Team Control Number

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Problem Chosen

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Summary

In order to improve road capacity and operation efficiency, a series of measures must be taken to remove snow and ice from road surface. The work of removing snow and ice in winter snowfall has become one of the most important tasks of the road maintenance department. All kinds of risks associated with the construction under harsh environment, as well as the delay caused by the amount of work and people's physical and mental fatigue to the resumption of traffic operation, are unavoidable problems at present. From the perspective of mathematical modeling, it will bring remarkable social and economic benefits to discuss the way to shorten the construction period reasonably.

For the first problem, instead of solving it strictly, we directly find the minimum spanning tree of the original graph and then find a circle in the graph. In this way, task assignment can be carried out through the closed circle in the figure, and the current policy can be flexibly changed according to the bifurcation information of the minimum spanning tree.

For the second problem, we mainly borrowed MATLAB functions. Meanwhile, by discussing the value of n and λ and conducting a series of lengthy calculations, we got the optimal solution in the relative sense, and provided a very convenient way for the sensitivity analysis of the model.

For the third question, we used the gray prediction method, respectively through verhulst and DGM(2,1) model, and compared and optimized their prediction results, and finally got the appropriate results.

In the fourth problem, we skillfully transformed the priority into the probability matrix in the form of negative logarithm, and at the same time, we used the minimum spanning tree in the first problem to provide the optimal solution from the perspective

of humanity. In our opinion, it is important to reduce the number of task assignments switching, because physical and mental fatigue is an important factor that restricts project rate.

Key word: ***Minimum spanning tree*** ***Linear programming*** ***Probability matrix and maximum reliability path*** ***The physical and mental fatigue***

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1. Introduction

1.1 Background

Recently, northern China saw a large-scale heavy snowfall. In some areas, the snowfall exceeds 20cm. The frequent occurrence of extreme weather in the world indicates that the probability of heavy snow in the future is increasing sharply. The large-scale heavy snow has caused a lot of inconveniences, especially to those areas in China with a high level of urbanization. The road blockage caused by the delay in snow removal has seriously affected the daily life of residents. Among the many places, the city of Changchun, for example, has suspended work and school due to the heavy snow.

In order to ensure the traffic order of the city and the safety of people's travel, we need to consider all aspects of the problem.Dealing with snow is a big project that seems to require only "moving the snow away". In fact, in addition to traffic conditions, consideration should be given to the allocation of snow, changes in the weather and the tiredness of the staff. In addition to dealing with all kinds of safety accidents, we should also consider the problem of power supply and water supply.

Among them, the most important is the priority of roads to deal with snow. The difference of urban road distribution and priority directly determines the change of optimal allocation and the problem of road icing on the safety level. In addition to dealing with snow in a timely manner during the specified time when there is less passenger flow (staggering the rush hour), it is also necessary to consider the physical and mental health of the staff and maintain the best work efficiency.

2. Problem analysis

2.1 Analysis of question one

When the snow is deep, you can simply sweep the snow in the middle of the road to the green belt or leisure area by the side of the road, or even ignore it, because the heat emitted by the car itself will melt the snow on the road (the environmental problem caused by the pollution of snow caused by the exhaust gas of the car). When the depth of snow increases, you need to use a sweeper (which can spray snow directly to both sides of the road) across each section (route between adjacent nodes). Suppose the size of the sweeper is the same, with a width of four meters, and a small sweeper can be used instead of a large car to clean a road with a width of 2 meters, which is the same as the cleaning rate on the length of a large car (meters per second).

If the number of cleaning cars is greater than the number of nodes. At this point, the problem is easy, a section of the road can be equipped with a cleaning car. In general, the number of cars is only about 30 or40, at this time, we have to assign snow clearance tasks (several sections in a row) for each car, and calculate the shortest overall time.

The longitude and latitude information of the nodes in the table, as well as the area and longitude and latitude position information of the potential snow cover area are used. Calculate the actual distance and distribution of the intersection and draw the map. Then the related parameters of the forklift are configured, and the linear modeling and analysis are carried out. In this case, we will discuss the optimal task allocation of different numbers of snow sweepers in order to facilitate the analysis of question 2.

2.2Analysis of question two

When the amount of snow is so large that we have to use the snow truck to transfer the location of the snow pile, unless the number of snow trucks is enough, we can only set up snow trucks at each node, or let the snow truck on one node manage multiple areas. Of course, we need to first consider the capacity of each snow truck, as well as, in the worst-case scenario, the snow capacity of the road or area and the optimal occupancy of the snow truck. At the same time, the number of snow trucks needs to be discussed.

2.3Analysis of question three

According to the meaning of the problem, we have to deal with the situation of snow ice (the pressure of the car will make the parking space on the road snow ice). It is impossible for us to concentrate the vehicles to a certain place and return after they have been disposed of. If necessary, we can only configure large carriers to flexibly manage the location of the vehicle. The melting rate of ice determines when the parking lot will resume use.Therefore, the establishment of a dissolution rate model of potassium chloride (cement-friendly snow melting agent) is helpful to solve the problem.

The commonly used ice-melting methods at home and abroad include removal method and melting method.The removal method can be divided into two kinds:artificial removal method and mechanical removal method. The fusion method includes chemical melting method and hot melting snow melting. At present, countries in the world mainly melt the ice layer by sprinkling salt.We will establish relevant physical models and explore the advantages and disadvantages of different dissolution methods.

