

# Interpretable Statistical Learning Methods

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## Abstract

Statistical learning methods, which are a flexible class of methods capable of estimating the parameters of complex data generating functions, offer an attractive alternative to conventional methods, which often rely on strong assumptions about the assumed functional form. Statistical learning methods make fewer assumptions about functional form and can estimate a much larger class of data generating functions. A key impediment to the use of statistical learning methods is that they are usually thought of as "black box," in that they cannot be directly interpreted in general. I show that this need not be the case by explaining and implementing methods that are capable of making any method that generates predictions interpretable. This allows researchers to learn about relationships in the data without having to prespecify their functional form. I illustrate this approach using an application to predictive policing in Chicago, wherein these interpretation methods can be used to show how using historical crime data to predict future crime reinforces socio-demographic divides.

## Introduction

The social world is often complex, messy, and difficult to describe with a simple set of rules or laws. In contrast, conventional methods *presume* data are generated by relatively simple processes (King 1989; R. A. Berk 2008). This hampers our ability to discover relationships in our data that we did not expect and reduces the predictive validity of our models (Miller and Page 2009; Beck, King, and Zeng 2000; Beck and Jackman 1998; Schrodт 2014; Fariss and Jones 2017). With conventional methods the data cannot tell us directly about relationships which were not prespecified in the model. Furthermore, the manner in which the data can inform our understanding of relationships which *were* prespecified in the model is constrained by the often strong assumptions made about their functional form, e.g., it is difficult to learn about nonlinear and interactive relationships from a linear additive model. A key benefit of making strong functional form assumptions is that models can be made to be directly interpretable, by, e.g., assuming linearity, simple forms of nonlinearity (e.g., polynomial terms), and/or the absence of interactions involving more than, say, two variables.<sup>1</sup> This comes at a substantial price, however. Conventional methods can miss interesting structure in data when that structure cannot be captured by the assumed model, which means that this information is not extracted from the data, damaging our ability to improve our understanding and degrading the quality of our predictions.

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<sup>1</sup>Interpretability can be thought of in a number of ways (See, e.g., Doshi-Velez and Kim (2017)), but here I use to to mean models which have additive components which can be visualized, e.g., for continuous inputs, functions of two or fewer covariates.

Advances in computing have allowed the development of statistical learning methods which allow us to make fewer assumptions about the structure in our data and consequentially to discover unexpected relationships in our data (J. Friedman, Hastie, and Tibshirani 2001). Statistical learning methods differ from conventional statistical methods in that the number of parameters estimated is a function of the data, rather than specified by the researcher. This gives statistical learning methods the ability to find relationships in data that were not prespecified. This ability to find the unexpected, however, frequently makes the outputs from these methods uninterpretable, which, I argue, has prevented their wide use in political science, where the focus is on substantive insights in addition to prediction (Fariss and Jones 2017; Ward, Greenhill, and Bakke 2010; Hill and Jones 2014). This is not generally a problem with conventional statistical methods because we *assume* that the function that generated the data has a relatively simple functional form which is usually easily interpretable.

I describe and implement general tools which can make any statistical learning method interpretable in ways that allow political scientists to answer questions about the importance of covariates, the nature of their estimated relationship with an outcome, and the presence of interactions amongst said covariates (and their shape) (J. H. Friedman 2001; J. H. Friedman and Popescu 2008; Hooker 2004; Goldstein et al. 2015). I have made these methods available in an accessible manner via three R packages: the **Machine Learning in R** (**mlr**) package, which provides infrastructure for fitting, tuning, and evaluating a wide variety statistical learning methods, the **Exploratory Data Analysis using Random Forests** (**edarf**) package, which contains interpretation methods specific to random forests, a particularly attractive statistical learning method for political scientists, and **Monte-Carlo Methods for Prediction Functions** (**mmpf**) which allows prediction functions to be marginalized to depend on a subset of the covariates (Jones and Linder 2016; Bischl et al. 2016; Z. Jones 2017).

The reason that statistical learning methods do not generally give interpretable outputs is that the manner in which they estimate a model, which is necessitated by their ability to learn complex relationships which were not presupposed, results in an output that, even if the true data generating process or model *is* human interpretable, would hide that fact: a “black box.” Throughout this paper I will use a simple simulated example

$$\begin{aligned} f(v, w, x, z) &= x + v^2 + xw + \sin(z) \\ y &= f + \epsilon \end{aligned} \tag{1}$$

where  $w$ ,  $v$ , and  $z$  are covariates uniformly distributed between  $-2$  and  $2$ ,  $x$  is a covariate equal to  $-1, 0, 1$  with equal probability, and  $\epsilon$  is drawn from a standard normal distribution. Implicitly the “weight” or  $\beta$  for each term is 1.

A simple example of a statistical learning method is a regression tree (Breiman et al. 1984). The way a regression tree estimates a model (i.e., how it arrives at predictions), is by subsetting (i.e., partitioning or grouping)  $y$  using the covariates repeatedly so that the variability of  $y$  within these subsets is minimized. This amounts to finding values of  $y$  which are similar using the covariates, i.e., estimating a smoothed relationship between the covariates and the outcome, which is the same task that conventional regression methods solve, albeit with stronger assumptions. The result of this partitioning process is a nested set of subsets, a hierarchical structure (a tree), which describes the series of subsets that were created: rules which describe the final subset an observation ends up in. As can be seen in fig. 1 this output is substantially more difficult to interpret than ordinary least squares.

This problem is exacerbated when the estimated tree is larger (more complex), or when it is combined with other trees as in ensemble methods like boosted trees or random forests, which are generally preferable for reasons described in later sections. While eq. 1 *could* be estimated with conventional methods, it would require the inclusion of the product of  $x$  and  $w$ , a third degree polynomial expansion of  $z$  to approximate the sine function between  $-2$  and  $2$ , a squared term to estimate the effect of  $v$  and a separate term to estimate the linear effect of  $x$ . Statistical learning methods can estimate this function without prespecification, and the interpretation methods I will describe herein can extract these terms.

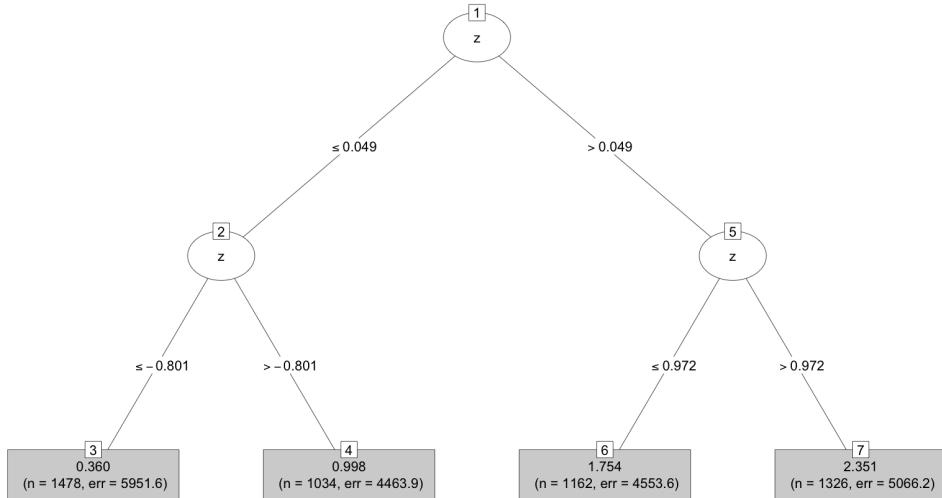


Figure 1: A model estimated via a regression tree from 5000 samples from the data generating function eq. 1. Each circle indicates a subset or partition defined on the labeled covariate, with the rule for creating the subset listed on the line below the circle. The final subsets, or terminal nodes, shown in grey, give them predictions of the tree, the number of observations that fall into that subset, and the mean square error of the prediction on those observations. This tree was constrained to be simple for this visualization.

