

# The Kinematic Bicycle Model

Course 1, Module 3, Lesson 2



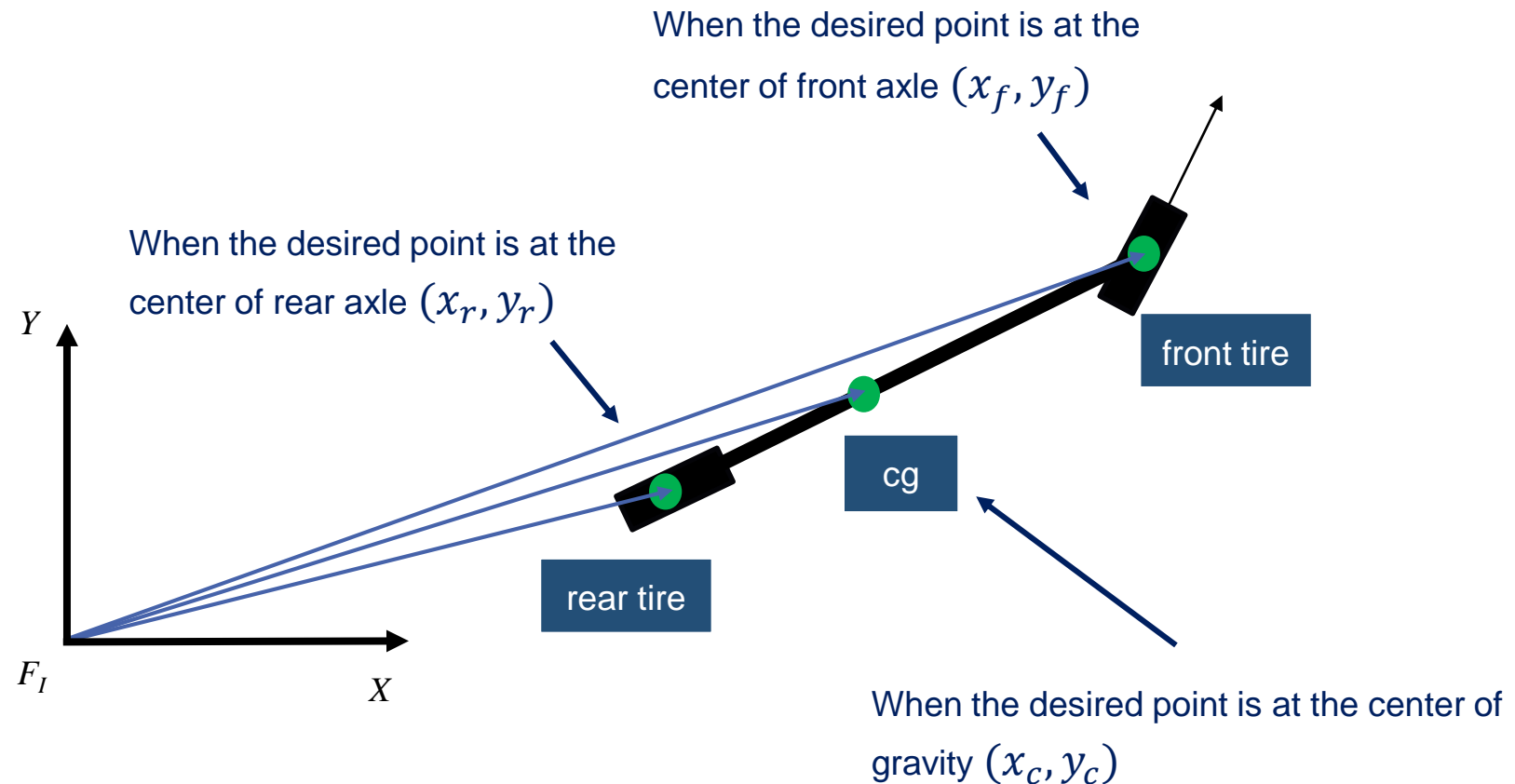
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# In this video...

- Learn about slip angle
- Develop the kinematic bicycle model

# Bicycle Kinematic Model

- 2D bicycle model (simplified car model)
- Front wheel steering



# Two-Wheeled Robot Kinematic Model

- Rear Wheel Reference Point
  - Apply Instantaneous Center of Rotation (ICR)