2.4Analysis of question four

According to the general understanding,the road priority is often prone to traffic accidents, or the ground is easy to freeze the road. Moreover, the processing rate of the unit distance of the high priority road section is higher than that of the general road section. This problem is only dealt with by weighted linear programming model, and the changes of different number of vehicle pairs are still considered.

3.Model

According to the Excel data in the appendix, the minimum points of longitude and latitude are selected by the node and reset to 0, which is recorded as long,lati. Then, the difference between the value of each latitude and longitude and the difference between long or lati is converted from degree to rad.

The transverse and longitudinal coordinates of each node are calculated according to the following formulas, and the final result is round to an integer. ("[]" is an round-off symbol).

$$x_i = [(long_i - long)^\circ \times r \cdot \pi / 180^\circ](i = 1,2,3,...,141)$$

$$y_i = [(lati_i - lati)^\circ \times r \cdot \pi / 180^\circ](i = 1,2,3,...,141)$$

First, we use the following MATLAB code to find the adjacent matrix and distance matrix that we may need to use.

```
%Adjacent Matrices of 141 points；
z=zeros(141,141);
x=x';y=y';
w=w';
for i=1:241
    k=x(i);
    m=y(i);
    if w(i)<=4
        z(k,m)=1;
    else
        z(k,m)=2;
    end
end
s=zeros(141);
flag=0;
sum=zeros(1,241);
for i=1:141
     for j=1:141
          s(i,j)=((x1(i)-x1(j))^2+(y1(i)-y1(j))^2)^0.5;
%Establishing a weight Matrix with distance as weight；
     end
end
s=s.*sign(z);
```

(Where x1, y1 is the column vector of141 values of the rounded transverse coordinates and longitudinal coordinates, respectively.

x, y is the column vector corresponding to the first and second column data in thesheet2 (links) in Appendix 1.

W is the width of the road, with a value of only 2, 4, 6, 8.

The data used are derived from the attachments provided.)

Combined with the links data in Appendix 1, a 141× 141 adjacent matrix is first created. Since the specification of the general forklift is2 to 4 meters,we have the following rules at this time.

1 the line segment with a width of 68 meters is recorded as 2 in the adjacent matrix.

2 roads with a width of less than 6 can only be cleaned by one vehicle

3 forklifts should drive along the continuous edge (that is, the path) in the sense of graph theory.

By observing the original data,it is found that the degree of each node does not exceed 4, that is to say, we can use the generating tree method of undirected graph.

For each feasible result (if the site size of the actual physical location of the node is not taken into account), it can be expected that the depth of the tree is close to the number of nodes.

3.1 The essential explanation of question one

For question 1, we give priority to the configuration of snow forklifts at the root node and at the node with less depth. If the width value of the root node and the line segment of the left sub tree or right sub tree is6, 8, configure two cars at this root node, otherwise configure one. Let them work at the same time at the beginning of the construction, and if one is finished first, find the tree generated along with his nearest car (if there is a car in front of the car) to carry out the construction (if). The so-called front of a car refers to the depth of the map from small to large direction. Of course, if there is no snow in front of us, we think that the time that the car will run to the location of the next car and the time planned for the reasonable allocation of tasks on the same side will not be recorded.

In this way, none of the cars are free until all the tasks are completed. So the completion time of the task depends on the sum of the weights of all the edges on the tree. We replace the weight with distance.

Such a scheme is also feasible in practical use, because we can ensure that the nodes before the position of the car at a certain time are snow-free.

However,when the whole graph is a tree, the problem has such a perfect solution, otherwise, we need to find out several cycles in the graph.After removing these cycles, the rest is the tree. Whether there is such a circle, and how to allocate part of the limited resources to the section on the circle, part to the section on the tree, requires additional model calculation.

At this point, the scheme of question one is completely given, and the specific numerical calculation is shown in question2.

3.2 The model of question two

According to the above,what this problem is actually solved is a minimum spanning tree problem. However,we also have to consider how to deal with sections with a width greater than four. In fact, these sections represent exactly twice the distance. As a result, the problem was simplified.

[1] The minimum spanning tree of connected weighted graph G=(V,E,W) is constructed, and two sets P and Q are set, where P is used to store the vertex in the minimum spanning tree of G, and the set Q stores the edges in the minimum spanning tree of G. Let the initial value of the set P be {v1} (assuming that the minimum spanning tree is constructed, starting from the vertex), and the initial value of the set Q is a empty set. The idea of prim algorithm is that from the edges of all $p \in P$,all $v \in V-P$ choosing the edge pv which with the minimum weight and add the vertex v to the set P, and the edge pr is added to the set Q. Such above is repeated until the minimum spanning tree is constructed when P = V, when all the edges of the minimum tree are wrapped in the collection.

