CS395T: Robot Learning from Demonstration and Interaction
Assignment 1 - Due at 11:59 PM on Sept 19th

In this assignment, you will be implementing Dynamic Movement Primitives (DMPs) to learn, generalize, and improve performance on a task. You should implement the modified version introduced by Pastor et al. (the paper from 8/30) described by equations 6-8 (and equation 4, which stays the same in this formulation). Instead of performing a task with a 6-DOF robot hand, you will be implementing a simpler version to control a point moving in 2D space.

Some things to keep in mind when implementing DMPs:

- Phase calculation: When calculating the phase variable $s(t)$, it may be helpful to recall that the differential equation $\dot{y}(t) = ky(t)$ can be solved as $y(t) = Ae^{kt}$, where $A = y(0)$, since $e^{k0} = 0$.
- Linked DMPs: To generate trajectories, you will use 2 different "linked" DMPs (one for each coordinate of the 2D pose) that all share the same phase variable, s (i.e. they are synchronized in time).
- Setting τ: When training the DMP, set τ to the length of the demonstration in seconds. When using the DMP to generate a new trajectory, set τ to the length of the desired trajectory in seconds.
- Setting α: Figure out how to set α such that the phase variable is 99% converged (i.e. $s = 0.01$) at time $t = \tau$ (the length of the trajectory in seconds).
- Setting K and D: You can experiment with different settings of K and D, but make sure that they are set relatively with respect to each other to be critically damped (i.e. $D = 2\sqrt{K}$).

Your main tasks are as follows:

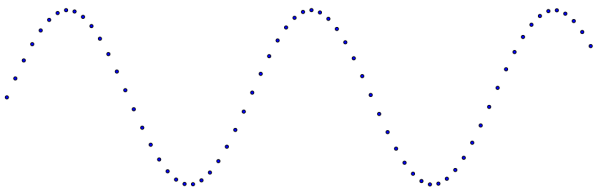
1. Implement two functions that perform the two main tasks of DMPs:
   (a) DMP learning: This function takes the following as input: a cartesian demonstration trajectory, and the spring / damping constants K and D. It then uses the demonstration to learn the parameters of a DMP, which are stored for later use.
   (b) DMP planning: This function takes the following as input: stored DMP parameters, a cartesian starting state, a starting velocity, a cartesian goal state, τ (the length of the desired plan in seconds), and dt (the time resolution of the plan). This function should return to the client a planned cartesian trajectory from the start to the goal that is τ seconds long and with waypoints spaced out every dt seconds. You only have to return a trajectory of time-stamped poses—you can ignore the corresponding velocities that the DMP calculates.
2. Function approximation: You will implement two different function approximators:

(a) When learning from a single demonstration, there is no need to use a regression-based function approximator for $f(s)$. Instead of using a weighted combination of basis functions, simply store values of $f_{target}(s)$. When looking up values of $f(s)$ during execution, simply perform linear interpolation between the 2 closest stored points (or when s corresponds to a stored value, simply return the stored value of $f_{target}(s)$ at that point.

(b) When learning from multiple demonstrations, it can be useful to use linear regression over a set of basis functions to learn the function $f(s)$. Implement Gaussian basis functions (also sometimes known a radial basis functions) as described in the Pastor et al. paper right below equation 3. **However, to simplify the problem to a standard linear regression problem, just learn $f(s) = \sum_i w_i \psi_i(s) s$ instead of the normalized version in equation 3**. You will have to choose an appropriate number of basis functions to use, as well as their spacing (in phase space) and width to optimize how well the function is approximated.

