

# KINETC: Koopman-Inspired Nonlinear Event-Triggered Control from Data

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**Abstract**—Event-triggered Control (ETC) presents a promising paradigm for efficient resource usage in networked and embedded control systems by reducing communication instances compared to traditional time-triggered strategies. This paper introduces a novel approach to ETC for discrete-time nonlinear systems using a data-driven framework. By leveraging Koopman operator theory, the nonlinear system dynamics are globally linearized (approximately in practical settings) in a higher-dimensional space. We design a state-feedback controller and an event-triggering policy directly from data, ensuring exponential stability in Lyapunov sense. The proposed Koopman-Inspired Nonlinear Event-Triggered Control from Data (KINETC) method is validated through extensive simulation experiments, demonstrating significant resource savings by reducing the communication instances by 40%.

## I. INTRODUCTION

ETC is an implementation strategy in which the plant and its controller only exchange data when certain output- or state-related conditions are met. Event-triggered control seeks to reduce communication instances by concentrating on the real needs of the system, in contrast to traditional conservative time-triggered strategies that depend on fixed time intervals for communication. In situations where efficient use of resources is essential, like networked control systems and embedded systems, this paradigm has become more and more popular. ETC strategies, which offer improved system performance and resource savings in a variety of setups and control problems, have been developed in the literature thanks to the early results by [1, 2] and the work of Tabuada [3], and Heemls et al. [4].

In most of the current research, parametric state-space models are the foundation of traditional control engineering, where the plant to be controlled must first be identified or modeled. These models make use of the data from the system (from the input and the output) and are frequently written down starting from physics first principles or architecturally constrained system identification techniques. But in cases when first-principles models are intricate or hard to derive, they are only approximate representations of real systems, which inevitably leads to modeling errors. These errors impede

accurate control design and spread through the phases of analysis and implementation, affecting the overall performance of the system.

By excluding the demand for explicit system identification and instead of leveraging data gathered from open-loop simulations/experiments for any system control analysis and design, data-driven control techniques serves as a promising alternative. Several data-driven techniques for creating state feedback controllers and illustrating system dynamics have been shown in recent works, such as those by da Silva et al. [5], and De Persis and Tesi [6]. These techniques greatly streamline the control design process and do not require constantly exciting input data. There are also numerous applications of data-driven control in fields such as robotics [7, 8], aerospace [9], and power systems [10].

However, an alternative option in the situation, when first principles or system identification techniques fail, would be to construct the controller directly using the input, state, and output data that are now accessible. This approach is known as direct data-driven control [11–13]. Although the literature is full of data-driven techniques for control, only a limited number of techniques exist in the current literature for data-driven event-based control, particularly for nonlinear systems such as [14–16].

There exists, therefore, a profound demand for comprehensive data-driven event-based control methods tailored for general nonlinear systems, particularly applicable to discrete-time systems in our case. In many cases, it is appropriate and feasible to formulate the control and triggering conditions as data-dependent Linear Matrix Inequality (LMI). Given that most of the existing literature on ETC is well developed for Linear Time Invariant (LTI) systems, we aim to globally linearize nonlinear systems by increasing the dimensional space in which they reside.

This is not totally a new idea, as Koopman [17] in 1931 presented a trade-off between the nonlinear nature of dynamical systems and infinite dimensional representation of the same nonlinear system but it will look linear in the lifted space. Koopman idea has been recently revisited in many research fields [18–21] due to the intersection between data science and the easy-to-access computational domain.

We consequently propose KINETC, an idea inspired by Koopman Operator (KO) to acquire (approximately) global linear systems but in a higher dimensional space. Afterwards, we design the controller and the triggering policy for ETC for discrete-time linear systems directly from data, all together ensuring performance metrics (i.e. Lyapunov exponential stability).

