Bearing-Only Formation Tracking Control for Multi-Agent Systems With Time-Varying Velocity Leaders

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***Abstract*—This letter studies the bearing-only formation tracking control problem for multi-agent systems in which the target formation moves with a time-varying reference velocity. We consider both single-integrator and double-integrator models and propose two control laws for them that can achieve this formation tracking control using only bearing measurements, without any other measurements or communication between agents. Moreover, these two control laws can be extended to deal with the system with bounded disturbances. The convergence of the systems under these two control methods is proven by rigorous mathematical derivations and simultaneously validated by numerical simulations.**

***Index Terms*—Bearing-only formation control, formation tracking, multi-agent systems.**

I. Introduction

FORMATION control for multi-agent systems (MASs) has become an active research area due to its extensive applications. Based on the sensing capabilities and the variables used in the control law, the existing results can be classified into displacement-based, distance-based, position-based, and bearing-based methods [1], [2]. Compared to the first three methods, the bearing-based method uses bearing measurements that are available from low-cost vision sensors or other passive sensors [3], which relaxes the requirements on the agent's sensing capabilities. Furthermore, we can refer to the control protocol as a *bearing-only control* law if it uses only inter-agent relative bearing measurements, without other measurements or communication between agents [4].

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These salient features of bearing-only control method have attracted extensive effort, and many advances have been made. Bearing rigidity theory in arbitrary dimensions was studied in [5]; this theory is a powerful tool for addressing the geometric uniqueness problem under bearing-only constraints and is the basis for bearing-only formation control. Subsequently, many bearing-only control laws for stationary target formation have been proposed, and various issues, such as prespecified time convergence [6], uncertain systems [7], orientation estimation [8], [9], attitude synchronization [5], [10], and special sense topology [10], have been addressed. In addition, compared to the stationary formation stabilization method mentioned above, formation tracking control, i.e., the target formation is moving, is a more practical subject. The bearing-only localizability was proposed in [11]; as a basis for formation tracking, it provides a condition for determining a unique formation with leaders' positions and necessary relative bearings. The bearing-only formation tracking control for MASs with constant-velocity leaders was studied in [12], [13], [14], [15]. Reference [12] studied the formation tracking for single-integrator, double-integrator, and unicycle and provided a stability analysis using the Lyapunov method. Reference [13] extended dynamic model to double integrators with uncertainties. And the attitude control was cascaded with bearing-only tracking control laws for constant-velocity MASs in [14], [15], which drops the need for global reference frames.

Despite these advances, they only consider the case where the target formation moves with a constant velocity, whereas the *time-varying velocity* is more realistic. Some existing work on *bearing-based* maneuvering seems to be able to meet this need, such as [16], [17], [18]. However, this is achieved by the *bearing-based* method rather than the *bearing-only* method, which means that additional measurements and communication are needed. Specifically, to estimate the desired velocity, distributed observers are used in [16], [18], [19], which means that some agents need to access leaders' velocity, and communication is necessary to exchange estimates. Similarly, control laws used in [17] and [20] also need the above requirements. Thus, we can see that the time-varying velocity tracking will become a common topic if we can get the leader's velocity or estimates via observers. However, this places higher demands on agents' sensing and communication capabilities, i.e., both leaders' velocity and communication are needed, thus it is out of the topic of *bearing-only* method. To the best of our knowledge, bearing-only formation tracking control for MASs with time-varying velocity leaders remains an open challenge.

Moreover, disturbance is inevitable in real systems, which may affect and even invalidate the control law. The constant disturbance is addressed in [12] by a PI control method. For the time-varying disturbance, [21] analyzed its impact on the formation control and gave the upper bound of the formation error. References [13], [22] proposed control laws to account for the time-varying bounded disturbance and achieve system convergence. Nevertheless, they did not account for the tracking control of formations with time-varying velocities.

Motivated by this, for the first time, we focus on bearing-only formation tracking control for MASs with time-varying velocity leaders. The main contributions are listed as follows. ***Firstly***, for a MAS with time-varying velocity leaders, we first propose bearing-only formation tracking control methods for both single and double integrators, which use only bearing measurements, without using communication between agents. ***Secondly***, with a minor modification, these two methods can be extended to address the single and double integrators with bounded disturbances.

Remark 1: The above results are achieved by introducing a term containing signum function. We all know that the signum function is a common tool used in topics such as finite-time control [23] and velocity estimator [24]. Also, some works in bearing-only formation use it to achieve finite-time control [9], [22]. However, despite using the signum function, they didn't achieve bearing-only time-varying formation tracking. Signum function in these works is more of a tool for finite time control rather than for time-varying tracking. When it comes to time-varying tracking, a common method is to estimate the leader's velocity and track it by using velocity error. However, this is difficult in bearing-only control since the velocity estimator is hard to implement due to the limited variables that can be used and the lack of communication. In this letter, signum function is used to connect the bearing between agents and agents' own states errors, instead of a well-known finite-time control tool.

II. Preliminaries

A. Model

Consider a multi-agent system consisting of n agents in $\mathbb{R}^d$, the first n_l agents are leaders moving with the following model:

$$\dot{p}_i = v_r(t),\ \ i \in \{1, \dots, n_l\}, \tag{1}$$

where $p_i \in \mathbb{R}^d$ is the i-th leader's position in the global frame and $v_r(t) \in \mathbb{R}^d$ is a continuous time-varying velocity that can be specified. The remaining n_f agents are followers; in this letter, we consider both single- and double-integrator models:

$$\dot{p}_i = u_i,\ \ i \in \{n_l + 1, \dots, n\}, \tag{2}$$

$$\dot{p}_i = v_i,\ \ \dot{v}_i = u_i,\ \ i \in \{n_l + 1, \dots, n\}, \tag{3}$$

where $p_i \in \mathbb{R}^d$ and $v_i \in \mathbb{R}^d$ are the i-th agent's position and velocity, respectively, and $u_i \in \mathbb{R}^d$ is the control input.

