CS1675 - Assignment 3

Zachary M. Mattis

February 7, 2019

I. Problem 1 - Bernoulli Trials

a. ML Estimate

$$\hat{\theta}(x) = 0.65$$

b. $Beta(\theta|1, 1)$

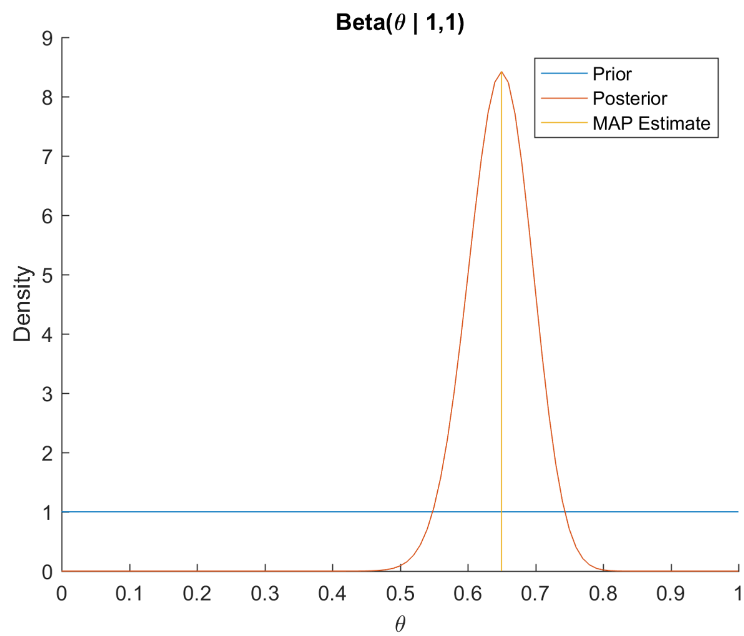

Figure 1: Prior = 1,1

c. MAP Estimate

$$\text{MAP Estimate of } \theta = 0.65$$

d. $Beta(\theta|4, 2)$

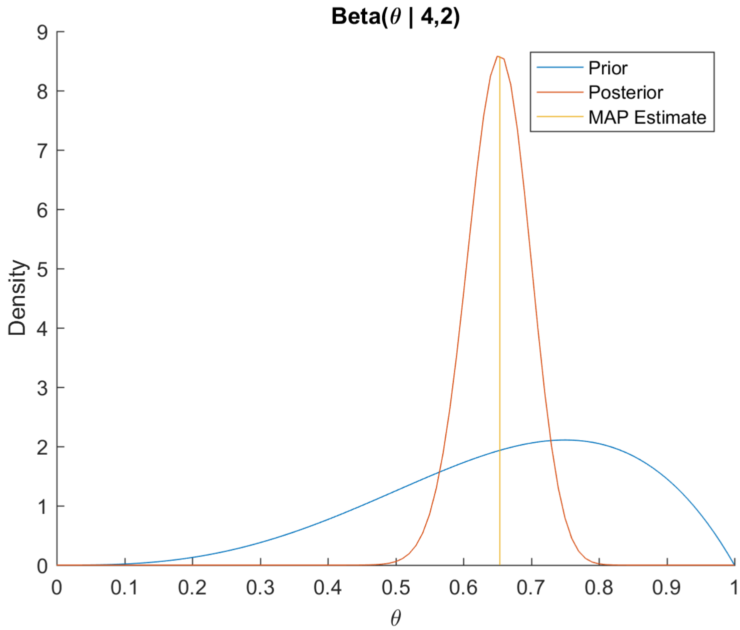

Figure 2: Prior = 4,2

MAP Estimate of $\theta = 0.6538$

II. Problem 2 - Multivariate Gaussian

a. Gaussian Scatter Plot

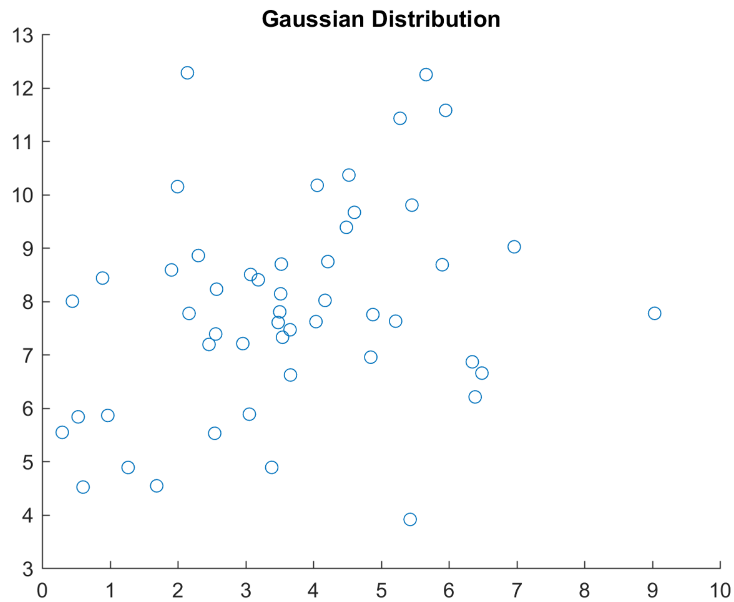

Figure 3: Gaussian Scatter Plot

b. ML Estimation

3.6377	7.8506

Table 1: Mean

3.6414	1.0779
1.0779	3.7831

Table 2: Covariance

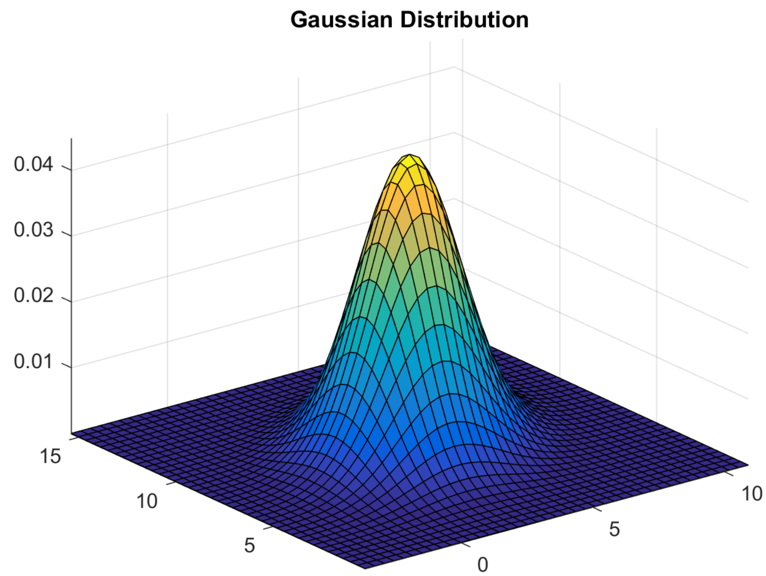

Figure 4: Gaussian 3-D

c. Individual Gaussian