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## II. PRELIMINARIES

### A. Notation

Let  $\mathbb{Z}_{\geq 0} := \{0, 1, 2, \dots\}$  denote the set of nonnegative integers, and let  $\mathbb{Z}_{> 0} := \mathbb{Z}_{\geq 0} \setminus \{0\}$  denote the set of positive integers. We denote by  $\mathbb{R}$  the set of real numbers and use a similar notation as for  $\mathbb{Z}$ . The  $\ell_2$  norm of a vector (a finite sequence) is denoted by  $\|\cdot\|$ . The symbols  $I$  and  $0$  denote the identity matrix and the zero matrix, respectively. Given a symmetric matrix  $A$ , the notation  $A \succ 0$  indicates that  $A$  is positive definite, while  $A \succeq 0$  means that  $A$  is positive semi-definite. Similarly,  $A \prec 0$  indicates that  $A$  is negative definite and  $A \preceq 0$  means that  $A$  is negative semi-definite. For any matrix  $A$ ,  $A^\top$  denotes the transpose of  $A$ . The symbol  $\mathcal{N}(\mu, \sigma^2)$  represents a normal distribution with mean  $\mu$  and variance  $\sigma^2$ . The symbol  $\lambda_i$  denotes an eigenvalue of a matrix.

### B. Problem Overview

Consider the discrete time dynamical system

$$x_{k+1} = f(x_k, u_k), \quad (1)$$

where  $x_k \in \mathbb{R}^n$  is the state and  $u_k \in \mathbb{R}^m$  is the control input, each at time instant  $k \in \mathbb{Z}_{\geq 0}$  with  $n, m \in \mathbb{Z}_{> 0}$ , and  $f$  is a transition map such that  $f : \mathbb{R}^n \times \mathbb{R}^m \mapsto \mathbb{R}^n$ , generally nonlinear, and assumed to be unknown, and stabilizable.

We consider a scenario where the system in (1) is connected to a controller via networked medium. Especially, the state measurements are sent to the controller via digital channel and the controller access the actuators directly. The goal is to design a data-driven event-triggered state-feedback controller with gain  $K \in \mathbb{R}^{m \times n}$  to stabilize the plant in (1) while abiding by a triggering policy that defines the instances  $\{k_i\}_{i \in \mathcal{Z}}$  at which a transmission happens, with  $\mathcal{Z} \subseteq \mathbb{Z}_{\geq 0}$ . At the time instant  $k = 0$ , we consider a transmission happens, so that  $k_0 = 0$ . In our settings, the controller is updated only upon the violation of some well-defined triggering policy in contrary to the nominal Time-triggered Control (TTC). The sequence  $\{k_i\}_{i \in \mathcal{Z}}$  leads to aperiodic updates of the controller. The controller then follows a zero-order hold implementation that takes the form of

$$u_k = Kx_{k_i}, \quad k \in [k_i, k_{i+1}]. \quad (2)$$

The zero-order hold nature of the presented controller is taken into consideration by the state error.

$$e_k = x_{k_i} - x_k, \quad (3)$$

which can be interpreted as the deviation between the current state and the last time an event happened. We consider the relative thresholding metric and events occur with the violation of the condition

$$\|e_k\| \leq \gamma \|x_k\| \quad (4)$$

where  $\gamma > 0$  is a relative parameter for the thresholding policy. The policy in (4) is checked every time instant  $k$ , and only upon violation of the policy, the control is updated.

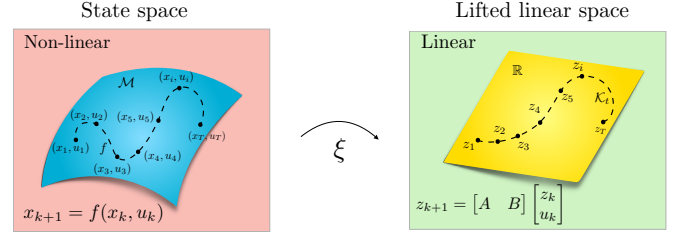


Fig. 1: Illustration of the Koopman Operator: The red panel represents the generic nonlinear state-space. Conversely, the green panel represents the linear space.