As a result, the code for MATLAB is as follows.

```
for i=1:241
    if w(i)>=6
        k=x(i);
        j=y(i);
            s(k,j)=2*s(k,j);
```
%When the width of a road is greater than or equal to six,the weight of the corresponding side is calculated at twice the distance.
```
    end
end
%The minimum spanning tree algorithm is carried out below.
a=s;
result=[];%Represented by result (3 × n) matrix,
%the first line represents the starting point,
%the second line represents the end point
%and the third line represents the weight.
p=1;tb=2:length(a);
a=a+a';
a(a==0)=inf;
while size(result,2)~=length(a)-1
    temp=a(p,tb);temp=temp(:);
    d=min(temp);
    [jb,kb]=find(a(p,tb) == d,1);
    j=p(jb);k=tb(kb);
    result=[result,[j;k;d]];p=[p,k];tb(find(tb==k))=[];
end
result
```

The results are as follows

```
result =

   1.0e+03 *

  1 至 15 列

    0.0010    0.0020    0.0050    0.0040    0.1310    0.1340    0.0040    0.0050    0.0070    0.1280    0.1290    0.1290    0.1260    0.1260    0.1080
    0.0020    0.0050    0.0040    0.1310    0.1340    0.1350    0.0030    0.0070    0.1280    0.1290    0.1250    0.1260    0.1270    0.1080    0.1070
    1.0283    1.4480    1.8442    0.9537    1.1109    0.6213    1.4664    2.0020    0.7488    1.5103    1.7429    1.9179    0.4545    1.2348    1.0440
```

......(Only partial results are displayed)

```
136 至 140 列

0.1220    0.0080    0.0060    0.1320    0.1370
0.1410    0.0060    0.0090    0.1330    0.1360
2.4208    2.4619    2.4551    2.6767    2.9129
```

At this point, we have found the minimum spanning tree of the graph.

Next, look at the circle of the graph.First of all, we examine the existence of cycles.

The MATLAB code is as follows (using graph theory toolbox).

```
s=sign(s);
s=sparse(s)%Convert a distance matrix into a sparse matrix
graphisdag(s)
```

The result:

```
ans =

  logical

   1
```

Explain that the circle exists.

Generally speaking, if there is both a nesting tree and a closed loop in a graph, it is unhindered for the car to migrate at any two points along the road section represented by the edge. If the graph does not have a closed loop, there may be two cars that are relatively unable to pass because the road width is not enough, because the tree has no closed loop. If a graph is not even connected, the situation considered is more complex. Above, we have explained that the task of the car can be assigned arbitrarily without traffic jam.

Next, count the number of snow plows as $n(n \le 141(=\text{the number of the nodes}))$ n and create a task assignment matrix $R_{n\times 241}$.

Where the element

$$r_{ij} = \text{the proportion of tasks assigned to the } i\text{th vehicle.}(i = 1,2,...,\text{n})$$

$$r_{ij} \le 1$$

(the reason of no restriction on the non-negative intent of RIj as explained below)

And

$$\sum_{i=1}^{n} r_{ij} = 1(j = 1,2,...,241);$$

Let $\mathbf{r_i}$ be the ith transversal vector in $R_{n\times 241}$,The sum of all the elements on each vector is ri

The objective function is

$$\min\{\max_{i\in\{1,2,...,n\}}\{\mathbf{r_i}\times\mathbf{T}\}\}$$

Where T is the column vector consisting of all the non-zero elements of the weight matrix we calculated before

Notice that the weighted sum of all the elements in R is constant, because the total amount of work is constant. If the weighted sum of any two rows in R is close, the value of the objective function is closer to the minimum.

On the one hand, we calculate the ratio of total quantity(counted in meters)to n, denoting q, and then use q-λ as the lower bound on the weighted sum of any line of elements, and q+ λ as the upper bound, by changing and adjusting and n until all rij are non-negative. So that's how model two was built.

With regard to the number of vehicles and the specific test of the optimal allocation model, we describe it in detail in Chapter 5.

3.3 The model of question 3

The key to this problem is to study how to remove the ice layer.

If the parking space on both sides of the road is small and the air temperature is near zero, the use of high temperature fluid such as boiling water can play a good melting effect. Of course,
the degree of ice looseness is positively correlated with the melting promoting effect of boiling water, because the greater the degree of looseness, the larger the contact area with the fluid. Next, we will consider the worst-case scenario. We assume that the parking space is large and that there are even situations where cars are frozen by ice and cannot be driven (and any other cases where physical methods cannot be used to remove the ice). At this time, we consider the effect of snow melting agent on ice and melting time.

According to the literature review[4], the melting data of three kinds of solid content of salt ice at different temperatures are as follows.

Table.4-1 The summary table of melting time

	0%	5%	8%	10%
10	4823	3024	2806	2407

20	3864	2300	1872	1742
30	2652	1470	1374	1231
40	1734	1054	935	864
50	1235	810	769	704
60	864	430	340	286
70	653	319	279	243
80	428	285	237	198

The first row represents the mass fraction of salt water (%), and the first column represents temperature (℃).

According to the freezing point of salt water with different mass fraction [4], we can know that when the ambient temperature is slightly less than zero, the salted ice can melt, and even the melting time of ice can be calculated by data simulation.

Table.4-2 The freezing point of water of different solids content salt water

Number of solid	Data number	time(min)	Freezing point(C°)
5%	55	18	-2.62
8%	46	15	-5
10%	31	10	-6.32

Build the following grey model

assume $x^{(0)}$ as Original sequence

$$x^{(0)} = (x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)),$$

is the One accumulation generates the sequence of $x^{(1)}$

we have

$$x^{(1)} = (x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(n)),$$

Its Mean-generating sequence is

$$z^{(1)} = (z^{(1)}(2), z^{(1)}(3), \ldots, z^{(1)}(n)),$$

Said

$$x^{(0)} + az^{(1)} = b(z^{(1)})^2$$

is Grey Verhulst model with a and B as parameters.

Said

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = b(x^{(1)})^2$$

is the whitening equation of gray Verhulst model, where T is time(the generalized x-coordinate)

Verhulst model is mainly used to describe processes with saturation state, such as population prediction, reproduction prediction, economic life prediction of products and so on.