While there have been some applications of statistical learning in political science, their use has been largely limited to instances in which forecasting (e.g., Goldstone et al. (2010)) is the primary goal, with a few notable exceptions (Beck, King, and Zeng 2000; Hainmueller and Hazlett 2013; Hill and Jones 2014). While the feed-forward artificial neural networks and kernel regularized least squares used in Beck, King, and Zeng (2000) and Hainmueller and Hazlett (2013) are powerful methods which could be appropriately applied to many problems they are not appropriate for *every* application, and thus it is desirable to be able to use, and interpret, the results of the best statistical learning method available for your problem. The methods described herein do precisely that, and, additionally, can be used with conventional methods. I will illustrate these methods using simulated data from eq. 1 and I will also

analyze the reported incidence of burglary in Chicago from 2001 to the present.

The prediction of crime is a high-stakes application because crime prediction systems are used in many cases to make decisions about where to allocate limited policing resources (Lum and Isaac 2016). Using a “black box” may suffice for some purposes, but understanding *why* specific predictions are made, as well as how the covariates used in these models relate to the predictions is important in developing explanations for reported patterns of crime as well as diagnosing any differential effects these predictions may have on economic and demographic groups (R. Berk 2009). These data are taken from the City of Chicago’s online data portal provided by the Chicago Police Department, and list reported burglaries from 2001 to the present. I aggregate these data to the “Beat” level (a geographic unit assigned to 2 patrol cars) for the morning and evening of each day, giving X-many observations over this period, with Y-many reported burglaries. I include “seasonality” covariates such as binary variables for AM/PM, the day of the week, the month of the year, and the year, the police beat, which is a geographic variable, and variables which measure temporal dynamics: the total number of burglaries reported in the previous day, the last week, and the last month, for each beat. Lastly, I include similar lagged totals of other reported crimes: ... This is a relatively small set of covariates, but is similar to that used by other crime forecasting systems and serves to illustrate the usefulness of interpreting models, even in cases where their primary use is forecasting [cite the orthogonal feature decomposition paper here?].

Before proceeding to the details of these methods for making statistical learning methods interpretable, I will review why statistical learning methods can outperform conventional methods and how they can be viewed in the same statistical framework.

## Statistical Learning

Statistical learning methods like artificial neural networks, decision trees, random forests, boosting, and others have seen use in political science, but are far from common (Beck, King, and Zeng 2000; Grimmer and Stewart 2013; Hill and Jones 2014; Montgomery et al. 2015; Green and Kern 2012; Montgomery and Olivella 2015; Hainmueller and Hazlett 2013). Statistical learning methods can be represented in the same framework as more conventional statistical methods. Here I review these equivalences and highlight both the difficulties and advantages of using statistical learning methods as compared to conventional methods.

Assume that the  $(\mathbf{X}, Y)$  are random variables drawn from some probability distribution  $\mathbb{P}$  over the sample space  $\mathcal{X} \times \mathcal{Y}$ , which represents all combinations of  $\mathbf{X}$  and  $Y$  that could be obtained, wherein  $Y$  represents an outcome and  $\mathbf{X}$  covariates.

We typically estimate the function which maps  $\mathbf{X}$  to  $Y$  by using sample data presumed drawn from  $\mathbb{P}$ . Often we are interested in the conditional expectation of  $Y|\mathbf{X}$ , and we assume that this function depends on some parameters  $\beta$ .

$$\mathbb{E}[Y|\mathbf{X} = \mathbf{x}] = f(\mathbf{x}, \beta)$$

Samples of  $Y$  are denoted  $\mathbf{y}$  and of  $\mathbf{X}$ ,  $\mathbf{x}$ . To estimate  $f$  we need to define a set of functions which can be searched for the best approximation to  $f$ , e.g., for a linear regression model the space of all possible values of the regression coefficients for the covariates included. We also

need a way to decide which values of these parameters are “best.” Finally, for computational reasons, we need a way to efficiently search through the set of possible models.

### Defining the Space of Possible Models

A *hypothesis space*, denoted  $\mathcal{H}$ , is a class which defines all possible functions that could be used to approximate the true data generating function  $f$ . In the case of a linear regression the hypothesis space is the set of functions which are linear and additive in the parameters $\$$ . Specifically, for a two parameter simple linear regression of the form  $y_i = \beta_0 + \beta_1 x_i + \epsilon$ , the hypothesis space is  $\mathbb{R}^2$ , because  $\beta_0$  and  $\beta_1$  could each be any number from negative to positive infinity. So  $\mathcal{H}$  contains an infinite number of models. We could estimate an  $f$  with nonlinear relationships between  $\mathbf{X}$  and  $Y$  if we expand the basis (i.e., the set of functions which represent  $\mathbf{X}$ ) by, for example, including polynomials of  $\mathbf{x}$ , but this must be specified in advance rather than being automatically learned from the data. Here the dimension of the space of possible parameters is fixed even though  $\mathcal{H}$  contains an infinite number of models.

$$h(\mathbf{x}, \boldsymbol{\beta}) = \boldsymbol{\beta}^T \mathbf{x} \quad (2)$$

Here each  $h \in \mathcal{H}$ , that is, eq. 2, is one of the possible models contained in the hypothesis space. Estimating, or learning, a model from data is searching over the hypothesis space to find the “best” candidate model.

In contrast to conventional methods like linear regression, statistical learning methods like boosting, decision trees, random forests, and some types of neural networks, utilize a data-adaptive number of parameters, that is, the number of parameters can grow depending on the data. For example a regression tree, as in fig. 1 can be written as

$$h(\mathbf{x}) = \sum_{m=1}^M w_m \mathbb{I}(\mathbf{x} \in R_m) \quad (3)$$

where  $\mathbb{I}(\cdot)$  is the indicator function which equals 1 if  $\mathbf{x} \in R_m$  and 0 if not,  $R_m$  is the  $m$ ’th of the final, disjoint, and exhaustive partitions of the data (the terminal nodes as in fig. 1), and  $w_m$  is the mean value of  $y$  in this region. Each region  $R_m$  is defined by which covariates, and which values in those covariates, were used to create the final set of partitions in the tree: so their number is data-dependent, generally growing with the size and complexity of the sample data.

