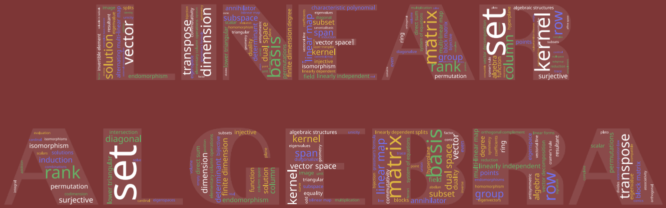

Wavelet Encoding

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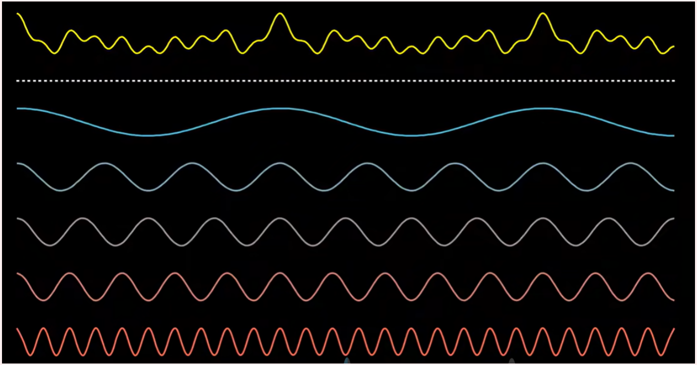
1. Background Information

Fourier Transform

$F(\omega) = \int_{-\infty}^{\infty} f(t) \cdot e^{-i\omega t}\, dt$:

- convert a function from the time domain to the frequency domain
- base function : sine and cosine



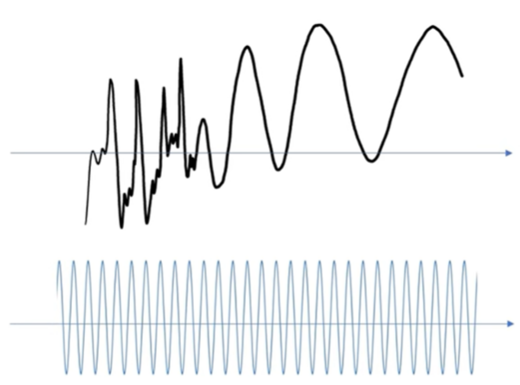
Fourier Transform

Advantage:

- provides information about the frequency components of a signal
- linearity makes it easy to analyze and process complex signals

Disadvantage:

- cannot provide precise time resolution for narrow pulse signals



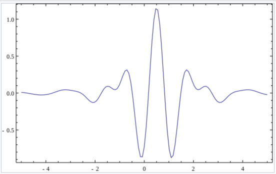
Wavelet Transform

Goal:

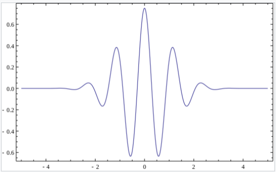
- better represent signal with abrupt change
- provide more precise time resolution

Solution:

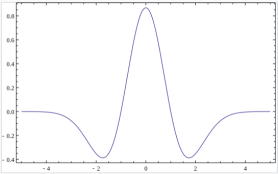
- change base functions to both time-localized and frequency-localized wavelet functions



Meyer



Morlet



Mexican hat

Limitation of Wavelet Transform

Time-Frequency Uncertainty:

- dilemma in signal processing where it is impossible to precisely determine both the time and frequency information of a signal

2. Wavelet transform in compression

Use Harr wavelet to compress a matrix

For a matrix $A \in M_8$,

$$H_1 = \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 & 0 & -\frac{1}{2} \end{pmatrix}$$

$$H_2 = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Use Harr wavelet to compress a matrix

$$H_3 = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & -\frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$H = H_1 H_2 H_3, B = H^T A H$$

Use Harr wavelet to compress a matrix

Choose a ϵ and turn every element in the matrix smaller than it into 0.
Denote this new matrix as B', $A' = (H^T)^{-1}B'(H)^{-1}$
In this way, we successfully compress matrix A.

If H_1, H_2, H_3 is turned into orthogonal matrixes,
the compression will be of better quality.