If $\mathbf{u}=[a,b]^{\mathrm{T}}$ is a parameter column, and

$$\mathbf{B}=\begin{bmatrix} -z^{(1)}(2) & (z^{(1)}(2))^2 \\ -z^{(1)}(3) & (z^{(1)}(3))^2 \\ \vdots & \vdots \\ -z^{(1)}(n) & (z^{(1)}(n))^2 \end{bmatrix} \mathbf{Y}=\begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix},$$

Then the least square estimation of parameter column $\mathbf{u}$ satisfies

$$\hat{\mathbf{u}}=[\hat{a},\hat{b}]^{\mathrm{T}}=(\mathbf{B}^T\mathbf{B})^{-1}\mathbf{B}^T\mathbf{Y}$$

From the solution of the whitening equation, the corresponding time sequence is

$$\hat{x}^{(1)}(k+1)=\frac{\hat{a}x^{(0)}(1)}{\hat{b}x^{(0)}(1)+(\hat{a}-\hat{b}x^{(0)}(1))e^{\hat{a}k}},$$

The reductive formula is

$$\hat{x}^{(0)}(k+1)=\hat{x}^{(1)}(k+1)-\hat{x}^{(1)}(k).$$

If the relative error of the above model is large, we consider the DGM(2,1) model.

Assume

$$\alpha^{(1)}x^{(0)}(k)=x^{(0)}(k)-x^{(0)}(k-1), k=2,3,\ldots,n,$$

And

$$\alpha^{(1)}x^{(0)}=(\alpha^{(1)}x^{(0)}(2),\ldots,\alpha^{(1)}x^{(0)}(n)),$$

then said

$$\alpha^{(1)}x^{(0)}(k)+ax^{(0)}(k)=b$$

as the DGM(2, 1) model
and due to

$$\mathbf{B}=\begin{bmatrix} -x^{(0)}(2) & 1 \\ -x^{(0)}(3) & 1 \\ \vdots & \vdots \\ -x^{(0)}(n) & 1 \end{bmatrix}, \mathbf{Y}=\begin{bmatrix} \alpha^{(1)}x^{(0)}(2) \\ \alpha^{(1)}x^{(0)}(3) \\ \vdots \\ \alpha^{(1)}x^{(0)}(n) \end{bmatrix}$$

Then the least square estimation of parameters in DGM(2.1 model)(15.1) satisfies $\hat{\mathbf{u}} = [\hat{a}, \hat{b}]^{\mathrm{T}} = (\mathbf{B}^T\mathbf{B})^{-1}\mathbf{B}^T\mathbf{Y}$

The corresponding time sequence is

$$\hat{x}^{(1)}(k+1) = \left(\frac{\hat{b}}{\hat{a}^2} - \frac{x^{(0)}(1)}{\hat{a}} \right) e^{-\hat{a}k} + \frac{\hat{b}}{\hat{a}} k + \frac{1+\hat{a}}{\hat{a}} x^{(0)}(1) - \frac{\hat{b}}{\hat{a}^2}.$$

reducing value for

$$\hat{x}^{(0)}(k+1) = \alpha^{(1)} \hat{x}^{(1)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k).$$

The specific idea is to first use the accuracy of the known data test model in table4.1, and then predict the melting time close to freezing point, so as to estimate the recovery time of parking lots.

3.4 model of question 4

Given that the integrity probability of the directed path (or path) between each vertex on the network G is $0 \leqslant \mathrm{p_{ij}} \leqslant 1$. If a directed path (or path) from vertex S to vertex T passes through vertices whose ordinal numbers are $\{S, S_1, S_2, \ldots, S_k, T\}$, then the intact probability of each arc (or edge) that the path passes through is $\{ \mathrm{p_{ss_1}}, \mathrm{p_{s_1 s_2}} \cdots, \mathrm{p_{s_k T}} \}$ The total good probability $\mathrm{p_{sr}}$ of this path is the product of the good probability of all arcs (or edges) it passes through. Therefore, the goal can be transformed into the problem of finding a directed path (or path) that maximizes the $\mathrm{p_{sr}}$. The maximum directed path is then deleted and the remaining maximum reliable path is found in the remaining subgraphs until any two edges of the remaining graphs have no common vertex. Then the remaining problem is trivial.

In network G, the intact probability $\mathrm{p_{ij}}$ has special provisions:

$$\mathrm{p_{ij}} = \begin{cases} 1, i = j \\ 0, v_i v_j \notin E \end{cases}$$

E stands for the edge set

In addition, let the probability $\mathrm{p_f}$ corresponding to the priority $\mathrm{f} \in \{1,2,\ldots,6\}$ satisfy:

$$p_1 = m;$$

$$\frac{p_i}{p_{i-1}} = \frac{1}{m^{0.2}};$$

$$p_6 = 1,$$

The model test in Chapter 5 can start with different m values.

In order to use the shortest path solution idea in the weight graph, the following transformation can be made (the probability is 0 when two points in the network are not adjacent)

$$a_{ij} = \begin{cases} -\ln p_{ij}, 0 < p_{ij} < 1; \\ \infty, p_{ij} = 0; \\ 0, p_{ij} = 1 or i = j \end{cases}$$

In this way, the problem is transformed into finding the shortest path from vertex S to vertex T on the matrix A=(aij).

