Geometric Steering Control I – Pure Pursuit

Course 1, Module 6, Lesson 2

UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE & ENGINEERING

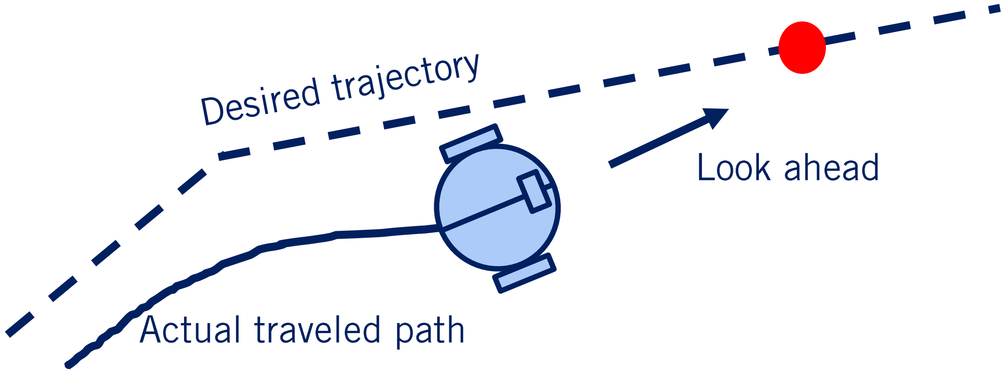
Learning Objectives

- In this video, you'll
  - Define the concept of a geometric path tracking controller
  - Develop a pure pursuit controller for path tracking

Geometric path tracking

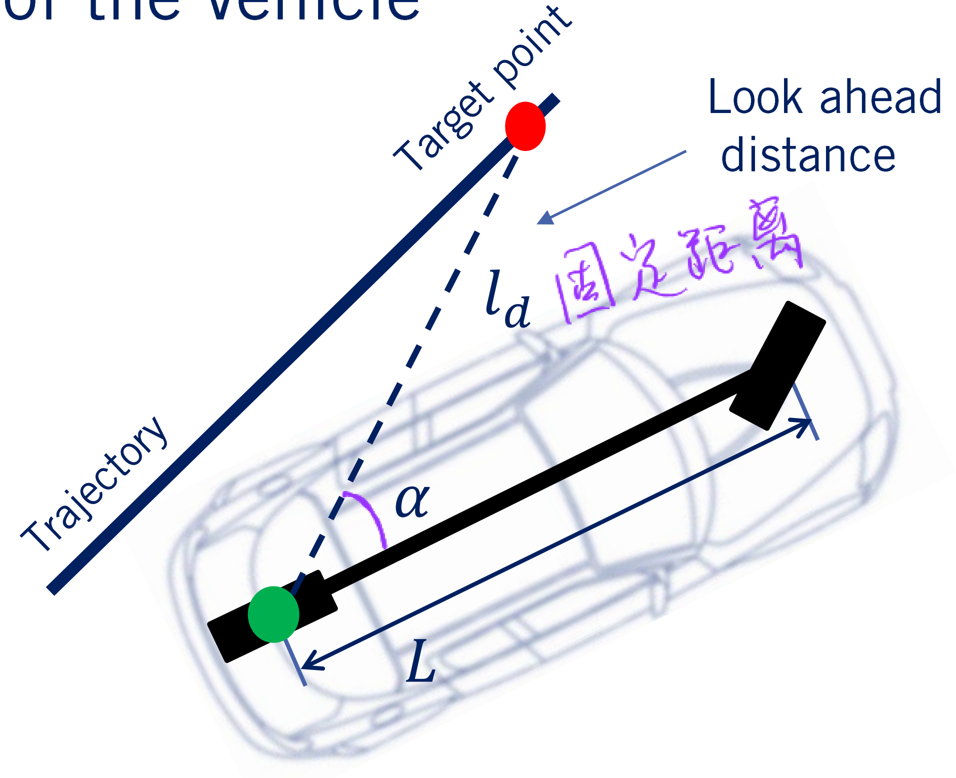
- One of the most popular classes of path tracking in robotics and autonomous vehicle
  - Exploits geometric relationship between the vehicle and the path resulting in compact control law solutions to the path tracking problem
  - Use of reference point on path to measure error of the vehicle, can be ahead of the vehicle

当无滑动时，几何轨迹跟踪效果也很不错.



