma}, y)$	$\text{Ext}(\sigma, \tilde{\sigma}, I_Y)$
$x := sk, (Y, \pi_Y) := I_Y$	$X := pk, (Y, \pi_Y) := I_Y$	$(r, \tilde{s}, K, \pi) := \tilde{\sigma}$	$(r, s) := \sigma$
$k \leftarrow_{\$} \mathbb{Z}_q, \tilde{K} := g^k$	$(r, \tilde{s}, K, \pi) := \tilde{\sigma}$	$s := \tilde{s} \cdot y^{-1}$	$(\tilde{r}, \tilde{s}, K, \pi) := \tilde{\sigma}$
$K := Y^k, r := f(K)$	$u := \mathcal{H}(m) \cdot \tilde{s}^{-1}$	<b>return</b> $(r, s)$	$y' := s^{-1} \cdot \tilde{s}$
$\tilde{s} := k^{-1}(\mathcal{H}(m) + rx)$	$v := r \cdot \tilde{s}^{-1}$		<b>if</b> $(I_Y, y') \in R'_g$
$\pi \leftarrow \text{P}_Y((\tilde{K}, K), k)$	$K' := g^u X^v$		<b>then return</b> $y'$
<b>return</b> $(r, \tilde{s}, K, \pi)$	<b>return</b> $((I_Y \in L_R)$		<b>else return</b> $\perp$
	$\wedge (r = f(K)) \wedge \text{V}_Y((K', K), \pi))$		



## 2.5 High-level Design of A<sup>2</sup>L<sup>+</sup>

**Puzzle-promise phase.** During this phase, the receiver  $P_r$  starts by sending a valid signature  $\sigma'_r$  on a transaction message  $m'$  to the hub  $P_h$ . The hub generates a statement/witness pair  $(A, \alpha)$  and creates a randomizable puzzle  $Z$  along with a zero-knowledge proof  $\pi_\alpha$  [15, 22] that proves  $\alpha$  is a valid solution to  $Z$ . The hub then produces an adaptor signature  $\hat{\sigma}'_h$  over the transaction  $m'$  using  $\alpha$  and shares both the puzzle and the adaptor signature with  $P_r$ . The receiver pre-verifies the signature and randomizes the puzzle  $Z$  to  $Z'$ , which is then shared with the sender  $P_s$ , completing the puzzle promise protocol.

**Puzzle-solver phase.** Here, the sender further randomizes the puzzle  $Z'$  to  $Z''$  and generates a pre-signature  $\hat{\sigma}_s$  on the transaction  $m'$  using  $Z''$ . This randomized puzzle and the pre-signature are sent to the hub, which then solves the puzzle  $Z''$  using the trapdoor information to obtain  $\alpha''$ . The hub uses  $\alpha''$  to adapt  $\hat{\sigma}_s$  into a valid signature  $\sigma_s$  and signs the transaction  $m'$  with its secret key, producing  $\sigma_h$ . After verifying the signature  $\sigma_s$ , the hub publishes both  $\sigma_s$  and  $\sigma_h$ . Finally, the secret  $\alpha''$  is extracted and shared with the receiver, allowing them to finalize the transaction by revealing the secret  $\alpha$ .



# **3 Linkable Randomizable Puzzle Scheme**

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## 3.1 Randomizable Puzzle Scheme

- Assuming that  $(KGen, Enc, Dec)$  is a linearly homomorphic encryption with statistical circuit privacy, there exists a randomizable puzzle with statistical privacy.

*Definition A.1 (Randomizable Puzzle).* A randomizable puzzle scheme  $RP = (PSetup, PGen, PSolve, PRand)$  with a solution space  $\mathcal{S}$  (and a function  $\phi$  acting on  $\mathcal{S}$ ) consists of four algorithms defined as:

$(pp, td) \leftarrow PSetup(1^n)$ : is a PPT algorithm that on input security parameter  $1^n$ , outputs public parameters  $pp$  and a trapdoor  $td$ .

$Z \leftarrow PGen(pp, \zeta)$ : is a PPT algorithm that on input public parameters  $pp$  and a puzzle solution  $\zeta$ , outputs a puzzle  $Z$ .

$\zeta := PSolve(td, Z)$ : is a DPT algorithm that on input a trapdoor  $td$  and puzzle  $Z$ , outputs a puzzle solution  $\zeta$ .