Since the number of edges in the minimum spanning tree in question 2 is similar to the number of edges in the graph, vertex S can be set as the root node of the minimum spanning tree, vertex T can be set first as the deepest point of the minimum spanning tree, and then the so-called "shortest path" can be removed, and so on, until only isolated edges are left.

Note that because the subject has been clear about the priority needs to be taken into consideration the fact that must let all the vehicle on a road or continuous several public endpoint, which is adjacent sections within the work, until this stretch of road and snow cleaned up, because in the actual work, also need to consider the management of the collection of snow, more complex traffic intersection terrain,cleared to maintenance and a series of problems. At this point, the model for problem 4 is complete.

4. Models testing and sensitivity Analysis

The following content is limited to space, only the specific model test to write out the situation in detail.

4.1Testing the model 1 for question1,2

4.1.1 Testing

Model 1 uses the following linear induction model.

$$\min_x \max_i F_i(\mathbf{x}),$$
$$\text{s.t.}\begin{cases} \mathbf{A}\cdot\mathbf{x}\le\mathbf{b}, \\ Aeq\cdot\mathbf{x}=beq, \\ \mathrm{c}(\mathbf{x})\le 0, \\ lb\le\mathbf{r_i}\times\mathbf{T}\le ub. \end{cases}$$

Its corresponding MATLAB function is

$$[\mathbf{x},\mathrm{fval}]=\mathrm{fminimax}(\mathrm{fun},\mathrm{x0},\mathrm{A},\mathrm{b},\mathrm{Aeq},\mathrm{beq},\mathrm{lb'},\mathrm{ub'})$$

$$\text{whereas } \mathrm{A}=[\,],\mathrm{b}=[\,].$$

$$lb'\le\mathbf{x}\le ub',lb',ub'\text{ are transformed from lb, ub and }\mathbf{ri}$$

What to remind is, x is a column matrix of length 241 by n

The key to model testing lies in the range of n value lb and the setting of in ub parameter.

We found that when λ = 0,n is rounded from1 to 5, the equation has a solution, which means that as long as there are fewer than 6 snowplows, we can get the job done in the shortest amount of time by properly allocating tasks. However, when 5< n< 53,there are no non-negative Numbers in the feasible solutions.After we've eliminated the software itself, we still don't find a non-negative solution in this range (λ is still 0),and the non-negative solution returns when n≥53, all the way to n=141.Obviously, as n increases, n provides more free variables r_{ij}.

The reason for this is that the model relies on a given average value. When n is taken as the units digit, an increase of one will cause a significant change in the ratio t/n, while the change of free variables cannot keep up with the change of the ratio, and the weight matrix differs greatly. The maximum value is about 100 times of the minimum value, so non-zero solutions will be caused.

4.1.2 sensitivity analysis

Now let's look at the solution by adjusting the value of λ.

To reduce the computational burden, we have modified the code so that the program can prompt an error message when a negative value occurs for a viable solution. When the feasible solution does not appear negative value, the program passes the result.

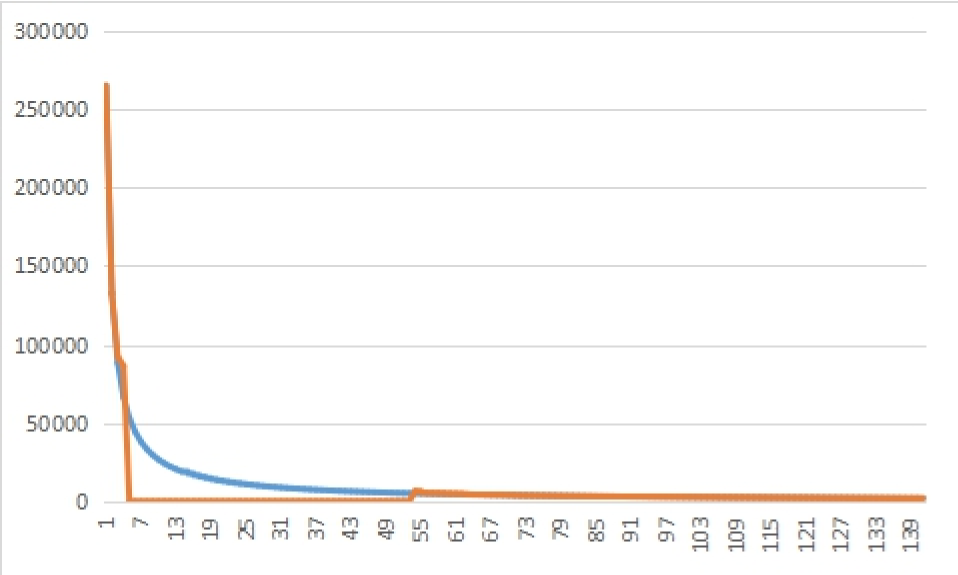
(The above feasible solution refers to all the solutions that satisfy the constraint (regardless of the objective function).)

First, we made λ = q (the loosest condition) so that only n= 5 is a viable solution, and then we gradually narrowed down the value of λ by, for example, following "for λ=q :20:0" inside of " for n=1:10". Where 20 represents the accuracy . And the purpose of this is to see what happens to the target function.

In order to consider the computational burden of the program, when the value of n is small and the corresponding value of q is large, the accuracy of our setting is also large (greater than 10 and less than 30). When the value of n is large, the setting accuracy can be adjusted to the units digit or one digit after the decimal point.

