Tire Slip & Modeling

Course 1, Module 4, Lesson 7

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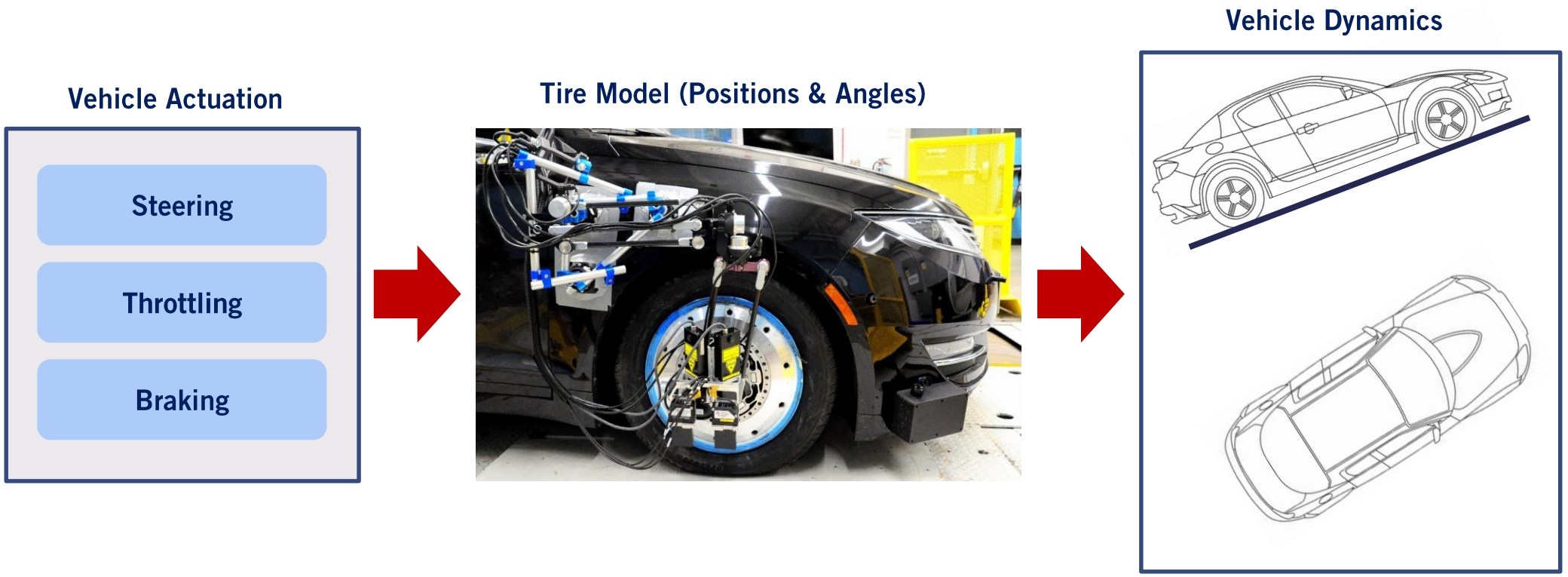
FACULTY OF APPLIED SCIENCE & ENGINEERING

