Event-triggered output feedback containment control for multi-agent systems

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***Abstract*—This paper proposes an event-triggered output feedback containment control method for multi-agent systems (MASs) with uncertainty and disturbance. First, a finite-time extended state observer (FTESO) is proposed to estimate the total uncertainty and recover the unmeasurable velocity. Second, the containment control law is derived in the backstepping framework, where the high-order state is obtained using a finite-time differentiator to reduce the computing burden. In addition, the event-triggered mechanism is integrated into control law to minimize the controller's update frequency. The simulation results verify the effectiveness of the proposed approach.**

Keywords—containment control, event-triggered control, multi-agent systems.

I. Introduction

Recently, the issue of cooperative control in MASs has garnered significant attention in research circles owing to its extensive application in various domains, such as sensor networks [1], autonomous underwater vehicles formation [2], and spacecraft coordination [3]. In particular, among the various forms of cooperative control for MASs, the containment control for MASs with several leaders has garnered the attention of researchers, driven by its significant value in practical applications [4, 5]. For example, collision avoidance and formation safety can be achieved by using containment control for MASs with only a few agents have obstacle detection capabilities.

The challenge of containment control stems from the existence of multiple leaders, and extensive efforts have been devoted to addressing this problem. [6] proposed a robust containment control for MASs with communication delay and input time-delay, a Lyapunov-Krasovskii function was presented to establish the stable conditions, which is formulated by two linear matrix inequalities. [7] introduced an innovative adaptive tracking control approach for MASs with unknown inputs, where the parameters in control law are determined using a novel method. [8] introduced a distributed control law for containment problem, utilizing the successive saturated loops to effectively handle velocity and acceleration saturations.

Another challenge in control law design for MASs is the uncertainty within the models, including model uncertainty, disturbances, and unmeasurable velocity. [9] proposed a distance-based formation control method in backstepping framework, where the dynamics uncertainty is compensated using RBFNN. [10] introduced consensus control method for MASs with exogenous disturbances, where a disturbance observer is designed using the dynamic gain technique to address the disturbances. The unmeasurable velocity is estimated by a cascaded extended state observers in [11].

Note that the control inputs of the aforementioned control law are continuously time-varying, and frequent updates of the control law may potentially result in mechanical losses in the practical scenarios. [12] introduced output consensus control approach with event-triggering mechanism, which is designed without using any global information and avoids the issue of continuous updates of the controller. [13] investigated the event-triggered coordination control problem by employing a Lyapunov equation, and the Zeno behavior was avoided via rigorous proof.

In this paper, we proposes an event-triggered output feedback containment control law for MASs with model uncertainty and disturbance. The main contributions are listed as follows

The event-triggered mechanism is seamlessly integrated into the containment control law, which effectively reduces the update frequency of the controller and mitigates mechanical losses.

The proposed FTESO is utilized to estimate the total uncertainty encompassing both model uncertainty and disturbance, while simultaneously recovering the velocity that is challenging to directly measure.

II. Preliminaries

A. Model Introduction

A MAS with n virtual leaders and m followers is considered in this paper, which can be expressed in mathematical form

$$\begin{aligned} \dot{\eta}_i &= R(\psi_i)\nu_i \\ M\dot{\nu}_i &= -C(\nu_i)\nu_i - D(\nu_i)\nu_i + d_i + \tau_i, \end{aligned} \tag{1}$$

where $\eta_i = [x_i, y_i, \psi_i]^T$ and $\nu_i = [u_i, \upsilon_i, r]^T$ stand for the position and the velocity, respectively. The matrices M, $C(\nu_i)$, and $D(\nu_i)$ denote inertia, Coriolis, and damping, respectively. τ_i and d_i refer to the control input and disturbance, respectively.

B. Assumptions

Assumption 1: For all the followers, there exists at least one directed spanning tree whose roots are located at the virtual leader.

Assumption 2: The disturbance d_i is bounded, and satisfies $|d_i| \le \bar{d}_i$.

Assumption 3: The state of virtual leaders and their first-order derivative are bounded.