### Evaluating Candidate Models

How can we evaluate different candidate models in the hypothesis space? A loss function  $\mathcal{L}$  describes how good a particular  $h$  is at predicting  $\mathbf{y}$ . Specifically, the loss  $\mathcal{L}$  is a function which maps predictions made by an approximation to  $f$ , a member of  $\mathcal{H}$ , which we’ve called  $h$  but is conventionally referred to as  $\hat{f}$ , and observed realizations of  $Y$ ,  $\mathbf{y}$ , to a positive real number. Herein lies a problem though, as we are trying to approximate  $f$ , not perfectly reconstruct  $\mathbf{y}$ .  $Y$  is assumed herein to be random, and so contains a random and a systematic component, i.e.,  $\mathbf{y} = f(\mathbf{x}) + \epsilon$ , as in eq. 1. If we could compute the discrepancy between  $\hat{f}$

and  $f$  rather than  $\hat{f}$  and  $\mathbf{y}$  we would be able to distinguish between candidate hypotheses  $h$  which have overadapted to the sample data and those that have more accurately estimated  $f$  and hence will generalize better to new data. However, since we do not know  $f$  we have to substitute in  $\mathbf{y}$ , which conflates systematic error, when the function learned from the sample data  $(\mathbf{y}, \mathbf{x})$  imperfectly represents  $f$ , and error due to the irreducible randomness in  $Y$ . Thus minimizing the loss over the observed data may result in models  $\hat{f}$  which are close approximations of  $\mathbf{y}$  rather than  $f$ : over-fitting. This is the opposite of the problem of picking a hypothesis space whose closest member is far from  $f$ : under-fitting. Both, however, leave us with models which can be rather useless in both learning about  $f$ , that is, explanations about how covariates are related to the outcome, which also means that predictions to new data  $\mathbf{y}$  drawn from  $Y$  will also be worse than they need be.

Like in the case of under-fitting, there are ways to avoid over-fitting such as minimizing the expected loss over the distribution of  $Y, \mathbf{X} : \mathbb{P}$ . The expected loss is usually called the risk, and minimizing it can eliminate optimism that comes from over-fitting the sample data. However, since  $\mathbb{P}$  is unknown this is not directly possible. The sample-average loss is an estimator of the risk, however, it is biased downwards, that is, it is optimistic, when applied to the same data on which the model was estimated (J. Friedman, Hastie, and Tibshirani 2001). A common solution to this issue is to estimate the risk by using Monte-Carlo methods like cross-validation or the bootstrap which randomly shuffle the data to make it more difficult for over-fit models to appear better than they are at generalizing beyond the sample data (J. Friedman, Hastie, and Tibshirani 2001).<sup>2</sup> Penalized regression models like the least absolute shrinkage and selection operator (LASSO) or ridge regression make use of this idea to shrink the regression coefficients of a conventional model (sometimes to 0), making it simpler, to avoid over-fitting.<sup>3</sup> Statistical learning methods use this idea to make the model as simple or complex as is necessary to minimize risk.

Choosing a loss function is an application specific task that depends on the costs of making different sorts of prediction errors. With a binary outcome, it might be the case that positive cases are more costly when they are not predicted than are negative cases that are not predicted, or, with a continuous outcome, over-prediction might be more costly than under-prediction (R. Berk et al. 2009; Muchlinski et al. 2016). Similarly, how the difference between the observed outcome and the prediction affects the loss also depends on how costly bigger or smaller mistakes are. For example mispredicting cases where persons convicted of violent crimes reoffend after parole is probably more costly than predicting recitivism amongst those convicted of non-violent crimes, and it is possible that in some cases over-predicting recitivism is less consequential than under-predicting it. While squared-loss or misclassification-loss is often used because it makes the third step, the search or optimization strategy, easier, it is not necessarily the most appropriate choice, and what loss function is used has an enormous impact on what function from the hypothesis space is chosen as the best approximation to  $f$  since it is *the* criteria for that choice.

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<sup>2</sup>It is also sometimes possible to derive an upper bound on the risk given the “size” (e.g., the cardinality or the complexity) of  $\mathcal{H}$ , the amount of data available, and the average loss on the sample data (V. Vapnik, Levin, and Le Cun 1994; V. N. Vapnik 1998).

<sup>3</sup>Another way of saying this is that regularization or shrinkage makes the model less sensitive to the sample data or that it makes the model smoother.

## Searching for the Best Model

In many cases, even for a simple linear regression, the hypothesis space contains an infinite number of models, and so it is impossible to evaluate every possibility. So a way to search through the hypothesis space must be devised. In some cases this can be done with relative ease, but for many hypothesis spaces even the best available optimization methods cannot guarantee that they will find the best approximation to the true function because of the shape of the loss function over the hypothesis space (i.e., due to non-convexity). However, if the best candidate in the hypothesis space for a linear, additive model is far from the truth:  $f$ , this guarantee means little, since it may be the case that a candidate taken from a larger hypothesis space, even if it is not known to be the best in said space, is much closer to  $f$  than the aforementioned optimal choice in the linear, additive hypothesis space. This is illustrated graphically in fig. 2 and discussed in more detail in Fariss and Jones (2017).

For illustrative purposes consider a linear additive model as in eq. 2 with a squared loss function. Then the function we want to minimize is the sum of squared errors. In this simple case where we have a convex loss function like squared loss gradient descent suffices to find the parameters which minimize the loss function (though it is not the most efficient method available for this problem).

$$\begin{aligned} h(\beta, \mathbf{x}) &= \sum_j \beta_j x_j \\ \mathcal{L}(\mathbf{y}, h) &= \sum_{i=1}^N \frac{1}{2} (y_i - h)^2 \\ \nabla \mathcal{L} &= \frac{\partial \mathcal{L}}{\partial \beta} = \frac{1}{N} \sum_{i=1}^N -x_i (y_i - h) \end{aligned}$$

Descending the gradient of the loss  $\nabla \mathcal{L}$  given a starting value for  $\beta$  (say  $\mathbf{0}$ ), converges to the global minimum. From the starting value  $\beta^0$  the sequential update for the next step is

$$\beta^{(k+1)} = \beta^{(k)} - \gamma_k \nabla \mathcal{L}(\beta^{(k)})$$

where  $\gamma$  controls the size of the steps along the gradient, which can itself be estimated by gradient descent. The procedure stops when  $\nabla \mathcal{L}$  is sufficiently small, that is, when the loss function is no longer decreasing.