Learning Objectives

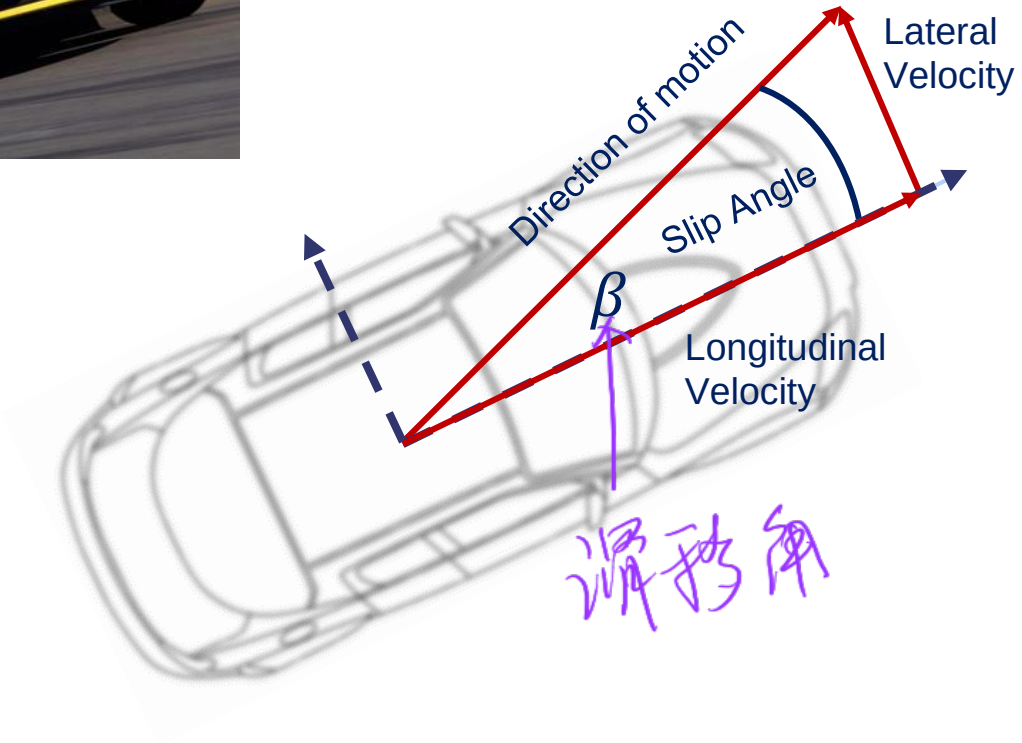
- Study tire slip angle and slip ratio in more detail
- Define tire models that capture forces produced by tires

