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Paper 1

Kant's argues in his essay *On the First Ground of the Distinction of Regions of Space* for the existence of absolute space through his thought experiment regarding a hand in empty space. Hands are objects called incongruent counterparts because left and right hands are mirror images of each other. The Stanford Encyclopedia of Philosophy defines an incongruent counterpart as an object that is "exactly equal and similar to another, but which cannot be enclosed in the same limits as that other, its incongruent counterpart," (Shabel). Intuitively, the reflection of a right hand is a left hand and vice versa. Kant's argument is summarized as follows:

1. There is a difference between a right hand and a left hand. This difference must be due to how
   i. The parts of the hand relate to one another internally
   ii. The hand relates to other objects- either other material objects or absolute space
2. It is not an internal relation between the parts of the hand.
3. It is not a relation to other material objects.
4. So, it must be a relation to absolute space; therefore, absolute space must exist.

Kant's argument presupposes the existence of an absolute space different than three-dimensional Euclidean space because his version of absolute space requires an additional primitive. In the following paragraphs I elaborate Kant's argument against other views of the incongruent counterpart issue, explain the ways in which four-dimensional spaces and non-orientable spaces refute his argument, and develop my own views regarding whether incongruent counterparts can prove the existence of absolute space.

In the second point of Kant's argument, he states that right and left hands cannot be different due to internal relations between parts of the hand. This view, called internalism, states that right and left consist in internal relations among the parts of a thing (van Cleve 35). Kant's argument against this is that internal relations are the same for a right and left hand. It is important to note that Kant was thinking about "internal relations" as those of distances and angles (van Cleve 40). One example would be the distance between the wrist and the tip of the index finger. Another one would be the angle between the thumb and index finger. Both these examples and other internal relations of right and left hands are the same. Therefore, right and left hands cannot be different due to internal relations.

Kant states in the third point of his argument that right and left hands cannot be different due to external relations between other material objects. Externalism holds that a hand being either left or right depends on relations to other objects such as the human body (van Cleve 38). Kant refutes this position by bringing up his thought experiment about a hand in empty space, which he states would be either right or left. There are no other material objects, but a hand must still be either right or left. If a human body were introduced, the hand would have to fit onto the wrist of the right or left arm. In addition, the introduction of the body does not actually change the hand but rather shows whether it is a right or left hand (van Cleve 39). Therefore, a hand in empty space must have a constant shape that does not depend on external relations.

Kant postulates the existence of absolute space, but his thought experiment requires a directional primitive along with congruence and betweenness (O' Pooley 2). Kantian absolute space is not Euclidean space, which solely contains the notions of congruence and betweenness. In a purely Euclidean space, the hand in empty space must be either right or left and cannot be definitively identified as either. This is because all corresponding points on right and left hands have the same congruence and betweenness, so there is no way to differentiate the two using these two notions. However, Kant believes that while a hand in empty space must be either left or right, its relation to absolute space may define it as right or left. He implicitly adds the directional primitive, or "rightness," which can be read as this: point w is on the right side of the plane defined by xyz. Point w is on the tip of the thumb, point y is on the tip of the index finger, x is on the base of the pinky finger, and z is on the lower palm. If the thumb is on the right side of the hand with the palm facing the viewer, then the hand is a right hand. Conversely, if the thumb is not on the right side of the hand with the palm facing the viewer, then the hand is a left hand. Here, Kant adds a geometrical primitive that defines the shape of a hand in terms of direction. By appealing to the notion of direction, this primitive appeals to the existence of absolute space: "the region towards which this ordering of the parts is directed involves reference to… universal space as a unity," (van Cleve 41). This conclusion posits that absolute space exists.

While Kant's argument may sound valid in three-dimensional absolute space, it breaks down if four-dimensional space exists. Incongruent counterparts in three-dimensional space are congruent, and left or right becomes a matter of orientation. The externalist way to understand a four-dimensional space is where it is possible to arrange four rods so that they all form right angles with one another (van Cleve 58). In three-dimensional space, three rods can be placed in such a manner to represent the mathematical dimensions of width, length, and height. In a four-dimensional space, the fourth rod would represent some fourth dimension. To understand why incongruent counterparts are congruent if four-dimensional space exists, it is necessary to provide an example in two-dimensional space.

First, an isometry is a mapping that preserves distances among the points (Maudlin 34). Imagine an L and a backwards L on a two-dimensional plane. These two L's are congruent by an isometry of reflection. However, a two-dimensional being would perceive the two L's as incongruent counterparts because they are equal and similar to one another but cannot be transposed onto each other by any isometry within two-dimensional space. The first L can be rotated in three-dimensional space to be transposed onto the backwards L. Therefore, a reflection in *n* dimension is a rotation in *n + 1* dimensions ("Incongruent Counterparts"). Right and left hands, which are incongruent counterparts in three-dimensional space, are intuitively congruent by reflection, as evident in a mirror. Right and left hands are also congruent by rotation in four-dimensional space. If four-dimensional space exists, then all incongruent counterparts in three-dimensional space must be congruent because they can undergo an isometry to be transposed onto each other.