Now perform the following data collection and experiments:

1. Create a "demonstration" by generating (x,y) samples from a sine curve, starting at any point that you want, and ending at any "goal" point that you choose. Sample somewhat densely—collect at least 20 samples per cycle of the sinusoid (i.e. crest to crest) and collect data for at least 2 full cycles of the sinusoid. Remember that each data point has to have a time stamp associated with it as well. Separate each data point by 0.1 seconds, starting a time $t = 0$. Plot your data and include it in your report. The data should look something like the following, but with proper axes, etc.:



2. Use your DMP learning code with the linear interpolation approximator to learn from this data. First, use the same start and goal as you did during the demonstration and generate a new trajectory with your DMP planning code, using the same τ, K, D, and α as you did for learning. Plot this new trajectory on a graph that contains the demonstration trajectory as well, showing that they are similar.

3. Do the same with a significantly different start and goal.

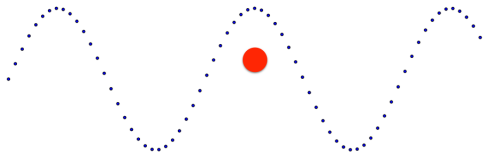
4. Repeat the same experiment, but with a trajectory that is half as fast and twice as fast, respectively. Describe how you did this and show a plot for each, similar to the ones above.

5. Create a second demonstration trajectory that is identical to the first, but with a small amount of zero-mean Gaussian noise added to each point (add noise to both the x and y coordinates). Plot this along with the original demonstration

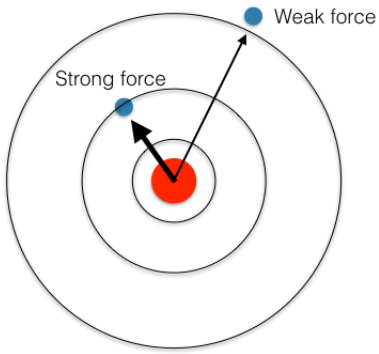
6. Use your DMP learning code with the Gaussian basis functions to learn from both demonstrations simultaneously by just combining both trajectories into a single list of data points to

learn $f(s)$. Test out different numbers of basis functions, different strategies for spacing them out (i.e. should they be spaced out evenly in phase space for the best performance? Why or why not?), and different widths. List the final parameters you chose and qualitatively describe how you went about choosing them and justify your spacing strategy. Use DMP planning to reproduce a trajectory from the same start and goal as the demo. How accurate is the reproduction? Change the start and goal. Does generalization still work as expected?

7. Introduce an "obstacle" into the original (non-noisy) demonstration such that the trajectory appears to avoid it, as shown here:



This obstacle can be introduced via a coupling term, similar to the one described by Pastor et al. in Equation 10. However, you will do a simplified 2D version of this—implement a coupling term that exerts an acceleration on the current point's location based on its distance from the center of the obstacle (this acceleration should weaken with distance as a Gaussian that is strongest at the obstacle center) and that exerts this acceleration in a direction along the vector from the obstacle to the current point location. The following image illustrates this:



After adding the coupling term, use DMP learning on the obstacle demonstration with the linear interpolation approximator. Plot 2 graphs to understand how this coupling term will change behavior of the DMP:

(a) Generate and plot a plan with no obstacle present. How has the trajectory changed due to having been learned with an obstacle and coupling term? Explain the reason for the change in your own words.

(b) Generate and plot a plan with the obstacle in a different position, and plot the location of the obstacle as well, in a different color. Describe how the trajectory changes and why.

Finally, answer the following additional questions:

1. Under what conditions did your DMP generalize properly to new starts and goals? Did it execute the motion in the same "style" that it was demonstrated? Why or why not?

2. Here, we simply have generated a static trajectory with the DMP. How would you go about implementing a DMP that calculates commands on-the-fly, so that the robot could respond to perturbations during execution? In this setting, how would the magnitude of constants K and D affect the behavior of the system under a perturbation?

3. If a learned DMP was used to solve a problem with a well-defined reward or cost function, policy search might be one way to iteratively improve performance. For example, you might randomly perturb DMP parameters, accepting the change is performance is better, and rejecting the change if it is worse. Under this approach, which parameters of the DMP would you randomly perturb, and why?

Be sure to include all relevant code as an appendix to your writeup.