# 第 4 节 分块矩阵

## 安徽财经大学

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有时候,我们用几条纵线与横线将矩阵分割,把一个大矩阵看成是由一些 小矩阵组成的,就如矩阵是由数组成的一样,构成一个分块矩阵,从而把 大型矩阵的运算化为若干小型矩阵的运算, 使运算更为简明. 这是处理 阶数较高的矩阵的重要方法.

若将 A 分块为

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ \hline a_{31} & a_{32} & a_{33} \end{pmatrix}$$





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## 则得四个子矩阵

$$A_{11} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}, \quad A_{12} = \begin{pmatrix} a_{13} \\ a_{23} \end{pmatrix},$$
 $A_{21} = \begin{pmatrix} a_{31} & a_{32} \end{pmatrix}, \quad A_{22} = \begin{pmatrix} a_{33} \end{pmatrix}.$ 

## 这样, A 就能表示为

$$A=\left(egin{array}{cc} A_{11} & A_{12} \ A_{21} & A_{22} \end{array}
ight)$$

于是, A 被看作是以矩阵为元的  $2\times 2$  型矩阵, 这样就能将行与列较多的矩阵根据需要简单地表出。



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$$oldsymbol{A} = \left(egin{array}{cc} oldsymbol{A}_{11} & oldsymbol{A}_{12} \ oldsymbol{A}_{21} & oldsymbol{A}_{22} \end{array}
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于是,  $m{A}$  被看作是以矩阵为元的  $2 \times 2$  型矩阵. 这样就能将行与列较多的矩阵根据需要简单地表出.



## 又如, 对矩阵 A 进行如下形式分块:

$$\boldsymbol{A} = \begin{pmatrix} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 & -3 \\ 0 & 0 & 1 & -1 & 0 \\ \hline 0 & 0 & 0 & 4 & 1 \end{pmatrix},$$

记

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, A_1 = \begin{pmatrix} 0 & 2 \\ 1 & -3 \\ -1 & 0 \end{pmatrix}, O = (0 0 0), A_2 = (4 1),$$

则

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$$m{A} = \left( egin{array}{cc} m{I} & m{A}_1 \ m{O} & m{A}_2 \end{array} 
ight).$$





当考虑一个矩阵的分块时,一个重要的原则是使分块后的子矩阵中有便于利用的特殊矩阵,如单位矩阵、零矩阵、对角矩阵、三角形矩阵等。常用的分块矩阵,除了上面的  $2\times 2$  分块矩阵,还有以下几种形式:将  $m\times n$  矩阵  $\mathbf{A}=\left(a_{ij}\right)_{m\times n}$  按行分块为  $m\times 1$  分块矩阵

$$oldsymbol{A} = \left(egin{array}{c} oldsymbol{lpha}_1 \ oldsymbol{lpha}_2 \ dots \ oldsymbol{lpha}_m \end{array}
ight),$$

其中  $\alpha_i = (a_{i1}, a_{i2}, \dots, a_{in})$   $(i = 1, 2, \dots, m)$ .

将  $m \times n$  矩阵  $A = (a_{ij})_{m \times n}$  按列分块为  $1 \times n$  分块矩阵

$$\mathbf{A}=(\boldsymbol{\beta}_1,\boldsymbol{\beta}_2,\cdots,\boldsymbol{\beta}_n),$$

其中  $\beta_j = (a_{1j}, a_{2j}, \cdots, a_{mj})^{\mathrm{T}}$   $(j = 1, 2, \cdots, n)$ 



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其中  $\beta_j = (a_{1j}, a_{2j}, \cdots, a_{mj})^{\mathrm{T}}$   $(j = 1, 2, \cdots, n).$ 



当矩阵  $A = (a_{ij})_{n \times n}$  中非零元都集中在主对角线附近时可将 A 分块成 下面的块对角矩阵 (又称为准对角矩阵):

其中 
$$A_i(i=1,2,\cdots,t)$$
 是  $r_i$  阶方阵  $\left(\sum_{i=1}^t r_i = n\right)$ .