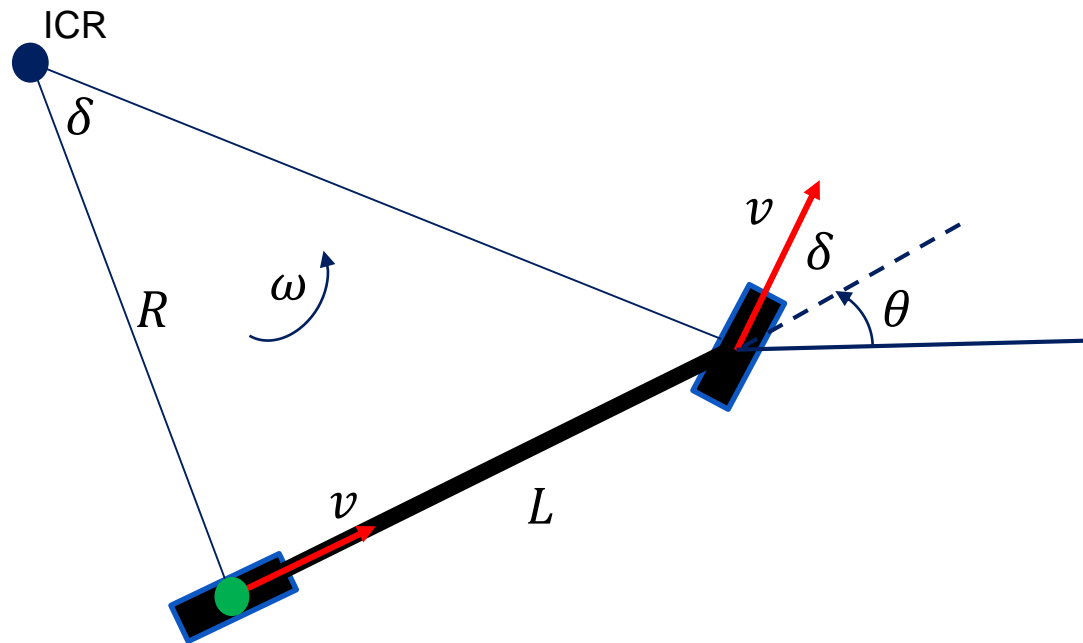
$$\dot{\theta} = \omega = \frac{v}{R}$$

- Similar triangles

$$\tan \delta = \frac{L}{R}$$

- Rotation rate equation

$$\dot{\theta} = \omega = \frac{v}{R} = \frac{v \tan \delta}{L}$$



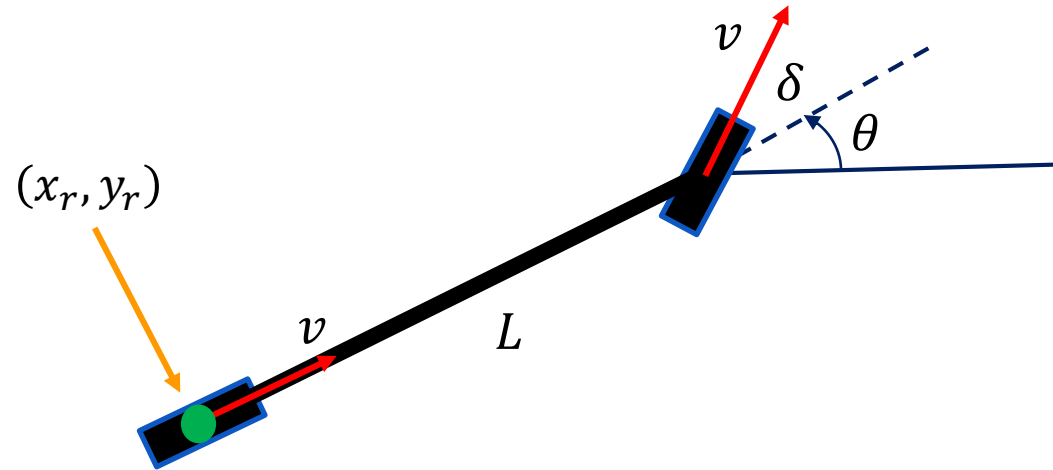
# Rear Axle Bicycle Model

- If the desired point is at the center of the rear axle

$$\dot{x}_r = v \cos \theta$$

$$\dot{y}_r = v \sin \theta$$

$$\dot{\theta} = \frac{v \tan \delta}{L}$$



# Bicycle Kinematic Model

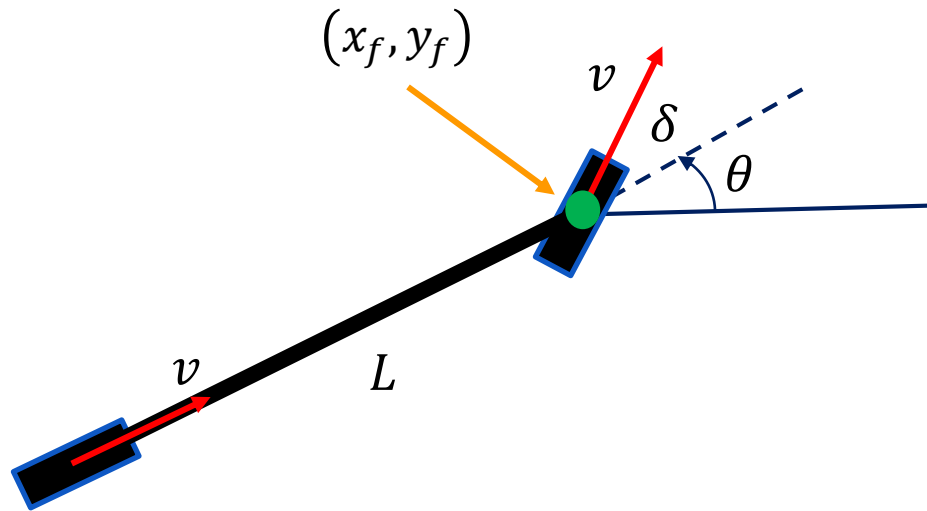
- If the desired point is at the center of the front axle