B. Graph Theory and Formation

In this letter, we consider an undirected graph $\mathcal{G}$ with a vertex set $\mathcal{V} = \{1, \dots, n\}$ and an edge set $\mathcal{E} = \mathcal{V} \times \mathcal{V}$, where $|\mathcal{V}| = n$ and $|\mathcal{E}| = m$ mean that $\mathcal{G}$ consists of n vertices and m directed edges. The edge (i, j) means that agent i can measure the relative bearing of j; then, the neighbor set of agent i can be defined as $\mathcal{N}_i = \{j \in \mathcal{V} | (i, j) \in \mathcal{E}\}$. For the edge (i, j), we define its edge vector as $z_{ij} = p_j - p_i$; then, we can index the edges in $\mathcal{E}$ such that $\mathcal{E} = \{z_1, \dots, z_m\}$. Subsequently, the incidence matrix can be defined as $H = [h_{ki}] \in \mathbb{R}^{m\times n}$, where $h_{ki} = -1$ if vertex i is the tail of edge z_k and $h_{ki} = 1$ if vertex i is the head of edge z_k; otherwise, $h_{ki} = 0$.

We denote $p = \mathrm{col}(p_L, p_F)$ as the formation configuration with $p_L = \mathrm{col}(p_1, \dots, p_{n_l})$ and $p_F = \mathrm{col}(p_{n_l+1}, \dots, p_n)$, and the velocity vector $v = \mathrm{col}(v_L, v_F)$ can be defined in the same way.

From the above definitions, we can know that $H\mathbf{1}_n = 0$ and $\mathrm{rank}(H) = n-1$ if graph $\mathcal{G}$ is connected. In addition, with the notation $z = \mathrm{col}(z_1, \dots, z_m)$, we can get $z = (H \otimes I_d)p = \overline{H}p$. Moreover, based on the edge vector z_{ij}, we can define the bearing vector as $g_{ij} = z_{ij}/\|z_{ij}\|$, and the orthogonal projection matrix for g_{ij} is defined as $P_{g_{ij}} = I_d - g_{ij}g_{ij}^T$. We can note that $P_{g_{ij}} \in \mathbb{R}^{d\times d}$ can project a vector onto the orthogonal space of $\mathrm{im}(g_{ij})$; moreover, we know that $P_{g_{ij}}$ is positive semidefinite and $\mathrm{Null}(P_{g_{ij}}) = \mathrm{span}\{g_{ij}\}$.

Different from the target formation defined by displacement $z_{ij} = p_j - p_i$, in this letter, the target formation $p^*(t)$ is given by the interneighbor bearings and the multiple leader positions. In other words, the bearing-only target formation $p^*(t)$ fulfills the following conditions:

$$\begin{aligned} &\text{i) } g_{ij} = g^*_{ij},\ \ \forall (i, j) \in \mathcal{E} \\ &\text{ii) } \dot{p}_k = v_r(t),\ \ k \in \{1, \dots, n_l\} \end{aligned} \tag{4}$$

where g^*_{ij} denotes the desired bearing between agents i and j.

To ensure that the target formation $p^*(t)$ can be determined uniquely by conditions in (4), we utilize the bearing Laplacian matrix $\mathcal{B}$ in [11] defined as $[\mathcal{B}]_{ij} = \mathbf{0}_{d\times d}$ for $i \neq j$, $(i, j) \notin \mathcal{E}$; $[\mathcal{B}]_{ij} = -P_{g^*_{ij}}$ for $i \neq j$, $(i, j) \in \mathcal{E}$; and $[\mathcal{B}]_{ij} = \sum_{k\in\mathcal{N}_i} P_{g^*_{ik}}$ for $i = j$, $i \in \mathcal{V}$. We partition matrix $\mathcal{B} \in \mathbb{R}^{dn\times dn}$ as $\mathcal{B} = \begin{pmatrix} \mathcal{B}_{ll} & \mathcal{B}_{lf} \\ \mathcal{B}_{fl} & \mathcal{B}_{ff} \end{pmatrix}$, where $\mathcal{B}_{ff} \in \mathbb{R}^{n_f d\times n_f d}$. Then, we introduce the following lemma.

Lemma 1 [11]: The conditions in (4) ensure the uniqueness of the desired formation $p^*(t)$ if and only if the block matrix $\mathcal{B}_{ff}$ in $\mathcal{B}$ is positive definite.

C. Problem and Assumptions

Problem: For a MAS with leaders (1) moving with a time-varying velocity $v_r(t)$, we design control laws for the single-integrator (2) and double-integrator (3) followers such that $p(t) \to p^*(t)$ and $v(t) \to v^*(t)$ as $t \to \infty$, using only relative bearing vectors $\{g_{ij}, \dot{g}_{ij}\}_{j\in\mathcal{N}_i}$ and without using communication.

Remark 2: Forming and tracking a formation moving with a time-varying velocity is common and can be easily achieved using a position-based control law. However, this is nontrivial in the bearing-only control. The challenge is that we can only use relative bearing between agents. The velocity error widely used in position-based control is not available, since followers have neither access to leaders' velocity nor their own velocity. Thus, significant effort is needed to build relationship between relative bearing between agents and agents' own states errors.

We now give some necessary assumptions:

Assumption 1: The desired formation determined by conditions in (4) is unique; i.e., $\mathcal{B}_{ff}$ is positive definite.

Assumption 2: In single-integrator agent formation tracking, the leader's velocity is bounded and satisfies $\|v_r(t)\| \le \xi_v$, and in the double-integrator agent formation tracking, the leader's acceleration is bounded and satisfies $\|\dot{v}_r(t)\| \le \xi_a$.