Importance of Tire Modeling

- The tire is the interface between the vehicle and road



Vehicle Slip Angle



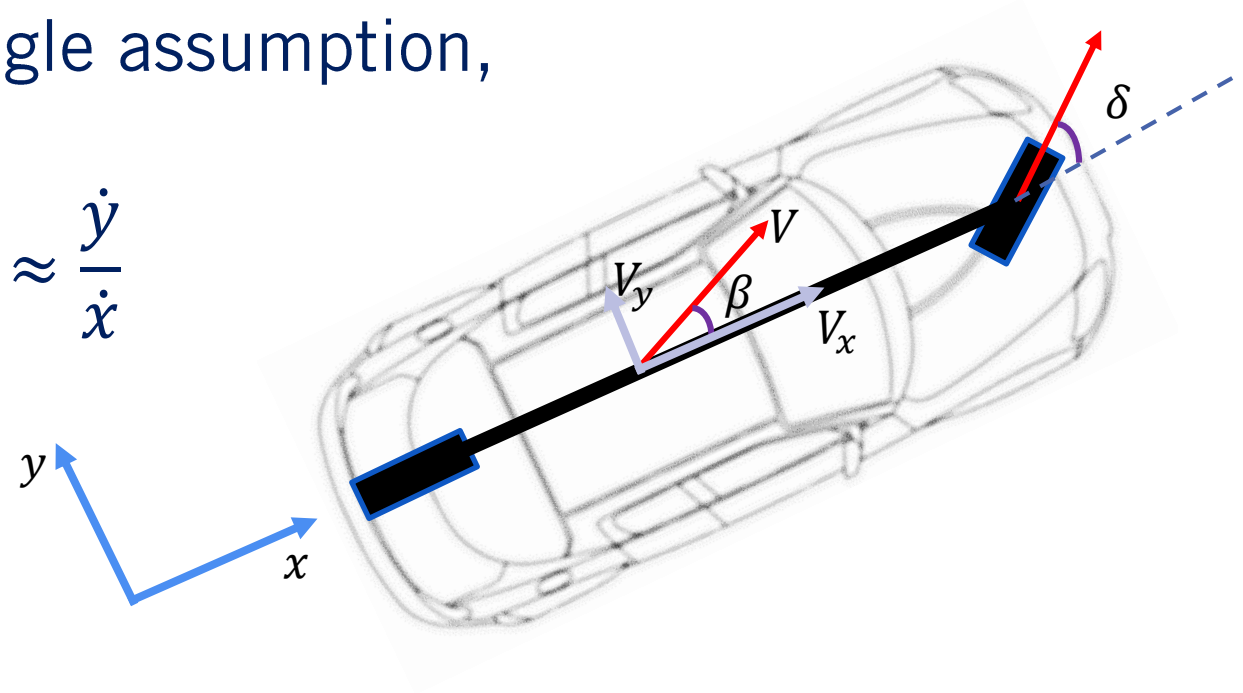
Vehicle (Bicycle) Slip Angle

- Slip angle

$$\beta = \tan^{-1}\frac{V_y}{V_x} = \tan^{-1}\frac{\dot{y}}{\dot{x}}$$

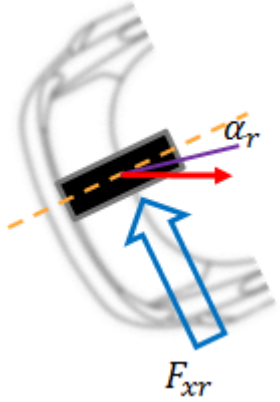
- Using small angle assumption,

$$\beta \approx \frac{\dot{y}}{\dot{x}}$$



Tire Slip Angles

- Tire slip angle is the angle between the direction in which a wheel is pointing and the direction in which it is actually travelling



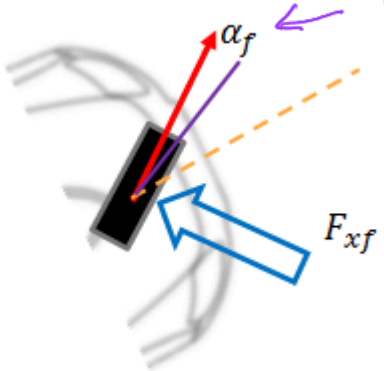
Rear tire slip angle

$$\alpha_r = -\beta + \frac{l_r\dot{\psi}}{V}$$

yaw rate

vehicle slip angle

forward velocity



Front tire slip angle

$$\alpha_f = \delta - \beta - \frac{l_f\dot{\psi}}{V}$$

steering angle

Slip Ratios

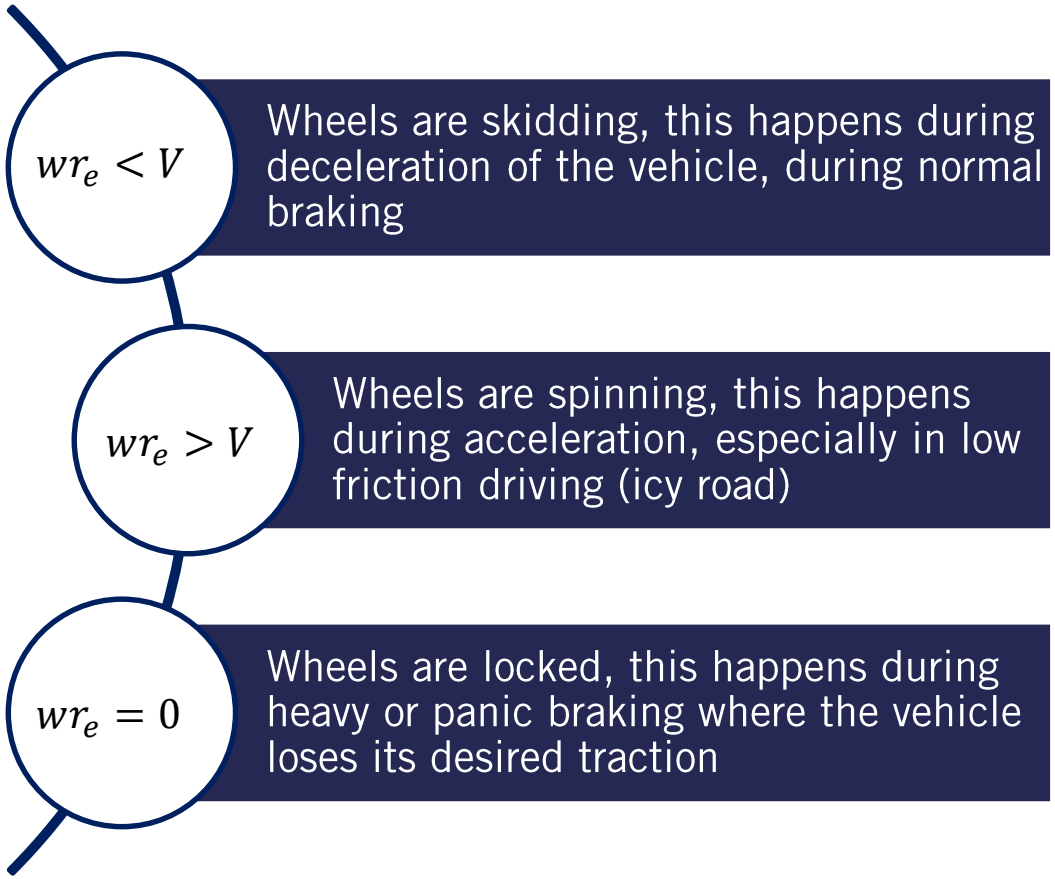
- Longitudinal slip (also called slip ratio)