#### 例如

$$m{A} = \left( egin{array}{cccccc} 1 & 3 & 0 & 0 & 0 & 0 & 0 \ 0 & 2 & 0 & 0 & 0 & 0 & 0 \ 0 & 0 & -1 & 0 & 0 & 0 & 0 \ 0 & 0 & 0 & 2 & 5 & 0 \ 0 & 0 & 0 & 0 & 1 & 1 \ 0 & 0 & 0 & 0 & 0 & 2 \end{array} 
ight) = \left( egin{array}{ccccc} m{A}_1 & & & & & \ & & m{A}_2 & & & \ & & & m{A}_3 \end{array} 
ight),$$

## 其中

$$A_1 = \begin{pmatrix} 1 & 3 \\ 0 & 2 \end{pmatrix}, \quad A_2 = (-1), \quad A_3 = \begin{pmatrix} 2 & 5 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{pmatrix}.$$





# 分块矩阵的运算

设分块矩阵

$$m{A} = \left(egin{array}{ccc} m{A}_{11} & \cdots & m{A}_{1s} \ dots & & dots \ m{A}_{r1} & \cdots & m{A}_{rs} \end{array}
ight), \quad m{B} = \left(egin{array}{ccc} m{B}_{11} & \cdots & m{B}_{1s} \ dots & & dots \ m{B}_{r1} & \cdots & m{B}_{rs} \end{array}
ight),$$

若 A, B 分块的办法相同, 即相应小矩阵  $A_{ij}$  和  $B_{ij}$  的行数、列数对应相等, 则

$$oldsymbol{A} + oldsymbol{B} = \left(egin{array}{ccc} oldsymbol{A}_{11} + oldsymbol{B}_{11} & \cdots & oldsymbol{A}_{1s} + oldsymbol{B}_{1s} \ dots & dots \ oldsymbol{A}_{r1} + oldsymbol{B}_{r1} & \cdots & oldsymbol{A}_{rs} + oldsymbol{B}_{rs} \end{array}
ight).$$



8/20



设 
$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & -1 & -4 \\ 3 & -1 & -2 & 2 \end{pmatrix}$$
,  $B = \begin{pmatrix} 2 & 5 & -6 & -1 \\ 4 & 7 & 3 & -2 \\ -1 & 2 & 4 & 5 \end{pmatrix}$ , 则

$$m{A} + m{B} = \left( egin{array}{ccc} m{A}_{11} + m{B}_{11} & m{A}_{12} + m{B}_{12} \ m{A}_{21} + m{B}_{21} & m{A}_{22} + m{B}_{22} \end{array} 
ight),$$

其中 
$$A_{11} + B_{11} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 2 \\ 4 \end{pmatrix} = \begin{pmatrix} 3 \\ 6 \end{pmatrix}$$
,

$$A_{12} + B_{12} = \begin{pmatrix} 2 & 3 & 4 \\ 3 & -1 & -4 \end{pmatrix} + \begin{pmatrix} 5 & -6 & -1 \\ 7 & 3 & -2 \end{pmatrix} = \begin{pmatrix} 7 & -3 & 3 \\ 10 & 2 & -6 \end{pmatrix},$$

$$\mathbf{A}_{21} + \mathbf{B}_{21} = (3) + (-1) = (2),$$

$$A_{22} + B_{22} = ( -1 \ -2 \ 2 ) + ( 2 \ 4 \ 5 ) = ( 1 \ 2 \ 7 ).$$

设分块矩阵  $\mathbf{A} = (\mathbf{A}_{ij})_{s \times t}$ , k 是数, 则  $\mathbf{A}$  的数乘为  $k\mathbf{A} = (k\mathbf{A}_{ij})_{s \times t}$ . 设  $\mathbf{A} = (a_{ij})_{m \times n}$ ,  $\mathbf{B} = (b_{ij})_{n \times p}$ , 如果把  $\mathbf{A}$ ,  $\mathbf{B}$  分别分块为  $r \times s$  和  $s \times t$  分块矩阵, 且  $\mathbf{A}$  的列的分法与  $\mathbf{B}$  的行的分法相同, 那么