III. Control Law Design

A. Uncertainty Estimation

(1) can be reformulated to

$$\begin{aligned} \dot{\eta}_i &= R_i(\psi_i)\nu_i, \\ \dot{\nu}_i &= \Xi_i + M_i^{-1}\tau_i \end{aligned} \tag{2}$$

where $\Xi_i = M_i^{-1}\left(-C(\nu_i)\nu_i - D(\nu_i)\nu_i + d_i\right)$ stands for the total uncertainty. The FTESO is proposed to estimate the unmeasurable velocity and total uncertainty.

$$\begin{aligned} \dot{\hat{\eta}}_i &= -3/\gamma \times R\,\mathrm{sig}^{\chi_1}\left(\gamma^2 R^T \tilde{\eta}_i\right) + R\hat{\nu}_i \\ \dot{\hat{\nu}}_i &= -3\,\mathrm{sig}^{2\chi_1-1}\left(\gamma^2 R^T \tilde{\eta}_i\right) + \hat{\Xi}_i + M_i^{-1}\tau_i \\ \dot{\hat{\Xi}}_i &= -\gamma\,\mathrm{sig}^{3\chi_1-2}\left(\gamma^2 R^T \tilde{\eta}_i\right) \end{aligned} \tag{3}$$

where $\gamma > 0$, $0 < \chi_1 < 0$, $\hat{\eta}_i$, $\hat{\nu}_i$, and $\hat{\Xi}_i$ denote the estimation value for η_i, ν_i, and Ξ_i, respectively.

Consider the observer error system as follows

$$\begin{aligned} \dot{\varsigma}_1(t) &= -3R\,\mathrm{sig}^{\chi_1}\left(R^T\varsigma_1(t)\right) + R\varsigma_2(t) \\ \dot{\varsigma}_2(t) &= -3\,\mathrm{sig}^{2\chi_1-1}\left(R^T\varsigma_1(t)\right) + \varsigma_3(t) \\ \dot{\varsigma}_3(t) &= -\mathrm{sig}^{3\chi_1-2}\left(R^T\varsigma_1(t)\right) - \dot{\Xi}_i/\gamma \end{aligned} \tag{4}$$

where $\varsigma_1(t)$, $\varsigma_2(t)$, and $\varsigma_3(t)$ are defined by

$$\begin{aligned} \varsigma_1(t) &= \gamma^2\tilde{\eta}(t/\gamma), \\ \varsigma_2(t) &= \gamma\tilde{\nu}(t/\gamma), \\ \varsigma_3(t) &= \tilde{\Xi}(t/\gamma). \end{aligned} \tag{5}$$

where $\tilde{\eta}$, $\tilde{\nu}$, and $\tilde{\Xi}$ are the estimate error defined by $(\tilde{\bullet}) = (\hat{\bullet}) - (\bullet)$.

Based on the results in [14], the error system (4) will converge in a finite time.

B. Control Law Design

In accordance with the containment control approach, the containment tracking error with respect to i-th follower can be expressed as

$$z_{1i} = R_i^T\left(\sum_{j=1}^{m} a_{ij}\left(\eta_i - \eta_j\right) + \sum_{k=m+1}^{m+n} b_{ik}\left(\eta_i - \omega_k\right)\right), \tag{6}$$

where ω_k denotes the k-th leader's state. $a_{ij} = 1$ if the state of the j-th follower is reachable for the i-th follower; otherwise, $a_{ij} = 0$. $b_{ik} = 1$ if the state of the k-th leader is reachable for the i-th follower; otherwise, $b_{ik} = 0$.

$\dot{z}_{1i}$ can be computed as

$$\dot{z}_{1i} = -r_i\mathcal{K}z_{1i} + \vartheta_i\hat{\nu}_i - \sum_{j=1}^{m} a_{ij}R_i^T R_j\hat{\nu}_j - \sum_{k=m+1}^{m+n} a_{ik}R_i^T\dot{\omega}_k, \tag{7}$$

where $\mathcal{K} = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, $\vartheta_i = \sum_{j=1}^{m} a_{ij} + \sum_{k=1}^{n} b_{ik}$. Note that $\hat{\nu}_i$ and $\hat{\nu}_j$ denote the estimate of ν_i and ν_j, which can be obtained by utilizing the FTESO (3).

Based on (7), we design the virtual control

$$\begin{aligned} \alpha_i = \vartheta_i^{-1}\Big(&-K_{1i}z_{1i} + r_i\mathcal{K}z_{1i} + \sum_{j=1}^{m} a_{ij}R_i^T R_j\hat{\nu}_j \\ &+ \sum_{k=m+1}^{m+n} a_{ik}R_i^T\dot{\omega}_k\Big) \end{aligned} \tag{8}$$

where $K_{1i} > 0$ is the gain diagonal matrix.