$(Z', r) \leftarrow PRand(pp, Z)$ : is a PPT algorithm that on input public parameters  $pp$  and a puzzle  $Z$  (which has a solution  $\zeta$ ), outputs a randomization factor  $r$  and a randomized puzzle  $Z'$  (which has a solution  $\phi(\zeta, r)$ ).

It is not hard to see that a linearly homomorphic encryption scheme  $(KGen, Enc, Dec)$  matches the syntax of a randomizable puzzle, setting  $pp$  to the encryption key and  $td$  to be the decryption key. For the  $PRand$  algorithm, we can sample a random  $r \leftarrow \mathbb{Z}_p$  and compute

$$Enc(ek, \zeta) \circ Enc(ek, r) = c$$

which is an encryption of  $\phi(\zeta, r) = \zeta + r$ . Next we recall the definition of security for randomizable puzzles.

## 3.2 Linkable Randomizable Puzzle Scheme

### The Linkable Randomizable Puzzle (LRP) Scheme

$(pp, sk_0) \leftarrow \text{Setup}(\lambda)$ : is a PPT algorithm (used by the auditor) that on input security parameter  $\lambda$ , outputs public parameters  $pp = (G, g, q, r_0, \Pi_{MP}, crs, \Pi_{El}, pk_0)$  and private key  $sk_0$ . The details of the outputs are as follows:

- $G$  is an elliptic curve group of order  $q$  with generator  $g$ .
- $r_0$  is a random number in  $Z_q$ .
- $\Pi_{MP}$  is a secure malleable non-interactive zero-knowledge proof scheme [16] under a common reference string  $crs$ .
- $\Pi_{El}$  is the ElGamal encryption scheme [18] of which  $\text{Enc}(pk_0, \cdot)$  is for encryption and  $\text{Dec}(pk_0, sk_0, \cdot)$  is for decryption, and  $(pk_0, sk_0)$  is a key pair of  $\Pi_{El}$ .

$(pk_1, sk_1) \leftarrow \text{KGen}(\lambda)$ : is a PPT algorithm (used by the hub) that on input security parameter  $\lambda$ , outputs a key pair  $(pk_1, sk_1)$  from the Castagnos-Laguillaumie (CL) encryption scheme  $\Pi_{CL}$  [29], of which  $\text{Enc}(pk_1, \cdot)$  is for encryption and  $\text{Dec}(pk_1, sk_1, \cdot)$  is for decryption.

$(pz, \pi) \leftarrow \text{PGen}(pp, pk_1, \mathfrak{N})$ : is a PPT algorithm (used by the hub) that on input public parameters  $pp$ , public key  $pk_1$ , and a puzzle solution  $\mathfrak{N} \in Z_q$ , outputs a puzzle  $pz = (\alpha, \beta, \gamma)$  and associated proof  $\pi$ . The details of the outputs are as follows:

- $\alpha = \Pi_{El} \cdot \text{Enc}(pk_0, r_0)$ .
- $\beta = \Pi_{CL} \cdot \text{Enc}(pk_1, \mathfrak{N})$ .
- $\gamma = g^{\mathfrak{N}}$ .
- $\pi$  is a malleable proof of existence of witness  $(\mathfrak{N}, r_0)$  for a statement  $\alpha \wedge \beta \wedge \gamma * g^{r_0}$  using  $\Pi_{MP}$ <sup>a</sup>.

$0/1 \leftarrow \text{PVerify}(pp, pz, \pi)$ : is a Deterministic Polynomial Time (DPT) algorithm that on input public parameters  $pp$ , a puzzle  $pz$ , and a proof  $\pi$ , outputs either 0 (failure) or 1 (success) for verifying  $pz$ . The process of generating the outputs is as follows:

- Parse  $pz = (\alpha, \beta, \gamma)$ .
- Check if proof  $\pi$  is correct for statement  $\alpha \wedge \beta \wedge \gamma \wedge \gamma * g^{r_0}$  using scheme  $\Pi_{MP}$ .
- Return 1 if the proof is correct and 0 otherwise.

$\mathfrak{N} \leftarrow \text{PSol}(pk_1, sk_1, pz)$ : is a DPT algorithm (used by hub) that on input public key  $pk_1$ , private key  $sk_1$ , and a puzzle  $pz$ , outputs  $\mathfrak{N} = \Pi_{CL} \cdot \text{Dec}(pk_1, sk_1, \beta)$ , where  $pz$  is parsed as  $pz = (\alpha, \beta, \gamma)$ .

