Peut-on entendre la forme d'un tambour ?

Zoïs Moitier

IRMAR, Université de Rennes 1

23 janvier 12018

CAN ONE HEAR THE SHAPE OF A DRUM?

MARK KAC, The Rockefeller University, New York

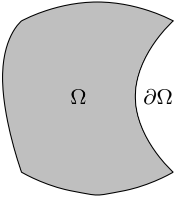
To George Eugene Uhlenbeck on the occasion of his sixty-fifth birthday

> "La Physique ne nous donne pas seulement l'occasion de résoudre des problèmes . . . , elle nous fait presentir la solution." H. POINCARÉ.

Before I explain the title and introduce the theme of the lecture I should like to state that my presentation will be more in the nature of a leisurely excursion than of an organized tour. It will not be my purpose to reach a specified destination at a scheduled time. Rather I should like to allow myself on many occasions the luxury of stopping and looking around. So much effort is being spent on streamlining mathematics and in rendering it more efficient, that a solitary transgression against the trend could perhaps be forgiven.

M. Kac, *Can One Hear the Shape of a Drum ?* The American Mathematical Monthly, Vol. 73, No. 4, Part 2 : Papers in Analysis (April, 1966), pp. 1-23.

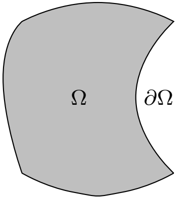
Tambour : Ω ouvert de $\mathbb{R}^2$



Entendre un tambour : trouver les $\lambda \in \mathbb{R}_+$ tels qu'il existe $u : \Omega \to \mathbb{R}$ non nul et

$$\left\{ \begin{array}{rcll} -\Delta u & = & \lambda u & \text{dans } \Omega \\ u & = & 0 & \text{sur } \partial\Omega \end{array} \right.$$

Tambour : Ω ouvert de $\mathbb{R}^2$



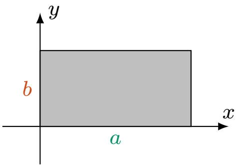
Entendre un tambour : trouver les $\lambda \in \mathbb{R}_+$ tels qu'il existe $u : \Omega \to \mathbb{R}$ non nul et

$$\begin{cases} -\Delta u &= \lambda u \quad \text{dans } \Omega \\ u &= 0 \quad \text{sur } \partial\Omega \end{cases}$$

Théorie spectrale : $0 < \lambda_1 \leq \lambda_2 \leq \ldots \leq \lambda_k \leq \lambda_{k+1} \leq \ldots \quad \nearrow +\infty$

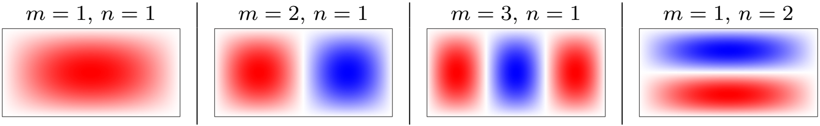
$$\text{Spectre}(\Omega) = \{\lambda_k,\, k \geq 1\}$$

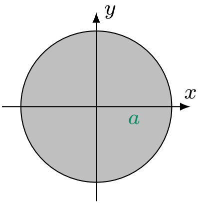
Spectre(Ω) est invariant par translation, rotation et réflexion.

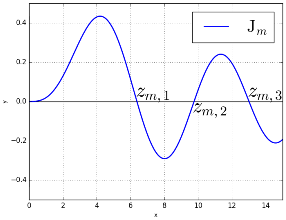


$$\text{Spectre(rectangle)} = \left\{ \left(\frac{m^2}{a^2} + \frac{n^2}{b^2} \right) \pi^2,\ m \geq 1,\ n \geq 1 \right\}$$

$$u_{m,n}(x,y) = \sin\left(m\pi \frac{x}{a} \right) \sin\left(n\pi \frac{y}{b} \right)$$



