Problem assignment 3
Due: Thursday, February 7, 2019

Problem 1. Bernoulli trials

Assume we have conducted a coin toss experiment with 100 coin flips. The results of the experiment are in file 'coin.txt' where 1 means a head and 0 means a tail. Assume that θ represents the probability of observing a head.

- (a) What is an ML estimate of θ?

- (b) Assume the prior on θ is defined by a Beta distribution $Beta(\theta|1, 1)$. Plot and report both the prior and the posterior distribution on θ.

- (c) Calculate and report the MAP estimate of the θ using the prior in Part b. Show (plot) the MAP estimate on the plot of the posterior of θ you have generated in part b.

- (d) Repeat part b and c by assuming that the prior on θ follows $Beta(\theta|4, 2)$.

Problem 2. Multivariate Gaussian

Assume the pairs of real valued measurements in file 'gaussian.txt'

- (a) Plot the data using the scatter plot matlab function.

- (b) Calculate and report the ML estimate of the mean and the covariance matrix from the data. Please use the unbiased estimate of the variance. Plot and report the resulting Gaussian distribution. (Note: you need to plot this in 3D).

- (c) Now consider each measurement in 'gaussian.txt' separately. Calculate the ML estimate of the mean and variance of these measurements. Plot and report the individual distributions.

- (d) Do you believe the mutlivariate Gaussian model is better than two separate univariate Gaussian models? Explain why yes or why not? How would you use the data to answer that question?

Problem 3. Exponential distribution

Assume that a random variable x follows an exponential distribution. The exponential distribution is defined as:

$$p(x|b) = \frac{1}{b}e^{-\frac{x}{b}},$$

where b is a parameter of the density model. Assume we have observed N independent examples $D = \{x_1, x_2, \cdots x_N\}$ from the distribution.

Answer the following questions:

- (a) Plot and report the density function for the exponential distribution with parameters $b = 1$, $b = 0.25$ and $b = 4$.

- (b) Given a set of independent observations $x_1, x_2, \ldots, x_n$ from an exponential distribution, derive the ML estimate of the parameter b.

Note: it is not neccessary to submit any Matlab code for this assignment, just include the plots in your report.