$(pz', \pi', r) \leftarrow \text{PRand}(pp, pk_1, pz, \pi)$ : is a PPT algorithm (used by users) that on input public parameters  $pp$ , public key  $pk_1$ , a puzzle  $pz$ , and a malleable proof  $\pi$ , outputs a randomized puzzle  $pz'$ , randomized proof  $\pi'$ , and associated random number  $r$ . The process of generating the outputs is as follows:

- Parse  $pz = (\alpha, \beta, \gamma)$ .
- Sample a random number  $r \in Z_q$ .
- Randomize puzzle  $pz$  to  $pz' = (\alpha', \beta', \gamma')$  using  $r$  such that  $\alpha' = \Pi_{El} \cdot \text{Enc}(pk_0, r_0 + r)$ ,  $\beta' = \Pi_{CL} \cdot \text{Enc}(pk_1, \mathfrak{N} + r)$ , and  $\gamma' = g^{\mathfrak{N}+r}$  (note that both  $\Pi_{CL}$  and  $\Pi_{El}$  are homomorphic).
- Randomize  $\pi$  to  $\pi'$  using  $r$  to prove the existence of witness  $(\mathfrak{N} + r, r_0 + r)$  for statement  $\alpha' \wedge \beta' \wedge \gamma' \wedge \gamma' * g^{r_0}$ . Note that  $T_{wit}((\mathfrak{N}, r_0)) = (\mathfrak{N} + r, r_0 + r)$  and  $T_{stmt}(\alpha \wedge \beta \wedge \gamma \wedge \gamma * g^{r_0}) = \alpha' \wedge \beta' \wedge \gamma' \wedge \gamma' * g^{r_0}$ .
- Return  $pz' = (\alpha', \beta', \gamma')$ ,  $\pi'$ , and  $r$ .

$0/1 \leftarrow \text{PLink}(pp, sk_0, pz, pz')$ : is a DPT algorithm (used by the auditor) that on input public parameters  $pp$ , (auditor's) private key  $sk_0$ , and two puzzles  $pz$  and  $pz'$ , outputs 1 (success) or 0 (failure). Respectively,  $pz$  and  $pz'$  are parsed as  $pz = (\alpha, \beta, \gamma)$  and  $pz' = (\alpha', \beta', \gamma')$ . The process of generating the output is as follows:

- If  $g^{\Pi_{El} \cdot \text{Dec}(pk_0, sk_0, \alpha')} / g^{\Pi_{El} \cdot \text{Dec}(pk_0, sk_0, \alpha)} = \gamma' / \gamma$ , return 1.
- Otherwise, output 0.

<sup>a</sup> Including  $\gamma * g^{r_0}$  in the statement is to bind  $\mathfrak{N}$  to  $r_0$ .