$$egin{aligned} oldsymbol{A}oldsymbol{B} = \left(egin{array}{cccc} oldsymbol{A}_{11} & oldsymbol{A}_{12} & \cdots & oldsymbol{A}_{1s} \ dots & dots & dots & dots \ oldsymbol{A}_{r1} & oldsymbol{A}_{r2} & \cdots & oldsymbol{A}_{rs} \end{array}
ight) \left(egin{array}{cccc} oldsymbol{B}_{11} & \cdots & oldsymbol{B}_{1t} \ oldsymbol{B}_{21} & \cdots & oldsymbol{B}_{2t} \ dots & & dots \ oldsymbol{B}_{s1} & \cdots & oldsymbol{B}_{st} \end{array}
ight) = oldsymbol{C}, \end{aligned}$$

其中 C 是  $r \times t$  分块矩阵, 且

$$C_{kl} = A_{k1}B_{1l} + A_{k2}B_{2l} + \dots + A_{ks}B_{sl}$$
  
=  $\sum_{i=1}^{s} A_{ki}B_{il}$   $(k = 1, 2, \dots, r; l = 1, 2, \dots, t).$ 





## 例 (1.4.2)

设 
$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 2 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 1 & 0 \\ -1 & 2 \\ 1 & 0 \\ -1 & -1 \end{pmatrix}, 求 \mathbf{AB}.$$





解

令 
$$A_1 = \begin{pmatrix} -1 & 2 \\ 1 & 1 \end{pmatrix}$$
, 则  $A = \begin{pmatrix} I & O \\ A_1 & I \end{pmatrix}$ . 再将  $B$  分块为

$$\boldsymbol{B} = \begin{pmatrix} 1 & 0 \\ -1 & 2 \\ \hline 1 & 0 \\ -1 & -1 \end{pmatrix} = \begin{pmatrix} \boldsymbol{B}_1 \\ \boldsymbol{B}_2 \end{pmatrix}.$$

于是

$$m{AB} = \left( egin{array}{cc} m{I} & O \ A_1 & m{I} \end{array} 
ight) \left( egin{array}{cc} B_1 \ B_2 \end{array} 
ight) = \left( egin{array}{cc} B_1 \ A_1 B_1 + B_2 \end{array} 
ight) = \left( egin{array}{cc} 1 & O \ -1 & 2 \ -2 & 4 \ -1 & 1 \end{array} 
ight).$$

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$$m{B} = \left( egin{array}{ccc} 1 & 0 \ -1 & 2 \ \hline 1 & 0 \ -1 & -1 \end{array} 
ight) = \left( egin{array}{c} m{B}_1 \ m{B}_2 \end{array} 
ight).$$

于是

$$m{A}m{B}=\left(egin{array}{cc}m{I} & m{O} \ m{A}_1 & m{I}\end{array}
ight)\left(egin{array}{cc}m{B}_1 \ m{B}_2\end{array}
ight)=\left(egin{array}{cc}m{B}_1 \ m{A}_1m{B}_1+m{B}_2\end{array}
ight)=\left(egin{array}{cc}1 & 0 \ -1 & 2 \ -2 & 4 \ -1 & 1\end{array}
ight).$$

## 例 (1.4.3)

若 n 阶矩阵 A, B 为同型块对角矩阵, 即

$$\mathbf{A} = \operatorname{diag}(\mathbf{A}_1, \mathbf{A}_2, \cdots, \mathbf{A}_t), \quad \mathbf{B} = \operatorname{diag}(\mathbf{B}_1, \mathbf{B}_2, \cdots, \mathbf{B}_t),$$