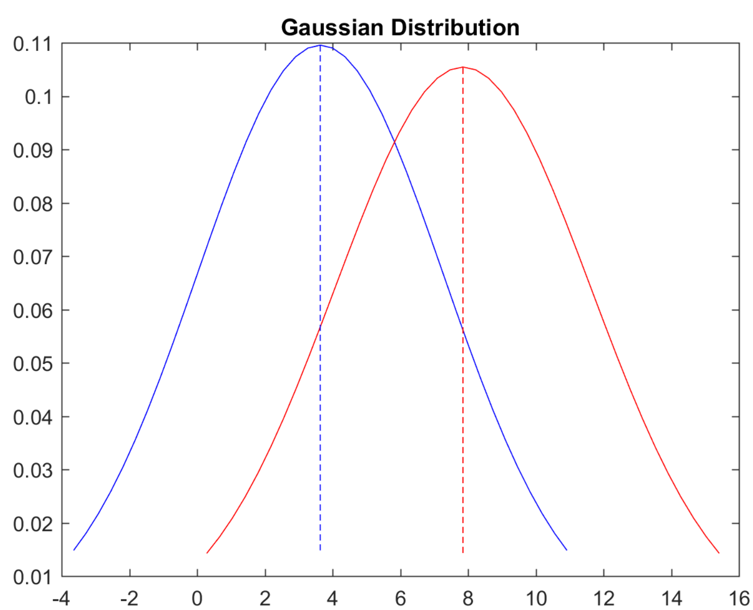

Figure 5: Individual Gaussian

The first column data of "gaussian txt" is in blue, while the second is in red. The corresponding means are 3.6377 and 7.8506. The corresponding variances are 3.6414 and 3.7831.

d. Multivariate vs. Univariate

I believe multivariate Gaussian models are a better model than two separate univariate models. Given a multivariate model, it is much easier to view any correlations between the data points as they are directly displayed together. Univariate models need to be interpolated side-by-side and any correlation must be interpreted by the viewer.

III. Problem 3 - Exponential Distribution

a. Density Function

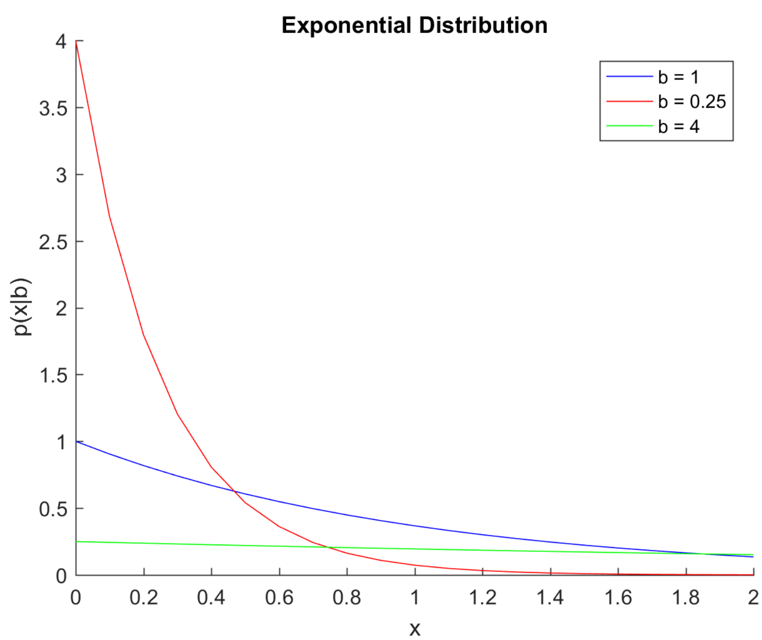

Figure 6: Exponential Distribution

b. ML Estimate

$$f(x;\theta) = \frac{1}{\theta} e^{\frac{-x}{\theta}}, 0 < x < \infty, \theta \in [0, \infty]$$

$$L(\theta) = L(\theta; x_1, x_2 ... x_n) = \left(\frac{1}{\theta} e^{\frac{-x_1}{\theta}}\right)\left(\frac{1}{\theta} e^{\frac{-x_2}{\theta}}\right) ... \left(\frac{1}{\theta} e^{\frac{-x_n}{\theta}}\right) = \frac{1}{\theta^n} exp\left(\frac{-\sum_1^n x_i}{\theta}\right)$$

$$lnL(\theta) = -(n)\,ln(\theta) - \frac{1}{\theta}\sum_1^n x_i, 0 < \theta < \infty$$

$$\frac{d\left[lnL\left(\theta\right)\right]}{d\theta}=\frac{-\left(n\right)}{\left(\theta\right)}+\frac{1}{\theta^{2}}\sum_{1}^{n}x_{i}=0$$

$$\theta=\frac{\sum_{1}^{n}x_{i}}{n}x`$$

$$\Theta=\frac{\sum_{1}^{n}X_{i}}{n}$$