III. Main Results

A. Single Integrators

We give the control law for single-integrator followers.

$$u_i = k_p \sum_{j\in\mathcal{N}_i}\left(g_{ij} - g_{ij}^*\right) + \vartheta \sum_{j\in\mathcal{N}_i} P_{g_{ij}^*}\mathrm{sign}\left(P_{g_{ij}^*} g_{ij}\right), \quad (5)$$

where $k_p > 0$ is the control gain, and $\vartheta > 0$ is the control parameter satisfying the following condition.

$$\vartheta \geq \sqrt{n\lambda_{\max}\left(\mathcal{B}_{ff}\right)}\xi_v / \lambda_{\min}\left(\mathcal{B}_{ff}\right) \quad (6)$$

Taking $g = \mathrm{col}(g_1, \ldots, g_m)$ and $g^* = \mathrm{col}(g_1^*, \ldots, g_m^*)$, we write (5) in a compact form as follows.

$$u = -k_p G\overline{H}^T\left(g - g^*\right) - \vartheta G\overline{H}^T \mathrm{diag}\left(P_{g_k^*}\right)\mathrm{sign}\left(\mathrm{diag}\left(P_{g_k^*}\right)g\right) \quad (7)$$

where $G = \begin{pmatrix} 0 & 0 \\ 0 & I_{dn_f}\end{pmatrix}$ and $\mathrm{diag}(P_{g_k^*}) = \mathrm{diag}\{P_{g_1^*}, \ldots, P_{g_m^*}\}$ is the block diagonal matrix. Then, we provide the following result.

Theorem 1: We consider a MAS with time-varying velocity leaders (1) and single-integrator followers (2), with the control law (5), formation tracking can be achieved using only relative bearing, without using communication or other measurements.

Proof: Consider the Lyapunov function $V = \frac{1}{2}\|\widetilde{p}\|^2$, where $\widetilde{p} = p - p^*$. Since control law (5) is discontinuous, we use the stability theory of discontinuous system in [25, Th. 2.2], which implies that $\dot{V} \in^{a.e.} \dot{\tilde{V}} = \bigcap_{\varpi\in\partial V} \varpi^T K[\dot{p}] = \nabla V^T K[\dot{p}]$. Then, we get

$$\begin{aligned}\dot{\tilde{V}} &= \widetilde{p}^T K[u] - \widetilde{p}^T\overline{v}_r \\ &= -k_p\widetilde{p}^T\overline{H}^T\left(g - g^*\right) - \widetilde{p}^T\overline{v}_r \\ &\quad - \vartheta\widetilde{p}^T\overline{H}^T\mathrm{diag}\left(P_{g_k^*}\right)K\left[\mathrm{sign}\left(\mathrm{diag}(P_{g_k^*})g\right)\right] \end{aligned} \quad (8)$$

where $\overline{v}_r = \mathbf{1}_n \otimes v_r$; the second equation in (8) results from $\widetilde{p}^T G = \widetilde{p}^T$ since $\widetilde{p}^T = [\mathbf{0}_{dn_l}^T, (p_F - p_F^*)^T]$. Moreover, we can get the following important equations, which create a relationship between relative bearings and agents' position errors by using the signum function and orthogonal projection matrix.

$$\begin{aligned}\mathrm{sign}\left(\mathrm{diag}\left(P_{g_k^*}\right)g\right) &= \mathrm{sign}\left(\mathrm{diag}\left(\|z_k\|P_{g_k^*}\right)g\right) \\ &= \mathrm{sign}\left(\mathrm{diag}\left(P_{g_k^*}\right)z\right) = \mathrm{sign}\left(\mathrm{diag}\left(P_{g_k^*}\right)\overline{H}(p - p^*)\right)\end{aligned} \quad (9)$$

where $\mathrm{diag}(\|z_k\|P_{g_k^*}) = \mathrm{diag}\{\|z_1\|P_{g_1^*}, \ldots, \|z_m\|P_{g_m^*}\}$; the first equation comes from the property of the signum function, and the last equation comes from $P_{g_k^*}z_k^* = P_{g_k^*}g_k^* = 0$.

With the property of the Filipov set $K[f(x)]$ defined in [25, Def. 2.1], we have $xK[\mathrm{sign}(x)] = \{|x|\}$, $\forall x \in \mathbb{R}$. Combining this with (9), we further calculate the last term in (8) as

$$\begin{aligned}&-\vartheta\widetilde{p}^T\overline{H}^T\mathrm{diag}\left(P_{g_k^*}\right)K\left[\mathrm{sign}\left(\mathrm{diag}(P_{g_k^*})g\right)\right] \\ &= -\vartheta\widetilde{p}^T\overline{H}^T\mathrm{diag}\left(P_{g_k^*}\right)K\left[\mathrm{sign}\left(\mathrm{diag}(P_{g_k^*})\overline{H}\widetilde{p}\right)\right] \\ &= -\vartheta\left\|\mathrm{diag}\left(P_{g_k^*}\right)\overline{H}\widetilde{p}\right\|_1 = -\vartheta\left\|\mathrm{diag}\left(P_{g_k^*}\right)\overline{M}(p_F - p_F^*)\right\|_1\end{aligned} \quad (10)$$

where $\overline{M} = M \otimes I_d$ and $M \in \mathbb{R}^{m\times n_f}$ is composed of the last n_f columns of H; i.e., $H = [\circ\, |M]$. Thus, we know that the last equation comes from $(p - p^*) = [\mathbf{0}_{dn_l}^T, (p_F - p_F^*)^T]^T$. Moreover, we know that $\mathcal{B} = \overline{H}^T\mathrm{diag}(P_{g_k^*}^T)\mathrm{diag}(P_{g_k^*})\overline{H}$ from [11, Lemma 2]. Combining this with the definition of $\mathcal{B}_{ff}$ in Section II, we have

$$\overline{M}^T\mathrm{diag}\left(P_{g_k^*}^T\right)\mathrm{diag}\left(P_{g_k^*}\right)\overline{M} = \mathcal{B}_{ff}. \quad (11)$$

In addition, we give the following lemma.

