1.1

If the image coordinates are normalized, the two corresponding points are x = [0,0,1] and x' = [0,0,1]. Because $x'Fx^T = 0$, we can get

$$\begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} F_{31} & F_{32} & F_{33} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = F_{33} = 0$$

1.2

We can assume the translation matrix is $t = \begin{bmatrix} t_1 \\ t_2 \\ t_3 \end{bmatrix}$

Because x' = R(x - t) and this transformation is only translation parallel to x axis, we can get

$$R = I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
 and $t_3 = t_2 = 0$

And
$$E = R[t_x] = I * \begin{bmatrix} 0 & -t_3 & t_2 \\ t_3 & 0 & -t_1 \\ -t_2 & t_1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_1 \\ 0 & t_1 & 0 \end{bmatrix}$$

So we can get the epipolar line for C2 is $l_2 = Ex_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_1 \\ 0 & t_1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -t_1 \\ t_1 y_1 \end{bmatrix}$ and this line

is parallel to x-axis and same to the epipolar line l_{1}

1.3

Use lower-case letters as image points and use upper-case letters as real world points. We can get

$$\begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} = K[R_1|t_1] \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = K \left(R_1 \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + t_1 \right) \text{ and } \begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} = K[R_2|t_2] \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = K \left(R_2 \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + t_2 \right)$$

and the real world point can be represented as $\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{pmatrix} K^{-1} \begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} - t_2 \end{pmatrix} * R_2^{-1}$

Then one image point can be represented by $\begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} = K \left(R_1 \left(K^{-1} \begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} - t_2 \right) * R_2^{-1} + t_1 \right) = K \left(R_1 \left(K^{-1} \begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} - t_2 \right) * R_2^{-1} + t_1 \right) = K \left(R_1 \left(K^{-1} \begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} - t_2 \right) * R_2^{-1} + t_1 \right) = K \left(R_1 \left(K^{-1} \begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} - t_2 \right) * R_2^{-1} + t_1 \right) = K \left(R_1 \left(K^{-1} \begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} - t_2 \right) * R_2^{-1} + t_1 \right) = K \left(R_1 \left(K^{-1} \begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} - t_2 \right) * R_2^{-1} + t_1 \right) = K \left(R_1 \left(K^{-1} \begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} - t_2 \right) * R_2^{-1} + t_1 \right) = K \left(R_1 \left(K^{-1} \begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} - t_2 \right) * R_2^{-1} + t_1 \right) = K \left(R_1 \left(K^{-1} \begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} - t_2 \right) * R_2^{-1} + t_1 \right) = K \left(R_1 \left(K^{-1} \begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} - t_2 \right) * R_2^{-1} + t_1 \right) = K \left(R_1 \left(K^{-1} \begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} - t_2 \right) * R_2^{-1} + t_1 \right) = K \left(R_1 \left(K^{-1} \begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} - t_2 \right) * R_2^{-1} + t_1 \right) = K \left(R_1 \left(K^{-1} \begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} - t_2 \right) * R_2^{-1} + t_1 \right)$

$$KR_1K^{-1}R_2^{-1}\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} - KR_1R_2^{-1}t_2 + Kt_1$$

Finally we can get the result

$$\begin{split} R_{rel} &= K R_1 K^{-1} R_2^{-1} \\ t_{rel} &= -K R_1 R_2^{-1} t_2 + K t_1 \\ E &= R_{rel} [t_{relx}] \\ F &= K^{-T} [t_{relx}] R_{rel} K^{-1} \end{split}$$

1.4

We can say an arbitrary point A on the object is $A = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

AndrewId:lizhiz 4/10/2018

And the
$$R = I - 2NN^T = \begin{bmatrix} 1 - 2x^2 & -2xy & -2xz \\ -2xy & 1 - 2y^2 & -2yz \\ -2xz & -2yz & 1 - 2z^2 \end{bmatrix}$$

$$t = k \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$
 So we can get the $E = [t_x]R = \begin{bmatrix} 0 & -kz & ky \\ kz & 0 & -kx \\ -ky & kx & 0 \end{bmatrix} \begin{bmatrix} 1 - 2x^2 & -2xy & -2xz \\ -2xy & 1 - 2y^2 & -2yz \\ -2xz & -2yz & 1 - 2z^2 \end{bmatrix} = \begin{bmatrix} 0 & -kz & ky \\ kz & 0 & -kx \\ -ky & kx & 0 \end{bmatrix}$ Finally, we can get $E = K^{-T}EK^{-1} = K^{-T}(-E^T)K^{-1} = -(K^{-T}EK^{-1})^T = -F^T$

$$\begin{bmatrix} 0 & -kz & ky \\ kz & 0 & -kx \\ -ky & kx & 0 \end{bmatrix}$$

 $\begin{bmatrix} \kappa Z & 0 & -kX \\ -ky & kx & 0 \end{bmatrix}$ Finally, we can get $F = K^{-T}EK^{-1} = K^{-T}(-E^T)K^{-1} = -(K^{-T}EK^{-1})^T = -F^T$ 2.1