$$\dot{x}_f = v \cos(\theta + \delta)$$

$$\dot{y}_f = v \sin(\theta + \delta)$$

$$\dot{\theta} = \frac{v \sin \delta}{L}$$

$$R = \frac{L}{\sin \delta}$$



# Bicycle Kinematic Model

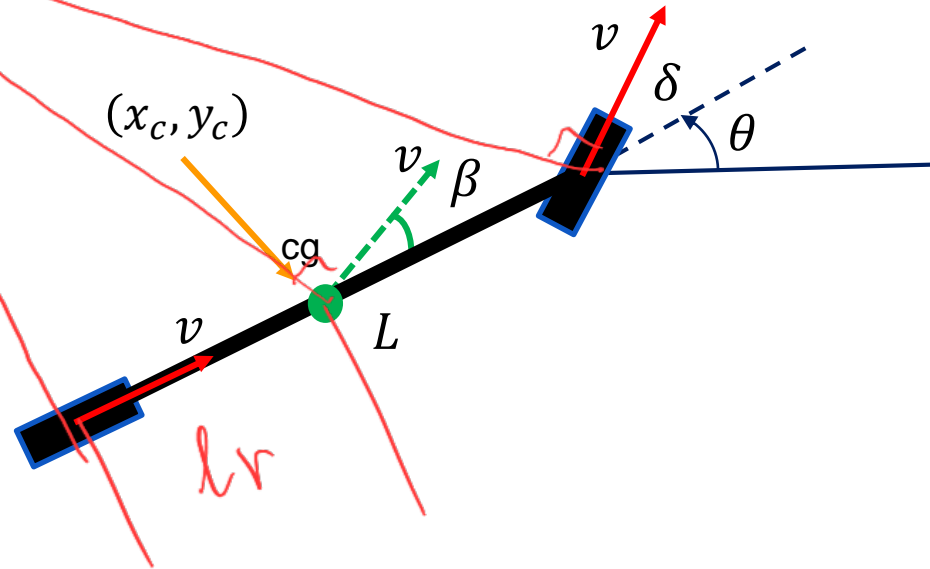
- If the desired point is at the center of the gravity (cg)

$$\dot{x}_c = v \cos(\theta + \beta)$$

$$\dot{y}_c = v \sin(\theta + \beta)$$

$$\dot{\theta} = \frac{v \cos \beta \tan \delta}{L}$$

$$\beta = \tan^{-1} \left( \frac{l_r \tan \delta}{L} \right)$$



# State-space Representation

- Modify CG kinematic bicycle model to use steering rate input

○ State:  $[x, y, \theta, \delta]^T$       Inputs:  $[v, \varphi]^T$

$$\dot{x}_c = v \cos(\theta + \beta)$$

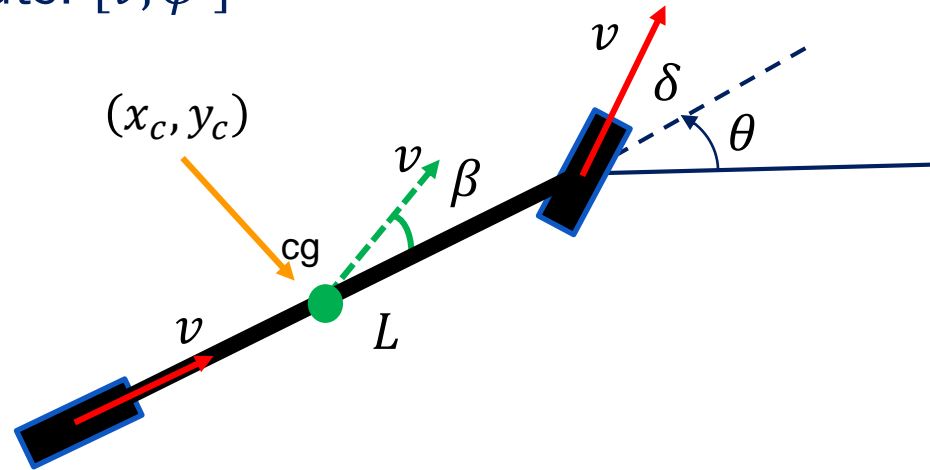
$$\dot{y}_c = v \sin(\theta + \beta)$$

$$\dot{\theta} = \frac{v \cos \beta \tan \delta}{L}$$

$$\dot{\delta} = \varphi$$

限制 转向角不能突变

Modified Input: rate of change of steering angle





# Summary

- What we have learned from this lesson:
  - Formulation of a kinematic bicycle model
  - State-space representation.
- What is next?
  - We will explore the Dynamics in 2D and how to define a vehicle model as a 2D dynamic system.