Pure pursuit

- Pure pursuit method consists of geometrically calculating the trajectory curvature
- Connect the centre of rear axle location to a target point on the path ahead of the vehicle



Pure pursuit - formulation

- Steering angle determined by target point location and angle between the vehicle's heading direction and lookahead direction.
- From the *law of sines*:

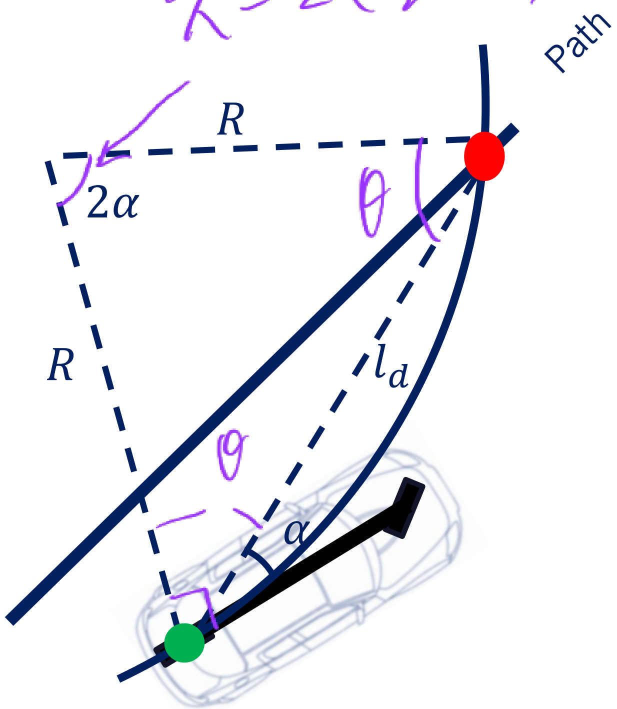
$$\pi - 2\left(\frac{\pi}{2} - \alpha\right) = \pi - \pi + 2\alpha = 2\alpha$$

$$\frac{l_d}{\sin 2\alpha} = \frac{R}{\sin\left(\frac{\pi}{2} - \alpha\right)}$$

$$\frac{l_d}{2\sin\alpha\cos\alpha} = \frac{R}{\cos(\alpha)}$$

$$\frac{l_d}{\sin\alpha} = 2R$$

$$\kappa = \frac{1}{R} = \frac{2\sin\alpha}{l_d}$$ Path curvature



$$\theta = \frac{\pi}{2} - \alpha$$

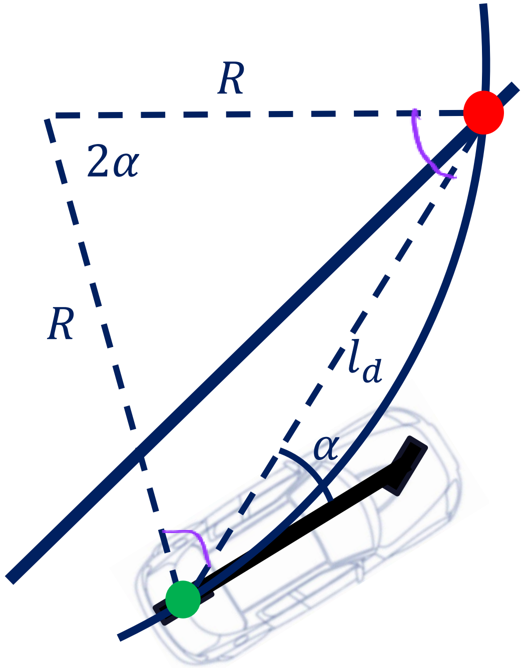
Pure pursuit - formulation

- Using the bicycle model the steering angle is calculated as:

$$\tan\delta = \frac{L}{R} \Rightarrow \delta = \arctan\frac{L}{R} \Rightarrow \delta = \arctan \kappa L$$

$$\underbrace{\kappa = \frac{2\sin\alpha}{l_d} \qquad \delta = \tan^{-1}\kappa L}$$

