1. Logic : Propositional - I^{st} Order - Modal

IA 301
Symbolic Ai

I. Propositional Logic :

- Syntax:
  - variables : p, q, r ...
  - connectives : $\wedge, \vee, \neg, \rightarrow, \leftrightarrow$
  - Formulas : combo of vars with conn.
- Semantics :
  - Interpretation of a Formula
    $v : \mathcal{F} \rightarrow \{0,1\}$
    v : truth value
- World :
  - Assignment to all variables
- $A \equiv B \leftrightarrow A, B$ have the same truth tables
- $\top$: Tautology : Always true
  $\bot$: Antilogy : Always false
- For Truth Value of a $\mathcal{F}$, use decomposition tree
- Other Connectives :
  - NOR : $p \downarrow q$: $\neg(p \vee q)$
  - NAND : $p \uparrow q$: $\neg(p \wedge q)$
  - XOR : $p \oplus q$: $(p \wedge \neg q) \vee (\neg p \wedge q)$
    $\neg(p \leftrightarrow q)$
- DNF : Disjunctive Normal Formula : $\bigvee q_i$
  CNF : Conjunctive Normal Formula : $\bigwedge q_i$
  * $(p \vee q) \wedge r$: CNF
  * $(p \wedge r) \vee (q \wedge r)$: DNF
- Knowledge Representation : In real words
  Knowledge Base : Set of sentences
  Models : Setting of vars that satisfy KB
- Axioms & Inference Rules

(A_1) : $A \rightarrow (B \rightarrow A)$

(A_2) : $A \rightarrow (B \rightarrow C) \rightarrow (A \rightarrow B) \rightarrow (A \rightarrow C)$

(A_3) : $(\neg B \rightarrow \neg A) \rightarrow (A \rightarrow B)$

. $A \vee B \equiv \neg A \rightarrow B$

. $A \wedge B \equiv \neg(A \rightarrow \neg B)$

(MODUS PONENS) $\dfrac{A,\ A \rightarrow B}{B}$

"Having A and $A \rightarrow B$, we deduce B"

- Consequence Relation $\vdash$
  $H \vdash C \Leftrightarrow$ C can be deduced from H
- Theorem : $A \vdash B \Leftrightarrow\ \vdash (A \rightarrow B)$

- Satisfiability :
  $m \models A \leftrightarrow$ A is true in world m
  $\leftrightarrow$ m is a model for A
  $\leftrightarrow$ m satisfies A
- KB is satisfiable :
  $\leftrightarrow \exists m, \forall \varphi \in KB, m \models \varphi$
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  A tautology $\leftrightarrow \forall m, m \models A$
  $A \rightarrow B$ tautology $\leftrightarrow \forall m, m \models A \rightarrow m \models B$
  $A \vdash B \leftrightarrow m \models A \rightarrow m \models B$
- Consistent formulas
  A consistent w. B $\leftrightarrow A \nvdash \neg B$
  $A \wedge B$ satisfiable
  $\exists m, m \models B \;\&\; m \models A$
  B consistent w. A

II Predicate / I^{st} order Logic :

- Syntax :
  - Constants : a, b ...
  - Variables : x, y, z ...
  - Function : f(..), g(..), r(..)
  - Logical Connectives : $\vee, \wedge, \neg, \rightarrow$ ...
  - Quantifiers : $\exists, \forall$
    * Universal : $\forall x P$: P holds for all x
    * Existential : $\exists x P$: P holds for some x
    * $\neg(\forall x P) \equiv \exists x \neg P$
    * $\neg(\exists x P) \equiv \forall x \neg P$
- Prenex Form : All quantifiers au début
  $\forall x F \rightarrow \exists x G \leftrightarrow \exists x (F \rightarrow G)$
- Axioms & Inference Rules :

(A4) : $\forall x F(x) \rightarrow F(t/x)$

(A5) : $\forall x (F \rightarrow G) \rightarrow F \rightarrow \forall x G$

(MP) : $\dfrac{F}{\forall x F}$

- Free var : has at least 1 non quantified
  Bound var : has at least 1 quantified
  Closed $\mathcal{F}$: has no free vars.
- Interpretation
  - $\mathcal{M} = (\mathcal{D}, \mathcal{I})$: Structure
  - $\mathcal{D}$: non empty domain
  - $\mathcal{I}$: Interpretat° in $\mathcal{D}$ of the symbols

