Article

RATIONALITY AND SOCIETY

Rationality and Society
2017, Vol. 29(3) 355–386

DOI: 10.1177/1043463117701127
journals.sagepub.com/home/rss
SAGE

Student sorting and implications for grade inflation

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Abstract

There is a sizable literature on higher education, both in the United States and beyond, that draws attention to the phenomenon known as grade inflation. We offer an interpretation of grade inflation that turns on the choices students have over academic departments, and we argue that patterns in grades cannot be considered in isolation from the incentives that students have to sort themselves strategically across departments. Our argument draws on a game-theoretic model in which students of varying abilities face a choice between enrolling in a department whose grades are inflated and thus ability-concealing versus enrolling in a department whose grades are ability-revealing. In equilibrium, all grades are high. Nonetheless, what appears to be grade inflation is a result of the fact that the ability-revealing department in our model attracts highly talented students seeking to distinguish themselves from students of lesser ability, who avoid said department because enrolling in it is costly. Our formalization shows how student sorting can confound grades, and it implies that a full understanding of university's grade distribution requires knowing which departments in the university are ability-concealing and which, in contrast, are ability-revealing.

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Keywords
Grade inflation, higher education, formal model

Introduction

Grade inflation appears by many measures to be endemic across the higher education landscape. Ray (2014) documents rising grades in the United Kingdom, and as of 2012, "80 percent of students [in Germany] graduate[d] with one of the top two grades."[1] A 2010 study of grade trends at over 160 American universities revealed that mean grade point average has risen by nearly a tenth of a point per decade (Rojstaczer and Healy, 2010), and the revelation that the most commonly awarded grade at Harvard University is an "A" unleashed a firestorm of media attention on the university's grading policies.[2] In 2001, 91% of Harvard's graduating class received honors, rendering this indicator of student ability essentially meaningless in the sense of distinguishing among Harvard undergraduates.[3] And at Yale University, approximately 62% of grades were "A" and "A" as of Spring, 2012, whereas in 1963 this proportion was only 10.[4] Empirical research on grading tends to show that the phenomenon of grade inflation is broad-based, although Kuh and Hu (1999) argue that, in the United States, grade inflation is disproportionately a feature of research universities and elite liberal arts colleges.

Grade inflation can be associated with numerous social problems. Because inflated grades can mask variance in student abilities (Pattison et al., 2013; Sabot and Wakeman-Linn, 1991), post-education institutions that must contend with universally high grades will have difficulty selecting the best students for the most demanding tasks. This will impose an efficiency cost on society. In addition, differing rates of grade inflation across departments can skew student enrollment decision; STEM (science, technology, engineering, and mathematics) departments are disproportionately less likely to grade inflate, and a student who receives an early and poor grade in an STEM class is less likely to major in an STEM field (Ost, 2010; Sabot and Wakeman-Linn, 1991; Strenta et al., 1994). Moreover, there are gender and socioeconomic differences in the types of students whose grades have increased over time; women in particular are disproportionately likely to enroll in disciplines prone to grade inflation (Riegle-Crumb et al., 2012; Riegle-Crumb and King, 2010) and in disciplines perceived as "soft" (Carnevale et al., 2011; Swift et al., 2013). This will make it difficult for the most talented women to stand out from the rest. There is also evidence that, among American high school students, grades have increased more slowly for Hispanic students than members of other ethnic groups (Sawtell et al., 2003), and in general minority students consistently receive lower grades than White students (Farkas and Hotchkiss, 1989; Van Laar et al., 1999). We return later to a discussion of social consequences of grade inflation, but for the moment, it suffices to note that differential rates of grade inflation across department and student types can reinforce stereotypes which affect some students more than others.

As we review shortly, scholars have offered a variety of explanations for observed upward trends in grades. Some explanations for grade inflation focus more heavily on (and tend to blame) the role of educational institutions and the incentives that their rules and procedures engender, while others draw attention to the behaviors of students and the extent to which their choices over departments and courses have downstream effects on measures like average grades. Certainly there are connections between these two ways of thinking—students make decisions in an environment crafted by universities—but it is nonetheless still useful to distinguish between incentives facing faculty members and incentives facing students.

Having noted this dichotomy, we offer an interpretation of grade inflation that turns on student choice over departments or, more broadly, courses of study. In particular, we present a formal model that allows students to choose whether they want to study in a department whose grades are inflated versus a department whose grades are accurate. Inflated grades can be thought of as *ability-concealing* and accurate grades as *ability-revealing*. The students in the model differ in their underlying levels of talent, and this induces students to have different preferences over the extent to which they want their grades to reveal their underlying abilities. Students in the model sort themselves strategically across departments, and this confounds the interpretation of grades. Among other things, the model shows that trends in grades—upward or downward—can be induced by changes in student sorting abilities. In light of this, caution should be exercised when drawing connections between trends in grades and potentially deleterious consequences for educational institutions and society at large.

In what follows, we discuss literature on grade inflation, and we then present our model and explain its various components. The basic model has two types of equilibria, and we show that, in equilibrium, strategic sorting by students leads to high grades. We then present an extension of the model which generalizes our initial, and coarse, characterization of student ability and also allows for what we call an education bonus. In the extension, we observe high grades in equilibrium, and the extension yields a somewhat counter-intuitive result: the more difficult a university's ability-revealing department, the higher are average grades and the more it appears that grade-inflation is ubiquitous. This extension has policy implications for efforts to rein in high grades, and we discuss such implications and others in the conclusion.

Grade inflation

Concerns over grade inflation in higher education are not new (e.g. Ekstrom et al., 1994), and as we noted in the introduction, one consequence of grade inflation is the masking of true student abilities. To the point, Sabot and Wakeman-Linn (1991) show that introductory course grades received in low-grading departments are better predictors of student performance in future classes compared to grades given in what appear to be grade-inflating departments. Similarly, they show that alternate predictors of student ability (e.g. standardized test scores, parental education levels, high school grades, and so forth) are associated with student grades in low-grading departments but not departments that routinely give high grades. These dual findings show that inflated grades effectively mask student abilities and diminish the extent to which grades signaling underlying skills and talent levels.[5]

One of the commonly cited reasons for an increase in grades across American colleges and universities has been the increasing weight placed on instructor evaluations in hiring and tenure decisions (Eiszler, 2002; Stratton et al., 1994). Students tend to give better evaluations to professors who award them higher grades (Johnson, 2003), and thus an increased reliance on teacher evaluations during evaluation processes can create incentives for high grades. Nelson and Lynch (1984) argue that the relationship between evaluations and grades can be exacerbated by stagnating faculty salaries, and Pressman (2007) notes that pressure for high grades will tend to be stronger for untenured but tenure-track professors and strongest for adjunct faculty, whose employment depends on enrollments.

Perrin (1998) and Kelly (2009) draw attention to the fact that a university professor may, in the course of grading, compare her students not just to other students at her own university but also to the typical American student. Perrin writes, "[Professors] imagine our students at a mythical Average U., and give the grades they would get there." If a faculty member believes that her institution's admission policies lead to a highly talented student body, then it follows that said faculty member should in general assign high grades. On this point, see Achen and Courant (2009) and their anecdote of

> a [University of Michigan] chemistry professor who had stuck to the standards of his own undergraduate work for decades, but who came to notice that incoming graduate students at Michigan often had better grades than graduates of [his] department with similar knowledge and skill.