In contrast consider boosting, a statistical learning method. Like with the aforementioned linear additive model we want to minimize the loss  $\mathcal{L}(\mathbf{y}, h) = \frac{1}{2} \sum_{i=1}^N (y_i - h)^2$ , however now each candidate  $h$  has more parameters. In this case it is not possible to  $h$  will be estimated sequentially, that is, it is composed of a sequence of models which are combined in the last step. We denote the first  $m$  components of  $h$  as  $h_m$ .

$$h_m(\gamma, \theta, \mathbf{x}) = \sum_{m=1}^M \gamma_m \phi_m(\mathbf{x}, \theta_m) \quad (4)$$

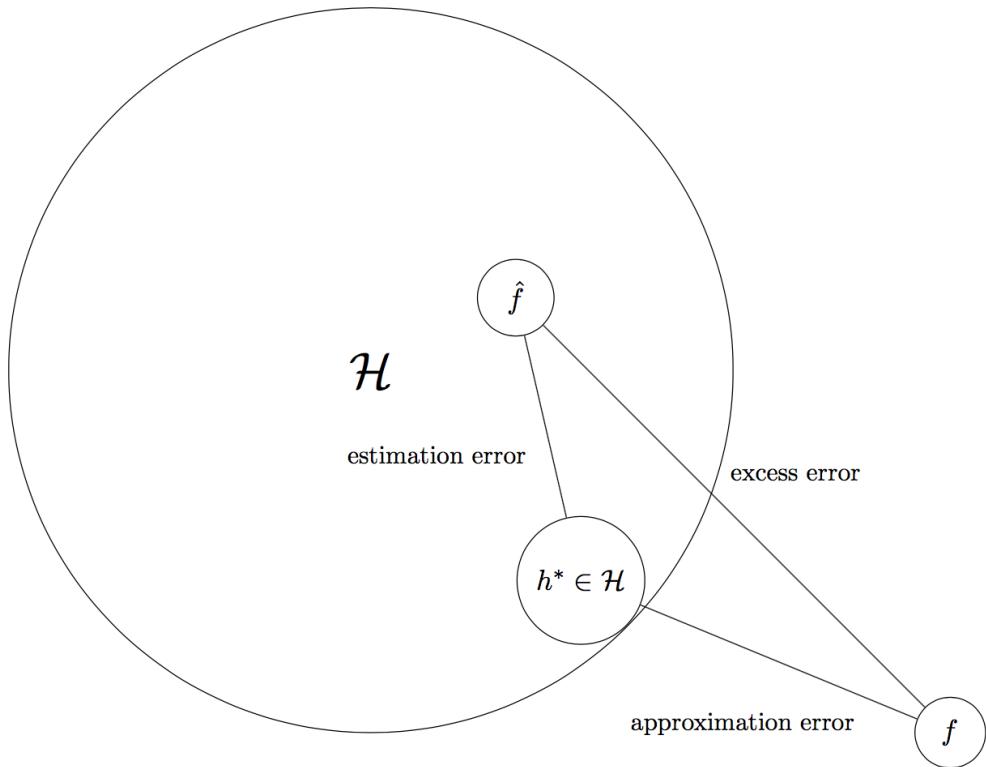


Figure 2: Here the circle represents the hypothesis space  $\mathcal{H}$ ,  $f$  is the true model, which in this case is not contained in the hypothesis space.  $h^*$  is the point (the model) in the hypothesis space closest to  $f$ , i.e., what could be achieved with an infinite amount of data. The difference between this point and the true model is approximation error that is due to restricting the hypothesis space to  $\mathcal{H}$ . Decreasing the “size” of  $\mathcal{H}$  increases approximation error, but decreases estimation error, which is error that comes from having to search  $\mathcal{H}$  for the best model:  $h^*$ , which generally becomes more difficult as  $\mathcal{H}$  grows. I am arguing herein that because  $f$  is likely very complex in many social systems, approximation error is substantially larger than estimation error.

Here  $\phi_m$  is a model itself, such as a regression tree like is shown in fig. 1. For each  $m$  the gradient of  $\mathcal{L}$  with respect to  $h_m$  is

$$\nabla \mathcal{L}(h_m) = \frac{\partial \mathcal{L}}{\partial h_m} = \sum_{i=1}^N (y_i - h_m)$$

and so, like in the case of gradient descent for linear regression

$$h_{m+1} = h_m - \gamma_m \nabla \mathcal{L}(h_m)$$

The scaling parameters  $\gamma$  are estimated by a separate application of gradient descent.<sup>4</sup>

So at each step  $m$  boosting using this loss function amounts to fitting  $\phi_m(\mathbf{x}, \theta_m)$  to  $\mathbf{y} - h_{m-1}$ : repeatedly refitting the residuals. This amounts to giving higher weight to observations where the sequence of models thus far has performed poorly, and by combining these models as in eq. 4 nonlinear relationships between  $\mathbf{x}$  and  $\mathbf{y}$  can be estimated without specifying  $\phi$ s which can estimate nonlinearity. It should also be clear that the resulting output  $h_M$ , which is a weighted sum of models  $\phi_m$ , which are themselves estimated on residuals which depend on  $h_{m-1}$  is not directly interpretable, even if the data generating function it is approximating is. This property is a consequence of statistical learning methods' capacity for automatically finding patterns (e.g., nonlinearity and/or interaction amongst the covariates) that were not prespecified.

## Interpreting Models

I argue that there are four main types of interpretation which are commonly sought. Functional form describes the shape of the relationship between covariates and the model. Magnitude describes how important covariates are in determining the model's predictions. Interaction detection relates to the estimation of the functional form and magnitude of the relationship between groups of covariates and the model. Reliability relates to how different sources of variability could possibly change the estimated effect.

### Functional Form

Interpreting the functional form of the relationship between the covariates and the model is one of the most common tasks. For a linear and additive model this is often described by a single number which is obtained by taking the partial derivative of the model with respect to said covariate; e.g.,  $\frac{\partial \hat{f}}{\partial \mathbf{x}_j} \sum_j \beta_j \mathbf{x}_j = \beta_j$ . The situation is complicated for models like logistic regression where the formulation of the model forces this derivative to depend on  $\mathbf{x}_{-j}$ , but the principle remains the same. For models which are nonlinear in  $\mathbf{x}_j$  this derivative is also not constant, but does not necessarily depend on  $\mathbf{x}_{-j}$ . For example if  $\hat{f} = \beta_1 \mathbf{x}_1 + \beta_2 \mathbf{x}_1^2 + \beta_3 \mathbf{x}_2$  then  $\frac{\partial \hat{f}}{\partial \mathbf{x}_1} = \beta_1 + \beta_2 \mathbf{x}_1$  and to interpret the relationship between  $\mathbf{x}_1$

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<sup>4</sup>Specifically,  $\gamma_m = \arg \min_\gamma \mathcal{L}(\mathbf{y}, h_{m-1} + \gamma h_m)$ .

and  $\hat{f}$  it is necessary to plot  $\mathbf{x}_1$  and this derivative at a grid of values on  $\mathbf{x}_1$ . This generalizes to the functional form of multiple variables as well. With statistical learning methods the estimated function  $\hat{f}$  is often a “black box,” and so this approach does not generally work analytically, though there are exceptions such as Hainmueller and Hazlett (2013).

## Magnitude

Magnitude describes how important covariates are in determining a models’ predictions. One way to measure importance in the case of a linear model is via the absolute size of the associated regression coefficients. For a nonlinear model this isn’t so simple. If the derivative of the model with respect to a covariate is not constant then there is no longer a single number summary of that covariate’s influence on the model’s predictions. Yet, this might be what is desired, as in Hill and Jones (2014) where the goal was to rank a large set of potentially relevant covariates in order to focus on ones that were identified as useful predictors by the model.

