

Recursion

Base Case of a Recursive Function

A recursive function should have a base case with a condition that stops the function from recursing indefinitely. In the example, the base case is a condition evaluating a negative or zero value to be true.

```
function countdown(value)
  if value is negative or zero
    print "done"
  otherwise if value is greater than
  zero
    print value
    call countdown with (value-1)
```

Recursive Step in Recursive Function

A recursive function should have a **recursive step** which calls the recursive function with some input that brings it closer to its base case. In the example, the recursive step is the call to `countdown()` with a decremented value.

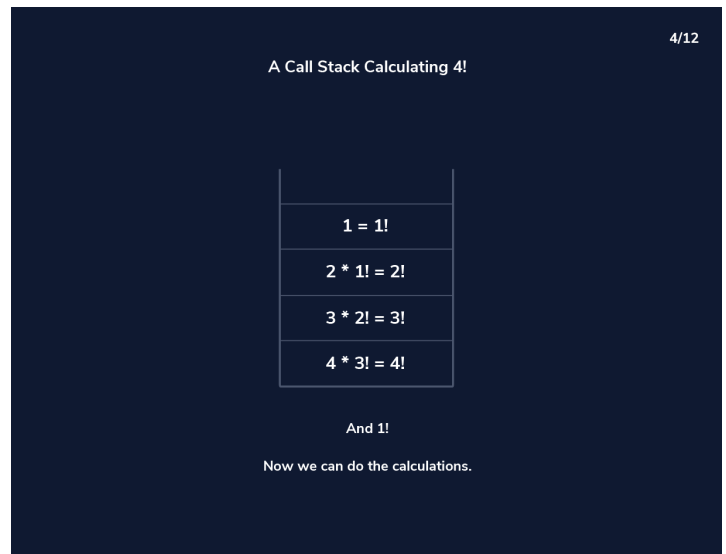
```
def countdown(value):
  if value <= 0:
    print("done")
  else:
    print(value)
    countdown(value-1) #recursive step
```

What is Recursion

Recursion is a strategy for solving problems by defining the problem in terms of itself. A recursive function consists of two basic parts: the base case and the recursive step.

Call Stack in Recursive Function

Programming languages use a facility called a **call stack** to manage the invocation of recursive functions. Like a stack, a call stack for a recursive function calls the last function in its stack when the **base case** is met.



Big-O Runtime for Recursive Functions

The big-O runtime for a recursive function is equivalent to the number of recursive function calls. This value varies depending on the complexity of the algorithm of the recursive function. For example, a recursive function of input N that is called N times will have a runtime of $O(N)$. On the other hand, a recursive function of input N that calls itself twice per function may have a runtime of $O(2^N)$.

Weak Base Case in Recursive Function

A recursive function with a weak base case will not have a condition that will stop the function from recursing, causing the function to run indefinitely. When this happens, the call stack will overflow and the program will generate a *stack overflow* error.

Execution Context of a Recursive Function

An execution context of a recursive function is the set of arguments to the recursive function call. Programming languages use execution contexts to manage recursive functions.

Stack Overflow Error in Recursive Function

A recursive function that is called with an input that requires too many iterations will cause the call stack to get too large, resulting in a stack overflow error. In these cases, it is more appropriate to use an iterative solution. A recursive solution is only suited for a problem that does not exceed a certain number of recursive calls.

For example, `myfunction()` below throws a stack overflow error when an input of 1000 is used.

```
def myfunction(n):
    if n == 0:
        return n
    else:
        return myfunction(n-1)
```

`myfunction(1000)` #results in stack overflow error

Fibonacci Sequence

A Fibonacci sequence is a mathematical series of numbers such that each number is the sum of the two preceding numbers, starting from 0 and 1.

Fibonacci sequence: 0, 1, 1, 2, 3, 5, 8, 13, 21, ...

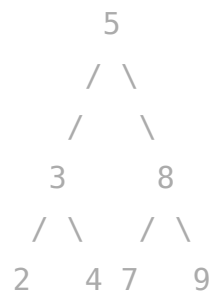
Call Stack Construction in While Loop

A call stack with execution contexts can be constructed using a `while` loop, a `list` to represent the call stack and a `dictionary` to represent the execution contexts. This is useful to mimic the role of a call stack inside a recursive function.

Binary Search Tree

In Python, a binary search tree is a recursive data structure that makes sorted lists easier to search. Binary search trees:

- Reference two children at most per tree node.
- The "left" child of the tree must contain a value lesser than its parent.
- The "right" child of the tree must contain a value greater than it's parent.



Recursion and Nested Lists

A nested list can be traversed and flattened using a recursive function. The base case evaluates an element in the list. If it is not another list, the single element is appended to a flat list. The recursive step calls the recursive function with the nested list element as input.

```
def flatten(mylist):
    flatlist = []
    for element in mylist:
        if type(element) == list:
            flatlist += flatten(element)
        else:
            flatlist += element
    return flatlist

print(flatten(['a', ['b', ['c', ['d']],
'e'], 'f']))
# returns ['a', 'b', 'c', 'd', 'e',
'f']
```

Fibonacci Recursion

Computing the value of a Fibonacci number can be implemented using recursion. Given an input of index N, the recursive function has two base cases – when the index is zero or 1. The recursive function returns the sum of the index minus 1 and the index minus 2. The Big-O runtime of the Fibonacci function is $O(2^N)$.

```
def fibonacci(n):
    if n <= 1:
        return n
    else:
        return fibonacci(n-1) +
fibonacci(n-2)
```

Modeling Recursion as Call Stack

One can model recursion as a `call stack` with `execution contexts` using a `while` loop and a Python `list`. When the `base case` is reached, print out the call stack `list` in a LIFO (last in first out) manner until the call stack is empty.