其中  $A_i$  和  $B_i$  是同阶方阵  $(i=1,2,\cdots,t)$ , 则





# 若块对角矩阵 $\mathbf{A} = \operatorname{diag}(\mathbf{A}_1, \mathbf{A}_2, \cdots, \mathbf{A}_t)$ , 其中 $\mathbf{A}_i (i=1,2,\cdots,t)$ 可逆. 因为

所以  $A^{-1} = \operatorname{diag}(A_1^{-1}, A_2^{-1}, \cdots, A_t^{-1}).$ 





同理, 若 
$$A_i (i=1,2,\cdots,t)$$
 可逆, 则

$$\begin{pmatrix} & & & & \boldsymbol{A}_1 \\ & & \boldsymbol{A}_2 \\ & & \ddots & & \\ \boldsymbol{A}_t & & & \end{pmatrix}^{-1} = \begin{pmatrix} & & & & \boldsymbol{A}_t^{-1} \\ & & & \ddots & \\ & & \boldsymbol{A}_2^{-1} & & \\ \boldsymbol{A}_1^{-1} & & & & \end{pmatrix}.$$





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## 若分块矩阵

$$m{A} = \left(egin{array}{cccc} m{A}_{11} & m{A}_{12} & \cdots & m{A}_{1s} \ m{A}_{21} & m{A}_{22} & \cdots & m{A}_{2s} \ dots & dots & dots \ m{A}_{r1} & m{A}_{r2} & \cdots & m{A}_{rs} \end{array}
ight),$$

#### 则不难验证

$$\boldsymbol{A}^{\mathrm{T}} = \begin{pmatrix} A_{11}^{1} & A_{21}^{2} & \cdots & A_{r1}^{r_{1}} \\ A_{12}^{\mathrm{T}} & A_{22}^{\mathrm{T}} & \cdots & A_{r2}^{\mathrm{T}} \\ \vdots & \vdots & & \vdots \\ A_{1s}^{\mathrm{T}} & A_{2s}^{\mathrm{T}} & \cdots & A_{rs}^{\mathrm{T}} \end{pmatrix}$$

即除了把子块的行与列对换外,每个子块还要进行转置。





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ight),$$

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$$oldsymbol{A}^{ ext{T}} = \left(egin{array}{cccc} oldsymbol{A}_{11}^{ ext{T}} & oldsymbol{A}_{21}^{ ext{T}} & \cdots & oldsymbol{A}_{r1}^{ ext{T}} \ oldsymbol{A}_{12}^{ ext{T}} & oldsymbol{A}_{22}^{ ext{T}} & \cdots & oldsymbol{A}_{r2}^{ ext{T}} \ dots & dots & dots & dots \ oldsymbol{A}_{1s}^{ ext{T}} & oldsymbol{A}_{2s}^{ ext{T}} & \cdots & oldsymbol{A}_{rs}^{ ext{T}} \end{array}
ight),$$

即除了把子块的行与列对换外,每个子块还要进行转置。





例 (1.4.4)

若乘法 AB 有意义, B 按列分块,  $B = (b_1, b_2, \dots, b_n)$ , 则

$$AB = A(b_1, b_2, \cdots, b_n) = (Ab_1, Ab_2, \cdots, Ab_n).$$





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可见, 若 AB = O, 则  $Ab_i = 0$ ,  $i = 1, 2, \dots, n$ . 即 B 的每一列  $b_i (i = 1, 2, \dots, n)$  都是齐次线性方程组 AX = 0 的解.





