CS1555 - Relational Algebra

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1. Arity / Cardinality

a) $\pi_{country}(Users)$

```
Arity       - 1
Cardinality - Min() = 1, Max() = 100
```

The arity is 1 due to only projecting the *Country* column from the *Users* relation. The min cardinality value occurs when all users live in the same country. The max cardinality occurs when each user lives in a different country (no duplicates). This could have an absolute max value of 195, as there are that many countries in the world today.

b) $Mail \bowtie_{Mail.mailID=Recipients.mailID} Recipients$

```
Arity       - 9
Cardinality - Min() = 3500, Max() = 5499
```

The arity is 9 due to selecting all columns from both the *Mail* and *Recipients* relations, even if they have the same value (*mailID*). The min cardinality value occurs when each piece of mail has been sent to at least 1 user. The max cardinality occurs if only 1 piece of mail has been sent to 3500 recipients, resulting in 1999 records with null recipients on the left outer join. However, if mail MUST have at least 1 recipient, then the max cardinality is 3500.

c) $Mail * Labels$

```
Arity       - 9
Cardinality - 5,000,000
```

The arity is 9 due to selecting all columns from both the *Mail* and *Labels* relations, even if they have the same value (*mailID*). The cardinality value is equal to the cardinality of each relation multiplied together.

2. Expressions

a) Urgent Mail

$$\pi_{fname,\ lname,\ userID,\ subject,\ timeSent}(\sigma_{urgent=1}(Mail \bowtie_{Mail.senderID=Users.userID} Users))$$

SQL:

```
SELECT
    u.fname,
    u.lname,
    u.userID,
```

```
    m.subject,
    m.timeSent
FROM
    Mail m
LEFT JOIN Users u
    ON m.senderID = u.userID
WHERE
    m.urgent = 1;
```

b) Outside USA

$$\pi_{userID}(\sigma_{urgent=1 \,\wedge\, country<>\text{"USA"}}(Mail \bowtie_{Mail.senderID=Users.userID} Users))$$

SQL:

```
SELECT  DISTINCT
    u.userID
FROM
    Mail m
LEFT JOIN Users u
    ON m.senderID = u.userID
WHERE
    m.urgent = 1 AND
    u.country <> "USA";
```

c) Important Label

$$G_{sum(mailID)}(\pi_{mailID}(\sigma_{label=\text{"important"} \,\wedge\, urgent<>1 \,\wedge\, timeSent=2015}(Mail \bowtie_{Mail.mailID=Labels.mailID} Labels)))$$

SQL:

```
SELECT
    COUNT( DISTINCT m.mailID)
FROM
    Mail m
LEFT JOIN Labels l
    ON m.mailID = l.mailID
WHERE
    l.label = "important" AND
    m.urgent <> 1 AND
    YEAR(m.timeSent) = 2015;
```

d) Ava Lovelace

$$\pi_{fname,\,lname}\big(\sigma_{recipientID=[\pi_{recipientID}(\sigma_{fname=\text{``Ava''}\,\wedge\,lname=\text{``Lovelace''}}(Mail \bowtie_{senderID=userID} Users \bowtie Recipients))]}$$

$$(Mail \bowtie Recipients \bowtie_{senderID=userID} Users))$$

SQL:

```
SELECT
    u.fname,
    u.lname
FROM
    Mail m
LEFT JOIN Users u
    ON m.senderID = u.userID
LEFT JOIN Recipients r
    ON m.mailID = r.mailID
WHERE
    r.recipientID = (

    SELECT DISTINCT
        r.recipientID
    FROM
        Mail m
    LEFT JOIN Recipients r
        ON m.mailID = r.mailID
    LEFT JOIN Users u
        ON m.senderID = u.userID
    WHERE
        u.fname = "Ava" AND
        u.lname = "Lovelace"

    );
```

e) Not Reply in USA

$$\pi_{mailID}\big(\sigma_{replyTo=NULL\,\wedge\,(u\_send.country=\text{``USA''}\,\vee\,u\_recv.country=\text{``USA''})}$$

$$(Mail \bowtie Recipients \bowtie_{senderID=u\_send.userID} Users\ u\_send \bowtie_{recipientID=u\_recv.userID} Users\ u\_recv))$$

SQL:

```
SELECT DISTINCT
```

```sql
    m.mailID
FROM
    Mail m
LEFT JOIN Users u_send
    ON m.senderID = u_send.userID
LEFT JOIN Recipients r
    ON m.mailID = r.mailID
LEFT JOIN user u_recv
    ON r.recipientID = u_recv.userID
WHERE
    m.replyTo IS NULL AND
    ( u_send.country = "USA" OR
      u_recv.country = "USA"
    );
```