Using another `while` loop, iterate through the call stack `list`. Pop the last item off the list and add it to a variable to store the accumulative result.

Print the result.

```
def countdown(value):
    call_stack = []
    while value > 0 :
        call_stack.append({"input":value})
        print("Call Stack:",call_stack)
        value -= 1
    print("Base Case Reached")
    while len(call_stack) != 0:
        print("Popping {} from call
stack".format(call_stack.pop()))
        print("Call Stack:",call_stack)
    countdown(4)
    ...

Call Stack: [{'input': 4}]
Call Stack: [{'input': 4}, {'input':
3}]
Call Stack: [{'input': 4}, {'input':
3}, {'input': 2}]
Call Stack: [{'input': 4}, {'input':
3}, {'input': 2}, {'input': 1}]
Base Case Reached
Popping {'input': 1} from call stack
Call Stack: [{'input': 4}, {'input':
3}, {'input': 2}]
Popping {'input': 2} from call stack
Call Stack: [{'input': 4}, {'input':
3}]
Popping {'input': 3} from call stack
Call Stack: [{'input': 4}]
Popping {'input': 4} from call stack
Call Stack: []
...
```

Recursion in Python

In Python, a recursive function accepts an argument and includes a condition to check whether it matches the base case. A recursive function has:

- Base Case - a condition that evaluates the current input to stop the recursion from continuing.
- Recursive Step - one or more calls to the recursive function to bring the input closer to the base case.

Build a Binary Search Tree

To build a binary search tree as a recursive algorithm do the following:

BASE CASE:

If the list is empty, return "No Child"

RECURSIVE STEP:

1. Find the middle index of the list.
2. Create a tree node with the value of
3. Assign the tree node's left child to
4. Assign the tree node's right child to
5. Return the tree node.

```
def countdown(value):
    if value <= 0:    #base case
        print("done")
    else:
        print(value)
        countdown(value-1)    #recursive case
```

```
def build_bst(my_list):
    if len(my_list) == 0:
        return "No Child"

    middle_index = len(my_list) // 2
    middle_value = my_list[middle_index]

    print("Middle index:
{0}".format(middle_index))
    print("Middle value:
{0}".format(middle_value))

    tree_node = {"data": middle_value}
    tree_node["left_child"] =
build_bst(my_list[ : middle_index])
    tree_node["right_child"] =
build_bst(my_list[middle_index + 1 : ])

    return tree_node

sorted_list = [12, 13, 14, 15, 16]
binary_search_tree =
build_bst(sorted_list)
print(binary_search_tree)
```

Recursive Depth of Binary Search Tree

A binary search tree is a data structure that builds a sorted input list into two subtrees. The left child of the subtree contains a value that is less than the root of the tree. The right child of the subtree contains a value that is greater than the root of the tree.

A recursive function can be written to determine the depth of this tree.

```
def depth(tree):
    if not tree:
        return 0
    left_depth =
    depth(tree["left_child"])
    right_depth =
    depth(tree["right_child"])
    return max(left_depth, right_depth) +
1
```

Sum Digits with Recursion

Summing the digits of a number can be done recursively. For example:

$$552 = 5 + 5 + 2 = 12$$

```
def sum_digits(n):
    if n <= 9:
        return n
    last_digit = n % 10
    return sum_digits(n // 10) +
    last_digit
```

```
sum_digits(552) #returns 12
```

Palindrome in Recursion

A palindrome is a word that can be read the same both ways - forward and backward. For example, abba is a palindrome and abc is not. The solution to determine if a word is a palindrome can be implemented as a recursive function.

```
def is_palindrome(str):
    if len(str) < 2:
        return True
    if str[0] != str[-1]:
        return False
    return is_palindrome(str[1:-1])
```

Fibonacci Iterative Function

A Fibonacci sequence is made up adding two previous numbers beginning with 0 and 1. For example:

0, 1, 1, 2, 3, 5, 8, 13, ...

A function to compute the value of an index in the Fibonacci sequence, `fibonacci(index)` can be written as an iterative function.

```
def fibonacci(n):
    if n < 0:
        raise ValueError("Input 0 or greater only!")
    fiblist = [0, 1]
    for i in range(2,n+1):
        fiblist.append(fiblist[i-1] +
fiblist[i-2])
    return fiblist[n]
```

Recursive Multiplication

The multiplication of two numbers can be solved recursively as follows:

Base case: Check for any number that is

Recursive step: Return the first number

```
def multiplication(num1, num2):
    if num1 == 0 or num2 == 0:
        return 0
    return num1 + multiplication(num1,
num2 - 1)
```

Iterative Function for Factorials

To compute the factorial of a number, multiply all the numbers sequentially from 1 to the number.

An example of an iterative function to compute a factorial is given below.

```
def factorial(n):
    answer = 1
    while n != 0:
        answer *= n
        n -= 1
    return answer
```


Recursively Find Minimum in List

We can use recursion to find the element with the minimum value in a list, as shown in the code below.

```
def find_min(my_list):  
    if len(my_list) == 0:  
        return None  
    if len(my_list) == 1:  
        return my_list[0]  
    #compare the first 2 elements  
    temp = my_list[0] if my_list[0] <  
my_list[1] else my_list[1]  
    my_list[1] = temp  
    return find_min(my_list[1:])  
  
print(find_min([]) == None)  
print(find_min([42, 17, 2, -1, 67]) ==  
-1)
```

 **Print**  **Share** ▼