Week 5: PID Controllers: Theory and Applications

AE 315 - Systems and Control

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Outline

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Introduction

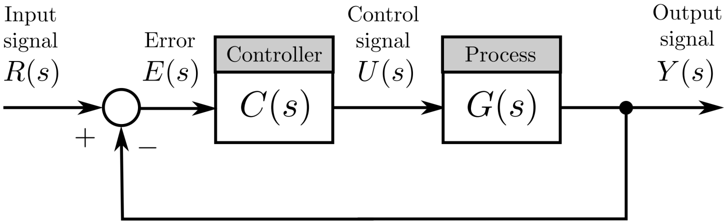

Figure: Block diagram of a process with feedback controller

1. The three-term controller, i.e. **proportional-integral-derivative (PID)** controller, is a control loop feedback mechanism widely used in many industrial control systems.
2. The PID controller algorithm's three parameters can be **tuned** to offer control action **tailored** to the needs of a particular process.

Outline

Basic control functions

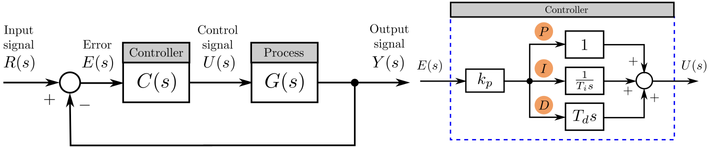

Figure: Block diagram of feedback control system with PID controller

The input/output relation for a **PID controller**

$$
\begin{aligned}
u(t) &= k_p \left[e(t) + \frac{1}{T_i} \int_0^t e(t)dt + T_d \frac{de(t)}{dt} \right] \\
&= k_p e(t) + k_i \int_0^t e(t)dt + k_d \frac{de(t)}{dt}
\end{aligned} \quad (1)
$$

- $e(t) = r(t) - y(t)$ – is the error signal, $u(t)$ – is the control signal.
- k_p – is the proportional gain, T_i – is the integral time, T_d – is the derivative time
- $k_i = \frac{k_p}{T_i}$ – is the integral gain, $k_d = k_p T_d$ – is the derivative gain.

Basic control functions

The general transfer function of the **PID** can be written as

$$\boxed{C(s) = \frac{U(s)}{E(s)} = k_p + \frac{k_i}{s} + k_d s} \tag{2}$$

1. **P Controller** $(k_d = k_i = 0)$

$$\boxed{C(s) = k_p}$$

2. **PI Controller** $(k_d = 0)$

$$\boxed{C(s) = k_p + \frac{k_i}{s}}$$

3. **PD Controller** $(k_i = 0)$

$$\boxed{C(s) = k_p + k_d s}$$

$\longrightarrow$ Gentle introduction to PID controllers: https://www.youtube.com/watch?v=4Y7zG48uHRo.

Effects of the PID gain values

CL RESPONSE	RISE TIME	OVERSHOOT	SETTLING TIME	S-S ERROR
Kp	Decrease	Increase	Small Change	Decrease
Ki	Decrease	Increase	Increase	Eliminate
Kd	Small Change	Decrease	Decrease	Small Change

Figure: The effect of PID gains on the system's step response

Outline

Solving ODEs Cont.

Example (Case Study)

Consider the mechanical system depicted in the figure. The input signal is given by the external force $F(t) = u(t)$ N for $t \geq 0$ acting on the mass $m = 1$ kg. The displacement $x(t)$ of the mass is the output signal. The displacement is measured from the equilibrium position in the absence of the external force. Let $k = 5\ N/m$ be the spring constant, $c = 2\ Ns/m$ be the damping coefficient.

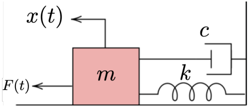

Figure: Mechanical system

Step response

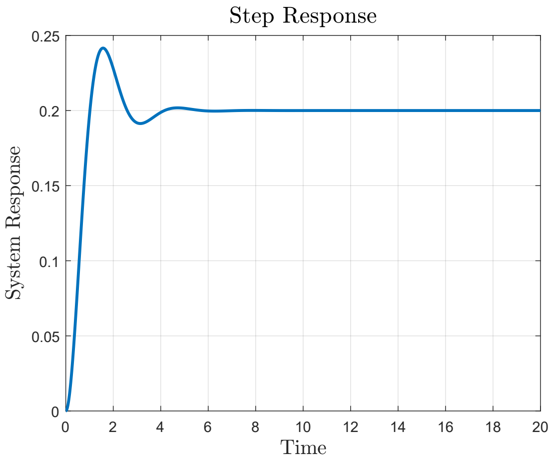

Figure: Step response of the system

P controller

Effects of P controller

1. Proportional term, P, causes a **corrective** control actuation proportional to the error.
2. The system with P controller will usually have **nonzero** steady-state errors. As k_p increases, then the static position error decreases. It is difficult to ensure both **good transient response** accuracy and **steadystate performance**.

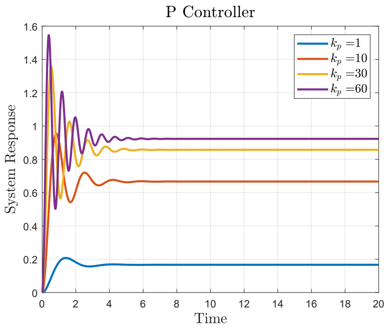

Figure: P controller characteristics

PD Controller

1. The derivative term, D, gives a **predictive capability**.
2. Derivative action tends to have a **stabilizing effect**.
3. PD controller **damps** the behaviour and **reduces maximum overshoot**
4. It **improves the transient response.**

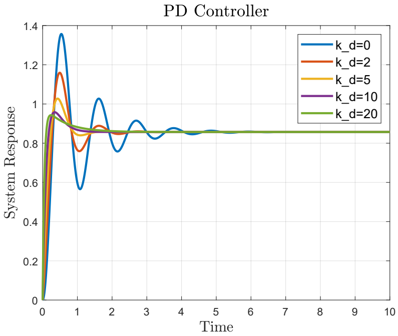

Figure: PD controller chs.

PI Controller

1. The integral term, I, gives a correction *proportional to the integral* of the error.
2. PI **reduces the steady-state error** as compared to P controller.
3. The transient response may be **slowed down** as an effect of PI controller.

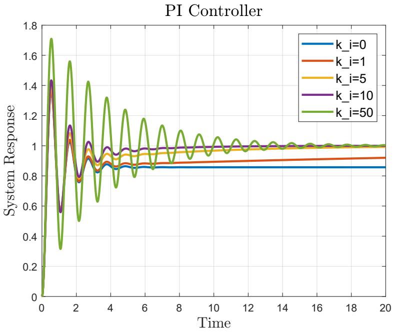

Figure: PI controller chs.

PID combined controller

1. The features of each of the PI and PD controllers are utilized.
2. Improves both steady-state errors as well as transient response specifications.
3. The PID controller algorithm involves three separate constant parameters that need to be tuned.

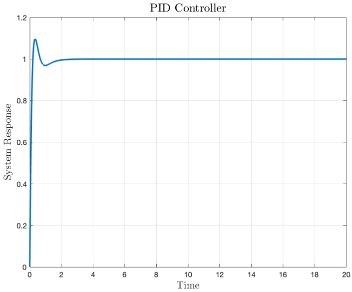

Figure: PID controller Chs.