3. Experiment

Experiment Target

Utilize wavelet encoding method the compress image.

Simplification: In the experiment, we first convert the sample images into grayscale before compressing them. This approach reduces the amount of data and simplifies the processing, thereby enhancing the efficiency of compression.

Metrics for measuring the degree of image compression:

1. Comparison of image size before and after compression
2. Comparison of image file size before and after compression
3. Peak signal-to-noise ratio (PSNR)

Procedure

1. **Initialization**: The original image is loaded into the program from a file. If the image is in color, it may be converted to grayscale to simplify the compression process.
2. **Preprocessing**: The image is processed row by row and column by column, applying the Haar wavelet transform. This step decomposes the image into a set of wavelet coefficients that represent the image in the wavelet domain.
3. **Quantization and Thresholding**: Wavelet coefficients are modified to compress the image data. Coefficients below a certain threshold are set to zero, effectively reducing the amount of data.
4. **Inverse Wavelet Transform**: An inverse wavelet transform is applied to the modified coefficients to reconstruct a compressed version of the original image.
5. **Performance Analysi**: After compression, metrics can be calculated to evaluate the quality and efficiency of the compression.

Codes for Wavelet Transformation

breaklines

```
function [approx, detail] = haar_wavelet_transform(signal)
    % Haar wavelet transformation function for one-dimensional signals

    N = length(signal);
    if mod(N, 2) ~= 0
        error('The length of the signal must be even.');
    end

    % Initialize coefficients
    approx = zeros(1, N/2);
    detail = zeros(1, N/2);

    % Haar filters
    for i = 1:2:N-1
        approx((i+1)/2) = (signal(i) + signal(i+1)) / sqrt(2);
        detail((i+1)/2) = (signal(i) - signal(i+1)) / sqrt(2);
    end
end
```

Codes for Inversed Wavelet Transformation

```
breaklines
function signal = ihaar_wavelet_transform(approx, detail)
    % Inverse Haar wavelet transformation for one-dimensional signals

    % Approx and Detail must be the same length
    if length(approx) ~= length(detail)
        error('Approximation and detail coefficients must be of the same length.
    end

    N = length(approx) * 2;
    signal = zeros(1, N);

    % Reconstruct the signal from approximation and detail coefficients
    for i = 1:length(approx)
        signal(2*i-1) = (approx(i) + detail(i)) / sqrt(2);
        signal(2*i) = (approx(i) - detail(i)) / sqrt(2);
    end
end
```

Results

Original image

Image Size: 65536 bytes
Image File Size: 43632 bytes

Threshold: 20

PSNR value: 33.4615 dB
Image Memory Size: 65536 bytes
Image File Size: 25706 bytes

Results

Threshold: 40

PSNR value: 29.6200 dB
Image Memory Size: 65536 bytes
Image File Size: 20288 bytes

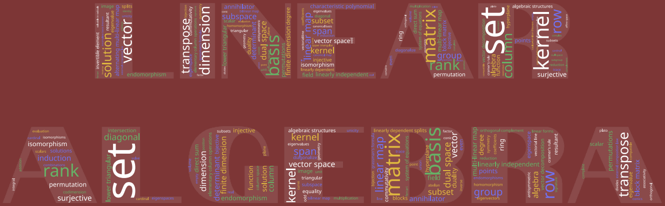
Threshold: 100

PSNR value: 22.1378 dB
Image Memory Size: 65536 bytes
Image File Size: 15667 bytes

Reference:

1. Wavelat Transformation, Wikipedia
2. Haar Wavelat, Wikipedia

Complete code can be found in the following Github repository


vector
solution
endomorphism
transpose
dimension
annihilator
subspace
basis
dual space
characteristic polynomial
subset
span
vector space
kernel
isomorphism
linearly independent
matrix
rank
group
permutation
set
kernel
row
column
algebraic structures
surjective
isomorphism
rank
induction
permutation
surjective
set
diagonal
injective
dimension
determinant
finite dimension
endomorphism
function
solution
column
algebraic structures
kernel
span
vector space
kernel
linearly dependent splits
linear map
matrix
basis
dual space
vector
annihilator
subset
linearly independent
group
row
algebra
transpose
block matrix

Thank you!