Other arguments on the subject of grade inflation focus specifically on student course selection. For example, in some American universities students are allowed to take classes without grades appearing on transcripts. Strategically minded students may seek to take advantage of this practice by ensuring that grades for their most difficult classes are not visible. If students strategically select certain classes to have non-visible grades, then average grade point may increase even in the face of fixed grading policies (Birnbaum, 1977). Foreshadowing the model that we present here, Prather et al. (1979) argue that changes in average grades can reflect changes in enrollment patterns; their empirical research finds that

> English majors tend to receive relatively higher grades in education courses than in their other courses, while the grades they receive for physical science and foreign language courses are, on the average, lower. Physical science courses generally record lower grades for all majors, while teacher education courses comparatively record higher grades for all majors. (pp. 21–22)

The implication of such a finding is that average grades reflect student selection into coursework of interest.

Bar et al. (2009) analyze data from Cornell University and find that publicly available median grades allow students to select into leniently graded classes. Strenta et al. (1994) and Ost (2010) have similar findings, and both note that low grades in first year science classes increases the probability that a student chooses a non-science major. There is also evidence that grading policies respond to the perceived value of a major (Freeman, 1999). When a department's graduates do not perform as well on the job market, they are forced to "buy" students with higher grades. Jewell et al. (2013) validate this, finding that there is substantial departmental variation in grade inflation.

In contrast to theories of grade inflation that consider student and institution incentives, Adelman (2008) argues that increasing grades may be explained by improved student ability and/or teaching quality. In this view, increasing grades are not inherently problematic. Brighouse (2008) emphasizes that, to assume that there has been no improvement in student quality over the past 30 years is to assume that have been no efficiency gains in higher education over this period.

There is some formal work on grade inflation, but the literature is not extensive. Four examples are Yang and Yip (2003), Chan et al. (2007), Franz (2010), and Popov and Bernhardt (2013). In the former, schools have incentives to give high grades because this helps weaker students obtain jobs; this leads to labor market inefficiencies. In Chan et al. (2007), employers cannot determine whether students with high grades are high quality or whether the university that granted said grades is an easy-grading institution; as in Yang and Yip (2003), this leads to inefficient labor market outcomes. Franz (2010) models professor–student interactions with an eye on the costs on faculty that students impose by requesting high grades. In equilibrium, the "nuisance" students in Franz (2010) lead professors to inflate grades. Finally, Popov and Bernhardt (2013) propose a model where universities compete for job market outcomes. They show that more selective universities have the strongest incentives to grade inflate.

Students and institutions in this limited formal literature are strategic, but extant models in the literature do not allow students to sort themselves across departments (or other academic units) in the way described here. Insofar as contemporary university students appear to be very attuned to grading policies and how they vary by field of study (and even by class and professor), our model of sorting fills a gap in the literature.

Model

We now describe a model that sheds light on the dynamics of student sorting across university departments and resulting patterns in grades. The model is set in a single university and includes a group of students and two departments. Its premise is that the students in the university have already been admitted but must choose a department or course of study in which to enroll. As will be clear, a student's choice between departments is informed by her interest, or lack thereof, in signaling her intrinsic ability level to a labor market that she will enter upon graduation.

Students

In the model, there are two types of students, low ability and high ability. A student's ability is fixed and exogenous, and let the proportion of high-ability students in our hypothetical university be $\pi_H \in (0,1)$. All students know their own abilities, either low or high, but face the problem of credibly signaling their talents to a labor market.[6] In the sense of Spence (1973), students attend university in order to earn a transcript which can subsequently be used to indicate ability. Like the students in Love and Kotchen (2010), our students value grades because of what they signal to a labor market.

We assume that the post-university labor market rewards high ability, and by implication being high ability is valuable to a student. In particular, if a student is known to be of high ability, then after her education is complete she earns a wage that we call w_H. In contrast, a student known to be of low ability earns in a post-education market a wage of w_L where $w_H > w_L$.[7]

If after graduation a student's ability is not known to the labor market, then said student receives a wage proportional to the probability that she is of high ability. For example, if the market believes that a student is of high ability with probability $1/4$, then her post-education wage will be $1/4 \times w_H + 3/4 \times w_L$. We do not model the post-education labor market explicitly, and implicit in our assumptions about post-education wages is that firms in the labor market are risk neutral and that all students have jobs after graduating.

This latter assumption is not binding; one could treat what we call the low wage w_L as an unemployment or welfare benefit. Moreover, one could also interpret what we call the high post-education wage w_H as a placement in a post-graduate institution like a medical school or law school. As long as $w_H > w_L$, which we assume throughout, being of high ability is better than being of low ability, *ceteris paribus*. All of our results that follow depend on the *difference* between w_H and w_L, and this means that we can simplify our presentation without any loss of generality. With this in mind, we henceforth assume that $w_H = 1$ and $w_L = 0$. This normalization is costless and reduces some of the algebra in our forthcoming equilibria.

Asserting that there are two types of students—low and high ability—is a simplification. We could have assumed that student ability exists on a continuum, and we consider such an extension to our model after we present initial results.

Departments

We assume that our hypothetical university contains departments whose grades are either *ability-revealing* or *ability-concealing*. These types of departments differ only in the manner in which they assign grades to their students. It is broadly accepted that grading practices differ between departments and that these differences impact student enrollment patterns (Bar et al., 2009; Ost, 2010; Sabot and Wakeman-Linn, 1991; Strenta et al., 1994).

An ability-revealing department is one that offers courses with regular and discriminating examinations, projects, assignments, and so forth. These examinations, say, allow the department to know whether a given student enrolled in the department is of low ability or is of high ability, and the department indicates this knowledge via grades. In particular, a grade-revealing department assigns an "A" grade to high-ability students (because these students did well on the department's examinations) and a "B" grade to low-ability students. Assuming that an ability-revealing department assigns grades of "A" and "B," as opposed to "A" and "C" or "A" and "D," is of no consequence. The key here is that an ability-revealing department assigns grades that discriminate between low- and high-ability students. The regular and discriminating examinations given in an ability-revealing department require an effort cost for enrolled students, who know that these examinations and related assignments will ascertain their underlying abilities. Let c denotes the effort cost associated with enrolling in an ability-revealing department, and we assume that $c > 0$. Recall that high-ability students identified as such earn a normalized wage of one post-graduation. If this wage of one is interpreted as the present value of a stream of wages, then c is presumably much smaller than one and in particular should be understood as being quite close to zero.

In contrast, an ability-concealing department is one whose courses do not discriminate between low- and high-ability students. The courses in an ability-concealing department are by definition not excessively challenging, and the key is that all students enrolled in them receive excellent grades, in particular, marks of "A." Moreover, the lack of discriminating examinations means that students in an ability-concealing department are not subject to the effort cost comparable to the cost c incurred by students in a grade-revealing department.[8]

We could have assumed that students enrolled in an ability-concealing department are forced to pay an effort cost akin to the cost required of students in a grade-revealing department. Had we done this, our model's equilibria, which follow shortly, would have been a function of the *difference* between the effort cost required of a student in an ability-revealing department and the cost required of a student in a grade-concealing department. Thus, the assumption that $c > 0$ is akin to assuming that it is more costly for a student to enroll in an ability-revealing department than in an ability-concealing department.

We also could have assumed that low- and high-ability students pay different costs for attending an ability-revealing department. That is, we could have posited that high-ability students pay c^H when enrolling in an ability-revealing department and low-ability students c^L. It would probably be natural to assume that $c^L > c^H > 0$, meaning, low-ability students have to work harder in ability-revealing departments than do high-ability students. Regardless, our main results do not depend on whether costs for attending an ability-revealing department vary by student type.[9]

Labor market

As we noted above, we do not formally model firms in a post-education labor market, nor do we model, say, admissions committees in graduate institutions. However, we assume that the market knows which types of departments are ability-revealing and which are ability-concealing. This does not strike us as a particularly strong assumption although we recognize that one could argue that firms, graduate schools, and other post-graduate institutions are not informed about which departments in universities give ability-discriminating grades.