### C. Persistence of Excitation

Consider a carried out experiment for the system in (1) and its states and input data are recorded in the following way

$$\mathcal{D} := \{x_k, u_k : k \in [0, (T-1)] \cap \mathbb{Z}_{\geq 0}\},$$

where  $T$  is the final time of the experiment. Let set  $\mathcal{D}$  exists. Then, we define

$$\Gamma_0 := [u_0 \ u_1 \ \dots \ u_{T-1}] \in \mathbb{R}^{m \times T}, \quad (5a)$$

$$X_0 := [x_0 \ x_1 \ \dots \ x_{T-1}] \in \mathbb{R}^{n \times T}, \quad (5b)$$

$$X_1 := [x_1 \ x_2 \ \dots \ x_T] \in \mathbb{R}^{n \times T}. \quad (5c)$$

*Assumption 1:* Taking into account  $T \geq n + m$ , the matrix  $\begin{bmatrix} X \\ \Gamma \end{bmatrix}$  has full row rank.  $\square$

Assumption 1 can be verified numerically for a given set  $\mathcal{D}$ . The results of Willems et al. [22] ensures, for discrete-time systems, the validity of assumption 1 as long as  $u$  is persistently exciting signal.

## III. FRAMEWORK

### A. Koopman operator theory

*Definition 1 (Koopman Operator (KO)):* Consider the system given in (1). The KO  $\mathcal{K}_t$  is an infinite dimensional operator

$$\mathcal{K}_t \xi = \xi \circ f(x_k, u_k).$$

acts on  $\xi \in \mathcal{H} : \mathbb{R}^n \times \mathbb{R}^m \mapsto \mathbb{R}$ , the observable functions of the state space, where  $\circ$  is the function composition.  $\square$

The KO acts on the Hilbert space  $\mathcal{H}$  of all scalar measurement functions  $\xi$  and is by definition a *linear operator*—since for  $\xi_1, \xi_2 \in \mathcal{H}$  and  $\beta_1, \beta_2 \in \mathbb{R}$

$$\begin{aligned} \mathcal{K}_t(\beta_1 \xi_1, \beta_2 \xi_2) &= \beta_1 \xi_1 \circ f + \beta_2 \xi_2 \circ f \\ &= \beta_1 \mathcal{K}_t \xi_1 + \beta_2 \mathcal{K}_t \xi_2, \end{aligned} \quad (6)$$

An infinite-dimensional space  $\mathcal{H}$  of observable functions is used to represent a nonlinear system linearly using this method [23]. This means that the dynamics are changed from nonlinear, finite-dimensional to linear, infinite-dimensional upon transitioning from the state-space model to the Koopman representation (see Fig. 1). However, we are interested in a finite-dimensional approximation of KO from a practical perspective. For those who may be interested, please refer to the following refs. [24, 25].

Also, KO provides (approximately, in a practical settings) global linear representation for nonlinear dynamics if the right, in a sense made precise in the sequel, set of observable functions is chosen. Generally speaking, the lifting observable

functions are hard to identify. They can be found either by brute-force trial and error in a specific basis for the Hilbert space (e.g., trying numerous polynomial functions or Fourier basis functions) or by prior knowledge about the system. Several efforts have been made on this matter [26–28].

Motivated by the preceding analysis, we will make use of the idea of lifting the nonlinear dynamics from its state-space to look linear in a higher-dimensional state-space.

*Remark 1:* Our method necessitates a blurry prior physical knowledge of the underlying plant, not necessarily a complete knowledge, but at least a knowledge that can describe the domain shape in which the system operates to design the lifting observable functions. Several methods can help in constructing analytical/heuristic observable functions can be found in [29, 30] – just to name a few.  $\square$

*Remark 2:* At this stage of the work, we design the controller directly from the data. This step requires a set of observable functions that are satisfactory to approximate KO with notes regarding that being discussed in remark 1. In the sense that we do not focus on the approximate identification of the KO itself, we did not include discussion on such a topic. However, in more general scenarios, one may need to identify the operator for any purpose. Readers can refer to [31, 32].  $\square$  Now, the collected set  $\mathcal{D}$  should be revised. Instead of having the system's states only, we must consider the additional observable functions taking the form of  $\Xi(x) = [\xi_1(x) \ \xi_2(x) \ \dots \ \xi_p(x)]^\top$ . The observable functions  $\Xi \in \mathbb{R}^p (p > n)$ . Note that we only lift the state not the control. So the set  $\mathcal{D}$  becomes

$$\Gamma_0 := [u_0 \ u_1 \ \dots \ u_{T-1}] \in \mathbb{R}^{m \times T}, \quad (7a)$$

$$Z_0 := \Xi(X_0) \in \mathbb{R}^{p \times T}, \quad (7b)$$

$$Z_1 := \Xi(X_1) \in \mathbb{R}^{p \times T}. \quad (7c)$$

*Remark 3:* In response to this change, a slight modification of assumption 1, where  $T \geq n + m$  will be replaced by  $T \geq p + m$ .  $\square$