Lemma 2 [12]: With no agents coinciding, we have

$$\widetilde{p}^T\overline{H}^T\left(g - g^*\right) \geq p^T\overline{H}^T\left(g - g^*\right) \geq \frac{\lambda_{\min}\left(\mathcal{B}_{ff}\right)}{2\max_k\|z_k\|}\|\widetilde{p}\|^2 \geq 0. \quad (12)$$

Based on (9), (10), (11), and Lemma 2, we have

$$\begin{aligned}\dot{V} &\leq -k_p\widetilde{p}^T\overline{H}^T\left(g - g^*\right) + \|\overline{v}_r\|_2\|\widetilde{p}\|_2 \\ &\quad - \vartheta\left\|\mathrm{diag}\left(P_{g_k^*}\right)\overline{M}(p_F - p_F^*)\right\|_2 \\ &\leq -k_p\widetilde{p}^T\overline{H}^T\left(g - g^*\right) + \|\overline{v}_r\|_2\|\widetilde{p}\|_2 \\ &\quad - \vartheta\frac{\left\|\overline{M}^T\mathrm{diag}\left(P_{g_k^*}\right)\mathrm{diag}\left(P_{g_k^*}\right)\overline{M}(p_F - p_F^*)\right\|_2}{\left\|\overline{M}^T\mathrm{diag}\left(P_{g_k^*}\right)\right\|_2} \\ &\leq -\frac{k_p\lambda_{\min}\left(\mathcal{B}_{ff}\right)}{2\max_k\|z_k\|}\|\widetilde{p}\|_2^2 + \sqrt{n}\xi_v\|\widetilde{p}\|_2 - \frac{\vartheta\lambda_{\min}\left(\mathcal{B}_{ff}\right)}{\sqrt{\lambda_{\max}\left(\mathcal{B}_{ff}\right)}}\left\|\left(p_F - p_F^*\right)\right\|_2 \\ &\leq -k_p\frac{\lambda_{\min}\left(\mathcal{B}_{ff}\right)}{2\max_k\|z_k\|}\|\widetilde{p}\|_2^2 - \left(\vartheta\frac{\lambda_{\min}\left(\mathcal{B}_{ff}\right)}{\sqrt{\lambda_{\max}\left(\mathcal{B}_{ff}\right)}} - \sqrt{n}\xi_v\right)\|\widetilde{p}\|_2\end{aligned} \quad (13)$$

where the third inequality arises from Assumption 1; i.e., $\mathcal{B}_{ff}$ is positive definite and $\|\overline{M}^T\mathrm{diag}(P_{g_k^*})\|_2 = \sqrt{\lambda_{\max}(\mathcal{B}_{ff})} \neq 0$.

Then, we can conclude that $\dot{V} \leq 0$ if ϑ satisfies condition (6), which implies that $p \to p^*$ and ends the proof. ■

Moreover, collision avoidance is necessary and we provide a sufficient condition for this along a similar line as in [12].

Proposition 1: If initial states of agents satisfy $\|\widetilde{p}(0)\| \leq \kappa := \frac{1}{\sqrt{n}}(\min_{i,j\in\mathcal{V}}\|p_i^* - p_j^*\| - \iota)$, we conclude that $\|p_i - p_j\| \geq \iota, \forall i, j \in \mathcal{V}$ always holds, and thus no collision occurs between agents, where ι is the inter-agent distance assigned to avoid collisions.

Proof: Based on (13) we know that $\dot{V} \leq 0$; thus $\|\widetilde{p}(0)\| \leq \kappa$ implies $\|\widetilde{p}(t)\| \leq \kappa$ for $t \geq 0$. Also, we have $\|p_i(t) - p_j(t)\| = \|(p_i^* - p_j^*) + (p_i(t) - p_i^*) - (p_j(t) - p_j^*)\| \geq \|p_i^* - p_j^*\| - \|p_i(t) - p_i^*\| - \|p_j(t) - p_j^*\| \geq \|p_i^* - p_j^*\| - \sum_{k=1}^n\|p_k(t) - p_k^*\| \geq \|p_i^* - p_j^*\| - \sqrt{n}\|\widetilde{p}(t)\|$. Then, combining the definition of κ in Proposition 1, we can further obtain $\|p_i(t) - p_j(t)\| \geq \|p_i^* - p_j^*\| - \min_{i,j\in\mathcal{V}}\|p_i^* - p_j^*\| + \iota \geq \iota$ and thus end the proof. ■

B. Double Integrators

When it comes to double integrator, the terms in (5) become invalid since the control input changes from velocity input to acceleration input. Also, the relationship mentioned in Remark 2 needs to be recreated. We give the following control law.

$$u_i = k_p\sum_{j\in\mathcal{N}_i}\left(g_{ij} - g_{ij}^*\right) + k_v\sum_{j\in\mathcal{N}_i}\dot{g}_{ij} + \alpha\sum_{j\in\mathcal{N}_i}P_{g_{ij}}\mathrm{sign}\left(\dot{g}_{ij}\right) \quad (14)$$

where $k_p > 0$ and $k_v > 0$, and $\alpha > 0$ is the control parameter that satisfies the condition given in (22).

We rewrite the control law (14) in the compact form.

$$u = -k_pG\overline{H}^T\left(g - g^*\right) - k_vG\overline{H}^T\dot{g} - \alpha G\overline{H}^T\mathrm{diag}\left(P_{g_k}\right)\mathrm{sign}(\dot{g}) \quad (15)$$

where $\mathrm{diag}(P_{g_k}) = \mathrm{diag}\{P_{g_1}, \dots, P_{g_m}\}$ is a block diagonal matrix.

Theorem 2: We consider a MAS with time-varying velocity leaders (1) and double-integrator followers (3). With control law (14), formation tracking can be achieved using only bearing measurements and without using communication.

Proof: Given that $p^T\overline{H}^T(g - g^*) > 0$ as indicated in Lemma 2, we propose a Lyapunov function as follows.