Average grades and grade inflation

As will be clear shortly, our model generates a distribution of grades across students, and from this distribution we can calculate average grades in equilibrium. Before we discuss equilibria, we need to define grade inflation. In our two-type model, high-ability students cannot receive inflated grades because they are of high ability; only low-ability students can have inflated grades. Thus, we say that a low-ability student receives an inflated grade if said student receives a grade that is equal to or greater than the grade received by a high-ability student. If a low-ability student receives a grade of "A," then we say that this student's grade is inflated. Why? A high-ability student who attends an ability-revealing department will by definition receive an "A" grade. Thus, a low-ability student who also receives an "A" grade has an inflated grade.

It is important to distinguish between the fraction of inflated grades in equilibrium and the fraction of high grades. These two quantities are distinct: a high grade is not necessarily inflated if it is earned by a high-ability student. Distinguishing between inflated versus high grades helps us understand the difference between situations where there is serious grade inflation (many students receive grades greater than their abilities) versus situation where there are many high yet accurate grades.

Beyond grade inflation

Key to our model is the opportunity for individuals—in our case, students—to signal their types by engaging in costly behavior—taking classes in an ability-revealing department. What makes grading particularly interesting is the fact that grading scales typically have upper bounds.[10] As we will see in our forthcoming equilibrium analysis, this can lead to situations where many students receive the same top grade even if said students differ in their underlying abilities.

If grades lacked an upper bound, then there would be no such thing as the maximum grade a student could receive. Still, an ability-concealing department in this situation could nonetheless give all of its students identical marks. This would be fully consistent with the department's being ability-concealing. In other words, in an ability-concealing department, the distribution of grades can be compressed even the absence of something like a top grade. With this in mind, even if grades were in theory unbounded, our model still would speak to the existence of different types of departments, some that offer grades that discriminate between students and others that do not.

Many evaluation systems have "top" grades, however, and our model thus applies to situations beyond university grading. Firms regularly have to assess their employees, for example. Some tasks in a firm are presumably ability-concealing and others are ability-revealing. If a firm's employees have a voice in choosing what they do within a firm, then the dynamics we have touched on in our model might lead to low-quality employees choosing tasks or career paths that are ability-concealing; high-quality employees will do the opposite.

Similarly, suppose that a set of legislators in a city council are of two types, low and high quality, and are faced with a policy reform problem, that is, how best to reform a municipal social welfare program. A legislator can choose between an easy reform, one that does not accomplish much but poses little risk, or a challenging reform, one that might expose a low-quality legislator to the critique that he or she did not design a reform very well. Suppose that after a reform effort, an interest group generates a rating—that is, a grade—of said reform. In this framework, high-quality legislators may seek to distinguish themselves by choosing challenging reforms if voters are sufficiently attuned to the interest group's post-reform ratings. However, if interest group ratings are bounded above like grades, then the notion of rating inflation is plausible if, say, all legislators choose easy reform projects that shed little light on underlying legislator quality.

Our general point here is that the study of grading extends beyond academic environments. The model we have offered is one of evaluation in an environment in which tasks differ in the extent to which they identify talented individuals. The equilibrium dynamics that are forthcoming below will be helpful guides in understanding all such environments.

Equilibria

We assume that a university has two departments, one that is ability-revealing and the other, ability-concealing; the assumption of two departments is not constraining and we comment on it later. The university's students, $1-\pi_H$ of whom are of low ability and π_H of whom are of high ability, choose simultaneously whether to enroll in one of two departments. Subsequent to this the departments assign grades, and then the students enter a post-education labor market. As we have noted above, the labor market pays each student based on her ability or her expected ability if the market cannot

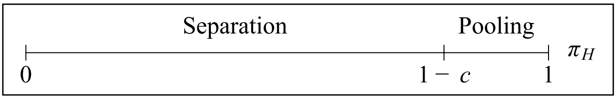

Figure 1. Model equilibria as a function of π_H.

discern based on grades whether the student is of low or high ability. Only the students in our model are strategic actors, and a student's utility is equal to her post-education wage minus the effort cost c if the student enrolled in an ability-revealing department.

The essence of a game-theoretic model is that the utility of a player selecting a given strategy is conditional on the strategies chosen by other players. One can see evidence of this type of mutual interdependence in the model described here when considering the value of a student's enrolling in an ability-concealing department. To make this clear, suppose that a low-ability student is considering such a department. If all other students also enroll in the ability-concealing department, and if there are mainly high-ability students because π_H is large, then our hypothetical low ability student will be pleased with her choice. Why? She will be grouped with both low- and high-ability students, and the post-education labor market will not know whether she is low ability. If, though, high-ability students all enroll in the ability-revealing department, then by enrolling in the ability-concealing department she is signaling that she is of low ability. This will be less pleasing as the post-education labor market will know for certain that she is of low ability. This example, which foreshadows the pooling and separating equilibria that we discuss below, shows why the value to any student (in this example, a low-ability student, but the same sort of example applies to high-ability students as well) of enrolling in an ability-concealing department is conditional on the choices made by others.

The (Bayesian) equilibria of the model depend on the fraction π_H of high-ability students and the cost c of enrolling in an ability-revealing department. As characterized in Lemma 1 and described visually in Figure 1, the model always has a separating equilibrium and has a pooling equilibrium if $\pi_H \geqslant 1-c$.

Lemma 1. *There is always a separating equilibrium wherein low-ability students enroll in an ability-concealing department and high-ability students enroll in an ability-revealing department. If $\pi_H \geqslant 1-c$, then in addition there is a pooling equilibrium wherein all students enroll in an ability-concealing department. Regardless of the values of* π_H and c, there is never a pooling equilibrium in which all students enroll in an ability-revealing department.

The proof of Lemma 1 is in Appendix 1, and the lemma characterizes the model's pooling and separating equilibria. We discuss these equilibria in this order.

Pooling equilibrium. When $\pi_H \geqslant 1-c$, the model has a pooling equilibrium in which all students enroll in an ability-concealing department. Consider the implication of the aforementioned (weak) inequality. When it holds, then the fraction π_H

of high-ability students is large and indeed almost all students are of high ability. The intuition for this is as follows. Recall that c denotes an effort cost, presumably short term, that students enrolling in an ability-revealing department pay as a consequence of having to endure discriminating examinations, assignments, and so forth. Recall as well that high-ability students identified as such earn a normalized wage of one post-graduation. Since this wage of one is best interpreted as the present value of a stream of wages, then c is presumably much smaller than one and in particular should be understood as being quite close to zero. And, when c is close to zero, then $1-c$ is close to one. Thus, $\pi_H \geqslant 1-c$ implies that we have a rarefied situation in which there are almost no low-ability students at all.

Continuing, when every student enrolls in the ability-concealing department, which is what happens in the pooling equilibrium under consideration, then all students receive the same grades, in particular, grades of "A." An outside observer assessing this situation—in which the fraction of "A" grades is one and there is no variance in awarded grades—might be inclined to say that this is a situation characterized by rampant grade inflation. Such a characterization would be inaccurate, however. Rather, the fraction of inflated grades in the pooling equilibrium is $1-\pi_H$, which is very small and actually quite close to zero given our earlier discussion of π_H and c. Intuitively, what our hypothetical observer is seeing is not a consequence of low-ability students flocking to an easy-grading department in order to hide their low ability levels, a pattern of behaviors that would in fact indeed induce high rates of grade inflation. Rather, the situation the observer sees is one in which high-ability students flock to such a department.

When $\pi_H \geqslant 1-c$, it would be hard to argue that grade inflation as we have defined it is troublesome. After all, when this inequality holds, almost every student is of high ability, and thus the fact that almost every student receives an "A" grade does not connote a serious mismatch between grades and underlying abilities. There is of course some mismatch here, as $1-\pi_H$ of the student body—the low-ability students—receives grades that are inflated. However, $1-\pi_H$ must be quite small for the pooling equilibrium to exist.