At the end of the day, after choosing the set of observable functions, the system in (1) can be now formulated as

$$z_{k+1} = Az_k + Bu_k, \quad (8a)$$

$$x_k = Cz_k \quad (8b)$$

### B. Event-triggered Control For the Lifted Representation of the Non-linear Dynamics

Let's consider the system given in (8), the globally linear version of the system in (1), subject to the controller (2) that result in

$$z_{k+1} = Az_k + BKz_{k_i} \quad (9a)$$

$$= Az_k + BKz_k + BKz_{k_i} - BKz_k \quad (9b)$$

$$= (A + BK)z_k + BKe_k, \quad \forall k \in [k_i, k_{i+1}[ , \quad (9c)$$

which can be understood as a closed-loop representation of the system in (8) with the state error.

An alternative representation of the event-triggered closed loop system should be derived to account for the data-driven nature of this work. In ref. [33] the authors derived

a data-driven representation of the closed loop system without considering the matrix  $BK$ . On the other hand, Digge and Pasumathy [15] arrived to a closed loop representation that allows dealing with the event-triggered formulation. We now borrow the work of [15], with a slight modification to account for the lifted linear representation of the nonlinear dynamics.

*Lemma 1 (Data-driven event-triggered closed loop representation [6, 15]):* The equivalent data-driven closed loop representation of the system (9) under the satisfaction of assumption 1 and where

$$\begin{bmatrix} I \\ K \end{bmatrix} = \begin{bmatrix} Z_0 \\ \Gamma_0 \end{bmatrix} L, \text{ and } \begin{bmatrix} 0 \\ K \end{bmatrix} = \begin{bmatrix} Z_0 \\ \Gamma_0 \end{bmatrix} N, \quad (10)$$

holds is given by

$$z_{k+1} = Z_1 L z_k + Z_1 N e_k, \quad (11)$$

where  $L$  and  $N$  are  $T \times p$  matrices.  $\square$

*Proof 1:* Let assumption 1 holds. Hence, by the Rouché-Capelli theorem, there exist  $L$  and  $N$  matrices that satisfy (10). So, another representation of (11) can be written as

$$z_{k+1} = \begin{bmatrix} A & B \end{bmatrix} \begin{bmatrix} I \\ K \end{bmatrix} z_k + \begin{bmatrix} A & B \end{bmatrix} \begin{bmatrix} 0 \\ K \end{bmatrix} e_k.$$

Using (10), the closed-loop system is given by

$$z_{k+1} = \underbrace{\begin{bmatrix} A & B \end{bmatrix} \begin{bmatrix} Z_0 \\ \Gamma_0 \end{bmatrix}}_{Z_1} L z_k + \underbrace{\begin{bmatrix} A & B \end{bmatrix} \begin{bmatrix} Z_0 \\ \Gamma_0 \end{bmatrix}}_{Z_1} N e_k.$$

Therefore, the data-driven representation of the closed-loop system (9) is given by (10).  $\blacksquare$

This formulation can be considered as a reparametrization of the system in (9) in terms of data. In other words, no need for the prior explicit system identification step. Since the formulation is derived, we move forward to derive the condition for system (11) to be exponentially stable in Lyapunov sense. A linear system described by  $z_{k+1} = Az_k$ , where  $A \in \mathbb{R}^p$ , is considered exponentially stable if there exists a function  $V : \mathbb{R}^p \mapsto \mathbb{R}$  defined by  $V(z_k) = z_k^\top S z_k$  with  $S \succ 0$  and symmetric, such that  $V(z_{k+1}) \leq \alpha V(z_k)$  along the system's trajectories for all  $k \geq 0$  and for some  $\alpha \in [0, 1] \subset \mathbb{R}$ .