F

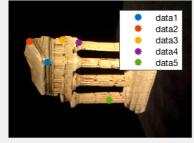
$$= \begin{bmatrix} -1.310951264370002e - 09 \\ -5.921318702546828e - 08 \\ -0.001080133996493 \end{bmatrix}$$

$$-1.308290656772906e - 07$$

 $3.564406722080434e - 09$

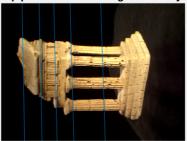
 $= \begin{bmatrix} -1.310951264370002e - 09 & -1.308290656772906e - 07 & 0.001124715246002 \\ -5.921318702546828e - 08 & 3.564406722080434e - 09 & -1.650299588884945e - 05 \\ -0.001080133996493 & 3.047751258987522e - 05 & -0.004165831804642 \end{bmatrix}$

Epipole is outside image boundary



Select a point in this image (Right-click when finished)

Epipole is outside image boundary



Verify that the corresponding point is on the epipolar line in this image

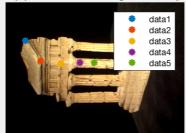
2.2

F

$$=\begin{bmatrix} 6.717780008346810e - 08 & 1.779027072066887e - 06 & -0.002469168327910 \\ -1.510702323985637e - 06 & -5.524069882753711e - 08 & 5.026249827532084e - 04 \\ 0.002367270966298 & -4.890216415683362e - 04 & 0.005120920625693 \end{bmatrix}$$

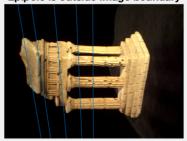
$$-0.002469168327910$$
 $5.026249827532084e - 04$
 0.005120920625693

Epipole is outside image boundary



Select a point in this image (Right-click when finished)

Epipole is outside image boundary



Verify that the corresponding point is on the epipolar line in this image

3.1

The E I got is

 $\begin{bmatrix} -0.003030416127690 & -0.303520601230333 & 1.660308953073076 \ -0.137373312525719 & 0.008299260566660 & -0.051154853735289 \ -1.665063449108456 & -0.012504423130672 & -0.001324288758394 \end{bmatrix}$

3.2

$$A = \begin{bmatrix} yp_3^T - p_2^T \\ p_1^T - xp_3^T \\ y'p_3'^T - p_2'^T \\ p_1'^T - x'p_3'^T \end{bmatrix}$$

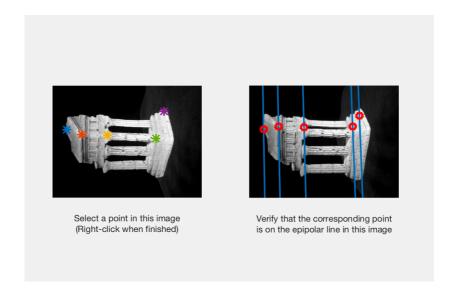
3.3

The sum of squared errors for both projections is 93.1082.

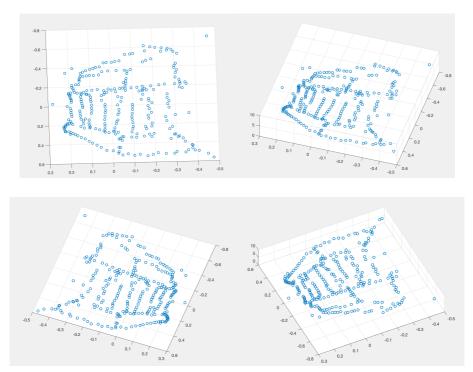
And the C2 I selected is

 $\begin{bmatrix} 1.5196696e + 03 & -22.0897060 & 3.051716523018233e + 02 & -21.784748553570704 \\ -57.946102 & 1.4080747e + 03 & 6.350415319094881e + 02 & -1.505518584331076e + 03 \\ 0.00125459 & -0.2601003135 & 0.965580785257230 & 0.082559305176506 \end{bmatrix}$

4.1



4.2



5.1 After I calculate the fundamental matrix F from each possible combination, I use the formula: $x'Fx^T=0$ to check if a pair of points are inlier.