Week 3 Tutorial Notes: Mathematical Modeling and Simulation

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Abstract

This document introduces focuses on the utilization of MATLAB and SIMULINK as for modeling and simulating control systems, with a specific emphasis on thermal and fluidic systems. The content includes figures illustrating the effect of various parameters on a the systems on hand

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1 Modeling Thermal Systems

A thermal system is one in which energy is stored and transferred as thermal energy ,commonly called heat. Thermal system may include heating, cooling or mixing processes where heat can be added or removed to maintain required temperature. Thermal systems operate due to temperature difference, allowing heat energy to move from a source at a higher temperature to a destination at a lower temperature. The foundation of thermal system models relies on the principle of heat energy conservation, coupled with the ideas of thermal resistance and thermal capacitance.

Given a thermal system shown in figure 1, this system involves a fluid-filled tank positioned on a heater that transfers energy to the fluid. The amount of energy delivered by the heater is directly influenced by the voltage it receives as an input. applying heat balance equation

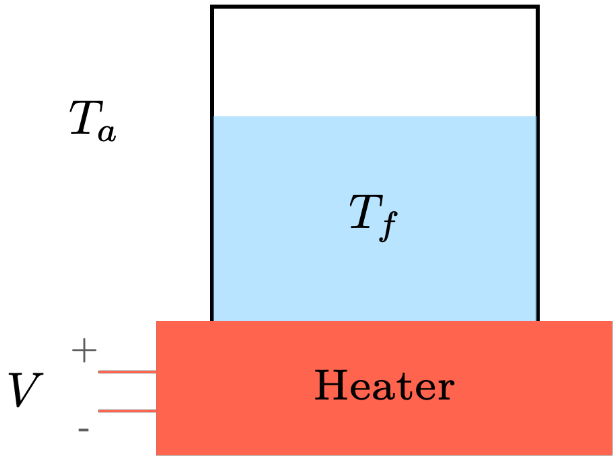

Figure 1: Thermal system

which states that the energy supplied by the heater should be equal to the energy stored by the fluid and the energy lost to the surrounding environment. This can be represented as shown in equation 1.

$$q_{\text{in}} = q_{\text{stored}} + q_{\text{lost}} \tag{1}$$

where q_{in} is the rate of heat provided by the heater, q_{stored} is the rate of heat stored in the fluid and q_{lost} is the rate of heat lost due to resistance. The rate of heat supplied by the heater is proportionally related to the voltage. It can be expressed as equation 2

$$q_{\text{in}} = kV \tag{2}$$

where k is a constant given by the heater manufacturer and V is the voltage drop. And q_{stored} can be represented as shown in equation 3

$$q_{\text{stored}} = c\frac{dT}{dt} \tag{3}$$

where c is the thermal capacitance. Also, q_lost is the energy dissipated in the thermal resistance. The thermal resistance (R_{tl}) can be represented as shown in equation 4

$$q_{\text{lost}} = \frac{1}{R_{\text{t}}}(T - T_a) \tag{4}$$

where R_t is the thermal resistance, T is the fluid temperature and T_a is the ambient temperature. By substituting equations 2, 3 and 4 in equation 1, we get the differential equation for this thermal system.

$$\begin{aligned} kV &= c\frac{dT}{dt} + \frac{T - T_a}{R_t} \\ \frac{dT}{dt} &= \frac{1}{c}\left(kV - \frac{T - T_a}{R_t}\right) \end{aligned} \tag{5}$$

Finally, equation 5 is a first order differential equation which can be modelled and simulated using ODE45 in MATLAB.

The MATLAB code used to simulate this system is shown below,

Listing 1: Fluid system Simulation

```
clear all; clc; close all;

% Initial condition
T0 = 25; % Initial current (in Amperes)

% Time span
tstart = 0; % Start time (in seconds)
tend = 10;   % End time (in seconds)

% System params
Vs   = [1 5 10 15 20]; % Volt
k    = 25;
T_a = 25;             % Degree c
c    = 1;
R_t = 1;


% Plotting
fig = figure(); % Initialize a figure
hold on
for idx = 1:length(Vs)
    V = Vs(idx);

    % Solve the differential equation using ode45
    [t, T] = ode45(@(t, T) fluid_heater(t, T, V, R_t, T_a, k, c),
        ...
        [tstart, tend], T0);

    % Plotting
    plot(t, T,'LineWidth', 2);
    legend_info{idx} = sprintf('$V = %.2f$', V);
end
legend(legend_info, 'Interpreter', 'latex', ...
    'FontSize', 16, 'location', 'best');

set(fig, 'color', 'w') % Set the background color to be white
xlabel('Time (s)', 'Interpreter', 'latex', 'FontSize', 18);
ylabel('T degree c', 'Interpreter', 'latex', 'FontSize', 18);
set(gca, 'FontSize', 16, 'TickLabelInterpreter', 'latex');


```

```
function dT_dt = fluid_heater(t, T, V, R_t, T_a, k, c)
    % states = T
    dT_dt = 1/c*((k*V) - (T - T_a)/R_t);
end
```

The code output is shown in figure 2.