## 例 (1.4.5)

设  $m \times n$  矩阵  $\mathbf{A} = (\boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, \cdots, \boldsymbol{\alpha}_n)$ , 则

$$m{A}m{A}^{ ext{T}} = (m{lpha}_1, m{lpha}_2, \cdots, m{lpha}_n) \left(egin{array}{c} m{lpha}_1^{ ext{T}} \ m{lpha}_2^{ ext{T}} \ m{arphi}_n \end{array}
ight) = m{lpha}_1 m{lpha}_1^{ ext{T}} + m{lpha}_2 m{lpha}_2^{ ext{T}} + \cdots + m{lpha}_n m{lpha}_n^{ ext{T}},$$

$$\boldsymbol{A}^{\mathrm{T}}\boldsymbol{A} = \begin{pmatrix} \boldsymbol{\alpha}_{1}^{\mathrm{T}} \\ \boldsymbol{\alpha}_{2}^{\mathrm{T}} \\ \vdots \\ \boldsymbol{\alpha}_{n}^{\mathrm{T}} \end{pmatrix} (\boldsymbol{\alpha}_{1}, \boldsymbol{\alpha}_{2}, \cdots, \boldsymbol{\alpha}_{n}) = \begin{pmatrix} \boldsymbol{\alpha}_{1}^{\mathrm{T}} \boldsymbol{\alpha}_{1} & \boldsymbol{\alpha}_{1}^{\mathrm{T}} \boldsymbol{\alpha}_{2} & \cdots & \boldsymbol{\alpha}_{1}^{\mathrm{T}} \boldsymbol{\alpha}_{n} \\ \boldsymbol{\alpha}_{2}^{\mathrm{T}} \boldsymbol{\alpha}_{1} & \boldsymbol{\alpha}_{2}^{\mathrm{T}} \boldsymbol{\alpha}_{2} & \cdots & \boldsymbol{\alpha}_{2}^{\mathrm{T}} \boldsymbol{\alpha}_{n} \\ \vdots & \vdots & & \vdots \\ \boldsymbol{\alpha}_{n}^{\mathrm{T}} \boldsymbol{\alpha}_{1} & \boldsymbol{\alpha}_{n}^{\mathrm{T}} \boldsymbol{\alpha}_{2} & \cdots & \boldsymbol{\alpha}_{n}^{\mathrm{T}} \boldsymbol{\alpha}_{n} \end{pmatrix} \cdot \right]$$





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m T} \ dots \ m{lpha}_n^{
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m T},$$

$$egin{aligned} oldsymbol{A}^{\mathrm{T}}oldsymbol{A} = \left(egin{aligned} oldsymbol{lpha}_{1}^{\mathrm{T}} & oldsymbol{lpha}_{1}^{\mathrm{T}} & oldsymbol{lpha}_{1}^{\mathrm{T}} lpha_{2} & \cdots & oldsymbol{lpha}_{1}^{\mathrm{T}} lpha_{n} \ & oldsymbol{lpha}_{2}^{\mathrm{T}} lpha_{1} & oldsymbol{lpha}_{2}^{\mathrm{T}} lpha_{2} & \cdots & oldsymbol{lpha}_{2}^{\mathrm{T}} lpha_{n} \ & oldsymbol{lpha}_{2}^{\mathrm{T}} lpha_{1} & oldsymbol{lpha}_{2}^{\mathrm{T}} lpha_{2} & \cdots & oldsymbol{lpha}_{2}^{\mathrm{T}} lpha_{n} \ & oldsymbol{lpha}_{2}^{\mathrm{T}} lpha_{1} & oldsymbol{lpha}_{2}^{\mathrm{T}} lpha_{2} & \cdots & oldsymbol{lpha}_{n}^{\mathrm{T}} lpha_{n} \ & oldsymbol{lpha}_{n}^{\mathrm{T}} lpha_{2} & \cdots & oldsymbol{lpha}_{n}^{\mathrm{T}} lpha_{n} \end{array} 
ight) \ .$$