## Interaction Detection

The identification and description of interactions is a common task as well (Brambor, Clark, and Golder 2006). In the case of a linear model, again, this is fairly straightforward, though the product terms commonly used to estimate interactions in these models are commonly misused in testing nonlinear hypotheses (Hainmueller, Mummolo, and Xu 2016; Pepinsky 2017). Using statistical learning methods avoids this problem by automatically detecting interactions while making few assumptions about their possible functional form. This allows researchers to ask these common questions, like, how does the effect of one variable change as a possibly nonlinear function of another. Interaction detection can be cast as the functional form of the joint effects of covariates or their joint importance.

## Reliability

Most of the above, when computed using conventional methods, are or can be accompanied by estimates of sampling variability. In some cases this can be a useful measure of reliability but in many cases the assumptions necessary for these reliability estimates to be valid are violated. This is particularly problematic in social data, where dependence is both complex and ubiquitous, so much so that it is often of primary interest, as it is in analyses of social networks. It may also be the case that uncertainty from non-random measurement error is substantially more important than random measurement error (i.e., sampling error) in many cases, and instances where non-random measurement error is identified or where multiple measures of the same (or similar) concepts produce substantially different conclusions constitute anecdotal evidence that this is the case (e.g., Lupu 2017; Baum and Zhukov 2015; Jerven 2013; Hegre and Sambanis 2006). This suggests that many measures of uncertainty might not be as trustworthy as their theoretical best-case properties suggest. Thus I have not focused on measuring reliability herein, however, all of the software referenced herein is capable of using estimates of the variability of a models’ predictions to produce reliability

bounds for the aforementioned types of interpretation tasks which correspond to the quantities that typically accompany the equivalent outputs from conventional methods.<sup>5</sup>

## Approximating and Interpreting Functional Form

When the most complex functional form capable of learned from the data is simple, as in a linear additive model, the parameters of said model are often directly interpretable, which, to reiterate, is the consequence of assumed simplicity, which may or may not be appropriate in a given circumstance. Some statistical learning methods have similar properties (See, e.g., Hainmueller and Hazlett 2013; Beck, King, and Zeng 2000). However, many such methods cannot be directly interpreted but may be otherwise desirable for a given application: due to, e.g., their ability to deal with a large number of samples or many covariates, their predictive performance in this or similar applications, etc. One attractive method is the random forest, which is used here to model data from eq. 1 and to predict the reported incidence of burglary using data from the Chicago Police at the police beat-day level [Breiman (2001); cite ranger]

One of the most common types of interpretation for conventional methods is functional form; i.e., for a linear model what is the sign and magnitude of the coefficient for a covariate, defining the slope of the line representing its relationship with the model's predictions. For conventional models where one or more of the covariates have a nonlinear relationship in the model, e.g., a logistic regression, the effect of a covariate on the model's predictions depends on the other covariates. To understand the effect of a covariate then, it is necessary to compute predictions at particular values of the other covariates. Choosing sample average values for these covariates is common practice, but with models which may have estimated interactions between the covariates, where the empirical distribution of the covariates is not unimodal, or when the distribution of the covariates is dependent, this approach makes it difficult to detect such interaction and may distort the marginal relationships as well. Further analysis of these problems are beyond the scope of this paper, but are elaborated in Z. Jones (2017). An average marginal effect gives the effect of a particular covariate computed at, rather than fixed values for the other covariates, every observed combination of the other covariates (or a subsample thereof for computational reasons). This is a form of Monte-Carlo integration which also works with statistical learning methods, where it is referred to as partial dependence. It allows the recovery of the partial relationship between particular covariates and  $\hat{f}$  regardless of how  $\hat{f}$  is estimated (J. H. Friedman 2001).

### Partial Dependence

If we knew the joint distribution  $\mathbb{P}$  of  $\mathbf{x}$  we could visualize the dependence between some some subset of  $\mathbf{x}$ , say  $\mathbf{x}_u$ , by integrating out the compliment  $\mathbf{x}_{-u}$  and computing, e.g., the conditional expectation of  $Y|\mathbf{x}_u$ . However, since we do not know  $\mathbb{P}$  and have instead estimated this expectation we can instead use Monte-Carlo integration using the sample data. We would compute

$$\mathbb{E}(f(\mathbf{x})|\mathbf{x}_u) = \int_{\mathbf{x}_{-u}} f(\mathbf{x}_u, \mathbf{x}_{-u}) \mathbb{P}(\mathbf{x}_u|\mathbf{x}_{-u}) \mathbb{P}(\mathbf{x}_{-u}) d\mathbf{x}_{-u}. \quad (5)$$

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<sup>5</sup>See, e.g., Sexton and Laake (2009) or Wager, Hastie, and Efron (2014) for examples of methods for estimating the variance of predictions from random forests.

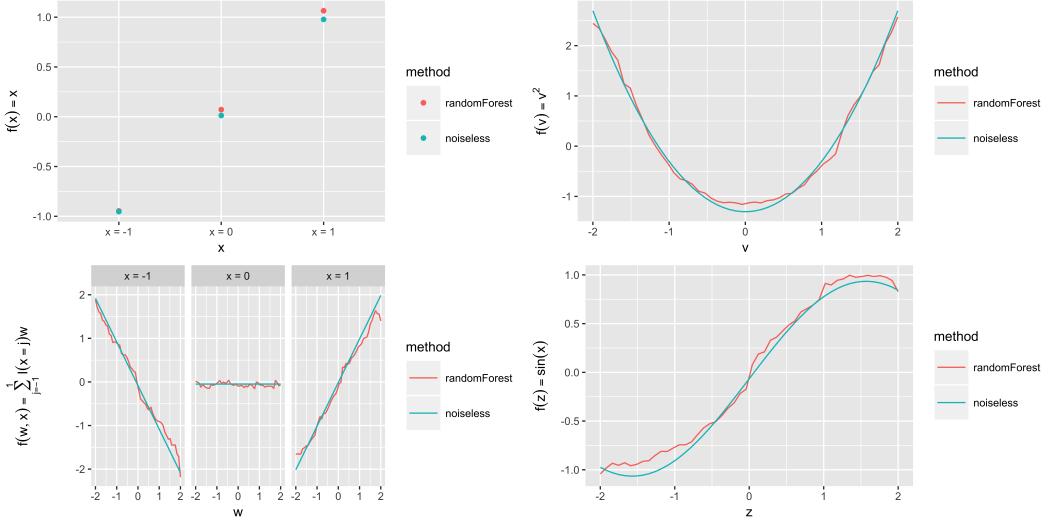


Figure 3: This figure shows the partial dependence: the marginalized prediction function, for each term in eq. 1, constructed from a random forest fit to 5,000 samples and the data generating function if it were known estimated exactly, but was still a “black box” (a “noiseless” estimate of eq. 1). In the top left panel is the partial dependence of the discrete covariate  $\mathbf{x}$ , and in the bottom left its interaction with the continuous covariate  $\mathbf{w}$ . In the upper and lower right panels the partial dependence of the covariates which have nonlinear effects,  $\mathbf{v}$  and  $\mathbf{w}$  are shown. While a conventional method like ordinary least squares could estimate eq. 1, the random forest estimated these relationships without prespecification, and with partial dependence we are able to recover the functional form of these relationships accurately.