Consider the error variable

$$z_{2i} = \hat{\nu}_i - \alpha_i \tag{9}$$

whose derivative can be computed as

$$\begin{aligned} \dot{z}_{2i} &= \dot{\hat{\nu}}_i - \dot{\alpha}_i \\ &= -3\,\mathrm{sig}^{2\chi_1-1}\left(\gamma^2 R^T\tilde{\eta}_i\right) + \hat{\Xi}_i + M_i^{-1}\tau_i - \dot{\alpha}_i \end{aligned} \tag{10}$$

Note that the direct calculation of $\dot{\alpha}_i$ will introduce high-order variable, which increases the computing burden. A finite-time differentiator is proposed to tackle this problem.

$$\begin{aligned} \dot{\wp}_i &= \zeta_i - \mu_1\mathrm{sig}^{1-1/\chi_2}(\wp_i - \alpha_i) \\ \dot{\zeta}_i &= -\mu_2\mathrm{sig}^{1-2/\chi_2}(\wp_i - \alpha_i) \end{aligned} \tag{11}$$

where $\wp_i$ and ζ_i are the estimates for α_i and $\dot{\alpha}_i$, respectively. $\mu_1 > 0$, $\mu_2 > 0$, and $\chi_2 > 2$.

Defining $\tilde{\wp}_i = \wp_i - \alpha_i$ and $\tilde{\zeta}_i = \zeta_i - \dot{\alpha}_i$, then the error system of (11) can be deduced as

$$\begin{aligned} \dot{\tilde{\wp}}_i &= \tilde{\zeta}_i - \mu_1\mathrm{sig}^{1-1/\chi_2}(\tilde{\wp}_i) \\ \dot{\tilde{\zeta}}_i &= -\mu_2\mathrm{sig}^{1-2/\chi_2}(\tilde{\wp}_i) - \ddot{\alpha} \end{aligned} \tag{12}$$

Assumption 4: The $\ddot{\alpha}$ is bounded.

According to [15], (12) will converges to 0 in a finite time, which means $\zeta_i \to \dot{\alpha}$ in a finite time.

Based on the above analysis, (10) can be rewritten as

$$\dot{z}_{2i} = -3\,\mathrm{sig}^{2\chi_1-1}\left(\gamma^2 R^T\tilde{\eta}_i\right) + \hat{\Xi}_i + M_i^{-1}\tau_i - \zeta_i + \tilde{\zeta}_i \tag{13}$$

The control law is designed as

$$\tau_i = M_i\left(-K_{2i}z_{2i} - \vartheta_i z_{1i} + 3\,\mathrm{sig}^{2\chi_1-1}\left(\gamma^2 R^T\tilde{\eta}_i\right) - \hat{\Xi}_i + \zeta_i\right) \quad (14)$$

C. Event-Triggered Control

An ETC algorithm will be integrated into control law (13) to minimize the controller's update frequency.

The actuator update protocol is

$$\overline{\tau}_i(t) = \lambda_i(t_k),\ \ t_k \le t < t_{k+1},\ \ k = 0,1,\ldots \quad (15)$$

where t_k stands for the k-th event-triggered time, $\lambda_i(t)$ denotes the intermediate control law.

The auxiliary error is calculated as

$$\xi_i(t) = \lambda_i(t) - \overline{\tau}_i(t) \quad (16)$$

Based on this, the update condition is specified as

$$t_{k+1} = \inf\left\{t > t_k \left| \left|\xi_i(t)\right| \ge \delta\left|\overline{\tau}_i(t)\right| + \sigma_1\right.\right\} \quad (17)$$

where $0 < \delta < 1$ and $\sigma_1 > 0$.

The intermediate control law is designed as

$$\lambda_i(t) = -(1+\delta)\left[\tau_i \tanh\left(\frac{z_{2i}^T\tau_i}{s}\right) + \sigma_2 \tanh\left(\frac{z_{2i}^T\sigma_2}{s}\right)\right] \quad (18)$$

where $s > 0$ and $\sigma_2 > \sigma_1/(1-\delta)$.

D. Stability Analysis

Theorem 1: Consider a MAS with n virtual leaders and m followers, with the FTESO (3), finite-time differentiator (11), control law (14), actuator update protocol (15), intermediate control law(18), the containment tracking error (6) will converge to a compact set.

