A Novel Model for Repeater Coordination Based on Density Dilution Algorithm

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Abstract

Due to the rapid development of high frequency radio telecommunication, repeater coordination needs to be optimized to cope with both increased users and limited spectrum. In addition, the interferences between repeaters, the distribution of users in the area and the topography of the district make repeater coordination more difficult. The CTCSS technology is useful to mitigate interference problems and it has been supplied to repeater coordination. To make an optimal solution for repeater coordination, a variety of factors need to be taken into consideration.

In this paper, we present a novel model for repeater coordination. We treat the circular area as a cellular radio network. Each repeater service area is called a cell. Our model is aimed to deploy the minimum number of repeaters and at the same time to achieve enough user capacity in the area without interferences. To impact some intuition about the model, we begin by presenting a simple model regardless of many influences. Next, we apply CTCSS technology to repeater coordination dealing with complex factors in our full model. We analyze the uniform distribution model and the random distribution model based on the distribution of users. We develop Density Dilution Algorithm(DDA) and Adjacent Removal Algorithm(ARA) to solve the problem. Finally, we discuss the case where there might be defects in line-of-sight propagation caused by mountainous areas.

Key Words: Density Dilution Algorithm(DDA);Adjacent Removal Algorithm(ARA); cellular radio network

Contents

1 Introduction

The main task of the repeater is to amplify the received signal and transmit it further forward. As repeaters can be added to the radio network to improve the overall coverage and capacity of the system[1], they have played a significant role in high frequency radio spectrum propagation. However, repeater location has less flexibility due to infrastructural, environmental, and governmental issues and rules. Besides, the interferences between repeaters are still a tricky affair in repeater coordination. With the help of the "continuous tone-coded squelch system"(CTCSS) technology, the interference problems can be mitigated. The number and location of repeaters are of paramount importance for the radio mobile communication in a particular area. Many professional operators are looking for the optimal scenario for repeater coordination. The optimal repeater coordination should generally have the following property:

- Applying CTCSS technology, the interferences are canceled,
- Deploying the minimum number of repeaters,
- Accommodating enough simultaneous users.

In this paper, we attempt to work out an optimal solution for practical repeater coordination in a particular area. The problem can be specifically depicted as

- Determine the minimum number of repeaters necessary to accommodate 1,000 or 1,0000 simultaneous users in a circular flat area of radius 40 miles,
- Discuss the case where there might be defects in line-of-sight propagation caused by mountainous areas.

Our main goal is to deploy the minimum number of repeaters and at the same time to achieve enough user capacity in the area without interferences. We solve the problem step by step. At the beginning,we present a simple model without considering difficult factors in repeater coordination. Next, we take the main factors into consideration and use the CTCSS technology to find the best solution for repeater coordination in our full model. We analyze the uniform distribution model and the random distribution model based on the distribution of users. We determine the minimum number of repeaters for the area in different cases. Finally, we discuss the case where there might be defects in line-of-sight propagation caused by mountainous areas.

2 Assumptions

- Every cell in the circular area has a single repeater at its center.
- Every cell can accommodate the same quantity of users.
- Every repeater is an ideal repeater, that is, it transmits radio signals equally well in all directions[2].
- The repeaters are completely the same, the same hight and the same coverage area.
- Ignore any other factors influenced signal(i.e., building, atmosphere) propagation except terrain.
- Movement of users does not affect the distribution of the user.
- It is line-of-sight transmission and reception between users and repeaters.

3 A Simple Model

For simplicity, we begin by presenting a special situation. We make some other specific assumptions.

- Each servicing area can contain enough simultaneous users.
- The CTCSS technology is not applied to repeater coordination.
- The distance between nearby repeaters is far enough to avoid interferences.

