Basic Dynamic Modeling in 2D

Course 1, Module 4, Lesson 3

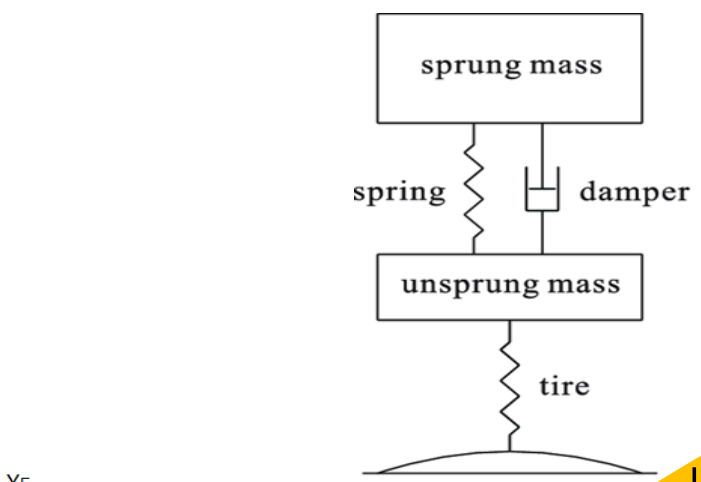
UNIVERSITY OF TORONTO
FACULTY OF APPLIED SCIENCE & ENGINEERING

Dynamic Modeling

- Why Dynamic Modeling is Important?
  - At higher speed and slippery roads, vehicles do not satisfy no slip condition
  - Forces such as drag, road friction govern required throttle inputs

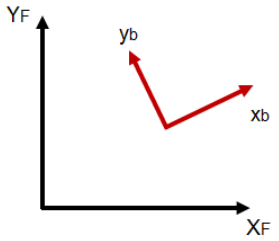
Dynamic Modeling

- Steps to build a typical dynamic model:



$$\sum F = ma$$

$$\sum \tau = J\alpha$$

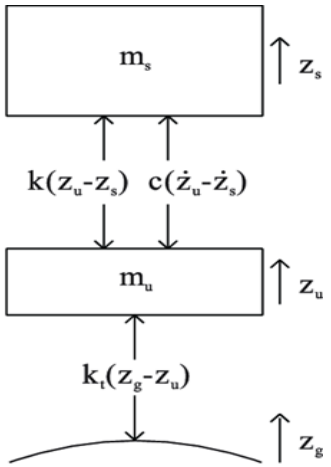


Coordinate frames

Lumped dynamic elements

Free body diagram

Dynamic equations

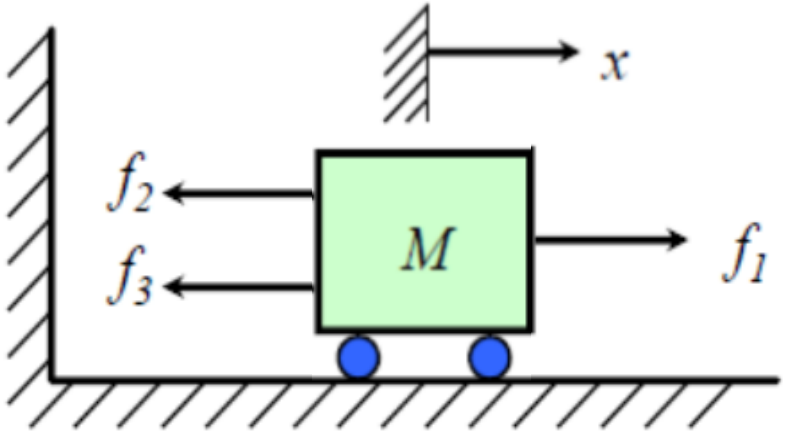


动力学建模的流程：

1. 确定坐标系统
2. 确定动态元素
3. 建立动态元素的受力图
4. 写出动力学方程

Dynamic Modeling - Translational System

- Deals with forces and torques
- Roughly, need to equate all forces
- Governed by Newton's second law



$$Ma = \sum F$$

$$M\ddot{x} = f_1 - f_2 - f_3$$

Example - Vehicle Shock Absorber (Suspension)