One might want to argue that the presence of all students' pooling on an ability-concealing department is an observable indication that almost every student is of high ability. Thinking empirically about actual trends in grades, this reverses the concern that many have articulated about inflated transcripts. To the point, in the pooling equilibrium discussed here, an abundance of students who enroll in an ability-concealing department means that almost every student is highly talented and not, say, that all students lack ability and are choosing an ability-concealing department because they fear being exposed as such by a grade-revealing department. In our pooling equilibrium, high-ability students do avoid paying the effort cost c, which could be criticized on normative grounds, that is, perhaps the students are lazy. However, these students are not enrolling in an ability-concealing department to hide their abilities, and this is the key point here. The small percentage of low-ability students who pool with the high-ability students do so in order to hide their (low ability) statuses; this group does end up with inflated grades, but the group is nonetheless very small.

In the pooling equilibrium here, the ability-revealing department has no students in it. Presumably this is not ideal for the department, and indeed one might conjecture that such a department would anticipate a lack of students and change its grading policy prior to student enrollment decisions. Department grading policies are probably sufficiently sticky so that changing grading norms is not a simple process, and from this perspective treating department grading policies as exogenous seems natural. Nonetheless, we are exploring the matter of strategic department grading policies in other research.

Separating equilibrium. The model always has a separating equilibrium in which low-ability students attend the ability-concealing department and high-ability students, the ability-revealing department. In this equilibrium, whose existence is not a function of the relationship between π_H and c, departments do not contain mixtures of low- and high-ability students. Rather, in the equilibrium all low-ability students attend one department and all high-ability students, the other.

This feature of the separating equilibrium has one rather notable consequence: grades *appear* inflated in both departments. To be precise, in the separating equilibrium all high-ability students receive top grades—because they are in fact of high ability and are enrolled in an ability-revealing department—and low-ability students receive top grades, too—because they enroll in an ability-concealing department which provides everyone with high grades. Thus, our model's separating equilibrium, which exists for all values of π_H and c, features a distribution of grades that looks on the surface to be highly inflated.

In fact, the distribution of grades in the separating equilibrium has literally zero variance because in it every student receives an "A." These numerous "A" grades, however, reflect fundamentally different dynamics. "A" grades received by high-ability students are accurate evidence of excellent students being willing to subject themselves to an ability-revealing process; thus, these “A” grades do not reflect grade inflation. In contrast, however, “A” marks received by low-ability students are evidence of low-ability students avoiding an ability-revealing process; “A” grades received by these students do reflect inflation, and thus the fraction of inflated grades in the separating equilibrium is $1-\pi_H$.

The model’s separating equilibrium is more compelling than the previously discussed pooling equilibrium because the latter only exists when the fraction of high-ability students is very large. With this in mind, we argue that our model shows that student sorting by itself is sufficient to lead to a situation in which all students receive identical grades, all of which are “A” marks; this situation looks like one in which grade inflation is a serious problem but, at least for high-ability students, it is not.

We earlier mentioned that our assumption about the existence of only two departments in a university is not binding. If there were more than two departments in our hypothetical university—some ability-concealing and others ability-revealing—the separating equilibrium we have described here would continue to exist as long as the ability-revealing department or departments imposed effort costs beyond those imposed by the ability-concealing departments. The key to the equilibrium is not the number of departments per se; rather, the key is the fact that ability-revealing departments impose more of a cost on enrolled students than do ability-concealing departments.

Another notable feature of the separating equilibrium is that it requires only that π_H be neither 0 nor 1. If π_H were 1, then all students would be of high ability and the only equilibrium that would exist in the model would be one in which students pooled on the ability-concealing department. Observationally speaking, grades would appear to be inflated in this scenario, but in reality they would not be because all students would be of high ability. If, on the other contrary, π_H were 0, then all students would again pool on the ability-concealing department. This would yield a situation with rampant grade inflation, one wherein all students of low ability are labeled by an ability-concealing department as high ability.

Extension: Continuous student ability and an education bonus

One might argue that our characterization of student ability as binary—either low or high—is too coarse and that this may be responsible for the result, above, that, when students separate, there is no variance in student grades. With this in mind, we now offer an extension of our model that allows us to explore the consequences of allowing student ability to exist on a continuum. Along with this change we also include in the extension an education bonus that a student receives if she enrolls in an ability-revealing department. As shown below, the extension of our model does lead to variance in student grades; however, it does not change our fundamental results about grade inflation and the effects of student sorting on the distribution of grades.

Let student ability be denoted θ. We assume that $\theta \sim U\left(0,1\right)$, but this assumption is not qualitatively necessary for the results that follow. We make the uniformity assumption because it allows for a closed-form equilibrium characterization.

When student ability exists on a continuum, we can no longer speak simply of "low" and "high" ability students. In addition, with continuous student ability, we need a more refined characterization of grades and of post-education wages. We continue to assume that an ability-revealing department is one that assigns grades based on underlying student abilities, and with a continuous distribution of abilities this is obviously a bit of an abstraction insofar as a finite number of class letter grades—"A," "A −," "B+," and so forth—cannot map one-to-one to a continuous range of student abilities. However, one can imagine that an ability-revealing department issues class grades, returns discriminating assignments, generates ability-revealing letters of recommendation, and so forth, in such a way that a student who enrolls in such a department has an overall record from her educational experience that perfectly reveals her ability level. Thus, in the extension of the model, a student with ability θ who attends an ability-revealing department receives a grade of θ.

With respect to ability-concealing departments, we continue to assume that such a department awards very high grades to all of its students. In particular, in the model extension, we assume that every student in it receives a grade of 1. This is parallel to our earlier assumption that ability-concealing departments award grades of "A" to their students.

In terms of wages, suppose that a student of ability θ whose ability is known to the labor market receives a base wage of θ in the post-education market. In other words, the more ability a student has, the greater the student's wage will be, assuming that the market knows the student's ability. If in addition this student attended an ability-revealing department, then she receives a wage boost of $e \geqslant 0$. Such an *education bonus* is intended to capture the fact that studying in an ability-revealing department and bearing the requisite effort cost can lead to increased knowledge and, accordingly, higher wages.

As was the case in our initial model formulation, a student whose wage is not known receives a base wage in the labor market corresponding to expected ability level where this expectation is taken given equilibrium student behavior. Such a student cannot receive the education bonus e because the only way to receive such a bonus is to enroll in an ability-revealing department, an action that would signal the student's ability.

Considering both effort cost, education, and the effect of ability on what we are calling base wages, if a student with ability θ enrolls in an ability-revealing department, her net utility is $\theta - c + e$. Similarly, if students with $\theta \in \Theta \subset (0,1)$ enroll in an ability-concealing department, then each student with $\theta \in \Theta$ receives net utility of $\int_{\Theta} \theta d\theta$.

The equilibrium of the model extension depends on a cutpoint that we call $\bar{\theta}$, and this cutpoint is characterized by student indifference between enrolling in an ability-concealing and an ability-revealing department. The equilibrium is described in Lemma 2, whose proof is in Appendix 1.

Lemma 2. *Let* $\bar{\theta} = 2(c - e)$.

If $\bar{\theta} \in (0,1)$, then the equilibrium of the model is semi-pooling. Namely, students with ability $\theta < \bar{\theta}$ enroll in an ability-concealing department and students with $\theta > \bar{\theta}$ enroll in the ability-revealing department. Students with $\theta = \bar{\theta}$ are indifferent between the two departments, and we ignore these students since they are of zero measure.