3.1 Repeater Service Area

In order to obtain the the coverage area of a repeater roughly, we use Okumura-Hata model[3] to calculate the path loss of the signal. Hata model was based on Okumuras field test results and predicted various equations for path loss with different types of clutter. The limitations on Hata Model due to range of test results from carrier frequency 100MHz to 1500MHz, the distance from the repeater ranges from 1Km to 20Km, the height of repeater antenna (hb) ranges from 30m to 200m and the height of mobile antenna (hm) ranges from 1m to 10m. Hata created a number of representative path loss mathematical models for each of the urban, suburban and open country environments. Here, we only discuss the path loss for urban clutter. It is given by the Hata empirical equation:

$$L = 69.55 + 26.16log(f) - 13.82log(h_b) - a(h_m) + [44.9 - 6.55log(h_b)]log(d), \quad (1)$$

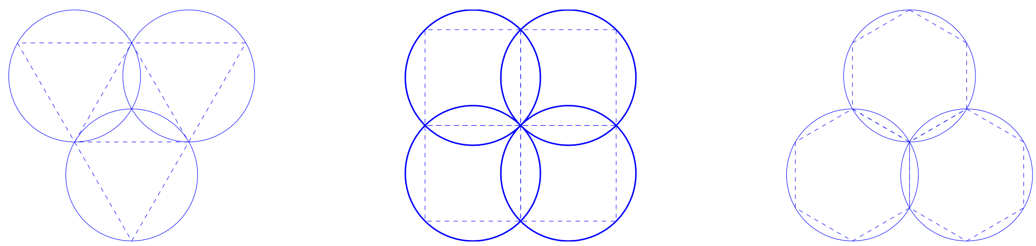
where f is frequency, d is distance, h_b is the height of repeater antenna, h_m is the height of mobile antenna, $a(h_m)$ is a correction factor of mobile antenna. $a(h_m)$ is described by the following equation:

$$a(h_m) = 8.29log^2(1.54)h_m - 1.1. \quad (2)$$

Learned from many literature, the path loss L is between 110 and 140dB when the spectrum available is 145 to 148 MHz. In this model, we let $L = 120dB$, $f = 146MHz$, $h_b = 50km$, $h_m = 1m$. Substituting these values into equation (1) and equation 2, we obtain $d = 18km$. Thus, we can get that the a repeater service area (a cell) is a circular area of radius $18km$. The repeater service area is large enough to avoid the interferences between repeaters.

3.2 Shape of a Cell

A repeater service area (a cell) is a circular area. In order to cover the whole area without leaving gaps, a lot of overlap must be contained between the circular radiation zone. Taking the overlap into consideration, we can know that the effective coverage of every radiation zone is a polygon. According to the different instance of overlap, the shape of a cell could be equilateral triangular, square and regular hexagonal, as illustrated in **Figure 1**.



(a) Equilateral triangular (b) Square (c) Regular hexagonal

Figure 1: Shape of a cell.

It is proven that only these three shapes are possible if filling a plane area with regular polygons without overlap. Then which is the best shape? In the case of the same radiation radius r, the distance between adjacent cells, the cell area, the overlap width and the overlap area have been calculated. The result is shown in **Table 1**. From **Table 1**, we can see that in the case of the same cell area, regular hexagonal cell is closest to the circular cell. Therefore, regular hexagonal cell can cover the whole service area with the minimum number of repeaters. As the network constituted by regular hexagonal cells looks like a beehive, it is called a cellular network[4].

Table 1: Comparison between three shapes.

shape of cell	equilateral triangular	square	regular hexagonal
adjacent distance	r	$\sqrt{2}r$	$\sqrt{3}r$
the cell area	$1.3r^2$	$2r^2$	$2.6r^2$
overlap width	r	$0.59r$	$0.27r$
overlap area	$1.27\pi r^2$	$0.73\pi r^2$	$0.35\pi r^2$

3.3 Solution to the Simple Model

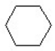
Under the assumptions, there are no interferences between the adjacent hexagonal cells. Thus, the problem turns to be easy. We only need to calculate how many cells could the circular flat area of radius 40 miles contain. To determine the number of repeaters, we illustrate how the regular hexagonal cell cover the area in **Figure 2**. To have a more clear cognition about the cellular network, we have the following denotations:

- i, the number of the layer of the cellular network;
- r, the length of the side of regular hexagonal cell;
- d, the distance between the central point and the border of the cellular network;
- n, the number of the cells.

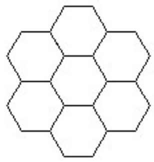
From **Figure 2**, We can find the relations between the above denotations. It is shown in **Table 2**.