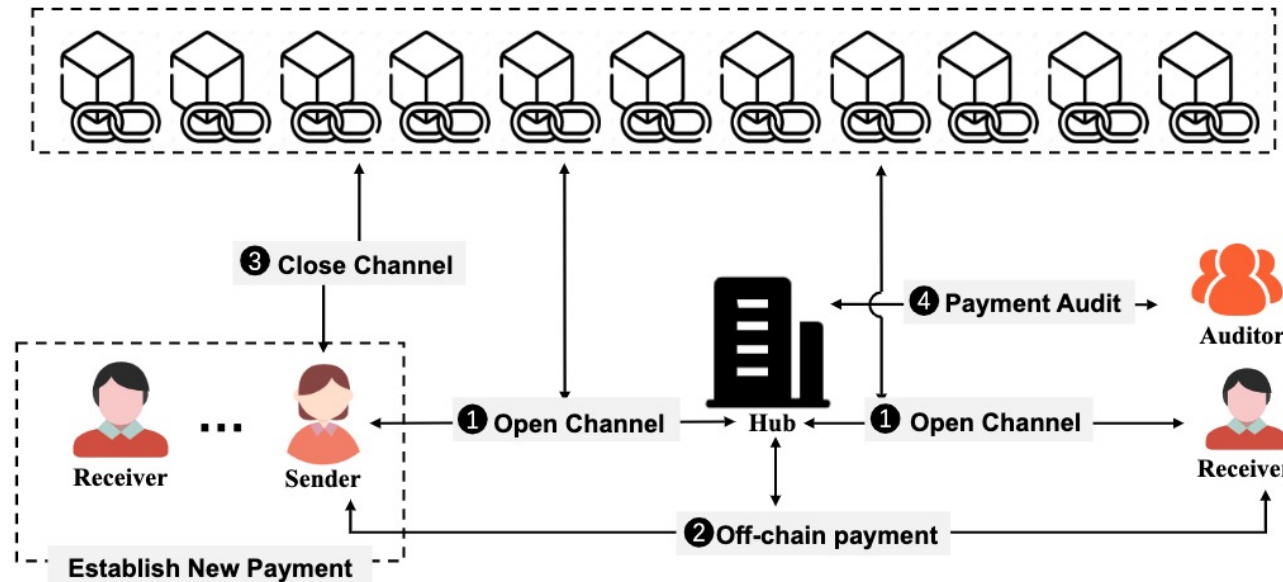
## **4 Auditable Anonymous PCH**

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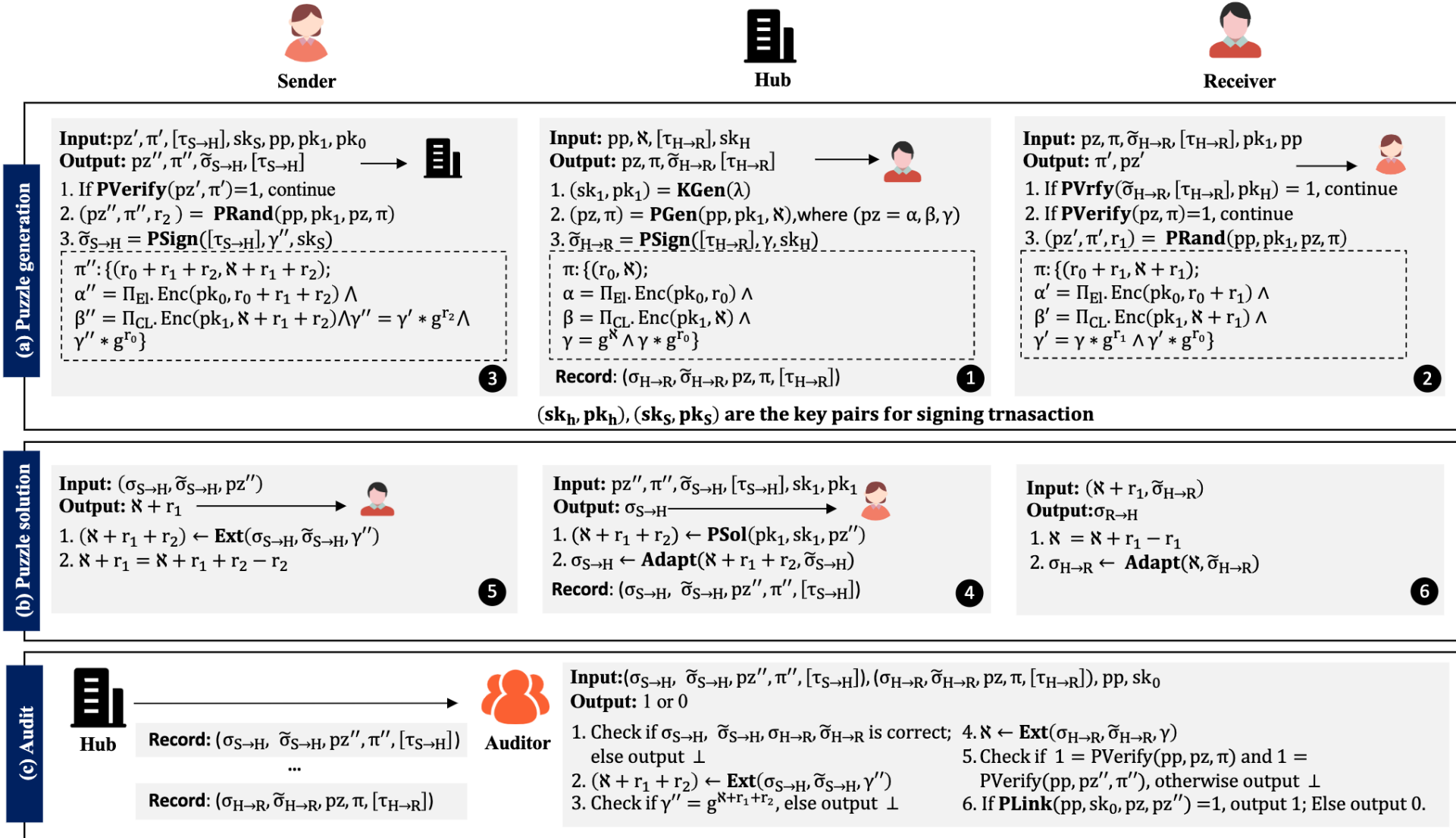