wheel angular speed

tire effective radius

$$s = \frac{w\, r_e - V}{V}$$

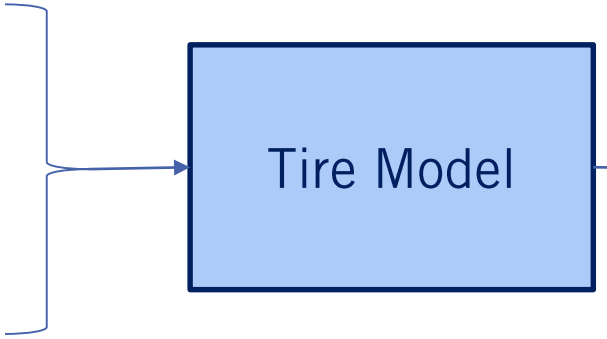
vehicle forward velocity



Tire Modeling

Inputs to the tire model

Tire Slip Angle
Slip Ratio
Normal Force
Friction Coefficient
Camber Angle
Tire properties



Outputs of the tire model

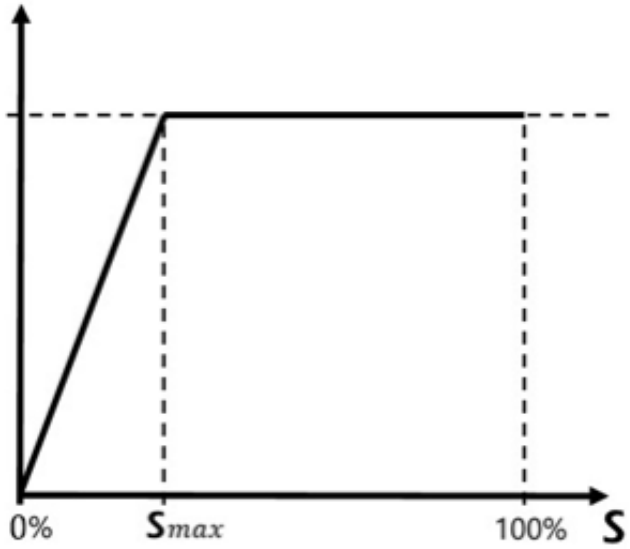
Lateral Force
Longitudinal Force
Self-Aligning Moment
Rolling Resistance Moment
Overturning Moment

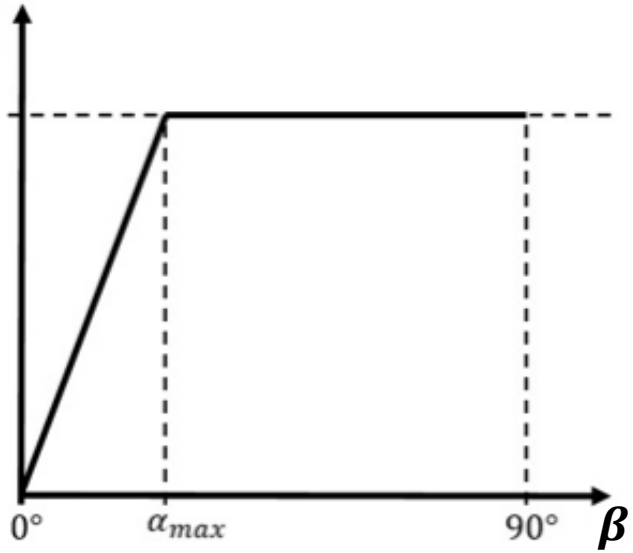
Tire Modeling

- Analytical - Brush, Fiala, Linear
  - Tire physical parameters are explicitly employed
  - Low precision, but simple
- Numerical
  - Look up tables instead of mathematical equations
  - No explicit mathematical form
  - Geometry and material property of tire are considered
- Parameterized – Linear, Pacejka, Dugoff
  - Need experiments for each specific tire
  - Formed by fitting model with experimental data
  - Match experimental data very well
  - Used widely for vehicle dynamics simulation studies and control design