$$V = k_p p^T\overline{H}^T\big(g - g^*\big) + \frac{1}{2}\widetilde{v}^T\widetilde{v} \tag{16}$$

where $\widetilde{v} = v - \bar{v}_r(t)$. We know $\dot{V} \in^{a.e.} \dot{\tilde{V}}$, and $\dot{\tilde{V}}$ is given by

$$\begin{aligned}\dot{\tilde{V}} &= k_p\dot{p}^T\overline{H}^T\big(g - g^*\big) + k_p z^T\dot{g} + \widetilde{v}^T\dot{\widetilde{v}} \\ &= k_p\big(g - g^*\big)^T\overline{H}v + \widetilde{v}^T\big(K[u] \ - \dot{\bar{v}}_r\big)\end{aligned} \tag{17}$$

where the second equation in (17) is due to $z_{ij}^T\dot{g}_{ij} = 0$ [12]. In addition, we have $\overline{H}v = \overline{H}\widetilde{v}$ since $\overline{H}\bar{v}_r = H\mathbf{1}_n \otimes v_r = \mathbf{0}_{md}$. Then, (17) can be further calculated as

$$\begin{aligned}\dot{\tilde{V}} &= k_p\big(g - g^*\big)^T\overline{H}v + \widetilde{v}^T K[u] \ - \widetilde{v}^T\dot{\bar{v}}_r \\ &= k_p\big(g - g^*\big)^T\overline{H}v - k_p\big(g - g^*\big)^T\overline{H}\widetilde{v} - k_v\widetilde{v}^T\overline{H}^T\dot{g} \\ &\quad - \widetilde{v}^T\dot{\bar{v}}_r - \alpha\widetilde{v}^T\overline{H}^T\mathrm{diag}\big(P_{g_k}\big)K\big[\mathrm{sign}(\dot{g})\big] \\ &= -k_v v^T\overline{H}^T\dot{g} - \widetilde{v}^T\dot{\bar{v}}_r - \alpha\widetilde{v}^T\overline{H}^T\mathrm{diag}\big(P_{g_k}\big)K\big[\mathrm{sign}(\dot{g})\big]\end{aligned} \tag{18}$$

where the second equation comes from $\widetilde{v}^T G = \widetilde{v}^T$ since $\widetilde{v} = [\mathbf{0}_{dn_l}^T, \widetilde{v}_F^T]^T$, with $\widetilde{v}_F = v_F - \mathbf{1}_{n_f} \otimes v_r$.

Furthermore, according to the face that $\dot{g}_{ij} = P_{g_{ij}}\dot{z}_{ij}/\|z_{ij}\|$, we obtain the following equations, which establish a relationship between relative bearings and agents' velocity errors by using the signum function and orthogonal projection matrix.

$$\begin{aligned}\mathrm{sign}(\dot{g}) &= \mathrm{sign}\big(\mathrm{diag}\big(P_{g_k}/\|z_k\|\big)\dot{z}\big) = \mathrm{sign}\big(\mathrm{diag}\big(P_{g_k}\big)\dot{z}\big) \\ &= \mathrm{sign}\big(\mathrm{diag}\big(P_{g_k}\big)\overline{H}(\dot{p} - \bar{v}_r)\big) = \mathrm{sign}\big(\mathrm{diag}\big(P_{g_k}\big)\overline{H}\widetilde{v}\big)\end{aligned} \tag{19}$$

where $\mathrm{diag}(P_{g_k}/\|z_k\|) = \mathrm{diag}\{P_{g_1}/\|z_1\|, \dots, P_{g_m}/\|z_m\|\}$ is a block diagonal matrix. Thus, the last term in (18) can be given by

$$\begin{aligned}&-\alpha\widetilde{v}^T\overline{H}^T\mathrm{diag}\big(P_{g_k}\big)K\big[\mathrm{sign}(\dot{g})\big] \\ &= -\alpha\widetilde{v}^T\overline{H}^T\mathrm{diag}\big(P_{g_k}\big)K\big[\mathrm{sign}\big(\mathrm{diag}\big(P_{g_k}\big)\overline{H}\widetilde{v}\big)\big] \\ &= -\alpha\big\|\mathrm{diag}\big(P_{g_k}\big)\overline{H}\widetilde{v}\big\|_1 = -\alpha\big\|\mathrm{diag}\big(P_{g_k}\big)\overline{M}\widetilde{v}_F\big\|_1\end{aligned} \tag{20}$$

where the last equation results from $\widetilde{v} = [\mathbf{0}_{dn_l}^T, \widetilde{v}_F^T]^T$. In addition, since $\mathrm{rank}(H) = n-1$ and $n_l > 2$, we know that $M \in \mathbb{R}^{m\times n_f}$ is a column full-rank matrix, i.e., $\mathrm{rank}(M) = n_f$. Based on this, we then provide the following important proposition.

Proposition 2: $\overline{M}^T\mathrm{diag}(P_{g_k})\mathrm{diag}(P_{g_k})\overline{M}$ is positive definite, and $\|\overline{M}^T\mathrm{diag}(P_{g_k})\|_2 \neq 0$.

Proof: We first prove $\overline{M}^T\mathrm{diag}(P_{g_k})\mathrm{diag}(P_{g_k})\overline{M}$ is positive definite. Note that $\overline{M}^T\mathrm{diag}(P_{g_k})\mathrm{diag}(P_{g_k})\overline{M}$ is positive semidefinite since A^TA is positive semidefinite for any $A \in \mathbb{R}^{m\times n}$; i.e., $x\overline{M}^T\mathrm{diag}(P_{g_k})\mathrm{diag}(P_{g_k})\overline{M}x \geq 0$ for any nonzero vector $x \in \mathbb{R}^{dn_f}$. In this way, if we demonstrate that there exists no nonzero vector x such that $x\overline{M}^T\mathrm{diag}(P_{g_k})\mathrm{diag}(P_{g_k})\overline{M}x = 0$, we can conclude that $\overline{M}^T\mathrm{diag}(P_{g_k})\mathrm{diag}(P_{g_k})\overline{M}$ is positive definite.

Along this line, we can assume that there exists a nonzero vector $y \in \mathbb{R}^{dn_f}$ which makes $y\overline{M}^T\mathrm{diag}(P_{g_k})\mathrm{diag}(P_{g_k})\overline{M}y = 0$, from which we get $\mathrm{diag}(P_{g_k})\overline{M}y = \mathbf{0}_{md}$. Note that $\mathrm{diag}(P_{g_k})\overline{M}y = \mathbf{0}_{md}$ only holds for the following two cases since $\mathrm{Null}(P_{g_{ij}}) = \mathrm{span}\{g_{ij}\}$. Case 1: $\overline{M}y = \mathbf{0}_{md}$; Case 2: $\overline{M}y = (K \otimes I_d)z = \overline{K}z$, where $K = \mathrm{diag}\{k_1, \dots, k_m\} \in \mathbb{R}^{m\times m}$ with $k_i \in \mathbb{R} \neq 0$. For Case 1, we can see that the equation $\overline{M}y = \mathbf{0}_{md}$ has only the zero solution, i.e., $y = \mathbf{0}_{dn_f}$, since coefficient matrix $\overline{M}$ is column full-rank and $\mathrm{rank}(\overline{M}) = dn_f$, which contradicts the fact that y is a nonzero vector.