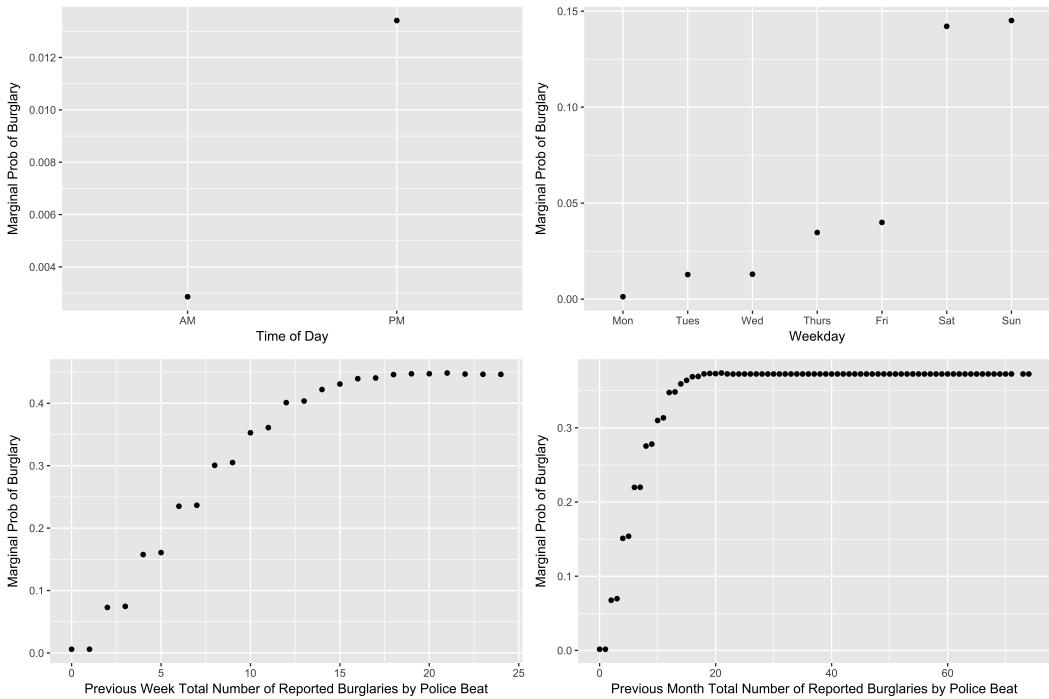


Figure 4: Partial dependence indicates that the marginal probability of a reported burglary in a police beat increases in the evening, on the weekend, and substantially for beats in which a large number of burglaries were reported in previous weeks and months.

However the conditional probability of  $X_u|X_{-u}$  introduces into the desired description of the relationship between  $\mathbf{x}_u$  and  $\hat{f}$  dependencies between  $\mathbf{x}_u$  and  $\mathbf{x}_{-u}$ . If this term is dropped, we get the *partial dependence* of  $\hat{f}(\mathbf{x})$  on  $\mathbf{x}_u$ , which can be estimated from the sample data (J. H. Friedman 2001 Z. Jones (2017)). The effect of the inclusion of this conditional probability on the estimate of the effect of how  $\hat{f}$  depends on  $\mathbf{x}_u$  can be substantial even when there is no interaction between  $\mathbf{x}_u$  and  $\mathbf{x}_{-u}$  (See Z. Jones (2017) for a more detailed explanation).

$$\mathbb{E}[f(\mathbf{x})] = \int_{\mathbf{x}_{-u}} f(\mathbf{x}) d\mathbb{P}(\mathbf{x}_{-u}) \approx \frac{1}{N} \sum_{i=1}^N \hat{f}(\mathbf{x}_u, \mathbf{x}_{-u}^{(i)}) = \bar{f}_{\mathbf{x}_u}(\mathbf{x}_u) \quad (6)$$

This gives the relationship between the prediction made by  $\hat{f}(\mathbf{x})$  and  $\mathbf{x}_u$  averaged across the observed values of  $\mathbf{x}_{-u}$ . A grid of points on  $\mathbf{x}_u$  must be chosen, which could be obtained by sampling from the observed values  $\mathbf{x}_u$ , by selecting values of interest, or by creating a uniform sequence of values.

How closely  $\bar{f}_{\mathbf{x}_u}(\mathbf{x}_u)$  corresponds to  $\hat{f}(\mathbf{x}_u)$  depends on the structure of  $\hat{f}$ . In the case  $\hat{f}(\mathbf{x})$  is additive or multiplicative in  $\{\mathbf{x}_u, \mathbf{x}_{-u}\}$ , that is,  $\hat{f}(\mathbf{x}) = \bar{f}_{\mathbf{x}_u}(\mathbf{x}_u) + \bar{f}_{\mathbf{x}_{-u}}(\mathbf{x}_{-u})$  or  $\hat{f}(\mathbf{x}) = \bar{f}_{\mathbf{x}_u}(\mathbf{x}_u)\bar{f}_{\mathbf{x}_{-u}}(\mathbf{x}_{-u})$  computing the partial dependence gives an approximation to  $\hat{f}(\mathbf{x}_u)$  that is accurate to an additive or multiplicative constant, respectively (J. H. Friedman 2001; Z. Jones 2017). Discovering when such (additive or multiplicative) factorizable structure is *not* present is a question about interaction, that is, non-separable dependence between  $\mathbf{x}_u$  and  $\mathbf{x}_{-u}$  in  $\hat{f}$ .

One way to detecting interaction amongst the covariates using partial dependence is to estimate the point-wise variability of eq. 6, as shown in fig. 5. This can be done again using Monte-Carlo methods by substituting the sample variance for the mean, i.e., the variance of  $\hat{f}$  for a fixed grid of values on  $\mathbf{x}_u$  across the sample values of  $\mathbf{x}_{-u}$  (or a subsample thereof). If there is interaction between  $\mathbf{x}_u$  and  $\mathbf{x}_{-u}$  which has been estimated in  $\hat{f}$  the the partial dependence of  $\mathbf{x}_{-u}$  will be non-constant, i.e., it will vary across the range of  $\mathbf{x}_u$ , and, its variability will in general increase as the strength of the interaction with  $\mathbf{x}_{-u}$  increases, which can be seen in fig. 5 with the two covariates  $x$  and  $w$  which interact in eq. 1.<sup>6</sup>

All of the above can be quickly and easily estimated for any prediction function  $\hat{f}$ , regardless of how it was estimated (i.e., both conventional and statistical learning methods) from the sample data using **Monte Carlo Marginalization of Prediction Functions** (**mmpf**) which is a publically available R package. The **Machine Learning in R** (**mlr**) package also makes all of this functionality available, and, additionally, makes available a wide variety of statistical learning methods available for use in a consistent manner, with a wide variety of tools for evaluating and tuning their performance (Bischl et al. 2016; Z. Jones 2017). Lastly, **Exploratory Data Analysis using Random Forests** (**edarf**) again provides similar functionality to what is described above along with additional interpretation methods specific to random forests, which are beyond the scope of this paper (Jones and Linder 2016).