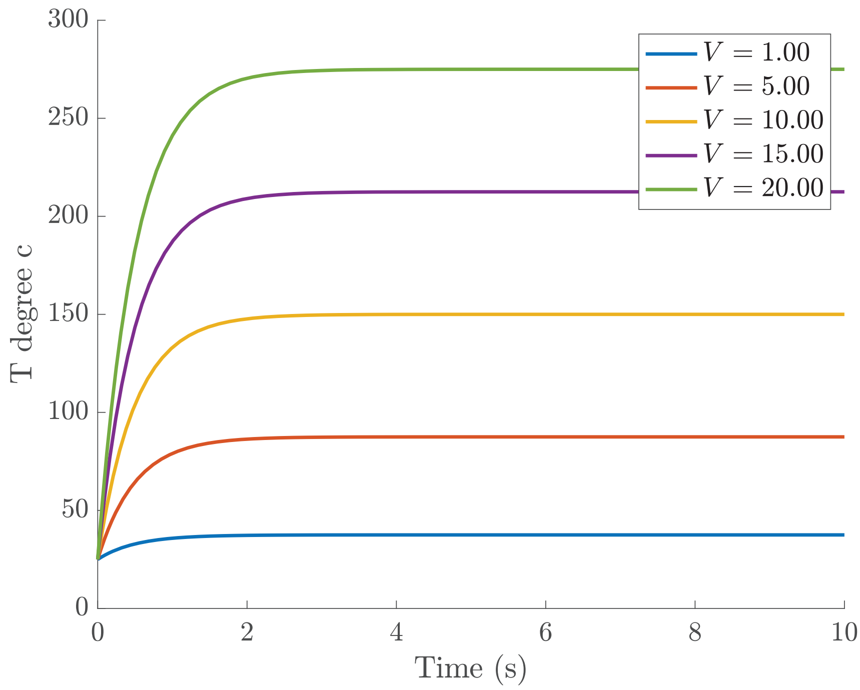

Figure 2: Thermal system simulation

Equation 5 can be modelled in SIMULINK as shown in Figure 3

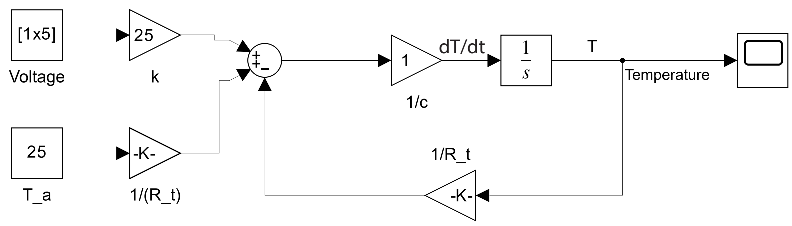

Figure 3: Modelling Thermal System on SIMULINK

2 Modeling Fludic Systems

Consider a fludic system which contains a storage tank with a specific cross-sectional area A holding a liquid at a particular height h. Liquid enters the tank from the top and exits through a valve at the bottom, which has a fluid resistance denoted as R. The rates at which liquid flows in q_i and out q_o of the tank are important parameters. The fluid density, represented as ρ, remains constant throughout the process. Figure 4 provides a visual illustration of this tank system. In this context, our goal is to estimate the resistance of various valves. It's important to note that we can control the water level by adjusting q_i, which serves as the input, while the output of the system is the water level, h. Fluid resistance R is a form of resistance that opposes the relative motion of fluids. It's encountered in various components like flow orifices,

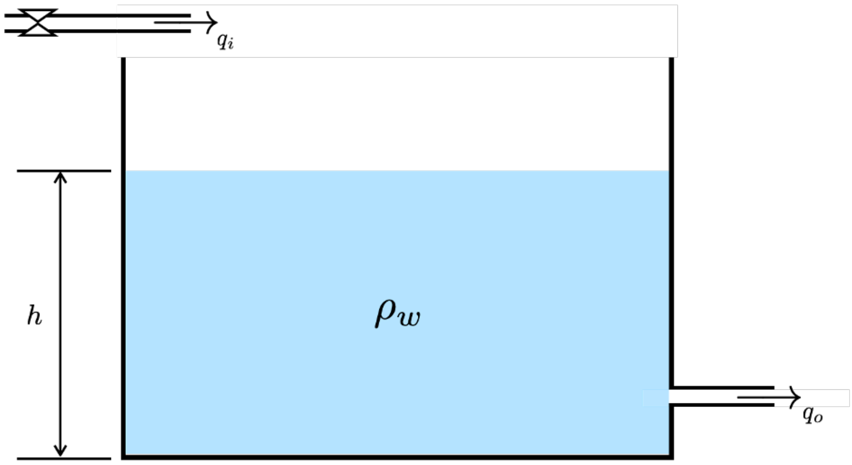

Figure 4: Fluid system

valves, and fluid lines. Similar to electrical systems, you can liken the pressure drop ($P1 - P2$) to a voltage drop (ΔV), the flow rate (q) to the current (I), and the fluid resistance (R) to the electrical resistance (R). So,