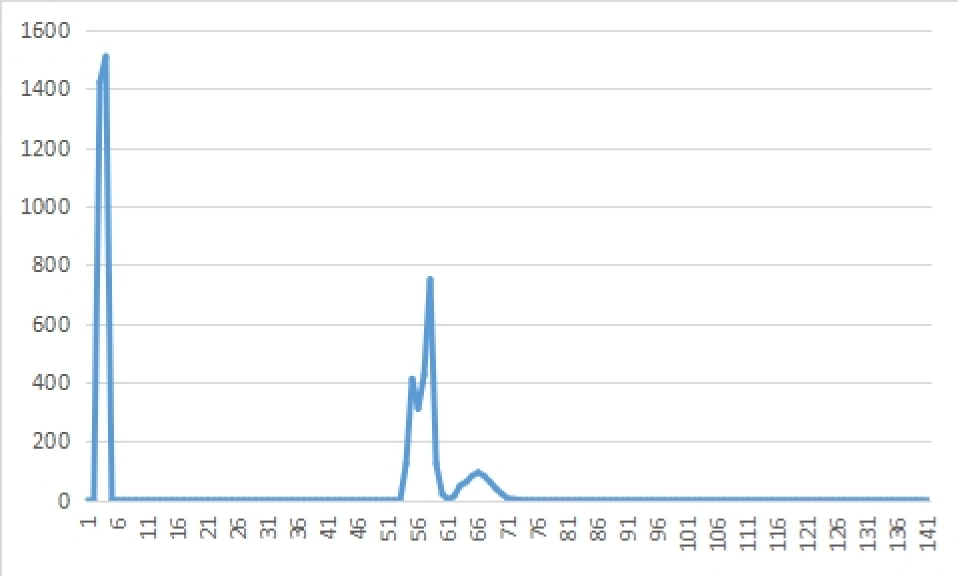
We found that as the value changes, as λ falls to a certain level, the value of the target function begins to change, until it converges to a larger number. When $n > 5$ and $n < 53$, the program even gives an error message, which matches the previous programming result.

So, we take n as the x-coordinate(except 6,7... ,52), draws a line graph with the final result of the objective function (denoted as z) (which is only an approximation since the accuracy of is limited) as the y-coordinate. (below)



The orange line represents the optimal z value under the most lenient conditions, and the blue line represents the function y=t/x, where the integer t is 265335 and represents the sum of all elements of the distance matrix. All invalid values are replaced by 0.

In the meantime, plot x as the horizontal axis and as the value that changes the result of the objective function for the first time when it changes from large to small.



In this way, we have reached the final conclusion.

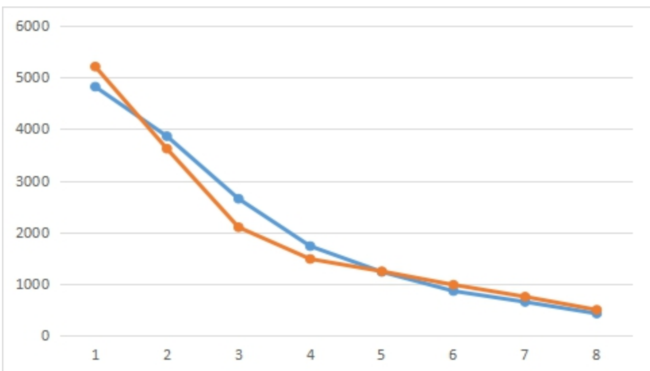
4.2 testing the model 2 for question 3

4.2.1 testing and sensitivity

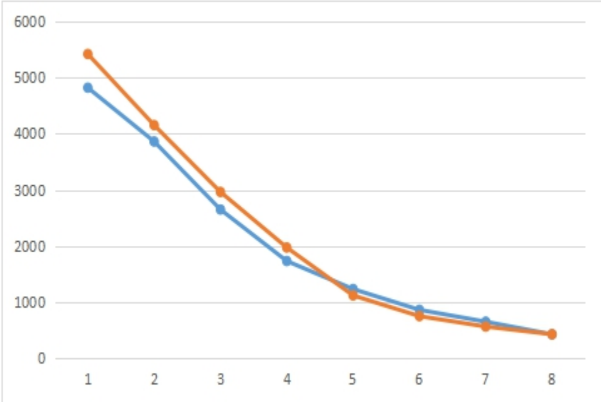
In chapter 6, we will discuss in detail the advantages, disadvantages and sensitivity of the grey model and the common elementary function fitting. In this chapter, we will only explain the fitting analysis process of the model given in Chapter 4.

The idea is to construct a fitting curve based on existing data and models, calculate their relative error values, and then make a prediction.

We use two models here, the verhulst model and the DGM(2,1) model (each model has a total of eight relative errors), and find that six of the former are within the range of [10.3%,23.7%], while five of the latter are within the range of [0, 10%]. Therefore, DGM(2,1) is used for prediction



Fitting results of verhulst



Fitting results of DGM(2,1)

The abscissa 1 in the above two figures represents 10%, 2 represents 20%and so on.