III. Modal Logics

- Modalities; modal operators:
  - $\Box$: necessity
  - $\Diamond$: Possibility

- Syntax : Same as propo logic

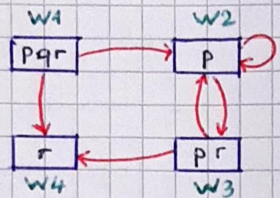
- Duality Constraint : $\Diamond A \equiv \neg \Box \neg A$

- Semantics:
  - P: Atoms of modal language
  - Structure $\mathcal{F} = (W, R)$
    * W: Universe of possible worlds
    * R: Relationship
  - Model $\mathcal{M} = (W, R, V)$

    $V : p \longrightarrow V(p)$

    $V(p)$: Subset of W where p is true
  - $\mathcal{M} \models_w A$: A is true at w in $\mathcal{M}$

- Example



- $P = \{p, q, r\}$
- $W = \{w_1, w_2, w_3, w_4\}$
- V as in figure
- $R = \{(w_1, w_2), (w_1, w_4), (w_2, w_2), (w_2, w_3), (w_3, w_2), (w_3, w_4)\}$
- $\mathcal{M} = (W, R, V)$

* $\mathcal{M} \models_{w_2} \Box p$: " $\forall w$, tq $w R w_2$ $\mathcal{M} \models_w p$ "

  $w_2 R w_2$, $w_2 R w_3$

  $V(p) = \{w_1, w_2, w_3\}$

  $w_2, w_3 \in V(p)$

  OK

* $\mathcal{M} \models_{w_1} \Diamond (r \wedge \Box q)$

  " $\exists w$, $w R w_1$ tq $\mathcal{M} \models_w r \wedge \Box q$ "

  " $\exists w$, $w R w_1$ tq $[\mathcal{M} \models_w r, \mathcal{M} \models_w \Box q]$ "

  . $w_1 R w_2$ and $w_1 R w_4$

  . $\mathcal{M} \not\models_{w_2} r$, w_2 eliminated

  . $\mathcal{M} \models_{w_4} r$, ok, Quid de l'autre

  $\mathcal{M} \models_{w_4} \Box q$?

Vu qu'aucun monde accessible de w_4, donc forcément

- Schemas.

T : Reflexive : $\forall s$, sRs

B : Symmetric : $\forall s,t$: $sRt \rightarrow tRs$

D : Reproductive : $\forall s$, $\exists t$ sRt

4 : Transitive : $\forall stu$, sRt, $tRu \rightarrow sRu$

5 : Euclidean : $\forall stu$, sRt, $sRu \rightarrow tRu$

S4 : KT4 , S5 : KT45

Normal Logics : Inference Rule RN : $\dfrac{A}{\Box A}$

- Examples:
  - $A \rightarrow \Diamond A$ is a theorem of S5.

    T : $\Box A \rightarrow A$ applied to $\neg A$

    $\Box \neg A \rightarrow \neg A \Leftrightarrow \neg \Box \neg A \vee \neg A$

    $\Leftrightarrow \neg A \vee \Diamond A$

    $\Leftrightarrow A \rightarrow \Diamond A$

Decidability

- Propositional : YES
- 1st order : NO IN GENERAL
- MODAL : YES if finite modal

2 - Decision Trees / Association Rules / Formal Concept Analysis / Revision Merging Abduction

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I - Decision Trees:

- Objective:
  - Find a model describing links { attribute / classes
  - Assign new items based on attributes
  - Interpretable by user

- Definition
  - Tree like graph
  - Vertices: Pick attribute and ask Q?
  - Edges: Answers to the Q?
  - Leaves: Actual output depending on edge

- How to choose best attribute?
  - Split based on purity.
  - Usual measures:
    * $GINI(node) = 1 - \sum_j p(j/t)^2$
      $p(j/t)$: relative frequency of class j at node t
      Should be minimized
    * ENTROPY: $-\sum_j p(j/t)\log(p(j/t))$
    * CLASSIF ERROR:
      $1 - \max_j p(j/t)$
  - Best split
    * Gain = G = Purity Bef - Purity After
      should be maximized
      normalize G by entropy