## 4.1 Security Model

- Both hub and users can be malicious while auditor is entrusted with auditing and verifying.
- PPT adversary  $\mathcal{A}$  adopts static corruption.
- Synchronous communication network  $\mathcal{F}_{syn}$  and secure transmission  $\mathcal{F}_{smt}$ .
- Blockchain is a global ledger  $\mathcal{F}_L$ .
- $\mathcal{F}_{AuditPCH}$  defines five operations: setup, open, pay, close, and audit.



## 4.2 Auditable Anonymous PCH



# **5 Performance Analysis**

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# 5.1 Performance Analysis

- 1.6GHz Intel Core i5-8265U with 8-Core and 8GB RAM.
- Java & C with JPBC and Relic.
- Secp256k1 elliptic curve.

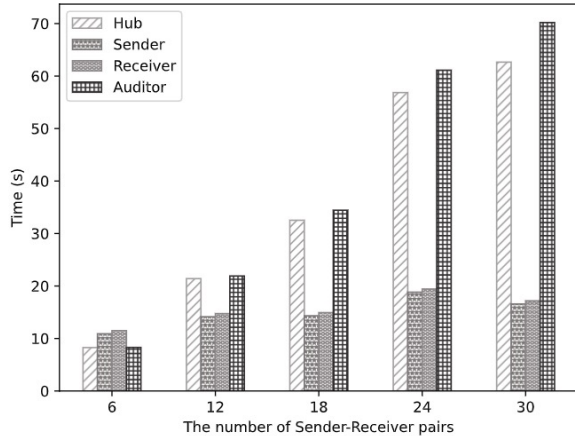


Fig. 4: The computation cost of each role while executing the AuditPCH protocol. The number of Sender-Receiver pairs ranges from 6 to 30.

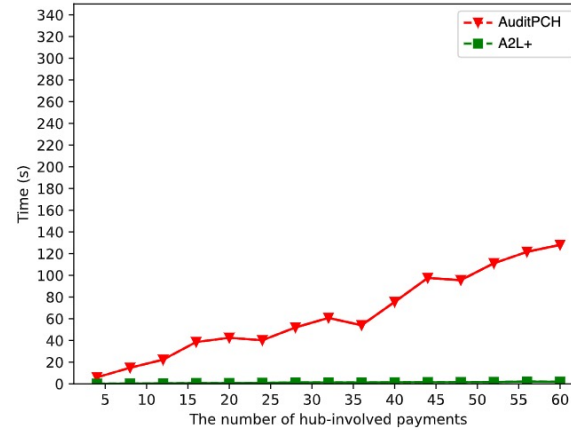


Fig. 5: The time cost comparison between AuditPCH and A<sup>2</sup>L<sup>+</sup>. The number of hub-involved payments ranges from 4 to 60.

TABLE II: The computation cost of the LRP scheme.

	Setup	KGen	PVerify	PGen	PSol	PRand	PLink
Cost(s)	0.419	0.074	6.504	0.615	0.070	0.622	0.013

TABLE III: The computation cost of AuditPCH. Time is shown in seconds.

	Pay			Open	Close	Audit	Total
	Channel Authentication	Puzzle Generation	Puzzle Solution				
Sender	0.002	7.279	0.008	–	–	–	7.289
Hub	0.004	0.768	6.582	0.004	–	–	7.358
Receiver	–	7.583	0.008	–	–	–	7.591
Auditor	–	–	–	0.485	–	13.258	13.716

TABLE IV: The communication cost of AuditPCH.  $n$  is the number of payments. Size is shown in KB.

	Pay			Open	Close	Audit
	Channel Authentication	Puzzle Generation	Puzzle Solution			
Sender	0.969	5.764	0.281	–	–	–
Hub	0.203	5.764	0.562	–	–	$11.52n + 2n[TX]$
Receiver	–	5.404	–	–	–	–
Auditor	–	–	–	7.230	–	–

# Thanks! Questions?

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