We use the notation D_i and N_i denote distance d and cell number n when the layer number is i respectively. According to **Table 2**, we have the relation between D_i and i, as described is the following equation:

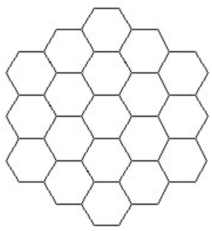
$$D_i = \begin{cases} D_i + r & \text{when } i \text{ is even} \\ D_i + 2r & \text{when } i \text{ is odd} \end{cases} \tag{3}$$



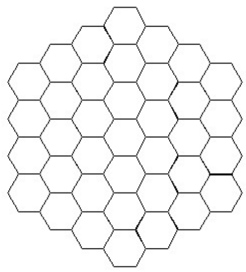
(a) one layer cell



(b) Two layer cells



(c) Three layer cells



(d) Four layer cells

Figure 2: Cellular network.

Table 2: Relations between i, r, and n.

layer number i	distance d	cell number n
1	r	1
2	2r	7
3	4r	19
4	5r	37
5	7r	61

From **Table 2**, we can see that when i increase by one, N_i increase by six. Then we can obtain an important relation between N_i and i, as shown in **Equation 4**.

$$N_i = 3i^2 - 3i + 1 \tag{4}$$

We let R stand for the radius of the flat circular area, then $R = 40$ miles. In order to make the cell group cover the circular area entirely, there must be

$d \geq R$. Since $r = 18km$, $R = 64.36km$, then $4r > R$. Hence, the situation in **Figure 2(c)** can meet the requirement of covering the area entirely. The number of repeaters is 19. Consequently, we get the solution to the simple model: the minimum number of the repeaters is 19 based on the assumptions in the simple model.

4 Our Model

Since the user capacity in each cell is limited. The user density affects the distribution of repeaters. High user density part of the area should have more repeaters than the low density part. Therefore, we have different solutions to the same problem when the user distribution is different. Here, we analyze the uniform distribution model and the random distribution model. We make an important assumption in our model: each repeater service area (a cell) can contain 30 simultaneous users[5].

4.1 Uniform Distribution Model

4.1.1 1,000 Simultaneous Users

The 1,000 users distribute uniformly in the flat circular area of radius 40 miles. When the area accommodate 1,000 simultaneous users, we need 34 cells at least. From **Table 2**, we can know that there are 37 cells in the area when four layers of cells distributed in the area, as shown in **Figure 3**.

Assuming that 37 cells filled the circular area exactly,we have $d = R$. Based on **Equation 3**, we calculate $r = 12.8km$. This value is less than the result $18km$ we calculate in **Section 3.1**. The cell area in **Figure 3** is smaller than the repeater service area. Therefore, there must be interferences between nearby repeaters. The CTCSS technology can be applied to mitigate the interference problems. There are 54 different PL tones available. We choose 37 different PL tones and assign them to the 37 repeaters in each cell. Since the 1,000 users distribute uniformly and interference problems are mitigated, 37 repeaters can meet the requirement. Thus, we obtain the minimum number of repeaters necessary accommodate 1,000 simultaneous users in uniform distribution model is 37.

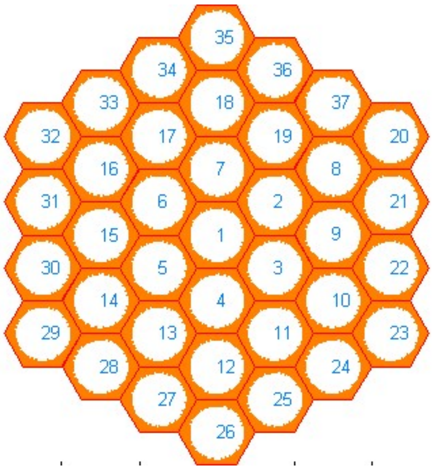


Figure 3: 37 cells accommodate 1,000 simultaneous users.

4.1.2 1,0000 Simultaneous Users

Now required simultaneous user capacity turns to be 1,0000. Still, each cell can contain 30 simultaneous users. Hence, 334 repeaters will be used at least. According to the relation between cell layer number i and repeater number N_i in**Equation 4**, we can calculate the minimum i. Let $3i^2 - 3i + 1 > 334$, we obtain the minimum i is 12. Substituting i to **Equation 4**,we can obtain the cell number is 397. That is to say, we need 397 cells to cover the entire area. Besides, we can calculate the length of the side of the regular hexagonal cell r based on **Equation 3**. Let $D_i = 17r = R$, we have $r = 3.8km$. Thus, interference problem between nearby repeaters will be severe when so many repeaters distribute in the same area. When the repeaters are far enough apart, interference will be canceled. Since the radius of a repeater service area is $18km$. We consider that when two repeaters are $36km$ apart, the interference problem can be ignored. If the distance between nearby repeaters are less than $36km$, we can use CTCSS technology to mitigate interference. There are 54 different PL tones available.