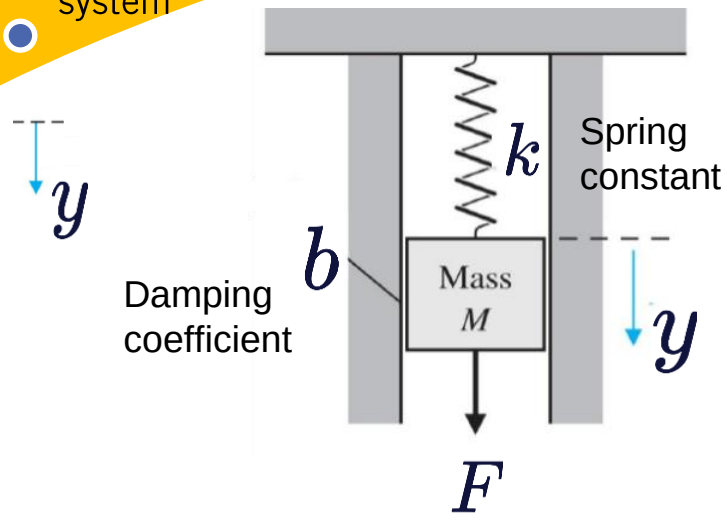
$$M\ddot{y} + b\dot{y} + ky = F$$

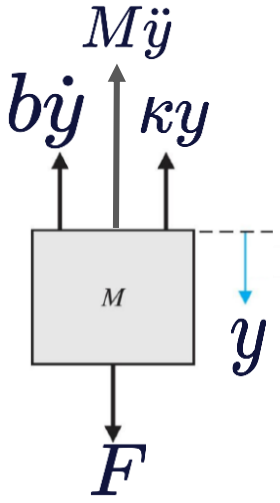
Coordinate system

Lumped dynamic elements

Free body diagram

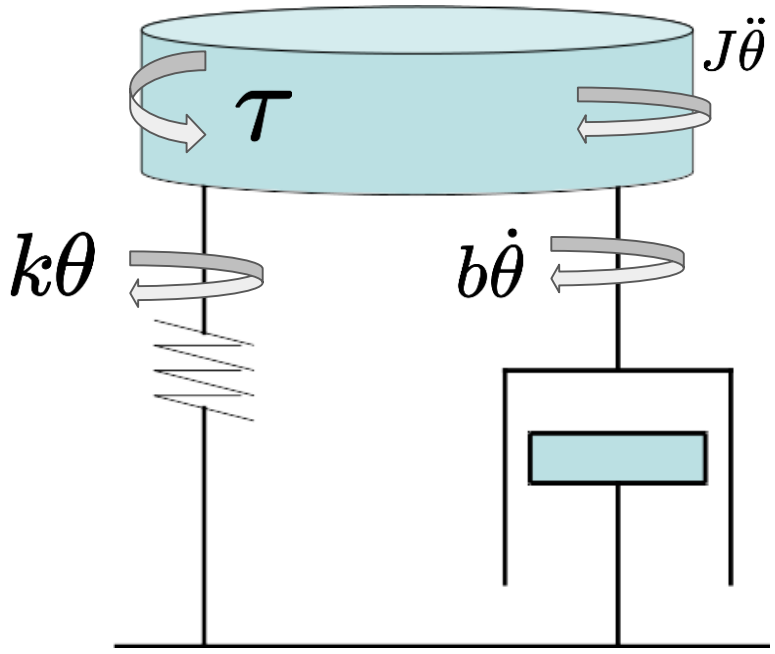
Dynamic Equation





Dynamic Modeling - Rotational Systems

- Inertia, J
- Torsional force, τ
- Forces resisting that torsional force
  - Spring force
  - Damping force
  - Inertia force



