Simulating Large Deformation Processes Using an Explicit Element-free Galerkin Method

With Applications to Material Forming

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I would like to dedicate this thesis to my loving parents ...

DECLARATION

I hereby declare that except where specific reference is made to the work of others, the contents of this dissertation are original and have not been submitted in whole or in part for consideration for any other degree or qualification in this, or any other university. This dissertation is my own work and contains nothing which is the outcome of work done in collaboration with others, except as specified in the text and Acknowledgements. This dissertation contains fewer than 80,000 words.

Stephen Patrick Smith
March 2020

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ABSTRACT

Computational methods are widely utilised in engineering to obtain the response of components and structures under loading. In the pioneering years, modest computational power limited the application of these computational techniques to relatively simple problems. However, with the substantial increase in computational power, the scope of these methods has broadened to simulate several physical processes used in industry to inform manufacturing strategies.

A forming technique that has benefited from this increased computational power is the stretch blow moulding (SBM) process, which is the primary manufacturing technique used to produce polymer bottles for the water and soft drinks industry. The finite element method is frequently used to simulate this process, to reduce the economic cost of bottle design and optimise for minimum material use. Despite its ubiquitous adoption, several issues are often encountered in finite element simulations, predominately related to the dependency of the results on the set of elements (mesh) used to solve the problem. In response to these issues, so-called meshfree methods have been developed. Despite twenty years of progress in meshfree technology, the application of these techniques to real-world applications is limited. This thesis aims to further widen the application of meshfree methods to material-forming, where the absence of a fixed computational mesh has the potential to provide numerous benefits.

In this work, an explicit element-free Galerkin method was used to simulate a range of large deformation problems. Initially, a total formulation was utilised, in which the neighbourhood information was kept constant throughout. Through several numerical examples, the benefits of meshfree methods in the simulation of large deformation problems were shown. The application of this total formulation culminated in a validated simulation of the stretch blow moulding process.

In the final part of this work, the formulation was enhanced to include updates of the neighbourhood information. This development includes a novel method to implement an evolving domain of influence into the general shape function scheme. This new formulation was validated through several examples, with the potential benefits of the approach outlined.

' I have no satisfaction in mathematical formula unless I feel their numerical magnitude'

Lord Kelvin

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INTRODUCTION

This thesis is concerned with the application of an alternative computational technique to solve large deformation problems within solid mechanics. Conventionally, the finite element method has been used to solve these problems with a computational mesh providing a discretisation of the problem. The mesh used within this method is typically based on either a Lagrangian description, where the mesh deforms with the deformation, or a Eulerian description, where the mesh is fixed in the spatial frame. In solid mechanics, Lagrangian finite element methods have emerged dominant. However, despite this dominance, difficulties can be encountered with the finite element methods when modelling large deformation, which can cause the numerical simulation to fail. In response to these issues, meshfree methods have been proposed by several authors [1–3], to reduce the dependency of the results on the quality of a computational mesh. This is how I reference a figure Fig. 1.1

$$a = b + c \tag{1.0.1}$$

I referenced equations like this (1.0.1)

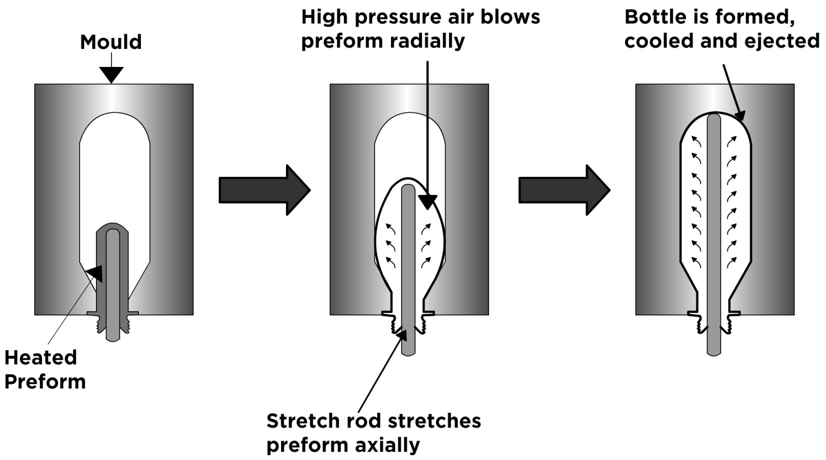
Mould
High pressure air blows preform radially
Bottle is formed, cooled and ejected
Heated Preform
Stretch rod stretches preform axially

Fig. 1.1 SBM

2

MESHFREE METHODS

Meshfree methods are good.

3

ELEMENT-FREE GALERKIN METHOD FOR LINEAR ELASTICITY

In this chapter, a methodology to apply the element-free Galerkin method is developed. This chapter aims to highlight the main features of a meshfree method, which forms the basis for the subsequent non-linear investigations. Firstly, an overview of the element-free Galerkin method is given, with the formulation of the shape functions discussed within a generalised framework, which allows for the simple transition between isotropic, and anisotropic shape functions. Next, the equations of motion for a linear elastic solid are introduced, followed by a discussion of the implementation within a discrete numerical framework. In the numerical examples an investigation of the influence of domain size, and numerical integration on the accuracy of the meshfree solution is discussed.

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ELEMENT-FREE GALERKIN METHOD FOR FINITE DEFORMATION ANALYSIS

Methods are particularly suitable in large deformation problems, where the finite element method encounters mesh-based difficulties. The developments within this chapter were motivated by the absence of material forming simulations within explicit meshfree methods literature. Based on the observations in this chapter, explicit meshfree methods are fully capable in large deformation loading and therefore have the potential to overcome limitations attributed to finite element based material forming simulations. To further explore the potential of this method, the formulation is utilised in the coming chapters to simulate both the free-blow and stretch blow moulding processes.

5

EXPERIMENTAL CHARACTERISATION OF THE STRETCH BLOW MOULDING PROCESS

My totally legit experimental chapter.

6

NUMERICAL SIMULATIONS OF THE STRETCH BLOW MOULDING PROCESS

In this chapter, a simulation of the stretch blow moulding is introduced. This simulation is composed of a validated free-blow simulation, which is followed by a full stretch blow moulding simulation. The developed simulation correspond to the inputs that were introduced in the previous chapter.

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SEMI-UPDATED ELEMENT-FREE GALERKIN FORMULATION

Within this chapter the development of an updated element-free Galerkin method is outlined. The benefits of an updated formulation, for material-forming applications, has been proposed by Cueto and Cinesta [4]. These benefits primarily relate to the ability to simulate large deformation and free surface flows. The updated formulation developed within this chapter is motivated by the development of updated formulations for *smooth particle hydrodynamics* [5], and the *optimal transport method* [6].

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CONCLUSIONS AND FUTURE WORK

8.1 THESIS SUMMARY

The objective of this thesis was to outline the developments of a meshfree method for large deformation applications, with a particular emphasis on material-forming. Typically, to model these material-forming problems the finite element method is used, which can lead to a host of difficulties relating to the finite element mesh required in the solution process. In response to these mesh-based issues, meshfree methods have been developed as a means to reduce the link between simulation accuracy and mesh quality.

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