**Proof Method-1: Intuitive**

Let’s take A = (n, n-1, ...1) as an list of numbers and the desired permutation B = (1, 2, 3, ...n). B can also be assumed as a ordered list of numbers for eg. 1,2, 3, 4, 5, 6 and 6, 5, 4, 3, 2, 1 are two array of numbers.

Now, we can see that A and B are actually reverse of each other. if we reverse A, we get B and vice versa.

For an n-length array (numbers starting from 1 upto n) to be reversed, we know we have to swap their corresponding elements/numbers in the following order:

=> swap first element with last element, then

=> swap second element with second last element

=> swap third element with third last element

=> …………

Upto n/2th element,

If n is odd, then we can stop here

If n is even , then swap n/2th element with (n/2+1)th element,

=> total number of swaps = ⌊n/2⌋

If we write this in mathematical terminology where we assume A[i] to be the ith number for permutation A , then swaps required for reversing A will be as follows:

A[1] swaps with A[n]

A[2] swaps with A[n-1]

A[3] swaps with A[n-2]

….

Upto n/2th element,

Or we can generalise this swap as follows

A[i] swaps with A[n-i+1]

=> The swap pair can be denoted by (i,j) = (i, n-i+1)

Now, to prove that (n, n-1, ...1) is (1,⌊n/2⌋)-reachable from permutation (1, 2, 3, ...n),

We can show that the swaps (i,n-i+1) where 1<=i <=n/2, exist in the swaps set Pd.

That is swaps (1,n), (2,n-1), (3, n-2).....(i, n-i+1) belong to set Pd.

For this, we will generate the set Pd to check whether the above swaps belong to the set or not.

For every element (i,j) of set Pd, we know

I = j or d<= j-i <= n-d,

Here d = 1,

Therefore,

i = j or 1 <= j - i <= n-1

That is swapping positions i and j can have a minimum difference of zero position (i = j) and maximum difference of n-1 position where j >= i

The swaps of the form (i, n-i+1) will always be present in this set Pd because the condition for j says that j = i or i+1 < = j <= n + i -1

So max value j can have is n + i - 1 and minimum is i +1 ,

We can see that swap (i, n-i + 1) will belong to set Pd because i+ 1 <= n - i + 1 < = n + i -1 for i<=n/2,

So the swaps of the form (i, n-i+1) will belong to Pd

=> There are ⌊n/2⌋ swaps which belong to set Pd which will generate the permutation B

Let’s take an example to demonstrate this proof

Let A = 1,2,3,4,5,6 and B = 6,5,4,3,2,1

Then B should be (1, 3)-reachable from A

Following our method to generate swap of the form (i, n-i+1) for all i <= ⌊n/2⌋, here ; n = 6, i<=3,

The swaps will be as follows

(1,6), (2, 5) (3, 4)

Now the set Pd where for (i,j) , i = j or d <= j-i <= n-d

=> i = j or 1 <= j-i <= 5

Pd = {(1,1), (1,2), (1,3) ,(1,4), (1,5), (1,6), (2,2), (2,3), (2,4), (2,5), (2,6), (3,3), (3,4), (3,5), (3,6), (4,4) ,(4,5), (4,6) ,(5,5) ,(5,6), (6,6) }

Now swaps (1,6), (2, 5) (3, 4) belong to the set Pd and when applied on A

T(1,6) A= 6,2,3,4,5,1

T(2,5) T(1,6) A = 6,5,3,4,2,1

T(3,4)T(2,5) T(1,6) A = 6,5,4,3,2,1 = B

Hence, we can see that B is (1, 3)-reachable from A

**Proof Method-2: Inductive**

To prove that permutation (n, n-1, ...1) is (1,⌊n/2⌋)-reachable from permutation (1, 2, 3, ...n) , we can use the principle of mathematical induction

Let P(n) : (n, n-1, ...1) is (1,floor(n/2))-reachable from permutation (1, 2, 3, ...n)

B**ase Case:** Checking for P(1),

P(1) :(1) is (1,0)-reachable from (1)

For d = 1, l = 0,

[1]x[1] = { (1,1) }

To generate the set Pd (for d = 1), d<= j-i <=n-d

=> 1 <= j-i <= 0 => no such i, j pair will be possible from this condition

For condition i = j, only pair (1,1) is possible

Therefore,

Pd = { (1,1) }

Where Pd is the set of allowable swaps

Now (1) is (1,0) reachable from (1)

=> there should be no element chosen from set Pd to perform the swap operation on permutation (1)

=> the permutation will remain as it is (unchanged)

=> (1) is (1,0 ) reachable from (1)

=> P(1) is true

**Assumption:** Now, assume for any positive integer k, P(k) is true./ then if P(k+1) is proved true/, this implies that P(n) is always true

P(k) : (k, k-1, ...1) is (1,⌊k/2⌋))-reachable from permutation (1, 2, 3, ...k)

Since P(k) is true, there are swap pairs (i1, j1), (i2,j2)...(i⌊k/2⌋, j ⌊k/2⌋) belonging to set Pd.

**Proof:**

Now for **P(k+1),**

P(k+1) : (k=1, k, k-1, ...1) is (1,⌊k+1/2⌋))-reachable from permutation (1, 2, 3, ...k, k+1)

The swap pairs for k+1-permutation will be (i1, j1), (i2,j2)...(i⌊k/2⌋, j ⌊k/2⌋), (i⌊k+1/2⌋, j ⌊k+1/2⌋)

Now since k+1- permutation has one more number than k- permutation, therefore the required swaps for k+1-permutation will be of the form

(ix,jx)k+1 = (ix, jx+1)k for all 1<=x

to reach the permutation (k+1, k,..2,1) .

We see that for Pd set of K+1-permutation,

1=< j-i<= K,

And for Pd set of k-permutation,

1=< j-i<= K-1

Now we see that the all the (i,j) pairs present in the Pd set of k-permutation will also be present in Pd set of k+1-permutation .

Therefore the swaps required will be available in set Pd and their count will be ⌊k+1/2⌋.

If k is even, ⌊(k+1)/2⌋ = ⌊k/2⌋,

If k is odd, ⌊(k+1)/2⌋ = 1 + ⌊k/2⌋, in this case , one extra swap for the middle element will be required which will be ((k+1)/2, k+1)/2).

Hence, we proved that P(k+1) is true; Therefore, P(n) is true .