For Case 2, we focus on showing that equation $\overline{M}y = \overline{K}z$ is unsolvable. To simplify the analysis, we let $d = 1$ and note that the following discussion can be extended to any $d \in \mathbb{N}^*$ by using the Kronecker product and block matrices. Consider $\overline{H} = [\beta_1, \dots, \beta_{n_l}, \beta_{n_l+1}, \dots, \beta_n]$, since $\mathrm{rank}(\overline{H}) = n-1$, without loss of generality, we consider $\beta_2, \beta_3, \dots, \beta_n$ as a maximal linearly independent group of $\overline{H}$ and we get that there exist nonzero constants $a_2, \dots, a_n, a_{n+1}$ such that $a_2\beta_2 + \cdots + a_n\beta_n + a_{n+1}z = 0$ since $\overline{H}p = z$ and $p_i \neq p_j$, $\forall i, j$. Since $n_l \geq 2$, $p_i \neq p_j$, $\forall i, j$, and $k_i \neq 0$, we can further conclude that $\beta_{n_l+1}, \dots, \beta_n, \overline{K}z$ is linearly independent. Thus, we can get that $\mathrm{rank}([\overline{M}|\overline{K}z]) = \mathrm{rank}([\beta_{n_l+1}, \dots, \beta_n, \overline{K}z]) = \mathrm{rank}(\overline{M}) + 1$. Therefore, the equation $\overline{M}y = \overline{K}z$ is unsolvable. From the above analyses for Cases 1 and 2, we can see that $x\overline{M}^T\mathrm{diag}(P_{g_k})\mathrm{diag}(P_{g_k})\overline{M}x > 0$ for any nonzero vector x, which implies that $\overline{M}^T\mathrm{diag}(P_{g_k})\mathrm{diag}(P_{g_k})\overline{M}$ is positive definite. Meanwhile, we can obtain $\|\overline{M}^T\mathrm{diag}(P_{g_k})\|_2 = \sqrt{\lambda_{\max}(\overline{M}^T\mathrm{diag}(P_{g_k})\mathrm{diag}(P_{g_k})\overline{M})} > 0$ and end the proof. ∎

According to (20) and Proposition 2, we can get

$$\begin{aligned}\dot{V} &\leq -k_v v^T\overline{H}^T\dot{g} - \alpha\big\|\mathrm{diag}\big(P_{g_k}\big)\overline{M}\widetilde{v}_F\big\|_2 + \big\|\dot{\bar{v}}_r\big\|_2\|\widetilde{v}\|_2 \\ &\leq -k_v\dot{z}^T\dot{g} - \frac{\alpha\Big\|\overline{M}^T\mathrm{diag}\big(P_{g_k}\big)\mathrm{diag}\big(P_{g_k}\big)\overline{M}\widetilde{v}_F\Big\|_2}{\Big\|\overline{M}^T\mathrm{diag}\big(P_{gk}\big)\Big\|_2} + \big\|\dot{\bar{v}}_r\big\|_2\|\widetilde{v}\|_2 \\ &\leq -k_v\dot{z}^T\dot{g} + \sqrt{n}\xi_a\|\widetilde{v}\|_2 \\ &\quad - \frac{\alpha\lambda_{\min}\Big(\overline{M}^T\mathrm{diag}\big(P_{g_k}\big)\mathrm{diag}\big(P_{g_k}\big)\overline{M}\Big)}{\Big\|\overline{M}^T\mathrm{diag}\big(P_{gk}\big)\Big\|_2}\|\widetilde{v}\|_2 \\ &\leq -k_v\dot{z}^T\mathrm{diag}\big(P_{g_k}/\|z_k\|_2\big)\dot{z} \\ &\quad - \left(\frac{\alpha\lambda_{\min}\Big(\overline{M}^T\mathrm{diag}\big(P_{g_k}\big)\mathrm{diag}\big(P_{g_k}\big)\overline{M}\Big)}{\Big\|\overline{M}^T\mathrm{diag}\big(P_{gk}\big)\Big\|_2} - \sqrt{n}\xi_a\right)\|\widetilde{v}\|_2\end{aligned} \tag{21}$$

Note that $-k_v\dot{z}^T\mathrm{diag}(P_{g_k}/\|z_k\|_2)\dot{z} \leq 0$ since P_{g_k} is positive semidefinite. Then, we can conclude that $\dot{V} \leq 0$ if α satisfies

$$\alpha \geq \frac{\sqrt{n}\xi_a\Big\|\overline{M}^T\mathrm{diag}\big(P_{g_k}\big)\Big\|_2}{\lambda_{\min}\Big(\overline{M}^T\mathrm{diag}\big(P_{g_k}\big)\mathrm{diag}\big(P_{g_k}\big)\overline{M}\Big)} \tag{22}$$

In this way, we can conclude that $p^T\overline{H}^T(g - g^*)$ and $\|\widetilde{v}\|$ are bounded since $V > 0$ and $\dot{V} \leq 0$. According to Lemma 2, $p^T\overline{H}^T(g - g^*)$ is bounded means that $\|\widetilde{p}\|$ is bounded. Then, there is a compact set of $\widetilde{v}$ and $\widetilde{p}$ that is invariant under the error dynamics. We can know that $\widetilde{v}$ and $\widetilde{p}$ converge to this compact set when $\dot{V} = 0$, which is equivalent to $\dot{g}_k = 0$, $\widetilde{v} = 0$. We further analyze this condition and find that there are two cases. Case 1: $\dot{g}_k = 0$ and g is equal to g^*; then, Theorem 2 is proved. Case 2: $\dot{g}_k = 0$ and g is not equal to g^*; then, the $-k_pG\overline{H}^T(g - g^*)$ in control law (15) is a nonzero constant vector, which will continue to change v. Thus, the $\widetilde{v}$ will increase to infinity, which contradicts $\widetilde{v} = 0$ in the condition. Therefore, we can conclude that only Case 1 is appropriate; thus, we end the proof. ■

Remark 3: The ideas in (13) and (21) seem similar. However, this is the final step of the proof, and their antecedent foundations are quite different. In particular, (9) is technically different from (19), and an effort is needed to prove the positive definiteness of $\overline{M}^T \mathrm{diag}(P_{g_k})\mathrm{diag}(P_{g_k})\overline{M}$ in Theorem 2.