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ight) = m{lpha}_1m{lpha}_1^{ ext{T}} + m{lpha}_2m{lpha}_2^{ ext{T}} + \cdots + m{lpha}_nm{lpha}_n^{ ext{T}},$$

$$egin{aligned} oldsymbol{A}oldsymbol{A}^{ ext{T}} &= (oldsymbol{lpha}_1, oldsymbol{lpha}_2, \cdots, oldsymbol{lpha}_n) egin{aligned} oldsymbol{lpha}_1^{ ext{T}} &= oldsymbol{lpha}_1^{ ext{T}} + oldsymbol{lpha}_2 oldsymbol{lpha}_2^{ ext{T}} + \cdots + oldsymbol{lpha}_n oldsymbol{lpha}_n^{ ext{T}}, \ oldsymbol{lpha}_1^{ ext{T}} oldsymbol{lpha}_1^{ ext{T}} oldsymbol{lpha}_2^{ ext{T}} oldsymbol{lpha}_1^{ ext{T}} oldsymbol{lpha}_2^{ ext{T}} oldsymbol{lpha}_2^{ ext{T}} oldsymbol{lpha}_1^{ ext{T}} oldsymbol{lpha}_2^{ ext{T}} oldsymbol{lpha}_2^{ ext{T}} oldsymbol{lpha}_1^{ ext{T}} oldsymbol{lpha}_2^{ ext{T}} oldsymbol{lpha}_1^{ ext{T}} oldsymbol{lpha}_2^{ ext{T}} oldsymbol{lpha}_2^{ ext{T}} oldsymbol{lpha}_1^{ ext{T}} oldsymbol{lpha}_2^{ ext{T}} oldsymbol{lpha}_1^{ ext{T}} oldsymbol{lpha}_1^{ ext{T}} oldsymbol{lpha}_2^{ ext{T}} oldsymbol{lpha}_1^{ ext{T}} oldsymbol{lpha}_2^{ ext{T}} oldsymbol{lpha}_1^{ e$$





## 应用案例: 图像压缩

相对矩阵运算, 矩阵分块运算在计算上能够更加高效地并行实现. 例如经典的 JPEG2000 图像压缩算法就是将如下图所示  $168 \times 168$  图像分块为  $21 \times 21$  个图像块, 然后分别处理每个  $8 \times 8$  小图像块.





# 小结

- 分块矩阵的加法和数乘:  $A = (A_{ij})_{s \times t}$ ,  $B = (B_{ij})_{s \times t}$ ,  $A + B = (A_{ij} + B_{ij})_{s \times t}$ ,  $kA = (kA_{ij})_{s \times t}$ .
- 分块矩阵的乘法 C=AB: A 的列的分法与 B 的行的分法相同.  $C_{kl}=A_{k1}B_{1l}+A_{k2}B_{2l}+\cdots+A_{ks}B_{sl}=\sum\limits_{i=1}^{s}A_{ki}B_{il}.$
- 分块矩阵的逆矩阵:

$$\begin{pmatrix} \mathbf{A} & \\ & \mathbf{B} \end{pmatrix}^{-1} = \begin{pmatrix} \mathbf{A}^{-1} & \\ & \mathbf{B}^{-1} \end{pmatrix}, \begin{pmatrix} \mathbf{A} & \\ \mathbf{B} & \end{pmatrix}^{-1} = \begin{pmatrix} \mathbf{B}^{-1} \\ \mathbf{A}^{-1} & \end{pmatrix}.$$

• 分块矩阵的转置:

$$\left(\begin{array}{cc} \boldsymbol{A}_{11} & \boldsymbol{A}_{12} \\ \boldsymbol{A}_{21} & \boldsymbol{A}_{22} \end{array}\right)^{\mathrm{T}} = \left(\begin{array}{cc} \boldsymbol{A}_{11}^{\mathrm{T}} & \boldsymbol{A}_{21}^{\mathrm{T}} \\ \boldsymbol{A}_{12}^{\mathrm{T}} & \boldsymbol{A}_{22}^{\mathrm{T}} \end{array}\right),$$