Linear Tire Model

- Assumption: the relationship between slip angle and force is linear
  - Piecewise linear curves: $F(x) = \begin{cases} Cx & if\ |x| < x_{max} \\ F_{max} & if\ |x| \geq x_{max} \end{cases}$





Pacejka Tire Model

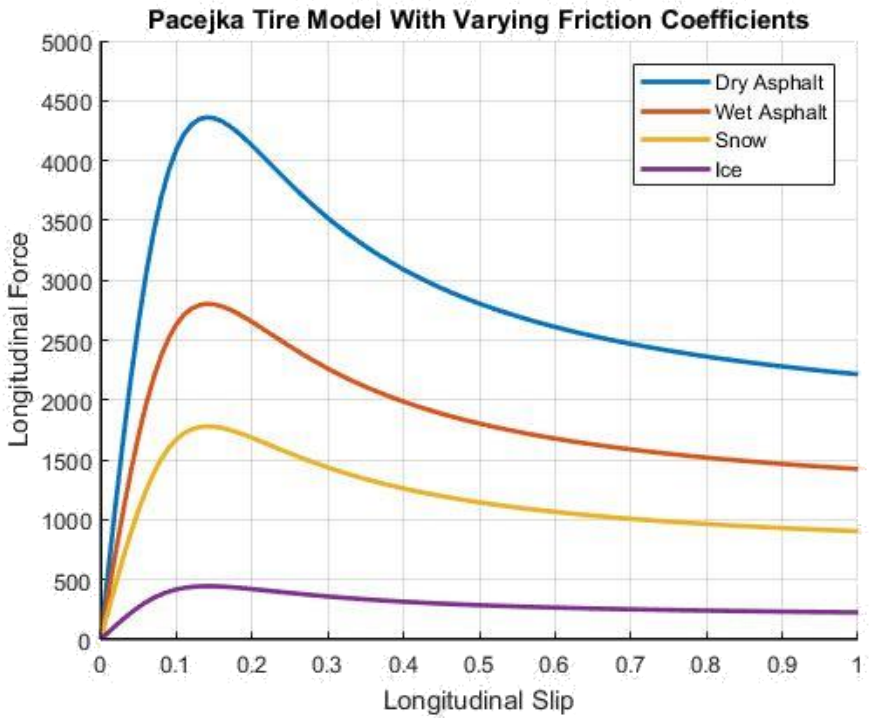
- Also called Magic Formula tire model
  - Widely used in model-based control development.

$$F(x, F_z) = D \sin(C \tan^{-1}(Bx - E(Bx - \tan^{-1}(Bx))))\mu F_z$$

tire vertical force

road friction coefficient

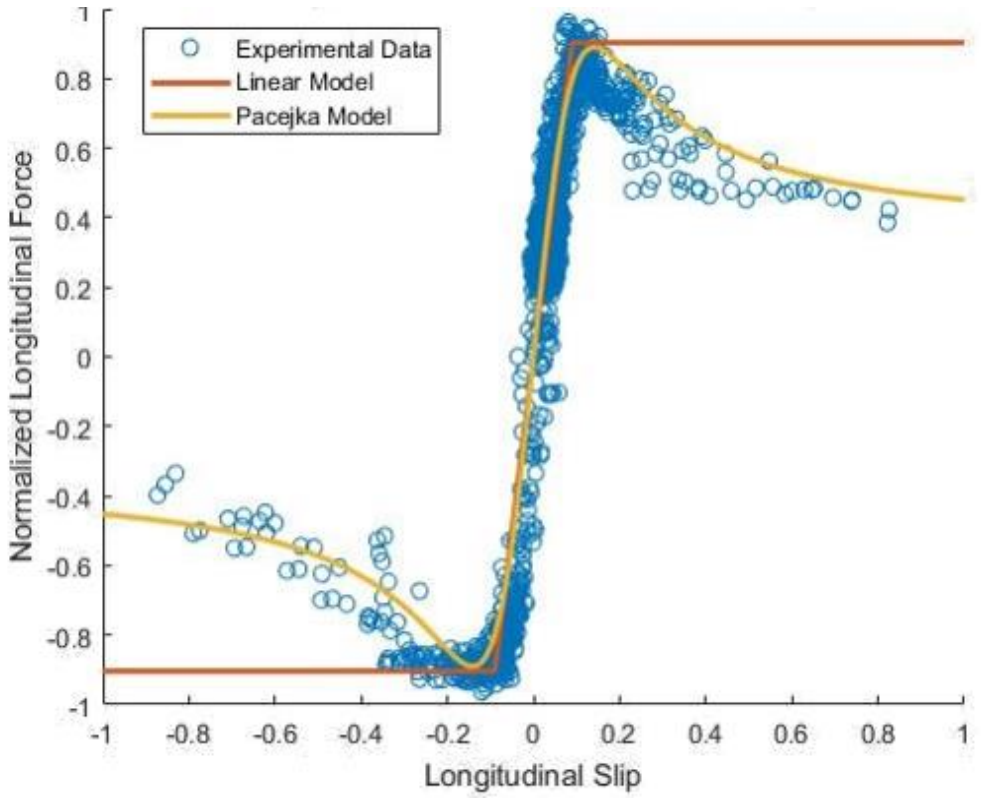
x could be either slip ratio or slip angle (in tire modeling)



