MAT 387/487 Project

Due Thursday, March 17th at 3pm

This is a project based on the paper Interior Point Methods and Linear Programming by Robert Robere (2012). One way to implement the interior point method is given by Algorithm 1 on the next page. Your task is to use MatLab to implement this algorithm.

MatLab

This is a mathematics project with a small amount of MatLab programming.
Use the template/skeleton of the MatLab code for the Project that I sent the class. By using the template, it probably makes this project much easier for you.

In the first part of the MatLab code, it creates the matrix A and other input to the LP. Where it says Main Steps, that is where you implement the interior point method. Where it says End of the Main steps, you should not need to type anything after where the code says End of Main steps. After the End of Main steps, the code does a few things to display the output. To implement the interior point method means that you are implementing Algorithm 1 on the next page.

Make sure you DO NOT change the way the matrices A0 and A are made. The commands `tic` and `toc` tell Matlab to start and to end the timing of the runtime of the code. The output of the code is to display two vectors as two columns. The first one is the true vector and second one is the optimal vector found by solving the optimization problem.

The features in MatLab you may find helpful are

1. To make an identity matrix B with 5 rows and 5 columns,
   type this: `Bmat = eye(5,5)`
2. Given a square matrix Q, to solve for x so that Qx = y, type this: `x = Q\y`

Summary of the Algorithm

Given a matrix A_0 with 52 rows and 128 columns, and a vector $\mathbf{b} \in \mathbb{R}^{52}$, find the sparse vector $\mathbf{x} \in \mathbb{R}^{128}$ such that $A_0\mathbf{x} = \mathbf{b}$. To solve this problem, we first enlarge the matrix A_0 to a matrix A with twice as many columns.

Algorithm 1 Interior Point Method

1: Input: vector $\mathbf{c}$, matrix A with 256 columns, vector $\mathbf{b}$, initial vector $\mathbf{x_0}$
2: Initialize parameters $\mu = 10, \rho = 0.6$
3: Initialize vector $\mathbf{x} = \mathbf{x}_0$
4: Set variable trials $= 0$
5: **for** iter $= 1, 2, 3, \ldots 20$ **do**
6: if iter > 1
7: $\mu \leftarrow \mu\rho$
8: end if
9: **for** $k = 1, 2, 3, \ldots, 100$ **do**
10: Set $X = \text{diag}(\mathbf{x}) + \epsilon I$, where $\epsilon = 0.001$ and I is the identity matrix
11: Solve for the vector $\underline{\lambda}$ such that

$$(AX^2A^T)\underline{\lambda} = AX^2\mathbf{c} - \mu AX\mathbf{e}$$

12: Update the vector $\mathbf{p}$ using the equation,

$$\mathbf{p} = X\mathbf{e} + \frac{1}{\mu}X^2\left(A^T\underline{\lambda} - \mathbf{c}\right)$$

13: Update the new estimate for the vector $\mathbf{x}$, set $\alpha = 1$,

$$\mathbf{x} = \mathbf{x} + \alpha\mathbf{p}$$

14: if $\|\mathbf{p}\|_2 < 1/(10^9)$
15: break
16: end if
17: **end for**
18: set trials $\leftarrow$ trials + 1
19: **end for**

(note that with $\alpha = 1$, we are using Pure Newton Iteration)
The break in line 15 will break out of the inner For Loop if the condition in line 14 is satisfied.

Problem

The true vector $\vec{x} \in \mathbb{R}^{128}$ is 5-sparse. So, only 5 entries in a vector of 128 numbers are NOT zero. For a given matrix A_0, we can only observe the vector $\vec{b}$, where $A_0\vec{x} = \vec{b}$. We want to recover $\vec{x}$ by solving the optimization problem,

$$\min_{\mathbf{x}} \|\mathbf{x}\|_1$$

subject to the constraint $A_0\mathbf{x} = \mathbf{b}$.

The goal of the Interior Point method is to use Linear Programming to recover the true vector $\vec{x}$, to within some acceptable error.

What to hand in

Hand in your MatLab program. Hand in the output of the program. The output is the display of two columns: the true vector and the vector found by the optimal solution to the LP. The output also shows the number of trials.

Put everything in one single pdf file. Hand in this one single pdf file. That means, the MatLab program and the output of the program are in one single pdf file.

Do not send me the Matlab code in email separately, unless I contact you and ask you to send me your code in email.

Remark about Algorithm 1

There are some parameters in this algorithm, μ and ρ on line 2, and ϵ in line 10. We can try to improve the algorithm by dynamically updating the value of ϵ on line 10, and by determining good criteria for choosing μ and ρ on line 2. However, these are BEYOND the scope of this project.

The matrix AX^2A^T is often a singular matrix, which is why we need to adjust X by adding ϵI to it.

Academic Integrity

It is important that you write your own MatLab code. This should not be code that you copy from other sources, such as code taken from the internet, or code written by other people.