- Show that, for mathematical statements P, Q, we have the equivalence:

$$\sim ((P \wedge \sim Q) \vee (Q \wedge \sim P)) \equiv (P \wedge Q) \vee ((\sim P) \wedge (\sim Q))$$

For space reasons, we first build the truth table for left hand side:

P	Q	$\sim P$	$\sim Q$	$P \wedge (\sim Q)$	$Q \wedge \sim P$	$(P \wedge (\sim Q)) \vee (Q \wedge \sim P)$	$\sim ((P \wedge (\sim Q)) \vee (Q \wedge \sim P))$
T	T	F	F	F	F	F	T
T	F	F	T	T	F	T	F
F	T	T	F	F	T	T	F
F	F	T	T	F	F	F	T

Now for the right hand side:

P	Q	$\sim P$	$\sim Q$	$P \wedge Q$	$(\sim P) \wedge (\sim Q)$	$(P \wedge Q) \vee ((\sim P) \wedge (\sim Q))$
T	T	F	F	T	F	T
T	F	F	T	F	F	F
F	T	T	F	F	F	F
F	F	T	T	F	T	T

The columns for the two compound statements agree, so the two are equivalent.

- Show that, for mathematical statements P, Q, R, we have the equivalence:

$$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$$

We first build the truth table for the left hand side:

P	Q	R	$Q \wedge R$	$P \vee (Q \wedge R)$
T	T	T	T	T
T	T	F	F	T
T	F	T	F	T
T	F	F	F	T
F	T	T	T	T
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

Now for the right hand side:

P	Q	R	$P \vee Q$	$P \vee R$	$(P \vee Q) \wedge (P \vee R)$
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	F	T	F
F	F	F	F	F	F

The columns for the two compound statements agree, so the two are equivalent.