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<sup>6</sup>Another possibility is to avoid Monte Carlo integration over the distribution of  $\mathbf{x}_{-u}$  and to instead visualize the expected value of the outcome at  $\mathbf{x}_u$  for each observation conditional on  $\mathbf{x}_{-u}$ . Goldstein et al. (2015) propose using this approach. **mmpf**, **edarf**, and **mlr** are capable of estimating this as well.

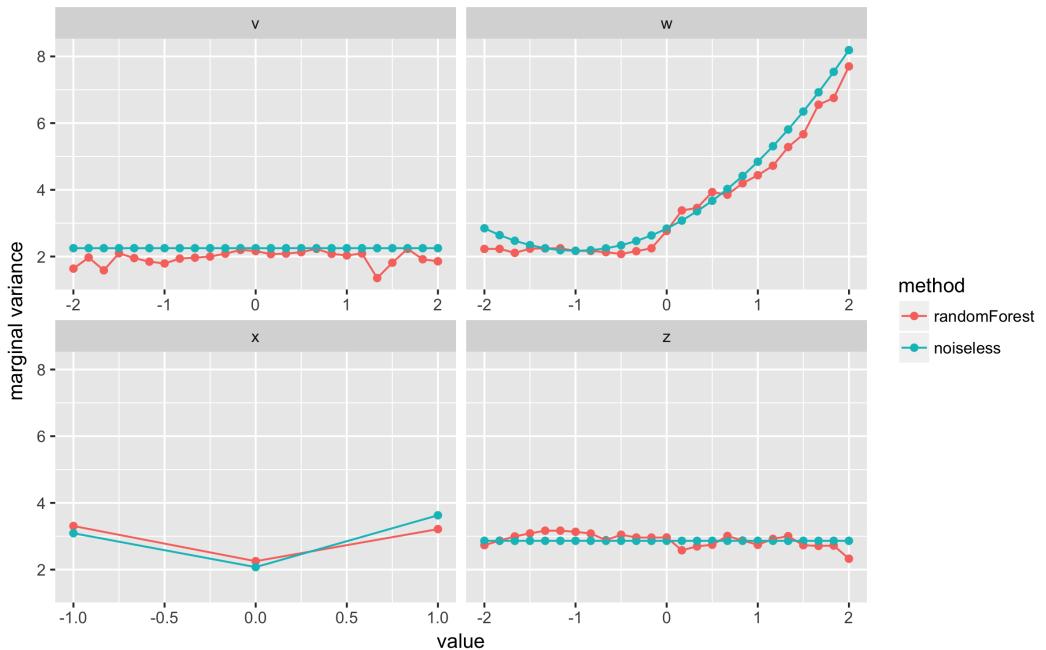


Figure 5: The marginal variance of the covariates indicated in the top of each panel. The covariates which do not interact in eq. 1 have a constant variance across their range, since in these cases changing the value of the indicated covariate does not modify the effects of any others: that is, there is no interaction, which the model has correctly estimated as can be seen in fig. 3. In the case of both  $w$  and  $x$ , which do interact, it is clear that the variance is non-constant, and that it increases when  $x = -1$  or  $x = 1$ , which are the two areas in which  $x$  interacts with  $w$ .

## Variable Importance

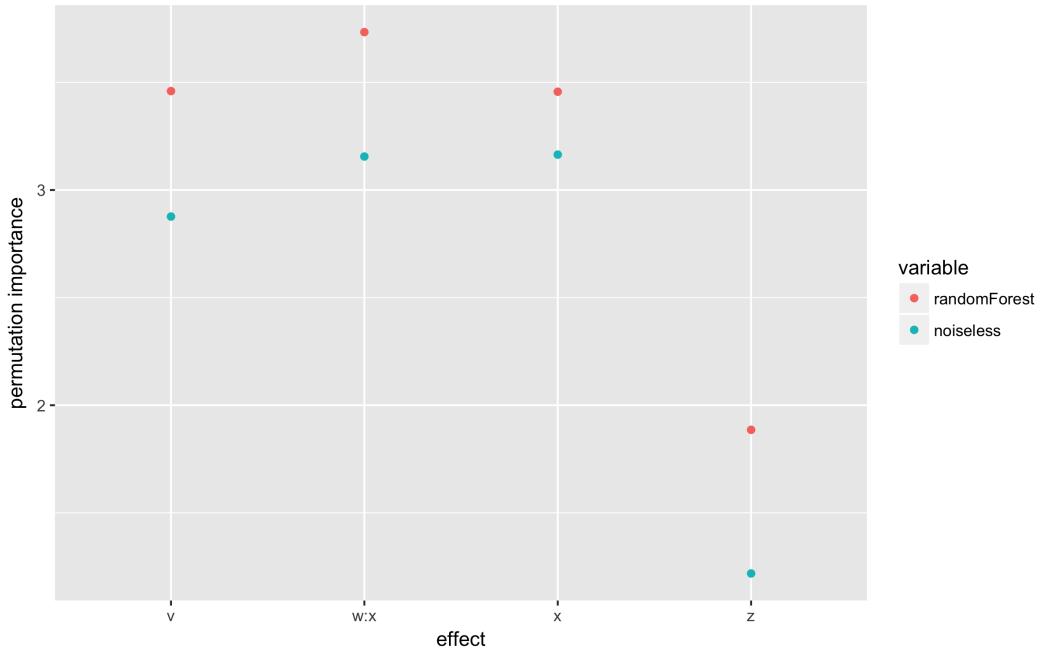


Figure 6: Permutation importance estimated from the data generating function in eq. 1. This indicates that the most important component of the model is the interaction of  $\mathbf{x}$  and  $\mathbf{w}$ . Since  $\mathbf{x}$  and  $\mathbf{w}$  interact in eq. 1 and the random forest estimated this interaction, as can be seen in fig. 3, permuting either or both covariates will increase prediction error substantially, as they introduce error from the individual components as well as their interaction. In this way permutation importance gives weight to covariates like  $\mathbf{w}$  which may only have a conditional effect. Components with less variability, like  $\sin(\mathbf{z})$  (which has a sample variance of 0.592) have less importance than those with more variability (e.g.,  $\mathbf{v}^2$ , which has a sample variance of 1.418).

The question of which and how important covariates are is a question that arises with predictive and exploratory tasks. In case of the former it is often advantageous to remove covariates which are not useful in predicting the outcome and in the latter case computing the importance of covariates is useful in finding strong predictive relationships for further study (Shmueli and others 2010). For example these methods were used by Hill and Jones (2014) to rank covariates according to their importance to accurately predicting different measures of human rights abuses. As previously mentioned in the case where the derivative of the model with respect to a covariate is constant the importance of that covariate in that model is summarized by a single number whose magnitude is often of primary interest. With models where this derivative is not constant there is no straightforward way to summarize the importance of a covariate with a single number; fortunately, there are methods which have generalized the notion of importance to models of an arbitrary form.