We treat 4 layer cells(37 cell) as a cell group. We choose 37 from the 54 different PL tones and assign them to the 37 repeaters in a cell group. Then there are no interference in a cell group. We calculate the length of the side of the regular hexagonal cell $r = 3.8km$. As $10r > 36$, when a repeater out a cell group is $10r$ away from the repeaters in the cell group, it has no influence on the cell group. We can distribute the cells as illustrated in **Figure 4**. For simplicity, we only put 8 layer cells in **Figure 4**.

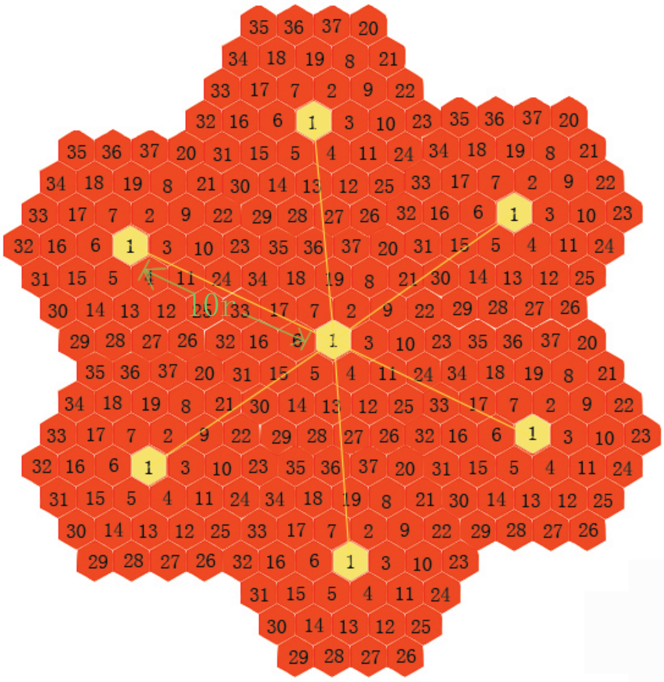


Figure 4: 397 cells accommodate 1,0000 simultaneous users.

In **Figure 4**, the distance between repeaters marked with the same number is $10r$. They do not interfere with each other. The repeaters marked with different numbers (1 to 37) are assigned with a separate PL. They do not influence each other either. Therefore, the interference and capacity problem are both solved. The scenario for this mode is feasible. We determine the minimum number of repeaters necessary to accommodate 1,0000 simultaneous users in uniform distribution model is 397.

4.2 Random Distribution Model

4.2.1 1,000 Simultaneous Users

When users distribute randomly in the circular area, the user density in different parts of the area are different. More repeaters should be deployed in high user density part of the area than low user density part. Simultaneously, the interference problem will be amplified in high user density part since more repeaters are deployed.

Taking both the interference and user capacity problem into consideration, we develop a dynamic algorithm to determine the minimum number of repeaters. We simulate the repeater coordination work in Matlab. The simulation is illustrated in **Figure 5**.

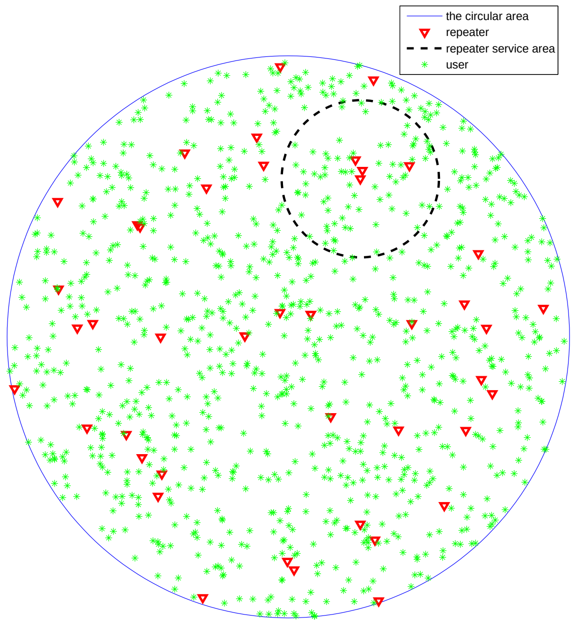


Figure 5: Simulation of repeater coordination.