Proof: According to (7), (8), and (9), the $z_{1i}^T\dot{z}_{1i}$ can be calculated as

$$z_{1i}^T\dot{z}_{1i} = -z_{1i}^T K_{1i} z_{1i} + z_{1i}^T\vartheta_i z_{2i} \quad (19)$$

According to (15), we know that the following inequality holds when $t_k \le t < t_{k+1}$.

$$\left|\lambda_i(t) - \overline{\tau}_i(t)\right| \le \delta\left|\overline{\tau}_i(t)\right| + \sigma_1 \quad (20)$$

Based on this, there exists continuous auxiliary variables $\varphi_1(t)$ and $\varphi_2(t)$ satisfying

$$\lambda_i(t) = \left(1 + \varphi_1(t)\delta\right)\overline{\tau}_i(t) + \varphi_2(t)\sigma_1 \quad (21)$$

where $\left|\varphi_1(t)\right| \le 1$ and $\left|\varphi_2(t)\right| \le 1$. Combining (15) and (21), the actuator update protocol can be calculated as

$$\overline{\tau}_i(t) = \frac{\lambda_i(t)}{1+\varphi_1(t)\delta} - \frac{\varphi_2(t)\sigma_1}{1+\varphi_1(t)\delta} \quad (22)$$

Combining (14) (18), (22), and (10), $z_{2i}^T\dot{z}_{2i}$ can be calculated as

$$\begin{aligned}
z_{2i}^T\dot{z}_{2i} &= z_{2i}^T\left(-3\,\mathrm{sig}^{2\chi_1-1}\left(\gamma^2R^T\tilde{\eta}_i\right) + \hat{\Xi}_i - \dot{\alpha}_i\right) \\
&\quad + \frac{z_{2i}^T M_i^{-1}\lambda_i(t)}{1+\varphi_1(t)\delta} - \frac{z_{2i}^T M_i^{-1}\varphi_2(t)\sigma_1}{1+\varphi_1(t)\delta} \\
&\le z_{2i}^T\left(-3\,\mathrm{sig}^{2\chi_1-1}\left(\gamma^2R^T\tilde{\eta}_i\right) + \hat{\Xi}_i - \dot{\alpha}_i + M_i^{-1}\tau_i\right) \\
&\quad + \left|z_{2i}^T M_i^{-1}\tau_i\right| + \frac{z_{2i}^T M_i^{-1}\lambda_i(t)}{1+\delta} - \left|\frac{z_{2i}^T M_i^{-1}\sigma_1}{1-\delta}\right| \\
&\le -z_{2i}^T K_{2i} z_{2i} - z_{2i}^T\vartheta_i z_{1i} + z_{2i}^T\tilde{\zeta}_i \\
&\quad + \left|z_{2i}^T M_i^{-1}\tau_i\right| - z_{2i}^T M_i^{-1}\tau_i \tanh\left(z_{2i}^T\tau_i/s\right) \\
&\quad + \left|z_{2i}^T M_i^{-1}\sigma_2\right| - z_{2i}^T M_i^{-1}\sigma_2 \tanh\left(z_{2i}^T\sigma_2/s\right)
\end{aligned} \quad (23)$$

Lemma 1: The tanh function has the following property:

$$0 \le |m| - m\tanh(m/s) \le 0.2785\,s \quad (24)$$

Consider the Lyapunov function

$$V_i = \frac{1}{2}z_{1i}^T z_{1i} + \frac{1}{2}z_{1i}^T z_{2i} \quad (25)$$

whose differential can be calculated as follows according to (19), (23), (24), and (25)

$$\begin{aligned}
\dot{V}_i &= z_{1i}^T\dot{z}_{1i} + z_{1i}^T\dot{z}_{2i} \\
&\le -z_{1i}^T K_{1i} z_{1i} - z_{2i}^T K_{2i} z_{2i} + \left\|z_{2i}\right\|^2/2 \\
&\quad + \left\|\tilde{\zeta}_i\right\|^2/2 + 0.557\lambda_{\max}(M_i^{-1})s \\
&\le \mathcal{C}_i V_i + \mathcal{D}_i
\end{aligned} \quad (26)$$

where $\mathcal{C}_i = \min\{\lambda_{\min}(2K_{1i}), \lambda_{\min}(2K_{2i}-1)\}$, $\mathcal{D}_i = \left(\left\|\tilde{\zeta}_i\right\|^2/2 + 0.557\lambda_{\max}(M_i^{-1})s\right)$.