If $\bar{\theta} > 1$, then the equilibrium of the model is pooling wherein all students enroll in an ability-concealing department. If $\bar{\theta} < 0$, then the equilibrium of the model is pooling wherein all students enroll in an ability-revealing department.

The knife-edge condition $\bar{\theta} = 1$ (equivalent to $c - e = 1/2$) is consistent with either semi-pooling as described above or students pooling on the ability-concealing department; without loss of generality, we assume that $\bar{\theta} = 1$ leads to the latter type of pooling. Similarly, the knife-edge condition $\bar{\theta} = 0$ (equivalent to $c = e$) is consistent with either semi-pooling or student pooling on the ability-revealing department; without loss of generality, we assume that $\bar{\theta} = 0$ leads to the latter type of pooling.

Lemma 2 shows that the indifference cutpoint $\bar{\theta}$, which determines whether the extended model's equilibrium is semi-pooling ($\bar{\theta} \in (0,1)$) or is pooling ($\bar{\theta} \leqslant 0$ or $\bar{\theta} \geqslant 1$), depends on the relationship between the effort cost c required of students enrolled in an ability-revealing department and the education bonus e. We earlier argued that c should be thought of as small compared to one because this parameter represents a short-term effort cost as opposed to a discounted stream of wages. Therefore, we assume that $c - e < 1/2$; this implies that the long-term benefit of education exceeds by a large amount of the short-term costs that a student faces when taking a

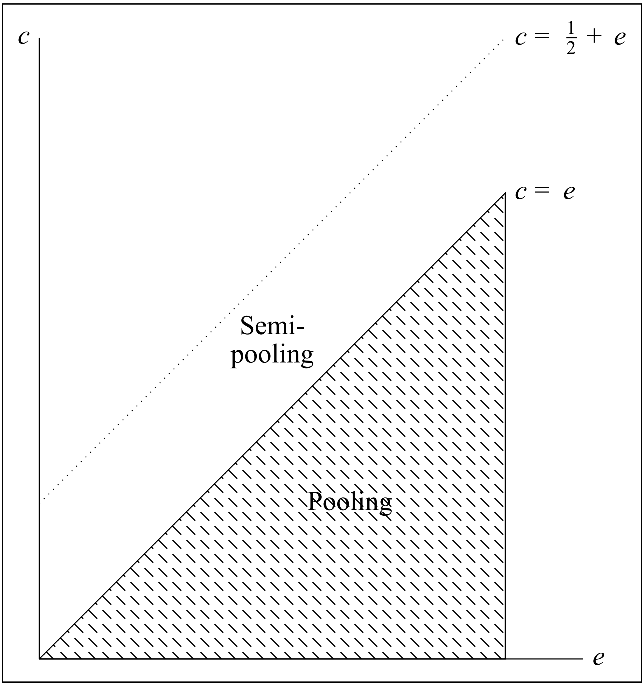

Figure 2. Extended model equilibria as a function of c and e.

large number of university tests. This assumption is intuitive, and thus we focus our attention on two possibilities for our cutpoint, either $\bar{\theta} \leqslant 0$ or $\bar{\theta} \in (0,1)$.[11] These regions are depicted visually in Figure 2.

Suppose first that $\bar{\theta} \leqslant 0$. Because $\bar{\theta} = 2(c - e)$, it follows that the education bonus e must be greater than or equal to the cost c that students must pay when taking classes in an ability-revealing department. When $e \geqslant c$, all students attend the ability-revealing department and, accordingly, all students' grades are accurate measures of their abilities. There is no grade inflation when $\bar{\theta} \leqslant 0$ because, in this situation, there are no students in the ability-concealing department.

Suppose one were to argue on normative grounds that student pooling on the ability-revealing department is a good thing, that is, that society benefits when $\bar{\theta} \leqslant 0$. One might then ask, how might students be induced to pool in this way? Based on Lemma 2, the answer here is simple: our hypothetical university should increase its education bonus e until the value of education overwhelms the cost of attending an ability-revealing department. As soon as this happens, then all students become willing to attend said department, an outcome that, as we noted, has no grade inflation at all. It is also true that making the effort cost c small will have the same effect.

Now consider the case $c > e$ with the previously noted proviso that the effort cost c remains small. Intuitively, the inequality $c > e$ implies that education is valuable to those students who enroll in an ability-revealing department—but not overly so. If one thinks of education like Spence (1973), solely in the sense of signaling, then $e = 0$, in which case the restriction $c > e$ certainly holds.

When $c > e$ the model's equilibrium is semi-pooling, and the first feature to notice about the equilibrium is that the fraction of students who enroll in the ability-revealing department is $1-\bar{\theta} = 1-2(c-e)$. This expression is decreasing in the cost term c, which is intuitive: the more effort required in the ability-revealing department, the fewer the number of students who are willing to enroll in it. It is also increasing in e: the greater the value of education for those students who work hard, the greater the number of students who are willing to enroll in an ability-revealing department.

In the semi-pooling equilibrium, the average student grade is

$$\int_0^{\bar{\theta}} d\theta + \int_{\bar{\theta}}^{1} \theta\, d\theta = 2(c-e) + \frac{1-4(c-e)^2}{2}$$

Differentiating this expression with respect to c yields $2-4(c-e)$, which is positive as long as $c-e < 1/2$. Thus, the more difficult the ability-revealing department becomes, the higher are average grades in equilibrium, *ceteris paribus*. However, the opposite result holds with respect to the education bonus e: the greater this bonus, the lower are average grades in equilibrium, *ceteris paribus*.

The result about the effect of the effort cost c on average grades is particularly notable, and there are two reasons that increases in c lead to higher grades on average. First, when the difficulty parameter c increases, the indifference cutpoint $\bar{\theta}$ increases as well. When c is large, that is, only high-ability students remain in the ability-revealing department; when their abilities are revealed, they are accurately revealed to be high. This is not grade inflation; to the contrary, it is accurate reporting. Second, and on the contrary, an increase in c causes some students to select out of the ability-revealing department and into the ability-concealing department; these students receive high grades on account of enrolling in the letter, indeed grades higher than they should based on their underlying abilities. This is a form of grade inflation, and the fraction of students with inflated grades is $2(c-e)$. Overall, the point here is that the increase in grades due to increasing effort cost c has two components which manifest themselves similarly but have different underlying causes, only one of which can be considered problematic.

When the education bonus e increases, then the effects described above move in the opposite difficulty will lower average grades is, however, a partial equilibrium assertion. Since students sort themselves conditional on department difficulty, making an ability-revealing department more difficult will drive students away from it and thus have the opposite effect of what the dean or other figure intended.

Such an argument holds conditional on the existence in our hypothetical university of an ability-concealing department, and this highlights the possibility that there is a collective action problem in university grading, one that the dean could overcome if she could simultaneously convince an ability-revealing department to be more difficult while convincing (or compelling?) an ability-concealing department to become ability-revealing. We will return to this point later. At the moment, though, it suffices to note that, when students have an ability-concealing department as an option, the more difficult the ability-revealing department, the higher are average grades and thus the more grades look like they are inflated.

One sees a similar point when examining the variance in grades in the extended model's semi-pooling equilibrium. This variance is

$$\int_0^{\bar{\theta}} d\theta + \int_{\bar{\theta}}^{1} \theta^2 \, d\theta - \left(2(c-e) + \frac{1-4(c-e)^2}{2} \right)^2 .$$

Algebra shows that this variance approaches $1/12$, the variance of the uniform distribution on the unit interval, as c approaches e. This is appropriate because, as c becomes smaller, more students enroll in the ability-revealing department and this leads to a grade distribution that gets close to the true ability distribution.