*Remark 4:* For unstable systems, the choice of  $\alpha$  is critical as it impacts the values of the controller gain  $K$  which must satisfy the necessary conditions and thresholds to stabilize the system.  $\square$

Considering a classical Lyapunov candidate function described in the later paragraph, the exponential Lyapunov stability criteria<sup>1</sup> is given by

$$\begin{bmatrix} z_k \\ e_k \end{bmatrix}^\top \begin{bmatrix} L^\top Z_1^\top S Z_1 L - \alpha S & L^\top Z_1^\top S Z_1 N \\ N^\top Z_1^\top S Z_1 L & N^\top Z_1^\top S Z_1 N \end{bmatrix} \begin{bmatrix} z_k \\ e_k \end{bmatrix} \leq 0 \quad (12)$$

In this work, the design of the ETC strategy should not violate the Lyapunov stability condition in (12) to ensure exponential stability.

<sup>1</sup>The full analysis is given in the appendix.

### C. Learning Controller From Data

Firstly, we design the controller gains to stabilize the globally linearized system. We consider the data-driven closed loop representation in (11) neglecting the error at this stage

$$z_{k+1} = Z_1 L z_k, \quad (13)$$

the controller gains can be designed directly from data, following the analysis in [6, Section IV. A], by the following theorem it ensures the Lyapunov stability condition.

*Theorem 1:* Let condition 1 hold. And by exploiting the results of lemma 1. Then any matrix  $G_1$  that satisfy the following LMI,

$$\begin{bmatrix} Z_0 G_1 & G_1^\top Z_1^\top \\ Z_1 G_1 & Z_0 G_1 \end{bmatrix} \succeq 0 \quad (14)$$

results in

$$K = \Gamma_0 G_1 (Z_0 G_1)^{-1} \quad (15)$$

which stabilizes the system (8).  $\square$

*Proof 2:* To certify stability in exponential decay of the system (13) with a rate  $\alpha$ , implies

$$L^\top Z_1 S Z_1 L - \alpha S \preceq 0, \quad (16)$$

with  $L$  satisfying (10). Let  $G_1 := L S^{-1}$ , and pre- and post-multiply (16) by  $S^{-1}$ , the stability of the system can be guaranteed if there exists two matrices  $G_1$  and  $S$  such that

$$\begin{aligned} G_1^\top Z_1^\top S Z_1 G_1 - \alpha S^{-1} &\preceq 0 \\ K S^{-1} &= \Gamma_0 G_1 \\ S^{-1} &= Z_0 G_1 \end{aligned}$$

Moreover, we use  $S^{-1} = Z_0 G_1$  and obtain

$$\begin{aligned} G_1^\top Z_1^\top (Z_0 G_1) Z_1 G_1 - \alpha Z_0 G_1 &\preceq 0 \\ Z_0 G_1 &\succ 0 \\ K &= \Gamma_0 G_1 (Z_0 G_1)^{-1} \end{aligned}$$

Using Schur's complement lemma on the first inequality, we reach to (14) which results in gains given from (15) that exponentially stabilize the system.  $\blacksquare$

### D. Learning the Triggering Policy from Data

In the interval  $[k_i, k_{i+1}[$ , it is essential that inequality (12), which ensures exponential convergence, is also satisfied. The following theorem derives a window for the parameter  $\gamma$  that ensures the stability of system (11).

*Theorem 2:* Assume that the condition 1 is satisfied. So, the relative threshold parameter  $\gamma$  for the event-triggered implementation (4) with the controller (15) can be calculated by solving for  $\gamma$  such that

$$\begin{aligned} \max_{q, G_2} \quad & \gamma \\ \text{s.t.} \quad & \begin{bmatrix} \alpha Z_0 G_1 & 0 & G_1^\top Z_1^\top & \gamma Z_0 G_1 \\ 0 & qI & G_2^\top Z_1^\top & 0 \\ Z_1 G_1 & Z_1 G_2 & Z_0 G_1 & 0 \\ \gamma Z_0 G_1 & 0 & 0 & qI \end{bmatrix} \succeq 0, \\ & q > 0, \quad Z_0 G_2 = 0, \quad U G_2 - qK = 0, \end{aligned} \quad (17)$$