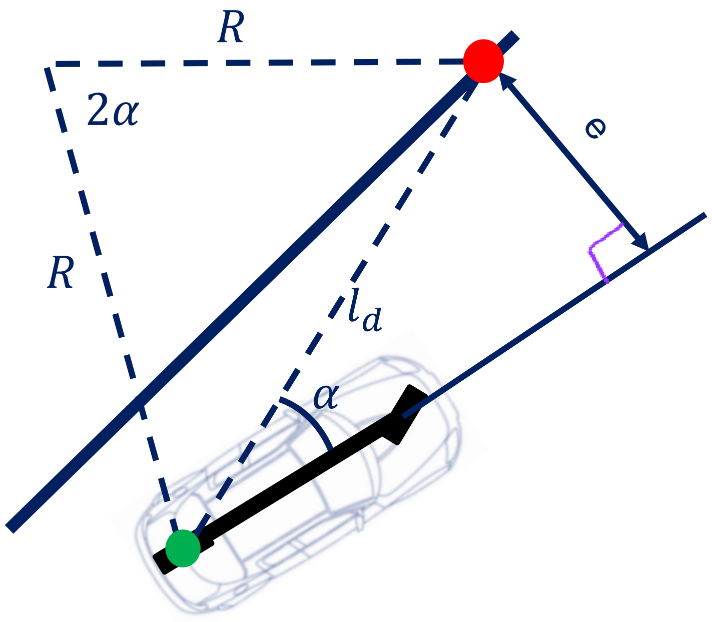
$$\delta = \tan^{-1}\left(\frac{2L\sin\alpha}{l_d}\right)$$



Pure pursuit - formulation

- Crosstrack error (e) is defined here as the lateral distance between the heading vector and the target point so:

$$\left.\begin{aligned} \sin\alpha &= \frac{e}{l_d} \\ \kappa &= \frac{2\sin\alpha}{l_d} \end{aligned}\right\} \quad \kappa = \frac{2}{l_d^2}e$$



Pure pursuit - formulation

- Pure pursuit is a proportional controller of the steering angle operating on a crosstrack error some look ahead distance in front of the vehicle

- The proportional gain $2/l_d^2$ can be tuned at different speeds (the l_d being assigned as a function of vehicle speed)

如果以恒定的增益，也就是ld不变的，
则车速为100和10的时转向角一样。

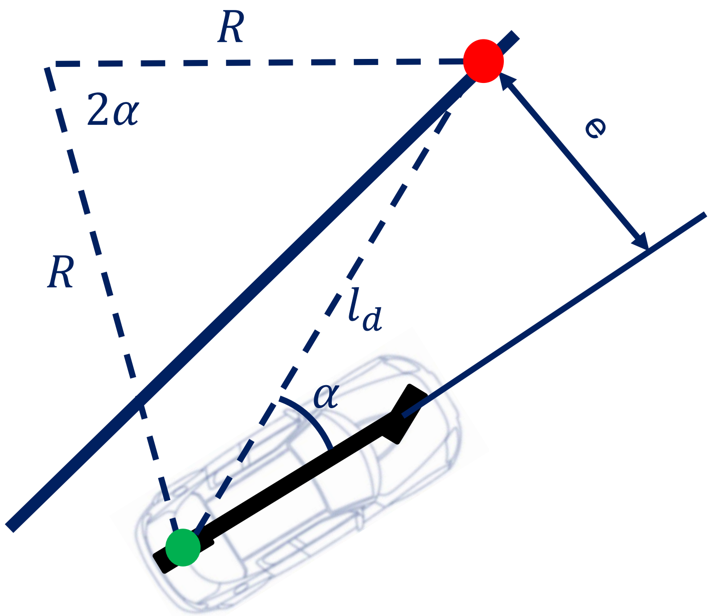
Pure pursuit - formulation

- Lookahead l_d is assigned as a linear function of vehicle speed: $l_d = K_{dd} v_f$

$$\delta = \tan^{-1}\left(\frac{2L\sin\alpha}{l_d}\right) \qquad \kappa = \frac{2}{l_d^2}e$$

$$\delta = \tan^{-1}\left(\frac{2L\sin\alpha}{K_{dd}v_f}\right)$$

Forward velocity



Summary

What we have learned from this lesson:

- The concept of geometrical path tracking and the pure pursuit method

What is next?

- A second geometrical path tracking method, the Stanley control approach