More importantly, the derivative of the above variance is negative for relevant parameter values. In other words, the more difficult the ability-revealing department becomes, the less variability there is in student grades. This is because increasing difficulty leads to an increasing number of students in an ability-concealing department and accordingly less grade variability. If, say, our aforementioned university dean or administrator were to argue that his or her institution should seek extensive variability in grades—because, say, variability in grades makes it easier to distinguish low and high ability students—the implication of our extended model is that the dean should insist that the ability-revealing department be as easy as possible on its students.

Given the definition of $\bar{\theta}$, another option open for a dean who wanted to increase grade variance would be to encourage the ability-revealing department to increase its education bonus. This would have the effect of inducing more students to enroll in said department, and this would lead to increased variance in grades and less grade inflation. This option may be more palatable than decreasing the effort cost c.

We motivated the extension of our model with the recognition that the coarse way in which we modeled student ability—low versus high—diminishes our ability to ascertain whether in equilibrium there is variance in student grades. Our extension shows that this concern was indeed valid. Namely, as long as the effort cost c associated with an ability-revealing department is not excessive compared to the education bonus (that is, $c - e < 1/2$, which we earlier justified by arguing that the short-term cost c is close to zero), then there is indeed variance in student grades. This variance is conditional on effort cost c and the education bonus e, and above we have explained how changing these two parameters changes average grades and grade variance.

Our final point about the model extension concerns the possibility of including a cost term for enrolling in an ability-concealing department. Were we to have done this, then we would have seen that what we call c in the model extension proxies for the *difference* in cost associated with an ability-revealing department and an ability-concealing department. Having said this, there are two ways to interpret the effects of an increase in the effort cost c. In particular, c in the extended model can increase because the ability-revealing department becomes more difficult, and it can increase because the ability-concealing departments become easier. These dual perspectives on c do not change any of our earlier interpretations or derivatives, but they do imply, for example, that increasing the cost required for a student to enroll in an ability-revealing department is equivalent to decreasing the cost associated with an ability-concealing department.

Discussion

We have offered a game-theoretic analysis of student grading, an analysis motivated by empirical studies documenting upward trends in grades in higher education institutions. Our formalizations shed light on the implications of student choice over departments, a key feature of grading processes in universities that is often neglected in discussions of grading trends in higher education. The students in our model are both effort-averse and forward-thinking, and this induces a dynamic in which the best students seek to distinguish themselves from their lower-ability counterparts and are willing to undertake costly behaviors so that their true abilities are revealed to a post-education labor market. The end result of this is that average grades are high but not because of, say, lax standards or enrollment pressures. Rather, grades are high because good students appropriately earn them from an ability-revealing department and lesser students garner them, so to speak, from an ability-concealing department.

To be clear, we are not arguing that our model should be thought of as a (or "the") comprehensive explanation of grade inflation in higher education. Our primary objective has not in fact been to offer a complete theory of grading in educational institutions but rather to encourage scholars interested in grading to consider assiduously the consequences of student sorting on grade distributions. Existing literature makes it clear that there are a variety of explanations for the types of grade inflation that empirically driven scholars of education have identified, and our models should remind those considering these explanations to be mindful of how sorting can manifest itself.

The model adduced here shows how student sorting at one level of the higher education landscape—students within universities—leads to distributions of grades that seem extensively inflated but, for high-ability students at least, are nonetheless accurate. There are additional levels of sorting that we have not specifically engaged, and these include sorting across universities and sorting within departments. Although we have not modeled choice of university, one could envision a more general model of education wherein a student selects into the best university that accepts him or her and then chooses a department within the chosen universities. If some universities are known for being ability-concealing and requiring little effort, then students of higher ability will presumably not apply to these institutions and instead pursue education in costly, ability-revealing institutions. These latter institutions will then be disproportionately populated by high-ability students, which will compound the grading dilemmas caused by within-university selection into departments. Given the recent increase in competition in the United States competition for admission into elite colleges and universities, it is conceivable that across-institution sorting may be a notable factor in explaining nationwide increases in average grades.[12]

What does our model say about contemporary trends in grades? Perhaps the most direct implication is as follows: within-institution trends in grades are hard, if not outright impossible, to interpret in isolation from trends in the extent to which students sort themselves strategically into departments. Put another way, student sorting confounds grading, and therefore analyses of grade trends that are executed independently of sorting dynamics can be misleading. In our basic model where parameters are reasonable (i.e. the cost of attending an ability-revealing department is not too high compared to long-term wage streams), literally all students receive high grades and there is correspondingly no variance in the distribution of grades. Viewed from the lens of grades only, this situation looks problematic and in need of remedy; it is problematic, however, only for low-ability students as the high grades received by top students correctly reflect these students' high-ability levels. Challenged to defend its plethora of high grades, an ability-revealing department in this situation might respond, "All of our students are excellent!" Due to student sorting driven by high-ability students seeking to distinguish themselves from low-ability students, this claim would be accurate.

Put another way, our models show that the grading practices of individual departments cannot be assessed simply by observing whether they assign many "A" marks or, say, "C" marks. Suppose that a university dean were to compare the grades across two departments in her jurisdiction, and suppose that she were to notice that both consistently give many (or perhaps exclusively) "A" grades. Should the dean insist that these two departments raise their grading standards or, say, ramp up the effort levels required for classes in said departments? Not necessarily. Of the two departments, if one is ability-concealing, then only top students choose the other, thus inducing this department to give a plethora of high grades that are accurate. If this department were to raise its standard, this would not alleviate the selection incentives that we have explored here.

Our model is not explicitly dynamic, but it nonetheless suggests that one source for observed trends in grades could be the emergence of one or two ability-concealing departments in a university. That is, suppose that many years ago all departments in a hypothetical university were ability-revealing and entailed effort costs. Were this the case, then we would expect these departments to have issued both low and high grades. Suppose then that an exogenous shock—the Vietnam War, as some have conjectured[13]—led one department in a university, or perhaps a small number of departments, to adopt grade-inflating practices and simultaneously reduce the effort needed to enroll in said department or departments. As soon as this were to have happened, we would be in a situation where the university had a combination of both ability-concealing departments and ability-revealing departments. In the presence of both types of departments, our model suggests that forward-looking students of high ability will seek to separate themselves from lower-ability students, the latter of whom will choose ability-concealing department and the former, ability-revealing departments. The result of this will be that all students earn high grades. In this example, the culprit for high average grades overall is the presence in a university of a small number (or even a non-small number) of ability-concealing departments. Indeed, one could argue that ability-revealing departments are somewhat at the mercy of ability-concealing departments: once some of the latter exist, the former will enroll only good students. This leads to a flattening of the grade distributions produced by grade-revealing departments.

This point highlights a collective action problem associated with grading. A department that by itself wanted to address institution-wide grade inflation can be stymied by the ability-concealing behaviors of other departments. If an ability-revealing department were to make its classes increasingly challenging in an attempt to mitigate inflation, then it would make the overall grade inflation problem worse and in so doing decrease its own enrollments. To the extent that low enrollments are problematic for departments who might want to use enrollment figures to argue for faculty positions, no department has an incentive on its own to increase the cost associated with its classes. This sort of collective action dilemma means that university administrators should not assume that individual departments will ever be able to coordinate themselves and form a solution to what administrators might consider a grade inflation problem.

Our model has implications for department enrollment patterns associated with gender, race, and socioeconomic status. We have assumed throughout this article that students know their own ability levels, but it is worth considering how it is that a student might learn whether she is of low or high ability and whether it is possible for a student to think, wrongly, that she is of low ability when she actually is of high ability. This sort of error is particularly pernicious for students because a high-ability student who believes that she is of low ability may prefer an ability-concealing department over an ability-revealing department—even though the latter would be more valuable. Moreover, if pre-university evaluations for a given group of students are systematically biased in this way so that students in said group regularly underestimate their abilities, then this group will enroll in ability-concealing department even though they should not. This will lead to labor market inefficiencies (high-ability individuals will not be treated as such) and depress the group's aggregate, post-education earnings in the long run.