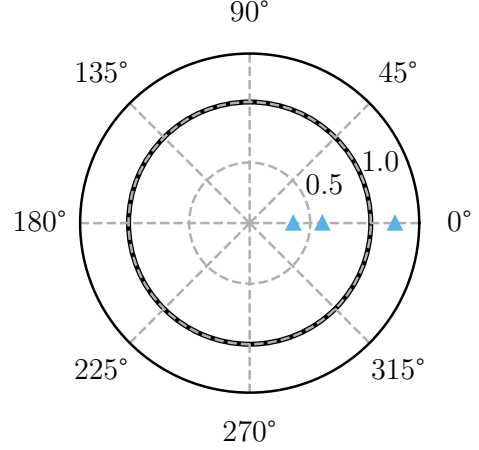


Fig. 2: The eigenvalues of the system given in (19). Clearly, all the eigenvalues are not contained inside the unit circle indicating the system instability.

which will result in stability of the system (11) in exponential behaviour.  $\square$

*Proof 3:* See ref. [15, Section IV. B].  $\blacksquare$

The result of theorem 2 allows to maximize  $\gamma$  over the variables  $G_2$  and  $q$ . The result also implies that any  $\gamma \in [0, \gamma^*[$  stabilizes the system, where  $\gamma^*$  is the solution for (17).

Now, we have all the components put together. A detailed algorithm for the entire process is given in algorithm 1.

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#### Algorithm 1 KINETC: Koopman-Inspired Nonlinear Event-Triggered Control

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**Require:**  $\alpha$ ,  $X_0$ ,  $X_1$ , and  $\Gamma_0$

- 1: Lift  $X_0$ , and  $X_1$  via (7 b, and c)
  - 2: Solve for  $G_1$  in the LMI given in (14)
  - 3: Solve for the controller gain  $K$  in (15)
  - 4: Maximize the threshold parameter  $\gamma$  to get  $\gamma^*$  in (17)
  - 5: Choose any  $\gamma \in [0, \gamma^*[$ , (typically the max. value gives wider inter-event time window)
  - 6: **return**  $\gamma^*$ , and  $K$
- 

## IV. ILLUSTRATIVE SIMULATIONS AND RESULTS

### A. Illustrative Example

We consider a case of nonlinear system with slow manifold inspired by an example from [34]:

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \mapsto \begin{bmatrix} \rho x_1 \\ \kappa x_2 + (\rho^2 - \kappa) x_1^2 + u \end{bmatrix} \quad (18)$$

In this scenario, there exists a polynomial stable manifold defined as  $x_2 = x_1^2$ . Within the Koopman-inspired framework, if the correct lifting observable functions were chosen such that  $\Xi(x) = [x_1, x_2, x_1^2]^\top$ , the nonlinear system in (18) can be expressed linearly as

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}_{k+1} = \begin{bmatrix} \rho & 0 & 0 \\ 0 & \kappa & (\rho^2 - \kappa) \\ 0 & 0 & \rho^2 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}_k + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u_k \quad (19)$$