$$P_1 - P_2 = qR \tag{6}$$

$$R = (P_1 - P_2)/q \tag{7}$$

By starting from continuity equation and applying conservation of mass law that states the time rate of change of fluid mass inside a tank equals the inlet mass flow rate minus the the outlet mass flow rate, we get:

$$\frac{d}{dt}(\rho_w A h) = \rho q_i - \rho_w q_o \tag{8}$$

$$\rho_w A \frac{dh}{dt} = \rho_w q_i - \rho_w \frac{\Delta P}{R} \tag{9}$$

where

$$A \frac{dh}{dt} = q_i - \frac{\rho_w g h}{R} \tag{10}$$

then, we have

$$\frac{dh}{dt} = \frac{q_i}{A} - \frac{h}{A R_f} \tag{11}$$

where $R_f = \frac{R}{\rho_w g}$.

Fortunately, equation 11 is a first order differential equation which can be modelled and simulated easily using ODE45 function in MATLAB.

The MATLAB code used to simulate this system is shown below,

Listing 2: Fluid system Simulation

```
clear all; clc; close all;

% Initial condition
h0 = 0; % Initial current (in Amperes)

% Time span
tstart = 0; % Start time (in seconds)
tend = 10;    % End time (in seconds)

% System params
R_fs = [0.25 0.5 1 1.25];
```

```
A = 2;
q = 1;

% Plotting
fig = figure(); % Initialize a figure
hold on
for idx = 1:length(R_fs)
    R_f = R_fs(idx);

    % Solve the differential equation using ode45
    [t, h] = ode45(@(t, h) fluidic_system(t, h, R_f, A, q), [
       tstart, tend], h0);

    % Plotting
    plot(t, h,'LineWidth', 1.5);
end

set(fig, 'color', 'w') % Set the background color to be white
xlabel('Time (s)', 'Interpreter', 'latex', 'FontSize', 18);
ylabel('h (m)', 'Interpreter', 'latex', 'FontSize', 18);
set(gca, 'FontSize', 16, 'TickLabelInterpreter', 'latex');
legend('$R_f = 0.25$', '$R_f = 0.5$', ...
    '$R_f = 1$', '$R_f = 1.25$', ...
        'Interpreter', 'latex', 'FontSize', 16, ...
        'location', 'best');

function dh_dt = fluidic_system(t, h, R_f, A, q)
    % states = h
    dh_dt = (q/A) - (h/(A*R_f));
end
```

The code output is shown in figure 5.

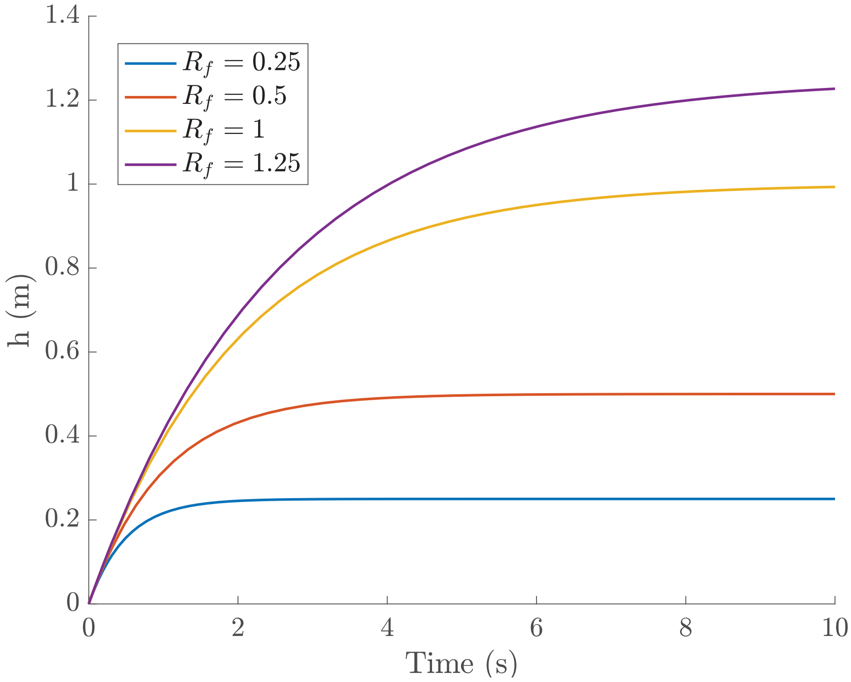

Figure 5: Fluid system simulation

Equation 11 can be modelled in SIMULINK as shown in Figure 6

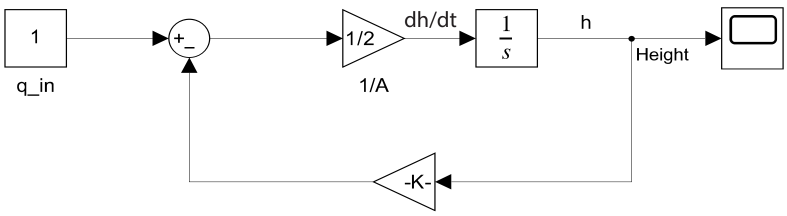

Figure 6: Modelling Fluid System on SIMULINK