There is evidence that these kinds of concerns shape student enrollment patterns across departments. Ehrlinger and Dunning (2003) show that women often underestimate their scientific knowledge and aptitude. According to our model, this will lead women disproportionately to select into ability-concealing departments, and this will have society-wide implications, namely, potentially wasted talent and a lack of women in certain scientific fields. Similar concerns have been raised about African-American students (Ewing et al., 1996; Nacoste, 1989), and there is evidence that, *ceteris paribus*, students of low socioeconomic status are less likely to pursue further academic study than students of higher socioeconomic status (Erikson et al., 2005; Jackson et al., 2007). Per our model, the culprit for this could be systematic biases in pre-university grades which lead low socioeconomic students to doubt their own abilities.

A final point on this subject concerns risk aversion. If a group of students is more risk averse than another, this could lead members of the group to enroll in ability-concealing departments rather than risking enrolling in the ability-revealing departments. The relationship between gender and risk aversion is not simple (e.g. Eckel and Grossman, 2008; Jianakoplos and Bernasek, 1998; Schubert et al., 1999), but it is worth pointing out that, if women are more risk averse than men, then this dynamic will compound the types of biases, noted above, that drive women disproportionately toward ability-concealing departments.

We conclude with comments that link our game-theoretic approach to empirical work on grade inflation, and here we pursue two approaches. First, we discuss our model's implications for the ways in which one might seek to understand whether grades in a given institution are inflated. And second, we consider how one might test to see whether our model's dynamics roughly approximate what one observes in higher education institutions.

With respect to the first point, suppose that a researcher at a university wanted to know about the extent to which her institution's departments were ability-concealing or ability-revealing. Knowledge of this type could in principle inform university policy decisions insofar as regulating ability-concealing departments by compelling them to issue ability-revealing grades could "solve" the problem of grade inflation to the extent that it is considered a problem. How would an interested party determine which departments in a university are ability-concealing? As is clear from our theoretical results, looking at which departments have high grades is not sufficient and can actually be misleading. Rather, ability-concealing departments can be identified because their students vary in ability yet receive similar grades. Presumably the administration in most if not all educational institutions knows, say, standardized test scores of all its enrolled students. Departments whose enrolled majors, say, have high variance in test scores yet have received disproportionately high grades may be ability-concealing.

To be precise, suppose that of two departments in a university, one had high variance in student test scores and low variance in grades, and suppose that the second had low variance in test scores and high variance in grades. This would presumably indicate differences in grading practices in a way that falls roughly along the ability-concealing versus ability-revealing dichotomy described here. These variances are still subject to a student sorting confound, but this is generically true unless a university were to compel all of its students to enroll in common course. Absent such a common course, combining variances in measures of ability like standardized tests with variances in grade distributions might yield a plausible picture of which departments in a university are ability-concealing and which are ability-revealing. In the long run, this will aid the general understanding of grading dynamics and how educators and researchers should interpret trends—both upward and downward—in grades.

Another approach to assessing whether grades in a university are inflated is to consider whether post-education institutions that in theory rely on these grades actually use them. Universities know the enrollment choices of their students and which students receive high grades; universities will often also know which students, say, receive interview opportunities and valuable job offers. Armed with this information, universities could in principle determine which grades are correlated with post-education success. For example, if there is a department that produces high grades which do not predict post-academic success, then it follows that this department may be guilty of grade inflation. Similarly, if there is another department such that grades from it—low or high—predict success, then this department is presumably ability-revealing and thus attracting the best students. Moreover, if a university observes that its students are subjected to grueling and extensive interviews, this might suggest that none of its grades is particularly meaningful. Long interviews are only necessary, one would think, if grades are not signaling abilities, that is, if there is university-wide grade inflation. Similarly, if student activities like internships and formal recommendations carry more weight in the post-education labor market than grades, then one might surmise that grades are lacking in their signaling ability.

With respect to our second empirical point, we now consider four empirically testable hypotheses based on our model.

First, *students will sort themselves based on self-perceived ability across types of departments.* This hypothesis could be tested by conducting a survey among undergraduate students prior to enrollment decisions. Our model suggests that student perceptions about the difficulty of grading in a department should be correlated with the perceived ability level of the students majoring or concentrating in that department. Students who perceive themselves as high ability should cluster in difficult concentrations, *ceteris paribus*, while students who believe they are low ability will do the opposite. Since an important implication of our model is that the grade distributions within a department cannot be, on their own, evidence of grade inflation, clustering in self-perceived student ability could be an important sign that grade inflation is occurring at disparate rates among departments.

Second, *employers' preferences for hiring students in departments perceived as ability-revealing will stem in part from the perceived higher ability level of students in those departments*. This hypothesis could be tested using a survey of employers who frequently recruit undergraduates on a particular campus. Employers could be asked about whether they prefer certain majors, how they interpret grades among students from different majors, and how they perceive the grading practices within those majors. Employer reactions to mock student profiles that only vary based on the major granting department could be used to measure how much of a difference perceptions of different majors make. These responses could then be used to assess the weights employers place on grades varies with the perceived grade inflation in the major. In the survey considered here, it would be important to focus on comparing departments that have similar course content and employers that do not have a strong preference for students with specific technical skills.

Third, *departments which place a high value on attracting the highest ability students will be the least likely to have inflated grades.* A key implication of our model is that high-ability students will select into non-grade inflating departments. Consequently, departments that want to attract these students will have an incentive to provide grades that accurately reflect student abilities. This hypothesis could be tested by conducting a survey of department faculty regarding both the types of students a department is seeking to attract and the department's grading policies, whether it mandates curves, mandates median grades, and considers grade distributions in the context of course evaluations. We would expect departments that put an emphasis on attracting the highest ability students to be the most aggressive in combating grade inflation, *ceteris parbus*, while departments that emphasize enrollment maximization regardless of student ability would be more tolerant of grade inflation.

Fourth, *student sorting will increase in the presence of increased information about department grading practices.* Students can only sort themselves across department types if they know which departments are ability-concealing and which are not. This said, it is probably rare that students have perfect knowledge about department grading practices. Consequently, in situations where information about department grading practices suddenly increases, we would expect a surge in student sorting. Suppose that an institution were to promulgate a policy by which class median grades are made public. Our model suggests that this sort of natural experiment will lead to increased student sorting.

With this last point in mind, we end by noting a previously cited study that draws on data from Cornell University. As we described earlier, Cornell began publishing information about course median grades in 1996, and Bar et al. (2009) discuss changes in student enrollment patterns in Cornell following the 1996 policy change. In particular, they show that enrollment in low-grading classes declined after Cornell began publishing median grades; these classes are presumably ability-revealing. However—and this is the key point—this effect was large for students with relatively low grade point averages and yet statistically indistinguishable from zero for students in the top 10% of Cornell's grade distribution. In other words, Cornell's policy change, which allowed students to determine which classes were ability-revealing and which were ability-concealing, produced a situation wherein low-ability student shifted out of the former in favor of the latter. High-ability students did not behave this way, and this is what our model would suggest. That the Cornell study produces results in line with our formalization is pleasing. This emphasizes the role that student sorting plays in grading and that incentives for sorting should be a part of the academic literature on grading practices.

Acknowledgements

The authors thank Craig Volden for the conversation that inspired this paper and Mark McPeek, Russell Muirhead, two anonymous referees, and seminar participants at Dartmouth College, the University of California at San Diego, the University of Virginia, and Yale University for helpful comments.

Funding

The authors received no financial support for the research, authorship, and/or publication of this article.