According to (26), we have

$$V_i \le V_i(0)e^{-\mathcal{C}_i t} + \mathcal{M}_i, \quad (27)$$

where $\mathcal{M}_i = \mathcal{D}_i/\mathcal{C}_i$ denotes a compact set by choosing proper parameters. According to (27) we get

$$\left\|z_{1i}\right\| \le \sqrt{2\mathcal{M}_i},\ \left\|z_{2i}\right\| \le \sqrt{2\mathcal{M}_i} \quad (28)$$

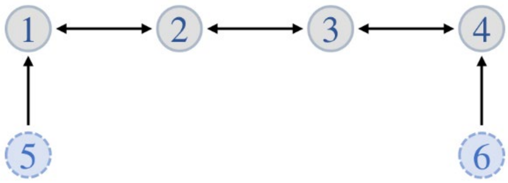

Fig. 1. Communication topology

Based on (28), we know that z_{1i} and z_{2i} are ultimately uniformly bounded, which means all agents will converge to the convex hull spanned by virtual leaders.

IV. Simulation Experiment

The simulation experiments are conducted using a MAS composed of 2 virtual leaders and 4 followers, whose dynamics are described in [16]. The initial state of 4 agents are specified as $\eta_1(0)=[75,99,\pi]^T$, $\eta_2(0)=[77,104,-\pi]^T$, $\eta_3(0)=[77,116,\pi]^T$, $\eta_4(0)=[75,121,-\pi]^T$, and $v_i=[0,0,0]^T, i=1,\ldots,4$. The trajectories of the virtual leaders are specified as $\eta_5(t)=[70-30\sin(\pi t/20), 70-30\cos(\pi t/20)$ $\pi t/20]^T$, $\eta_6(t)=[70-50\sin(\pi t/20), 70-50\cos(\pi t/20),$ $\pi t/20]^T$. The disturbances are specified as $\tau_d=[4+\sin(t+\pi/3), 3+0.8\cos(0.5t+\pi/6), 0.8+0.75\cos(t+\pi/4)]^T$. Communication topology is shown in Fig. 1.

The parameters in control law is $\gamma=2$, $\chi_1=0.8$, $K_{1i}=\text{diag}[35,35,10]$, $K_{1i}=\text{diag}[45,45,15]$, $\mu_1=6$, $\mu_2=6$, $\chi_2=3$, $\delta=0.1$, $\sigma_1=0.5$, and $\sigma_2=3$.