Predictive values are:(temperature=-10℃)

0%	5%	8%	10%
6893	4211	3642	2971

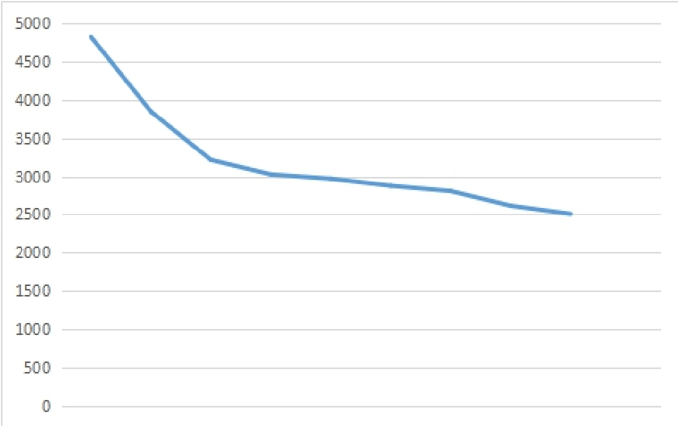
Here only gives a reasonable prediction of the melting of the fixed concentration time, to get the optimal concentration (i.e., melting time), we tried to temperature as the independent variable model inspection (limited to time, here only consider temperature equal to 10,20,and,30), found that the simulation results of the two models and its predictive value difference is very big. Obviously, too few samples resulted in too large relative error range of data fitting results, and the model sensitivity was abnormal.

In order to reduce the sensitivity, we treat the more stable fitting result as the actual result. So, for example, when the temperature is 10 degrees Celsius, the concentration at 0%, 5%, 8%, 10% is known, and we use:

$$\hat{x}^{(1)}(t+1)=\frac{\hat{a}x^{(0)}(1)}{\hat{b}x^{(0)}(1)+(\hat{a}-\hat{b}x^{(0)}(1))e^{\hat{a}t}},$$

$$\hat{x}^{(0)}(t+1)=\hat{x}^{(1)}(t+1)-\hat{x}^{(1)}(t).$$

The formula of the two models, respectively, results in the concentration of 1%, 2%, 3%, 4%, 6%, 7%, 9% data (rounded to the integer)



Finally, we refer to relevant literature [6] and re-list the data in the above table to predict their melting time near the freezing point.

11%	12%	13%	14%	15%	16%	17%	18%	19%	20%	21%
2311	2246	2208	2134	2119	2101	2087	2067	2049	2034	2027

Because too high a concentration of solution may damage the road surface, we chose the salt solution of %15.

4.3 testing the model 3 for 4

Here, we first convert the probability matrix to the weight matrix in the sense of negative logarithm (as described in the previous chapter), and then find the minimum spanning tree containing all vertices. This is an algorithm to observe the maximum reliable path generated by Model 3.

Psychological research has found that more task switching and assignment will aggravate staff's physical and mental fatigue [5], especially those requiring priority consideration. So, in order to find the minimum spanning tree, we also have to consider which nodes are rooted to maximize the depth of the tree. After selecting an optimal root node and the corresponding deepest child node, the maximum reliable path is obtained in the original diagram, and then the matching degree of the maximum reliable path and the minimum spanning tree is observed.

If the degree of matching is too small, we consider the other optimal root nodes, if there is only one optimal root node, we consider the other deepest child nodes, if there is only one root node and the corresponding deepest child node, we consider the sub optimal child nodes, until we find the maximum reliable path that matches well.

Delete this path in the original diagram, and the remaining diagram may be connected, or it may be composed of two unrelated sub-trees. Continue until the rest of the diagram consists of five or more unrelated sub-trees, stopping the program.

In our opinion, under this mode of task assignment, physical and mental fatigue can be minimized while tasks can be completed as soon as possible.

From the running results, we get 17 maximum reliable paths in turn.

Of course, different m values (i.e. the original weight of the edge with priority 1) will result in different results. The number of maximum reliable paths we can get is 15,17,18,19,20,23. We chose the case with the fewest Numbers (the value of m at this point is 0.2375). (Due to space limitation, we do not give any description of the code operation results and the code results.)

And that's where we end up.

5. Strengths and Weakness

5.1 model 1

Model 1 gives a good global representation of each task assignment, and by controlling the upper and lower bounds (the value of), the global optimal solution for the different n values (that is, the number of plows) is obtained. However, the model itself was so computationally intensive that we had to use a better-configured computer to run the program because it crashed during the process. At the same time, the definition of upper and lower bounds is not flexible. In the case of not considering the workload, it is also necessary to consider the case of no upper bounds. Therefore, at this point, the model is flawed, and the resulting global optimal solution may not be optimal in the true sense.

One idea for simplifying the model is the integer programming model, where we only allow rij is 0 or 1, that is to say, only one snowplow is allowed to construct in a section. Although the solution obtained at this time is not optimal, it greatly reduces the difficulty of task assignment and reduces the physical and mental fatigue of staff. At the same time, it also greatly reduces the program computation.

5.2 model 2

[7] The gray model is suitable for small model modeling, but not for large model modeling. In this paper, different versions of the gray model, Verhulst model and DGM(21) model, are used respectively to conduct data fitting and prediction and compare the results with each other. The final conclusion is quite accurate.

Moreover, [7] the model can tolerate the actual numerical disturbance caused by the change of weather pressure, and can be applied to parking Spaces with different terrain.

However, with Autoregressive Integrated Moving Average Model, model 2 would be more robust and useful for practical use with consideration for atmospheric pressure and related parameters for different impurities or crystal shapes (or for treating their variations as minor perturbations).

5.3 model 3

In this model, priority is well included into the weight matrix, and a humanized scheme is presented from the perspective of neuroscience.

It should be noted that this figure has 141 nodes but only 241 edges, so the difficulty of generating the minimum spanning tree and the change of the number of the maximum reliable paths with the change of m value of the model always remain within a stable range.

