A Matlab/Octave toolbox for the computation of Bessel/Hankel functions with large complex order

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Abstract

We code a Matlab/Octave toolbox to compute the Bessel and Hankel functions with large complex order.

1 How to use the toolbox

The goal of the toolbox is to compute the Bessel functions $\mathsf{J}_\nu/\mathsf{Y}_\nu$ and the Hankel functions $\mathsf{H}^{(1)}_\nu/\mathsf{H}^{(2)}_\nu$ when the order ν is a large complex number in modulus. The main functions of the toolbox are:

Function	How to call it.
$\mathsf{J}_\nu(z)$	`J  = besselj_cplx(v, z)`
$\mathsf{J}'_\nu(z)$	`Jp = besseljp_cplx(v, z)`
$\mathsf{Y}_\nu(z)$	`Y  = bessely_cplx(v, z)`
$\mathsf{Y}'_\nu(z)$	`Yp = besselyp_cplx(v, z)`
$\mathsf{H}^{(1)}_\nu(z)$	`H1  = besselh_cplx(v, 1, z)`
$\mathsf{H}^{(1)'}_\nu(z)$	`H1p = besselhp_cplx(v, 1, z)`
$\mathsf{H}^{(2)}_\nu(z)$	`H2  = besselh_cplx(v, 2, z)`
$\mathsf{H}^{(2)'}_\nu(z)$	`H2p = besselhp_cplx(v, 2, z)`

Table 1: Link between Bessel/Hankel functions and their name in the code.

The implementation is base on uniform asymptotic expansions for large order as describe in [1]. For now, we should expect good accuracy for $|\nu| \geq 20$.

References

[1] N. M. Temme. Numerical algorithms for uniform airy-type asymptotic expansions. *Numerical Algorithms*, 15(2):207–225, 1997. doi:10.1023/A:1019197921337.