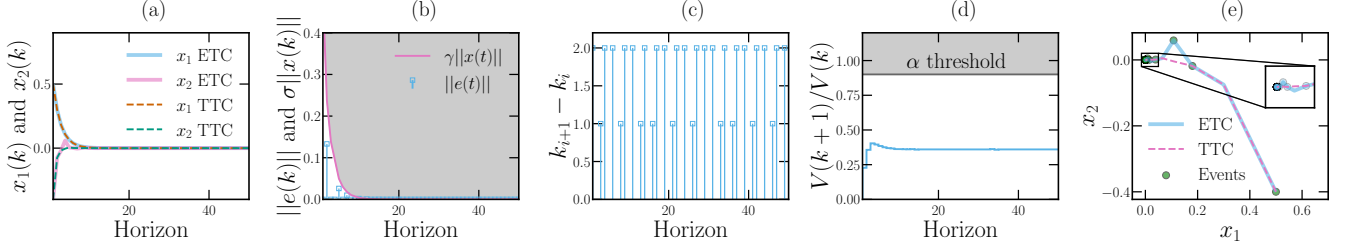


Fig. 3: Results of the illustrative example. (a) Behaviour of state  $x_1$  and  $x_2$  for both ETC and TTC over the horizon. (b) Norms of the error  $\|e_k\|$  and the threshold parameter  $\gamma\|x_k\|$ . (c) Inter-event times  $k_{i+1} - k_i$  showing the intervals between successive events. (d) Lyapunov function ratio  $V(x_{k+1})/V(x_k)$  with stability threshold  $\alpha$ . (e) The phase portrait of the system trajectory in the state space showing both the ETC and TTC behaviour, it also shows the instances at which an event happens along the trajectory in the ETC framework.

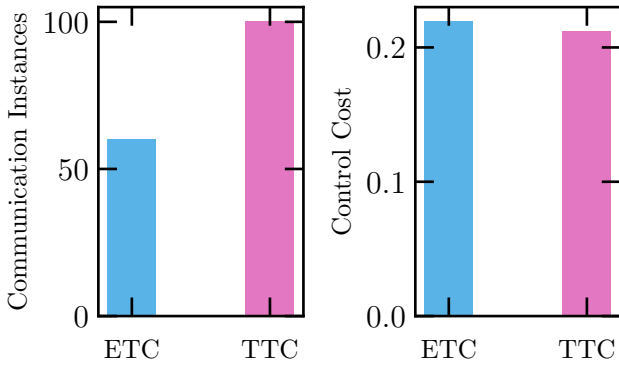


Fig. 4: Comparison between ETC and TTC methodologies. The plot on the left shows the number of communication instances for each control strategy, where ETC significantly reduces the communication overhead compared to TTC. The plot on the right shows the corresponding control costs for both methods.

Considering the parameters for the system,  $\rho = 0.6$ , and  $\kappa = 1.2$ , the corresponding eigenvalues (displayed in Fig. 2)

are  $\lambda_1 = 0.6$ ,  $\lambda_2 = 1.2$ , and  $\lambda_3 = 0.36$ . Since  $\lambda_2 > 1$ , the system exhibits instability and the goal is to stabilize the trajectory around the origin. We collected the data for  $T = 45$  which is enough for assumption 1 to hold – on a theoretical note,  $T \geq m+p$  samples should be enough (i.e. in this example  $T \geq 4$ ) to obey assumption 1. Therefore,  $T = 4$  should work. The input signal is drawn from a normal distribution following  $u \sim \mathcal{N}(0, 1)$ .

Then, after deploying the steps in algorithm 1, we obtain  $K = [0.0206 \quad -1.1109 \quad -0.1530]$ , which in turn gives  $\gamma^* = 0.7664$ . We simulated the system for both ETC, and TTC and illustrated the behaviour in Fig. 3. All the results depicted in Fig. 3, are acquired after pulling the states back from the higher-dimensional space, in this case from  $\mathbb{R}^3$  to  $\mathbb{R}^2$ , by applying (8b) with  $C = \begin{bmatrix} I_2 & 0 \\ 0 & 0 \end{bmatrix}$ .

Fig. 3(a), illustrates the state evolution  $x_1$  and  $x_2$  against time under both ETC and TTC techniques. The trajectories for ETC demonstrate excellent tracking performance in comparison with the nominal TTC. This highlights the efficacy of

the developed event-triggered approach in maintaining system stability while minimizing unnecessary updates.

Also, in Fig. 3(b), the graph shows that  $\|e_k\|$  remains consistently below  $\gamma\|x_k\|$ , satisfying the triggering condition. As shown in Fig. 3(c), the substantial reduction in communication instances (40%) addresses potential concerns regarding communication overhead in practical implementations.

Additionally, results in Fig. 3(d) depicts the ratio of the Lyapunov function  $V(x_{k+1})/V(x_k)$  compared to the stability decaying rate  $\alpha$ . The ratio remains below  $\alpha$ , verifying that the proposed event-triggered control law ensures exponential stability of the system. This gives another certificate of the theoretical guarantees provided by the Lyapunov-based stability analysis.

The results in Fig. 3(e) show the trajectory of the system in the state space and the instances on the trajectory in which the event occurs.