Example - Tire Model

$$J\ddot{\theta} + b\dot{\theta} + k\theta = \tau_{Drive} - \tau_{Load}$$

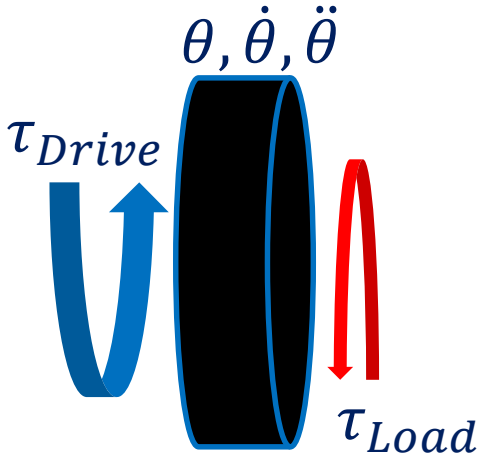
Coordinate system

Lumped dynamic elements

Free body diagram

Dynamic Equation

Tire inertia: J
Tire torsional stiffness: k
Tire torsional damping: b

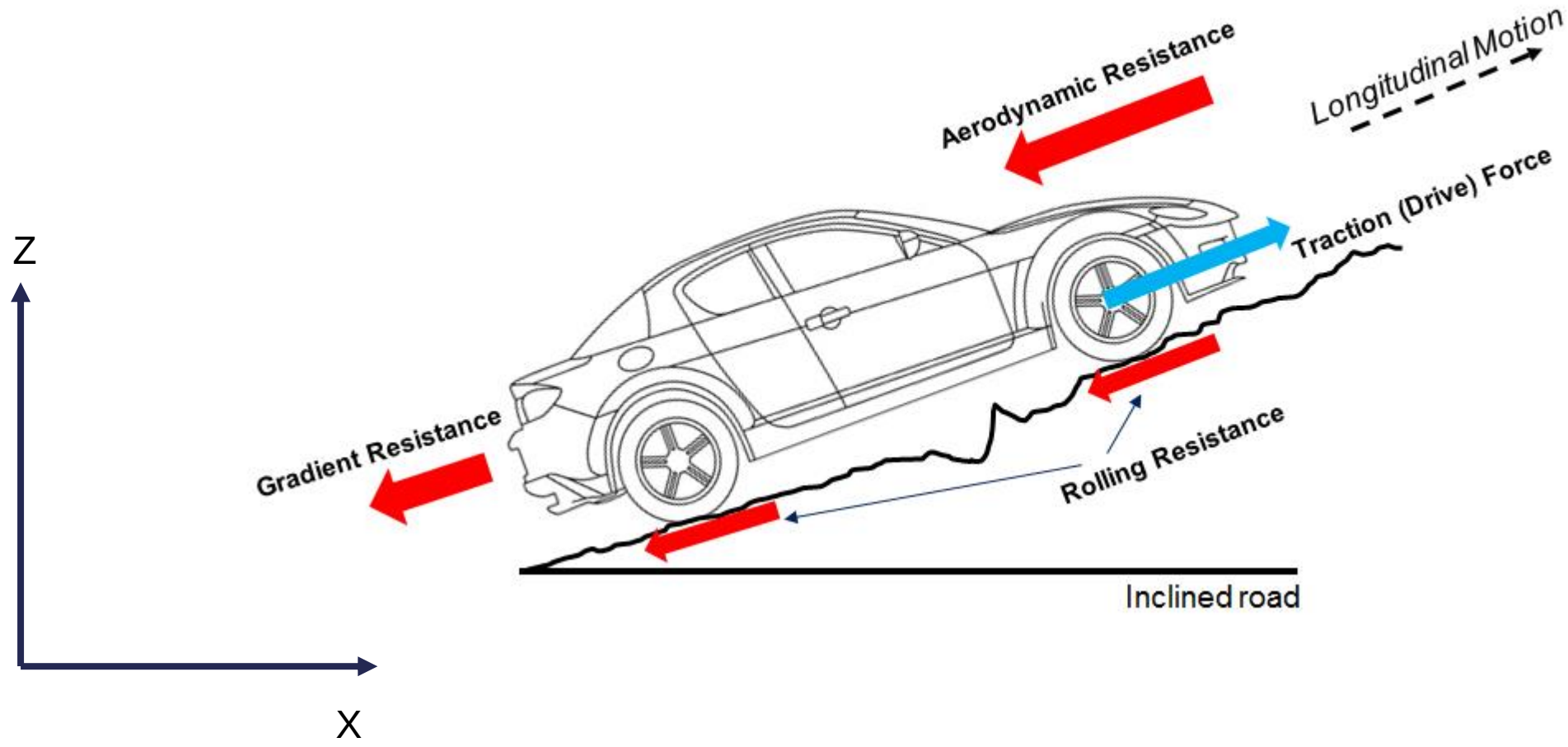


Full Vehicle Modeling

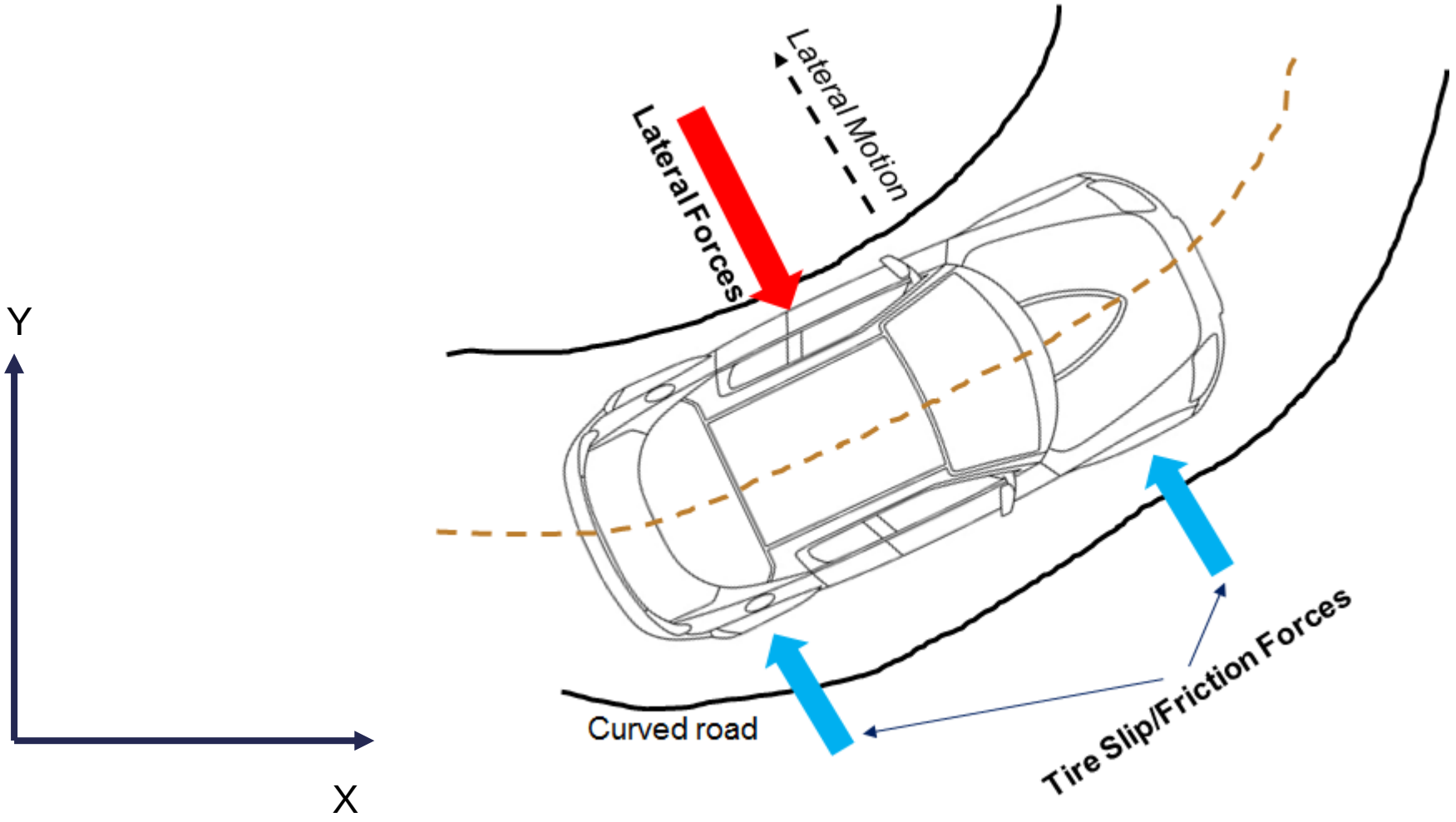
- All components, forces and moments in 3D
  - Pitch, roll, normal forces
  - Suspension, drivetrain, component models

Project Chrono

2D Dynamics - Vehicle Longitudinal Motion



2D Dynamics - Vehicle Lateral Motion



Summary

- What we have learned from this lesson?
  - Basics of 2D dynamic and how to start modeling a dynamical system, along with some application in automotive.

- What is next?
  - We will take a look at the vehicle longitudinal dynamics and drivetrain system