Permutation importance is a general method for estimating variable importance which is agnostic to the learning method used. It is the Monte Carlo expectation of the change in

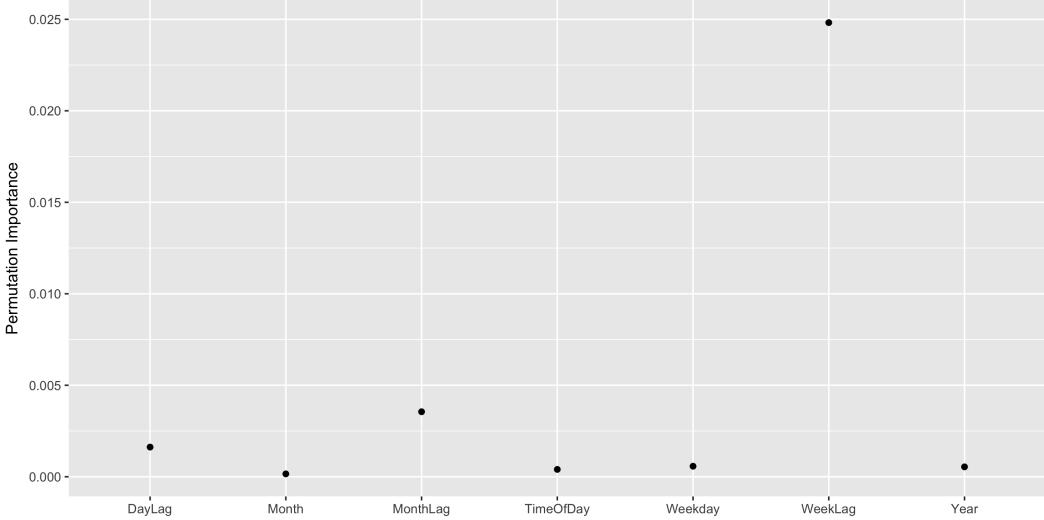


Figure 7: The permutation importance of covariates used to predict the reported incidence of Burglary between 2001 and present in Chicago at the police beat level. The covariates which are most important are those that measure temporal dependence: beats wherein there were a high number of reported burglaries in the previous weeks and months are more likely to have burglaries reported on any given day.

prediction error that occurs when  $\mathbf{x}_u$  is permuted. If  $\mathbf{x}_u$  was not useful to  $\hat{f}(\mathbf{x})$  in making predictions then we would expect no change or a decrease in prediction error, whereas if  $\mathbf{x}_u$  is important there should be an increase in prediction error when it is permuted. Due to Monte-Carlo error from the permutation, this is repeated  $M$  times to give the expectation of the change in the loss  $\mathcal{L}$  from permuting  $\mathbf{x}_u$ .

$$I_{\mathbf{x}_u} = \frac{1}{M} \sum_{j=1}^M \mathcal{L} \left( \hat{f}(\mathbf{x}_{u\pi(j)}, \mathbf{x}_{-u}), \hat{f}(\mathbf{x}) \right) \quad (7)$$

Take for example a linear model of the form  $\hat{f}(\mathbf{x}) = \beta_u \mathbf{x}_u + \beta_{-u} \mathbf{x}_{-u}$ . Suppose that  $\beta_{-u}$  was 0. Then if we randomly permuted  $\mathbf{x}_{-u}$   $M$  times, and, for each one of those times we computed  $\hat{f}$  using this randomly shuffled version of  $\mathbf{x}_{-u}$ , denoted  $\mathbf{x}_{u\pi}$  in eq. 7 we would find that the prediction error did not change at all, because  $\mathbf{x}_{-u}$  does not contribute to the prediction made by  $\hat{f}(\mathbf{x})$ . Conversely, if we permuted  $\mathbf{x}_u$ , and  $\beta_u$  was non-zero, perhaps relatively large, we would be multiplying randomly shuffled covariates by large numbers  $\beta_u$ , and, unsurprisingly, this would make  $\hat{f}$  also basically random, and consequently its predictions would degrade accordingly, which would allow us to conclude that  $\mathbf{x}_u$  is important in determining  $\hat{f}$ . Clearly this is unnecessary in this simple case where we could simply inspect the parameters  $\beta$ , but with statistical learning methods this is not usually possible.

Measuring the change in the loss that results from permuting certain covariates as a way of estimating predictive importance is agnostic to the choice of loss function. With categorical (ordered or unordered) outcome variables category-specific or overall (e.g., average agreement

across categories) importance can be computed, which can be useful in cases where categories are imbalanced or the costs of making prediction errors differs across the categories, e.g., as may be the case when predicting violence, where an absence of violence is more common than violence but violence is often costly when not anticipated (R. Berk et al. 2009). With a continuous outcome the permutation importance can be computed for the mean, i.e., how permuting a covariate affects the conditional expectation function, or any other summary of the distribution of outcomes, such as a quantile. Lastly the permutation importance may not be aggregated at all, giving an observation specific measure of importance. This can, for example, be used to estimate the importance of covariates across the distribution of the outcome (a continuous analogue to category-specific importance).

Permutation importance is certainly not the only method for computing predictive importance with statistical learning methods. There are many method specific measures for Random Forests have been subject to study (Strobl et al. 2007; Strobl et al. 2008; Altmann et al. 2010; Nicodemus et al. 2010; Schwarz, König, and Ziegler 2010; Janitza, Strobl, and Boulesteix 2013; Grömping 2009; Louppe et al. 2013). J. H. Friedman (2001) proposed permutation importance as well as other generic methods such as that of Laan (2006) which may prove useful for political scientists but which are beyond the scope of this paper. At this point the theoretical study of this method is limited (Louppe et al. 2013).

Permutation importance as described above is implemented in `edarf`, `mmpf`, and `mlr`.

## Summary and Future Work

Future empirical work can make use of statistical learning methods which allow us to avoid making unwarranted assumptions and learn about unexpected interactions and nonlinearities. Statistical learning methods will generally estimate models that generalize to new data better and extract more information about the relationships between the covariates and outcomes. Additionally, using statistical learning methods in cases where many conventional methods would have been fit and compared in a semi-formal manner can help reduce researcher degrees of freedom (i.e., arbitrary but consequential data analysis decisions) that can have substantial effects on conclusions (Gelman and Loken 2013).

Statistical learning methods can be used on their own for data analysis and can also be used in conjunction with conventional methods (See, e.g., Muchlinski et al. 2016). For example the partial dependence of a covariate can be compared to the average marginal effect from a conventional model with a more restricted functional form, and the permutation importance of the covariates could be compared to the size of the coefficients from a conventional method (or their ranks). The expected prediction error could also be compared in this instance; if statistical learning methods do substantially better than conventional methods this would indicate that conventional methods under-fit the data, i.e., the models that they represent are under-theorized.

The methods for interpreting the “black box” functions which are commonly the output of statistical learning methods described herein have all relied on integration. Another class of interpretation methods relies on projecting the estimated function onto a more interpretable set of functions, which is the subject of ongoing research (Hooker 2007; Gaëlle Chastaing and Le Gratiet 2015; Gaëlle Chastaing, Gamboa, and Prieur 2015). Further study of the statistical properties of these methods is also underway.

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