Finally, despite the huge reduction in the communication instances, it came on the expense of the control cost. The control cost of ETC compared to TTC became larger by 3.3150%. Although the increment is not drastically large as seen in Fig. 4, it is noted that there is a trade-off between the efficiency of communication resources and the cost of control. ETC achieves comparable performance to TTC but with fewer communication resources. These findings indicate the high effectiveness of ETC in optimizing resource utilization while maintaining effective. Another note in our experiment, both Koopman based linearization ETC and TTC have control cost much lower than the nominal Taylor linearization technique, consistent with the results of Brunton et al. [34].

## V. CONCLUSION

To sum up, this study suggests an event-triggered control approach based on data-driven methods for discrete-time nonlinear systems. By lifting the nonlinear dynamics into a higher-dimensional linear representation inspired by the KO theory, the method makes it possible to create an event-triggered controller driven by data. Through the development of a closed-loop system and the implementation of a triggering strategy, the suggested method stabilizes the plant with less frequent control updates.

The event-triggered closed-loop system's exponential stability is guaranteed by the stability analysis, which is based on the Lyapunov criterion. Numerical simulations and theoretical analysis are used to show how effective the suggested strategy is. This work creates opportunities for real-world applications in networked control systems and advances event-triggered control techniques for nonlinear systems.

The foundations provided by this work shall allow dealing with many other scenarios including, when the plant (discrete or continuous) include time varying parameters, when the full state measurements are not available, or when policies other than the zero-order hold is used. Additionally, examining the various lifting techniques available in the literature is something important, as well as, testing the scalability of the solution.

## APPENDIX

### LYAPUNOV STABILITY ANALYSIS

This appendix presents the exponential stability of the system given by

$$z_{k+1} = Z_1 L z_k + Z_1 N e_k \quad (20)$$

in a Lyapunov sense with the candidate  $V(k) = z_k^\top S z_k$ . First,  $V(z_{k+1})$  can be computed as follows

$$\begin{aligned} V(z_{k+1}) &= (Z_1 L z_k + Z_1 N e_k)^\top S (Z_1 L z_k + Z_1 N e_k) \\ &= z_k^\top L^\top Z_1^\top S Z_1 L z_k + z_k^\top L^\top Z_1^\top S Z_1 N e_k + \dots \\ &\dots e_k^\top N^\top Z_1^\top S Z_1 L z_k + e_k^\top N^\top Z_1^\top S Z_1 N e_k \end{aligned} \quad (21)$$

Lyapunov exponential stability condition with convergence rate  $\alpha$  can be reached by defining  $V(z_{k+1}) \leq \alpha V(z_k)$ . This leads to the following identity based on the candidate Lyapunov function

$$\begin{aligned} &z_k^\top L^\top Z_1^\top S Z_1 L z_k + z_k^\top L^\top Z_1^\top S Z_1 N e_k + \dots \\ &\dots e_k^\top N^\top Z_1^\top S Z_1 L z_k + e_k^\top N^\top Z_1^\top S Z_1 N e_k \leq \alpha z_k^\top S z_k \end{aligned} \quad (22)$$

By defining  $v = \begin{bmatrix} z_k \\ e_k \end{bmatrix}$ , Eqn. (22) can be written in the form of  $v^\top \Psi v \leq 0$ , where

$$\Psi = \begin{bmatrix} L^\top Z_1^\top S Z_1 L - \alpha S & L^\top Z_1^\top S Z_1 N \\ N^\top Z_1^\top S Z_1 L & N^\top Z_1^\top S Z_1 N \end{bmatrix} \quad (23)$$

Therefore, the Lyapunov stability condition for the system can be written as

$$\begin{bmatrix} z_k \\ e_k \end{bmatrix}^\top \begin{bmatrix} L^\top Z_1^\top S Z_1 L - \alpha S & L^\top Z_1^\top S Z_1 N \\ N^\top Z_1^\top S Z_1 L & N^\top Z_1^\top S Z_1 N \end{bmatrix} \begin{bmatrix} z_k \\ e_k \end{bmatrix} \leq 0 \quad (24)$$

If condition (24) is satisfied, it then guarantees exponential stability of the system with convergence rate  $\alpha$ .

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