Non-orientable spaces allow right hands to become left hands while staying in three-dimensional space. Non-orientable spaces are spaces with a "twist" so that objects are reflected as they travel through the twist. For example, a mobius strip is a two-dimensional loop that has a twist. An L passing through the twist on the mobius strip becomes a backward L. When it travels in another loop, it passes through the twist and becomes the original L. A three-dimensional non-orientable space is also possible and functions similarly to the two-dimensional mobius strip. If a right hand travels through the twist of this non-orientable space, it becomes a left hand and vice versa. Kant's argument is therefore invalid because the difference between right and left hands is simply a matter of orientation. The one caveat is that a three-dimensional non-orientable space is only possible in an ambient four-dimensional space (van Cleve 45). If four-dimensional space exists, then right and left hands are congruent by rotation. In both cases, the problem of incongruent counterparts need not appeal to the existence of absolute space.

Besides four-dimensions and non-orientable spaces refuting Kant's argument, his directional primitive of rightness in absolute space is also problematic. The primitive "w is on the right side of the plane defined by xyz" can also be applied to points of matter rather than points of space. An internalist can say w is on the tip of the thumb, y is on the tip of the index finger, x is on the base of the pinky finger, and z is on the lower palm. If the thumb is on the right side of the hand with the palm facing the viewer, then the hand is a right hand. The use of Kant's rightness primitive can define right and left hands from the internalist perspective, so it cannot be a concrete notion in absolute space. This point was brought up by Professor Bacon during the class lecture on February 20, 2024.

Taking all these factors into consideration, Kant's argument was correct in the fact that a hand in empty space must be either right or left. However, the understanding of four-dimensional and non-orientable spaces adds the condition that the right hand in empty space can become a left hand. Likewise, if it were a left hand in empty space, it could become a right hand. The existence of incongruent counterparts cannot prove the existence of absolute space as he concluded in his argument.

Now I will provide my own argument and justification for why a hand in empty space must be either left or right. First, "right" and "left" refer to the shape of the hand rather than describing their direction. I am also assuming that four-dimensional space and non-orientable space are not considered, so right and left hands are not congruent.

1. A hand in empty space must be either left or right shaped. In other words, it must fit onto either the right arm or left arm of a human body.
2. The introduction of another object does not change the shape of the hand.
3. Space itself cannot determine whether a hand is right or left.

4. The introduction of another object can determine whether a hand is right or left.
5. Therefore, a hand in empty space must be either right or left, which can be determined only when another material object is introduced into the empty space.

One counterexample to this logic is this: suppose a ball is in empty space. When another ball is introduced, the original ball can be characterized as small or large. Therefore, the original ball must have been small or large in the first place, and it required another object to determine so. My point against this is that the traits of "small" and "large" are qualitative properties because they are relative to other objects. "Right" and "left" are not qualitative because they are objective characteristics of a hand and do not relate to some other object. These properties are best revealed by "showing and not telling," (O' Pooley 5). Intuitively, a hand must be either right or left, and pointing towards either the idea of a hand being both or neither is complete nonsense (O' Pooley 5). Therefore, this hand must be able to fit on either the right arm or left arm of a human body.

Kant's argument fails to prove the existence of absolute space because four-dimensional and non-orientable spaces prove that incongruent counterparts are congruent and "right" versus "left" is a matter of orientation. In addition, Kant posits the primitive of direction in absolute space that can also define incongruent counterparts from an internalist point of view. Regarding the thought experiment about a hand in empty space, I argue that Kant's conclusion was correct in that it must be either right or left. The hand must be either right or left as an unchanging condition which is revealed by the introduction of another material object.

Bibliography

"Incongruent Counterparts." Oxford Reference, Oxford University Press, https://www.oxfordreference.com/display/10.1093/oi/authority.20110803100000547.

Maudlin, Tim. *Philosophy of Physics: Space and Time*. Princeton University Press, 2012.

O' Pooley. "Lecture 7: Incongruent Counterparts." Advanced Philosophy of Physics, 2003, https://users.ox.ac.uk/~ball0402/teaching/handout7.pdf.

Shabel, Lisa, "Kant's Philosophy of Mathematics", *The Stanford Encyclopedia of Philosophy* (Fall 2021 Edition), Edward N. Zalta (ed.), https://plato.stanford.edu/entries/kant-mathematics/.

Van Cleve, James. "Right, Left, and the Fourth Dimension." *The Philosophical Review*, vol. 96, no. 1, 1987, pp. 33-68.