We develop a algorithm in the model. We call it Density Dilution Algorithm. In our simulation, we deploy repeaters in the circular area one bye one. Every time, we search the highest user density part in the area at first. Then we deploy one repeater in this part. Since each repeater service area can contain 30 users. After we deploy one repeater in the area, the user capacity reduce by 30. When the repeater service area cover every user of 1,000 simultaneous users, the deployment work is finished. Then we can calculate the number of repeaters we have deployed in the area. To make the result more accurate, we simulate 1,000 times. The detail steps of the simulation program are described as follows:

1. Step 1: Distribute the 1,000 users randomly in the circular area.
2. Step 2: Set the radius of a repeater service area to be $18km$.

3. Step 3: Mark the 1,000 users with different numbers.
4. Step 4: Search the highest user density part in the area.
5. Step 5: Deploy one repeater in this part and record its location. If the number of users in this part is more than 30, then subtract 30 from the total number of users, else, subtract the number of users in this part from total number of users.
6. Step 6: Calculate the remaining number of users in the area. If the remaining users more than 0, then jump to Step 4 and finish the following steps, else exit the program and return the number of repeaters.

The simulation result is shown by hist. (see in **Figure 6**). The horizontal axis of the hist represents the number of repeaters deployed in the whole circular area. The vertical axis represents the times of result appeared in the simulation of 1,000 times.

From **Figure 6**, the result appears most frequently at 45 in the simulation of 1,000 times. Since there are 54 PL available, we choose 45 or 46 different from 54 PL for the repeaters, then the interference between repeaters can be canceled. Finally, we determine the minimum number of repeaters necessary to accommodate 1,000 simultaneous users in random distribution model is 45.

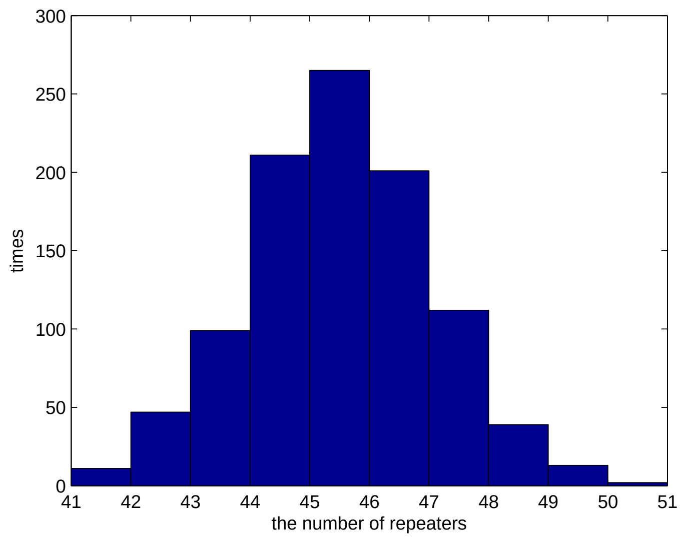


Figure 6: Result of simulation of 1,000 times.

4.2.2 1,0000 Simultaneous Users

According to the analysis in **section 4.1.2**, 334 repeaters are needed to accommodate 1,0000 simultaneous users at least. The distance between some of the repeaters is less than the $36km$ (2R). Therefore, we must use CTCSS technology to mitigate the interferences between nearby repeaters. However, there are 54 PL available. In some cases, the user density in some part of the area is so high that more than 54 nearby repeaters are needed for this part. The number of PL is not enough to mitigate the interference completely. We do not take these cases into consideration because they are rare and complex. We only discuss the situation that 54 PL can solve the interference problem. In this case, this model is very similar to the model of 1,000 simultaneous users. We can applied the program in the model of 1,000 simultaneous users to this model. But if 54 PL are distributed unreasonably, the interference problem still exits. Therefore, the distribution of 54 PL is significant in our model. Our main task is to solve the distribution problem of 54 PL. We mark the 54 PL with integers from 1 to 54. In this model, we have the following denotations:

- A, the set of PL mark (from 1 to 54).
- B, the set of PL mark in range of the a repeater service area (a circular area of radius $18km$).
- i, a variable.
- P_i, the PL mark of ith repeater.
- S, the total number of repeaters.

