Summary of Heat Transfer Diffusion Equation Discretization

This document summarizes the transition from a continuous form of the heat transfer diffusion equation to its discretized form in the context of a Linear Parameter Varying Thermal Network (LPTN), and the subsequent application of first-order Euler discretization.

1. Continuous Heat Diffusion Equation:

The original continuous form of the heat transfer equation, as per Bergman et al. (2007), is given by:

rho * c_p * (partial theta / partial t) = p + nabla * (lambda * nabla * theta)

Where:

- rho is the mass density.
- c_p is the specific heat at constant pressure.
- theta is the temperature.
- p represents thermal energy generation.
- nabla is the del operator indicating spatial derivatives.
- lambda is the thermal conductivity.

2. Discretized Form for LPTN:

The discretized form of the equation for an LPTN is:

C_i(zeta(t)) * d(theta_i)/dt = P_i(zeta(t)) + sum_{j in Omega \ {i}} ((theta_j - theta_i) / R_{i,j}(zeta(t))) + sum_{j=1}^{n} ((tilde{theta}_j - theta_i) / R_{i,j}(zeta(t)))

Where:

- C_i is the thermal capacitance.
- P_i is the power loss.
- R_{i,j} is the bidirectional thermal resistance.
- theta represents the temperature at time t.

- Omega is the set of nodes in the network.

3. First-Order Euler Discretization:

Applying first-order Euler discretization, the time derivative is approximated as (theta_i^{(k+1)} - theta_i^{(k)}) / Delta t. The discretized equation becomes:

hat{theta}_i[k + 1] = hat{theta}_i[k] + T_s * kappa_i[k] * (pi_i[k] + sum_{j in Omega \ {i}} ((hat{theta}_j[k] - hat{theta}_i[k]) * gamma_{i,j}[k]) + sum_{j=1}^{n} ((tilde{theta}_j[k] - hat{theta}_i[k]) * gamma_{i,j}[k]))

Where:

- hat{theta}_i[k] is the estimated temperature at node i at time step k.
- T_s is the time step size.
- kappa_i[k] is the inverse of the thermal capacitance at node i.
- pi_i[k] represents power loss or heat generation.
- gamma_{i,j}[k] is related to the thermal resistance.
- tilde{theta}_j[k] represents external temperatures influencing the node.