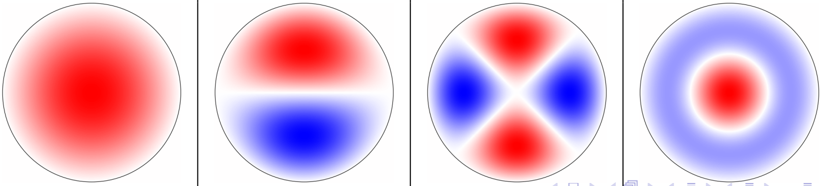




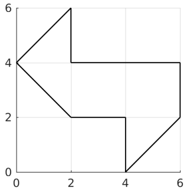
$$\text{Spectre(disque)} = \left\{ \frac{z_{m,n}^2}{a^2},\ m \geq 0,\ n \geq 1 \right\}, \qquad u_{m,n}(r,\theta) = \mathrm{J}_m\left(z_{m,n} \frac{r}{a} \right) \cos(m\theta)$$

$$\mathrm{J}_m(z_{m,n}) = 0$$

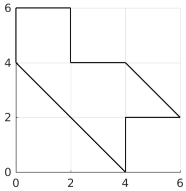
$m = 0,\ n = 1$ | $m = 1,\ n = 1$ | $m = 2,\ n = 1$ | $m = 0,\ n = 2$



Non !



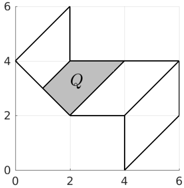
(a) Cocotte



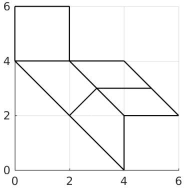
(b) Flèche

C. Gordon, D. Webb et S. Wolpert, *Isospectral plane domains and surfaces via Riemannian orbifolds*, Inventiones mathematicae, December 1992, Volume 110, Issue 1, pp. 1–22.

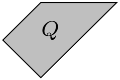
On utilise la méthode des éléments finis :



(a) Cocotte



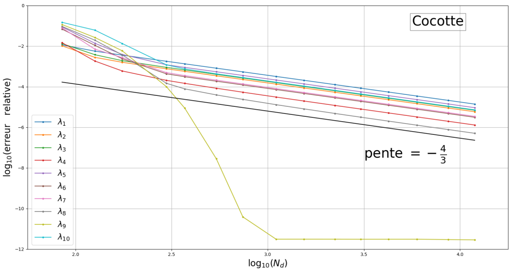
(b) Flèche

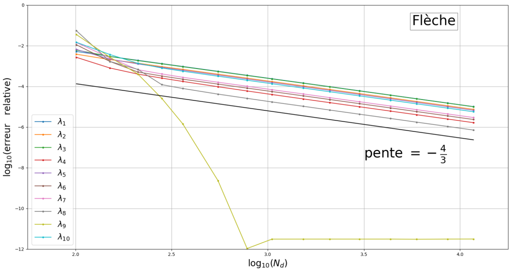


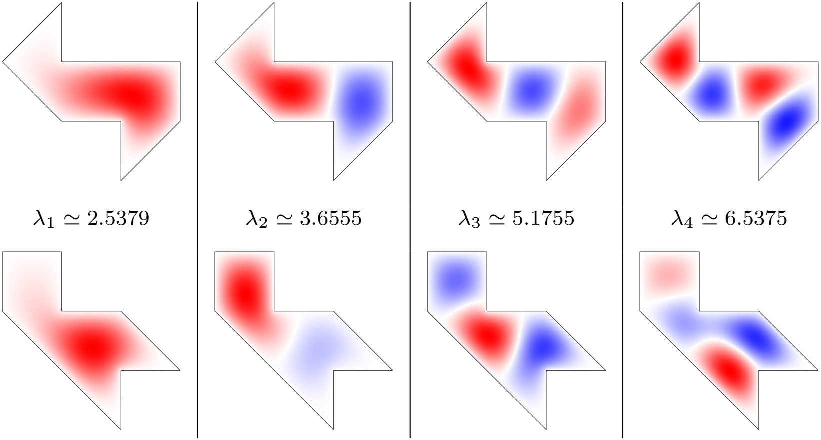
$$u_Q(x,y) \approx \sum_{0\leq i,j\leq d} c_{i,j} x^i y^j, \quad c_{i,j} : 1 \text{ degré de liberté}$$