II - Association Rules

- Objective: Data-mining, Freq. Patterns
  Automatic rule construction ...
- Example:

1	Novel, Newspaper
2	Novel, Film, Comics, Music
3	NP, Film, Comics, Music
4	Novel, NP, Film, Comics
5	Novel, NP, Film, Music,

$\Rightarrow$: co-occurrence
$X \Rightarrow Y$: "if we have X, we'll have Y"
{Comics} $\Rightarrow$ {Film} , {Music} $\Rightarrow$ {Film}

- Definitions:
  - Itemset: Collection of items
  - k-itemset: Itemset with k items
  - σ: Support count: # Occurences of item
  - S: Support: σ / # observations
  - Frequent itemset: Itemset, $S \geq$ threshold

Ex* Itemset = {NP, Novel, Film}
σ({Itemset}) = 2
S({Itemset}) = 2/5

- Association Rule: "$X \Rightarrow Y$"
  - Support of Rule:
    $S(X \Rightarrow Y) = \frac{\sigma(X,Y)}{|T|}$, T = # Obs
    "Relative frequency of co-occurence of X, Y"
  - Confidence in a Rule
    $C(X \Rightarrow Y) = \frac{\sigma(X,Y)}{\sigma(X)}$
    "How Often Items in Y appear in records containing X"

EX: * {NP, Film} $\Rightarrow$ {Comics} "R"
$S(R) = \frac{2}{5}$, $C(R) = \frac{2}{3}$

- Rule mining
  - Either Brute Force, but hard
    d items $\rightarrow$ 2^d itemsets $\rightarrow$ $R = \sum_{k=1}^{d-1} C_d^k \left(\sum_{j=1}^{d-k} C_{d-k}^j\right)$
    6 items $\rightarrow$ 64 itemsets $\rightarrow$ 602 Rules
  - Either Based on frequent itemsets
    Still computationally expensive
  - Use the A-Priori Algorithm
    "If an itemset is frequent, so must be its subsets"
    Results of the montony rule
    $X \subseteq Y \Rightarrow S(X) \geq S(Y)$
    Or if an item is infrequent, we drop all subitemsets containing it
    N.B. Frequency is given in Exam

III - Formal Concept Analysis

- Objective: Symbolic Learning / Datamining ..
- Input: Table: objects x attributes
- Output: Concept Lattice
  Attribute implications
- Example:

	y_1	y_2	y_3
x_1	X	X	X
x_2	X		X
x_3		X	X

  - Formal Concept: $(\{x_1, x_2\}, \{y_1, y_3\})$
  - Attribute Implication $\{y_1\} \Rightarrow \{y_3\}$

- (X, Y) is a formal Concept if
  $\alpha(X) = Y$, $\beta(Y) = X$

- Galois Connexion
  (α, β) G.N btwn $(\mathcal{P}(G), \subseteq)$ and $(\mathcal{P}(M), \subseteq)$
  if $\forall X, Y$, $X \subseteq \beta(Y) \Leftrightarrow Y \subseteq \alpha(X)$

- Equivalence: $X_1 \subseteq X_2 \Rightarrow \alpha(X_2) \subseteq \alpha(X_1)$
  $Y_1 \subseteq Y_2 \Rightarrow \beta(Y_2) \subseteq \beta(Y_1)$
  $X \subseteq \beta(\alpha(X))$, $Y \subseteq \alpha(\beta(Y))$
  $\alpha(X) = \alpha(\beta(\alpha(X)))$,
  $\beta(Y) = \beta(\alpha(\beta(Y)))$

- $\alpha\beta$ and $\beta\alpha$ are closure operators increasing, extensive & idempotent

- $\alpha(\bigcup_i X_i) = \bigcap_i \alpha(X_i)$
  $\beta(\bigcup_i Y_i) = \bigcap_j \beta(Y_i)$

- Clarification : Remove redundant columns

- Construction of lattice
  - Start at $(\{\phi\}, \alpha(\{\phi\}))$
  - Singletons, Duplets, Triplets
  - (Whole Itemset, α(W.I.))

- Attribute Implication :
  - Description of dependencies btw data

IV – Reasoning, Revision, Merging, Abduction

1 – Revision