A Matlab/Octave toolbox for the computation of Bessel/Hankel functions with large complex order

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Abstract

We code a Matlab/Octave toolbox to compute the Bessel and Hankel functions with large complex order.

1 How to use the toolbox

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The goal of the toolbox is to compute the Bessel functions J_{ν}/Y_{ν} and the Hankel functions $H_{\nu}^{(1)}/H_{\nu}^{(2)}$ when the order ν is a large complex number in modulus. The main functions of the toolbox are:

Function	How to call it.
$\frac{J_{\nu}(z)}{J'_{\nu}(z)}$	<pre>J = besselj_cplx(v, z) Jp = besseljp_cplx(v, z)</pre>
$egin{aligned} Y_{ u}(z) \ Y_{ u}'(z) \end{aligned}$	Y = bessely_cplx(v, z) Yp = besselyp_cplx(v, z)
${\sf H}_{ u}^{(1)}(z) \ {\sf H}_{ u}^{(1)'}(z)$	<pre>H1 = besselh_cplx(v, 1, z) H1p = besselhp_cplx(v, 1, z)</pre>
${\sf H}_{ u}^{(2)}(z) \ {\sf H}_{ u}^{(2)}(z)$	<pre>H2 = besselh_cplx(v, 2, z) H2p = besselhp_cplx(v, 2, z)</pre>

Table 1: Link between Bessel/Hankel functions and their name in the code.

The implementation is base on uniform asymptotic expansions for large order as describe in [1]. For now, we should expect good accuracy for $|\nu| \geq 20$.

References

[1] N. M. Temme. Numerical algorithms for uniform airy-type asymptotic expansions. Numerical Algorithms, 15(2):207–225, 1997. doi:10.1023/A:1019197921337.