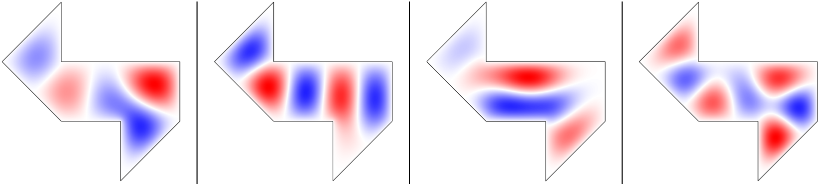
N_d = degré de liberté total

Algorithme $\rightarrow (\lambda_k, u_k)$

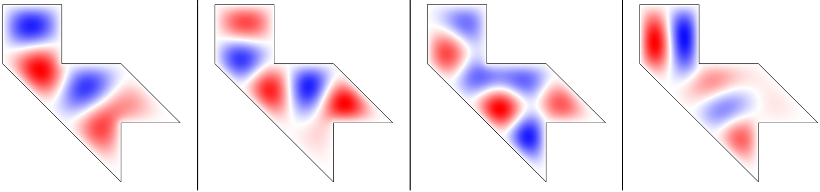

Cocotte
pente $= -\frac{4}{3}$
$\log_{10}$(erreur relative)
$\log_{10}(N_d)$
λ_1
λ_2
λ_3
λ_4
λ_5
λ_6
λ_7
λ_8
λ_9
λ_{10}


Flèche
pente $= -\frac{4}{3}$
$\log_{10}$(erreur relative)
$\log_{10}(N_d)$
λ_1
λ_2
λ_3
λ_4
λ_5
λ_6
λ_7
λ_8
λ_9
λ_{10}


$\lambda_1 \simeq 2.5379$
$\lambda_2 \simeq 3.6555$
$\lambda_3 \simeq 5.1755$
$\lambda_4 \simeq 6.5375$



$\lambda_5 \simeq 7.2480$ | $\lambda_6 \simeq 9.2092$ | $\lambda_7 \simeq 10.596$ | $\lambda_8 \simeq 11.541$



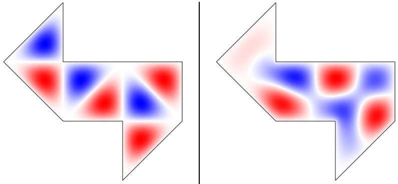
Mais...

$$\Lambda(R) = \mathrm{Card}\{k,\ \lambda_k \leq R\}$$

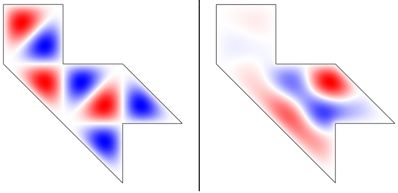
$A = \mathrm{Aire}(\Omega)$ $\qquad$ $L = \mathrm{Longueur}(\partial\Omega)$

$$\Lambda(R) = \frac{A}{4\pi} R - \frac{L}{4\pi}\sqrt{R} + \mathcal{O}(1) \qquad \text{quand } R \to +\infty$$

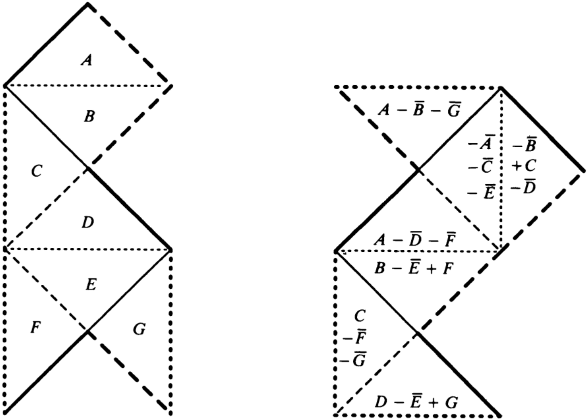
H. P. Baltes et E. R. Hilf, *Spectra of Finite Systems*, Bibliographisches Institut, Mannheim, 1976.



$\lambda_9 \simeq 12.337$ $\lambda_{10} \simeq 13.053$



Merci de votre attention



(a) (b)

Figure 2

S. J. Chapman, *Drum That Sound the Same* The American Mathematical Monthly, Vol. 102, No. 2 (February, 1995), pp. 124-138.