Systems and Control - AE 315, 231

Week 4: Assignment No. 5 (Due 4 October 11:59 p.m.)

King Fahd University for Petroleum and Minerals - Aerospace Dept.

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Assignment Instructions

1. Attempt all the presented questions for partial grades.
2. Deliverables:
   (a) The **MATLAB/SIMULINK** file.
   (b) A **report** showing your work (.pdf). Please stick to the formal report format (cover page, table of contents, introduction, ...)
   (c) Name your files according to this format: `AE_315__Your_Name__HW_#.(pdf/m)`

1 Pendulum introduction

The pendulum in figure 1 has stable and unstable equilibria - upright and inverted positions. Upright yields stable behaviour against perturbations. Inverted configuration results in unstable falling from disturbances, posing a control challenge. Simulating the pendulum's dynamics shows this behavior. Analysis provides control insights to stabilize the unstable inverted configuration, addressing a classic problem.

Consider the following figure 1 with the following equations of motion

$$ml^2\ddot{\theta} + b\dot{\theta} + mgl\sin\theta = \tau \tag{1}$$

where $m = 0.5\ kg$, $l = 1.5\ m$, $b = 2\ Nms/rad$, and $g = 9.8\ m/s^2$.

The system can be linearized using small angle approximation to take the form of

$$ml^2\ddot{\theta} + b\dot{\theta} \pm mgl\theta = \tau \tag{2}$$

where, reference to eqn. (2), indicate that '+' refers to the upright pendulum and '-' to the inverted.

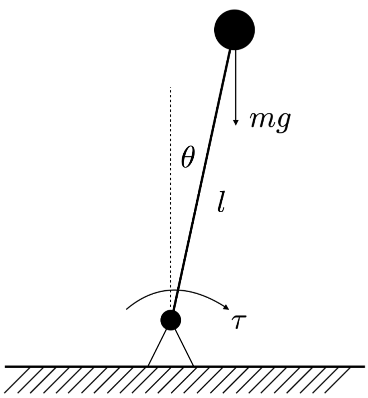

Figure 1: Pendulum schematic

2 Objectives

1. (1 point) Derive the transfer function of the system in figure 1 based on the linearized equations of motion.

2. (1 point) Discuss the stability of the upright and inverted configurations using the system's transfer functions poles. Plot and comment on the poles of the system with the help of MATLAB function `pzmap`.

3. (2 point) Check the open loop step response of the two configurations and discuss the findings using plots.

4. (6 points) Design a PID controller to stabilize the unstable configurations, then examine the step response of the over all closed loop transfer function and report the findings as follows:

   (a) Proportional Controller

Proportional Gains	Rise Time (ms)	Overshoot (%)	Steady state error
10			
20			
30			
50			
70			

Table 1: Proportional controller

   (b) Proportional – Derivative Controller

Proportional Gains	Derivative Gains	Rise Time (ms)	Overshoot (%)	SS error
10	10			
20	10			
30	10			
50	10			
70	10			

Table 2: Proportional-Derivative controller

   (c) Proportional – Integral Controller

Proportional Gains	Integral Gains	Rise Time (ms)	Overshoot (%)	SS error
10	40			
20	40			
30	40			
50	40			
70	40			

Table 3: Proportional-Derivative controller

   (d) Proportional-Integgral-Derivative Controller

Proportional Gains	Integrator Gains	Derivative Gains	Rise Time (ms)	Overshoot (%)	SS error
10	40	10			
20	40	10			
30	40	10			
40	40	10			
50	40	10			

Table 4: Proportional-Derivative-Integral controller

Use any preferred tool (MATLAB/SIMULINK) to tackle the problem.