Similarly, we provide the condition for collision avoidance.

Proposition 3: If the Lyapunov function (16) satisfies $V(0) \leq k_p\omega$, with ω being a positive constant selected to satisfy $(\eta\omega + \sqrt{\eta^2\omega^2 + 4\eta\omega\|\widetilde{p}^*\|})/2 \leq \kappa$, where $\eta = 2\|\overline{H}\|/\lambda_{\min}(\mathcal{B}_{ff})$, $\widetilde{p}^* = p^* - \mathbf{1}_n \otimes (\sum_{i=1}^n p_i^*/n)$, and κ is defined in Proposition 1, then we can conclude $\|p_i - p_j\| \geq \iota$, $\forall i, j \in \mathcal{V}$ for all time, and thus no collision occurs.

Proof: We can first get $V(t) \leq k_p\omega$ since $V(0) \leq k_p\omega$ and $\dot{V} \leq 0$, which implies that $p^T\overline{H}^T(g - g^*) \leq \omega$ according to (16). Moreover, [12, Corollary 2] shows that $\|\widetilde{p}\|^2/\eta(\|\widetilde{p}\| + \|\widetilde{p}^*\|) \leq p^T\overline{H}^T(g - g^*)$. Thus, we can get $\|\widetilde{p}\|^2 \leq \eta\omega(\|\widetilde{p}\| + \|\widetilde{p}^*\|)$, which indicates that $\|\widetilde{p}(t)\| \leq (\eta\omega + \sqrt{\eta^2\omega^2 + 4\eta\omega\|\widetilde{p}^*\|})/2 \leq \kappa$. Then, the rest of proof can proceed as in Proposition 1 and is omitted.

Remark 4: We notice that conditions in (6) and (22) contain some interaction topology information, i.e., $\mathcal{B}_{ff}$ and incidence matrix H. Since they need to be determined before control law design in order to meet Assumption 1, we consider them to be a priori knowledge that can be accessed by agents in advance.

C. Extension to Models With Disturbances

For further use, disturbance needs to be considered. Single and double integrators with matching disturbance are given by

$$\dot{p}_i = u_i + \Delta_i, \tag{23}$$

$$\dot{p}_i = v_i, \;\; \dot{v}_i = u_i + \Delta_i, \tag{24}$$

where Δ_i is the disturbance. Suppose that Δ_i is bounded and satisfies $\|\Delta_i\|_2 \leq \xi_\Delta$, we have the following conclusion.

Theorem 3: For a MAS in which followers are modeled by single integrators with disturbances (23), time-varying formation tracking can be achieved under control law (5) with parameter ϑ satisfying $\vartheta \geq \sqrt{n\lambda_{\max}(\mathcal{B}_{ff})}(\xi_v + \xi_\Delta)/\lambda_{\min}(\mathcal{B}_{ff})$.

Proof: Consider the Lyapunov function $V = \frac{1}{2}\|\widetilde{p}\|^2$; tacking $\Delta_F = \mathrm{col}(\Delta_{n_l+1}, \ldots, \Delta_n)$, we can obtain

$$\begin{aligned}\dot{\widetilde{V}} = &-k_p\widetilde{p}^T\overline{H}^T\left(g - g^*\right) - \widetilde{p}^T\overline{v}_r + \left(p_F - p_F^*\right)^T\Delta_F \\ &- \vartheta\widetilde{p}^T\overline{H}^T\mathrm{diag}\left(P_{g_k^*}\right)K\left[\mathrm{sign}\left(\mathrm{diag}(P_{g_k^*})g\right)\right]\end{aligned} \tag{25}$$

Following the similar analysis in Theorem 1, we have

$$\dot{V} \leq -\frac{k_p\lambda_{\min}\left(\mathcal{B}_{ff}\right)}{2\max_k\|z_k\|_2}\|\widetilde{p}\|_2^2 - \left(\frac{\vartheta\lambda_{\min}\left(\mathcal{B}_{ff}\right)}{\sqrt{\lambda_{\max}\left(\mathcal{B}_{ff}\right)}} - \sqrt{n}(\xi_v + \xi_\Delta)\right)\|\widetilde{p}\|_2$$

Thus, we can conclude that $\dot{V} \leq 0$ if ϑ satisfies the condition in Theorem 3, which means $p \to p^*$ and ends the proof. ■

Along a similar method, we arrive at the below conclusion.

Corollary 1: For a MAS in which followers are modeled by double integrators with disturbances (24), time-varying formation tracking can be achieved under control law (14) with α satisfying

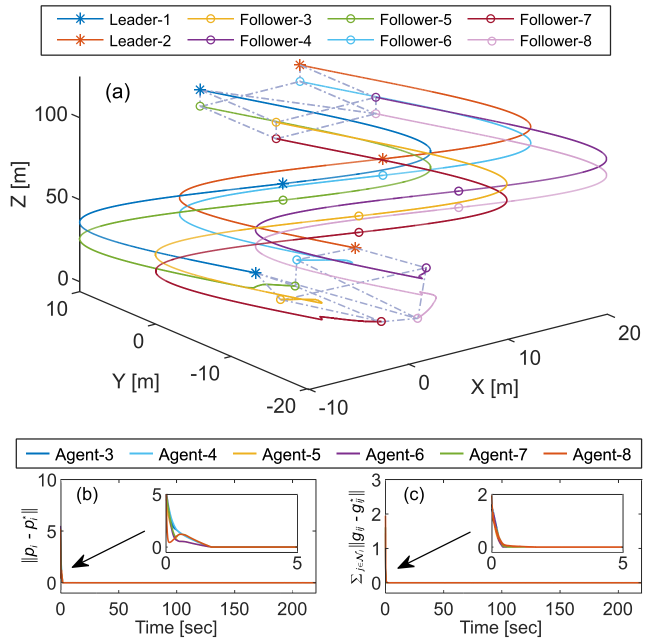

Fig. 1. Single integrators. (a) Tracking trajectories, (b) position errors, (c) bearing errors.