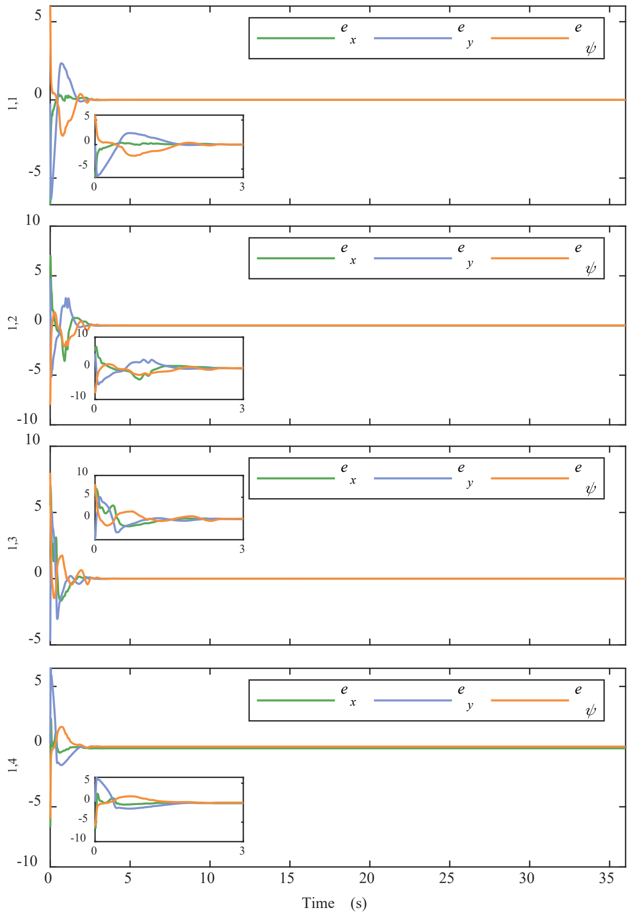

Fig. 2. The containment tracking error

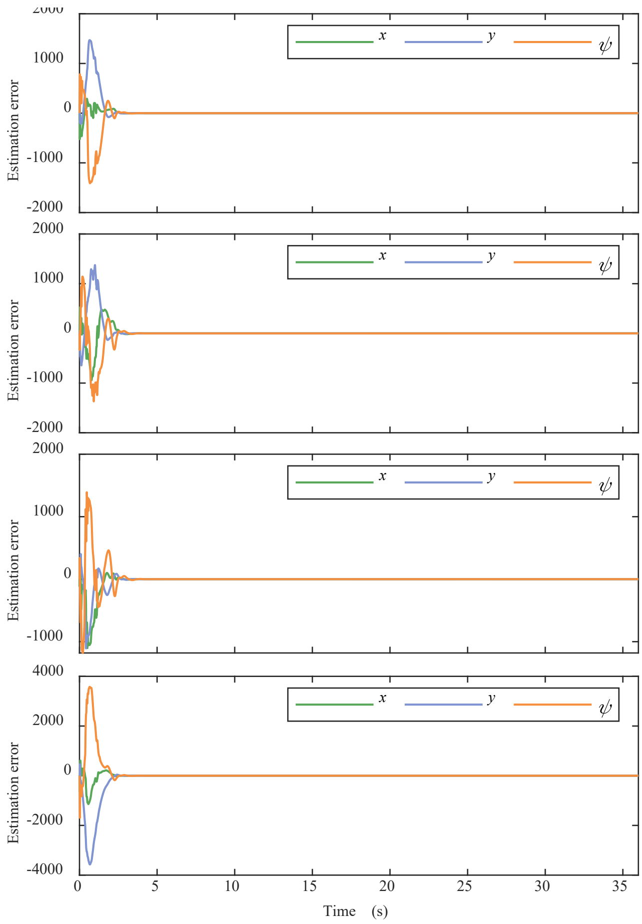

Fig. 3. The estimation error

The results are shown in Fig. 2-Fig. 3. Fig. 2 reveals the containment error converge to the neighbor near 0 using proposed control law, which means all the followers are driven to a convex hull spanned by virtual leaders. Fig. 4 shows the estimation error converges to a compact set using proposed FTESO. Fig. 3 visualizes the tracking path of four agents, which visually validates the feasibility of the algorithm.

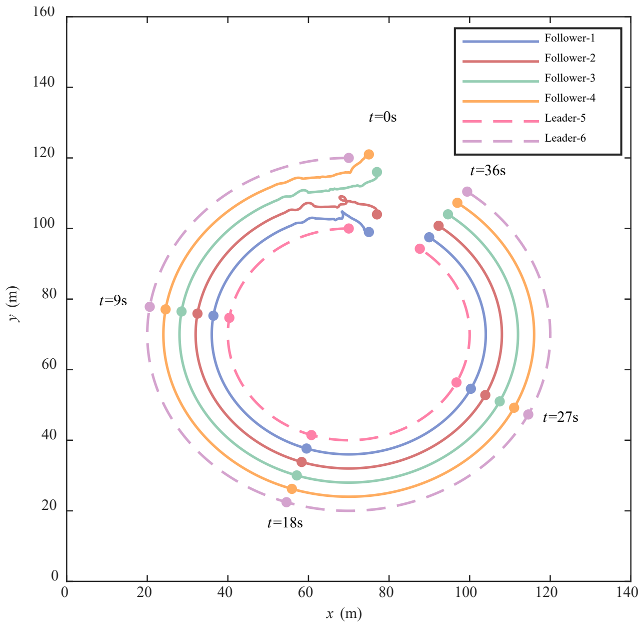

Fig. 4. The moving path

V. Conclusion

This paper focused on the event-triggered containment control problem for MAS. The FTESO is utilized to estimate the total uncertainty and recover the unmeasurable velocity. The control law is derived in backstepping framework, seamlessly integrating an event-triggered mechanism into the containment control law. Meanwhile, the high-order state is obtained using a finite-time differentiator, which reduce the computing burden.

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