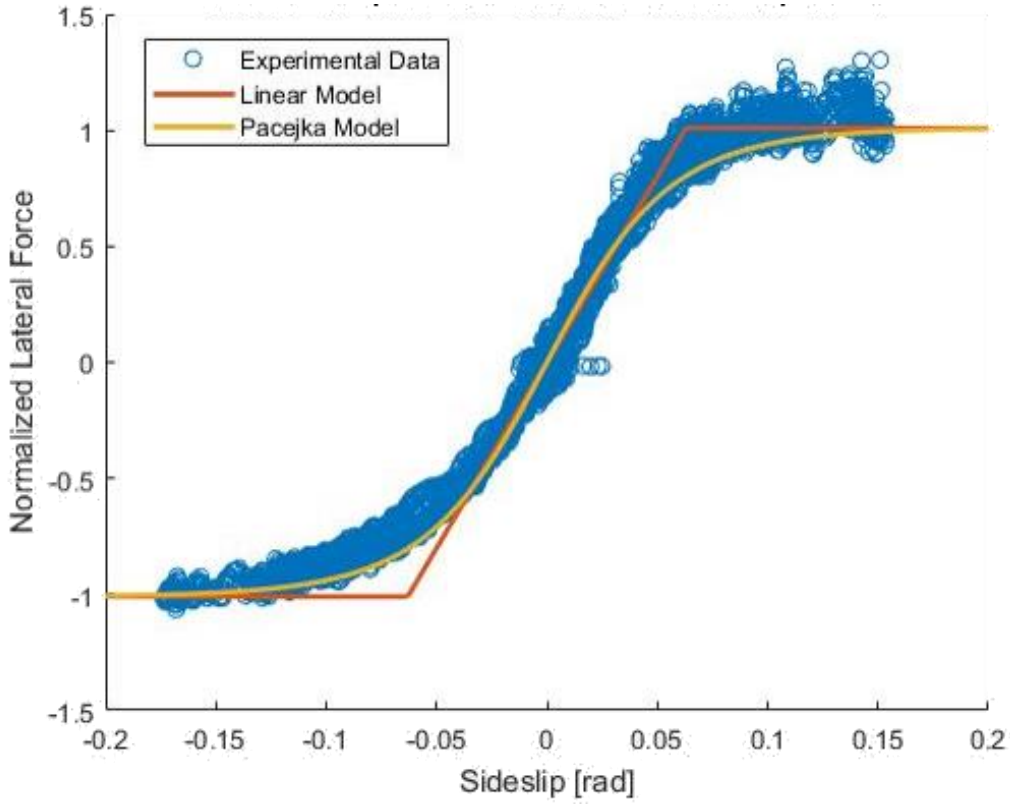
B – Stiffness Factor
C – Shape Factor
D – Peak Factor
E – Curvature Factor

Forces vs Slips

Normalized Longitudinal Force vs. Slip Ratio



Normalized Lateral Force vs. Slip Angle



Lesson Summary

What we have learned from this lesson:

- The role of tire in vehicle dynamics
- The terminology used in tire modelling such as slip angle and slip ratio
- The linear and Pacejka tire models

Module Summary

What we have learned from this module:

- Kinematic and dynamic modeling of vehicles
- Kinematic bicycle model
- Lateral and longitudinal dynamic modeling
- Actuator and tire subsystem modeling

What is next?

- The basics of controller design and its application to vehicle longitudinal control