We develop a algorithm in the model. We call it Adjacent Removal Algorithm. Our algorithm is described as follows:

1. Step 1: Get the number and location of repeaters based on the program in the model of 1,000 simultaneous users.
2. Step 2: Let $i = 1$.
3. Step 3: Calculate B.
4. Step 4: Set P_i as a random integer, $P_i \in A$ and $P_i \notin B$.
5. Step 5: $i = i + 1$.
6. Step 6: If $i > S + 1$, exit, else ,jump to Step 3.

In order to test the feasibility of our model, we will apply this method to the model of 1,0000 simultaneous users. However, the number of users is so great that it is very hard to do the simulation. We take 1,000 users as an example to do simulation. The result of the simulation is shown in **Figure 7**.

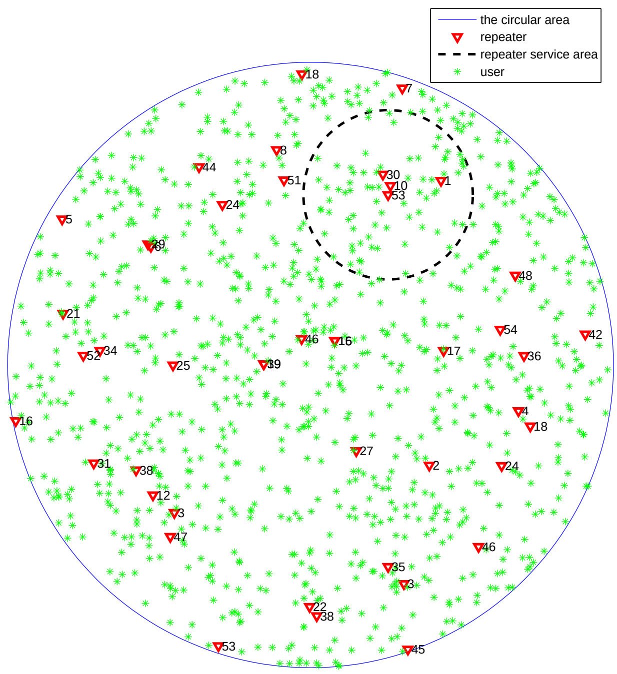


Figure 7: Result of simulation in this model.

Figure 7 illustrate the simulation in some extent. But the number of users do not reach 1,0000. We do the simulation for 10 times. The results show the number of repeaters necessary are 342, 345, 343, 345, 341, 342, 341, 344, 340. So we determine the minimum number of repeaters necessary to accommodate 1,0000 simultaneous users in random distribution model is about 342.

5 Defects Caused by Mountainous Areas

According to some literature[5,6], we summarize the main defects caused by mountainous areas. We described the defects as follows:

- Mountains will block the transmission of signal, therefore, the coverage area of a repeat decreases and the energy of the signal will get weak.
- Signal will propagate in multi-path when it meets mountains. It will cause interferences between different signals.
- Mountains divide the area to separate parts, disordering the distribution of cells in the cellular radio network.
- The forest on mountains will absorb some energy of signals, the signal transmission distance will reduce. Therefore, the communication between repeaters and users will be obstructed.

6 The Model Results

Based on the analysis and calculation in **section 4**, we determine the minimum number of repeaters in different cases. The results are represented in **Table 3**.

Table 3: The model results.

user distribution	1,000 users	1,0000 users
uniform distribution	37	397
random distribution	45	342

From **Table 3**, more repeaters in the area can accommodate more users. The distribution of users has an impact on repeater coordination. To the operators, repeater coordination in uniform distribution area is obviously more easy compared to random distribution area. However, the minimum number of necessary repeaters to accommodate 1,0000 users in uniform distribution area is less than that in random distribution area.

7 Strengths and Weaknesses of the Model

Like any model, our model presented above has its strengths and weaknesses. Some of the major points are presented below.

7.1 Strengths

- We propose different models according to different user distribution. We present uniform distribution model and random distribution model respectively.
- In the simulation of random distribution model, the distribution of repeaters and users is produced randomly by computer.
- We do the simulation for many times, the accuracy of the results is improved.

7.2 Weaknesses

- Our model are based on many assumptions, the results may change if the assumptions are different.
- We do not develop a model considering the effect of terrain.
- We only use regular hexagonal cells to fill the area.The solution will be different when the shape of cell is different.