However, if the diagram's construction is more complex, the computation of the code can be greatly increased. We can look at this problem from the perspective of linear programming and from the perspective of practical application, including how to efficiently adjust the probability (or otherwise) corresponding to the priority, so that the first maximum reliable path generated has the maximum number of edges, and reduce the physical and mental fatigue caused by task switching.

6Conclusion

Question 1

Find the minimum spanning tree corresponding to the figure, put enough vehicles near the root node, and the number of vehicles at the root node should not be less than the degree of the root node (the number of edges connected to the root node).

Question 2

When the number of vehicles is less than 5 but more than 53, the task can be assigned directly according to a solution result of rij. When the number of vehicles is between 5 and 53, the restriction of simultaneous completion of two vehicles is lifted and the task is assigned again according to the solution result of the model. Of course, if the number of cars is too large, say more than 60, we need to shelve some of the cars to reduce the additional oil consumption.

Question 3

When the weather is stable, we estimate the thickness of the ice, and then salt the ice at a rate of 15%, or increase the concentration of the brine, and spray it carefully in case the vehicle freezes. When the weather is unstable, the mathematical model in literature [7] can be used.

Problem four

Taking the distance matrix as the weight matrix, we first obtain the minimum spanning tree of the map, then work out the maximum reliable way of the weight matrix based on priority and the longest number of overlapping edges with the tree in turn, and delete one for each one, until the remaining graph is composed of at least five subtrees. Based on the number of configurations available, the longest maximum reliable path is processed first, and then the processing sequence of the remaining maximum reliable paths is planned. Finally, when you have at least five subtrees left, copy the solution to problem Number one. (The nodes of the largest reliable tree we found were 10(adjacent edges were (10, 12) and 91(adjacent edges were 90)).

References

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[2]**Research on the application of deicing and snowmelt technology in expressway** Doctoral thesis of Hefei University of Technology ,Xie Kang
[3] **Study on physical and chemical effects of melting snow and deicing asphalt mixture** ,Master's degree thesis of Chongqing Jiaotong University ,Zou Mengqiu
[4]**Effect of Sodium Chloride on Water freezing Point and lce melting rate Sound and experimental study,** Master's degree thesis of Qingdao University of Science and Technology, Liang Chang
[5]**A study on the effects of mental fatigue on pre-attention and attention-processing ability under prolonged vigilance task.** The Fourth Military Medical University. Doctoral dissertation .Yang Bo
[6]**Freezing point comparison table for different concentrations of chlorine brine solutions** https://wk.baidu.com/view/10f855b469dc5022aaea0003?pcf=2
[7]**Discrete grey model based on fractional order accumulate** System engineering Theory and Practice, Vol. 34, No. 7 Wu Lifeng,Liu Sifeng,Yao Ligen

Appendix：Part of the running code on MATLAB.

MATLAB Code to find the most reliable path (part)

命令行窗口 编辑器

as.m m534.m n14523.m Untitled4.m +

```
B=[-x0(2:end),ones(n-1,1)];
u=B\a_x0
u=B\Y
syms x(t)
x=dsolve(diff(x)+u(1)*x==u(2)*x^2,x(0)==x0(1));
xt=vpa(x6)
yuce= subs(x,'t',[0:n-1]);
yuce=double(yuce)
x0_hat=[yuce(1),diff(yuce)]
epsilon=x0-x0_hat
delta=abs(epsilon./x0)

x0=[3024,2300,1470,1054,810,430,319,285];
n=length(x0);
a_x0=diff(x0)';
B=[-x0(2:end),ones(n-1,1)];
u=B\a_x0
u=B\Y
syms x(t)
x=dsolve(diff(x)+u(1)*x==u(2)*x^2,x(0)==x0(1));
xt=vpa(x6)
yuce= subs(x,'t',[0:n-1]);
yuce=double(yuce)
x0_hat=[yuce(1),diff(yuce)]
epsilon=x0-x0_hat
delta=abs(epsilon./x0)


x0=[2806,1872,1375,935,769,340,279,237];
n=length(x0);
a_x0=diff(x0)';
B=[-x0(2:end),ones(n-1,1)];
u=B\a_x0
u=B\Y
syms x(t)
x=dsolve(diff(x)+u(1)*x==u(2)*x^2,x(0)==x0(1));
xt=vpa(x6)
yuce= subs(x,'t',[0:n-1]);
yuce=double(yuce)
x0_hat=[yuce(1),diff(yuce)]
```

MATLAB Code to find the most reliable path (part)

```
function [P,u]=as(W,k1,k2)

n=length(W);
U=W;
m=1;
while m<=n
    for i=1:n
        for j=1:n
            if U(i,j)>U(i,m)+U(m,j)
                U(i,j)=U(i,m)+U(m,j);
            end
        end
    end
    m=m+1;
end
u=U(k1,k2);
P1=zeros(1,n);
k=1;
P1(k)=k2;
V= ones(1,n)* inf;
kk= k2;
while kk ~= k1
    for i=1:n
        V(1,i)=U(k1,kk)-W(i,kk);
        if V(1,i)==U(k1,i)
            P1(k+1)=i;
            kk= i;
            k=k+1;
        end
    end
end
k=1;
wrow=find(P1~=0);
for j= length(wrow):(-1):1
    P(k)=P1(wrow(j));
    k=k+1;
end
P;
```