Notes

1. On the German case, see "Grade inflation blows up German universities," *The Local.de*, available at http://www.thelocal.de/20121219/46847 (last accessed 19 September 2016).
2. For press coverage of grade inflation in American higher education, see "Doesn't Anybody Get a C Anymore?" *The Boston Globe*, October 5, 2008, available at http://www.boston.com/bostonglobe/magazine/articles/2008/10/05/doesnt_anybody_get_a_c_anymore (last accessed 15 February 2015); "How to Fix College Grade Inflation," *U.S. News & World Report*, December 26, 2013, available at http://www.usnews.com/opinion/blogs/economic-intelligence/2013/12/26/why-college-grade-inflation-is-a-real-problem-and-how-to-fix-it (last accessed 14 February 2015); "Harvard professor says grade inflation rampant," *The Boston Globe*, December 44, 2013, available at http://www.bostonglobe.com/metro/2013/12/03/harvard-professor-raises-concerns-about-grade-inflation/McZHfRZ2RxpoP5Xvwged1N/story.html (last accessed 14 February 2015); and, "The Most Commonly Awarded Grade at Harvard Is an A," *The Atlantic*, December 4, 2013, available at http://www.theatlantic.com/education/archive/2013/12/the-most-commonly-awarded-grade-at-harvard-is-an-a/282020/ (last accessed 28 April 2015).
3. See "Harvard's dirty, little secret is out—grade inflation / Graduating with honors is a breeze," *Boston Globe*, October 14, 2001, available at http://www.sfgate.com/news/article/Harvard-s-dirty-little-secret-is-out-grade-2868775.php (last accessed 23 April 2015).
4. See "Grade expectations," *Yale Alumni Magazine*, September/October 2013, available at https://www.yalealumnimagazine.com/articles/3735/grade-expectations (last accessed 20 April 2015).

5. Millman et al. (1983) argue that grade inflation through 1983 had not in the United States eliminated the signaling value of grades, and Pattison et al. (2013) makes similar claims with contemporary data.
6. A plausible way for students to know their own abilities is through introductory “weed-out” classes that are highly discriminatory and do not require commitment to a major or course of study. One could also argue that students know their abilities via secondary education coursework, but this might be troubling as secondary education may suffer from the same sorts of grading dynamics that are present in universities.
7. This parallels an assumption made by Popov and Bernhardt (2013).
8. The key here is that an ability-concealing department gives all enrolled students “A” grades. One could posit that the examinations in such a department are ability-revealing but that the faculty in said department simply ignore this when assigning grades.
9. Our model assumes that a student’s being of low or high ability is a generic statement about underlying talent. However, some academic departments, civil engineering for example, teach specific skills that a high-ability student who focuses in a non-engineering area will not have, despite being of high ability. As long as the types of departments that teach specific skills are ability-revealing, then our model applies to them. Departments to which our model might not apply are those that teach specific skills yet are ability-concealing. If testing for the presence of skills requires careful examinations, it is hard to imagine that many departments with specific skill requirements—engineering, medicine, and so forth—are like this. Finally, it is worth emphasizing that there may be differences in grading policies between departments that teach similar skills and provide similar credentials. For example, electrical engineering and computer science might have tracks that qualify students for similar technical jobs but nonetheless that have different grading practices.
10. Another feature of grades is that they usually are discrete. However, our model does not have anything to say about this feature of grading.
11. If the effort cost c is close to zero, then $c - e$ will be small as well, even if e is negligible.
12. Not all schools may be subject to sorting in the same way; the University of Texas, for example, is subject to a state law that mandates admissions policies for freshman from Texas high schools. This law may lead to a larger variance in underlying student ability levels than would be expected otherwise. See “Report to the Governor, the Lieutenant Governor, and the Speaker of the House of Representatives on the Implementation of SB 175, 81st Legislature For the period ending Fall 2014,” The University of Texas at Austin, available at https://www.utexas.edu/student/admissions/research/SB_175_Report_for_2014.pdf (last accessed 12 May 2015).
13. For example, see “A History of College Grade Inflation,” *The New York Times*, July 14, 2011, available at http://economix.blogs.nytimes.com/2011/07/14/the-history-of-college-grade-inflation/ (last accessed 13 February 2015).

14. We need not specify the off-equilibrium path belief of the labor market that a deviating student in an ability-concealing department is of high ability conditional on all students pooling on enrolling in an ability-revealing department. This is because any such conditional belief will contradict the existence of the conjectured pooling equilibrium.

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Appendix 1

Proof of Lemma 1

Proof. Suppose that $\pi_H \geq 1-c$. If all students enroll in a grade-inflating department, then each receives π_H as a post-education wage. This situation obtains because the labor market cannot distinguish between the students based on their transcripts, all of which contain identical marks (all “A” grades, as the ability-concealing department by construction issues only “A” marks) and all of which were issued by one department. A low-ability student who deviates to a grade-revealing department will pay a cost of c and, by virtue of enrolling in an ability-revealing department, will be identified as having low ability. Such a deviating student will thus receive utility of $-c$, which is strictly less than π_H. In contrast, a high-ability student who deviates to the ability-revealing department will receive a wage of one, which is better than receiving π_H, but will also have to pay a cost of c. If $\pi_H \geqslant 1-c$, then this deviation is not optimal.

Now let π_H and c be given such that $\pi_H \in (0,1)$ and $c \in (0,1)$, and consider the existence of a separating equilibrium as posited in the lemma. In such an equilibrium, low-ability students receive utility of zero. If a low-ability student were to deviate to the ability-revealing department, she would continue to receive a wage of zero and in additional would pay the effort cost c. Since $c > 0$, deviating in this way is not optimal. Similarly, with separation a high-ability student receives a wage of one and an overall utility of $1-c$. Deviating to an ability-concealing department would lead to a utility of zero because the student would be considered low ability. Since $0 < 1-c$, deviating is not optimal.

To show that there is no pooling equilibrium in which all students enroll in a grade-revealing department, suppose that such an equilibrium were to exist. A low-ability student would earn utility of $-c < 0$ in such an equilibrium. Were such a student to deviate to an ability-concealing department, the student would not pay the effort cost c and would receive a wage between zero and one, inclusive. This would lead to positive utility greater than $-c$, thus contradicting the conjectured equilibrium.[14]

As an aside, this proof does not depend on the value of c being identical for both low- and high-ability students. This point supports the claim made earlier in the body of the paper that, as long as enrolling in an ability-revealing department imposes an effort cost, the results of the model do not depend on whether c varies by student ability.

Proof of Lemma 2

Proof. If in equilibrium a student with ability θ_1 prefers an ability-revealing department, then it is straightforward to show that a student with ability $\theta_2 > \theta_1$ does as well. This follows from the fact that a student's utility for an ability-revealing department is increasing in underlying student ability. Similarly, if in equilibrium a student with ability θ_1 prefers an ability-concealing department, then it is straightforward to show that a student with ability $\theta_2 > \theta_1$ does as well. Thus, if in equilibrium students separate in their choices between departments, they must do so where a cutpoint, which we denote $\bar{\theta}$, partitions students based on ability.

We now derive the value of $\bar{\theta}$. Let a student with ability $\theta = \bar{\theta}$ be indifferent between enrolling in an ability-concealing department and enrolling in an ability-revealing department. The former nets the student $\bar{\theta}/2$ and the latter, $\bar{\theta} - c + e$. Equating these two utilities yields $\bar{\theta} = 2(c-e)$.

If $c - e \geqslant 1/2$, then $\bar{\theta} \geqslant 1$ and all students pool on the grade-revealing department. Similarly, if $c \leqslant e$, then $\bar{\theta} \leqslant 0$ and all students pool on the ability-revealing department. If $\bar{\theta} \in (0,1)$, the equilibrium is semi-pooling.