$$\alpha \geq \frac{\sqrt{n}(\xi_a + \xi_\Delta)\left\|\overline{M}^T\mathrm{diag}\left(P_{g_k}\right)\right\|_2}{\lambda_{\min}\left(\overline{M}^T\mathrm{diag}\left(P_{g_k}\right)\mathrm{diag}\left(P_{g_k}\right)\overline{M}\right)} \tag{26}$$

IV. Simulation Results

We present two simulation examples to verify the validity of the proposed control laws. The MAS used in the examples consists of eight agents, with two leaders denoted as $\{1, 2\}$, and six followers designated as $\{3, \ldots, 8\}$. The desired formation is a cube as in [12, Fig. 1(c)], with the bearing constraints given by $g_{34}^* = g_{56}^* = g_{78}^* = [1, 0, 0]^T$, $g_{51}^* = g_{62}^* = g_{73}^* = g_{84}^* = [0, 0, 1]^T$, $g_{31}^* = g_{42}^* = g_{75}^* = g_{86}^* = [0, 1, 0]^T$, and $g_{81}^* = -\frac{1}{\sqrt{3}} \times [1, -1, -1]^T$. The initial positions and velocities are set to be $p_3(0) = [-3, -7, 12]^T + \delta_1$, $p_4(0) = [11, -5, 6]^T + \delta_2$, $p_5(0) = [2, -2, 3]^T + \delta_3$, $p_6(0) = [6, 3, -2]^T + \delta_4$, $p_7(0) = [2, -13, 3]^T + \delta_5$, $p_8(0) = [6, -12, -3]^T + \delta_6$, and $v_i(0) = \mathbf{0}_3 + \delta_7$, $i = 3, \ldots, 8$, where $\delta_i = [\sigma_{i,1}, \sigma_{i,2}, \sigma_{i,3}]^T$ with $\sigma_{i,j}$ being the noise random series in $(-2, 2)$. The disturbance is introduced as $\Delta_i = [0.11\sin(0.5t), 0.12\cos(0.6t), 0.18\sin(0.85t)]^T$.

A. Simulation 1: Single Integrators

The time-varying reference velocity is set to $v_r = [-0.3 \times \sin(0.03t + \pi/2), 0.3\cos(0.03t), 0.5]^T$, and control gain is set to $k_p = 10$. The results are presented in Fig. 1. We see that the single-integrator followers can track the time-varying velocity leaders with tracking errors converge to zero.

B. Simulation 2: Double Integrators

The time-varying velocity is set to $v_r = [0.35, 0.3\cos(0.03t), 0.3]^T$ and control gains are $k_p = 30$, $k_v = 15$. The results are presented in Fig. 2. We can see that the followers can track the time-varying velocity leaders with position and velocity errors converge to zero, and Fig. 2(e) shows that the constraint on parameter α in (22) is bounded.

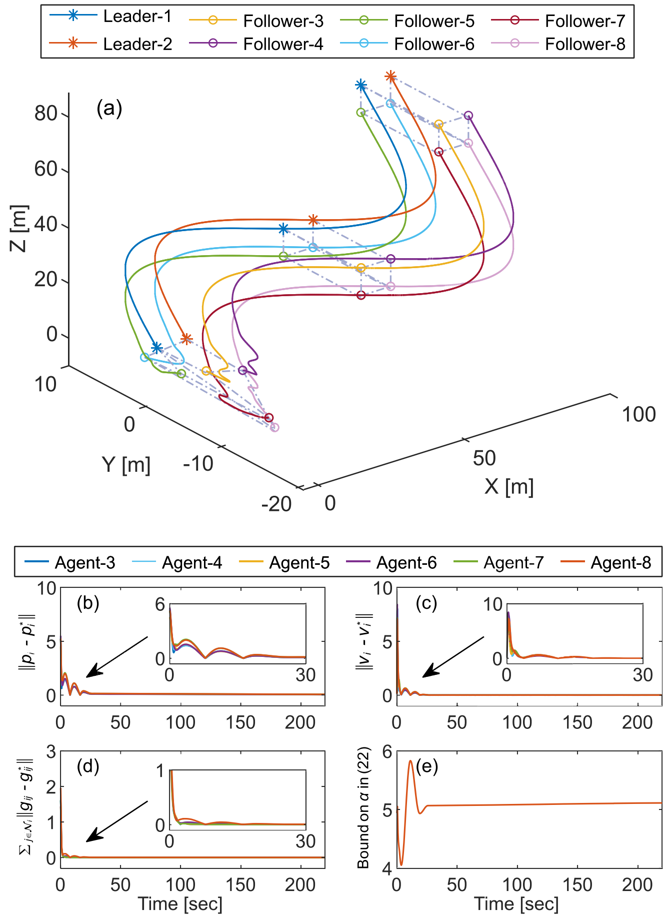

Fig. 2. Double integrators. (a) Tracking trajectories, (b) position errors, (c) velocity errors, (d) bearing errors, (e) bound constraint on α in (22).

V. Conclusion

In this letter, we focused on bearing-only formation tracking control problem and removed its restriction that the leaders' velocity can only be a constant. Two control laws have been proposed to address this topic for single and double-integrator agents, using only bearing measurements and without communication between agents. In addition, these two methods can be extended to deal with integrators with bounded disturbance with only minor changes. Moreover, sufficient conditions for collision avoidance are provided. Achieving these goals using bearing measurements in a local framework would be an interesting topic. Also, some interesting research directions include studying the models with unmatching disturbance and avoiding using the knowledge of interaction topology.

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