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Appendices

Appendix A First appendix

Here are simulation programme we used in random distribution model for 1,000 simultaneous users as follow.

Input matlab source:

```
clc
clear;
clf;
s=zeros(1,100);
%p=1:10;
for p=1:100
%% a circular flat area of radius 40 miles radius
RD=40; %the area's radius is 40 miles
R=RD*1.609; % Miles converted into kilometers
MRN=30; % The maximum reception relay station
%% plot the area
theta=0:0.01*pi:2*pi;
x0=R*cos(theta);
y0=R*sin(theta);
plot(x0,y0)
%% plot the random user
num=1000;
x=R*(2*rand(1,sqrt(2)*num)-1);
y=R*(2*rand(1,sqrt(2)*num)-1);
z=x+1i*y;
z=z(abs(z)<=R);
initial_user=z(1:num);
hold on
plot(initial_user,'g*');
hold off
axis equal

%% the number of repeaters under the condition of Uniform distribution
RR=18;% repeater's recieve radius
RI=18;% repeater's interference radius
r=RR;%
%NL=1+ceil((R-RI)/(1.5*RI)); % total number of layers
t=ceil((R-r)/r);
if mod(t,3)==0  %calculate the layers of the entire regular hexagon
    NL=t/3*2+1;
else
    NL=floor(t/3)*2+2;
end
```

```
N=3*NL*(NL-1)+1;% the number of repeaters
ii=0;
t=1;

%% Problem of random distribution
% show the density of users
QQ=0+1i*0;
% p=length(initial_user);
while t>0
   DS=zeros(size(initial_user)); % initial density vector
% Traverse calculations the density
 % find_radius=RI; %Set the search radius
for ind=1:length(initial_user);
     find_radius=RI; %Set the search radius
     find_distance=abs(initial_user-initial_user(ind)); % distance of user to user
L=(find_distance<=find_radius); %
DS(ind)=sum(sum(L));
end
%%%%
[M,V]=max(DS);
find_radius=RI; %Set the search radius
     find_distance=abs(initial_user-initial_user(V));
     QQ(ii+1)=initial_user(V);
     L=(find_distance<=find_radius);
     temp=L;
     for ind=1:MRN;
     I=find(temp,1);
     temp(I)=0;
     end
     A=initial_user(temp);
     initial_user(L)=[];
     initial_user=[initial_user,A];
     t=length(initial_user);
%%
theta=0:0.01*pi:2*pi;
x0=R*cos(theta);
y0=R*sin(theta);
plot(x0,y0)
hold on
   plot(initial_user,'g*');
   hold off
axis equal
ii=ii+1;
end
s(p)=ii;
end
hold on;
plot(QQ,'rv','linewidth',2);
x1=real(QQ(1))+RR*cos(theta);
y1=imag(QQ(1))+RR*sin(theta);
plot(x1,y1,'k--','linewidth',2);
%% a circular flat area of radius 40 miles radius
```

```
RD=40; %the area's radius is 40 miles
R=RD*1.609; % Miles converted into kilometers
MRN=30; % The maximum reception relay station
%% plot the area
% theta=0:0.01*pi:2*pi;
% x0=R*cos(theta);
% y0=R*sin(theta);
% plot(x0,y0)
%% plot the random user
num=1000;
x=R*(2*rand(1,sqrt(2)*num)-1);
y=R*(2*rand(1,sqrt(2)*num)-1);
z=x+1i*y;
z=z(abs(z)<=R);
initial_user=z(1:num);
hold on
plot(initial_user,'g*');
hold off
axis equal
axis off;
```

Appendix B Second appendix

Here are simulation programme we used in random distribution model for 1,0000 simultaneous users as follow.

Input matlab source:

```
qq=QQ;
PL=1:54;
NameR=zeros(size(qq));
NameR(1)=randi(length(PL),1);
for ii=2:length(qq);
 DR=(qq(1:ii-1)-qq(ii));
L=(DR<RR);
temp=setdiff(PL,NameR(L));
ind=randi(length(temp),1);
NameR(ii)=temp(ind);
end
NameR
 hold on



for i=1:length(NameR);
    b=NameR(i);
```

```
    c=num2str(b);
    c=[' ',c];
    text(real(qq(i)),imag(qq(i)),c);
end
```